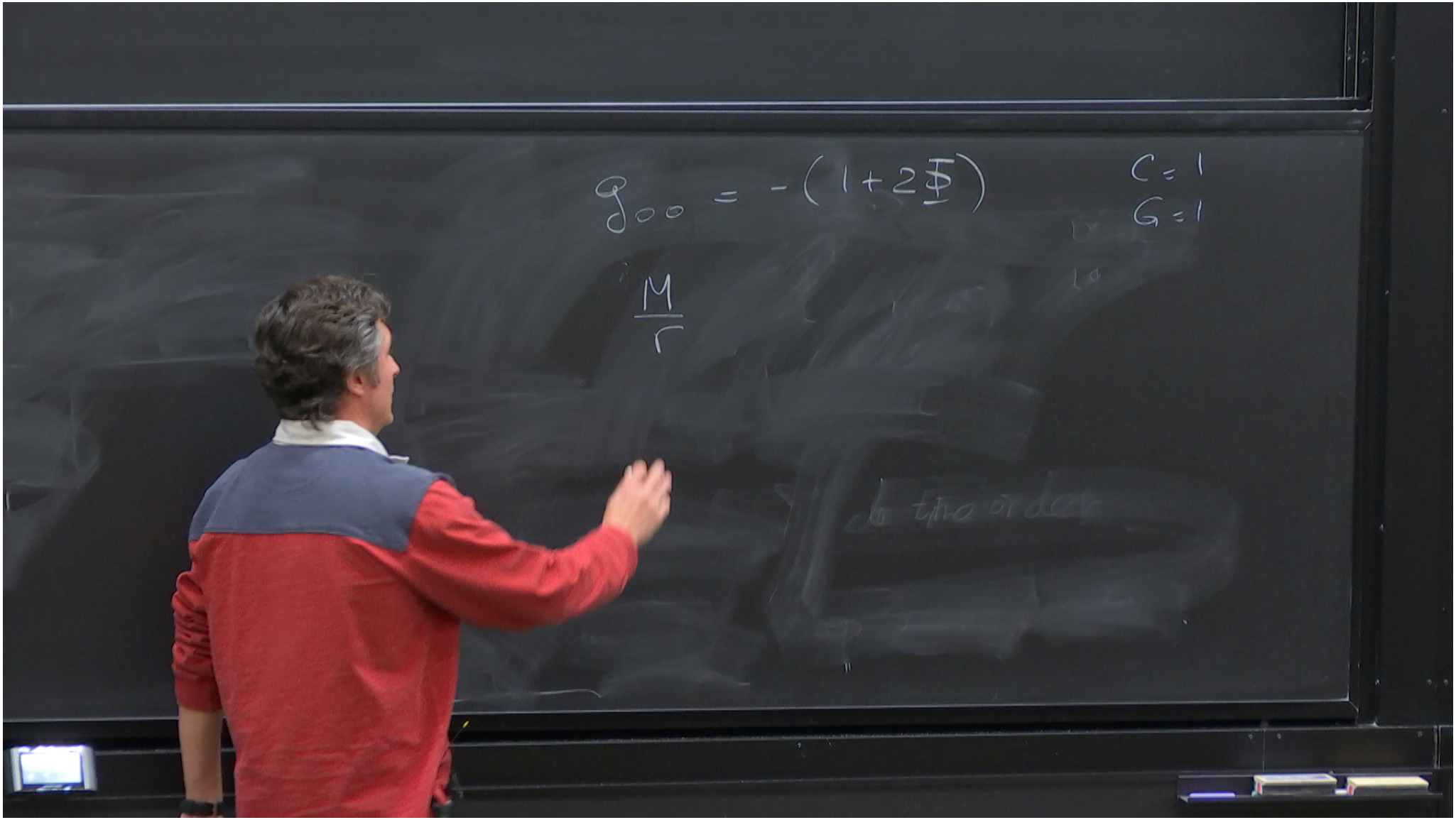
Title: Relativity - Lecture 221018

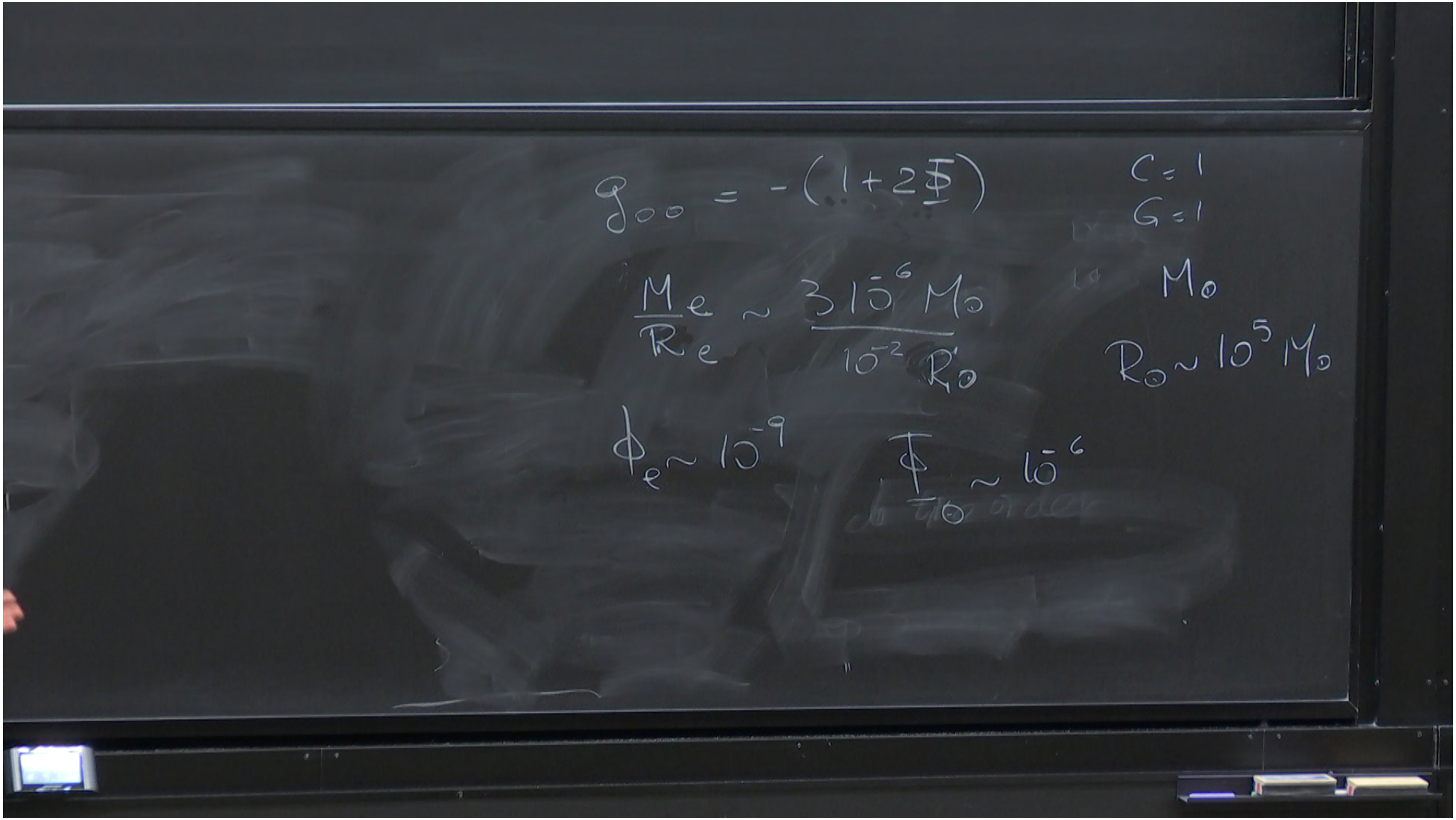
Speakers:

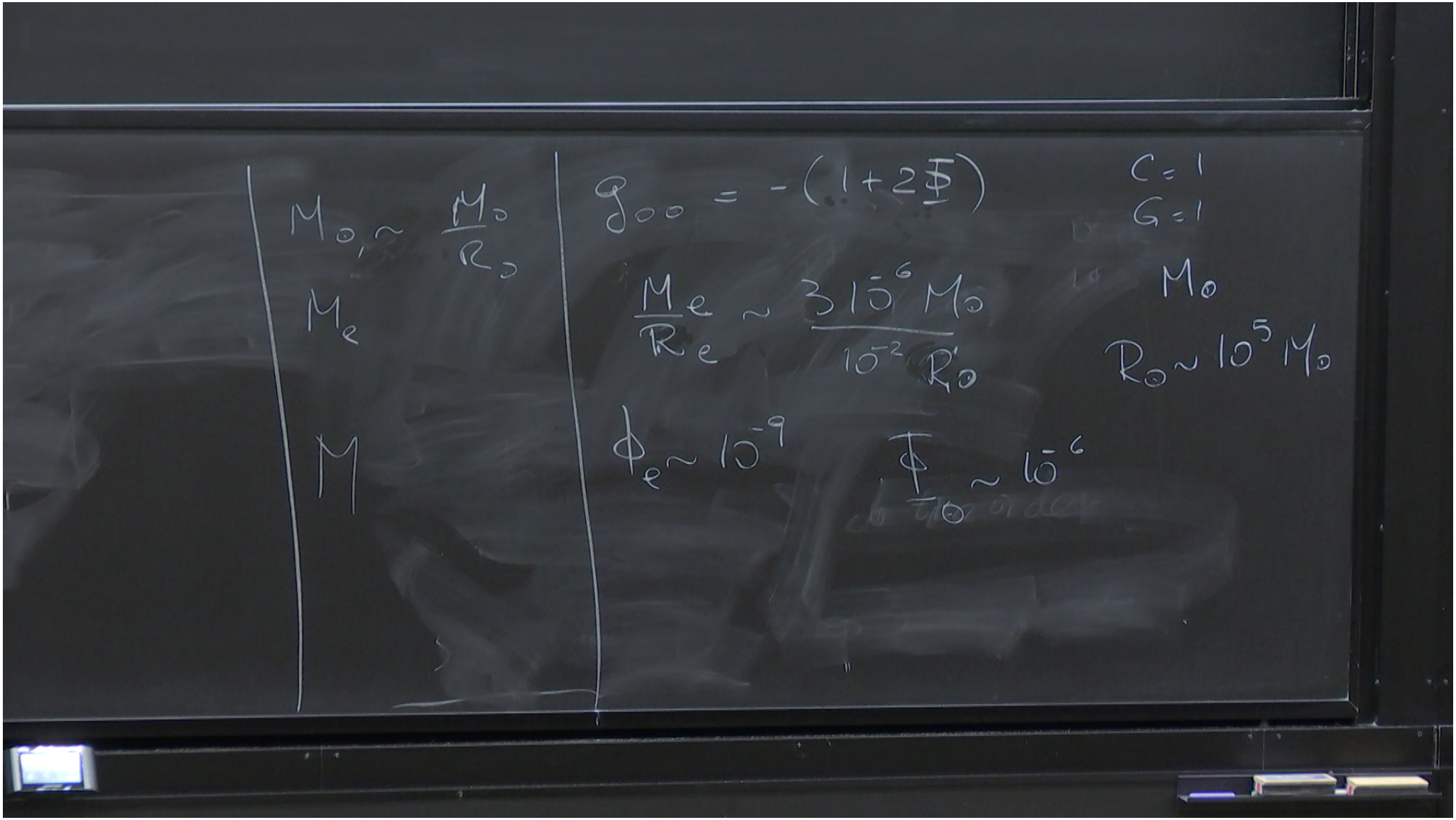
Collection: Relativity (2022/2023)

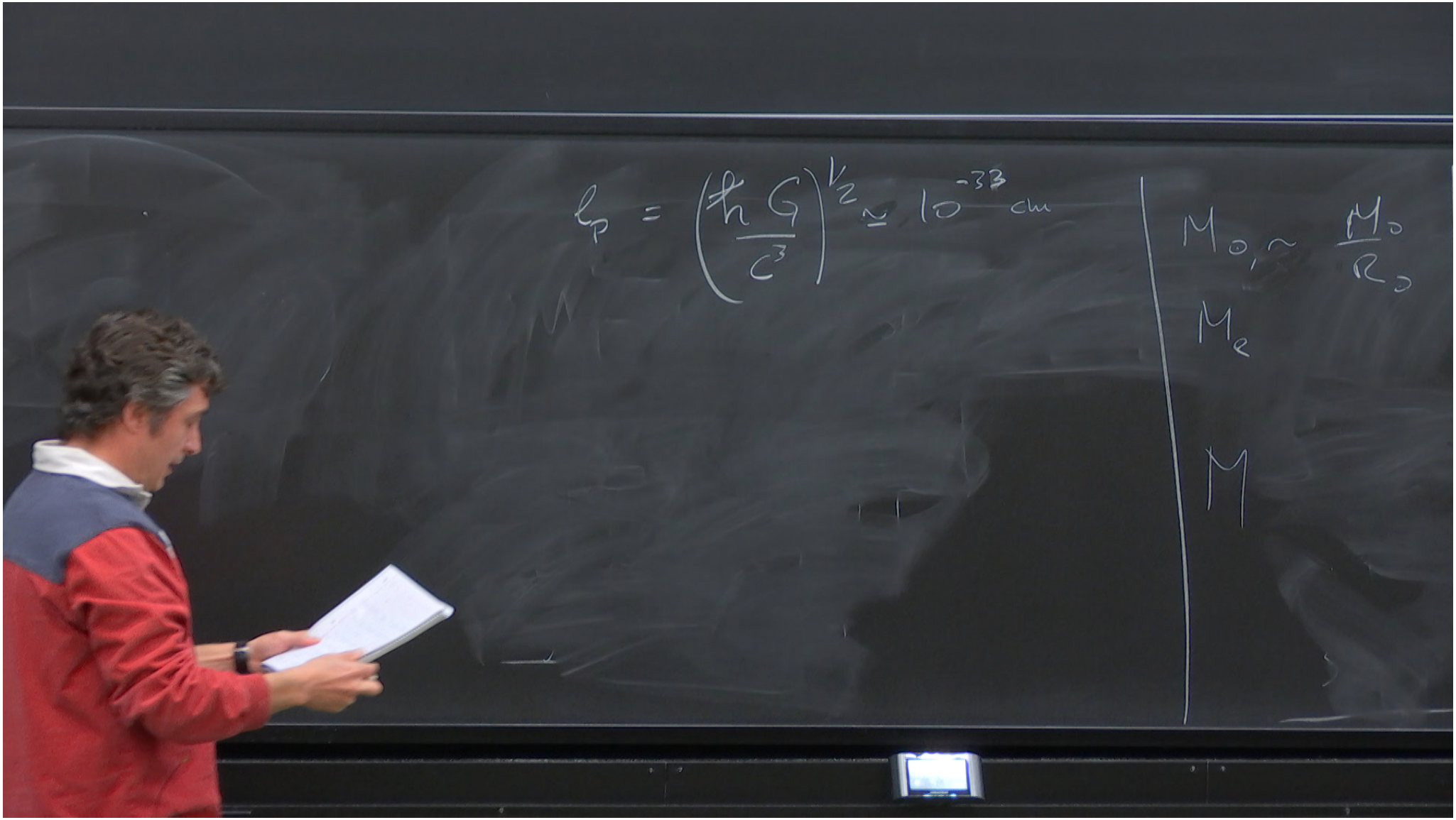
Date: October 18, 2022 - 2:00 PM

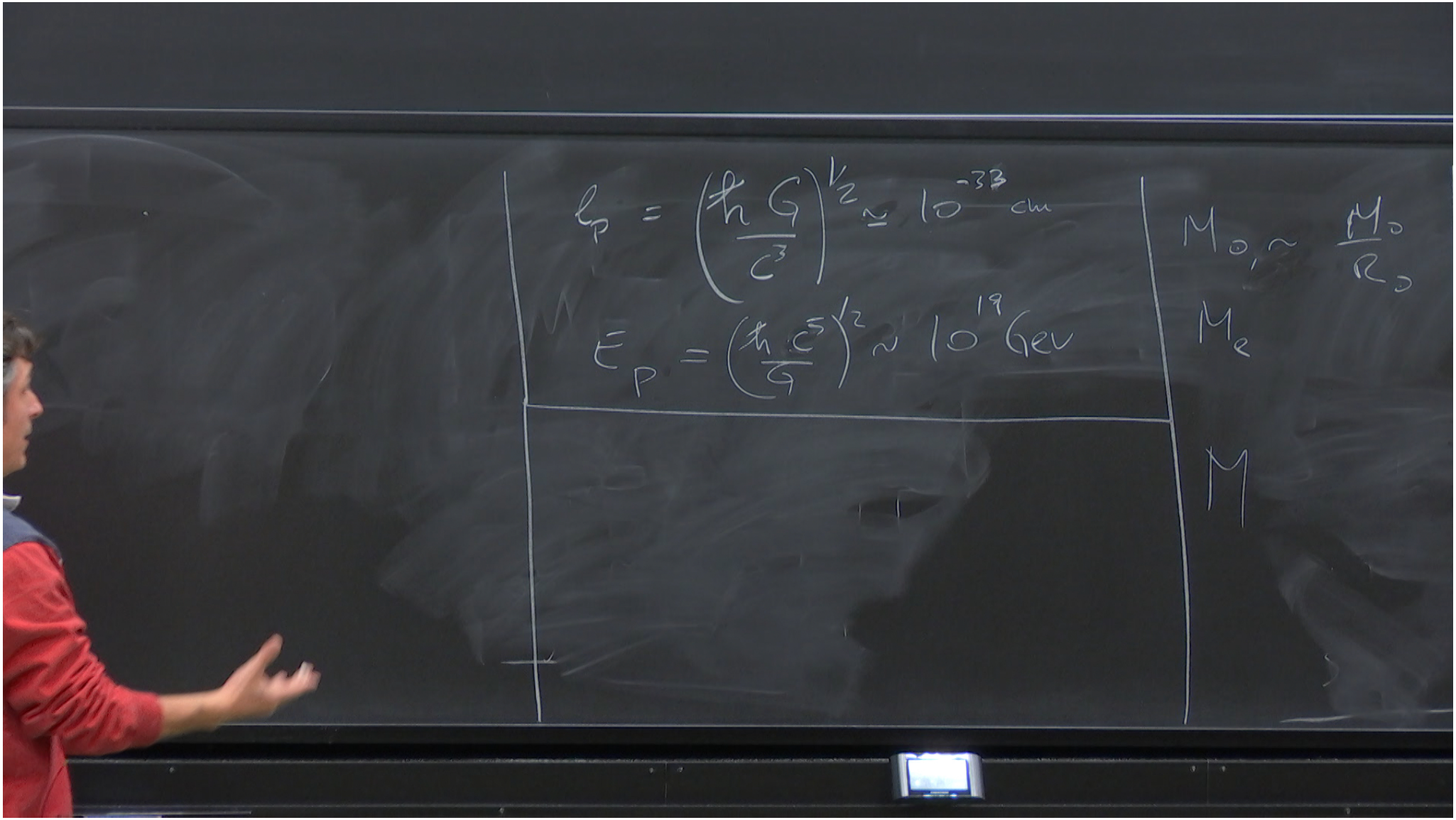
URL: https://pirsa.org/22100079

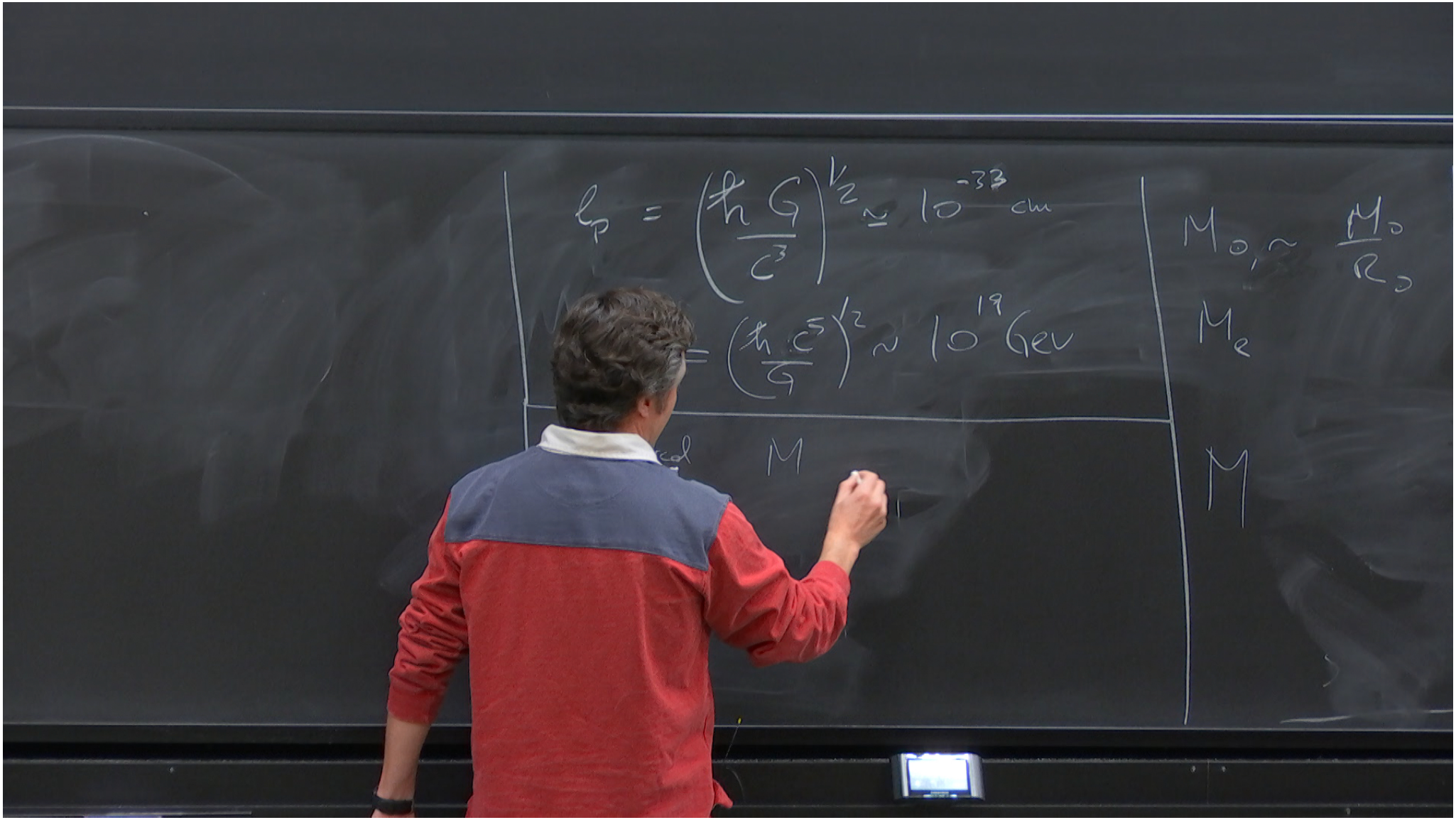

$g_{00} = -(1 + 2\frac{M}{r})$
$C = 1$
$G = 1$
$\frac{M}{r}$

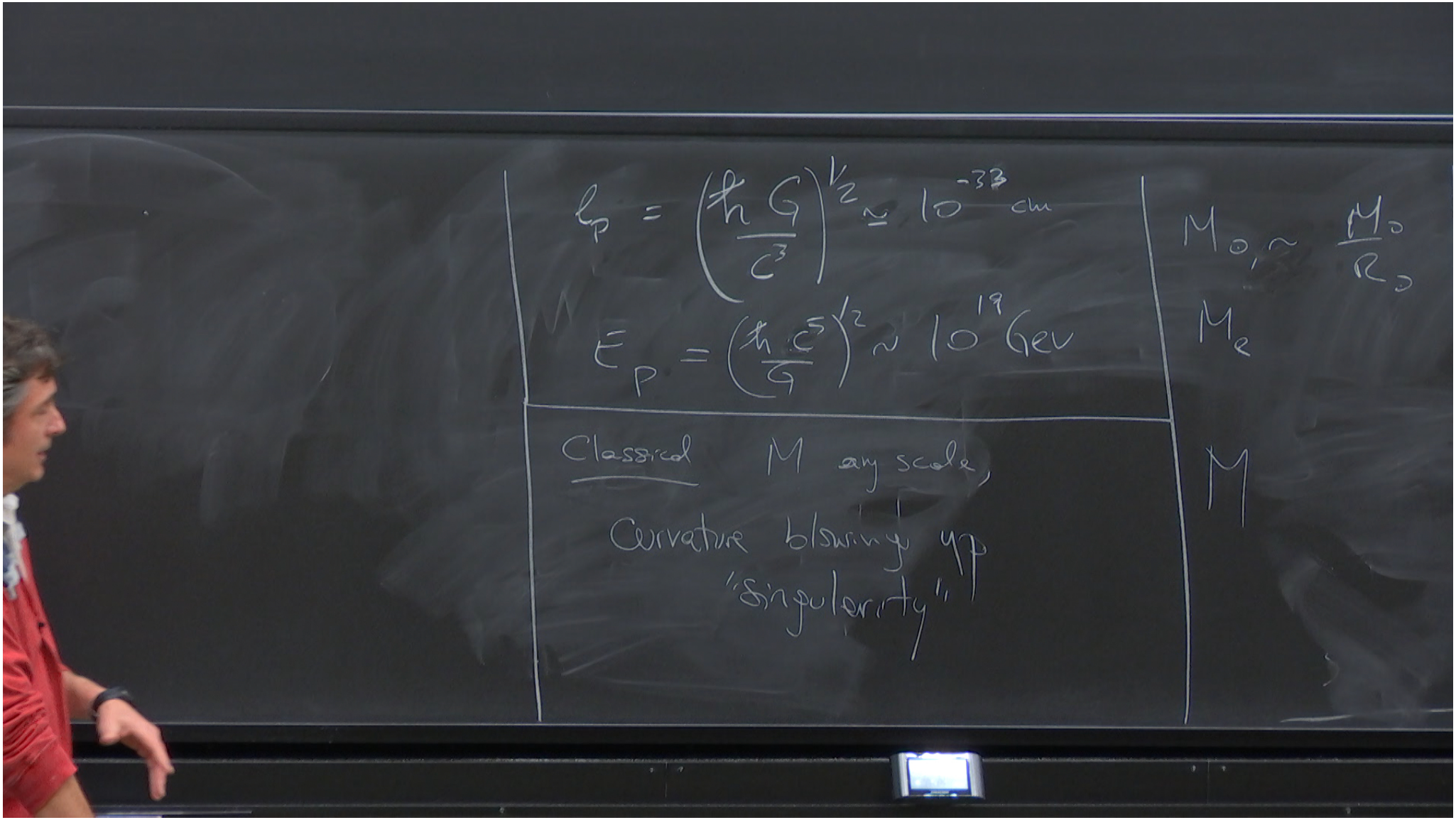

$g_{00} = -(1+2\Phi)$
$C = 1$
$G = 1$
$\frac{M_e}{R_e} \sim \frac{3\,10^{6}\,M_\odot}{10^{2}\,R_\odot}$
$M_\odot$
$R_\odot \sim 10^{5}\,M_\odot$
$\phi_e \sim 10^{-9}$
$\Phi \sim 10^{-6}$

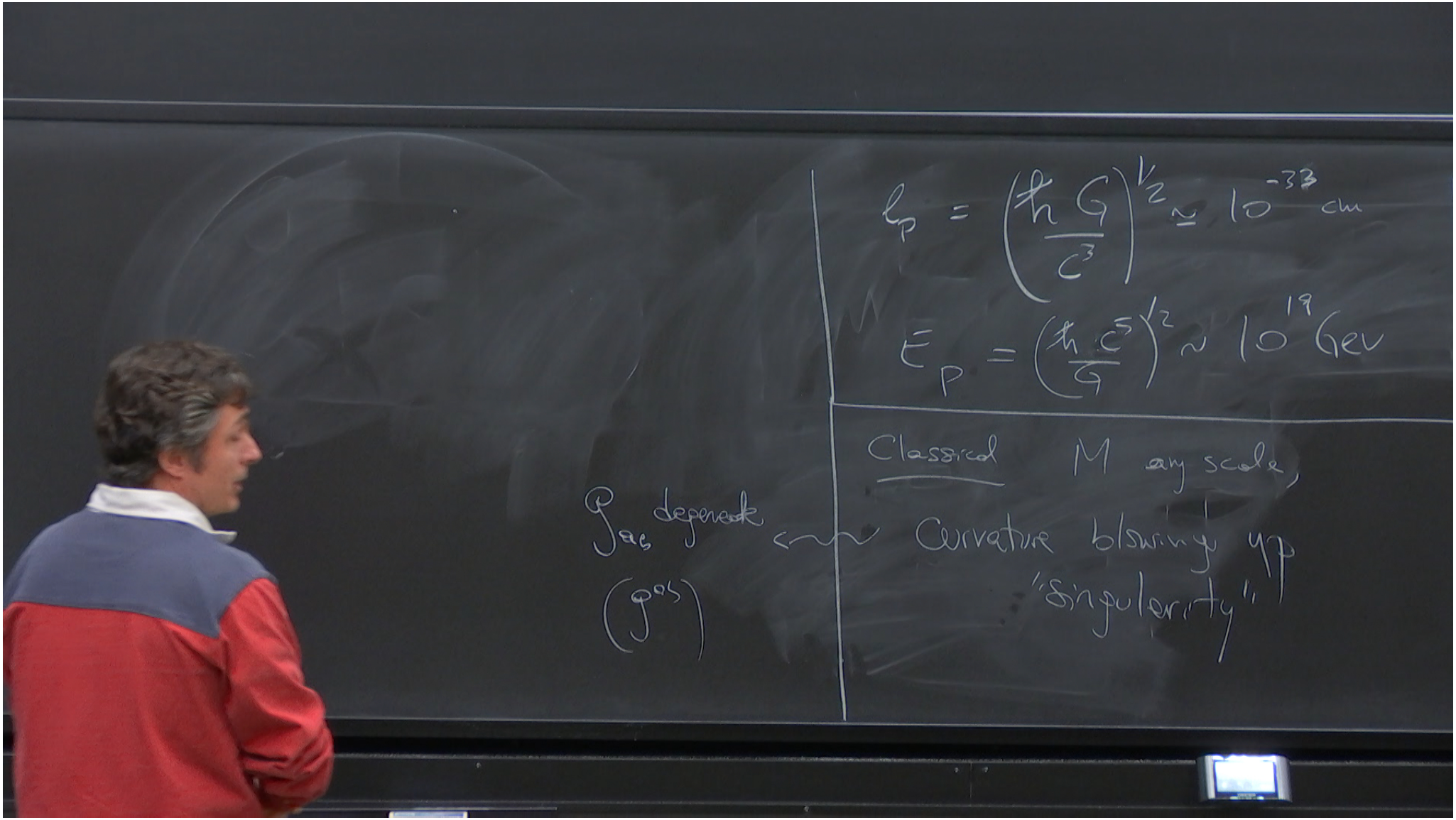

$M_{0,\sim} \frac{M_0}{R_0}$
M_e
M
$g_{00} = -(1+2\Phi)$
$\frac{M_e}{R_e} \sim \frac{3 10^{6} M_0}{10^{-2} R_0}$
$\phi_e \sim 10^{-9}$
$\Phi_{\text{Sun}} \sim 10^{-6}$
$C = 1$
$G = 1$
M_0
$R_0 \sim 10^{5} M_0$

