Title: Discrete Holography

Speakers: Johanna Erdmenger

Series: Colloquium

Date: October 05, 2022 - 2:00 PM

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Abstract: The AdS/CFT correspondence (Anti-de Sitter gravity/ conformal field theory correspondence), also referred to as holography, provides the first example of a duality relating a gravity theory to a quantum field theory without gravity. The gravity theory involved describes the hyperbolic bulk spacetime and the quantum field theory its boundary. This duality has its origin within string theory. Recent developments based on both quantum information theory and the physics of black holes raise the question if dualities of this type exist more generally, even beyond string theory. As a specific example, I will describe recent progress towards establishing a duality based on a discretisation of hyperbolic Anti-de Sitter space that is obtained by a regular tiling with polygons. I will explain how to obtain a dual Hamiltonian on the boundary that reflects properties of the bulk tiling, and describe its properties. This research direction is related to recent developments in mathematics, quantum information, condensed matter physics and electrical engineering, making it truly interdisciplinary. I will conclude by giving an outlook on the next steps to be followed in view of obtaining a full discrete duality.

Zoom link: https://pitp.zoom.us/j/95553458965?pwd=bHZIamd3Q1BNRjBhZGk5Y1BPK0d6QT09

Discrete Holography

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Complexity and Topology in Quantum Matter

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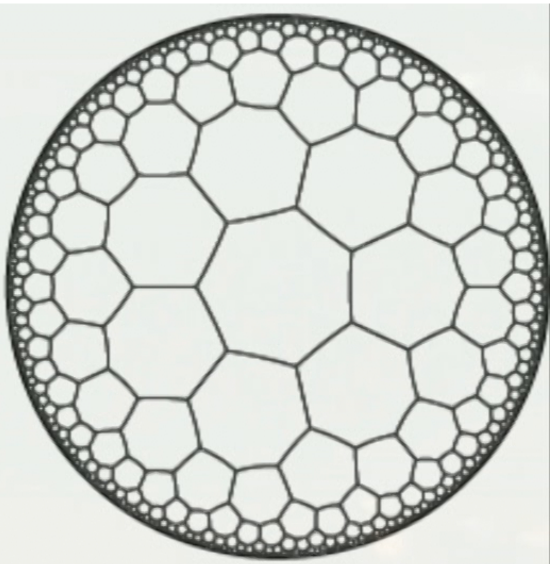
Overview



I. Motivation and review of AdS/CFT

II. Hyperbolic tilings

III. Breitenlohner - Freedman bound

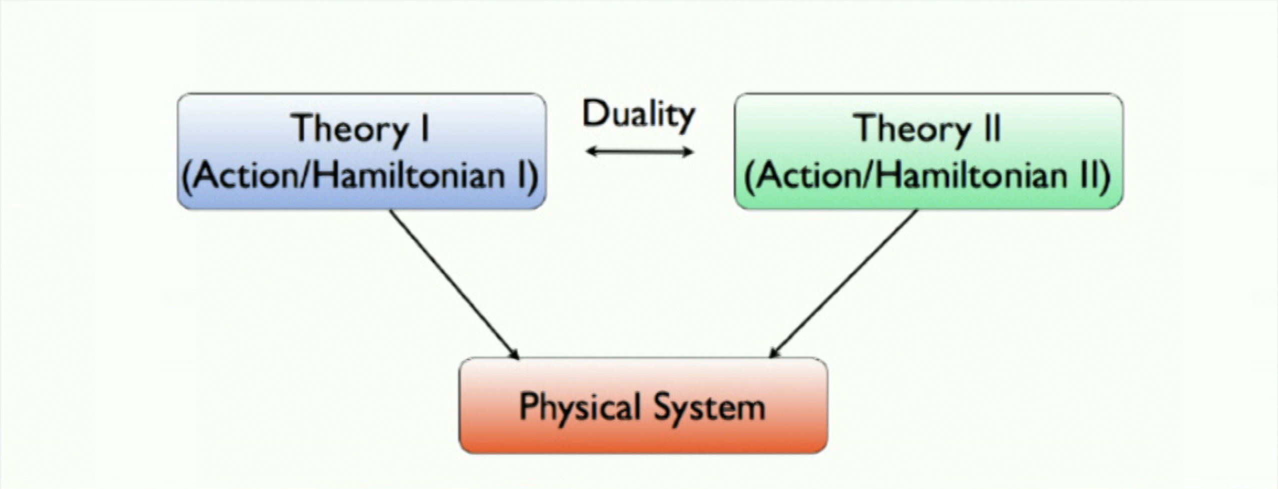
IV. Boundary spin chain Hamiltonian reflecting bulk tiling

V. RG study, tensor networks and entanglement entropy

VI. Connections to mathematics, condensed matter theory, quantum gravity

VII. Outlook

Duality



Example within quantum field theory: Sine-Gordon/Massive Thirring duality

AdS/CFT correspondence: First duality between gravity theory and quantum field theory

Gauge/Gravity Duality

Generalizations of AdS/CFT to scenarios with less global symmetries

- Duality:

  Quantum field theory at strong coupling

  $\Leftrightarrow$ Theory of gravitation at weak coupling

- Holography:

  Quantum field theory in d dimensions

  $\Leftrightarrow$ Gravitational theory in $d + 1$ dimensions

Quantum field theory defined at the boundary of the d+1-dimensional space

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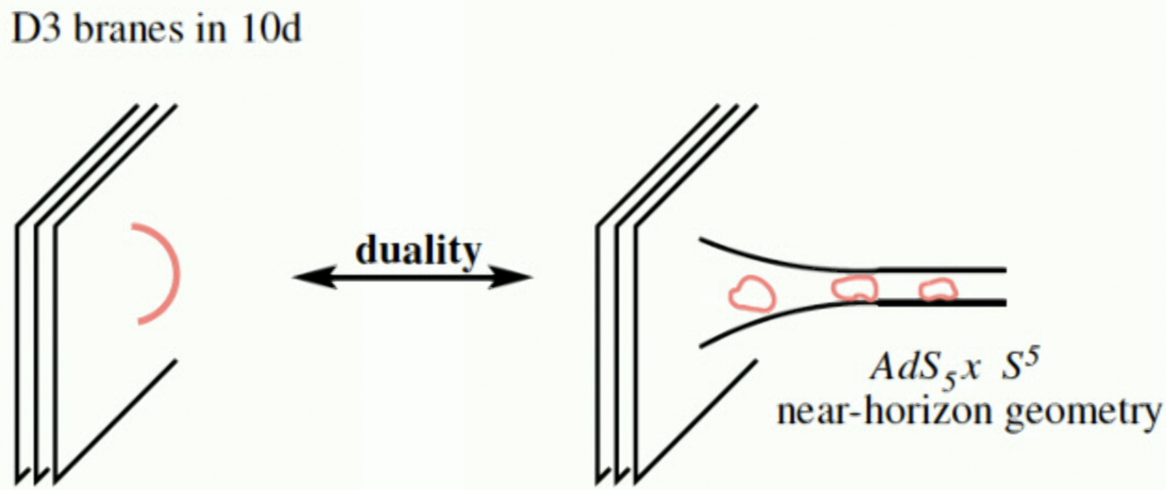
String theory origin of the AdS/CFT correspondence

