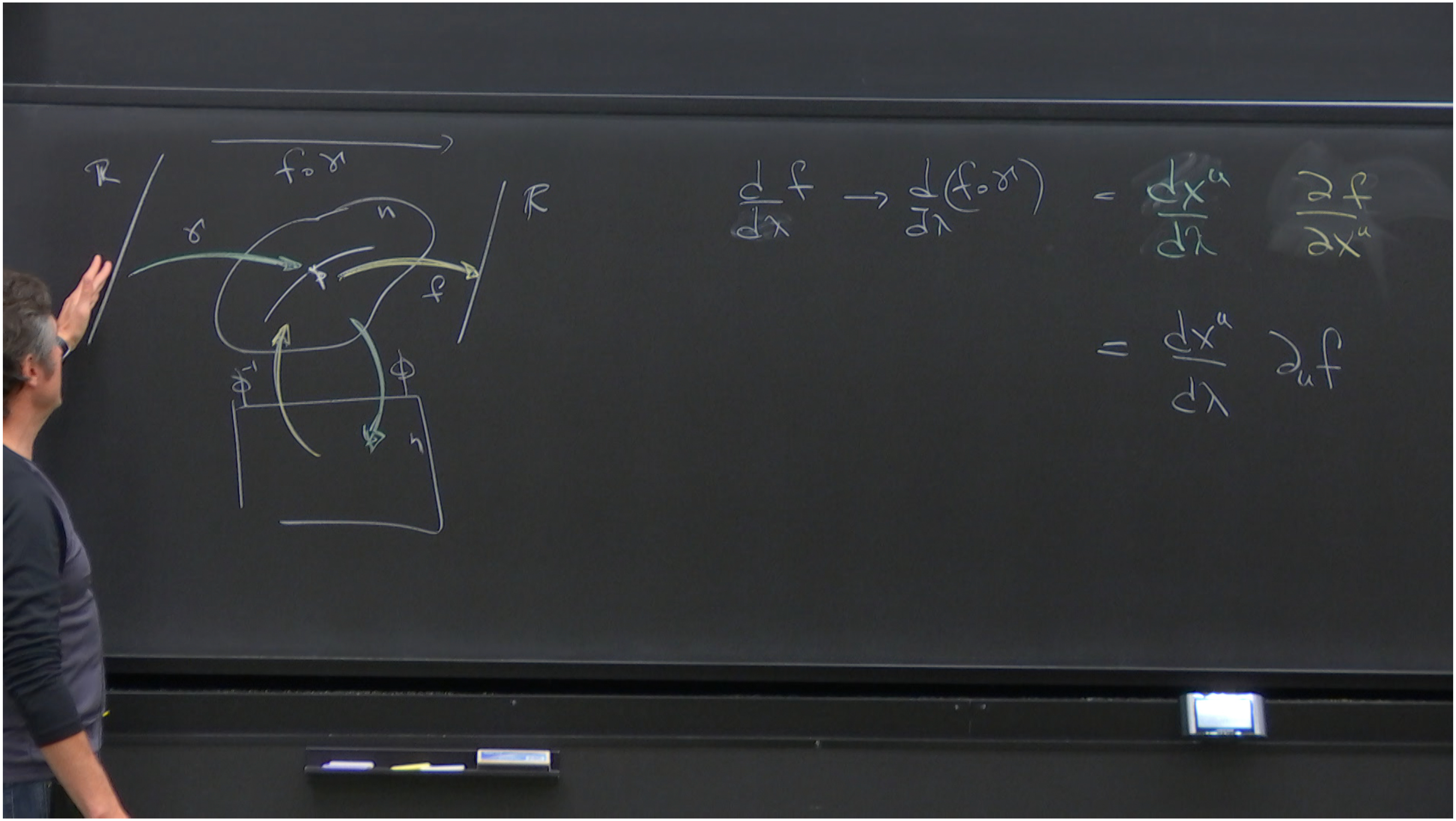
Title: Relativity - Lecture 221012

Speakers:

Collection: Relativity (2022/2023)

Date: October 12, 2022 - 9:00 AM

URL: https://pirsa.org/22100076



$f \circ \gamma$
$\mathbb{R}$
γ
f
ϕ^{-1}
ϕ
$\frac{d}{dx} f \longrightarrow \frac{d}{d\lambda}(f \circ \gamma) = \frac{dx^{\mu}}{d\lambda} \frac{\partial f}{\partial x^{\mu}}$
$= \frac{dx^{\mu}}{d\lambda} \partial_{\mu} f$

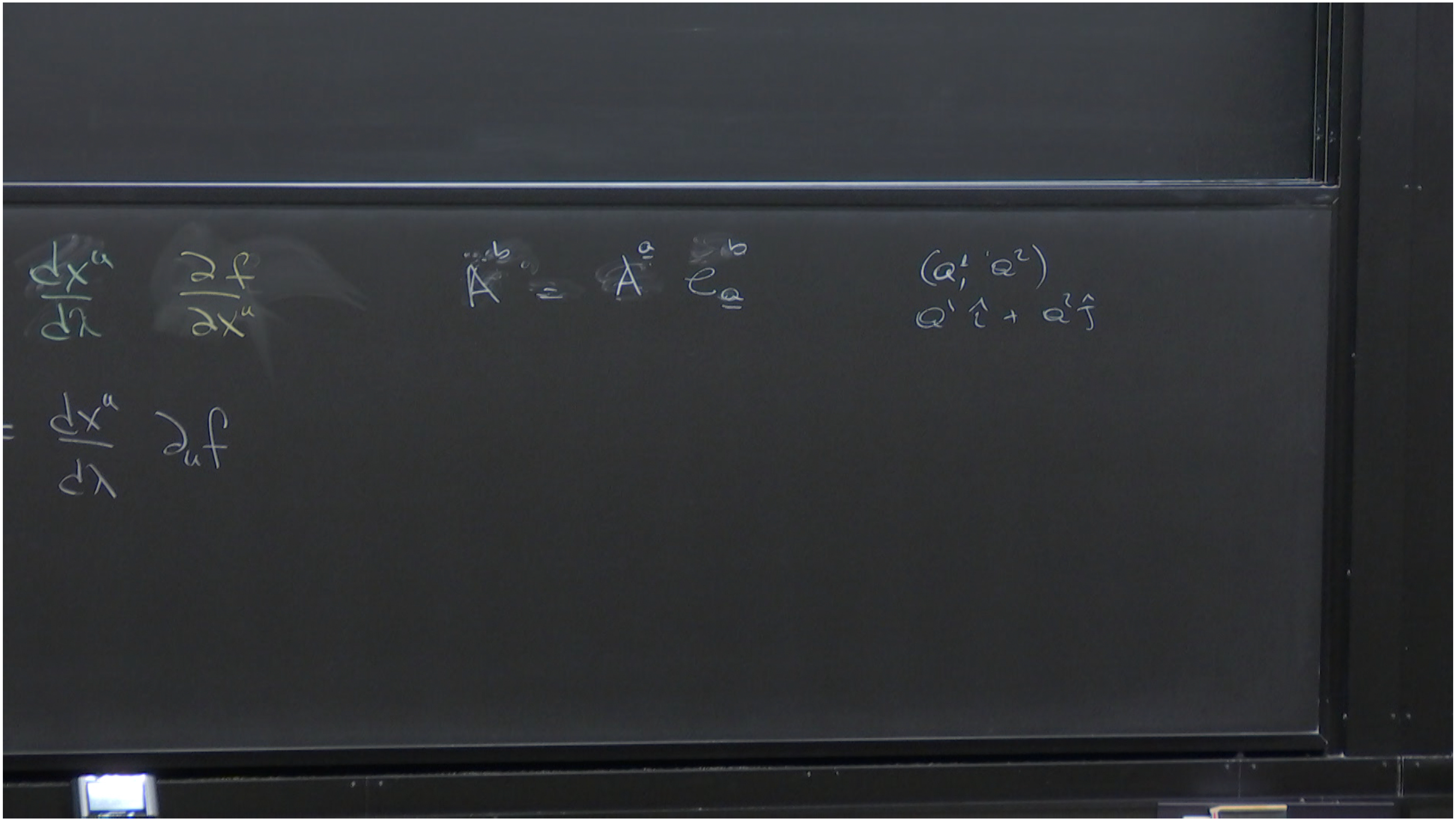
$$\frac{dx^\mu}{d\lambda} \frac{\partial f}{\partial x^\mu}$$

$$= \frac{dx^\mu}{d\lambda} \partial_\mu f$$

$$A = a^i \hat{e}_i$$

$$(a^1, a^2)$$

$$a^1 \hat{i} + a^2 \hat{j}$$


$\frac{dx^a}{d\lambda} \frac{\partial f}{\partial x^a}$
$= \frac{dx^a}{d\lambda} \partial_a f$
$A^b = A^a e_a{}^b$
(a^1, a^2)
$a^1 \hat{i} + a^2 \hat{j}$

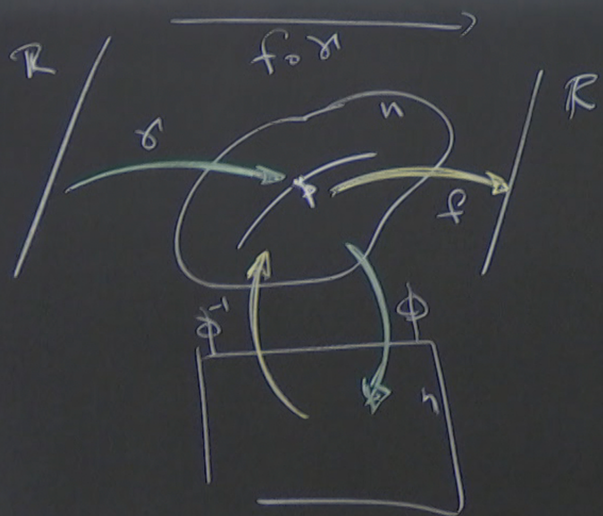
$$\frac{d}{d\lambda} f \longrightarrow \frac{d}{d\lambda}(f \circ \gamma) = \frac{dx^a}{d\lambda} \frac{\partial f}{\partial x^a}$$

$$A^{b} = A^{a} e_{a}^{\ b}$$

$$(a^1, a^2)$$

$$a^1 \hat{i} + a^2 \hat{j}$$

$$\frac{d}{d\lambda} = \frac{dx^a}{d\lambda} \partial_a$$



Tangent space $\quad \frac{d}{d\lambda} f \longrightarrow \frac{d}{d\lambda}(f\circ\gamma) = \frac{dx^\mu}{d\lambda}\frac{\partial f}{\partial x^\mu}$ $\qquad A^{\cdots b}$

T_p

$$\frac{d}{d\lambda} = \underbrace{\frac{dx^\mu}{d\lambda}}\,\partial_\mu$$

Cotangent space $^*T_p \qquad \omega: T_p \longrightarrow \mathbb{R}$

$$\frac{dx^a}{d\lambda} \quad \frac{\partial f}{\partial x^a}$$

$$\frac{dx^a}{d\lambda} \quad \partial_a$$

$$\rightarrow \mathbb{R}$$

$$A^{b} = A^{a} \, e_{a}^{\ b}$$

$$(a^1, a^2)$$

$$a^1 \hat{\imath} + a^2 \hat{\jmath}$$

Notation

$$V_b = v_a \, e^{a}_{\ b}$$

$\frac{dx^a}{d\lambda} \quad \frac{\partial f}{\partial x^a}$

$\frac{dx^a}{d\lambda} \, \partial_a$

$\rightarrow \mathbb{R}$

$A^b = A^a \, e_a{}^b$

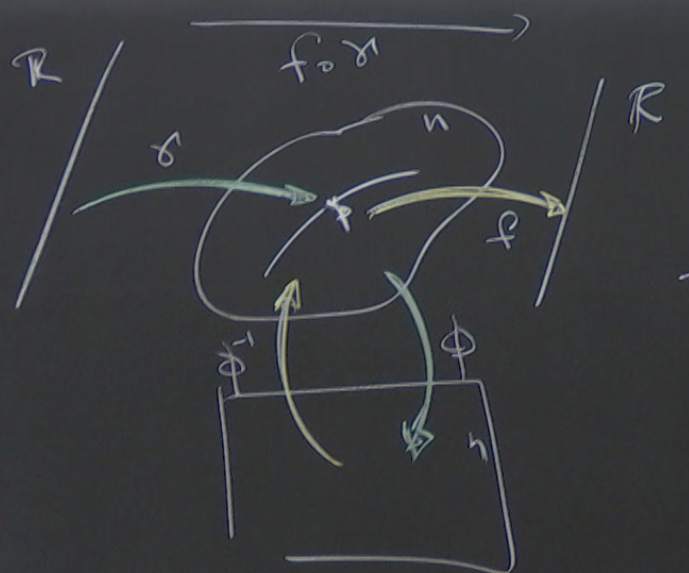
Notation

$V_b = v_a \, e^a{}_b$

(a^1, a^2)