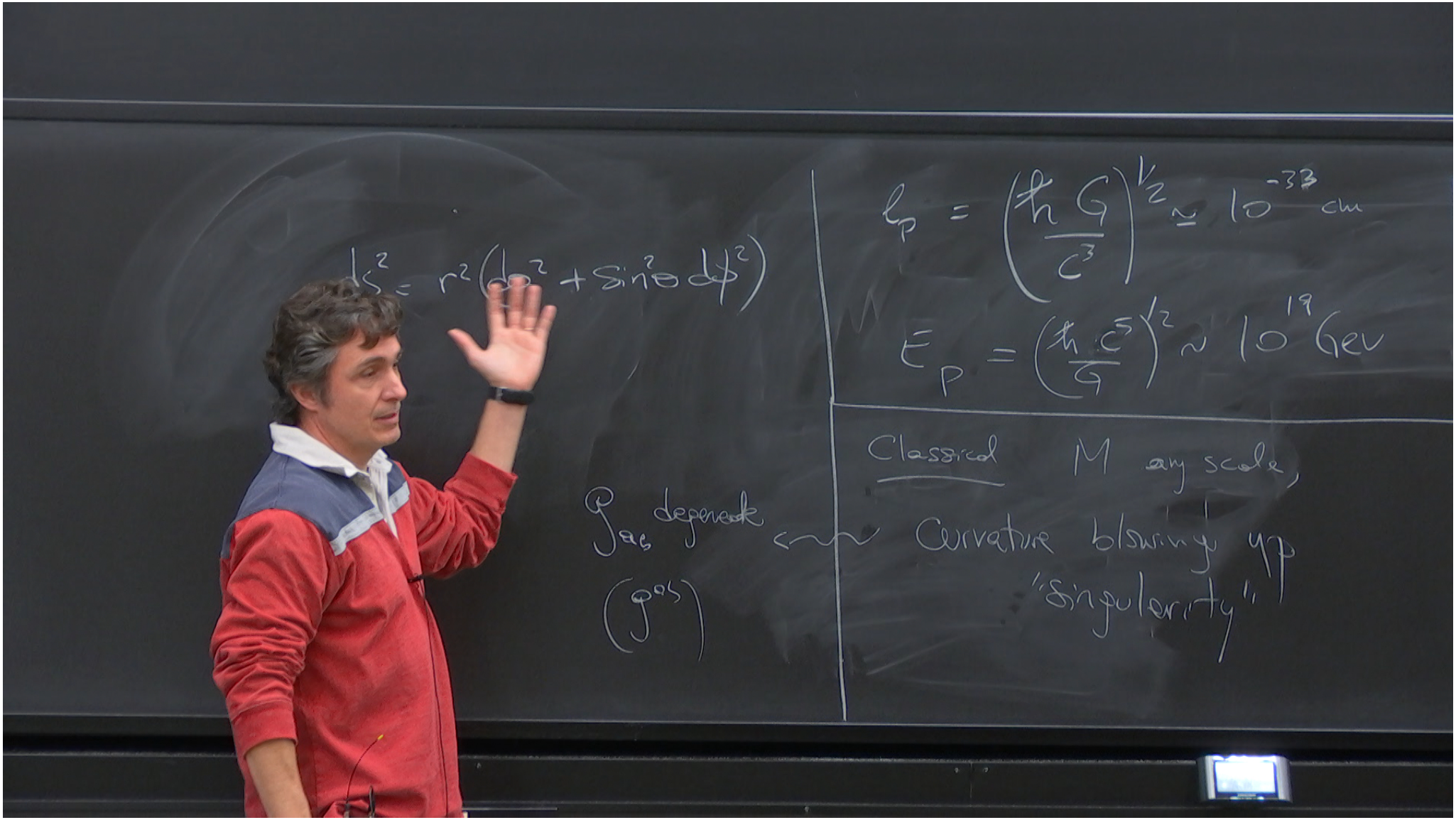

$\ell_P = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}$ cm
$M_{0,\sim} \frac{M_\odot}{R_\odot}$
M_e
M

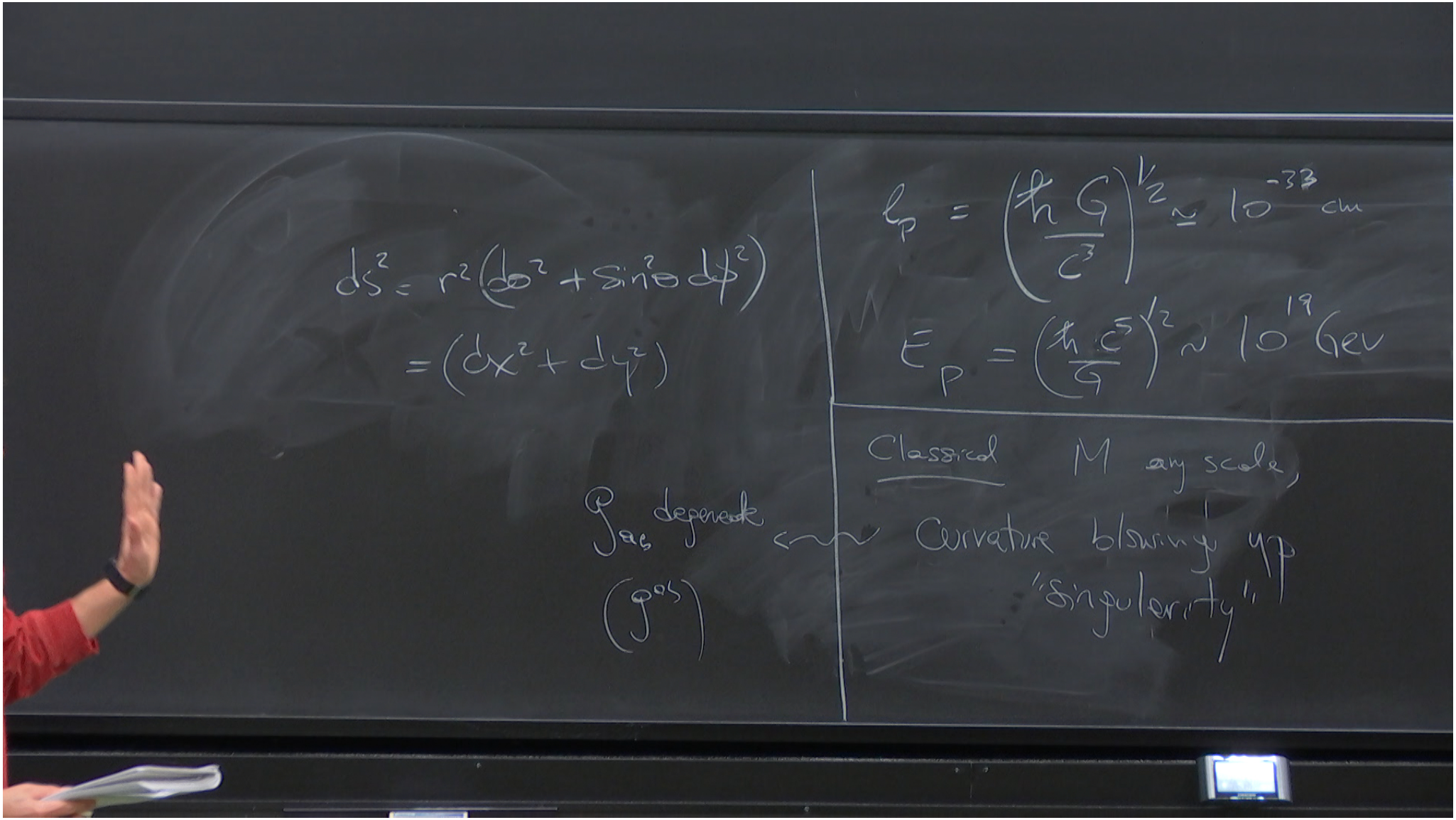

$\ell_P = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}$ cm
$E_P = \left(\frac{\hbar c^5}{G}\right)^{1/2} \simeq 10^{19}$ Gev
$M_{\odot} \sim \frac{M_{\odot}}{R_{\odot}}$
M_e
M


$l_P = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}$ cm
$= \left(\frac{\hbar c^5}{G}\right)^{1/2} \sim 10^{19}$ GeV
M
$M_{0,\odot} \sim \frac{M_\odot}{R_\odot}$
M_e
M


$\ell_P = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}$ cm
$E_P = \left(\frac{\hbar c^5}{G}\right)^{1/2} \sim 10^{19}$ GeV
Classical M any scale,
Curvature blowing up
"singularity"
$M_{0,\odot} \sim \frac{M_\odot}{R_\odot}$
M_e
M


$\ell_P = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}$ cm
$E_P = \left(\frac{\hbar c^5}{G}\right)^{1/2} \sim 10^{19}$ Gev
Classical M any scale,
Curvature blowing up
"singularity"
$g_{\mu\nu}$ degenerate
$(g^{\mu\nu})$


$ds^2 = r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$
$\ell_p = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}$ cm
$E_p = \left(\frac{\hbar c^5}{G}\right)^{1/2} \sim 10^{19}$ GeV
$g_{\mu\nu}$ degenerate
$(g^{\mu\nu})$
Classical M any scale,
Curvature blowing up
"singularity"


$ds^2 = r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$
$= (dx^2 + dy^2)$
g_{as} degenerate
(g^{as})
$\ell_p = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}$ cm
$E_p = \left(\frac{\hbar c^5}{G}\right)^{1/2} \sim 10^{19}$ GeV
Classical M any scale,
Curvature blowing up
"singularity"

$$ds^2 = r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$= (dx^2 + dy^2)$$

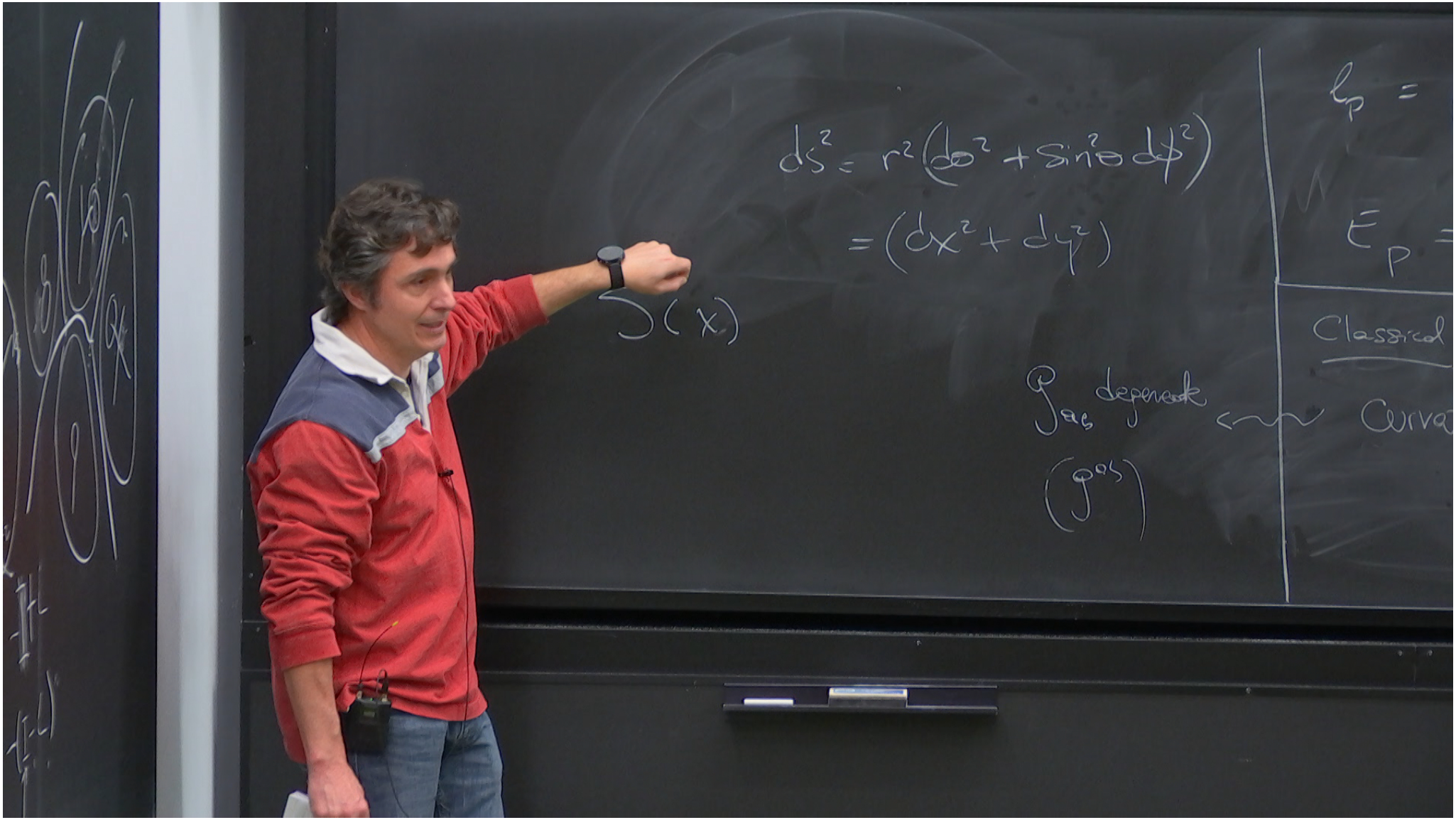
g_{ab} dependent $\rightsquigarrow$ Curvature blowing up

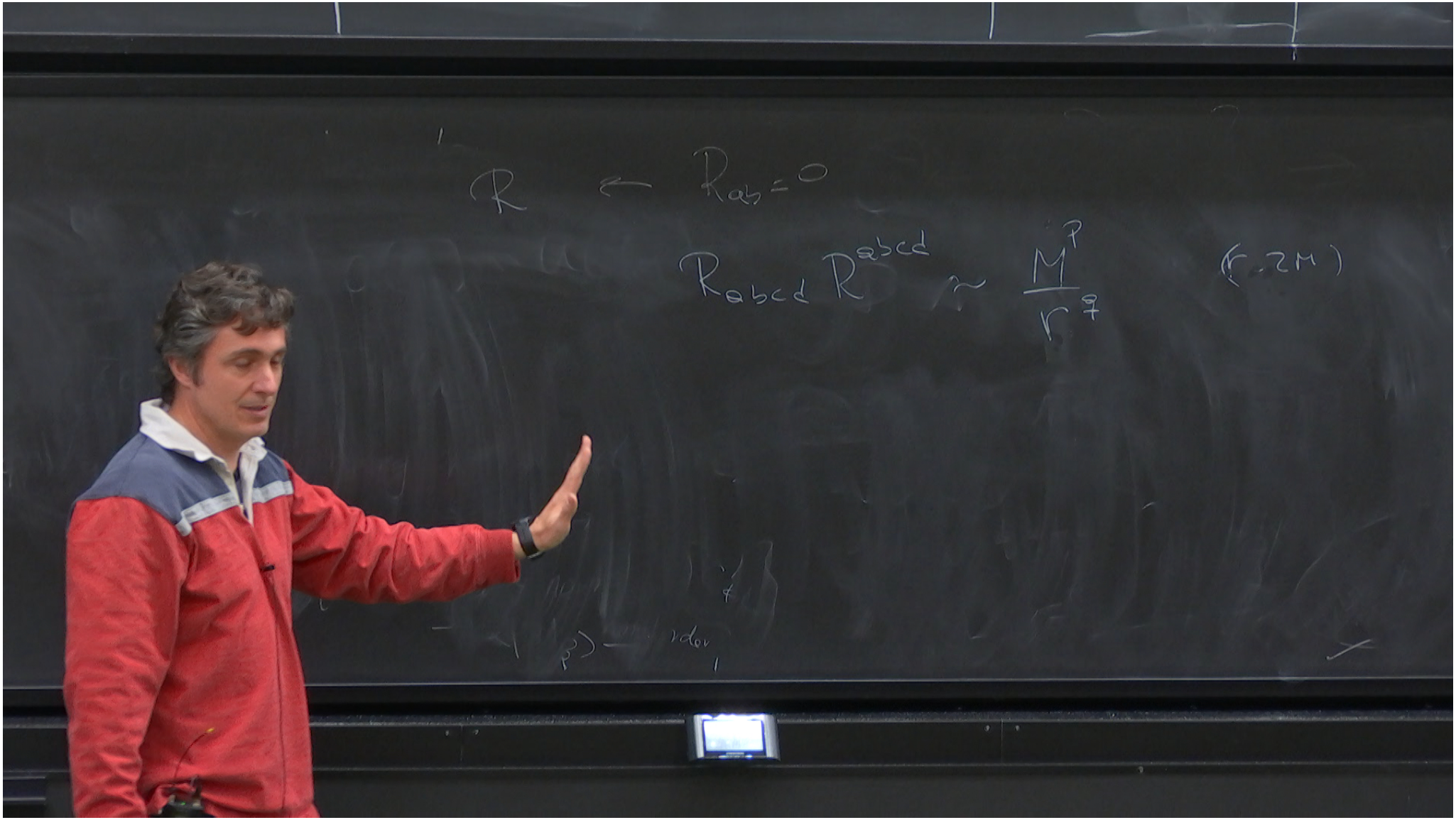
(g^{ab}) "singularity"

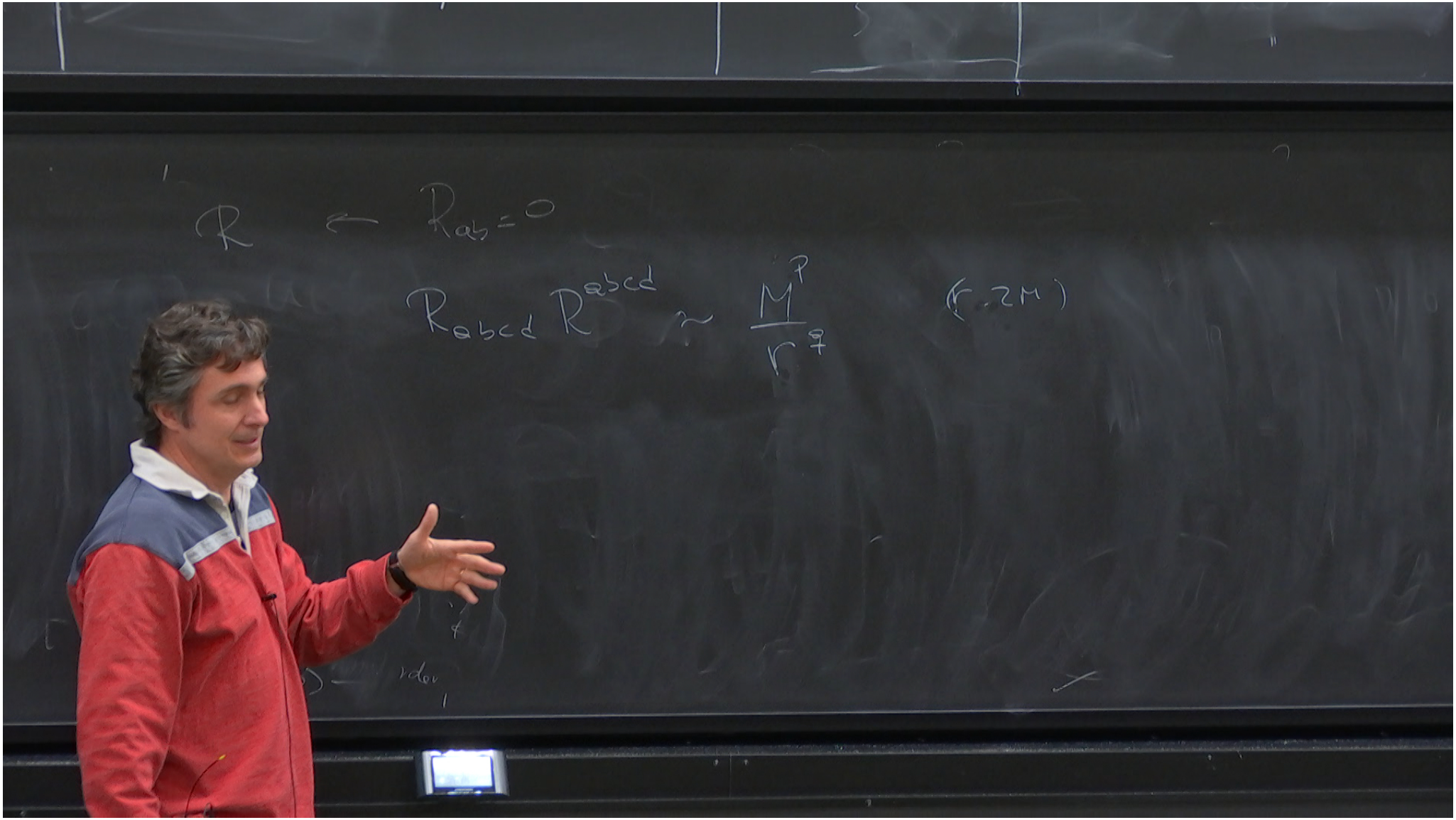
$$\ell_p = \left(\frac{\hbar G}{c^3}\right)^{1/2} \simeq 10^{-33}\ \text{cm}$$

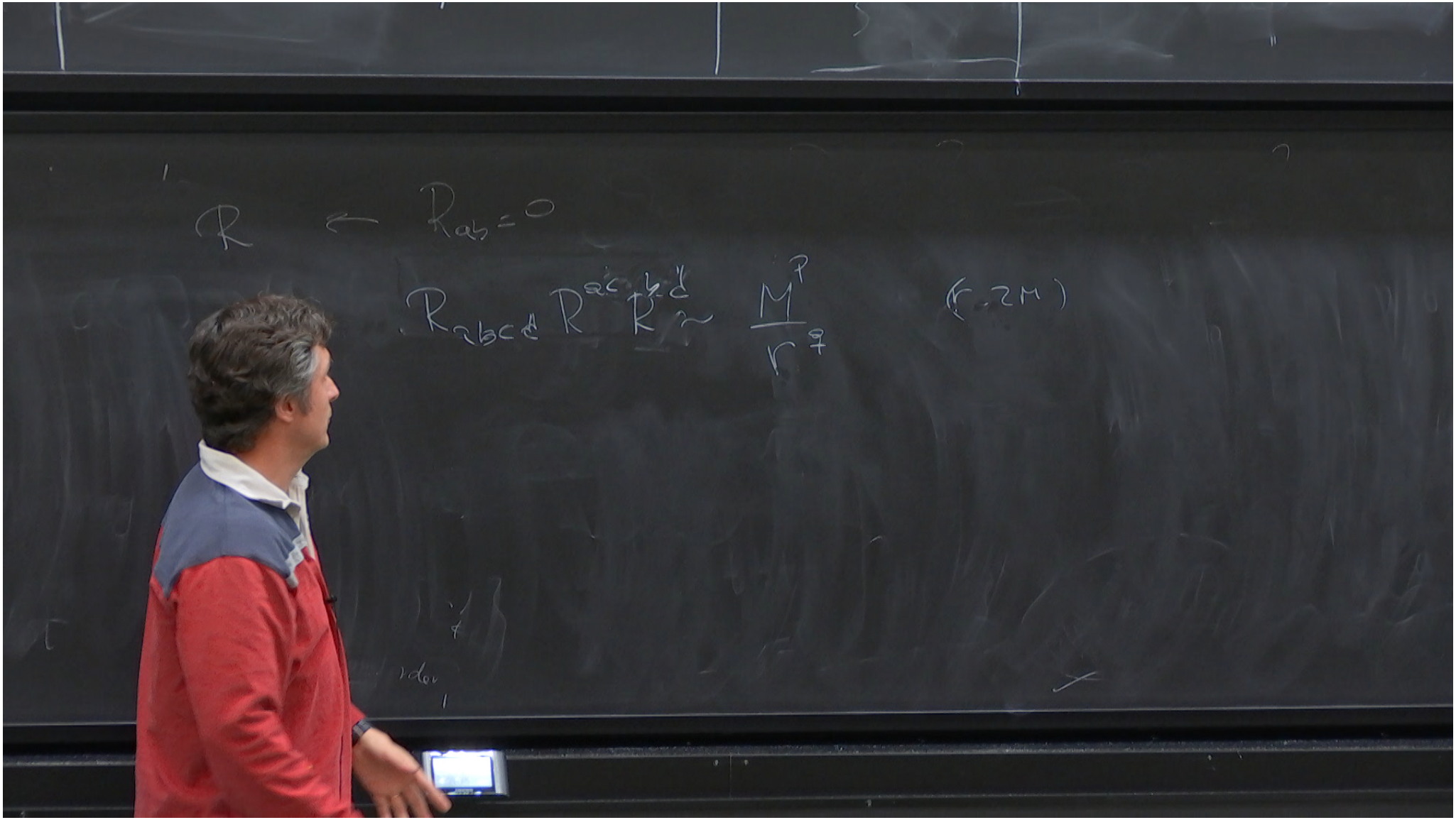
$$E_p = \left(\frac{\hbar c^5}{G}\right)^{1/2} \sim 10^{19}\ \text{GeV}$$

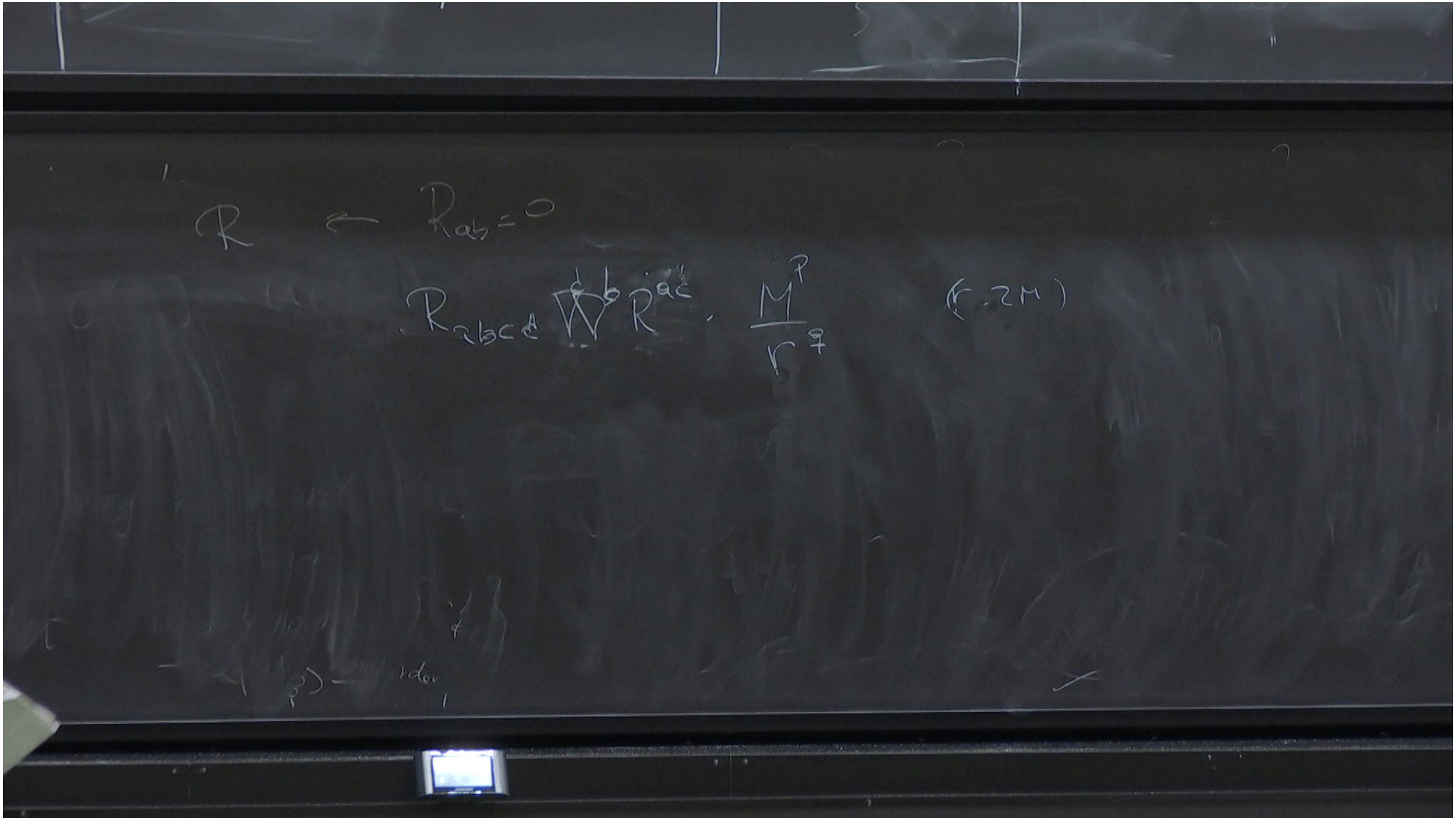
Classical M any scale,

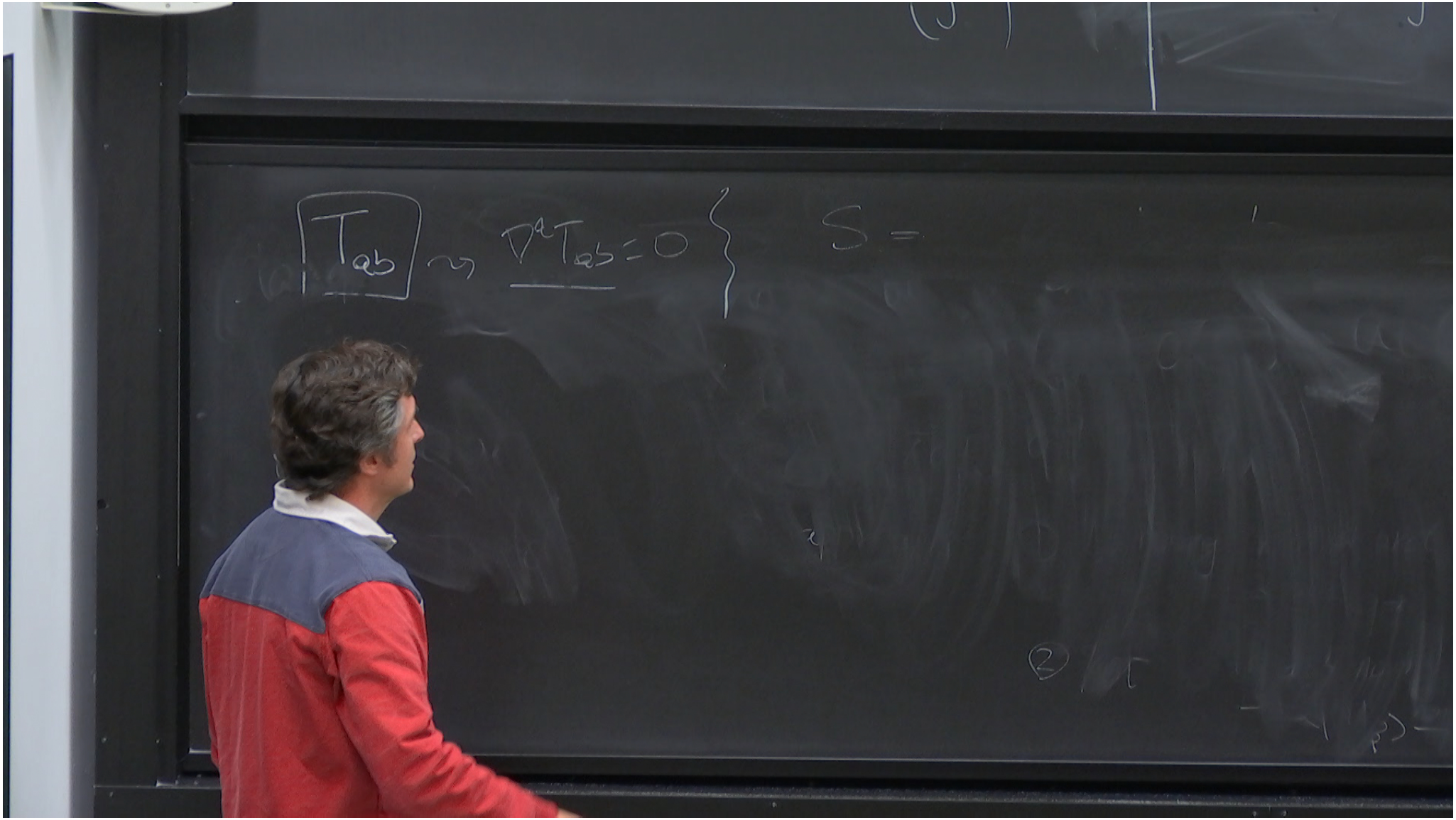

$ds^2 = r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$
$= (dx^2 + dy^2)$
$S(x)$
$g_{\mu\nu}$ degenerate
$(g^{\mu\nu})$
$\ell_p =$
$E_p =$
Classical
Curva