Maldacena 1997



⇓ Low energy limit

Supersymmetric $SU(N)$ gauge theory in four dimensions $(N \to \infty)$

Supergravity on the space $AdS_5 \times S^5$

Limits in AdS/CFT

SU(N) Super Yang-Mills theory for any N, any λ ⟺ Type IIB string theory on $AdS_5 \times S^5$

↓ $\lambda = {g_{\mathrm{YM}}}^2 N$, $N \to \infty$ ↓ $g_s = {g_{\mathrm{YM}}}^2 = \dfrac{\lambda}{N} \to 0$

SU(N) Super Yang-Mills theory in large N limit ⟺ Classical string theory on $AdS_5 \times S^5$

↓ $4\pi\lambda = \dfrac{L^4}{{\ell_s}^4} \to \infty$ ↓

SU(N) Super Yang-Mills theory for large N, large λ ⟺ Type IIB supergravity on $AdS_5 \times S^5$

Global symmetries and asymptotic behaviour in AdS/CFT

D3-brane example: Bosonic symmetries of both SU(N) Super Yang-Mills and $AdS_5 \times S^5$:

SO(4,2) x SU(4)

Asymptotic near-boundary solution of scalar field:

$$\varphi(z) \sim \varphi_0 z^{d-\Delta} + \langle O \rangle z^{\Delta}$$

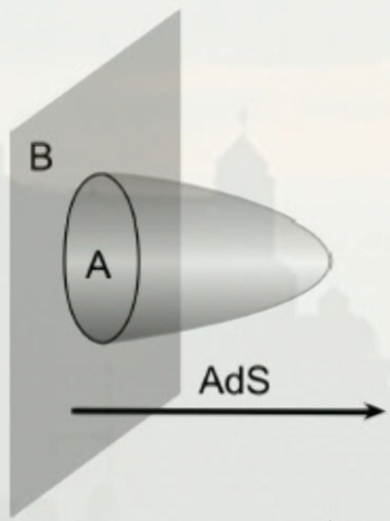
$$m^2 L^2 = \Delta(\Delta - d)$$

AdS/CFT and quantum information

CFT entanglement entropy dual to minimal surface in the bulk

(Ryu, Takayanagi Phys.Rev.Lett. 96 (2006) 181602)

Leading term in entanglement entropy given by
area of minimal surface in holographic dimension



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Examples for field theory/gravity theory beyond string theory?

Motivation:

- Holographic principle
- Entanglement = geometry
- Bulk reconstruction

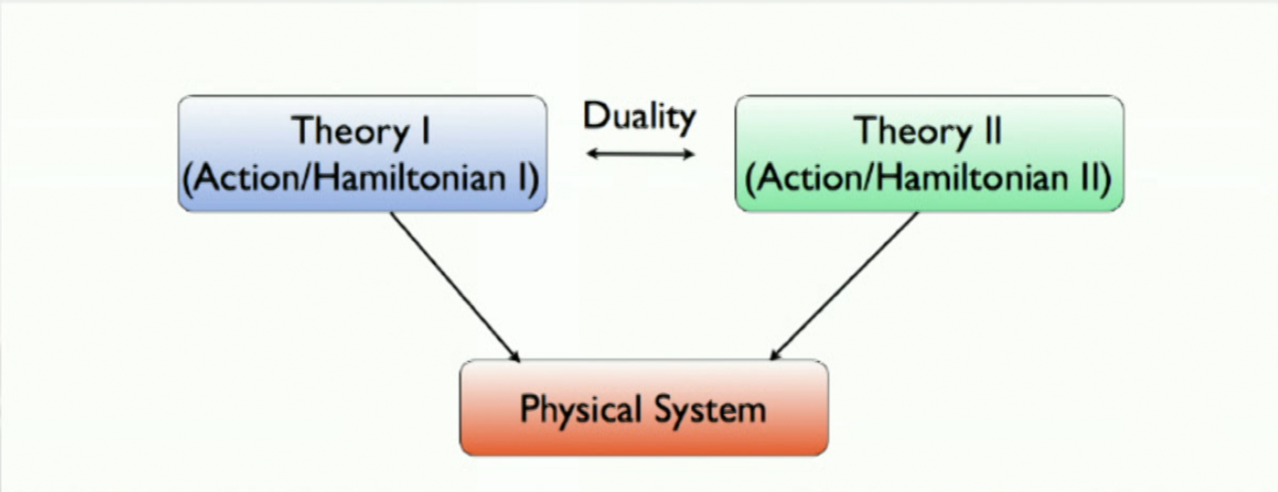
$$N^2 \propto \frac{L^3}{G_N^5}$$

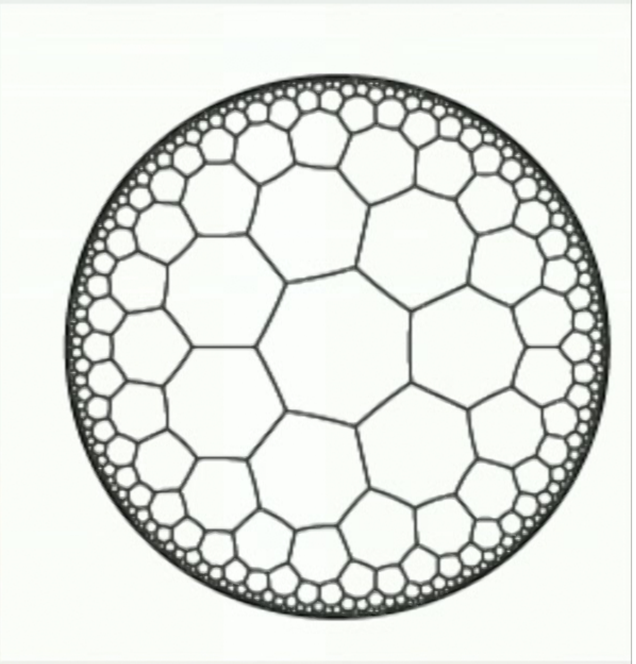
$$S = \frac{A}{4G_N}$$

Bekenstein-Hawking entropy

Discrete Holography

Goal: Establish holographic duality on hyperbolic tiling





arXiv:2205.05081 (Basteiro, Dusel, J.E., Hinrichsen, Meyer, Herdt, Schrauth)

arXiv:2205.05693 (Basteiro, Di Giulio, J.E., Karl, Meyer, Xian)