$a^1 \hat{i} + a^2 \hat{j}$

$e^b \cdot e_c = \delta^b{}_c$



Tangent space

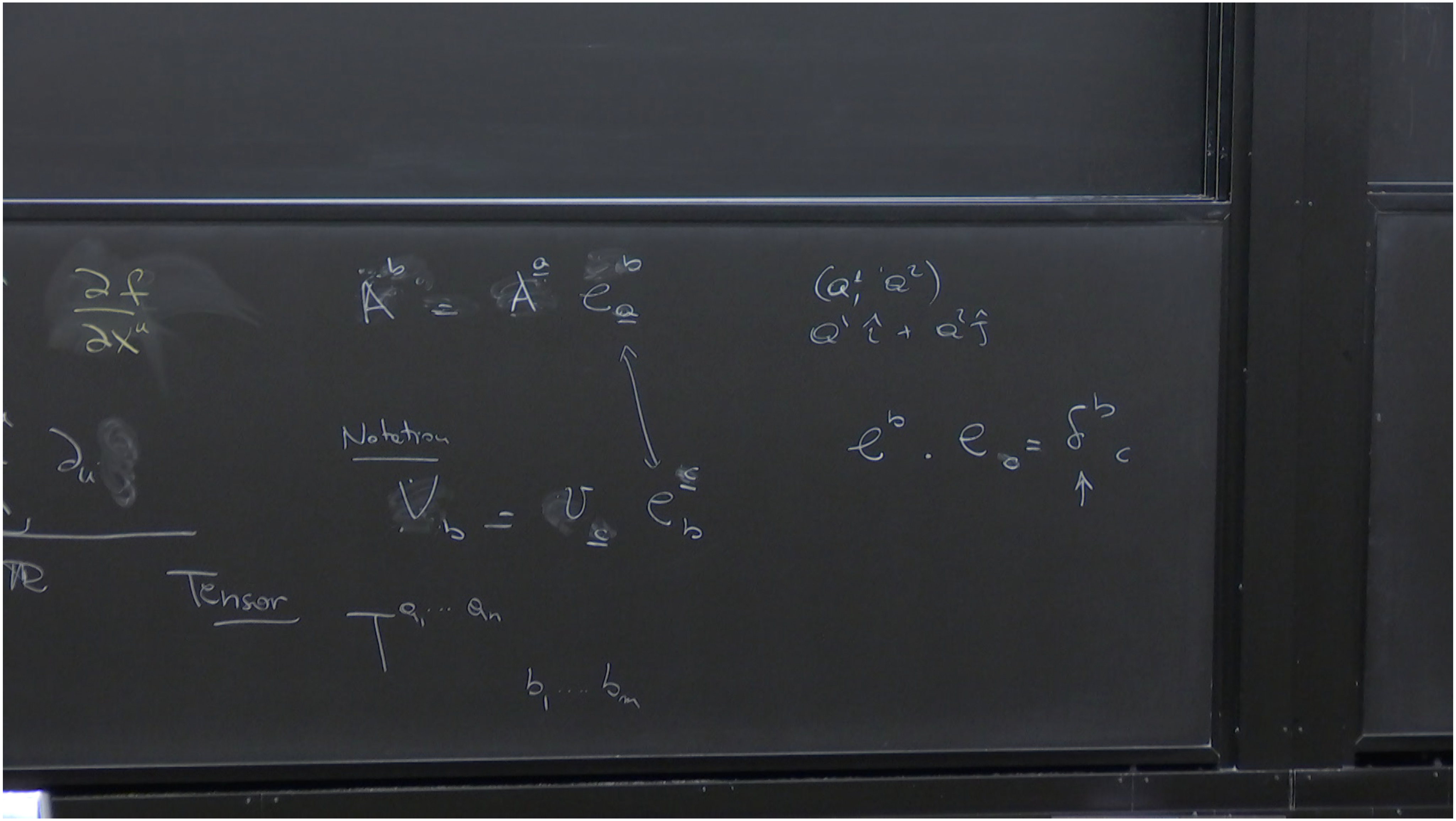
T_p

$T_p \times \bar{T}_p \times \bar{T}_p$

$$\frac{d}{dx} f \longrightarrow \frac{d}{d\lambda}(f\circ\gamma) = \frac{dx^\mu}{d\lambda}\frac{\partial f}{\partial x^\mu}$$

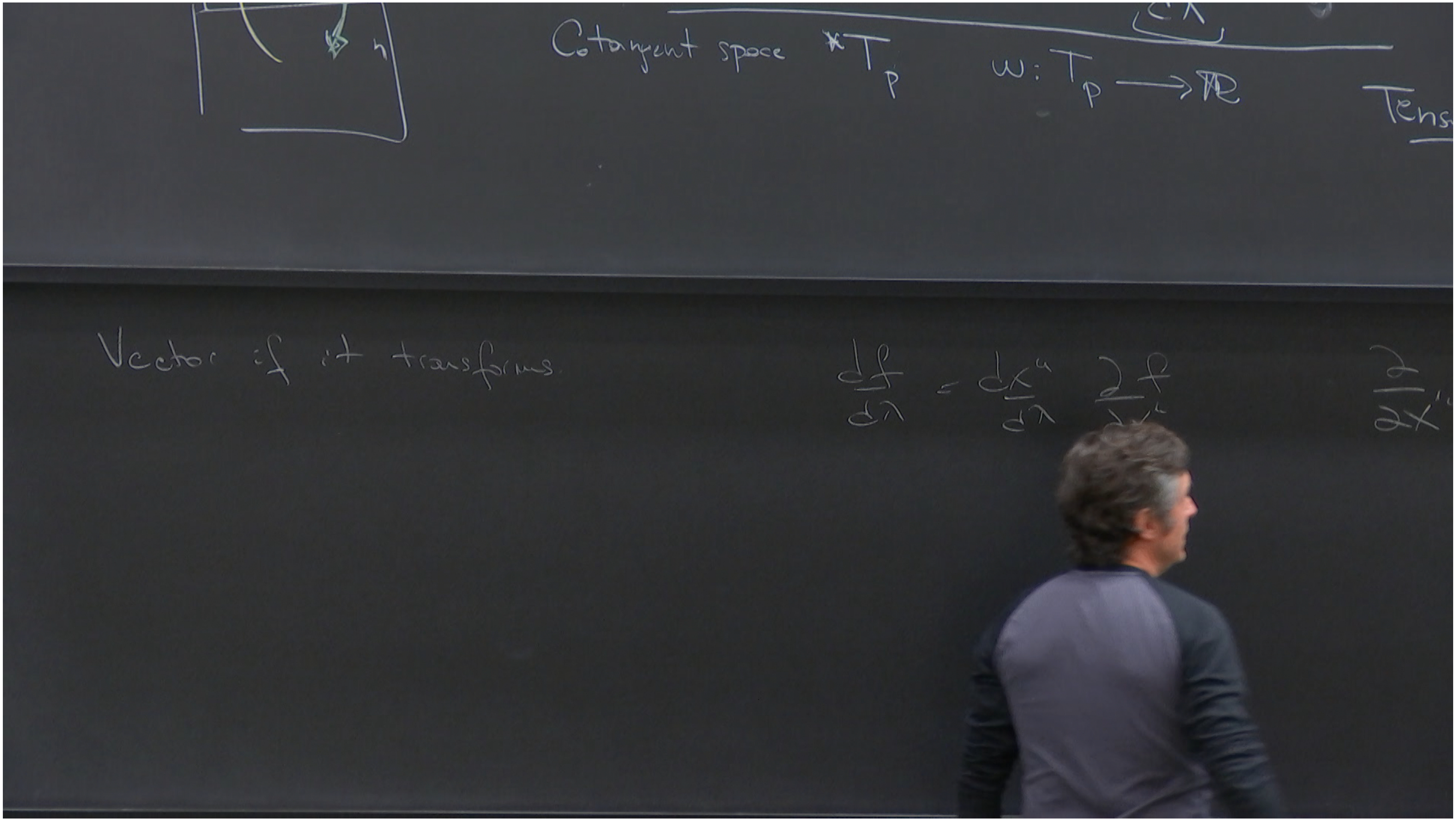
$$\frac{d}{d\lambda} = \frac{dx^\mu}{d\lambda}\partial_\mu$$

Cotangent space $^*T_p$ $\quad \omega : T_p \longrightarrow \mathbb{R}$


$\frac{\partial f}{\partial x^\mu}$
$A^b = A^a e_a{}^b$
(a^1, a^2)
$a^1 \hat{i} + a^2 \hat{j}$
Notation
∂_μ
$V_b = V_c e^c{}_b$
$e^b \cdot e_c = \delta^b{}_c$
Tensor
$T^{a_1 \cdots a_n}{}_{b_1 \cdots b_m}$

Tensor

$$T^{a_1 \cdots a_n}{}_{b_1 \cdots b_m} \qquad (n, m)$$

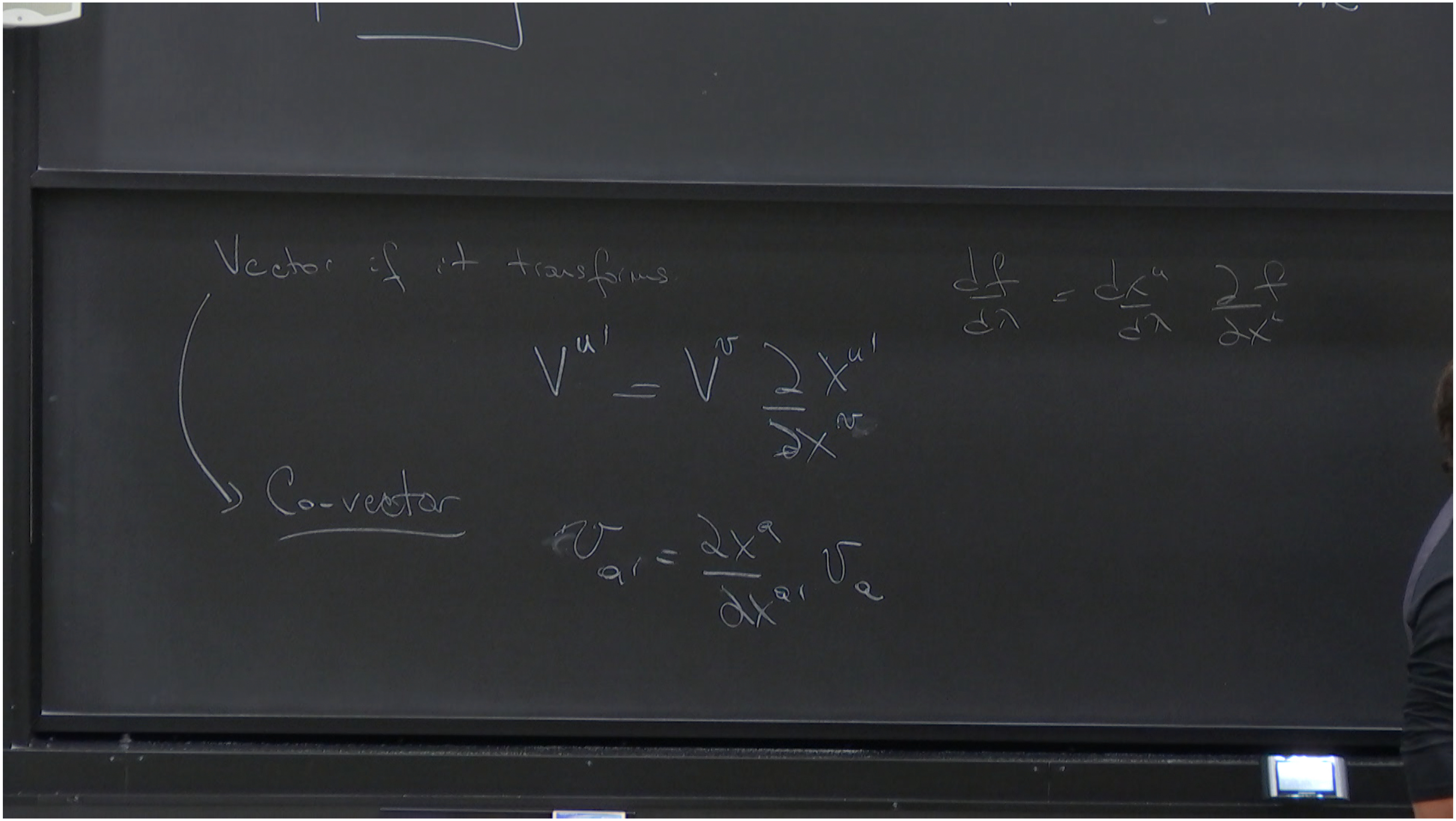

Cotangent space *T_p
ω: T_p → ℝ
Vector if it transforms.