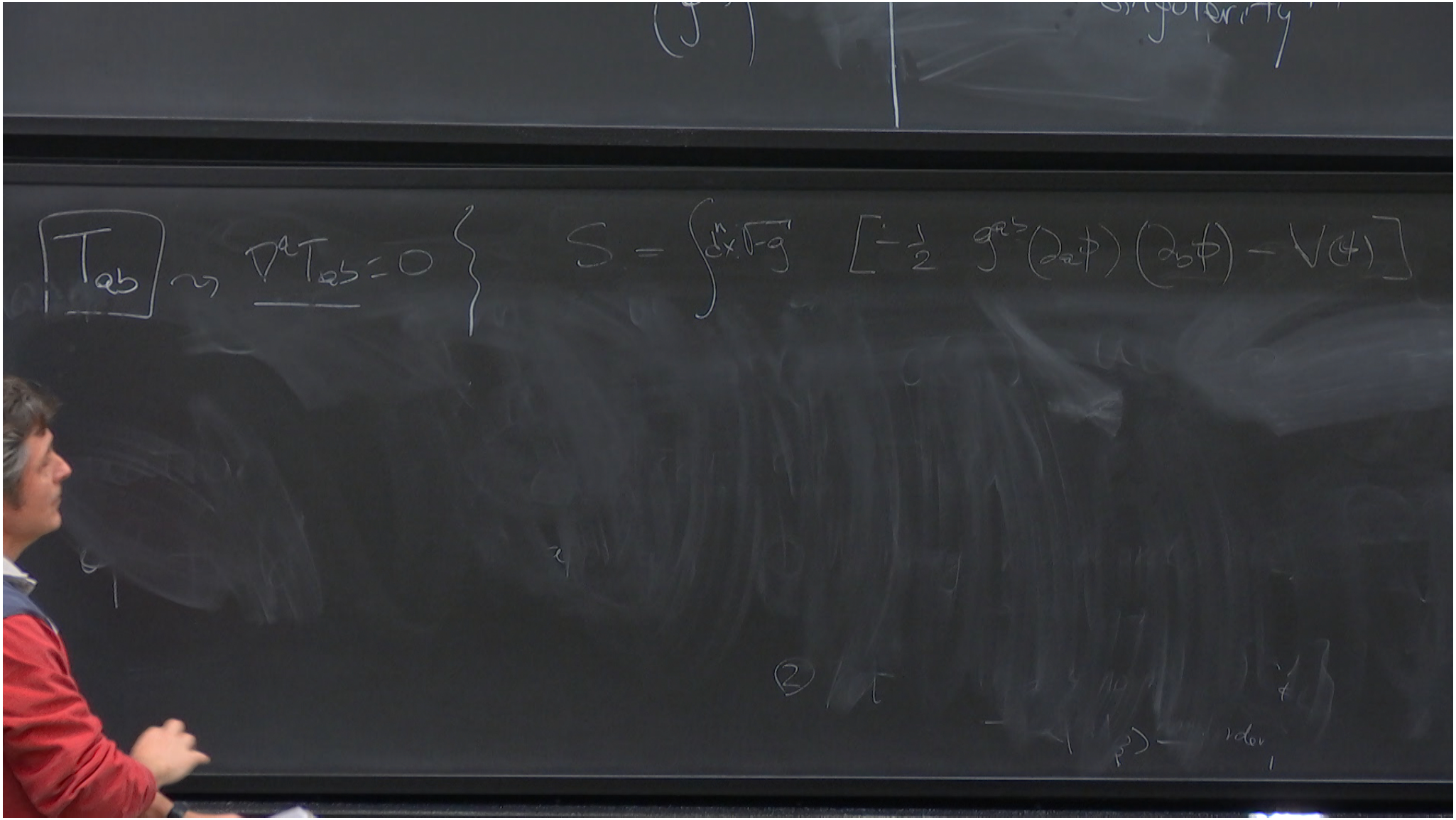

R ← R_ab = 0
R_abcd R^abcd ~ M^p / r^q
(r > 2M)

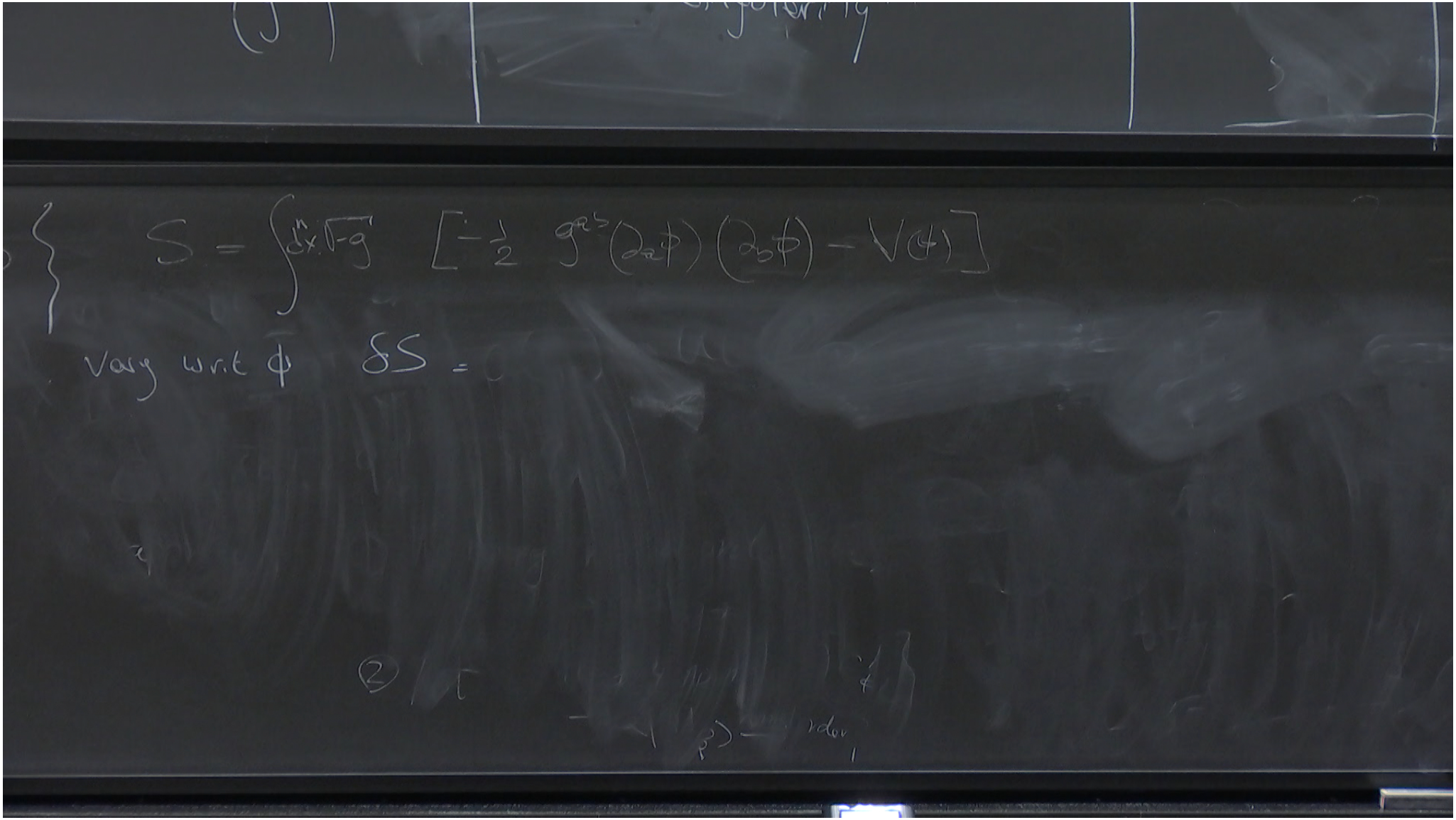

R ← Rab = 0
Rabcd R^abcd ~ M^2 / r^8
(r > 2M)


R ← Rab = 0
Rabcd Rabcd ~ M²/r⁶
(r > 2M)


R ← Rab = 0
Rabcd W^b R^ac . M^7 / r^7 (r > 2M)

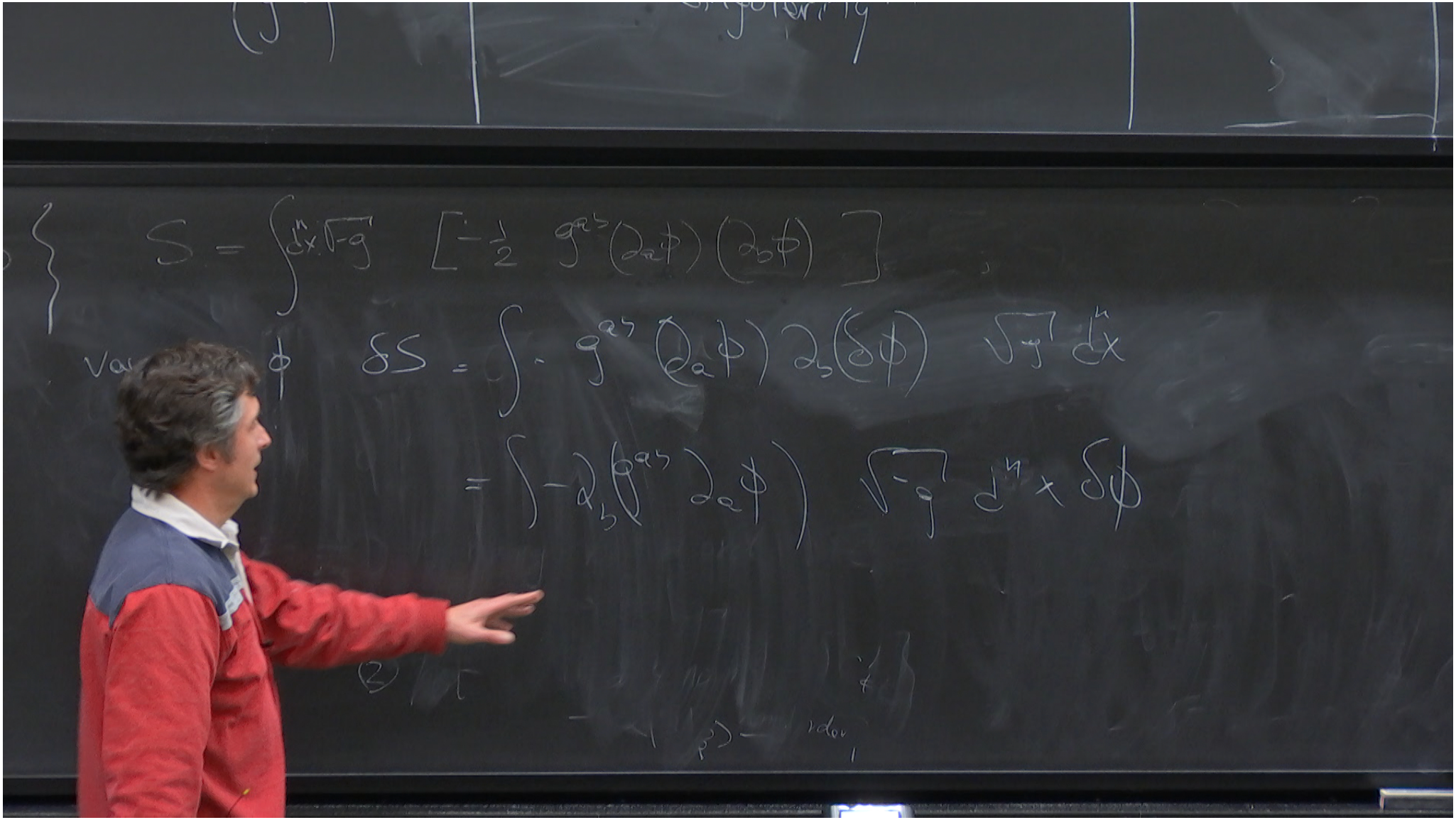

T_{ab}
$\nabla^a T_{ab} = 0$
$S =$


T_{ab} ⇝ $\nabla^a T_{ab} = 0$
$S = \int d^n x \sqrt{-g} \left[-\frac{1}{2} g^{ab} (\partial_a \phi)(\partial_b \phi) - V(\phi) \right]$


$S = \int d^n x \sqrt{-g} \left[-\frac{1}{2} g^{ab} (\partial_a \phi)(\partial_b \phi) - V(\phi) \right]$
Varg wrt ϕ $\delta S =$

$$S = \int d^n x \sqrt{-g} \left[-\frac{1}{2} g^{ab} (\partial_a \phi)(\partial_b \phi) \right]$$

vary w.r.t ϕ $\quad \delta S = \int \cdot\, g^{ab} (\partial_a \phi) \partial_b (\delta\phi) \sqrt{-g}\, d^n x$

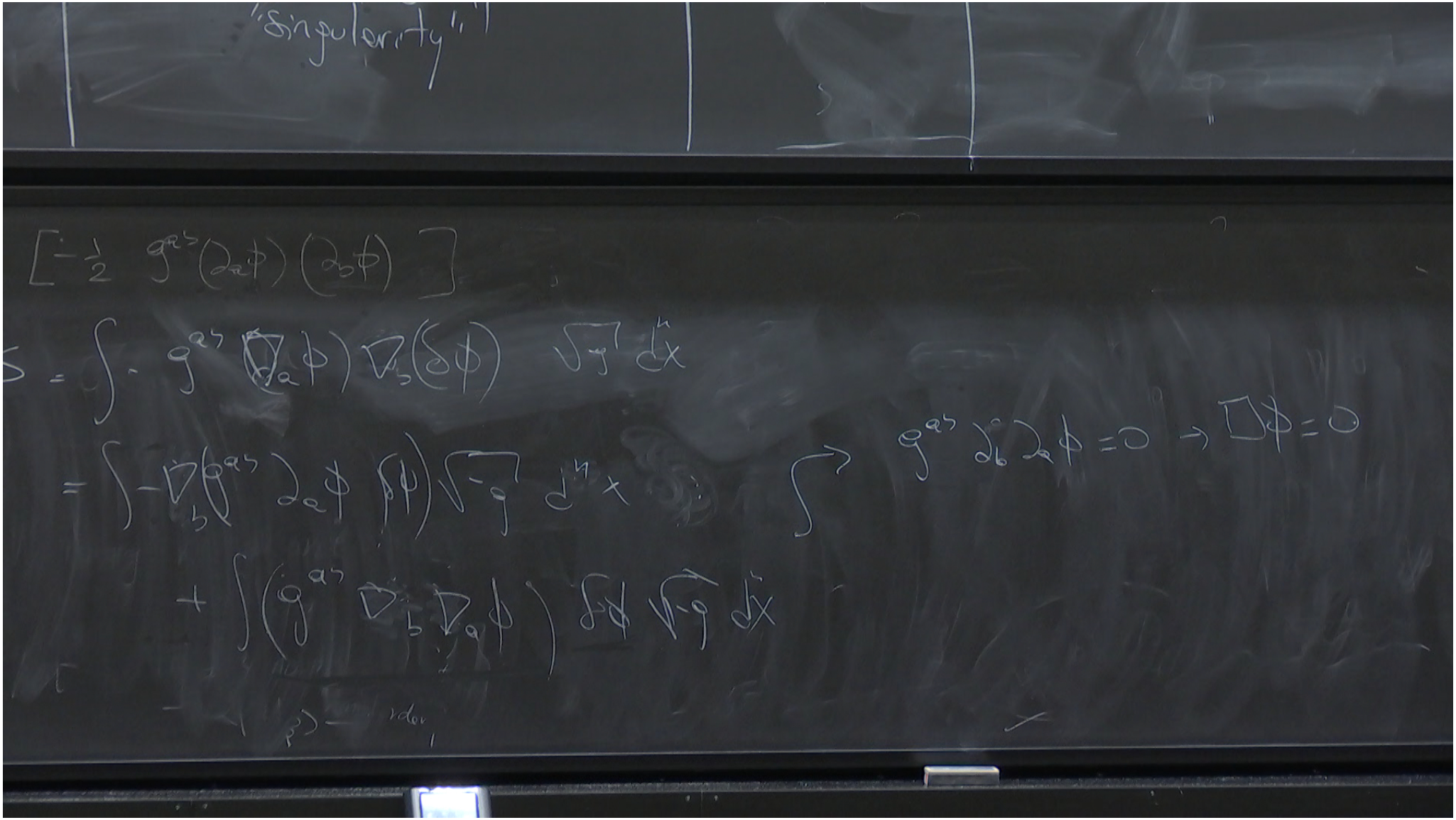

S = ∫ d^n x √-g [-1/2 g^ab (∂_a φ)(∂_b φ)]
Var φ δS = ∫ - g^ab (∂_a φ) ∂_b (δφ) √-g d^n x
= ∫ - ∂_b (g^ab ∂_a φ) √-g d^n x δφ

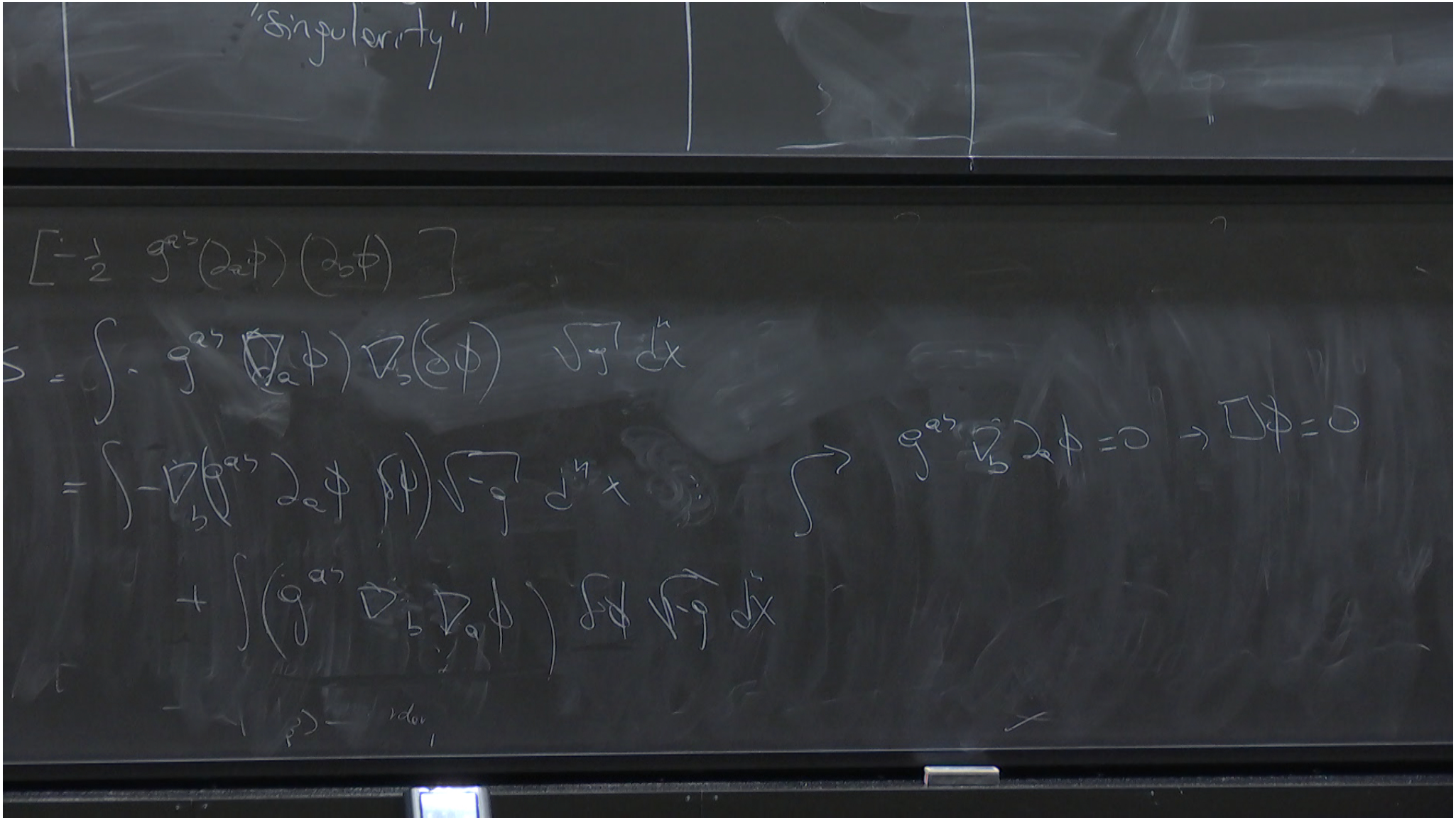
$\Gamma^{ab}_{ab} = 0$ $\left\{ \right.$ $S = \int d^n x \sqrt{-g} \left[-\frac{1}{2} g^{ab} (\partial_a \phi)(\partial_b \phi) \right]$

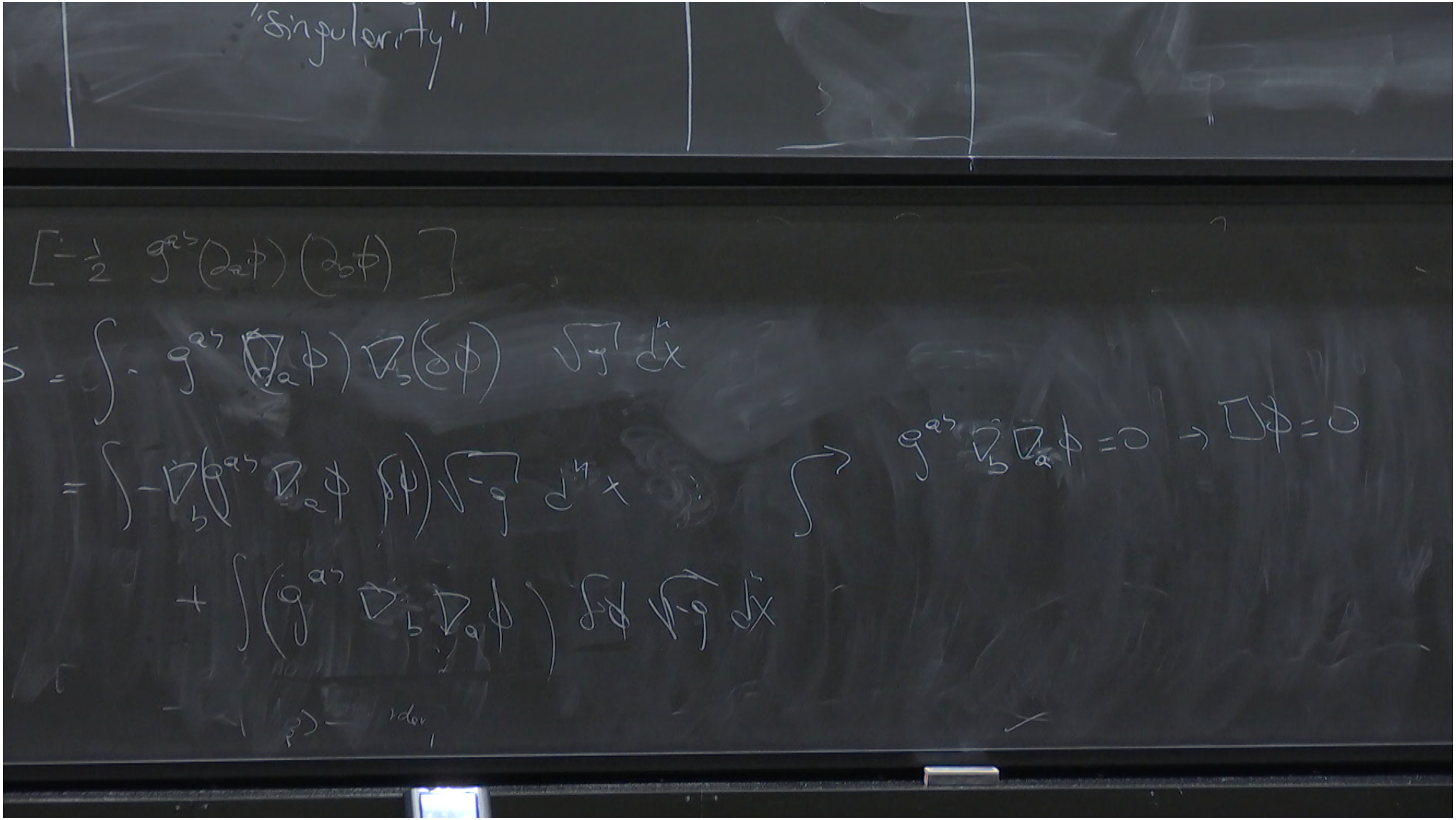
vary wrt ϕ $\quad \delta S = \int \cdot\, g^{ab} (\partial_a \phi)\, \partial_b(\delta\phi)\, \sqrt{-g}\, d^n x$

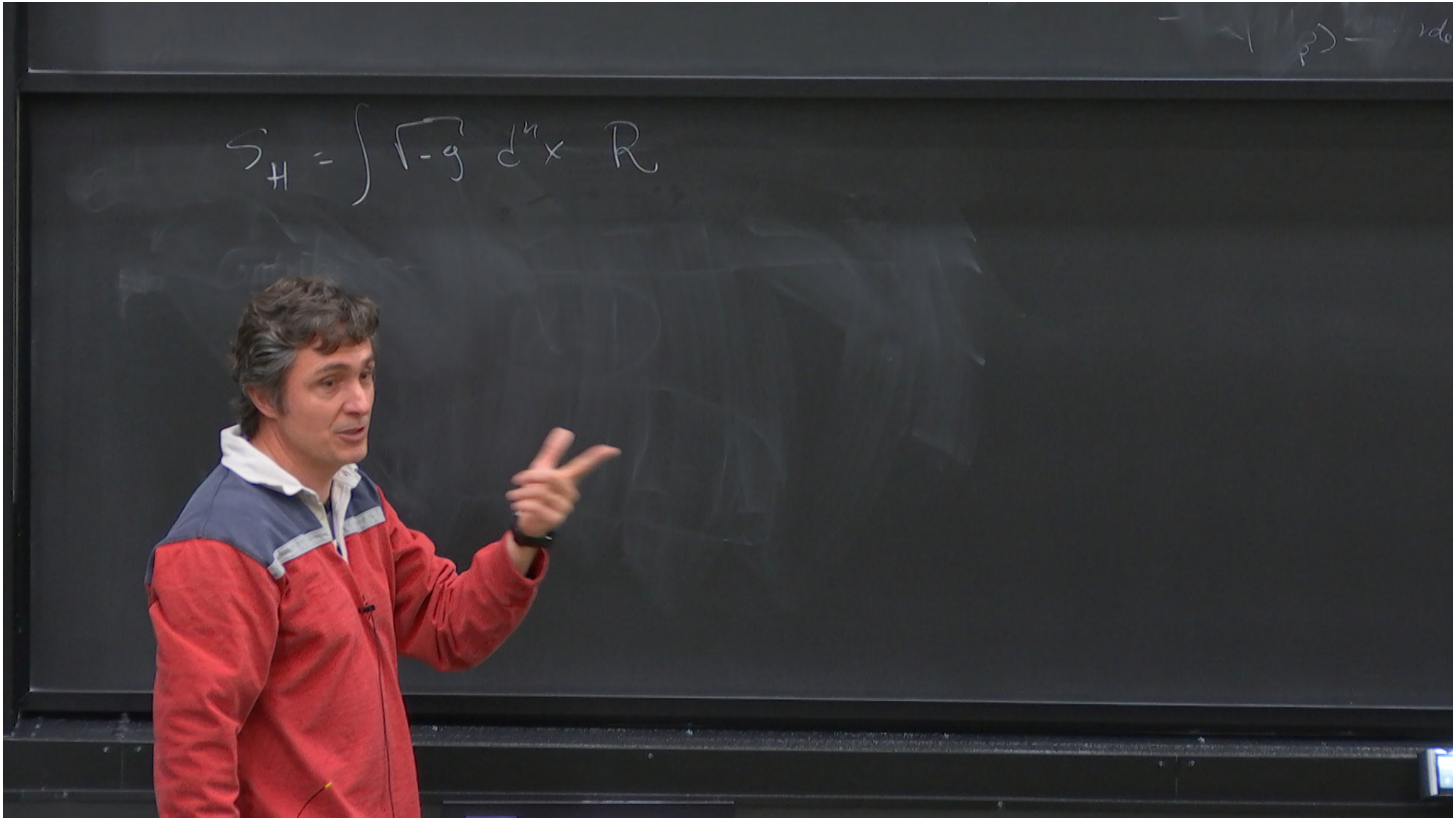
$= \int -\partial_b (g^{ab} \partial_a \phi\, \delta\phi) \sqrt{-g}\, d^n x + \int$