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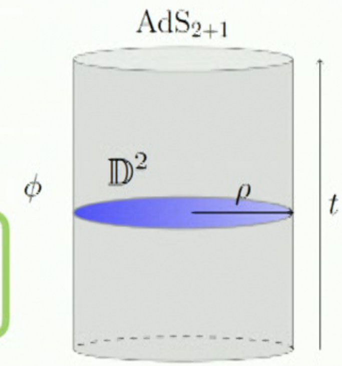
Regular hyperbolic tilings of the Poincaré disk

Consider AdS_{2+1} and fix a constant time slice $\longrightarrow$ $$ds^2 = (2R)^2 \frac{d\rho^2 + \rho^2\, d\phi^2}{(1-\rho^2)^2}$$

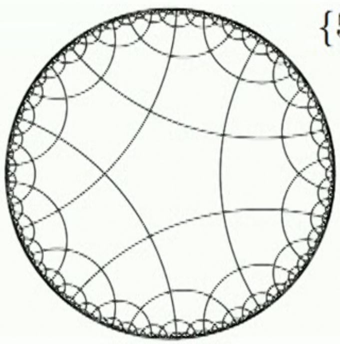


The metric induces the geodesic distance between two points $z_1 = \rho_1 e^{i\phi_1}, z_2 = \rho_2 e^{i\phi_2}$

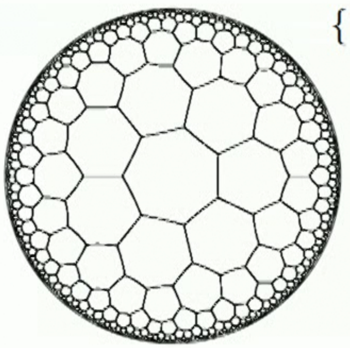
$$d(z_1, z_2) = R\,\mathrm{arccosh}\left(1 + \frac{2(\rho_1^2 + \rho_2^2 - 2\rho_2\rho_1 \cos(\phi_1 - \phi_2))}{(1-\rho_1^2)(1-\rho_2^2)}\right)$$

Tiling of hyperbolic space if $(p-2)(q-2) > 4$

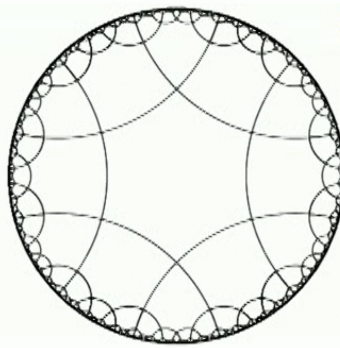
Regular hyperbolic tilings $\{p,q\}$: q regular p-gons meet at each vertex



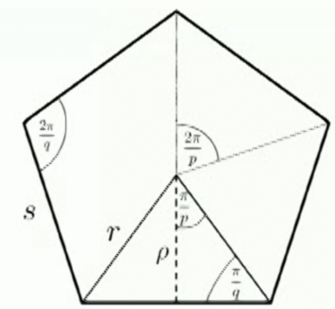
$\{5,4\}$



$\{7,3\}$



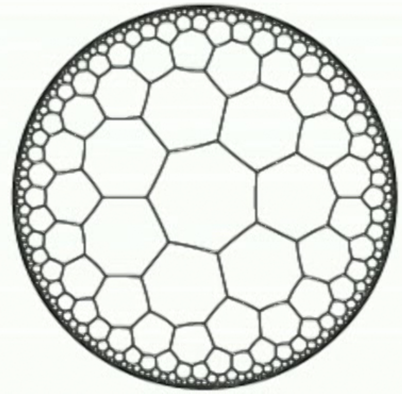
$\{6,4\}$



Hyperbolic trigonometry + distance d $\longrightarrow$ $r(p,q),\ s(p,q),\ \rho(p,q)$

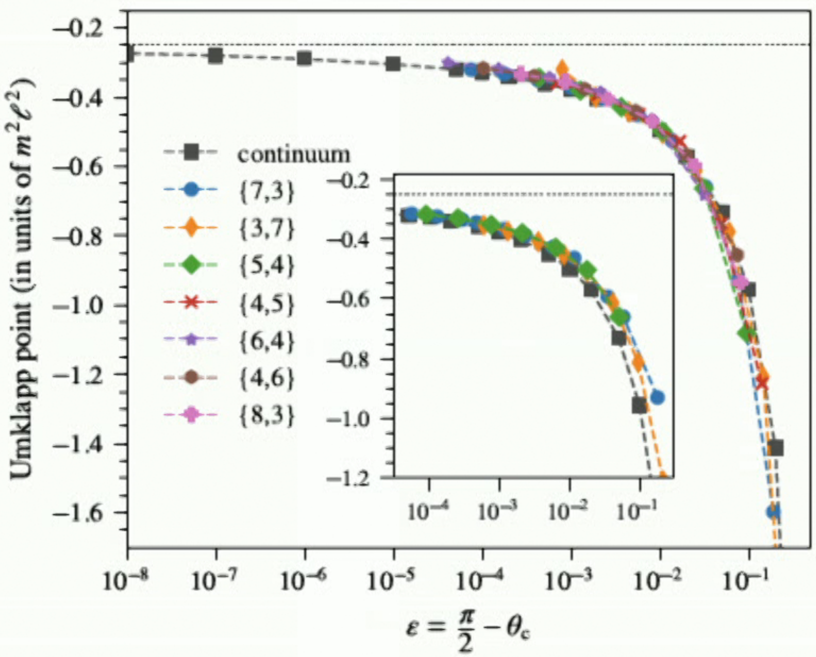
Breitenlohner-Freedman bound on hyperbolic tilings $m^2\ell^2 \geq -\frac{1}{4}$