ngent space $^*T_p$ $\quad \omega : T_p \longrightarrow \mathbb{R}$

$v_b = v^c e_b$

Tensor $\quad T^{a_1 \cdots a_n}{}_{b_1 \cdots b_m} \quad (n, m)$

$$\frac{df}{d\lambda} = \frac{dx^\mu}{d\lambda} \frac{\partial f}{\partial x^\mu} \qquad \frac{\partial}{\partial x'^\mu} = \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu}$$

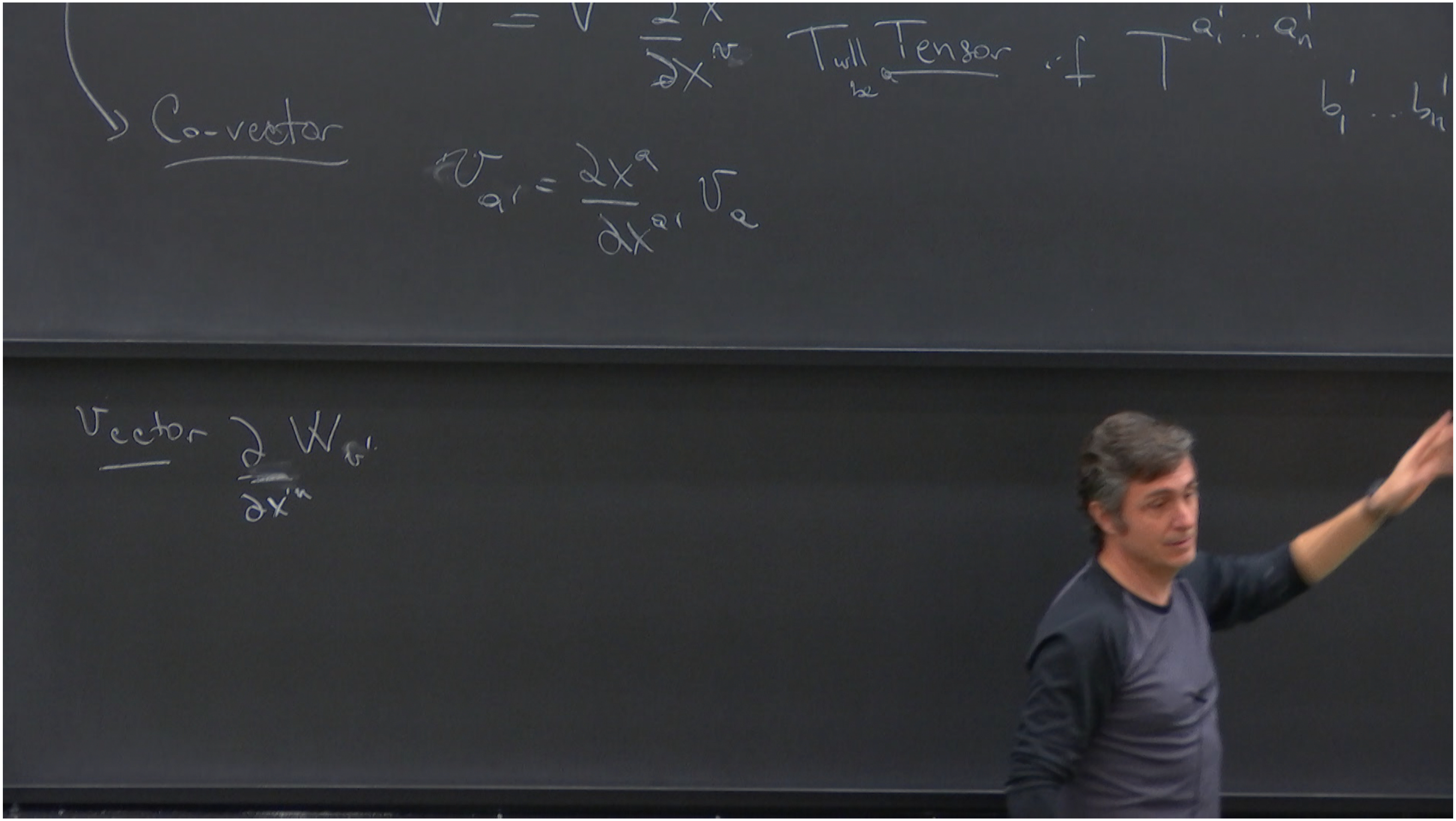

Vector if it transforms
$V^{\mu'} = V^{\nu} \frac{\partial x^{\mu'}}{\partial x^{\nu}}$
Co-vector
$v_{a'} = \frac{\partial x^{a}}{\partial x^{a'}} v_{a}$
$\frac{df}{d\lambda} = \frac{dx^{\mu}}{d\lambda} \frac{\partial f}{\partial x^{\mu}}$

$T_p \qquad \omega : T_p \longrightarrow \mathbb{R}$

Tensor $\quad T^{a_1 \cdots a_n}{}_{b_1 \cdots b_m} \quad (n, m)$

$$\frac{df}{d\lambda} = \frac{dx^{\mu}}{d\lambda} \frac{\partial f}{\partial x^{\mu}} \qquad \frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}}$$

Tu be a Tensor if $T^{a_1' \cdots a_n'}{}_{b_1' \cdots b_n'} = T^{\alpha_1 \cdots \alpha_n}{}_{\beta_1 \cdots \beta_n} \left(\frac{\partial x'^{a_1'}}{\partial x^{\alpha_1}} \cdots \frac{\partial x'^{a_n'}}{\partial x^{\alpha_n}} \right) \left(\frac{\partial x^{\beta_1}}{\partial x'^{b_1}} \cdots \frac{\partial x^{\beta_n}}{\partial x'^{b_n}} \right)$


Co-vector
Tensor
Vector

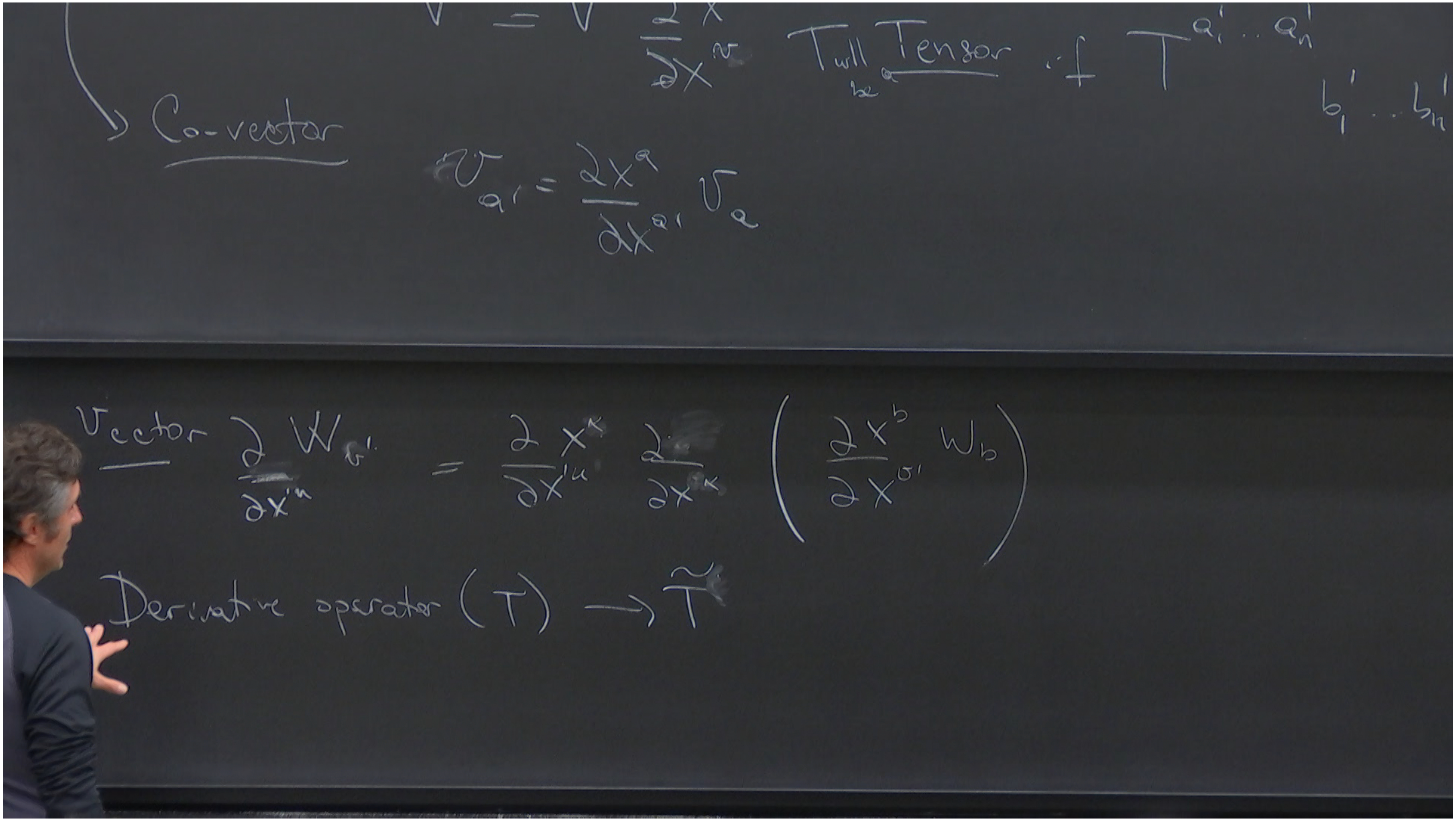
$$\frac{\partial x}{\partial x^{\nu}}$$ Tull Tensor of $T^{a_1' \dots a_n'}_{\quad b_1' \dots b_n'}$

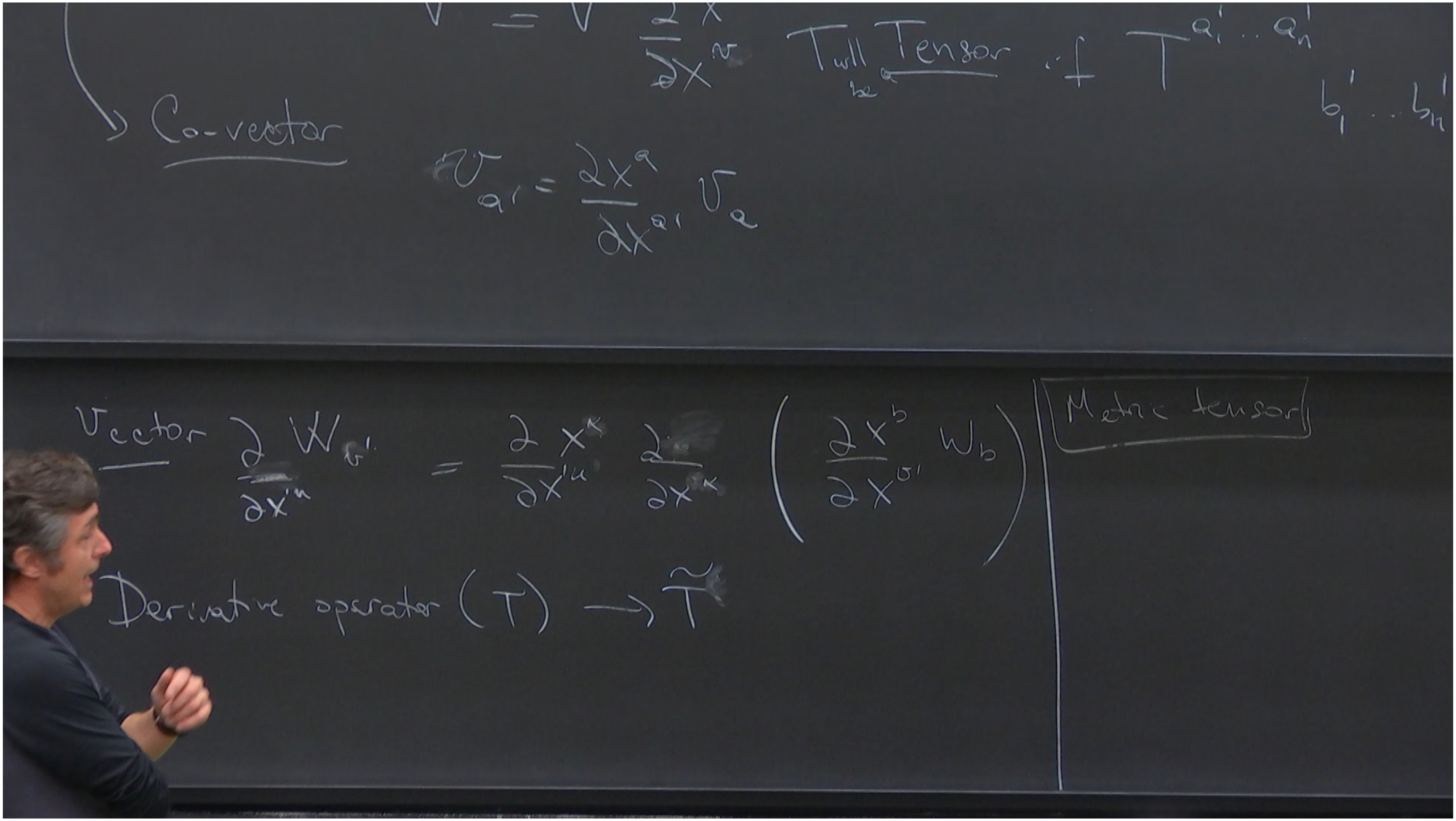
Co-vector

$$v_{a'} = \frac{\partial x^a}{\partial x^{a'}} v_a$$

Vector

$$\frac{\partial W_{b'}}{\partial x'^{\mu}} = \frac{\partial x^{\kappa}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\kappa}} \left(\frac{\partial x^b}{\partial x^{b'}} W_b \right)$$


