

Title: Relativity - Lecture 221021

Speakers:

Collection: Relativity (2022/2023)

Date: October 21, 2022 - 9:00 AM

URL: <https://pirsa.org/22100080>

$$\underbrace{\frac{d^2 x^a}{d\lambda^2}}_{L^{-1}} + \underbrace{\Gamma^a_{bc}}_{L^{-1}} \underbrace{\frac{dx^b}{d\lambda}}_{L^0} \underbrace{\frac{dx^c}{d\lambda}}_{L^0} = \alpha \underbrace{(\nabla_a R)}_{L^2} \underbrace{\frac{dx^a}{d\lambda}}_{L^0} \underbrace{\frac{dx^d}{d\lambda}}_{L^0}$$

$$\boxed{l_p = 10^{-33} \text{ cm}}$$

$$l_{\oplus}$$

$$\underbrace{\frac{d^2 x^a}{d\lambda^2}}_{L^{-1}} + \underbrace{\int_{\mathcal{C}}}_{L^{-1}} \underbrace{\frac{dx^b}{d\lambda}}_{L^0} \underbrace{\frac{dx^c}{d\lambda}}_{L^0} = \underbrace{\alpha}_{L^2} \underbrace{(\nabla_a R)}_{\frac{1}{L^3}} \underbrace{\frac{dx^a}{d\lambda}}_{L^0} \underbrace{\frac{dx^d}{d\lambda}}_{L^0}$$

$$\boxed{l_P = 10^{-33} \text{ cm}}$$

$$\boxed{l_{\oplus} = \frac{c^2}{a} = 10^{18} \text{ cm}}$$

$$a \sim \frac{\text{m}}{\text{s}^2} ; c \sim \frac{\text{m}}{\text{s}}$$

$$\begin{aligned}
 & \delta \left[G_{ab} \delta g^{ab} + \nabla^\mu \overline{\nabla_\mu (\delta g)} \right] \\
 & \left[f(b) G_{ab} \right] \delta g^{ab} + \int \sqrt{-g} \left[f(b) \nabla^\mu \overline{\nabla_\mu (\delta g)} \right] - \\
 & \int \sqrt{-g} \left[\begin{matrix} \xi \\ \end{matrix} \right]
 \end{aligned}
 \quad \left| \quad \begin{aligned}
 & \delta R_{ab} \\
 & \hookrightarrow R \rightarrow S^\mu \rightarrow \delta g
 \end{aligned} \right.$$

Scalar Tensor theories $(M, \{g, \phi\})$

$$S_{ST} \approx \int \sqrt{-g} f(\phi) R \, d^4x \quad \xrightarrow{\delta S(\delta g)} \quad \sqrt{-g} f(\phi) [G_{ab} \delta g^{ab} +$$

$$\int \sqrt{-g} [f(\phi) G_{ab}] \delta g^{ab} +$$

$$\xrightarrow{EOM.} \quad G_{ab} = \frac{1}{f(\phi)} \left[T_{ab}^M + \frac{1}{2} T_{ab}^{(\phi)} + \underbrace{\nabla_a \nabla_b \phi - g_{ab} \square \phi}_{\text{...}} \right]$$

$$\square \phi = \dots + \frac{\delta f}{\delta \phi} R$$

$$\begin{aligned}
 & \left[G_{ab} \delta g^{ab} + \nabla^\mu \overline{\nabla_\mu (\delta g)} \right] \\
 & \left[\sqrt{-g} G_{ab} \right] \delta g^{ab} + \int \sqrt{-g} \left[f(\phi) \nabla^\mu \nabla_\mu (\delta g) \right] - \\
 & \left[\phi - g_{ab} \nabla^\mu \phi \right] \int \sqrt{-g} \left[\begin{matrix} \delta g \\ \end{matrix} \right]
 \end{aligned}
 \quad \left| \begin{array}{l} g^{ab} \delta R_{ab} \\ \hookrightarrow R \rightarrow \delta R \rightarrow \delta g \end{array} \right.$$

$$[G_{ab} \delta g^{ab} + \nabla^\mu \overline{\nabla}(\delta g)_\mu]$$

$$+ \int \sqrt{-g} [f(\phi) \nabla^\mu \nabla(\delta g)_\mu] - \int \sqrt{-g} [\xi]$$