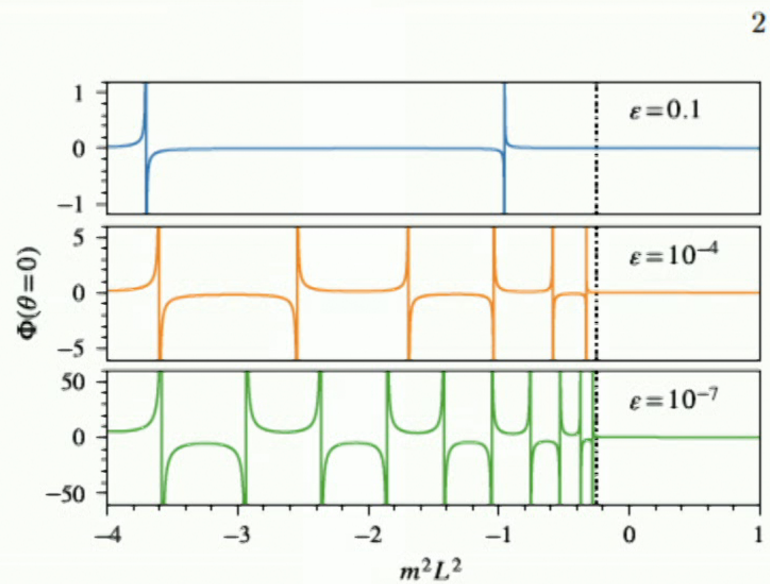
arXiv:2205.05081 Implications from gravity theory for mode stability



$$S = \frac{1}{2}\int d^2x\sqrt{g}\left(\partial^\mu\Phi\partial_\mu\Phi + m^2\Phi^2\right) \qquad \left(\tilde{\Box}\Phi\right)_j = \sum_{k|j} w_{jk}\ell^{-2}(\Phi_k - \Phi_j)$$

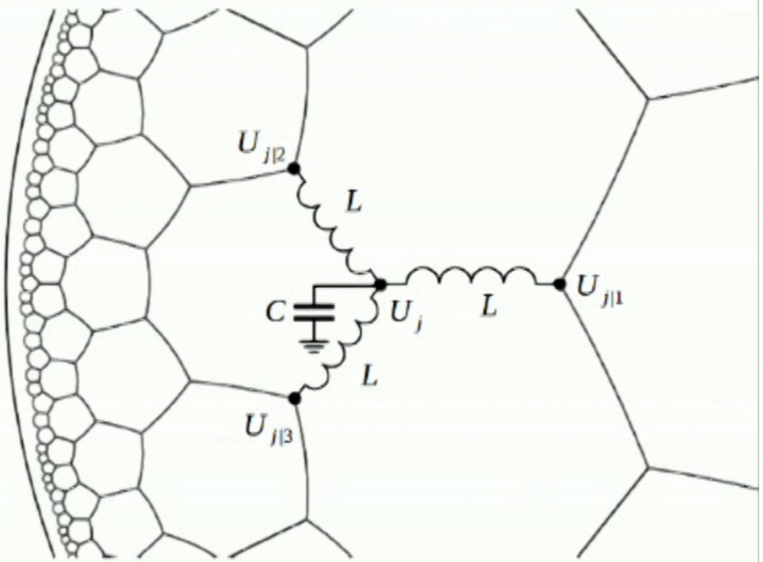
$$E = \int d\theta\left((\partial_\theta\tilde{\Phi})^2 + \Delta^2\tilde{\Phi}^2\right)$$





arXiv:2205.05081, Basteiro, J.E., Herdt, Hinrichsen, Meyer, Schrauth

Breitenlohner-Freedman bound on hyperbolic tilings



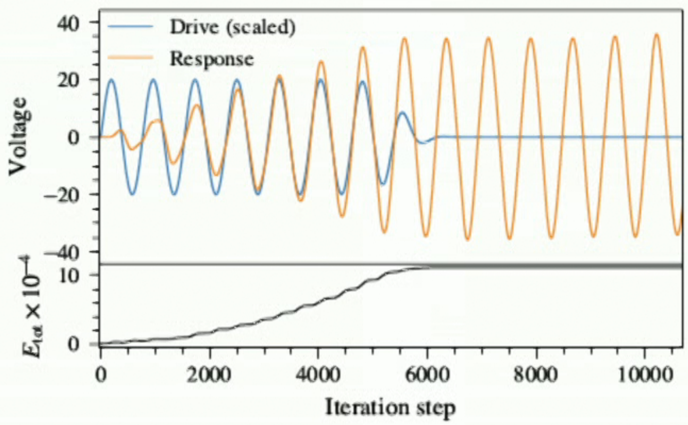
Map to electric circuit

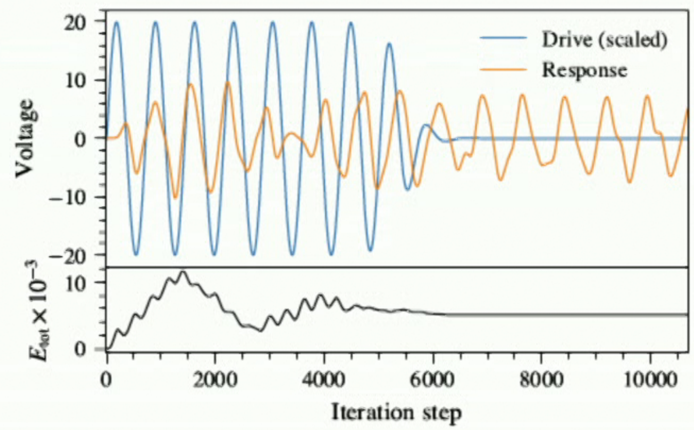
$$I_j = C\dot{U}_j \qquad w_{jk}(U_k - U_j) = L\dot{I}_{jk}$$

$$\ddot{U}_j = \frac{1}{LC}\sum_{k|j} w_{jk}(U_k - U_j)$$

$$U_j(t) = u_j e^{i\omega t} \qquad -\omega^2 U_j = \frac{1}{LC}\sum_{k|j}(U_k - U_j) = \frac{1}{LCw^{(p,q)}}\tilde{\Box} U_j$$