Tensor of $T^{a_1' \ldots a_n'}_{b_1' \ldots b_n'}$
Co-vector
$v_{a'} = \frac{\partial x^a}{\partial x^{a'}} v_a$
Vector
$\frac{\partial W_{\sigma'}}{\partial x'^{\mu}} = \frac{\partial x^{\kappa}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\kappa}} \left(\frac{\partial x^b}{\partial x^{\sigma'}} W_b \right)$
Derivative operator $(T) \longrightarrow \tilde{T}$


Full Tensor of $T^{a_1' \dots a_n'}{}_{b_1' \dots b_n'}$
Co-vector
$v_{a'} = \frac{\partial x^a}{\partial x^{a'}} v_a$
Vector
Metric tensor
Derivative operator $(T) \longrightarrow \tilde{T}$

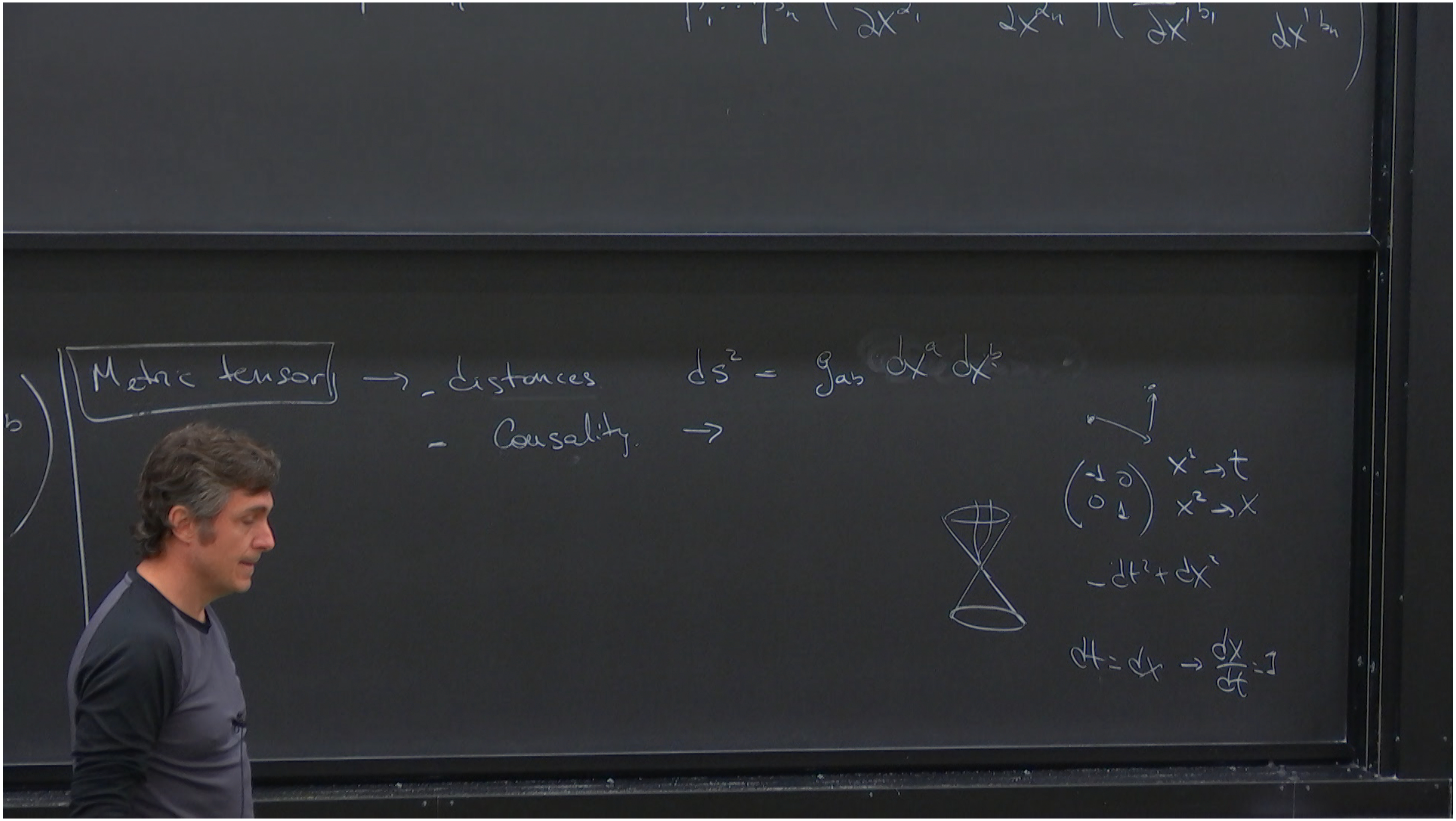
$$\left(\frac{\partial x^{a_1}}{} \cdots dx^{a_n} \right) \left(\frac{}{\partial x'^{b_1}} \cdots dx'^{b_n} \right)$$

$\boxed{\text{Metric tensor}}$ $\rightarrow$ - distances $\quad ds^2 = g_{ab}\, dx^a dx^b$

- Causality $\rightarrow$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{array}{l} x^1 \rightarrow t \\ x^2 \rightarrow x \end{array}$$

$$-dt^2 + dx^2$$


Metric tensor → - distances
$ds^2 = g_{ab}\, dx^a dx^b$
- Causality →
$x^1 \to t$
$x^2 \to x$
$-dt^2 + dx^2$
$dt = dx \to \frac{dx}{dt} = 1$

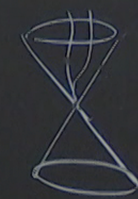
$$\left(\frac{\partial}{\partial x^{a_1}} \cdots \frac{\partial}{\partial x^{a_n}}\right)\left(dx'^{b_1} \cdots dx'^{b_n}\right)$$

pseudo Riemannian

Lorentzian

$\boxed{\text{Metric tensor}}$ → - distances $ds^2 = g_{ab}\, dx^a dx^b$

- Causality →



$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{array}{l} x^1 \to t \\ x^2 \to x \end{array}$$

$$-dt^2 + dx^2$$

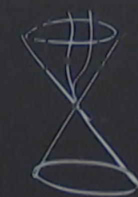
$$dt = dx \to \frac{dx}{dt} = 1$$

$\boxed{\text{Metric tensor}}$ → - distances $ds^2 = g_{ab}\, dx^a dx^b$

- Causality →

- extrema of distances

→ replace grav. field



pseudo Riemannian

Lorentzian

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $x^1 \to t$, $x^2 \to x$

$-dt^2 + dx^2$

$dt = dx \to \frac{dx}{dt} = 1$

$$\partial x^{a_1} \quad dx^{a_n} \Big) \Big(\partial x'^{b_1} \quad dx'^{b_n} \Big)$$

$\boxed{\text{Metric tensor}}$ → - distances $\quad ds^2 = g_{ab}\, dx^a dx^b$

$\omega_a(v^a) \to \mathbb{R}$

- Causality →
- extrema of distances

→ replace grav. field

→ "dot products"

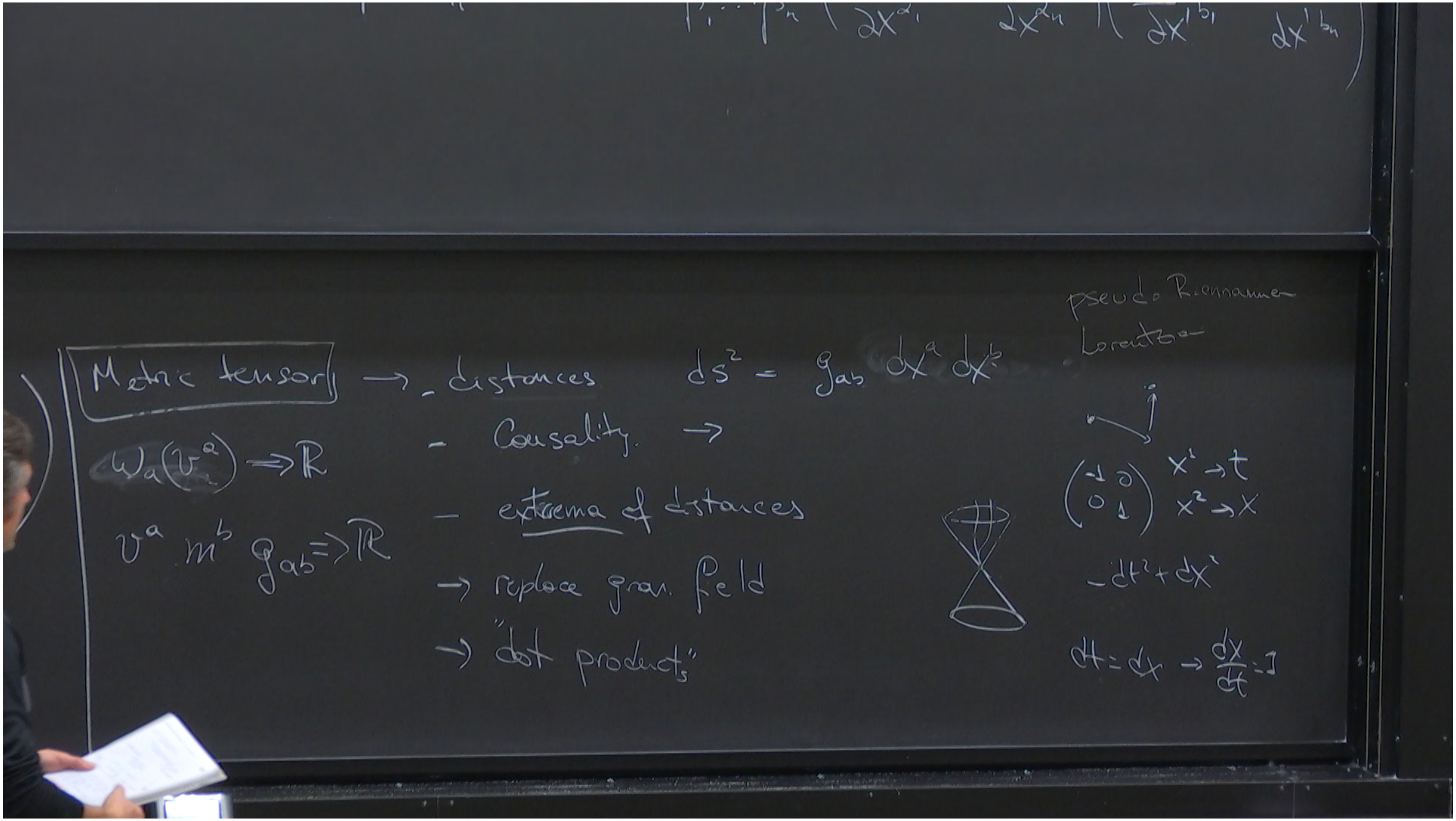
pseudo Riemannian

Lorentzian

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad x^1 \to t, \quad x^2 \to x$

$-dt^2 + dx^2$

$dt = dx \to \frac{dx}{dt} = 1$


Metric tensor → - distances
$ds^2 = g_{ab} dx^a dx^b$
- Causality →
- extrema of distances
→ replace grav. field
→ "dot products"
$w_a(v^a) \Rightarrow \mathbb{R}$
$v^a m^b g_{ab} \Rightarrow \mathbb{R}$
pseudo Riemannian
Lorentzian
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
$x^1 \to t$
$x^2 \to x$
$-dt^2 + dx^2$
$dt = dx \to \frac{dx}{dt} = 1$

$\ldots \left(\frac{\partial x^{a_1}}{\ldots} \ldots dx^{a_n} \right) \left(\frac{\ldots}{\partial x'^{b_1}} \ldots dx'^{b_n} \right)$

pseudo Riemannian

Lorentzian

$\boxed{\text{Metric tensor}}$ → - distances $ds^2 = g_{ab}\, dx^a dx^b$

- Causality →

- extrema of distances

→ replace grav. field

→ "dot products"

$W_a(V^a) \Rightarrow \mathbb{R}$

$V^a m^b g_{ab} \Rightarrow \mathbb{R}$

Map $T_p \rightleftarrows T^*_p$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $x^1 \to t$, $x^2 \to x$