$$\delta R_{ab} \rightarrow R \rightarrow \delta R \rightarrow \delta g$$

$$\phi_{,tt} = \phi_{,xx} + \dots \phi_{,xxx}$$

$$\frac{d^2 x}{dt^2} = () + \frac{d^3 x}{dt^3}$$

$(\eta, \{g\}, \phi)$

$\delta S(\delta g)$

$$\sqrt{-g} f(\phi) [G_{ab} \delta g^{ab} + \nabla \nabla (\delta g)]$$

$$\int \sqrt{-g} [f(\phi) G_{ab}] \delta g^{ab} + \int \sqrt{-g} [\nabla \nabla (\delta g)]$$

$$= \frac{1}{f(\phi)} \left[T_{ab} + \frac{1}{2} T_{ab}^{(\phi)} + \nabla_a \nabla_b \phi - g_{ab} \nabla^2 \phi \right]$$

$$= \dots + \frac{\delta \mathcal{L}}{\delta \phi} R$$

$$G_{ab} = -R_{ab} + \frac{1}{2} g_{ab} R$$

$$\begin{aligned}
 & + \frac{\delta \mathcal{L}}{\delta \phi} R \\
 & G_{\mu\nu} = \underbrace{-D_\mu \phi D_\nu \phi - \frac{1}{2} g_{\mu\nu} (D_\rho \phi)^2}_{\text{DBT}} + \frac{1}{2} g_{\mu\nu} [M_{\text{Pl}}^2]
 \end{aligned}$$

$$1 - \frac{1}{2} \frac{d^2 \phi}{d\tau^2} \cdot \frac{d\phi}{d\tau}$$

$$G_{ab} = -D_a \phi D_b \phi - \frac{1}{2} g_{ab} (D_c \phi)^2$$

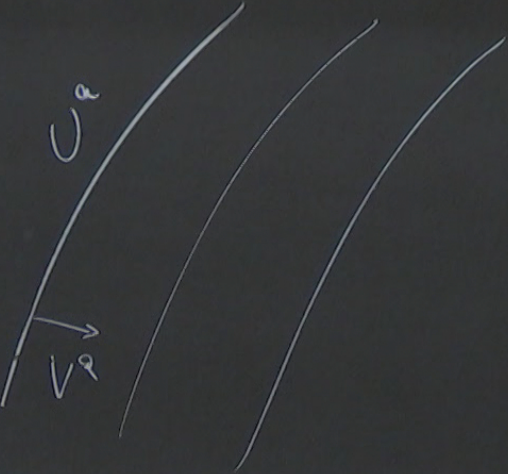
(D_b ↗
DTU ↘)

Geodesic deviation [Appendix F Carroll]

$$U^a \rightarrow U^a \nabla_a U^b = 0$$

$$U^a U_a = -1$$

Prop
1



define: $B_{ab} = \nabla_a U_b$

props: $U^a B_{ab} = 0 \rightarrow$ geod $U^a \nabla_a U_b$

$U^b B_{ab} = 0 \rightarrow$ normalized $U^b \nabla_a U_b = \frac{1}{2} \nabla_a$

$$\rightarrow U^a \nabla_a U^b = 0$$

$$U^a U_a = -1$$

Projector
 $\perp$ to U_a

$$h_{ab} U^b = [g_{ab} + U_a U_b] U^b$$

$$= g_{ab} U^b + U_a U_b U^b$$

$$h_{ab} U^b = U_a - U_a = 0$$

$$\rightarrow \text{geod} \quad U^a \nabla_a U_b$$

$$\rightarrow \text{normalized} \quad U^b \nabla_a U_b = \frac{1}{2} \nabla_a (U^b U_b)$$

Projector
 $\perp$ to U_a

$$h_{ab} U^b = [g_{ab} + U_a U_b] U^b$$

$$= g_{ab} U^b + U_a U_b U^b$$

$$h_{ab} U^b = U_a - U_a = 0$$

$$U^b \nabla_a U_b$$

$$U^b \nabla_a U_b = \frac{1}{2} \nabla_a (U^b U_b)$$

$$[U, V] \Rightarrow U^a \nabla_a V^b = V^a \nabla_a U^b$$

DBT
DTU

Appendix F Carroll

$$U^a \rightarrow U^a \nabla_a U^b = 0$$

$$U^a U_a = -1$$

Projector
↓ to U_a

define: $B_{ab} = \nabla_b U_a$

Props: $U^a B_{ab} = 0 \rightarrow$ geod

$$U^a \nabla_a U_b$$

$U^b B_{ab} = 0 \rightarrow$ normalized

$$U^b \nabla_a U_b = \frac{1}{2} \nabla_a (U^b U_b)$$

$$= \underbrace{U^c \nabla_b \nabla_c U_a}_{\nabla_b [U^c \nabla_c U_a]} + R^d{}_{cba} U^c U_d$$