$$-m^2\ell^2 = \omega^2 LC w^{(p,q)}$$





Boyle, Dickens, Flicker 1805.02665

Hyperbolic tilings through inflation rules

Starting point: central tile = 0-th layer

↓

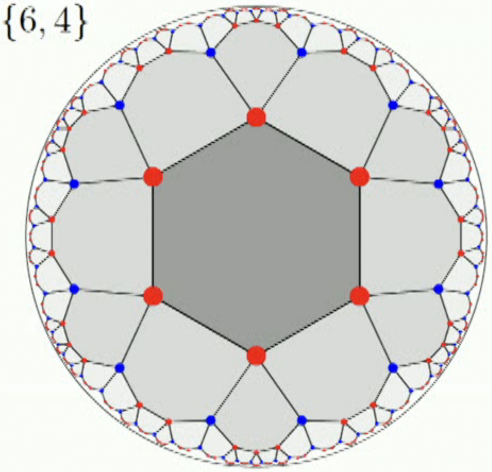
we construct concentric layers of tiles

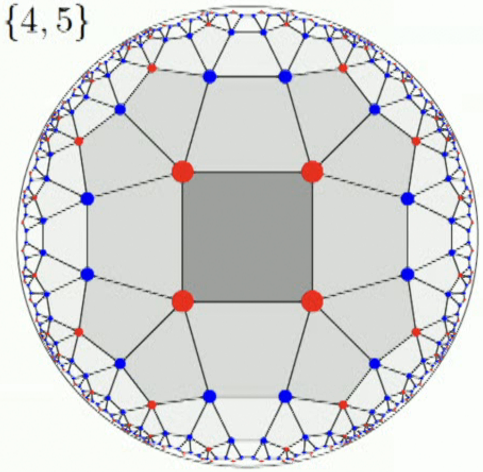
↓

After n layers: outermost set of edges and vertices = **boundary of the tiling**

2 internal neighbours = ● $\longleftrightarrow a$

3 internal neighbours = ● $\longleftrightarrow b$





Inflation rule

$$\sigma_{\{p,q\}} : \begin{cases} a \to w_a(a,b) \\ b \to w_b(a,b) \end{cases}$$

$$\sigma_{\{6,4\}} : \begin{cases} a \to aabaaab \\ b \to aab \end{cases}$$

$$\sigma_{\{4,5\}} : \begin{cases} a \to babab \\ b \to bab \end{cases}$$

The boundary of the tiling after a large number of inflation steps is characterised by a long **aperiodic sequence**, whose size grows by a factor $\lambda_+(p,q)$ at each step arXiv:2205.05693 (Basteiro, Di Giulio, J.E., Karl, Meyer, Xian)

A new step towards a discrete duality

Quantum gravity theory in AdS_{2+1} — dynamically dual — (1+1)-dimensional CFT at $\partial\mathrm{AdS}_{2+1}$

Empty AdS_{2+1} space

CFT vacuum state

Fix a constant time slice and regularly discretize the Poincaré disk via {p,q} tiling

Theory on the discrete geometry of the $\{p,q\}$ tiling — dynamically dual ? — Quantum chain at the boundary of the $\{p,q\}$ tiling

Empty tiling of the Poincaré disk

Ground state of the quantum chain

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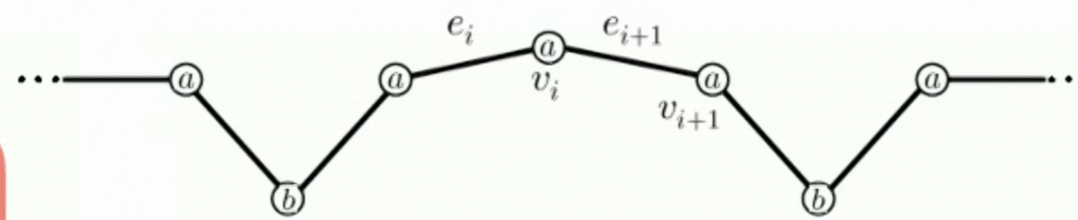
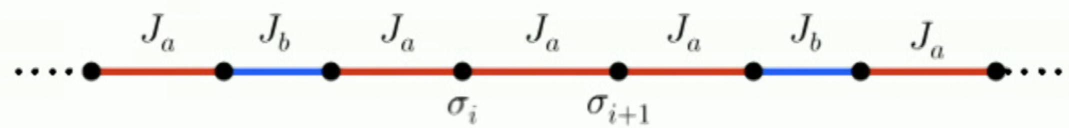
Aperiodic spin chains

Theory at the boundary of the tiling

Edge →	Spin 1/2
Vertex →	Bond
Letter →	Coupling





Aperiodic spin chains **[Luck '93; Hermission '97; Vidal, Mouhanna, Giamarchi '99]**

Consider a homogeneous chain in a gapless regime and introduce aperiodic modulation $\sigma_{\{p,q\}}$ on it

- **Relevant**: system in an aperiodicity-induced fixed point
- **Marginal**: critical properties dependent on J_a, J_b
- **Irrelevant**: same critical properties as homogeneous model

We focus on **aperiodic XXZ chain**

$$H = \sum_{i\in\mathbb{Z}} J_i \left[\sigma_i^{(x)}\sigma_{i+1}^{(x)} + \sigma_i^{(y)}\sigma_{i+1}^{(y)} + \Delta_0 \sigma_i^{(z)}\sigma_{i+1}^{(z)} \right] \qquad J_i \in \{J_a, J_b\}$$

Physical parameters: $\Delta_0,\ r \equiv J_a/J_b$

Interacting model and gapless when $0 \leqslant \Delta_0 \leqslant 1$

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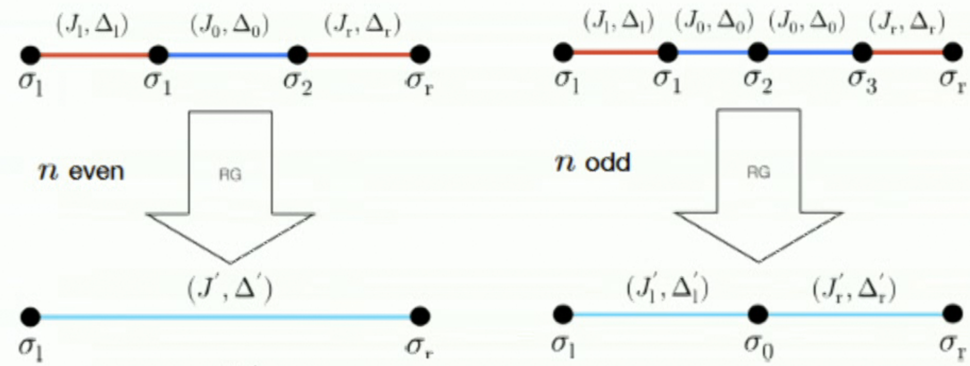
Strong disorder renormalisation group (SDRG)