$-dt^2 + dx^2$

$dt = dx \to \frac{dx}{dt} = 1$

$$v_{a'} = \frac{\partial x^a}{\partial x^{a'}} v_a$$

$$W_{\mu'} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha} \left(\frac{\partial x^b}{\partial x^{\sigma'}} W_b \right)$$

operator $(T) \longrightarrow \tilde{T}$

$v^a \to v_a = v^a g_{ab}$

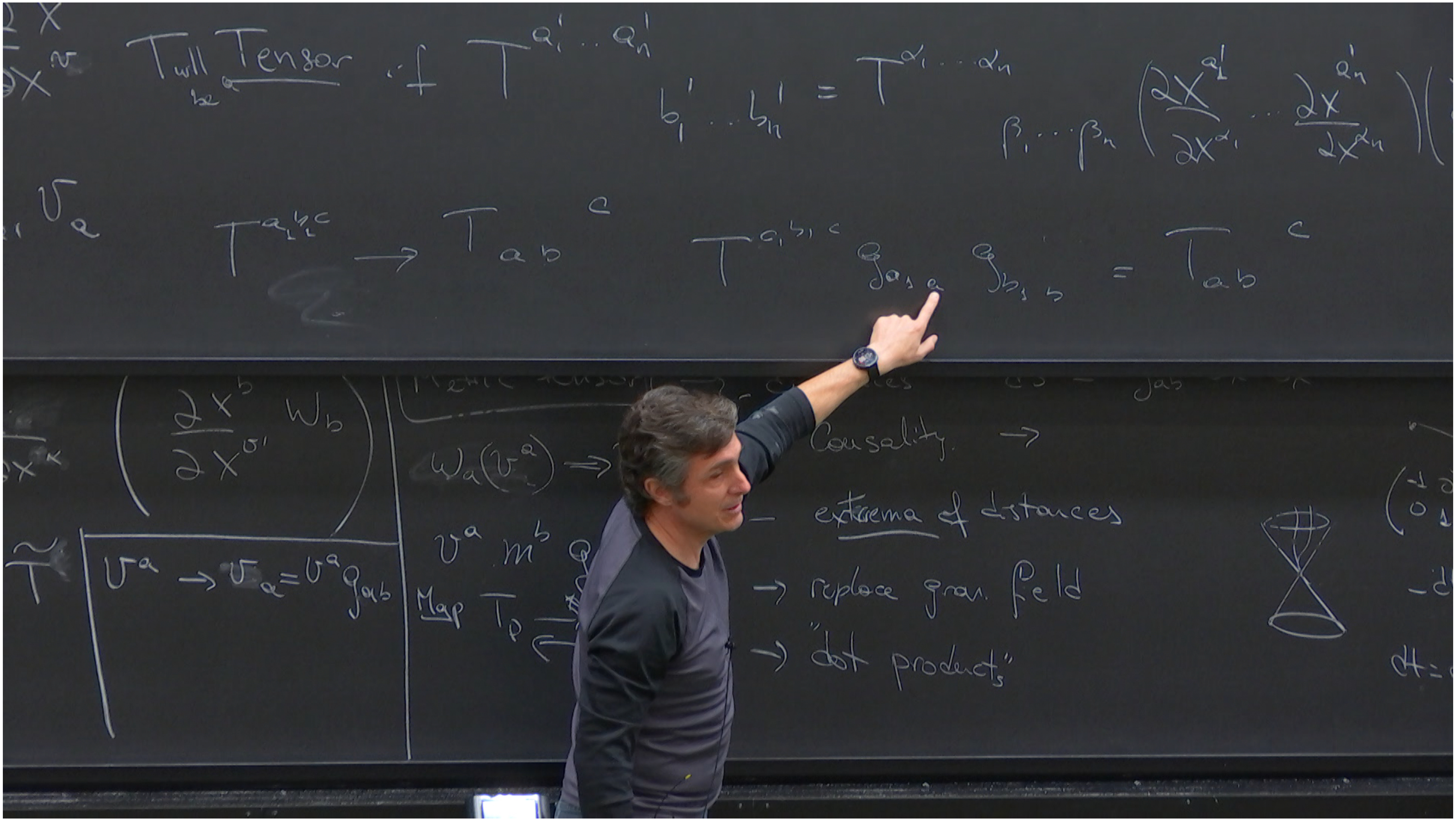
$\boxed{\text{Metric tensor}}$ $\longrightarrow$ - distances ds^2

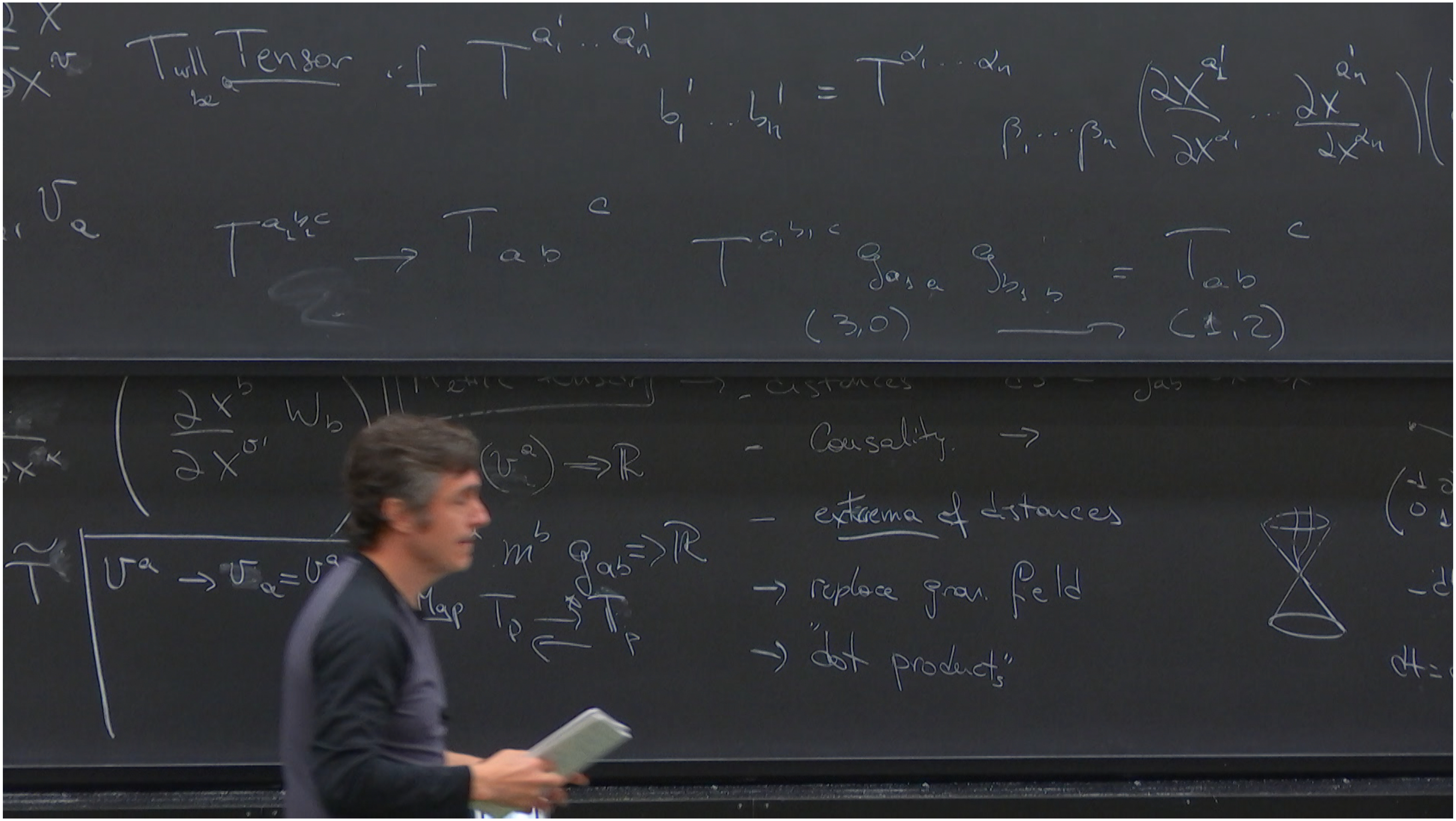
- Causality $\to$
- extrema of distance
- $\to$ replace grav. field
- $\to$ "dot products"

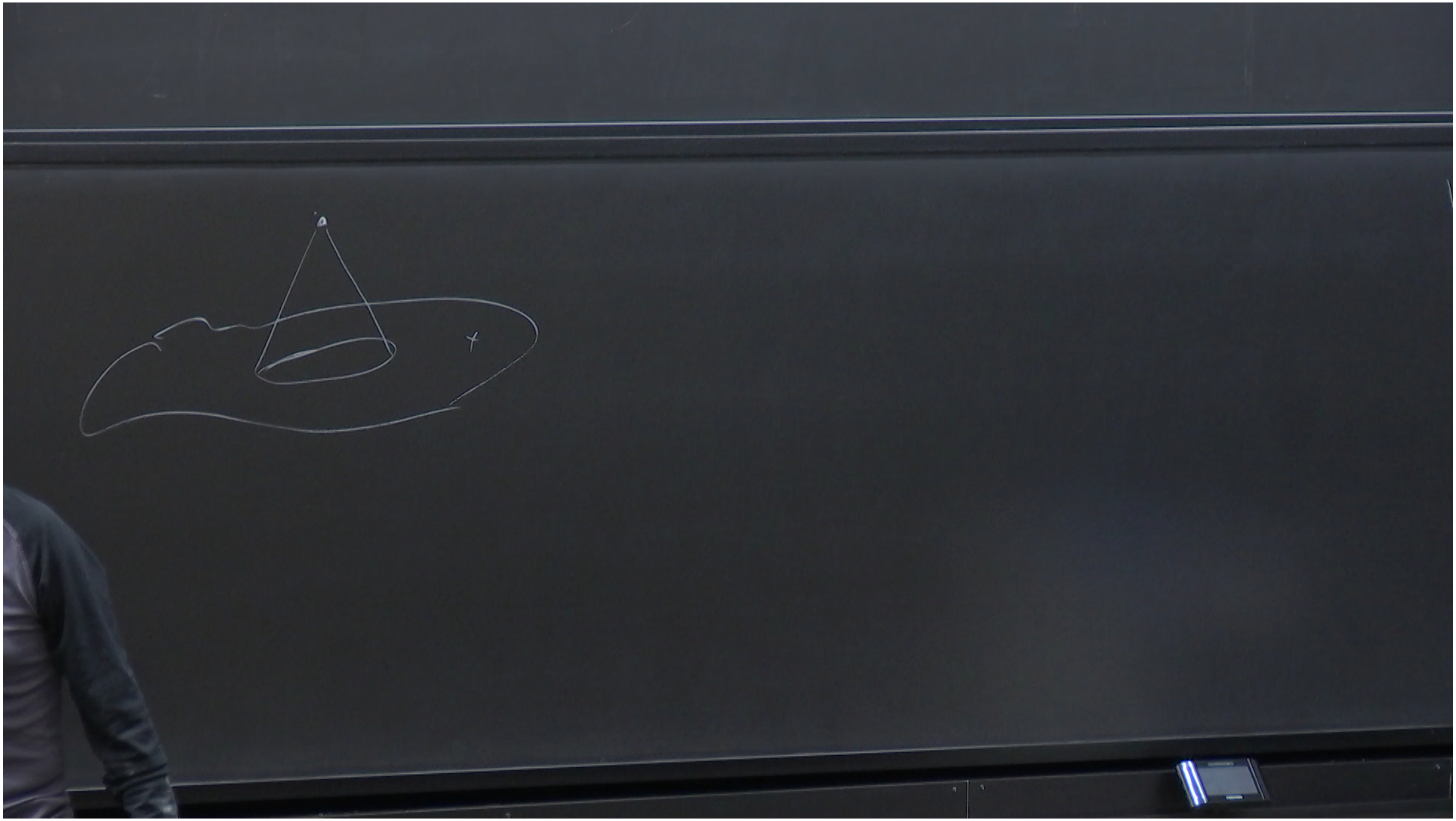
$W_a(v^a) \Rightarrow \mathbb{R}$

$v^a\, m^b\, g_{ab} \Rightarrow \mathbb{R}$

Map $T_p \rightleftarrows T^*_p$


Full Tensor
Causality
extrema of distances
replace grav. field
"dot products"
Map


Tensor
Causality
extrema of distances
replace grav. field
"dot products"
(3,0)
(1,2)
Map



$\boxed{\text{Curvature}}$ " $\frac{\partial}{\partial x^a}$ " not good for "staying within tensors"

…nsors" . Connection ; Christoffel symbols not good for "staying within tensors" . Connection ; Chr

$$\Gamma_{ab}{}^{c} = \frac{1}{2} g^{cd} \left(g_{bd,a} + g_{ad,b} - g_{ab,d} \right)$$

$\boxed{\text{...ature}}$ "$\dfrac{\partial}{\partial x^a}$"

not good for "staying within tensors".