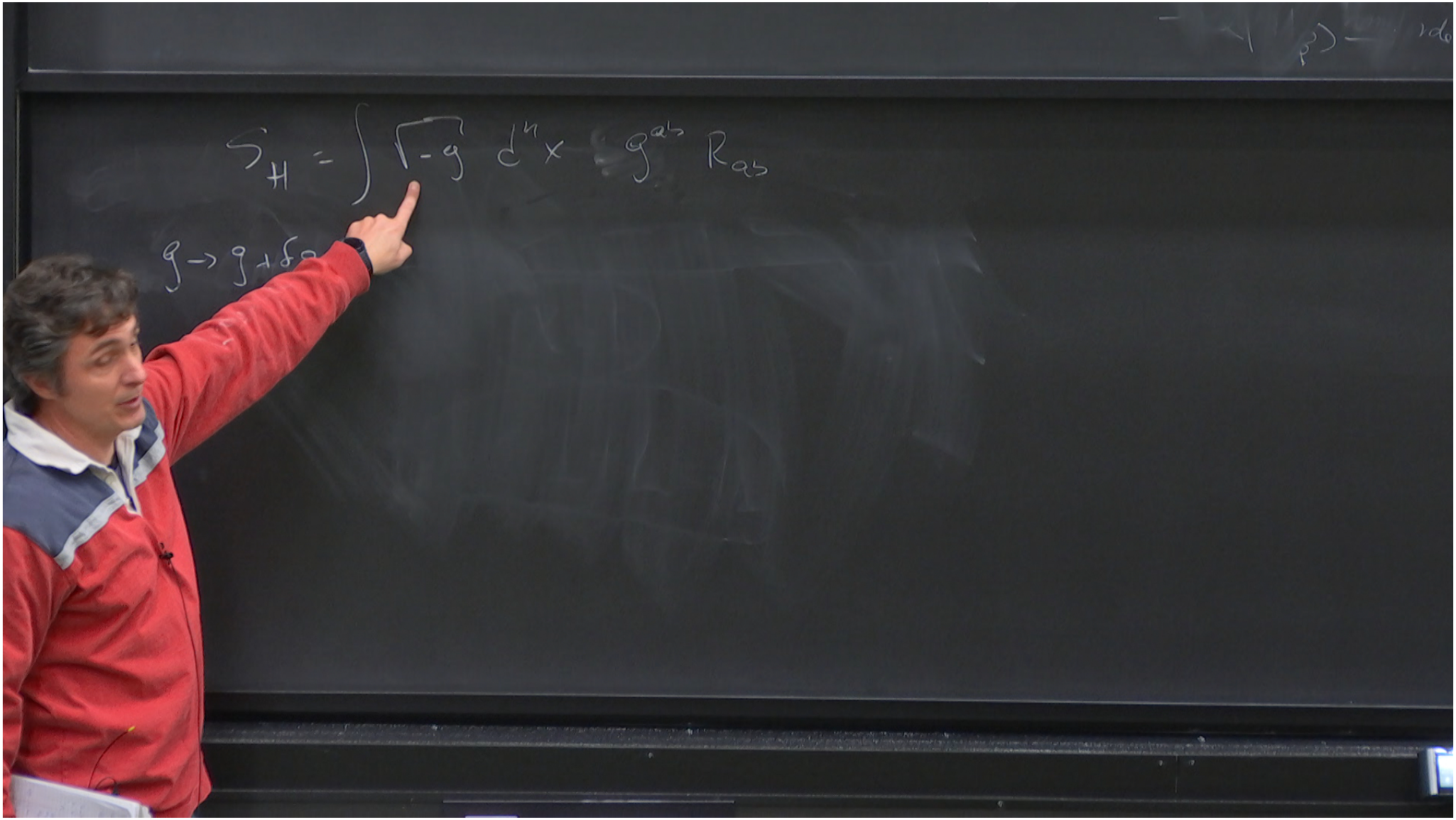
$+ \int (g^{ab} \partial_b \partial_a \phi)\, \delta\phi \sqrt{-g}\, dx$

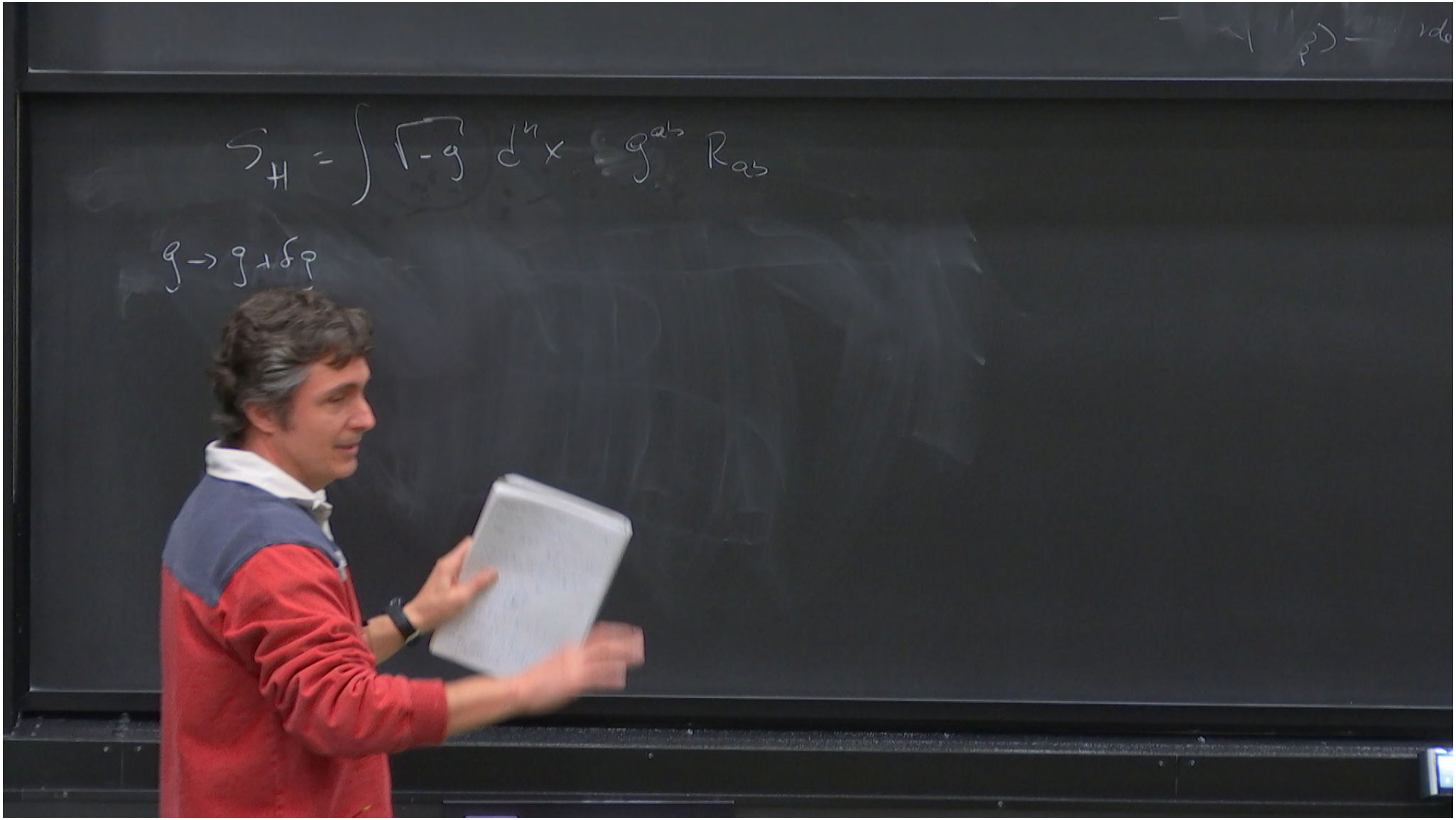

"singularity"
$\left[-\frac{1}{2} g^{ab} (\partial_a \phi)(\partial_b \phi)\right]$
$\delta = \int \cdot g^{ab} (\nabla_a \phi) \nabla_b (\delta\phi) \sqrt{-g}\, d^4x$
$= \int -\nabla_b (g^{ab} \partial_a \phi)\, \delta\phi \sqrt{-g}\, d^4x + \int (g^{ab} \nabla_b \nabla_a \phi)\, \delta\phi \sqrt{-g}\, d^4x$
$\int g^{ab} \partial_b \partial_a \phi = 0 \rightarrow \Box\phi = 0$

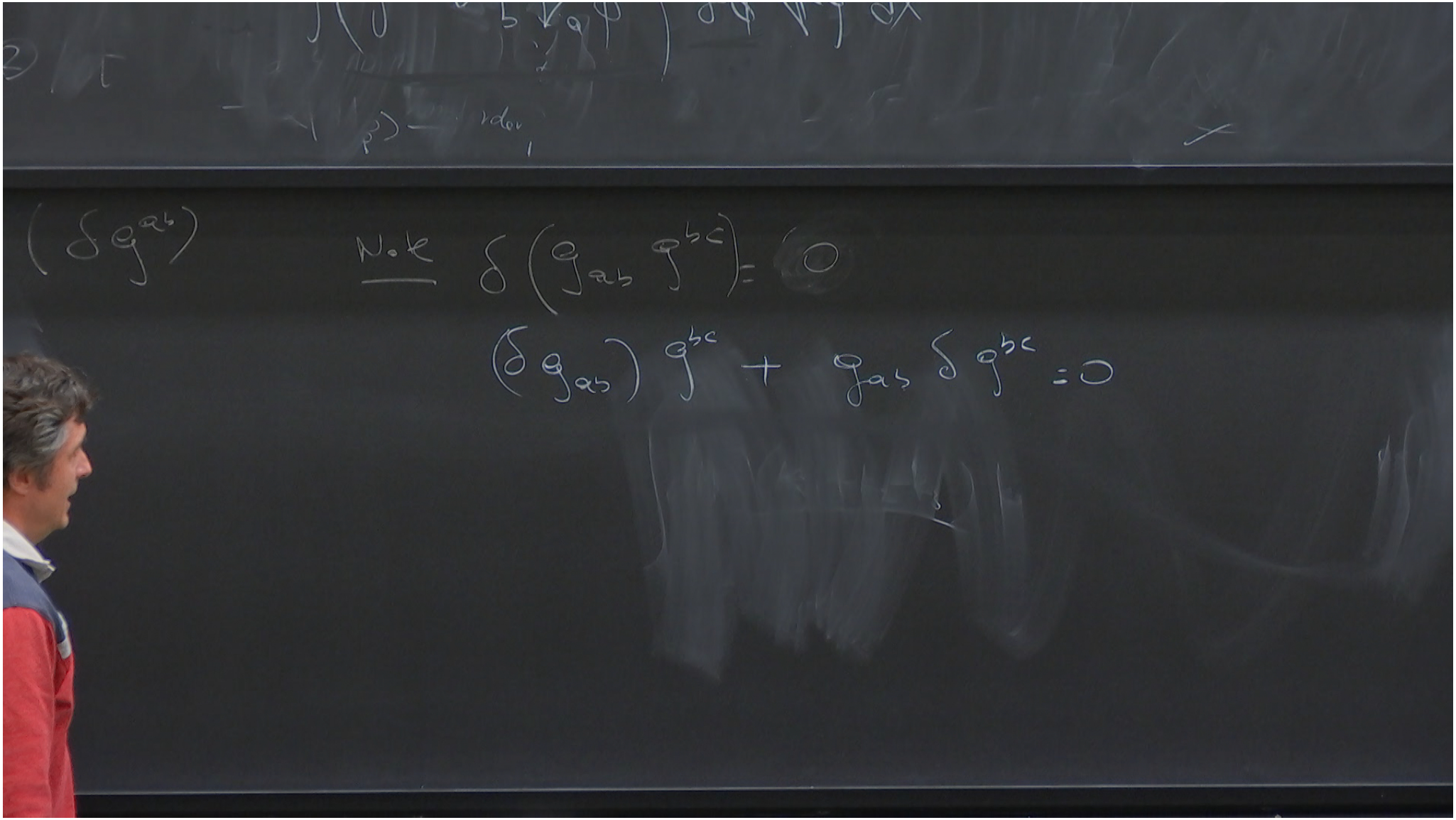

"singularity"
$\left[-\frac{1}{2} g^{ab} (\partial_a \phi)(\partial_b \phi)\right]$
$S = \int \cdot g^{ab} (\nabla_a \phi) \nabla_b (\delta\phi) \sqrt{-g}\, d^4x$
$= \int -\nabla_b (g^{ab} \partial_a \phi)\, \delta\phi \sqrt{-g}\, d^4x$
$+ \int (g^{ab} \nabla_b \nabla_a \phi)\, \delta\phi \sqrt{-g}\, d^4x$
$\int g^{ab} \nabla_b \partial_a \phi = 0 \rightarrow \Box\phi = 0$

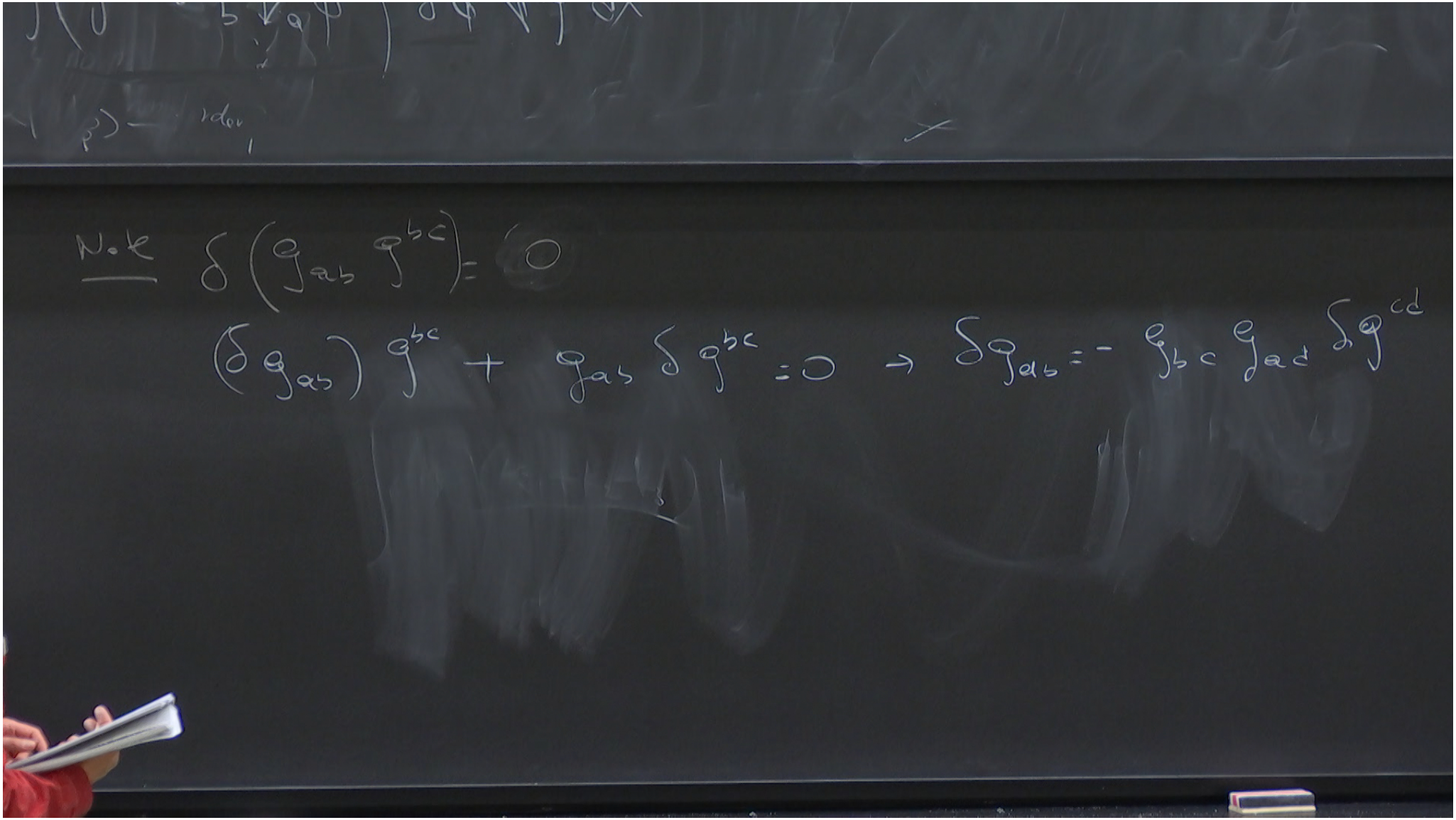

"singularity"
$\left[-\frac{1}{2} g^{ab} (\partial_a \phi)(\partial_b \phi)\right]$
$S = \int -g^{ab} (\nabla_a \phi) \nabla_b (\delta\phi) \sqrt{-g}\, d^4x$
$= \int -\nabla_b (g^{ab} \nabla_a \phi \, \delta\phi) \sqrt{-g}\, d^4x$
$+ \int (g^{ab} \nabla_b \nabla_a \phi) \delta\phi \sqrt{-g}\, dx$
$\int g^{ab} \nabla_b \nabla_a \phi = 0 \rightarrow \Box\phi = 0$


$S_H = \int \sqrt{-g}\, d^n x\, R$


$S_H = \int \sqrt{-g}\, d^n x\, g^{ab} R_{ab}$
$g \to g + \delta g$


$S_H = \int \sqrt{-g}\, d^n x\, g^{ab} R_{ab}$
$g \to g + \delta g$


(δg^{ab})
Note $\delta(g_{ab} g^{bc}) = 0$
$(\delta g_{ab}) g^{bc} + g_{ab} \delta g^{bc} = 0$


N.B $\delta(g_{ab} g^{bc}) = 0$
$(\delta g_{ab}) g^{bc} + g_{ab} \delta g^{bc} = 0 \rightarrow \delta g_{ab} = - g_{bc} g_{ad} \delta g^{cd}$

Note $\delta\left(g_{ab}\, g^{bc}\right) = 0$

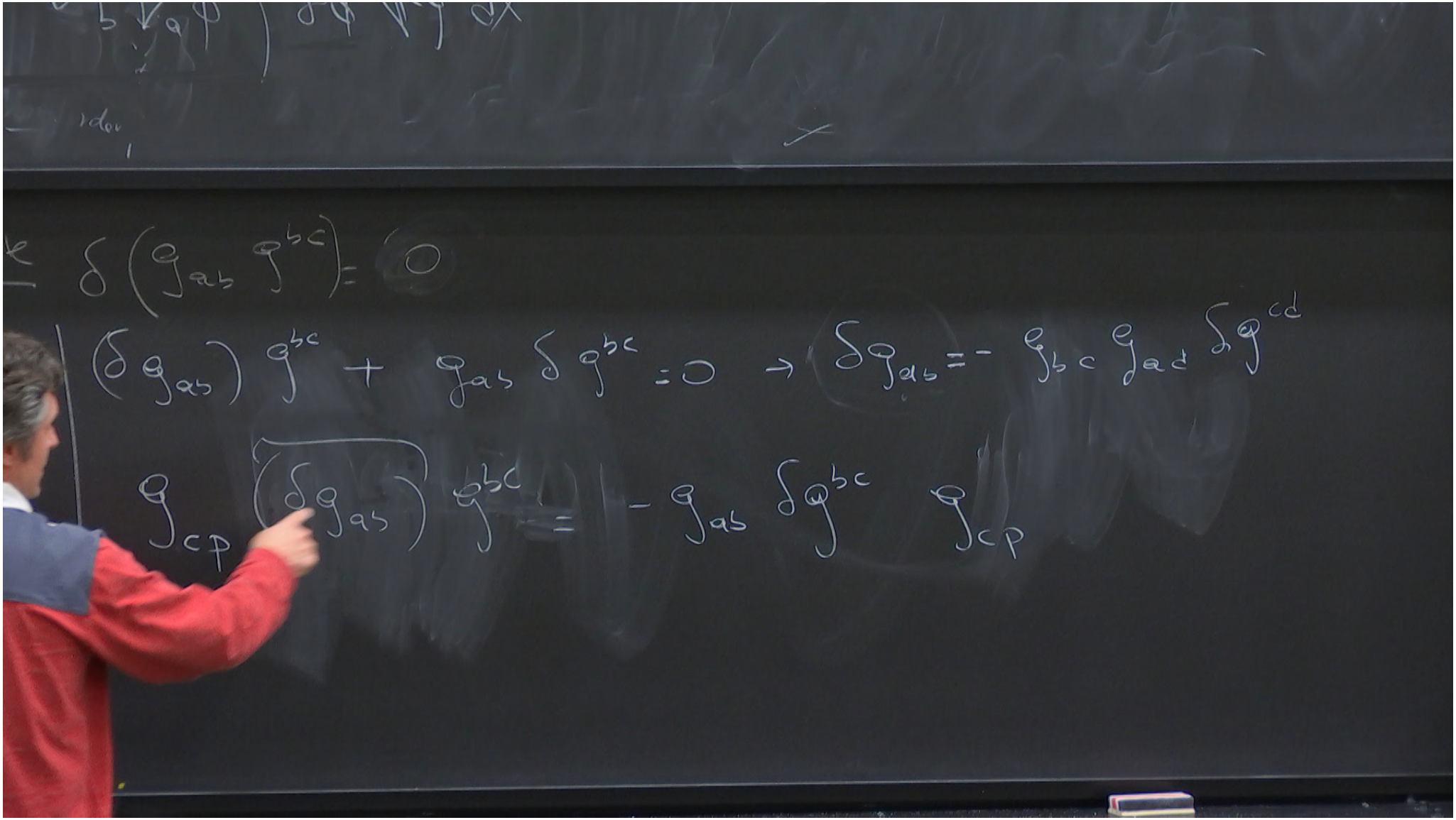
$$(\delta g_{ab})\, g^{bc} + g_{ab}\, \delta g^{bc} = 0 \rightarrow \delta g_{ab} = - g_{bc}\, g_{ad}\, \delta g^{cd}$$

$$(\delta g_{ab})\, g^{bc} =$$

Note $\delta\left(g_{ab}\, g^{bc}\right) = 0$

$$(\delta g_{ab})\, g^{bc} + g_{ab}\, \delta g^{bc} = 0 \rightarrow \delta g_{ab} = - g_{bc}\, g_{ad}\, \delta g^{cd}$$

$$(\delta g_{ab})\, g^{bc} = - g_{ab}\, \delta g^{bc}$$

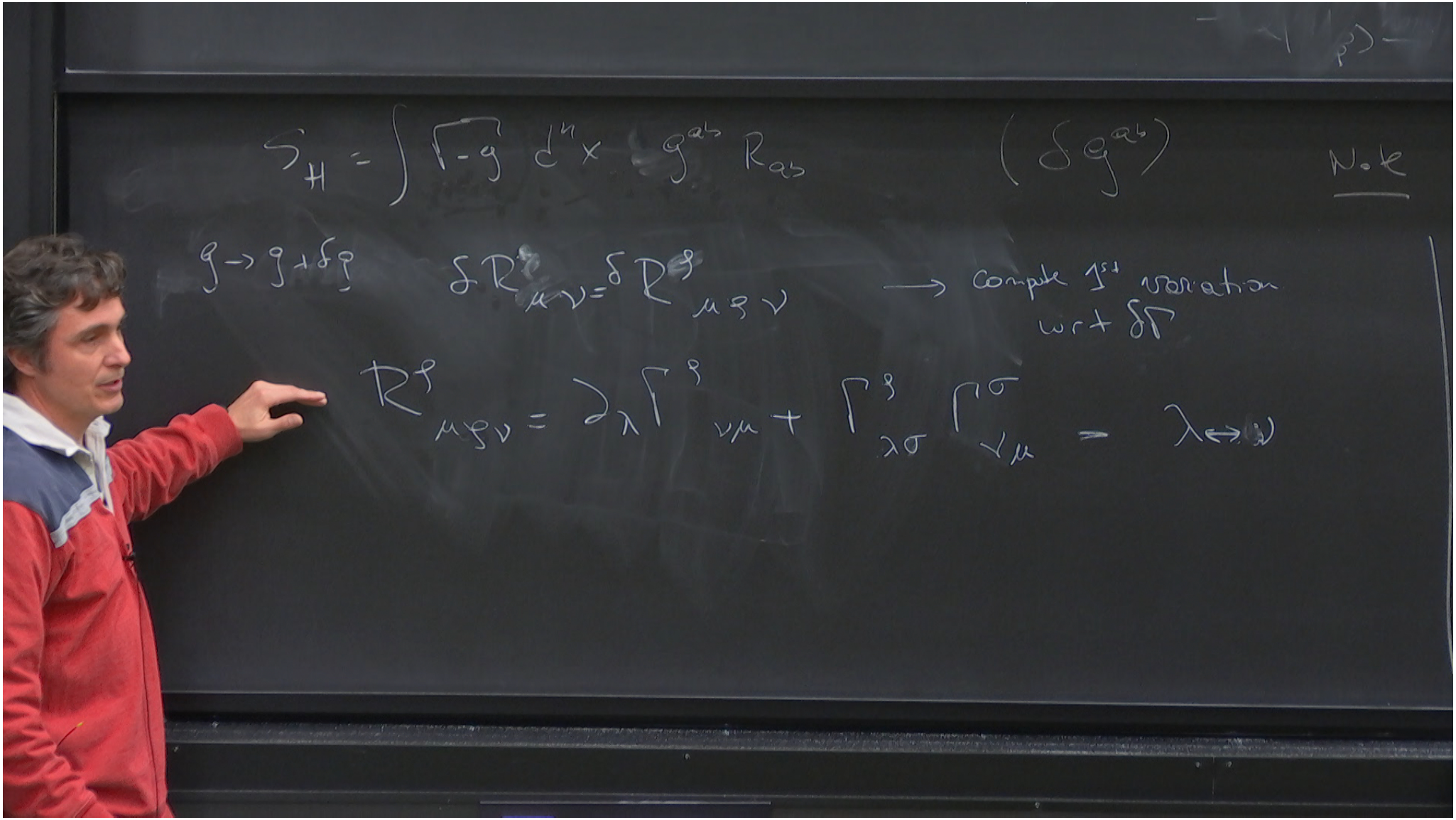

$\delta(g_{ab} g^{bc}) = 0$
$(\delta g_{ab}) g^{bc} + g_{ab} \delta g^{bc} = 0 \rightarrow \delta g_{ab} = - g_{bc} g_{ad} \delta g^{cd}$
$g_{cp} (\delta g_{ab}) g^{bc} = - g_{ab} \delta g^{bc} g_{cp}$

$$\delta\left(g_{ab}\, g^{bc}\right) = 0$$

$$(\delta g_{ab})\, g^{bc} + g_{ab}\, \delta g^{bc} = 0 \rightarrow \delta g_{ab} = - g_{bc}\, g_{ad}\, \delta g^{cd}$$

$$g_{cp}\,(\delta g_{ab})\, g^{bc} = - g_{ab}\, \delta g^{bc}\, g_{cp}$$

$$\delta g_{ap} = - g_{ab}\, g_{cp}\, \delta g^{bc}$$

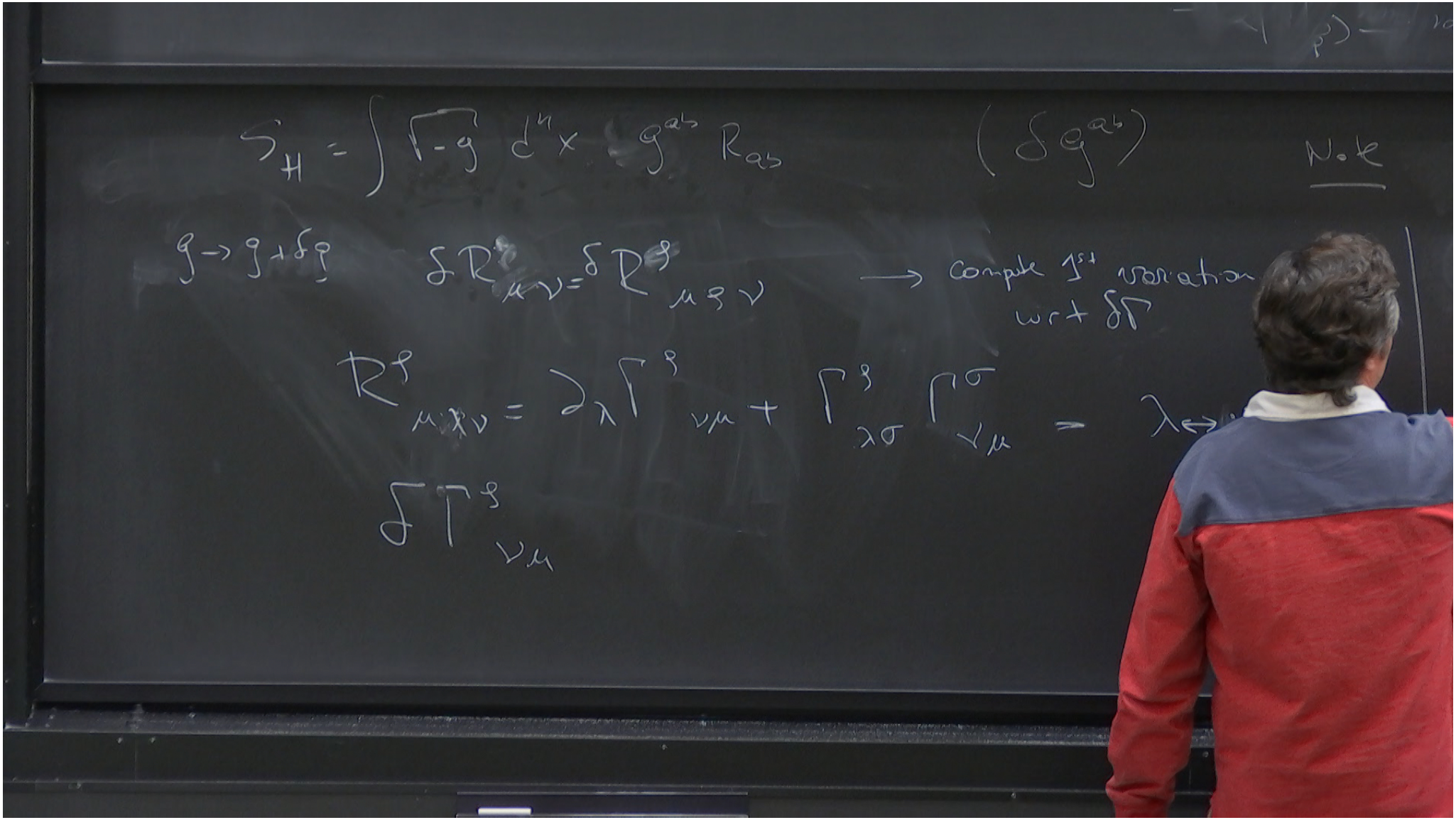

$S_H = \int \sqrt{-g}\, d^4x\, g^{ab} R_{ab}$
(δg^{ab})
N.B
$g \to g + \delta g$
$\delta R^{\rho}_{\mu\nu} = \delta R^{\rho}_{\mu\rho\nu}$
$\longrightarrow$ Compute 1st variation wrt $\delta\Gamma$
$R^{\rho}_{\mu\rho\nu} = \partial_\lambda \Gamma^{\rho}_{\nu\mu} + \Gamma^{\rho}_{\lambda\sigma}\Gamma^{\sigma}_{\nu\mu} - \lambda \leftrightarrow \nu$