$$= \nabla_b [U^c \nabla_c U_a]$$

$$- (\nabla_b U^c) (\nabla_c U_a)$$

$$U^c \nabla_c B_{ab} = - B^c{}_b B_{ac} + R^d{}_{cba} U^c U_d$$

$$\begin{aligned}
 U^c \nabla_c (B_{ab}) &= U^c \nabla_c (\nabla_b U_a) \\
 &= \underbrace{U^c \nabla_b \nabla_c U_a} + R^d{}_{cba} U^c U_d \\
 &= \nabla_b [\cancel{U^c \nabla_c U_a}] \\
 &\quad - (\nabla_b U^c) (\nabla_c U_a)
 \end{aligned}$$

$$U^c \nabla_c B_{ab} = -B^c{}_b B_{ac} + R^d{}_{cba} U^c U_d$$

$$\Theta = B_{ab} \underbrace{h^{ab}}_{\Theta = (\nabla_b u^b) + 0} = (\nabla_b u^b) (g^{ab} + u^a u^b)$$

$$g^{ab} g_{ab} = 4$$

$$h^{ab} h_{ab} = 3$$

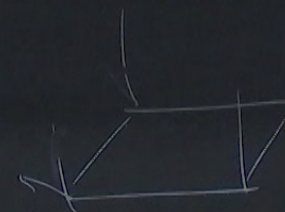
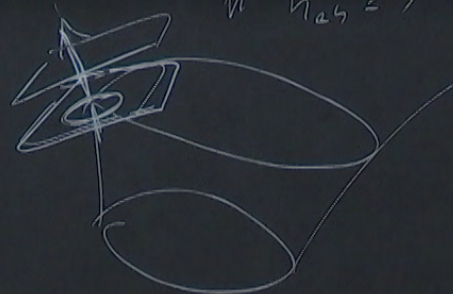
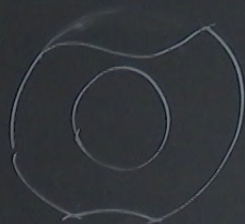
$$\sigma_{ab} = P_{ab} - \frac{1}{3} \Theta h_{ab}$$

shear

$$\omega_{ab} = P_{ab}$$

twist

$$g = h - u u$$



$$B_{ab} = \frac{1}{3} \Theta h_{ab} + \sigma_{ab} + \omega_{ab}$$

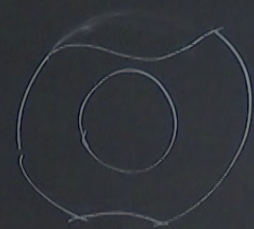
$$\Theta = B_{ab} h^{ab} = (\nabla_b u^b) (g^{ab} + u^a u^b)$$

$$\Theta = (\nabla_b u^b) + 0$$

$$\sigma_{ab} = B_{(ab)} - \frac{1}{3} \Theta h_{ab} \quad \text{shear}$$

$$\omega_{ab} = B_{[ab]} \quad \text{twist}$$

$$g = h -$$



trace with g^{ab}

$$U^c \nabla_c B_{ab} = - B^c_b B_{ac} + R^d_{cba} U^c U_d$$

$$B_{ab} = \frac{1}{3} \Theta h_{ab} + \sigma_{ab} + \omega_{ab}$$

$$\frac{d\Theta}{d\tau} = -\frac{1}{3} \Theta^2 - \sigma^{ab} \sigma_{ab} - \omega^{ab} \omega_{ab} - R_{cd} U^c U^d$$

$$\Theta = B_{ab} h^{ab} = (\nabla_b U_a) (g^{ab} + U^a U^b)$$

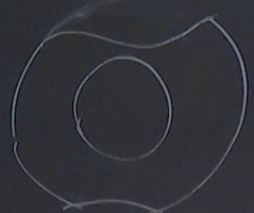
$$\Theta = (\nabla_b U^b) + 0$$

$$\sigma_{ab} = B_{(ab)} - \frac{1}{3} \Theta h_{ab}$$

$$\omega_{ab} = B_{[ab]}$$

shear

twist



$$B_{ab} = \frac{1}{3} \Theta h_{ab} + \sigma_{ab} + \omega_{ab}$$

$$\frac{d\Theta}{d\tau} = -\frac{1}{3} \Theta^2 - \sigma^{ab} \sigma_{ab} - \omega^{ab} \omega_{ab} - R_{ab} U^a U^b$$