$$\Gamma_{ab}{}^{c} = \frac{1}{2} g^{cd} \left(g_{bd,a} + g_{ad,b} - g_{ab\ldots} \right.$$

if acting on a vector.

$$\nabla_a v^b = \partial_a v^b$$

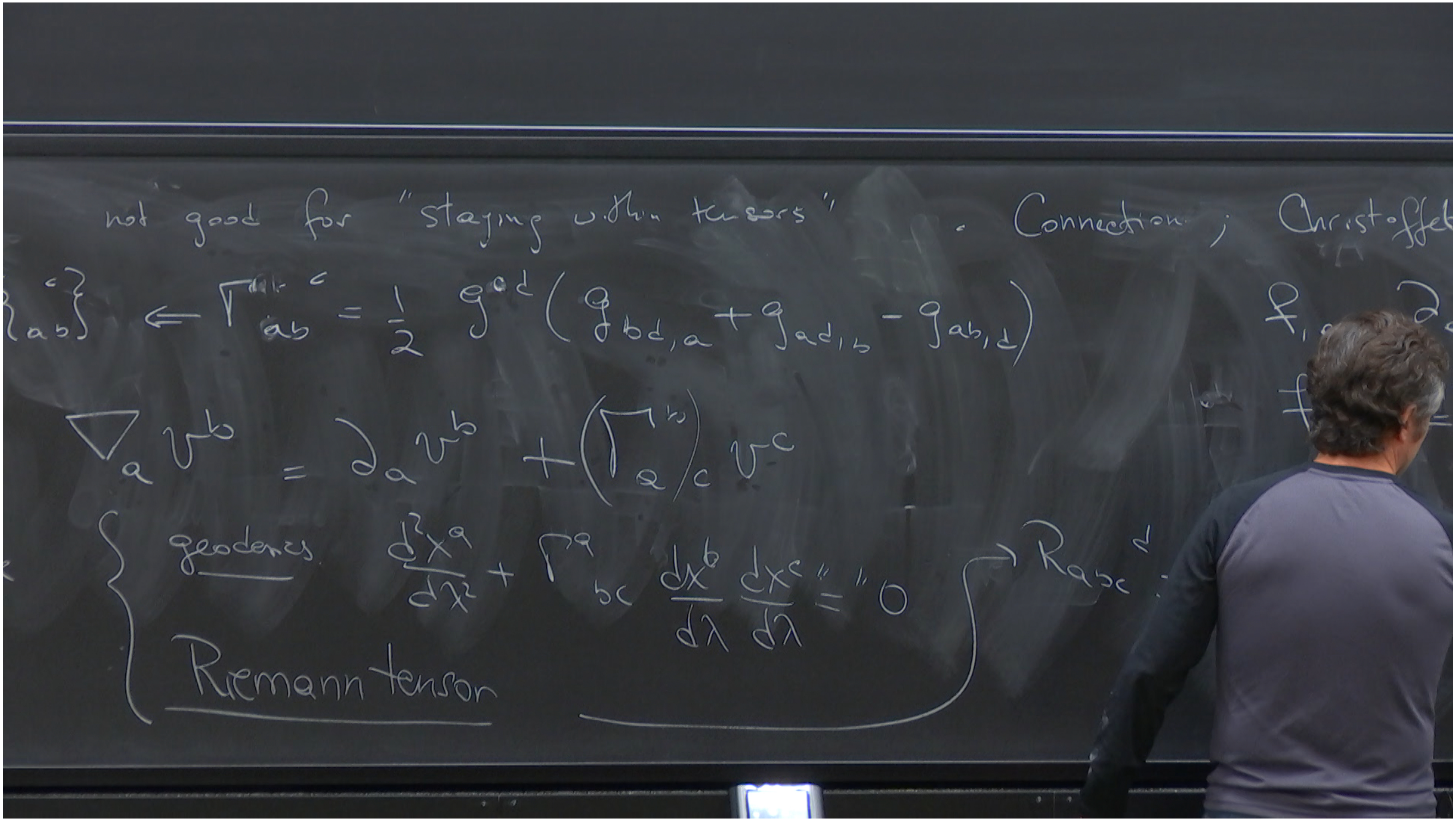
Covariant derivative

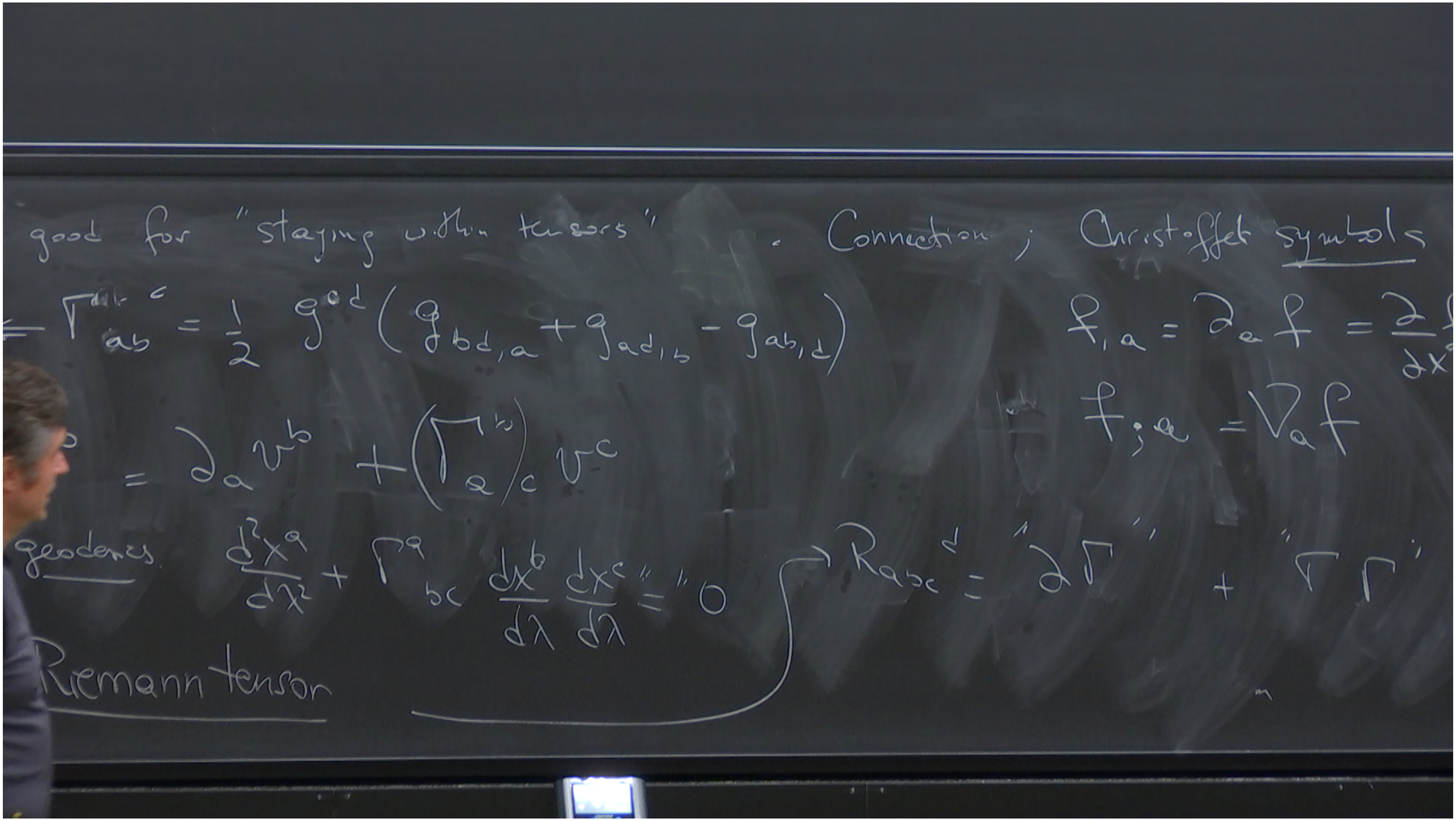
s" · Connection ; Christoffel symbols

$$\ldots{}_{ad,b} - g_{ab,d})$$

$$f_{,a} = \partial_a f = \frac{\partial f}{\partial x^a}$$

$$f_{;a} = \nabla_a f \qquad f_{||a} = \nabla_a f$$


not good for "staying within tensors"
Connection ; Christoffel
$\{^{c}_{ab}\} \Leftarrow \Gamma_{ab}{}^{c} = \frac{1}{2} g^{cd}(g_{bd,a} + g_{ad,b} - g_{ab,d})$
$\nabla_a v^b = \partial_a v^b + (\Gamma_a{}^b)_c v^c$
geodesics $\frac{d^2x^a}{d\lambda^2} + \Gamma^a{}_{bc} \frac{dx^b}{d\lambda}\frac{dx^c}{d\lambda} \text{"="} 0$
Riemann tensor
$R_{abc}{}^{d}$


good for "staying within tensors"
Connection ; Christoffel symbols
$\Gamma_{ab}{}^{c} = \frac{1}{2} g^{cd}(g_{bd,a} + g_{ad,b} - g_{ab,d})$
$f_{,a} = \partial_a f = \frac{\partial f}{\partial x^a}$
$f_{;a} = \nabla_a f$
$= \partial_a v^b + (\Gamma_a{}^b{}_c) v^c$
geodesics $\frac{d^2x^a}{d\lambda^2} + \Gamma^a{}_{bc} \frac{dx^b}{d\lambda}\frac{dx^c}{d\lambda} = 0$
$R_{abc}{}^{d} = \text{"}\partial\Gamma\text{"} + \text{"}\Gamma\Gamma\text{"}$
Riemann tensor

Riemann Tensor

Covariant derivative, (∇_a)

0) $\nabla_a f = \partial_a(f)$

1) Linearity. $\nabla_a(F+G) = (\nabla_a F) + (\nabla_a G)$

2) Leibnitz $\nabla_a(F \cdot G) = (\nabla_a F)G + F(\nabla_a G)$

$(U^a W_e)$

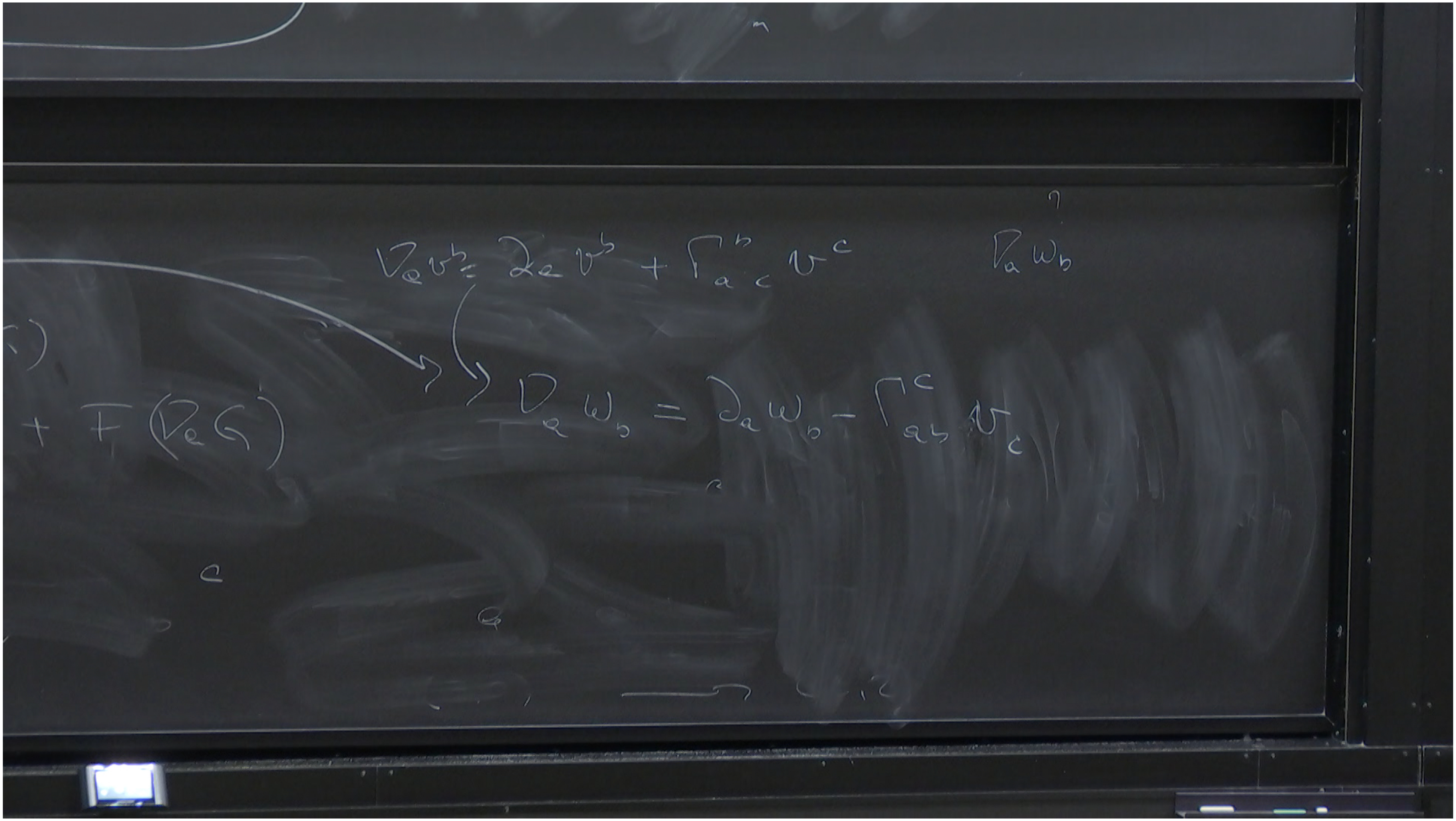
$$\nabla_a v^b + \Gamma^b{}_{ac} v^c \qquad \nabla_a w_b$$

$$+ F(\nabla_a G)$$

$$\nabla_a v^b + \Gamma^b_{ac} v^c \qquad \nabla_a \omega_b$$

$$+ F(\nabla_a G)$$

$$\nabla_a \omega_b = \partial_a \omega_b - \Gamma^c_{ab} v_c$$


$\nabla_a v^b = \partial_a v^b + \Gamma^b_{ac} v^c$
$\nabla_a \omega_b$?
$+ F(\nabla_a G)$
$\nabla_a \omega_b = \partial_a \omega_b - \Gamma^c_{ab} v_c$