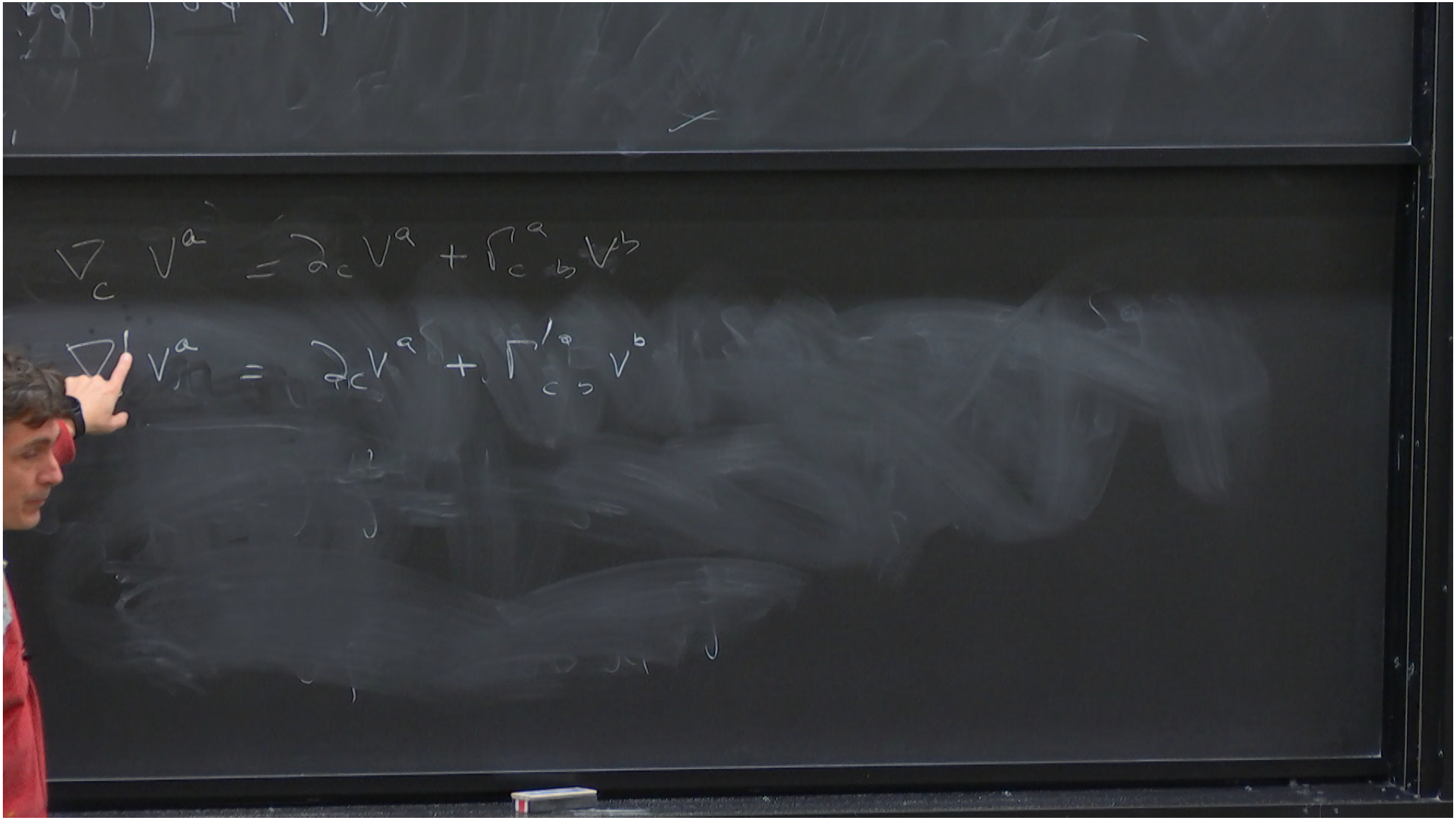
$$S_H = \int \sqrt{-g}\, d^4x\, g^{ab} R_{ab} \qquad (\delta g^{ab})$$

N.B

$g \to g + \delta g$ $\quad \delta R^{\rho}_{\mu\nu} = \delta R^{\rho}_{\mu\rho\nu}$ $\longrightarrow$ Compute 1st variation wrt $\delta\Gamma$

$$R^{\rho}_{\mu\lambda\nu} = \partial_\lambda \Gamma^{\rho}_{\nu\mu} + \Gamma^{\rho}_{\lambda\sigma}\Gamma^{\sigma}_{\nu\mu} - \lambda \leftrightarrow \nu$$


$S_H = \int \sqrt{-g}\, d^n x\, g^{ab} R_{ab}$
(δg^{ab})
N.B
$g \to g + \delta g$
$\delta R^{\rho}_{\mu\nu} = \delta R^{\rho}_{\mu\rho\nu}$
$\longrightarrow$ Compute 1st variation wrt $\delta\Gamma$
$R^{\rho}_{\mu\lambda\nu} = \partial_\lambda \Gamma^{\rho}_{\nu\mu} + \Gamma^{\rho}_{\lambda\sigma}\Gamma^{\sigma}_{\nu\mu} - \lambda \leftrightarrow$
$\delta\Gamma^{\rho}_{\nu\mu}$

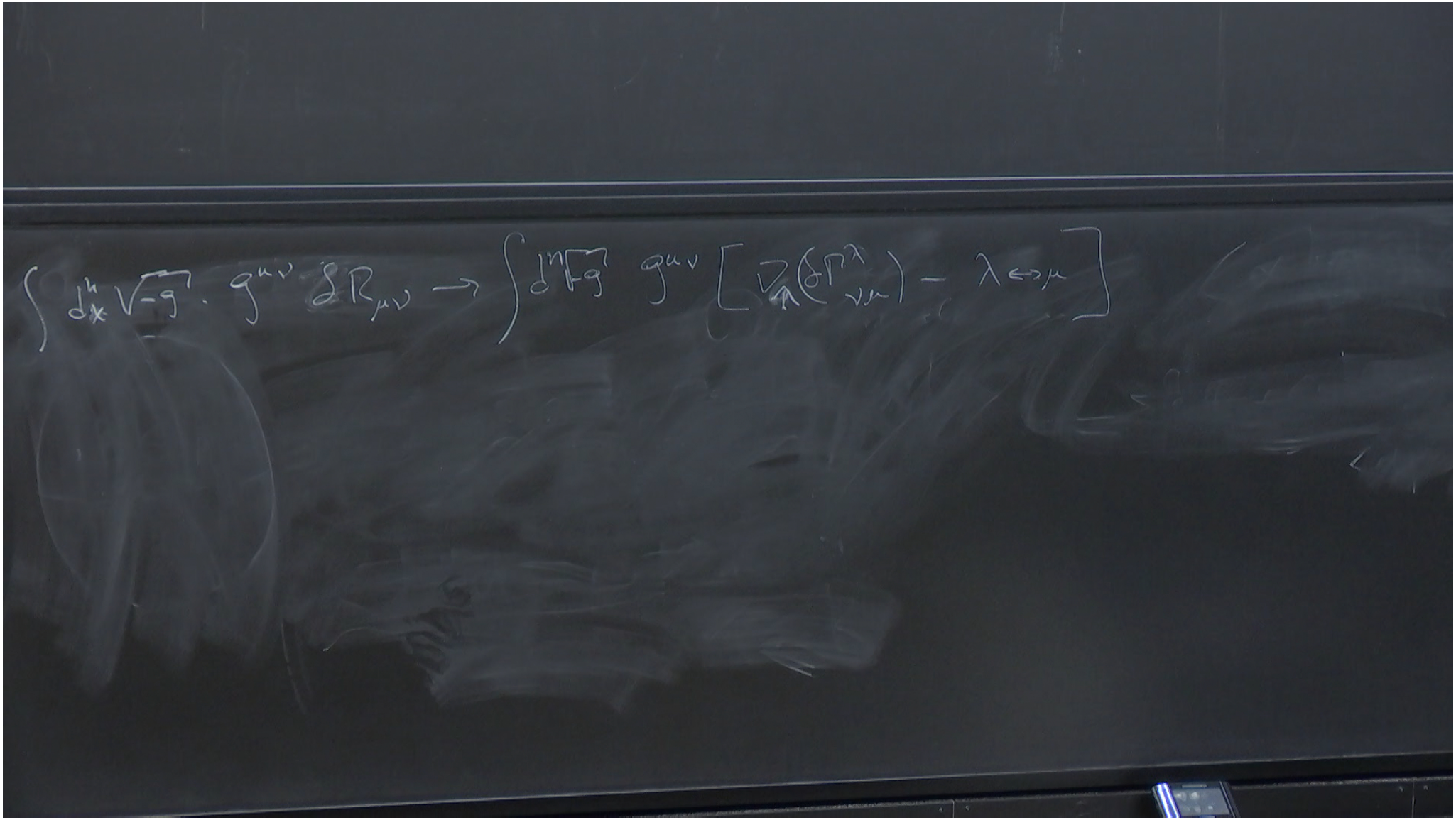

$\nabla_c V^a = \partial_c V^a + \Gamma^a_{cb} V^b$
$\nabla'_c V'^a = \partial_c V'^a + \Gamma'^a_{cb} V'^b$

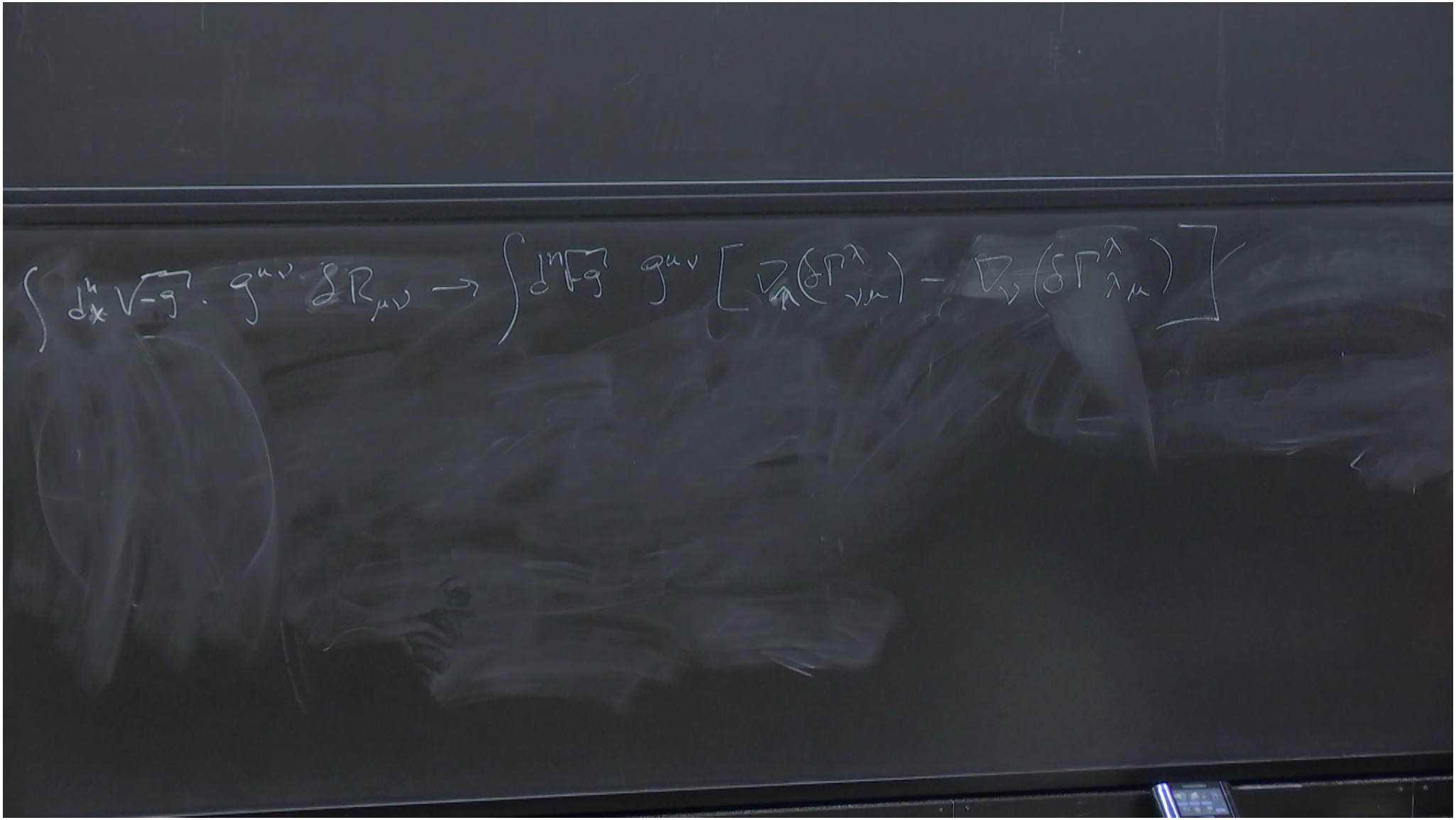
$$\nabla_c V^a = \partial_c V^a + \Gamma^a_{cb} V^b$$

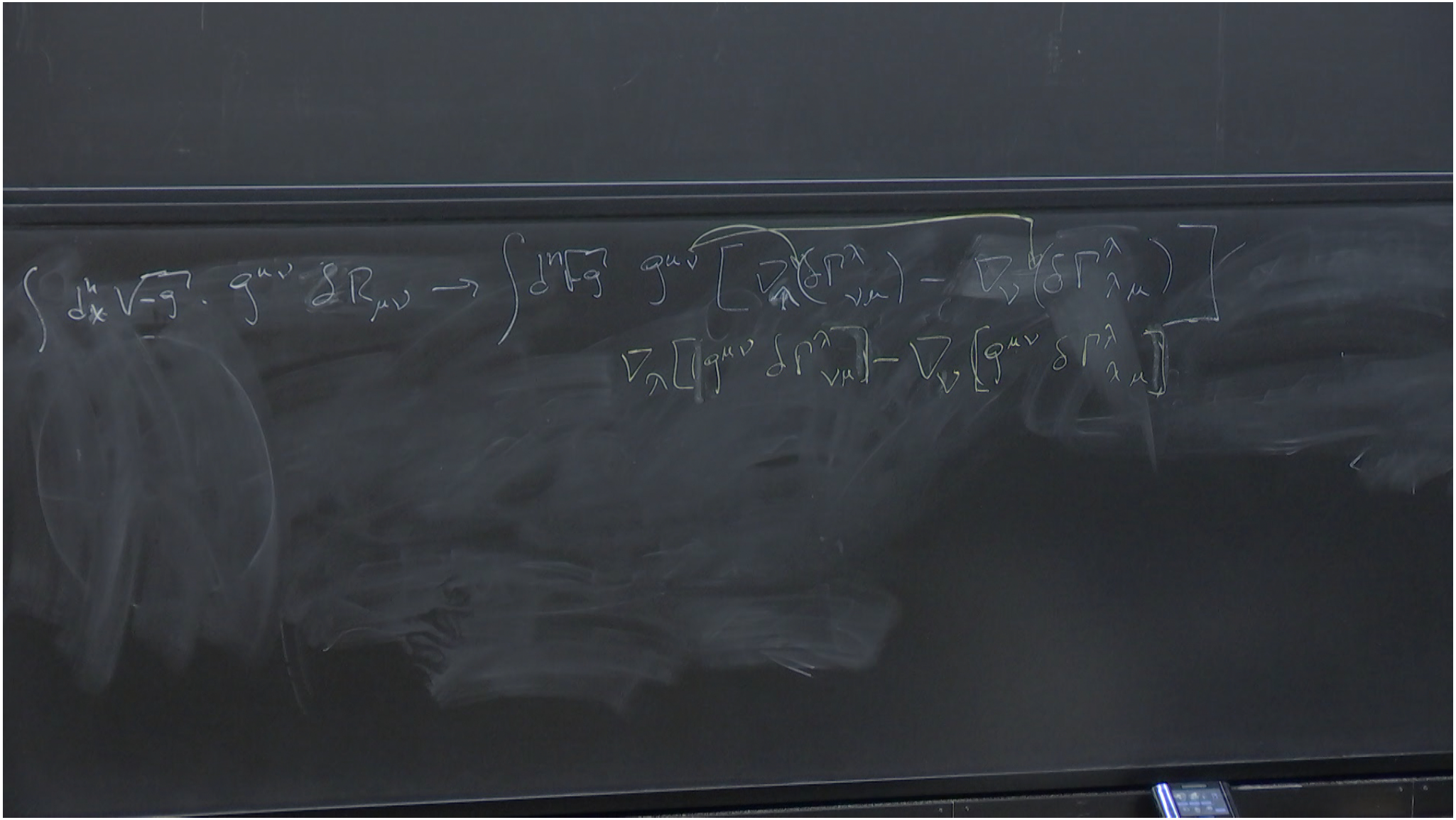
$$\nabla'_c V^a = \partial_c V^a + \Gamma'^a_{cb} V^b \qquad \left(\Gamma^a_{cb} - \Gamma'^a_{cb}\right) V^b$$

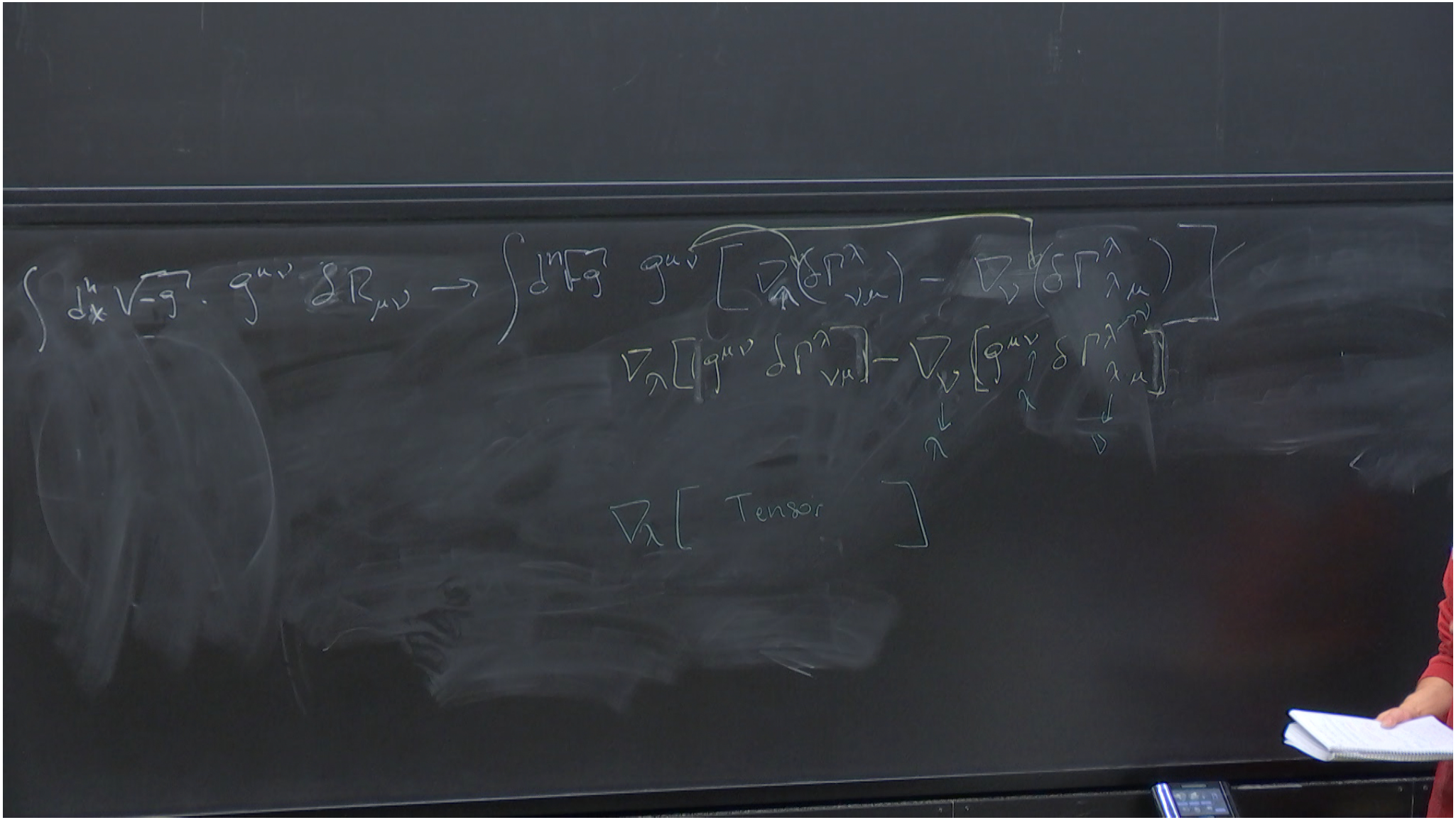
$$\rightsquigarrow \boxed{\delta R^\rho_{\mu\nu\gamma} = \nabla_\nu \left(\delta \Gamma^\rho_{\gamma\mu}\right) - \nabla_\gamma \left(\delta \Gamma^\rho_{\nu\mu}\right)}$$

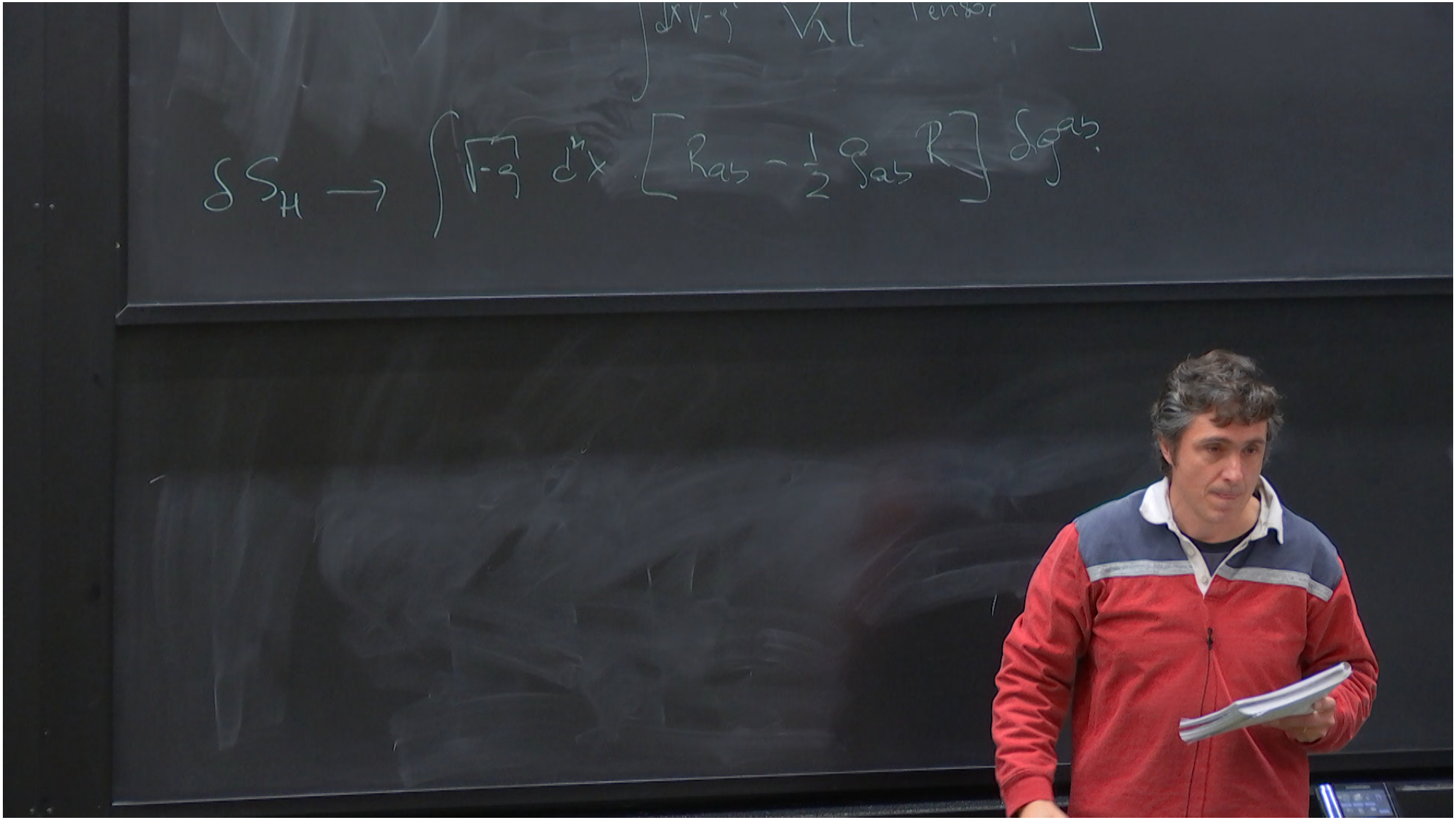
Pave!

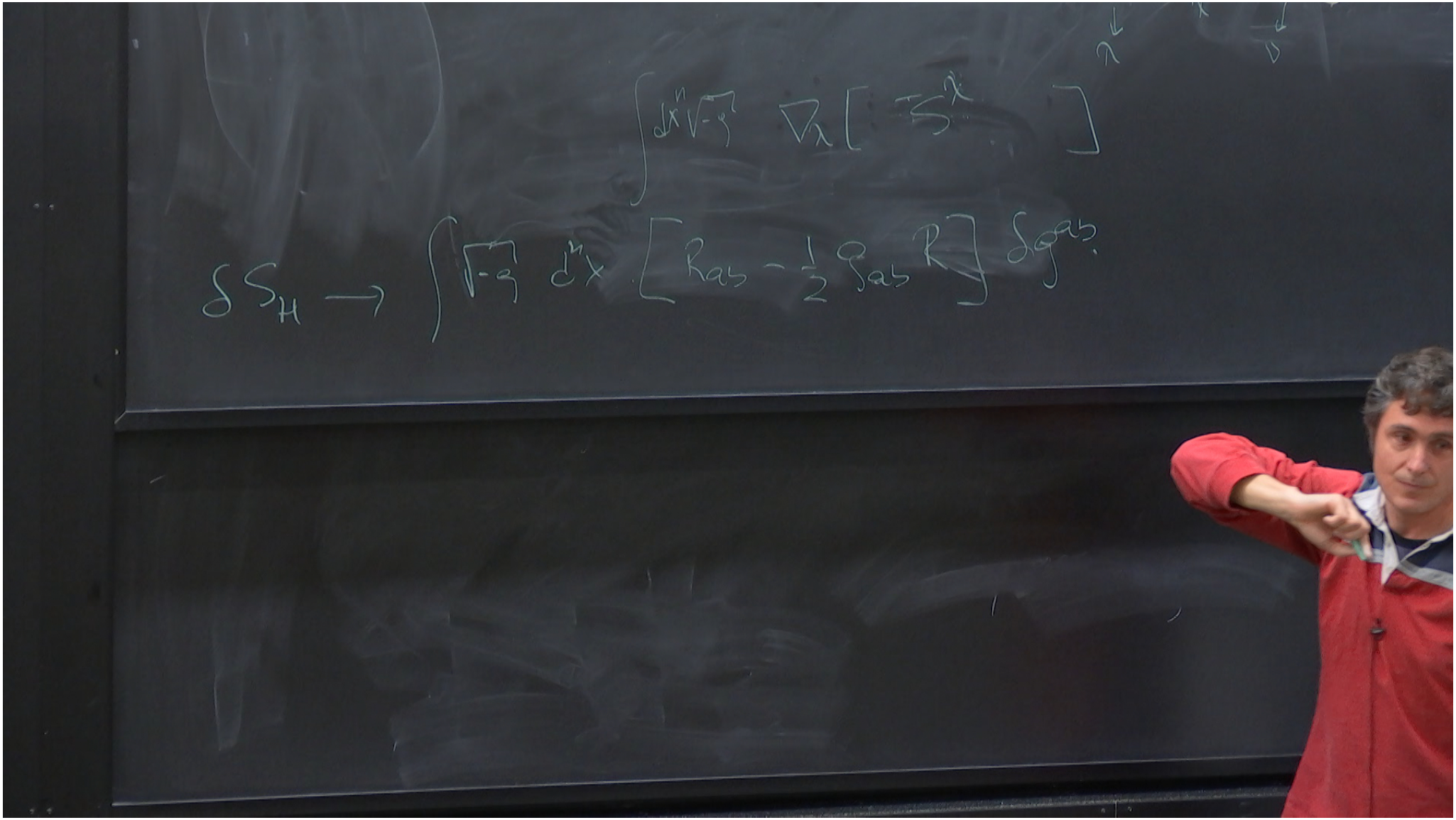

$\int d^n x \sqrt{-g} \cdot g^{\mu\nu} \delta R_{\mu\nu} \rightarrow \int d^n x \sqrt{-g}\; g^{\mu\nu} \left[\nabla_\lambda (\delta \Gamma^\lambda_{\nu\mu}) - \lambda \leftrightarrow \mu \right]$