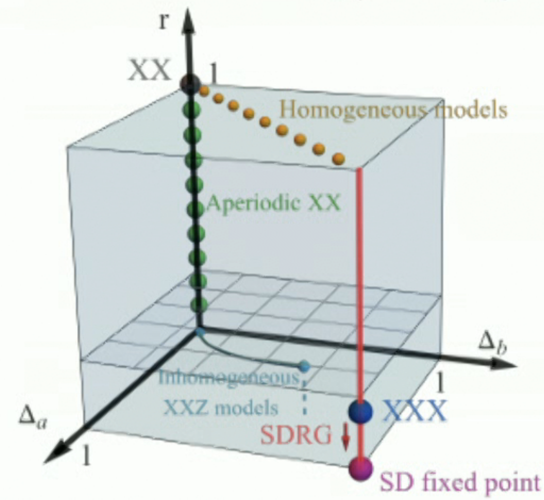
Developed for random disorder in [Dasgupta, Ma ‘79] and applied to aperiodic XXZ chains in [Vieira ’04; Hida ‘04]

At low energy, block of n consecutive spins with (J_0, Δ_0) can be decimated out if $J_0 \gg J_{\mathrm{l,r}},\ \Delta_0 \gg \Delta_{\mathrm{l,r}}$

We apply these rules along the whole chain: **flows of the couplings and critical properties of the chain**

Aperiodic XXX chain ($\Delta_0 = 1$) : the modulations generated by $\sigma_{\{p,q\}}$ for any $\{p, q\}$, drive the chain to an aperiodicity-induced fixed point (relevant modulations)





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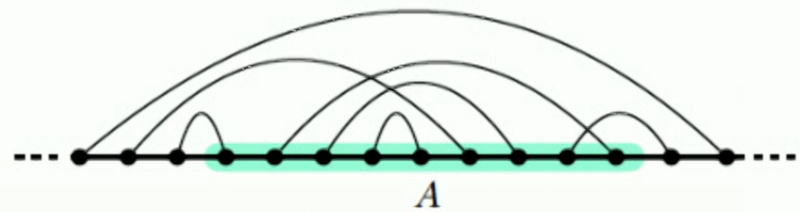
Entanglement entropy in aperiodic XXX chain

Aperiodic XXX chain with modulation generated by $\sigma_{\{p,q\}}$: the critical properties do not depend on r



SDRG + asymptotic sequence from $\sigma_{\{p,q\}}$

↓

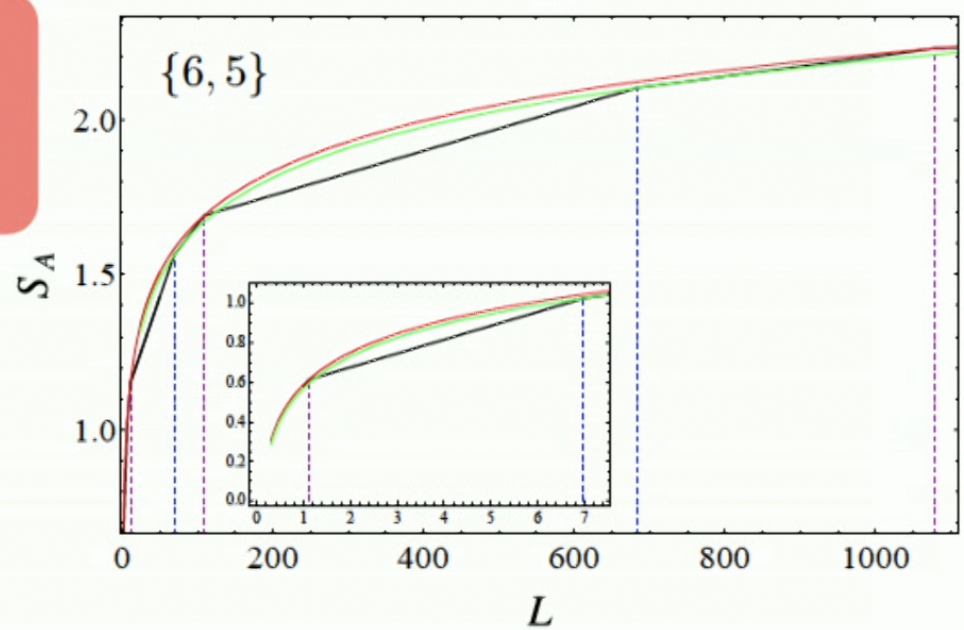
Analytic expression for S_A : piecewise linear behaviour
(generalising [Iglói, Juhász, Zimborás '07])

$$S_{\text{env},1} = \frac{c_{\text{eff}}}{3} \log L + \kappa_1$$

Effective central charge dependent on p and q

For modulations generated by $\sigma_{\{6,q\}}$, $q \geqslant 4$

$$c_{\text{eff}}(6,q) = \frac{6 - 2q + \sqrt{q^2 - 5q + 6}}{\ln\left(2q - 5 + 2\sqrt{q^2 - 5q + 6}\right)} \frac{6 \log 2}{6 - 2q}$$



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Tensor networks provide natural construction for holographic dimension

Swingle '09

Holographic quantum-error correction codes from perfect tensor networks

Pastawski, Harlow, Preskill, Yoshida '15 (HaPPY), Hayden, Nizami, Qi, Thomas, Walter '16

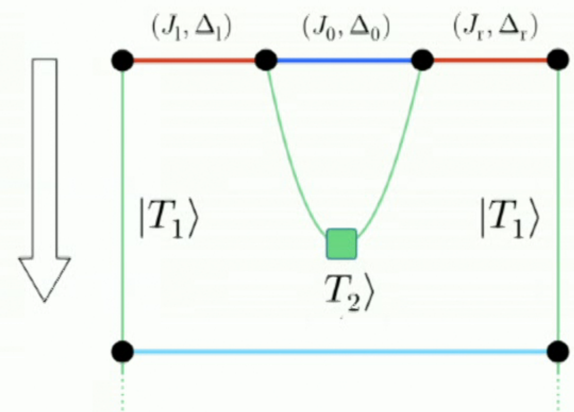
Use tensor network to implement the SDRG transformation for the aperiodic spin chain:

$\sigma^{(z)} |\pm\rangle = \pm |\pm\rangle$

Ground state of our Hamiltonian: $$|T\rangle = \sum_{\{m_i=\pm\}} |m_1\rangle \dots |m_n\rangle T_{m_1 \dots m_n}$$

n number of spins to be renormalised:

$$|T_n\rangle = \begin{cases} \frac{1}{\sqrt{2}} \sum_{\{m_i=\pm\}} \delta^{m_0}_{m_1} |m_0\rangle |m_1\rangle \,, & n=1\,, \\ \sum_{\{m_i=\pm\}} T_{m_1 \dots m_n} |m_1\rangle \dots |m_n\rangle \,, & n \text{ even}\,, \\ \frac{1}{\sqrt{2}} \sum_{\{m_i=\pm\}} T^{m_0}_{m_1 \dots m_n} |m_0\rangle |m_1\rangle \cdots |m_n\rangle \,, & n \text{ odd } \neq 1 \end{cases}$$



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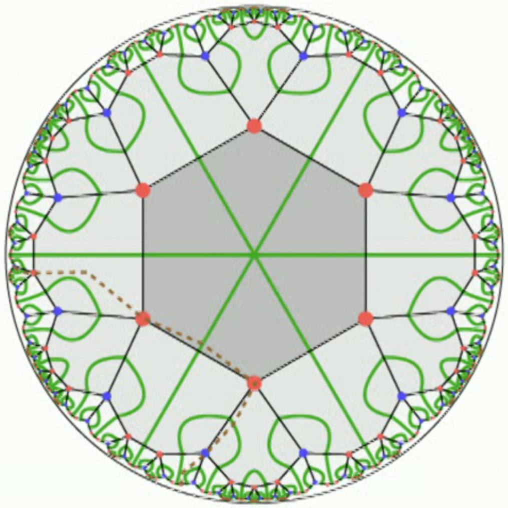
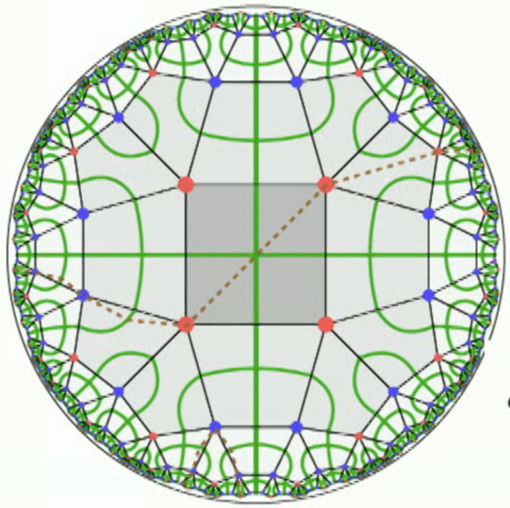
Tensor networks for the ground states of aperiodic spin chains



$\{6,4\}$



$\{4,5\}$

A bound for the effective central charge

$$c_{\text{eff}}^{\{2k,q\}} \leq \frac{3\ln 2}{\ln \lambda_+}\frac{(k-1)x+(k-3)(k-2)}{(k-2)+x}$$

$$x = 2-k+\sqrt{(k-1)\left(k-\frac{q}{q-2}\right)}$$

Bound saturated when the tensor states in the TN are perfect, e.g. for $\{6,q\}$

Replacing the decimated blocks with tensor states throughout the whole SDRG flow and contracting the shared legs of the tensors, we obtain the TN which reproduces exactly the ground state of aperiodic XXX chains

Entanglement entropy for the bulk

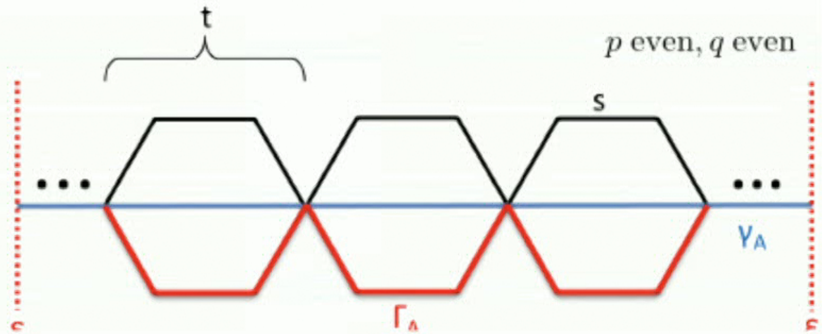
Adapted Ryu-Takayanagi formula

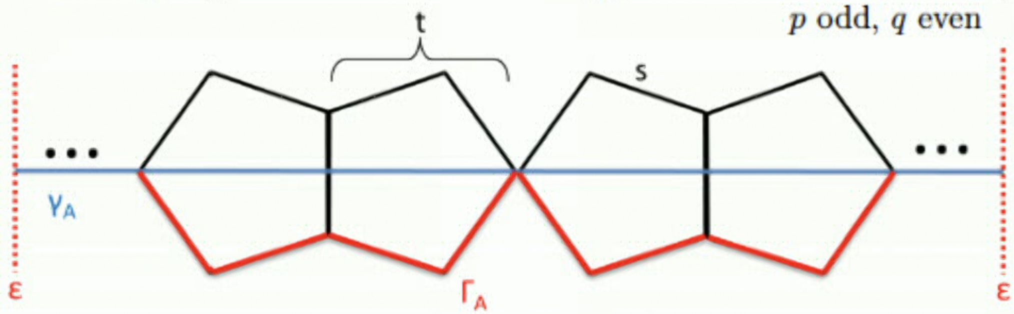
Starting from continuous setup: holographic CFT on the boundary

$\downarrow \longrightarrow \{p, q\}$ tiling

A = boundary subregion containing L vertices of the tiling

We assume the validity of Ryu-Takayanagi formula, but $\gamma_A \to \Gamma_A$





$$S_A = \frac{|\Gamma_A|}{4G_N} = \frac{c_{\text{eff, bulk}}}{3} \ln L$$

The coefficient $c_{\text{eff,bulk}}$ depends on the discretisation through p and q

Bulk coefficient differs from boundary coefficient

Similar analysis in the context of tensor networks in [Jahn, Zimborás, Eisert '20]