Riemann Tensor

Covariant derivative (∇_a)

0) $\nabla_a f = \partial_a(f)$

1) Linearity. $\nabla_a(F + G) = (\nabla_a F) + (\nabla_a G)$

2) Leibnitz $\nabla_a(F \times G) = (\nabla_a F) G + F (\nabla_a G)$

$(U^a W_a)$

Restriction

$\boxed{3}$ "Torsion free" $\longrightarrow \Gamma^c_{ab} = \Gamma^c_{ba}$

$$\nabla_a v^b = \partial_a v^b + \Gamma^b_{ac} v^c \qquad \nabla_a \omega_b$$

Ex)

$$\nabla_a \omega_b = \partial_a \omega_b - \Gamma^c_{ab} \omega_c$$

$+ F(\nabla_a G)$

4 $\nabla_a g_{cd} = 0$

Ex 2 Show that $\nabla_a v^a = \frac{1}{\sqrt{|g|}} \partial_a \left(\sqrt{|g|}\, v^a \right)$

Ex 2 Show that $\nabla_a v^a = \frac{1}{\sqrt{|g|}} \partial_a\left(\sqrt{|g|}\, v^a\right)$

$\Gamma^{a}{}_{a\lambda}$

Ex 2 Show that $\nabla_a v^a = \frac{1}{\sqrt{|g|}} \partial_a\left(\sqrt{|g|}\, v^a\right)$

$$\Gamma^a{}_{a\lambda} = \frac{1}{\sqrt{|g|}} \partial_\lambda\left(\sqrt{|g|}\right)$$

Jacobi's exp.

$$\delta(\det g) = (\det g)\, g^{ab}\, \delta g_{ab}$$

(Ex 2) Show that $\nabla_a v^a = \frac{1}{\sqrt{|g|}} \partial_a \left(\sqrt{|g|}\, v^a \right)$

$$\Gamma^a{}_{a\lambda} = \frac{1}{\sqrt{|g|}} \partial_\lambda \left(\sqrt{|g|} \right)$$

Jacobi's exp.

$$\delta(\det g) = (\det g)\, g^{ab}\, \delta g_{ab}$$

$$g_{ab}\; g^{bd} = \delta_a{}^b$$

(Ex 2) Show that $\nabla_a v^a = \frac{1}{\sqrt{|g|}} \partial_a \left(\sqrt{|g|}\, v^a \right)$

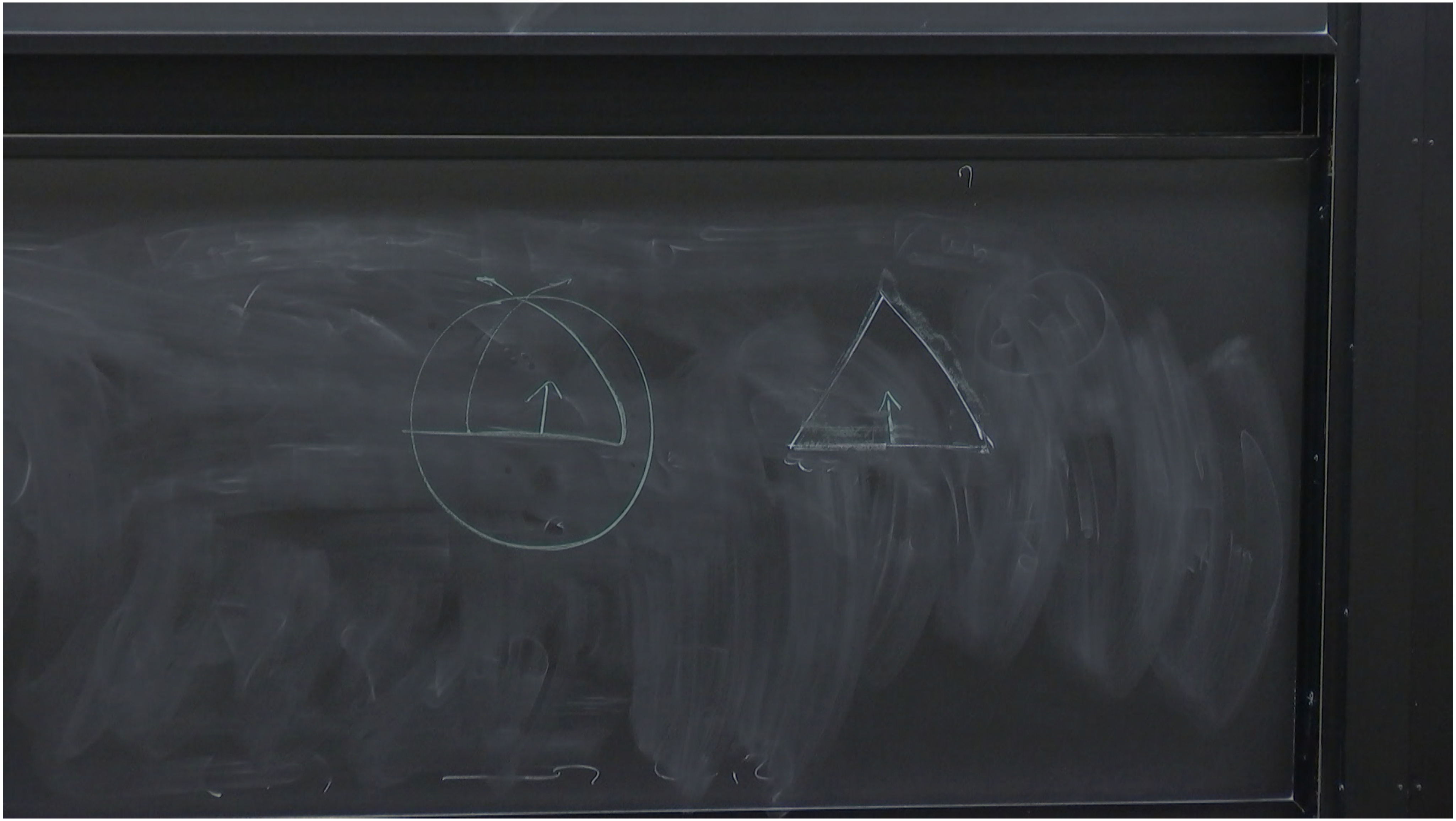
$$\Gamma^a{}_{a\lambda} = \frac{1}{\sqrt{|g|}} \partial_\lambda \left(\sqrt{|g|} \right)$$

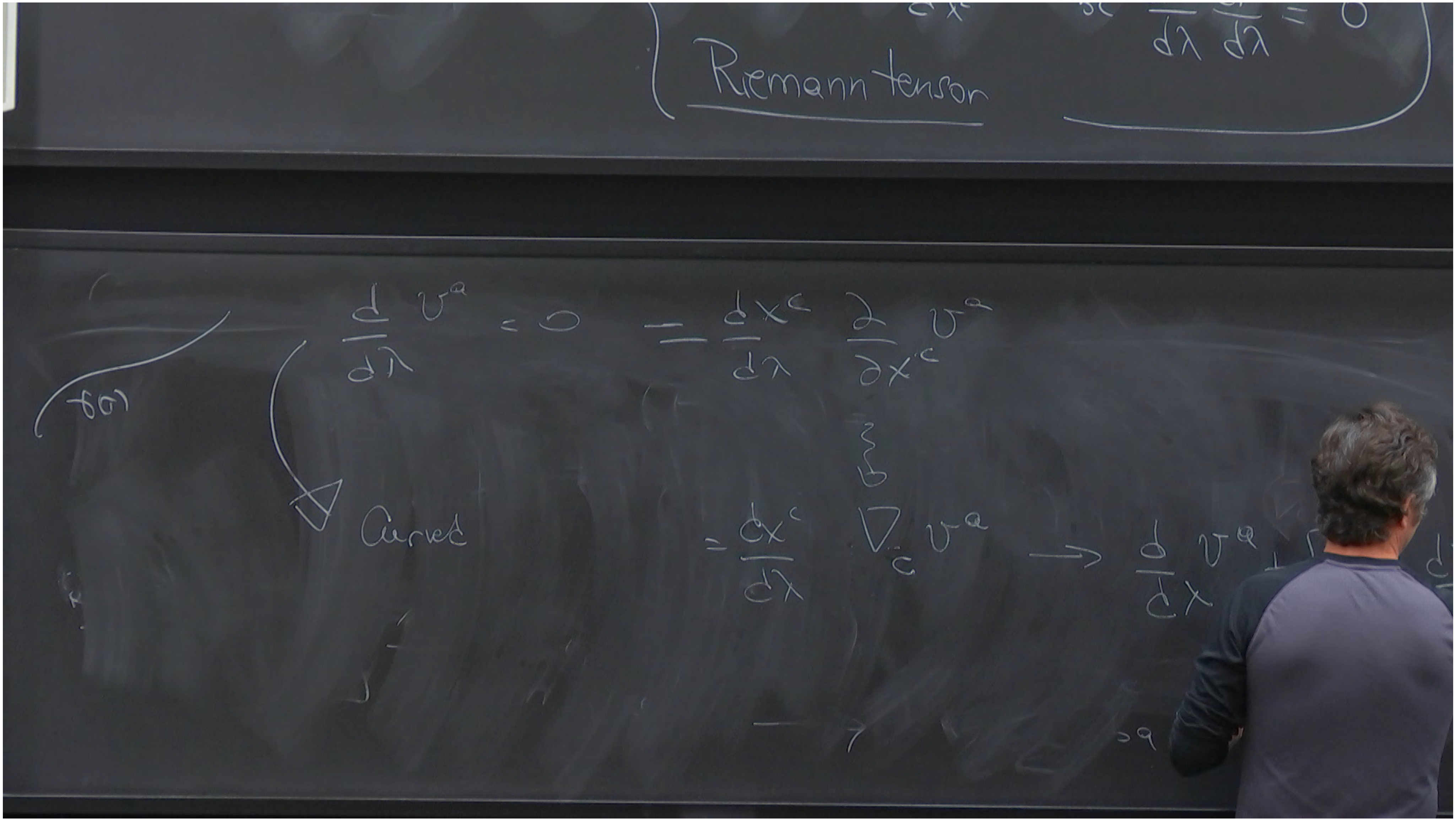
$$\eta_{ab} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