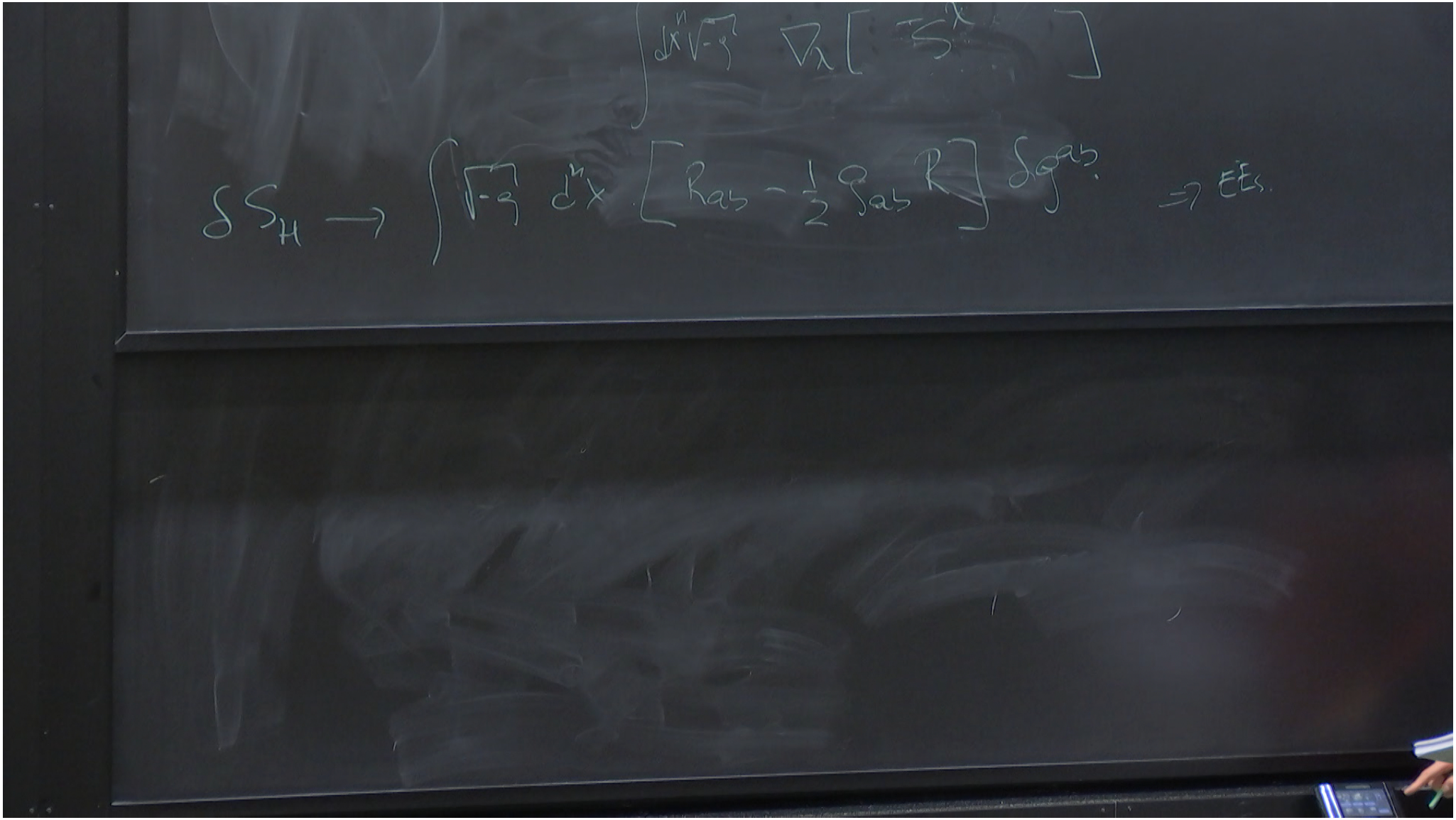

$\int d^n x \sqrt{-g} \cdot g^{\mu\nu} \delta R_{\mu\nu} \rightarrow \int d^n x \sqrt{-g}\; g^{\mu\nu} \left[\nabla_\lambda (\delta \Gamma^\lambda_{\nu\mu}) - \nabla_\nu (\delta \Gamma^\lambda_{\lambda\mu}) \right]$

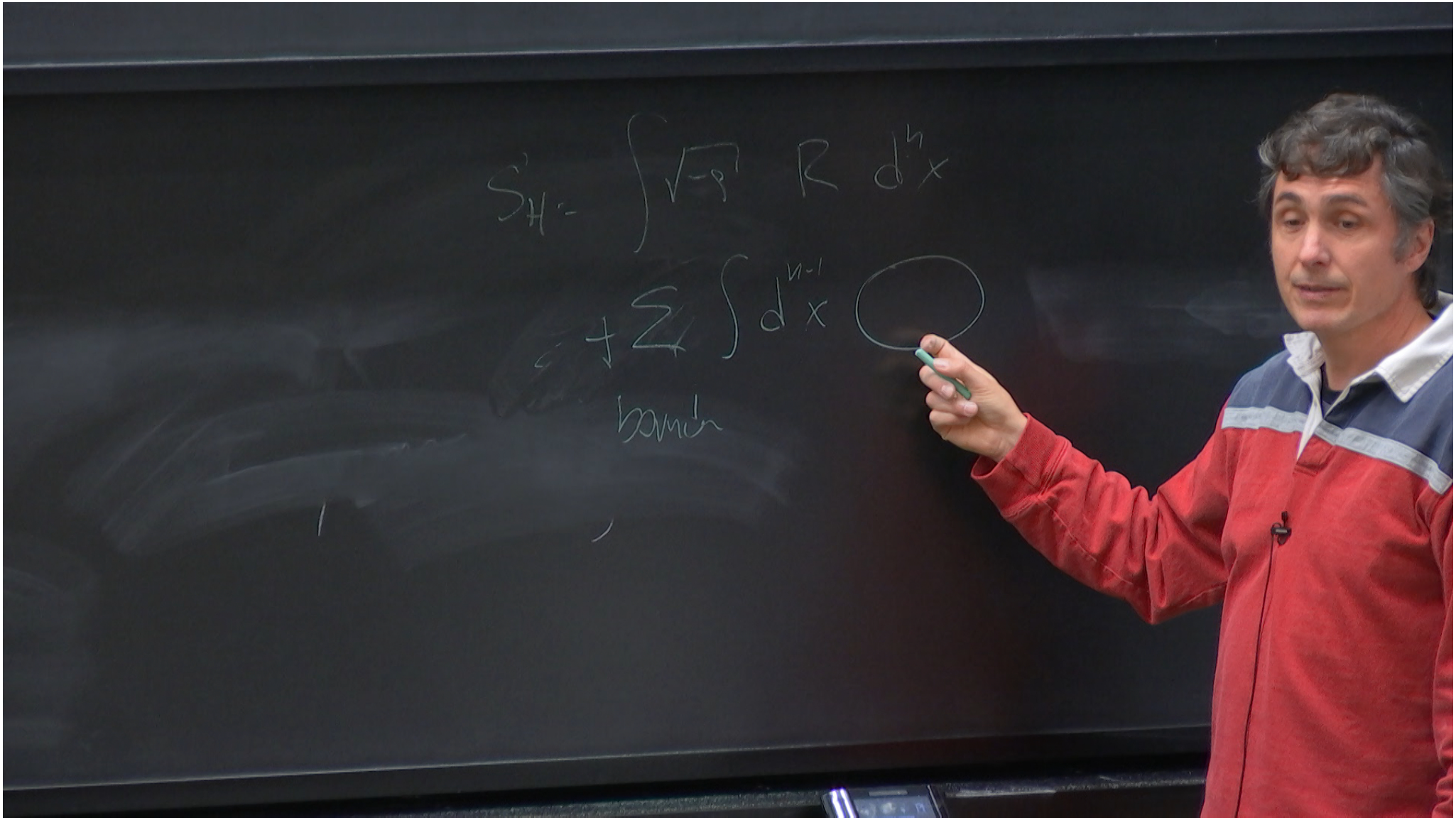

$\int d^4x \sqrt{-g} \cdot g^{\mu\nu} \delta R_{\mu\nu} \rightarrow \int d^n x \sqrt{g}\; g^{\mu\nu} \left[\nabla_\lambda (\delta \Gamma^\lambda_{\nu\mu}) - \nabla_\nu (\delta \Gamma^\lambda_{\lambda\mu}) \right]$
$\nabla_\lambda [g^{\mu\nu} \delta \Gamma^\lambda_{\nu\mu}] - \nabla_\nu [g^{\mu\nu} \delta \Gamma^\lambda_{\lambda\mu}]$

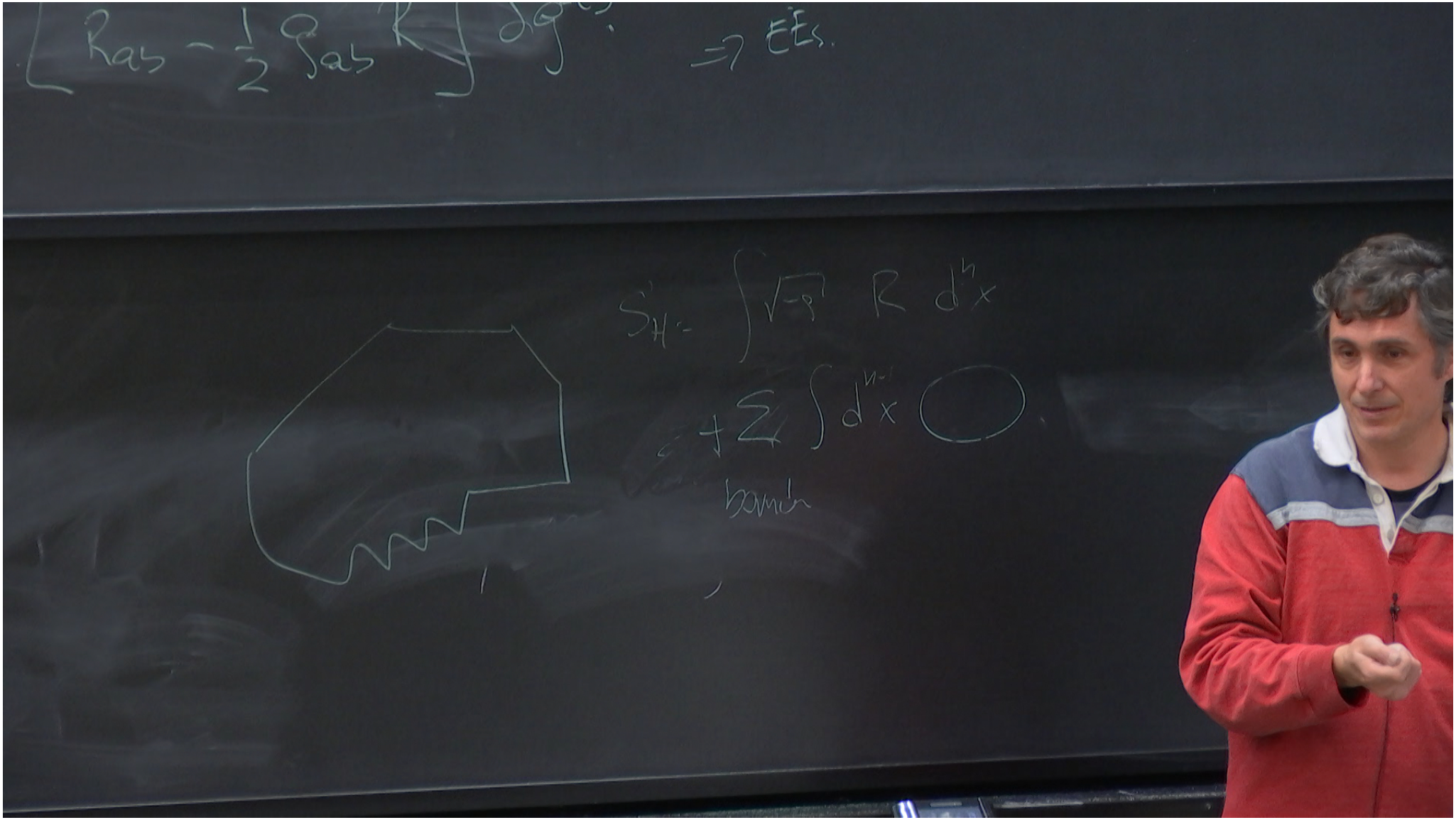

$\int d^4x \sqrt{-g} \cdot g^{\mu\nu} \delta R_{\mu\nu} \rightarrow \int d^n x \sqrt{g}\, g^{\mu\nu} \left[\nabla_\lambda(\delta\Gamma^\lambda_{\nu\mu}) - \nabla_\nu(\delta\Gamma^\lambda_{\lambda\mu}) \right]$
$\nabla_\lambda [g^{\mu\nu} \delta\Gamma^\lambda_{\nu\mu}] - \nabla_\nu [g^{\mu\nu} \delta\Gamma^\lambda_{\lambda\mu}]$
$\nabla_\lambda [\ \text{Tensor}\ ]$

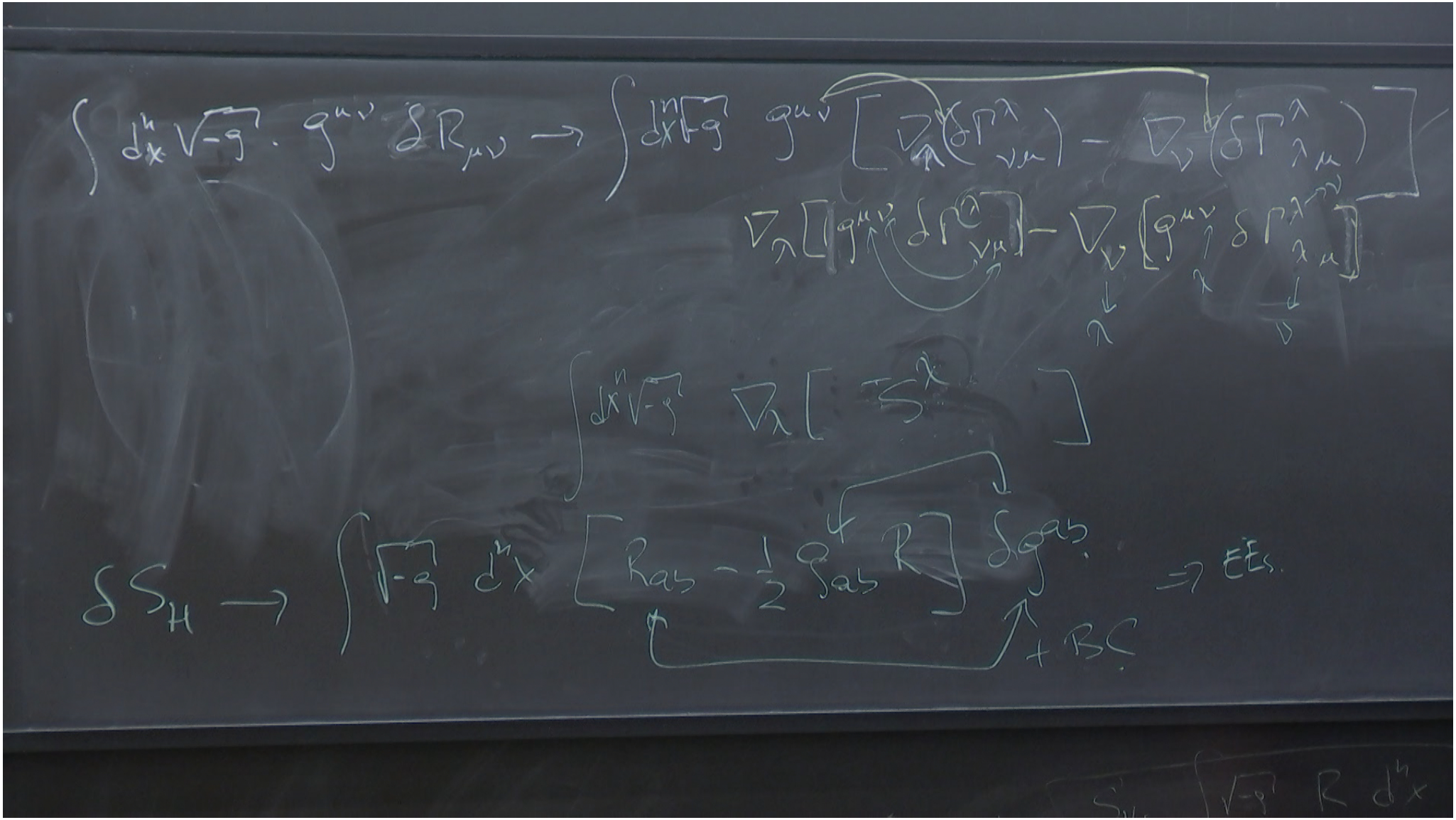

$\delta S_H \rightarrow \int \sqrt{-g}\, d^4x \left[R_{ab} - \frac{1}{2} g_{ab} R \right] \delta g^{ab}$

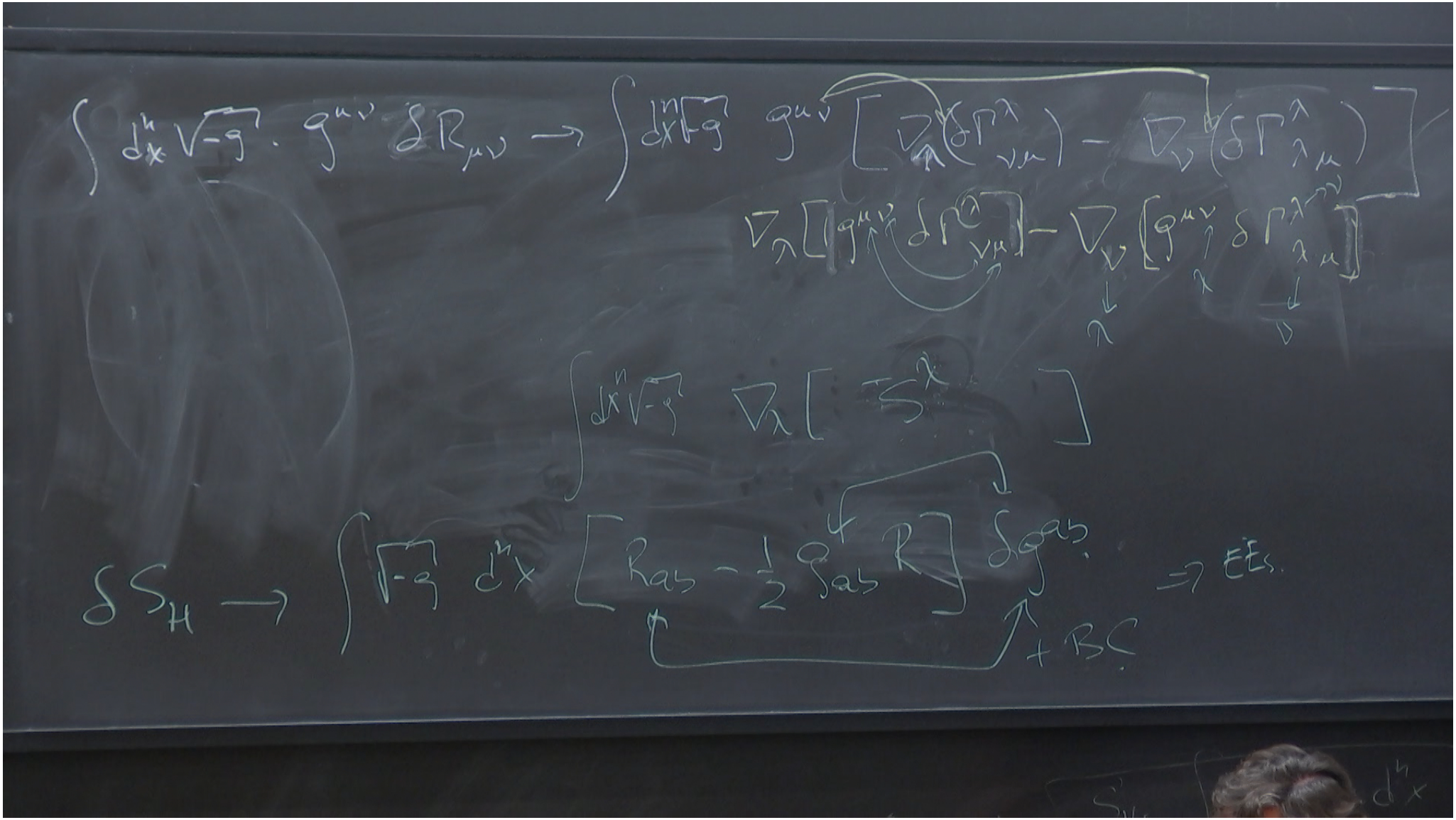

$\int d^n x \sqrt{-g}\, \nabla_\lambda [\quad]$
$\delta S_H \to \int \sqrt{-g}\, d^n x \left[R_{ab} - \frac{1}{2} g_{ab} R \right] \delta g^{ab}$

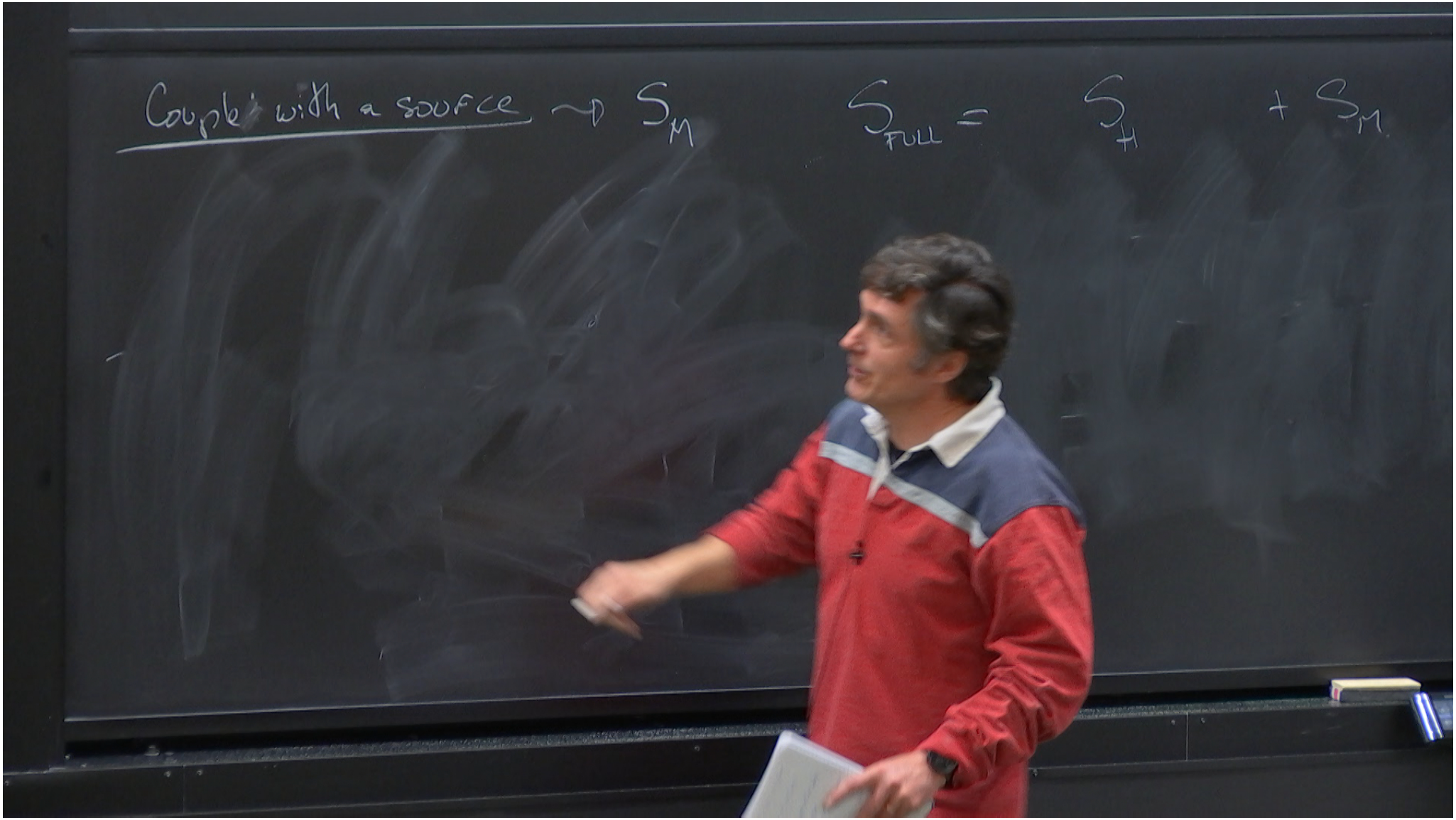

$\int d^n x \sqrt{-g}\ \nabla_\lambda [\ \ S^\lambda \ \ ]$
$\delta S_H \to \int \sqrt{-g}\ d^n x \left[R_{ab} - \frac{1}{2} g_{ab} R \right] \delta g^{ab}$
$\Rightarrow$ EEs.

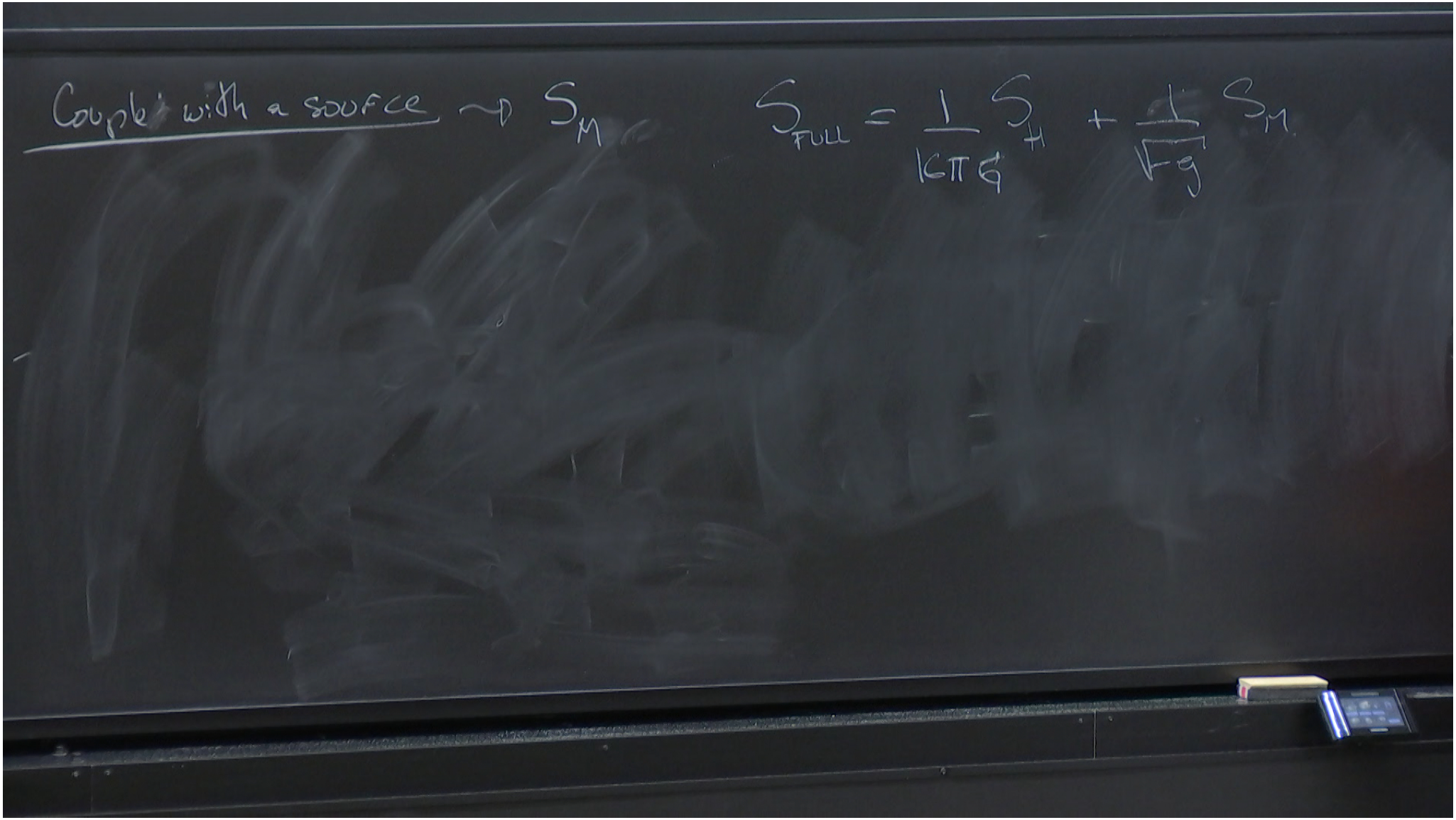

$S_H = \int \sqrt{-g}\, R\, d^n x$
$+ \sum_i \int d^{n-1} x$
boundary

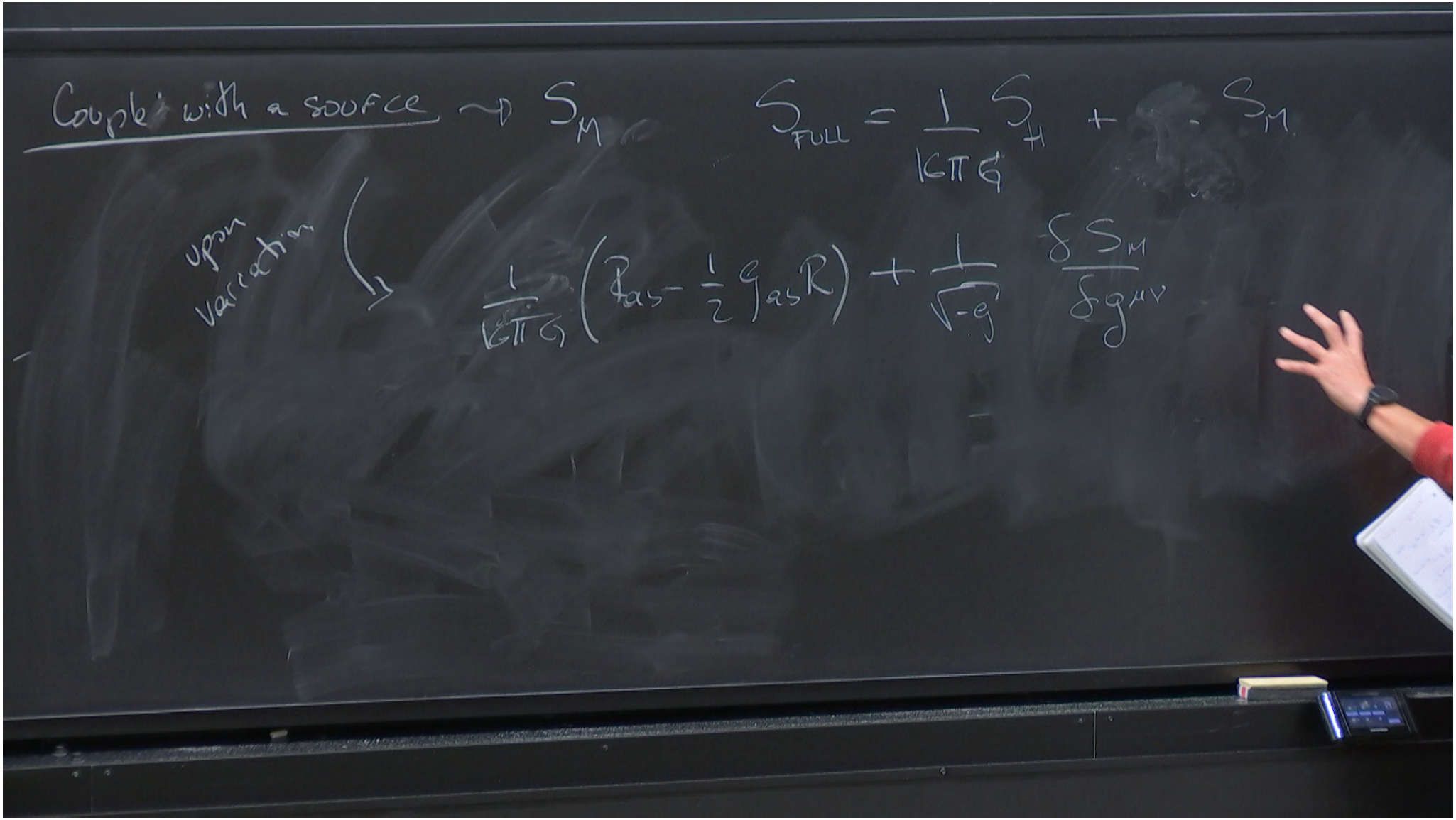

$R_{ab} - \frac{1}{2} g_{ab} R$
$\Rightarrow$ EEs.
$S_H = \int \sqrt{-g}\, R\, d^n x$
$+ \Sigma \int d^{n-1} x$
boundary