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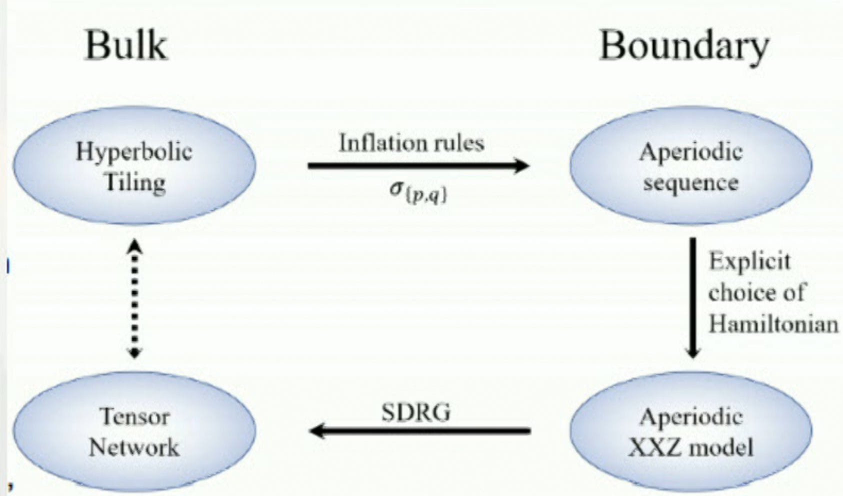
Summary of aperiodic sequence model

- Through the inflation procedure, the $\{p, q\}$ tiling induces an aperiodic sequence on the boundary of the Poincaré disk
- Aperiodic spin chain on the boundary: aperiodic XXZ chain
- Entanglement properties of aperiodic XXX chain: piecewise linear behaviour and logarithmic envelops with c_{eff} depending on p and q
- Exploiting SDRG we construct a tensor network in the bulk, which realize the ground state of the aperiodic XXX chain



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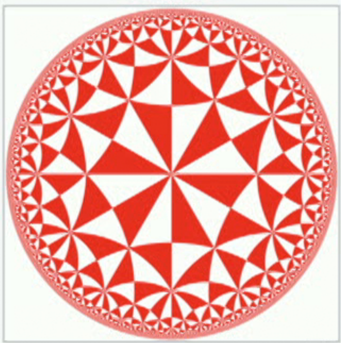
Hyperbolic lattices in mathematical physics

Gromov boundary

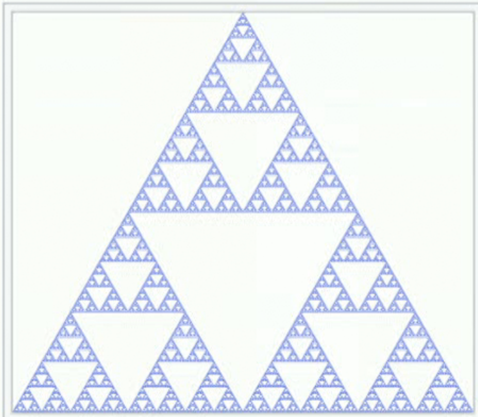
Wikipedia: The Gromov boundary of a δ-hyperbolic space (especially a hyperbolic group) is an abstract concept generalizing the boundary sphere of hyperbolic space. Conceptually, the Gromov boundary is the set of all points at infinity.



The (6,4,2) triangular hyperbolic tiling. The triangle group corresponding to this tiling has a circle as its Gromov boundary.

Source: Wikipedia

Higher dimensions:
Gromov boundaries are fractals



Sierpiński triangle

Gesteau, Marcolli, Parikh 2202.01788:

Systematic construction of (perfect) tensor networks and RT formulae using generalized hyperbolic spaces

Open question: CFTs on fractals?

Hyperbolic lattices in condensed matter physics

Tight-binding Hamiltonian with nearest-neighbour hopping on hyperbolic lattice

$$H_{\mathrm{TB}} = \omega_0 \sum_i a_i^\dagger a_i - t \sum_{<i,j>} (a_i^\dagger a_j + a_j^\dagger a_i)$$

Energy spectrum
Band theory
Fourier transform on hyperbolic space
Representations of Fuchsian group
Generalization of Bloch's theorem

Kollar, Fitzpatrick, Houck,
Hyperbolic lattices in circuit electrodynamics, Nature 2019

Maciejko, Rayan: Hyperbolic band theory, Science 2021

Boettcher, Gorshkov, Kollár, Maciejko, Rayan, Thomale:
Crystallography of hyperbolic lattices, PRB 2022

Cheng, Serafin, McInerney, Rocklin, Sun, Mao:
Band theory and boundary modes, PRL 2022

Hyperbolic electric circuits

Hyperbolic lattice from coplanar wave-guide resonators

Electric circuit networks (inspired by topolectric circuits)

Eigenvalues of Laplacian

Signal propagation along geodesics

Hyperbolic lattices in circuit electrodynamics,
Kollar, Fitzpatrick, Houck Nature **571**, 45 (2019)

Simulating hyperbolic space on a circuit board,
Lenggenhager, Stegmaier, Upreti, Hofmann, Helbig, Vollhardt, Greiter,
Lee, Imhof, Kießling, Boettcher, Neupert, Thomale, Bzdusek Nature Comm. 2022

Quantum simulation of hyperbolic space with circuit quantum electrodynamics,
Boettcher, Bienias, Belyansky, Kollár, Gorshkov , PRA 2020

Lattice gauge theory and quantum gravity

Lattice gauge theory studies of scalar fields in hyperbolic lattices

Calculation of correlation functions - agreement of bulk results with CFT expectations

Brower, Cogburn, Fitzpatrick, Howarth, Tan 2019, PRD

Asaduzzaman, Catterall, Hubisz, Nelson, Unmuth-Yockey 2021, PRD

Quantum gravity:

Use discrete approaches to quantum gravity for 1/N corrections from bulk side

Freidel et al 2004, Dittrich et al 2017

Conclusion and Outlook

Discrete Holography - towards a new example of holography duality

of relevance in quantum gravity, condensed matter physics, mathematical physics

Boundary spin chain Hamiltonian reflecting the tiling structure (inflation rule)

RG / tensor network study

Entanglement entropy: Comparison with modified Ryu-Takayanagi formula

Outlook:

Large N, beyond nearest-neighbour interaction and averaging

implementing global symmetries

Quantum gravity - adapted version of discrete approaches