$$[T_{ab} U^b U^b = \dots]$$

$$\Theta = B_{ab} h^{ab} = (\nabla_b U_a) (\nabla^a U^b + U^a U^b)$$

$$\Theta = (\nabla_b U^b) + \Theta$$

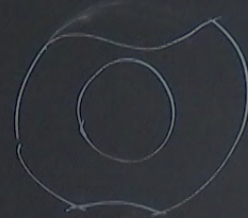
$$\sigma_{ab} = B_{(ab)} - \frac{1}{3} \Theta h_{ab}$$

shear

$$\omega_{ab} = B_{[ab]}$$

twist

$$(-, +, +, +)$$



$$g = h -$$

g

$$R = -2T \quad \left[R_{ab} - \frac{1}{2} g_{ab} R = T_{ab} \right] \rightarrow R_{ab} = T_{ab} - \frac{1}{2} S_{ab}$$

$$B_{ab} = \frac{1}{3} \Theta h_{ab} + \sigma_{ab} + \omega_{ab}$$

$$\frac{d\Theta}{d\tau} = -\frac{1}{3} \Theta^2 - \sigma^{ab} \sigma_{ab} - \omega^{ab} \omega_{ab} - R_{ab} U^a U^b$$

$$\left[T_{ab} U^b U^b + \frac{1}{2} T \right]$$

g

70

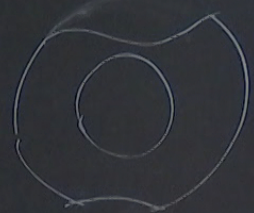
$$\Theta = B_{ab} h^{ab} = (\nabla_b U_a) (g^{ab} + U^a U^b)$$

$$\Theta = (\nabla_b U^b) + 0$$

$$\sigma_{ab} = B_{(ab)} - \frac{1}{3} \Theta h_{ab} \quad \text{shear}$$

$$\omega_{ab} = B_{[ab]} \quad \text{twist}$$

$$(-, +, +, +)$$



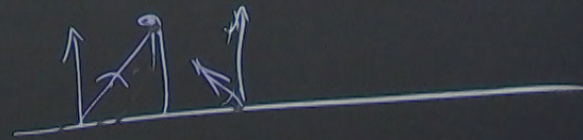
$$g = h$$

Projector
 $\perp$ to U_a

$$h_{ab} U^b = [g_{ab} + U_a U_b] U^b$$

$$= g_{ab} U^b + U_a U_b U^b$$

$$h_{ab} U^b = U_a - U_a = 0$$



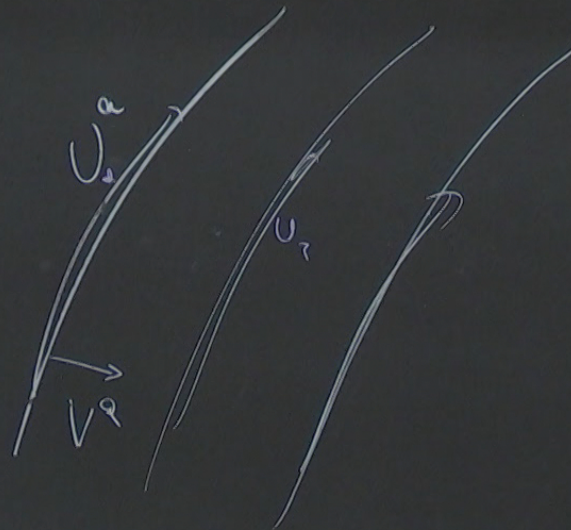
U_b

$$\nabla_a U_b = \frac{1}{2} \nabla_a (U^b U_b)$$

$$[U, V] \Rightarrow U^a \nabla_a V^b = V^a \nabla_a U^b$$

Geodesic deviation [Appendix F Carroll]

$U^a \rightarrow$



define: $B_{ab} = \nabla_b U_a$

props: $U^a R_{ab} = 0 \rightarrow$
 $U^b R_{ab} = 0$