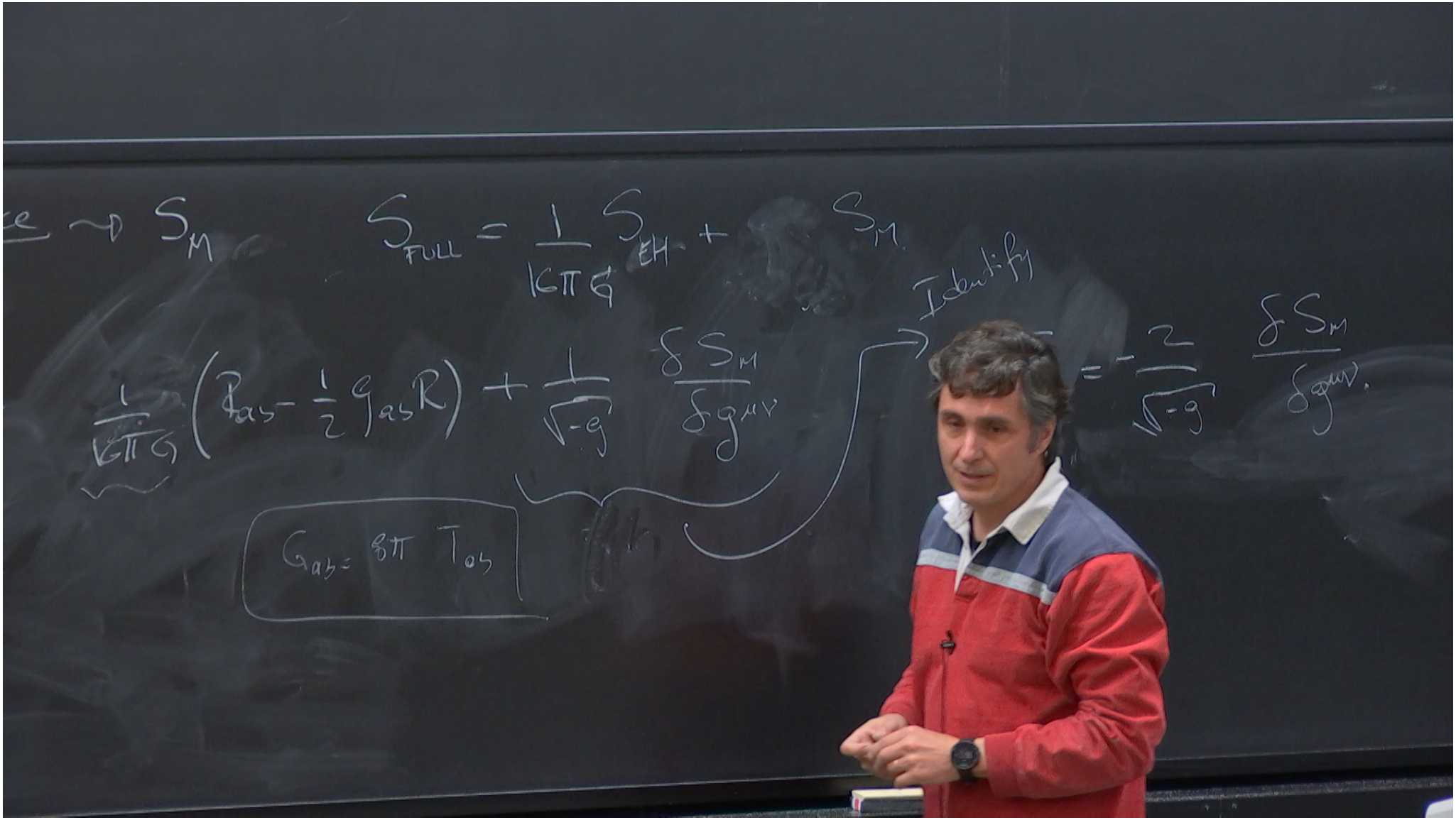

$\int d^4x \sqrt{-g}\, g^{\mu\nu}\, \delta R_{\mu\nu} \to \int d^4x \sqrt{-g}\, g^{\mu\nu} \left[\nabla_\lambda (\delta\Gamma^\lambda_{\nu\mu}) - \nabla_\nu (\delta\Gamma^\lambda_{\lambda\mu}) \right]$
$\nabla_\lambda [g^{\mu\nu} \delta\Gamma^\lambda_{\nu\mu}] - \nabla_\nu [g^{\mu\nu} \delta\Gamma^\lambda_{\lambda\mu}]$
$\int d^4x \sqrt{-g}\, \nabla_\lambda [\ S^\lambda \ ]$
$\delta S_H \to \int \sqrt{-g}\, d^4x \left[R_{ab} - \frac{1}{2} g_{ab} R \right] \delta g^{ab}$
$\Rightarrow$ EEs.
+ BC
$S_H = \int \sqrt{-g}\, R\, d^4x$

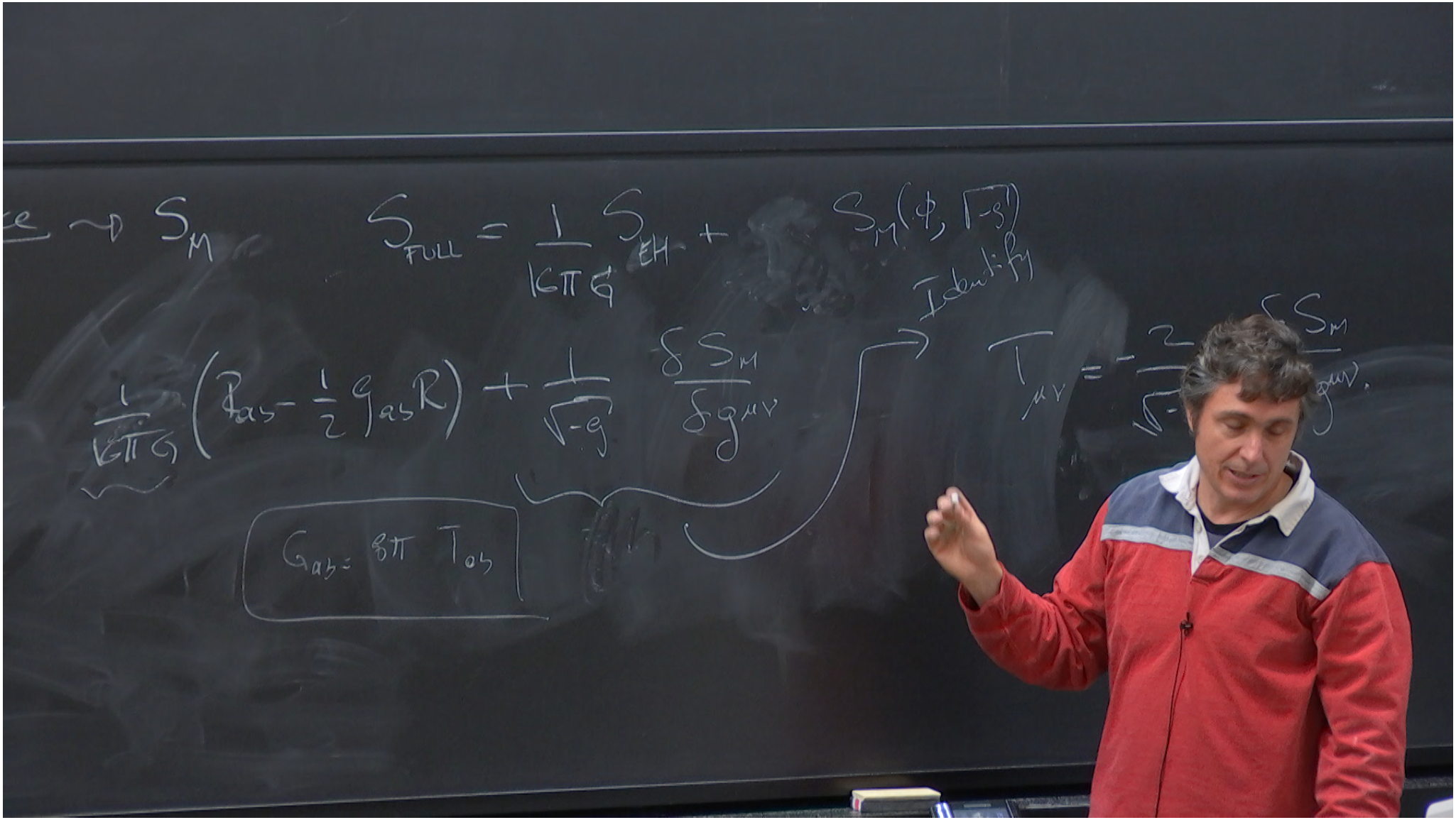

$\int d^4x \sqrt{-g}\, g^{\mu\nu}\, \delta R_{\mu\nu} \to \int d^4x \sqrt{-g}\, g^{\mu\nu} \left[\nabla_\lambda (\delta\Gamma^\lambda_{\nu\mu}) - \nabla_\nu (\delta\Gamma^\lambda_{\lambda\mu}) \right]$
$\nabla_\lambda [g^{\mu\nu} \delta\Gamma^\lambda_{\nu\mu}] - \nabla_\nu [g^{\mu\nu} \delta\Gamma^\lambda_{\lambda\mu}]$
$\int d^4x \sqrt{-g}\, \nabla_\lambda [\ S^\lambda\ ]$
$\delta S_H \to \int \sqrt{-g}\, d^4x \left[R_{ab} - \frac{1}{2} g_{ab} R \right] \delta g^{ab}$
$\Rightarrow$ EEs.
+ BC

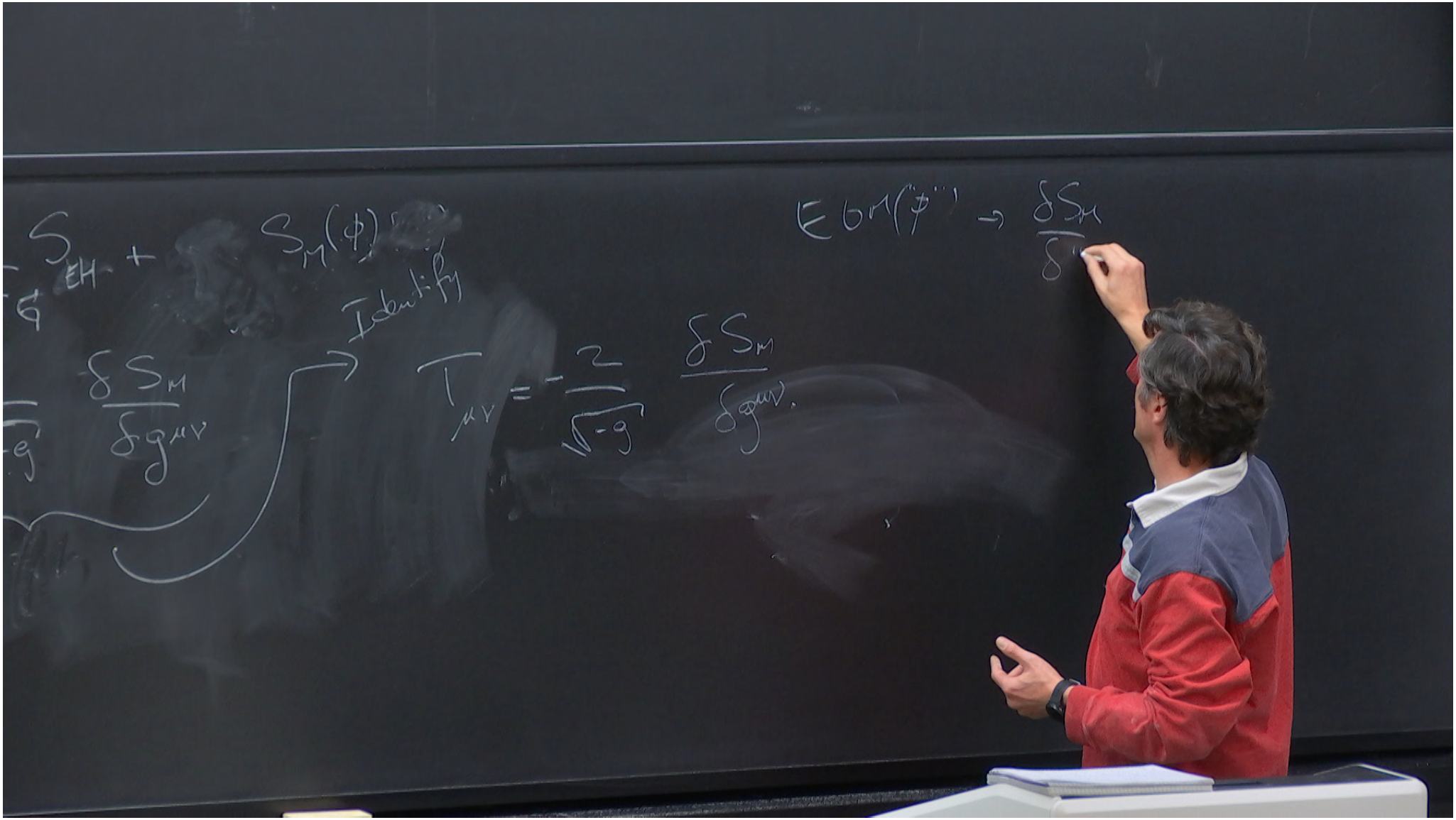

Couple with a source → S_M
$S_{FULL} = S_H + S_M$

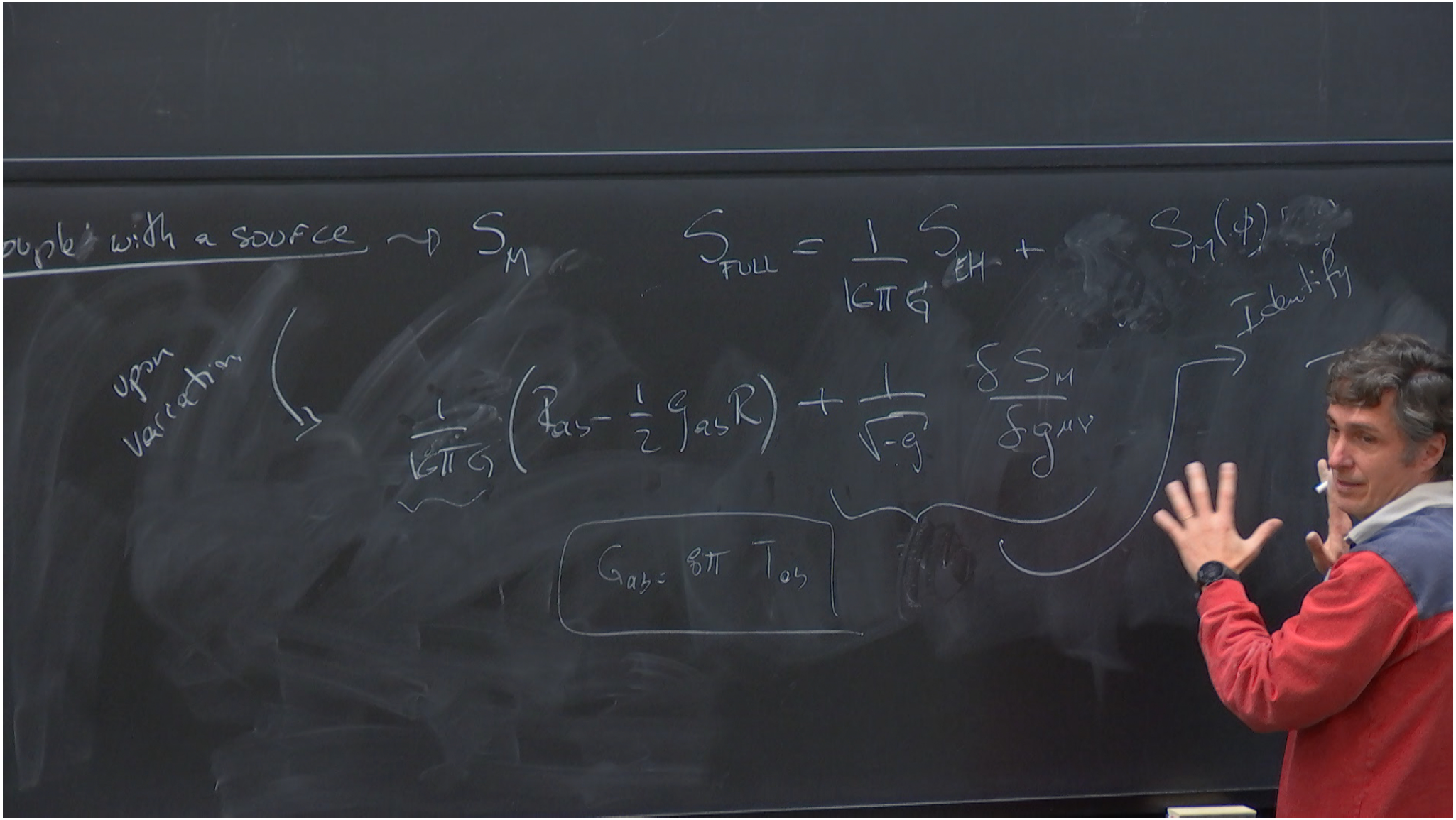

Couple with a source ⇝ S_M
$S_{FULL} = \frac{1}{16\pi G} S_H + \frac{1}{\sqrt{-g}} S_M$

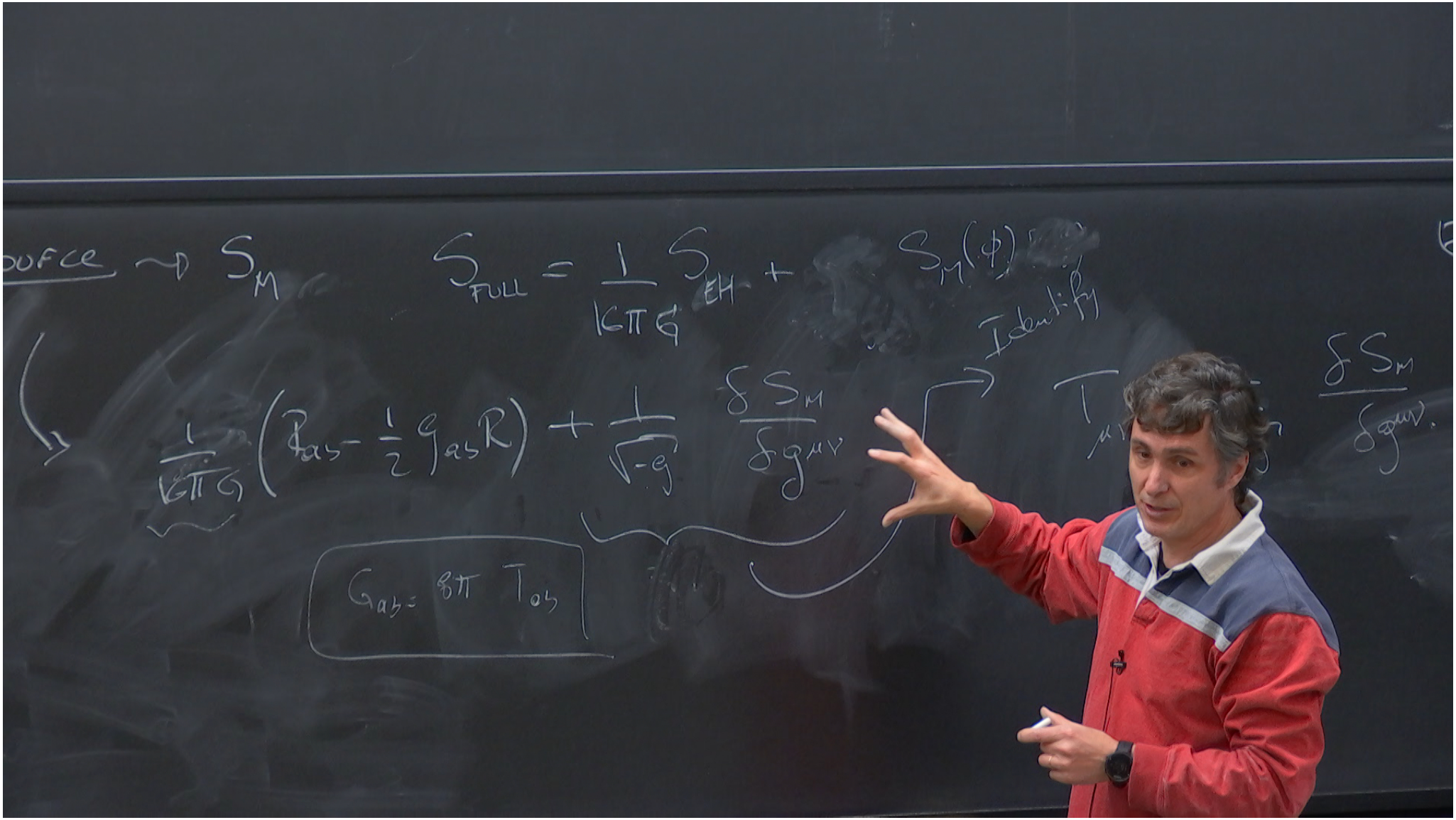

Couple with a source → S_M
$S_{FULL} = \frac{1}{16\pi G} S_H + S_M$
upon variation
$\frac{1}{16\pi G}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}$

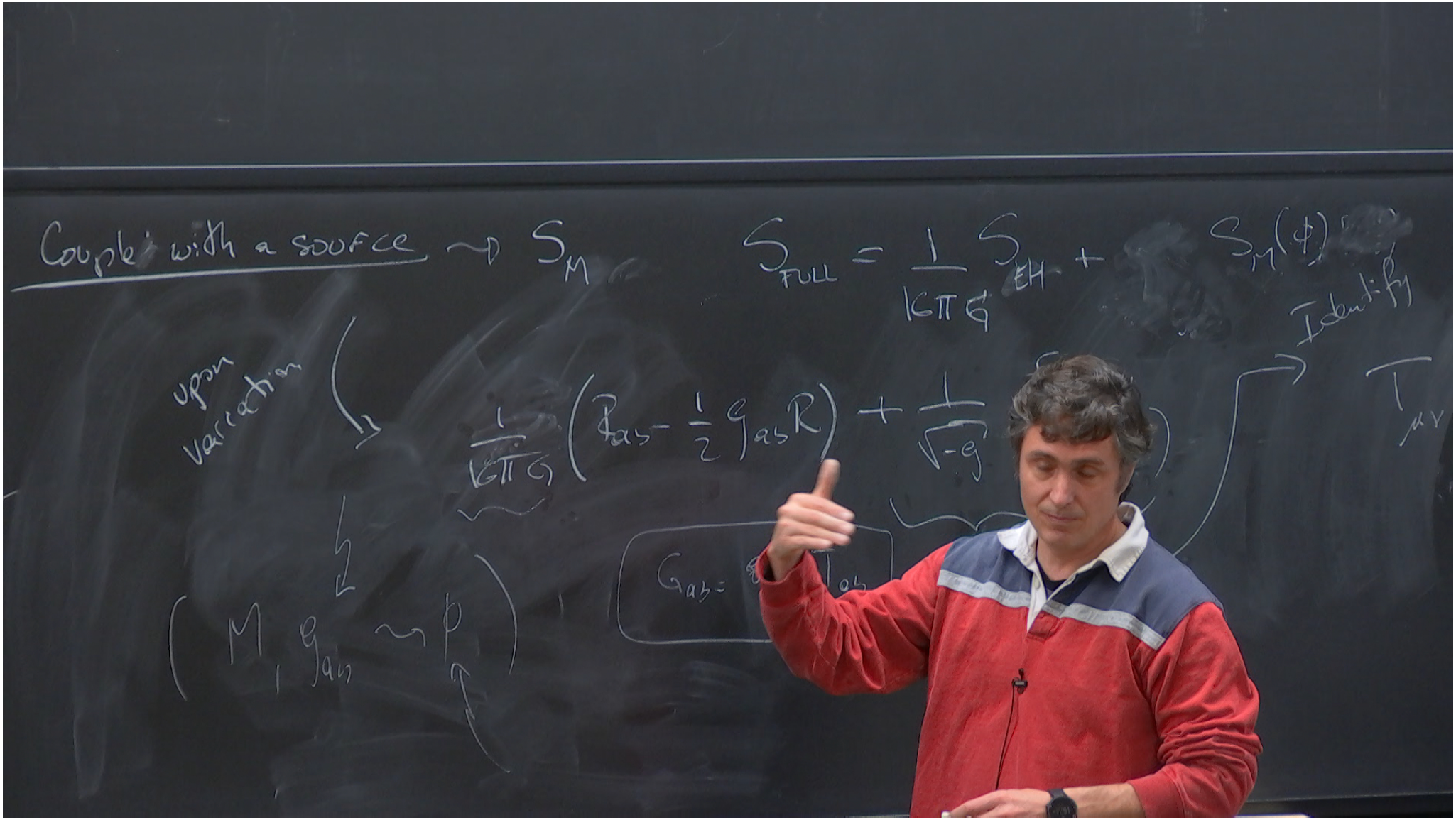

S_M
$S_{FULL} = \frac{1}{16\pi G} S_{EH} + S_M$
Identify
$\frac{1}{\sqrt{16\pi G}}\left(R_{ab} - \frac{1}{2} g_{ab} R\right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}$
$= -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}$
$G_{ab} = 8\pi T_{ab}$

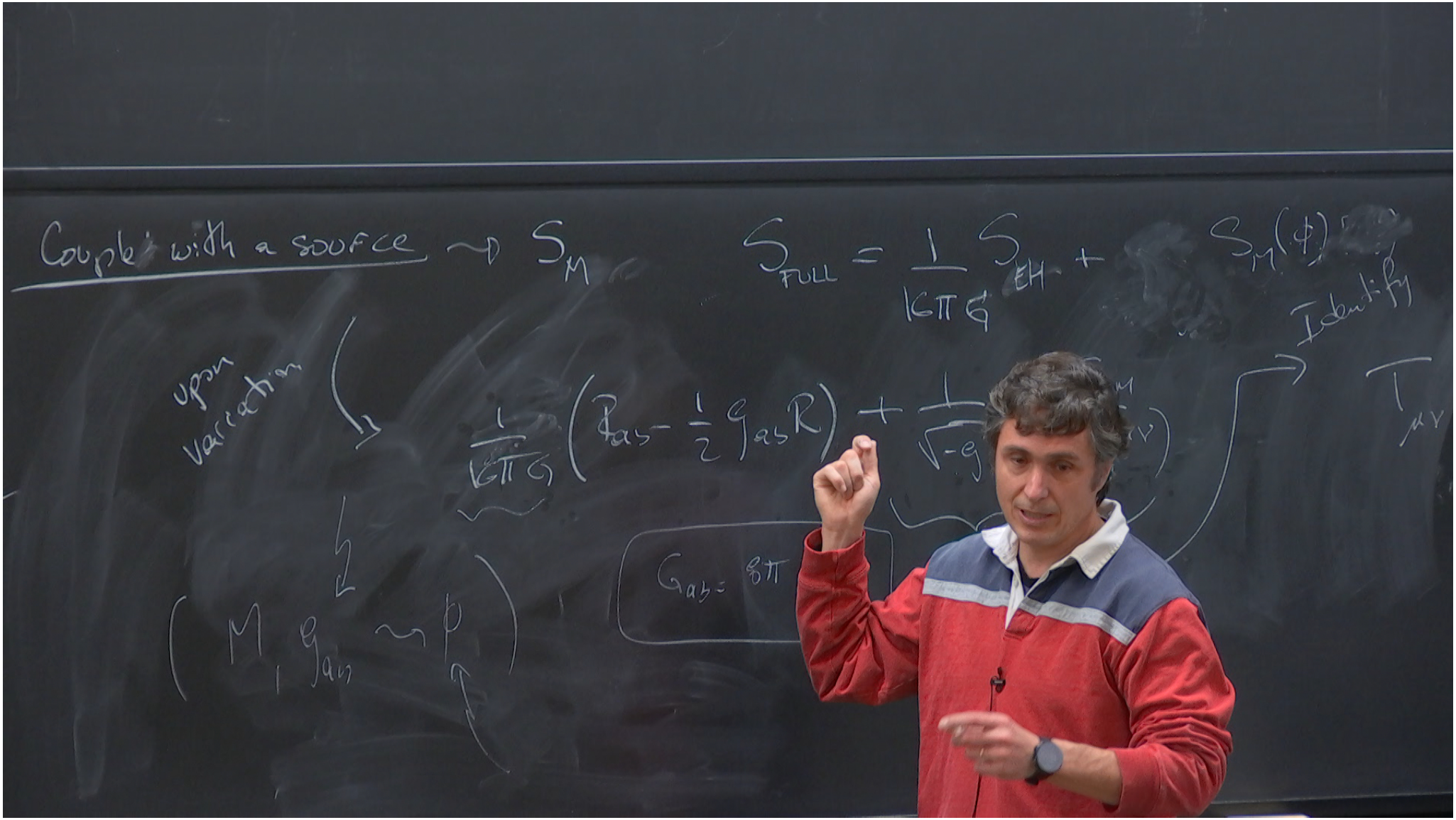

$S_{FULL} = \frac{1}{16\pi G} S_{EH} + S_M(\phi, \sqrt{-g})$
Identify
$\frac{1}{\sqrt{16\pi G}}\left(R_{ab} - \frac{1}{2} g_{ab} R\right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}$
$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}$
$G_{ab} = 8\pi T_{ab}$

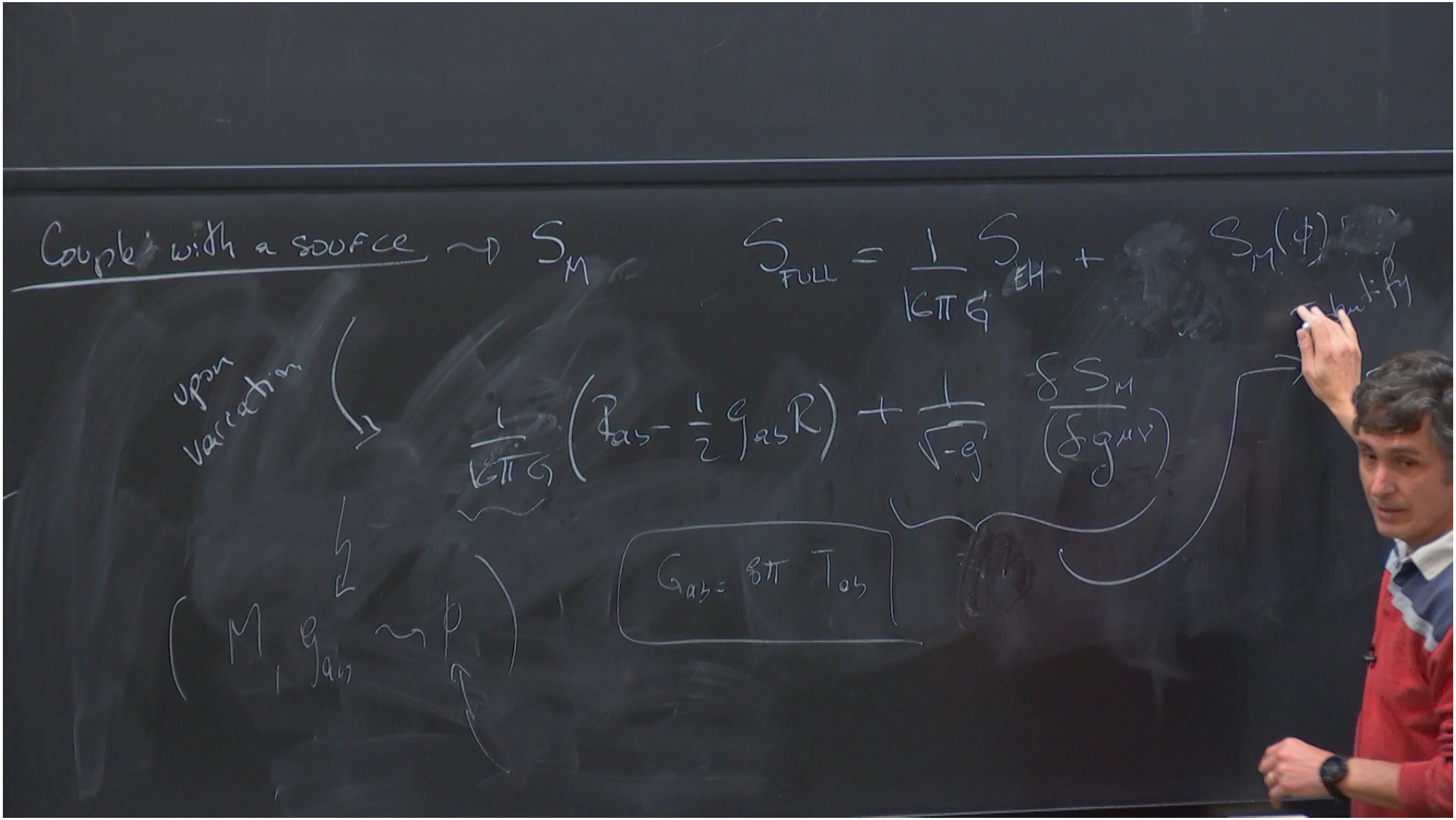

S_{EH} +
$S_M(\phi)$
Identify
$\frac{\delta S_M}{\delta g_{\mu\nu}}$
$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}$
EOM(ϕ) → $\frac{\delta S_M}{\delta$

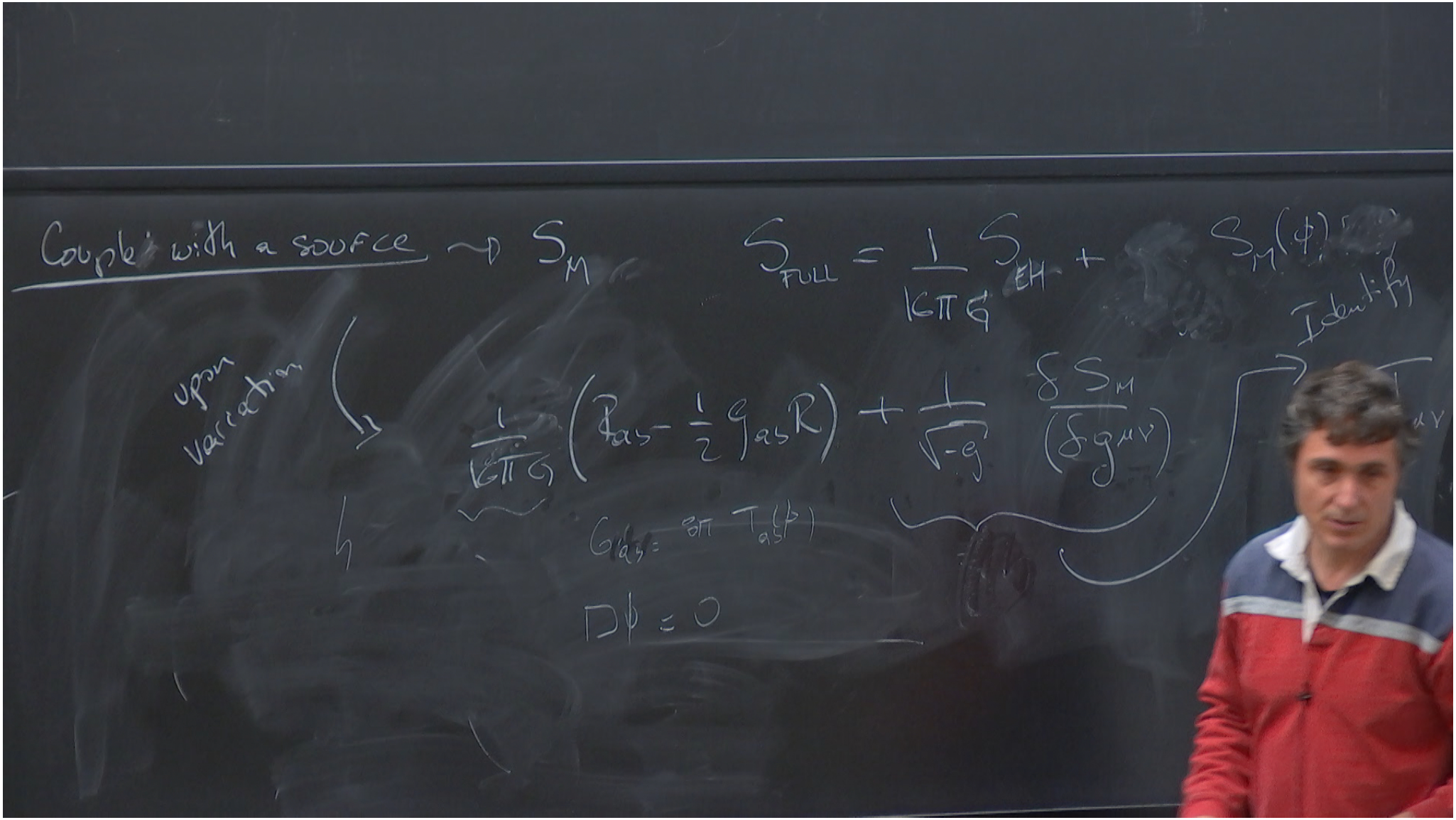

coupled with a source → S_M
$S_{FULL} = \frac{1}{16\pi G} S_{EH} + S_M(\phi)$
upon variation
$\frac{1}{16\pi G}\left(R_{ab} - \frac{1}{2} g_{ab} R\right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}$
Identify
$G_{ab} = 8\pi T_{ab}$

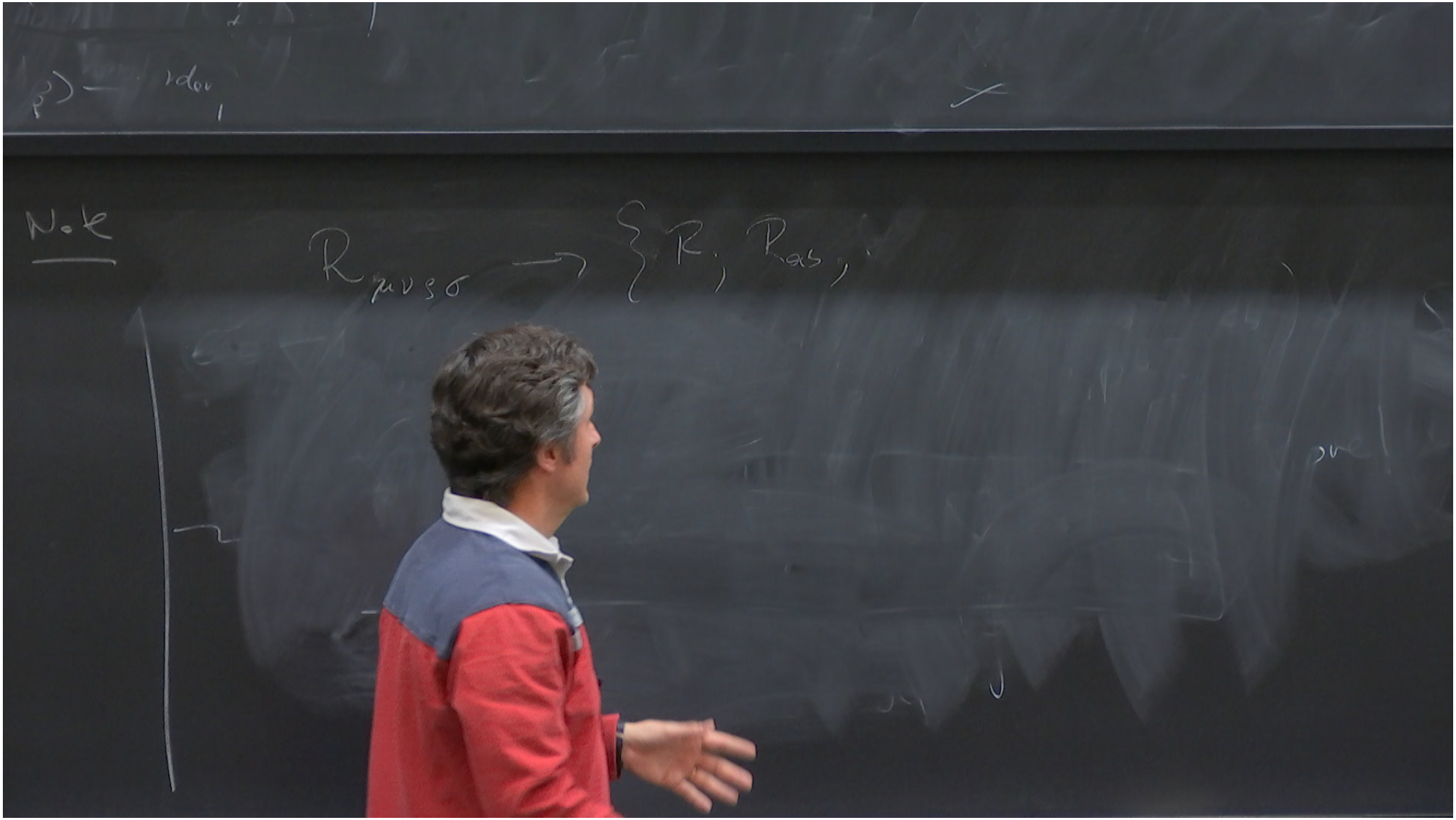

$S_{FULL} = \frac{1}{16\pi G} S_{EH} + S_M(\phi)$
Identify
$\frac{1}{16\pi G}\left(R_{ab} - \frac{1}{2} g_{ab} R\right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}$
$\frac{\delta S_M}{\delta g^{\mu\nu}}$
$G_{ab} = 8\pi T_{ab}$

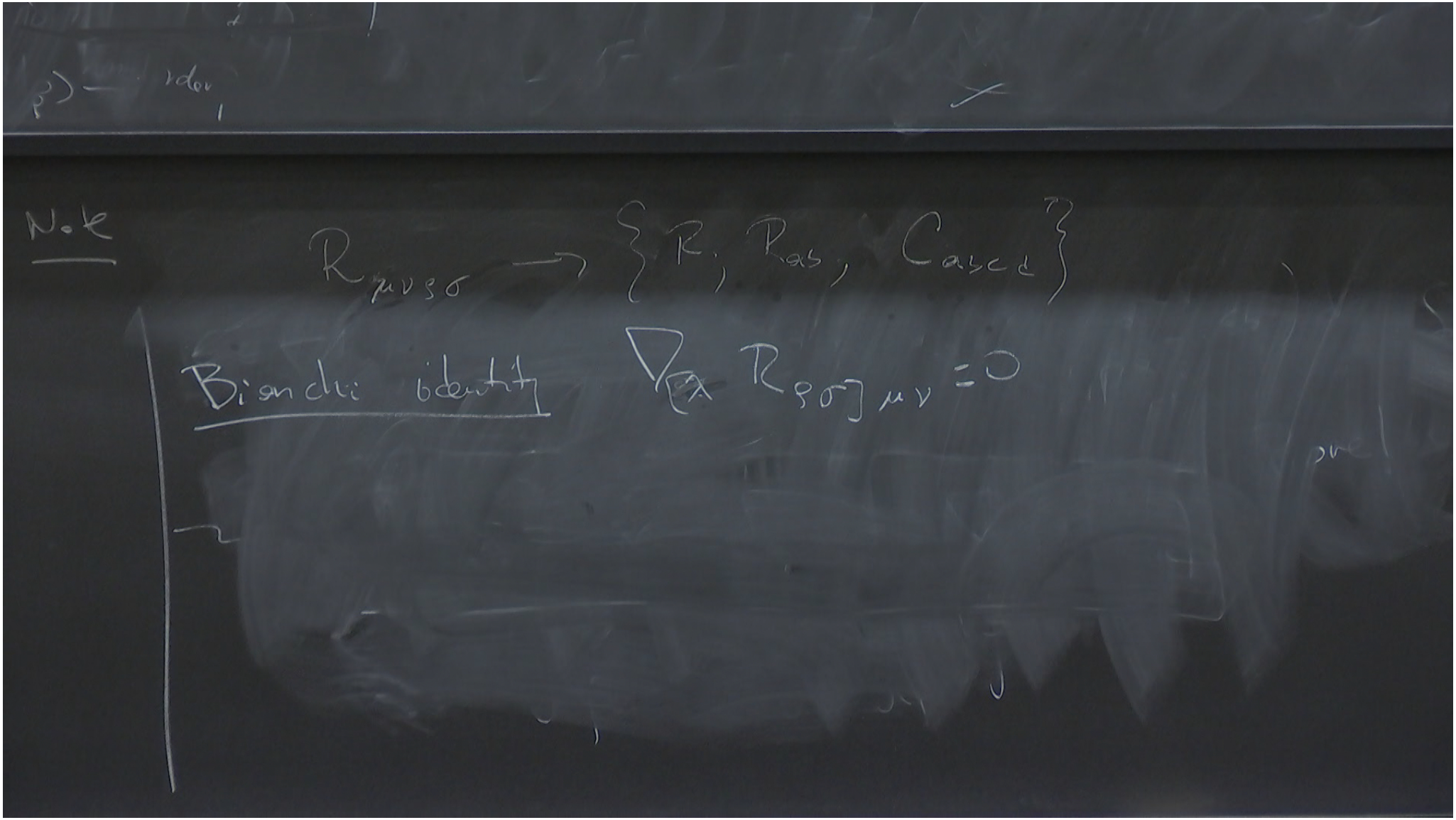

Couple with a source
upon variation
Identify

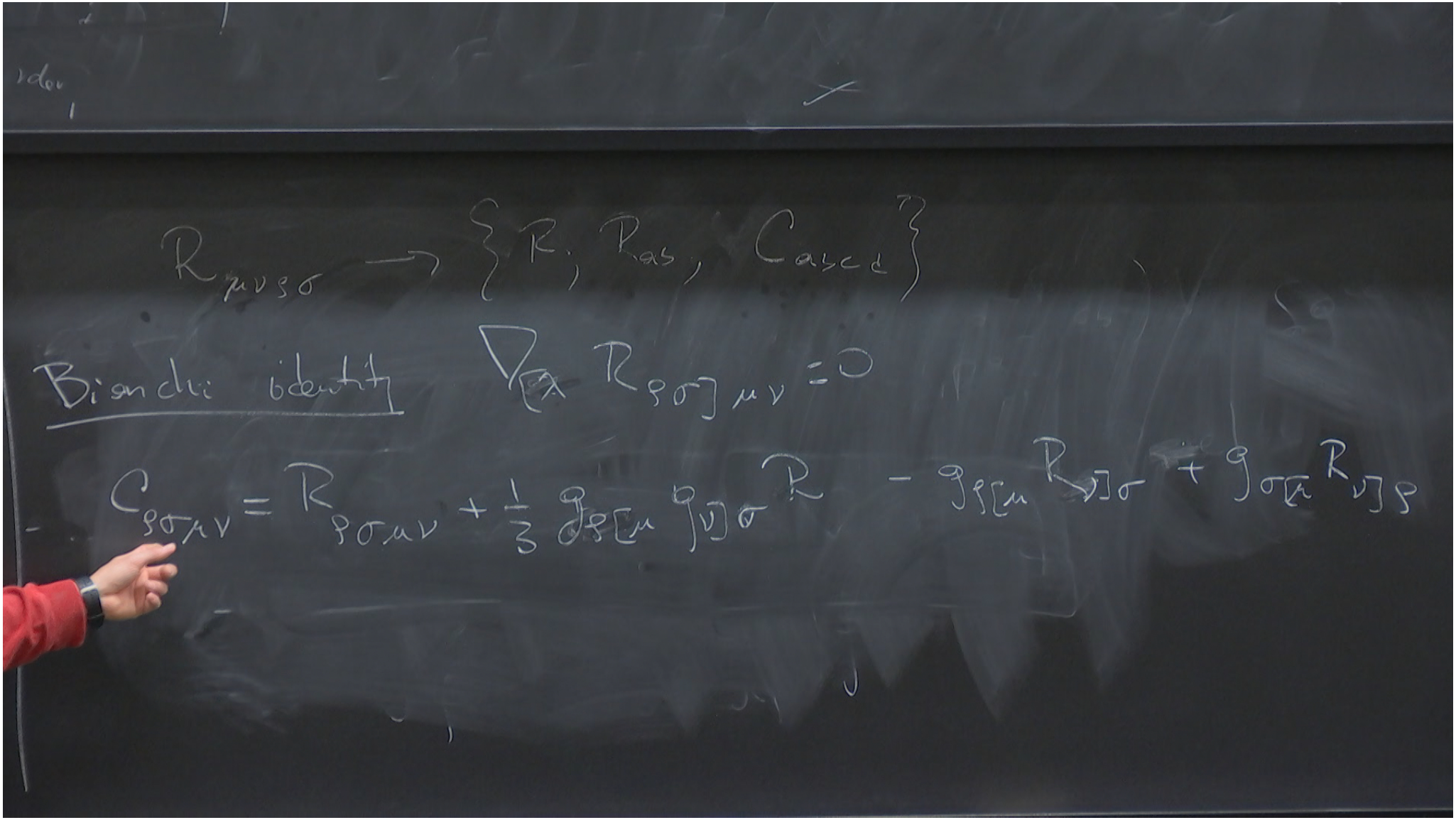

Couple with a source ⇝ S_M
$S_{FULL} = \frac{1}{16\pi G} S_{EH} + S_M(\phi)$
upon variation
$\frac{1}{16\pi G}\left(R_{ab} - \frac{1}{2} g_{ab} R\right) + \frac{1}{\sqrt{-g}}$
Identify
$T_{\mu\nu}$
$G_{ab} = 8\pi$
$M, g_{ab} \rightsquigarrow p$

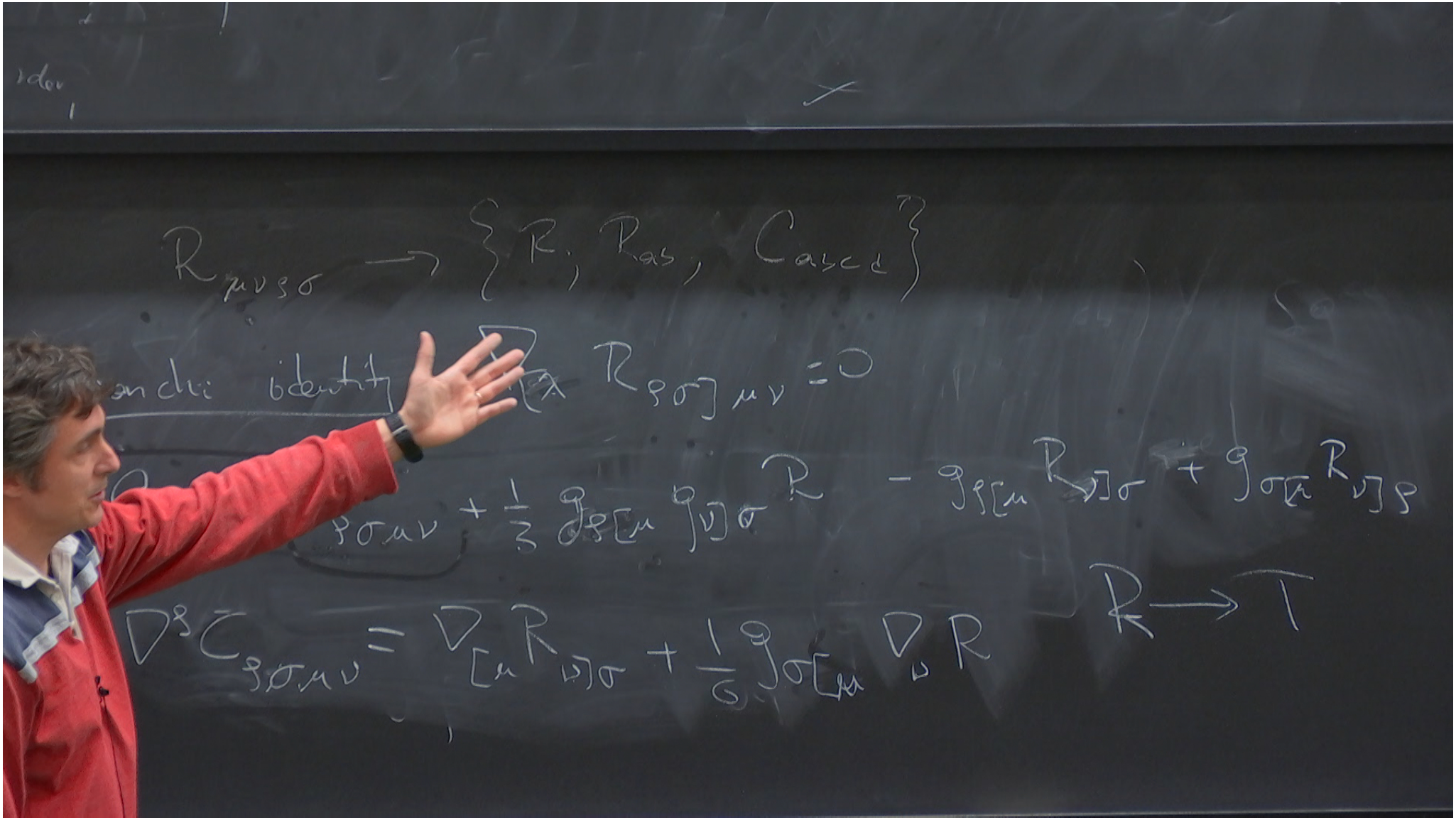

Couple with a source $\rightsquigarrow S_M$
$S_{FULL} = \frac{1}{16\pi G} S_{EH} + S_M(\phi)$
upon variation
$\frac{1}{16\pi G}\left(R_{ab} - \frac{1}{2} g_{ab} R\right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}$
$G_{ab} = 8\pi T_{ab}$
$M, g_{ab} \rightsquigarrow p$


Couple with a source → S_M
$S_{FULL} = \frac{1}{16\pi G} S_{EH} + S_M(\phi)$
upon variation
$\frac{1}{16\pi G}\left(R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R\right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}$
Identify
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}(\phi)$
$\Box\phi = 0$


N.B
R


N.B
$R_{\mu\nu\sigma} \rightarrow \{R, R_{ab}, C_{abcd}\}$
Bianchi identity $\nabla_{[\lambda} R_{\rho\sigma]\mu\nu} = 0$


$R_{\mu\nu\rho\sigma} \rightarrow \{R;\ R_{ab},\ C_{abcd}\}$
Bianchi identity $\nabla_{[\lambda} R_{\rho\sigma]\mu\nu} = 0$
$C_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} + \frac{1}{3} g_{\rho[\mu} g_{\nu]\sigma} R - g_{\rho[\mu} R_{\nu]\sigma} + g_{\sigma[\mu} R_{\nu]\rho}$

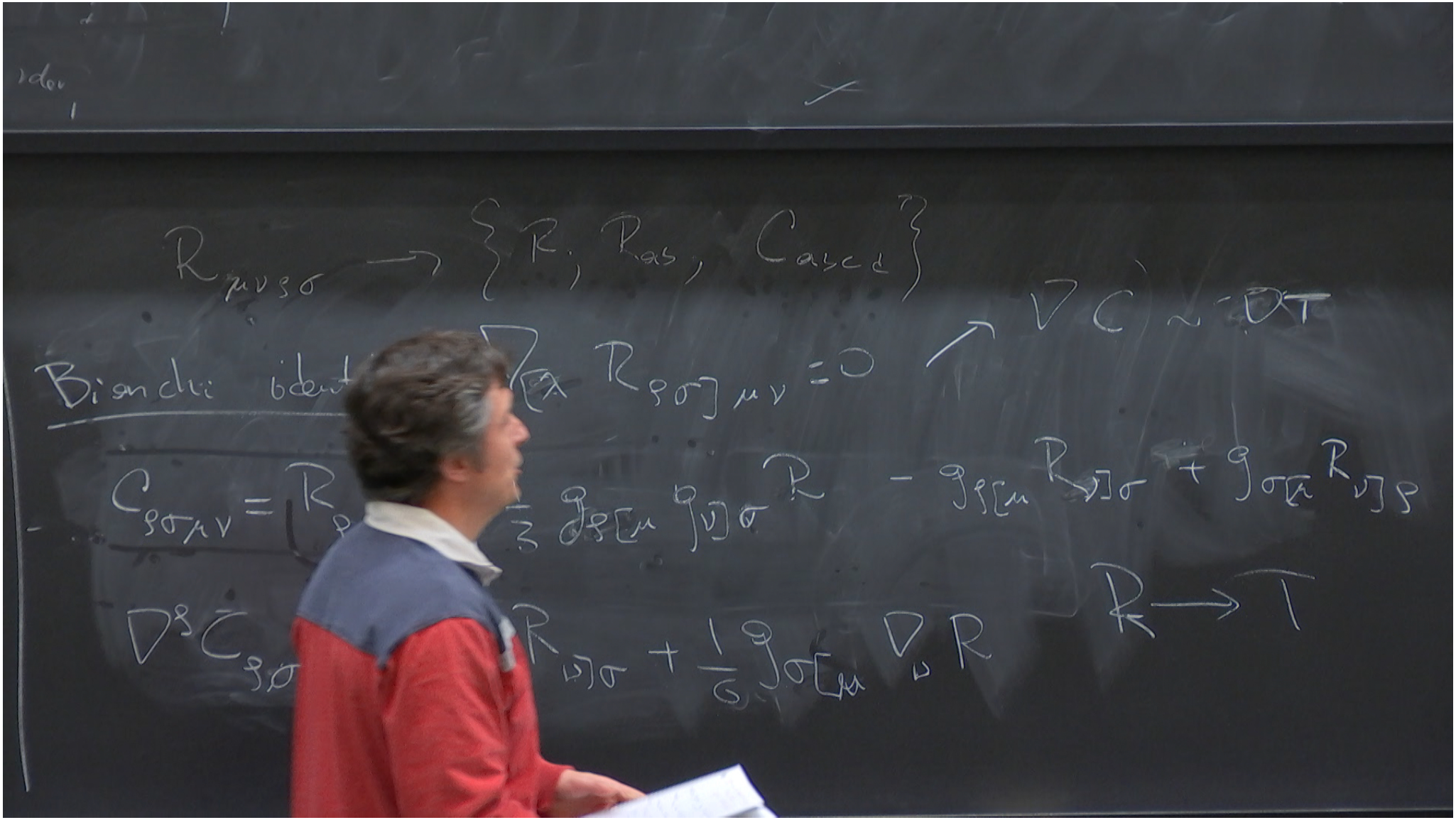

$R_{\mu\nu\rho\sigma} \longrightarrow \{R, R_{ab}, C_{abcd}\}$
$\nabla_{[\lambda} R_{\rho\sigma]\mu\nu} = 0$
$+ \frac{1}{3} g_{\rho[\mu} g_{\nu]\sigma} R - g_{\rho[\mu} R_{\nu]\sigma} + g_{\sigma[\mu} R_{\nu]\rho}$
$\nabla^\rho C_{\rho\sigma\mu\nu} = \nabla_{[\mu} R_{\nu]\sigma} + \frac{1}{6} g_{\sigma[\mu} \nabla_{\nu]} R$
$R \longrightarrow T$

$$R_{\mu\nu\rho\sigma} \longrightarrow \{R;\ R_{ab},\ C_{abcd}\}$$

Bianchi identity $\nabla_{[\lambda} R_{\rho\sigma]\mu\nu} = 0$ $\longrightarrow \nabla C \sim T$

$$C_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} + \frac{1}{3} g_{\rho[\mu} g_{\nu]\sigma} R - g_{\rho[\mu} R_{\nu]\sigma} + g_{\sigma[\mu} R_{\nu]\rho}$$

$$\nabla^\rho C_{\rho\sigma\mu\nu} = \nabla_{[\mu} R_{\nu]\sigma} + \frac{1}{6} g_{\sigma[\mu} \nabla_{\nu]} R \qquad R \longrightarrow T$$


$R_{\mu\nu\rho\sigma} \rightarrow \{R, R_{ab}, C_{abcd}\}$
Bianchi ident
$\nabla_\lambda R_{\rho\sigma]\mu\nu} = 0$
$\nabla C \sim \partial T$
$C_{\rho\sigma\mu\nu} = R_{\rho}$
$\frac{1}{3} g_{\rho[\mu} g_{\nu]\sigma} R$
$- g_{\rho[\mu} R_{\nu]\sigma} + g_{\sigma[\mu} R_{\nu]\rho}$
$D^\rho C_{\rho\sigma}$
$R_{\nu]\sigma} + \frac{1}{6} g_{\sigma[\mu} D_{\nu]} R$
$R \rightarrow T$