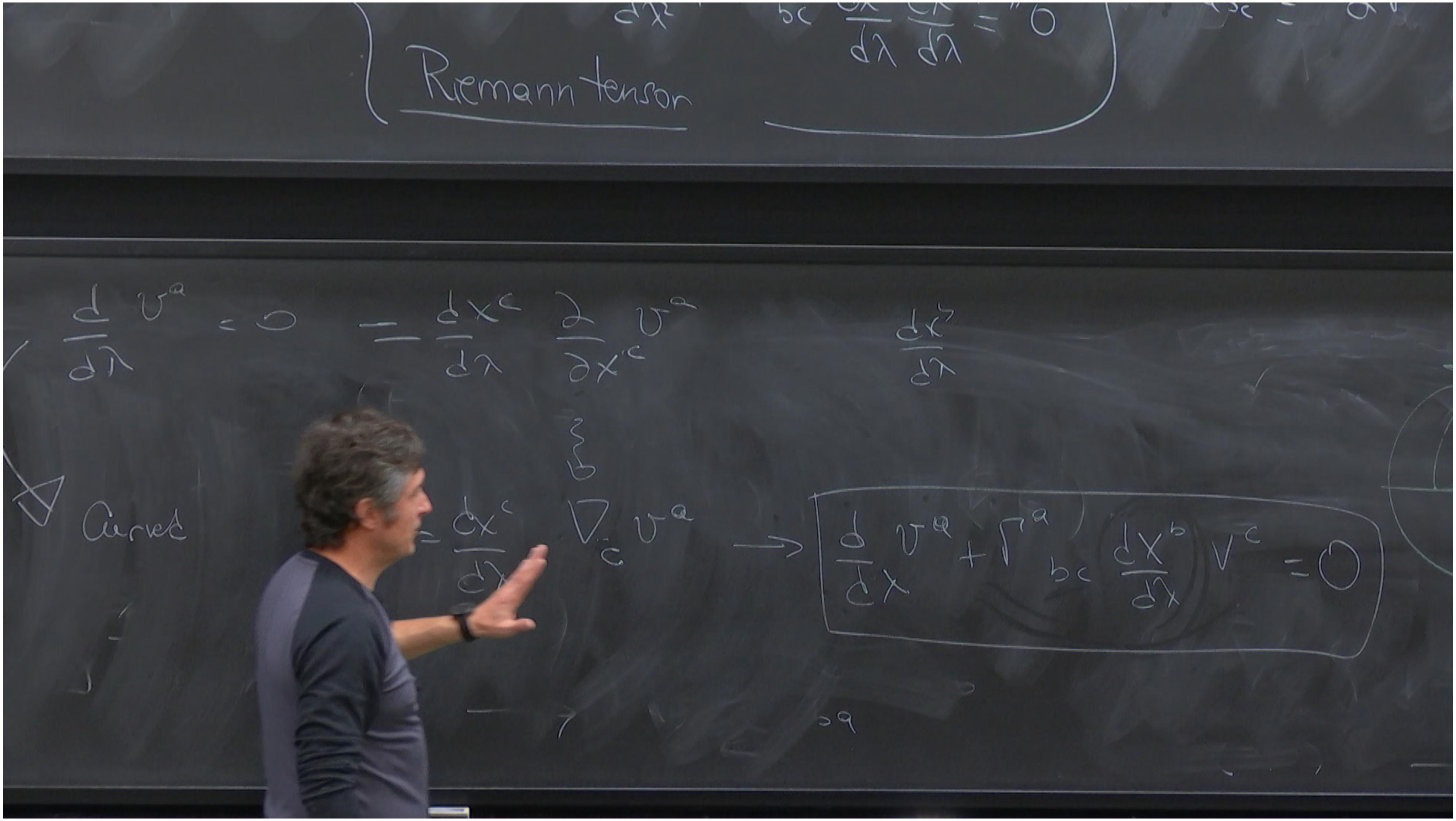
Jacobi's exp.

$$\delta(\det g) = (\det g)\, g^{ab}\, \delta g_{ab}$$

$$g_{ab}\, g^{bd} = \delta_a{}^b$$



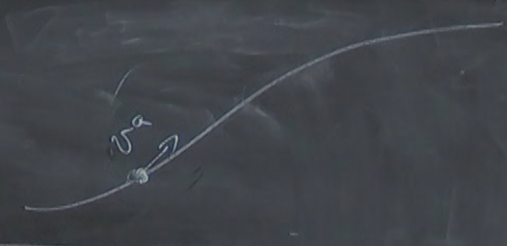

Riemann tensor
$\frac{d}{d\lambda} v^a = 0$
$= \frac{dx^c}{d\lambda} \frac{\partial}{\partial x^c} v^a$
curved
$= \frac{dx^c}{d\lambda} \nabla_c v^a$
$\longrightarrow \frac{d}{d\lambda} v^a$


Riemann tensor
$\frac{d}{d\lambda} v^a = 0$
$= \frac{dx^c}{d\lambda} \frac{\partial}{\partial x^c} v^a$
Curved
$= \frac{dx^c}{d\lambda} \nabla_c v^a$
$\frac{d}{d\lambda} v^a + \Gamma^a{}_{bc} \frac{dx^b}{d\lambda} v^c = 0$

$$\Gamma^a{}_{bc} \frac{dx^b}{d\lambda} \frac{dx^c}{d\lambda} = 0$$

Riemann tensor

$$= \frac{dx^c}{d\lambda} \frac{\partial}{\partial x^c} v^a \qquad \frac{dx^?}{d\lambda}$$

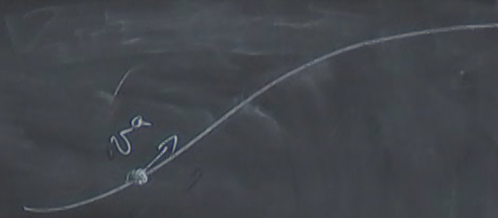


$$= \frac{dx^c}{d\lambda} \nabla_c v^a \longrightarrow \boxed{\frac{d}{d\lambda} v^a + \Gamma^a{}_{bc} \frac{dx^b}{d\lambda} v^c = 0}$$

$$\Gamma_{bc}\frac{dx}{d\lambda}\frac{dx}{d\lambda} = 0$$

Riemann tensor

$$0 = \frac{dx^c}{d\lambda}\frac{\partial}{\partial x^c} v^a \qquad \frac{dx^?}{d\lambda}$$

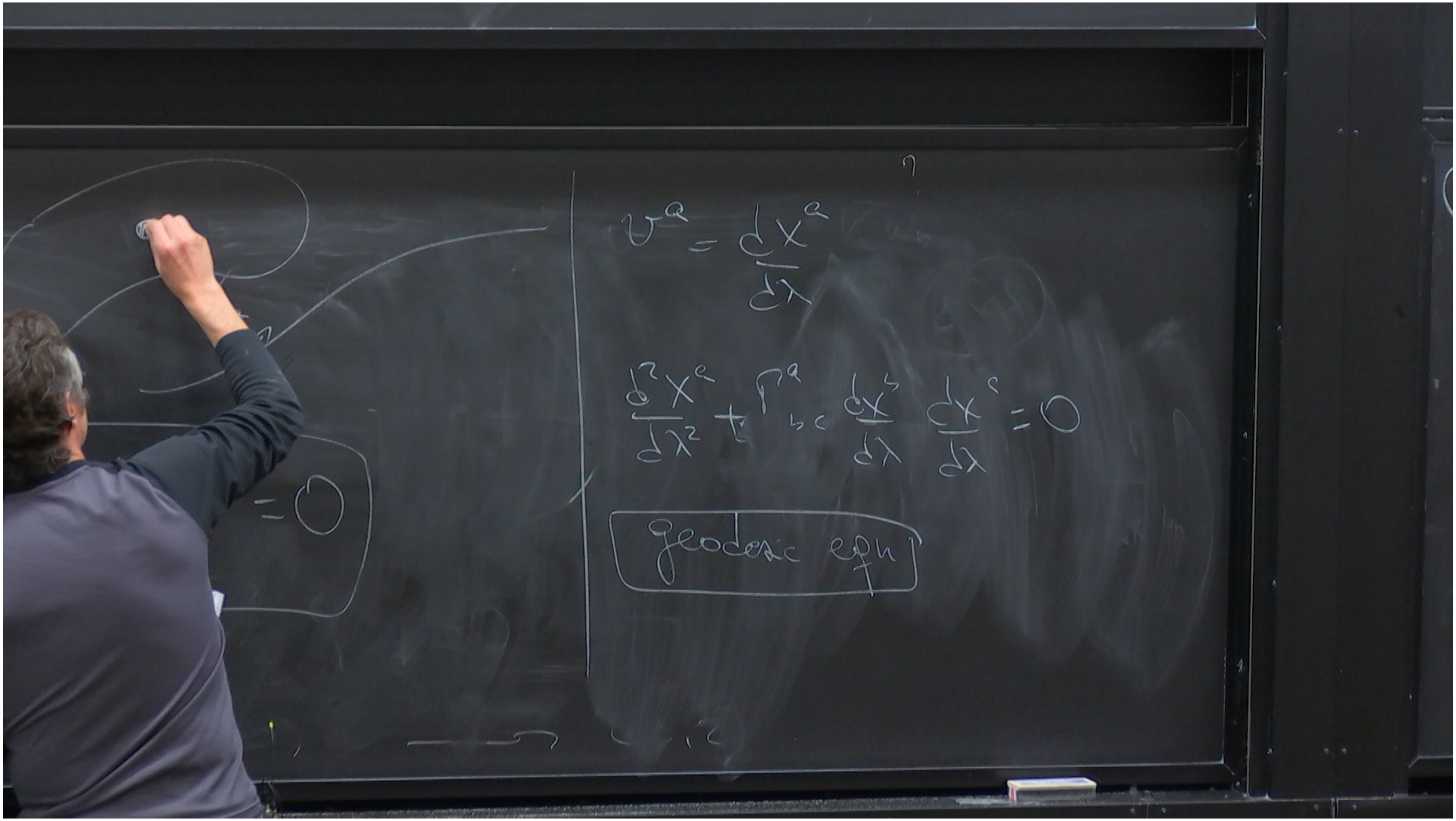


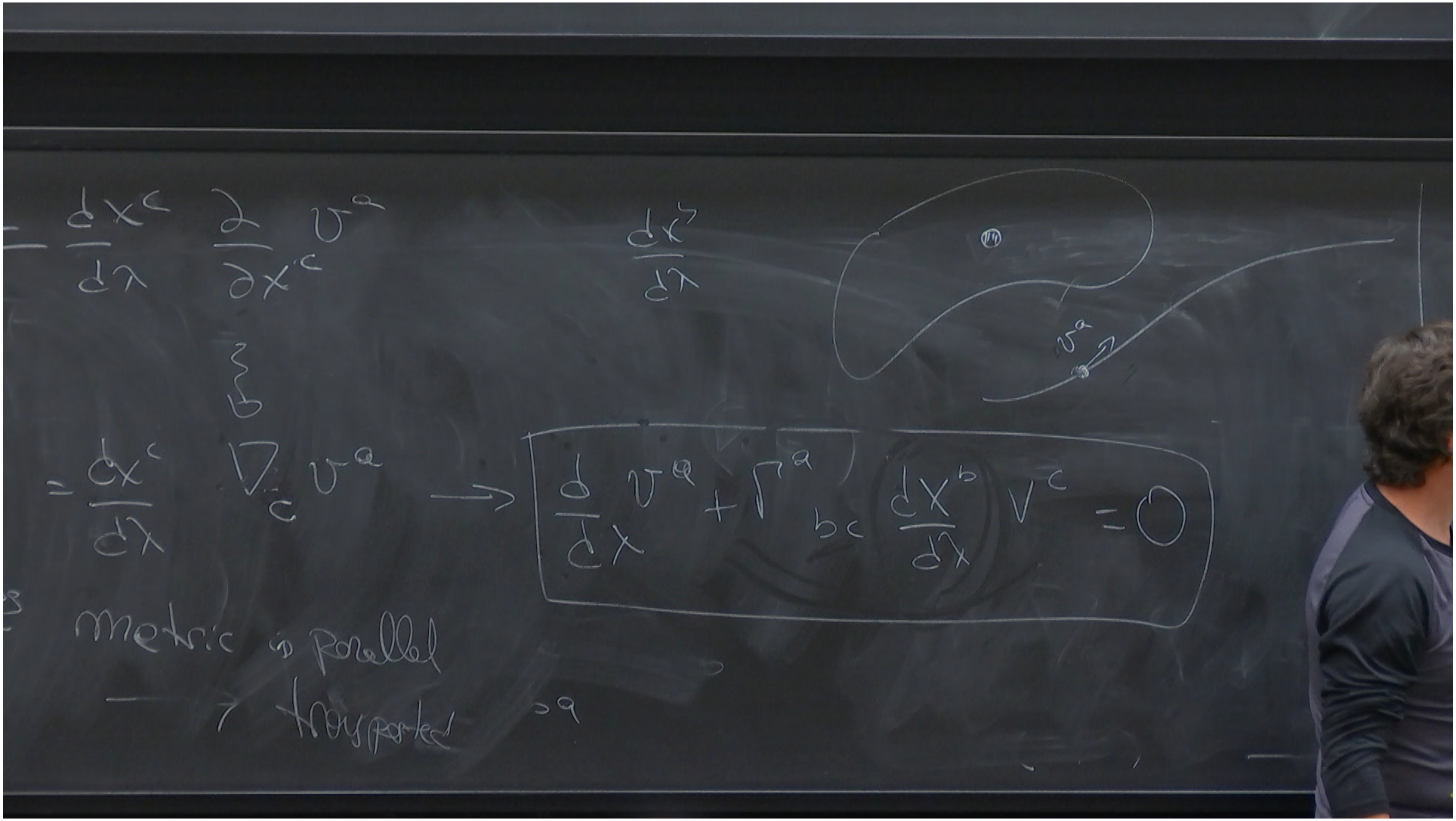
$$= \frac{dx^c}{d\lambda}\nabla_c v^a \longrightarrow \boxed{\frac{d}{d\lambda}v^a + \Gamma^a{}_{bc}\frac{dx^b}{d\lambda}v^c = 0}$$

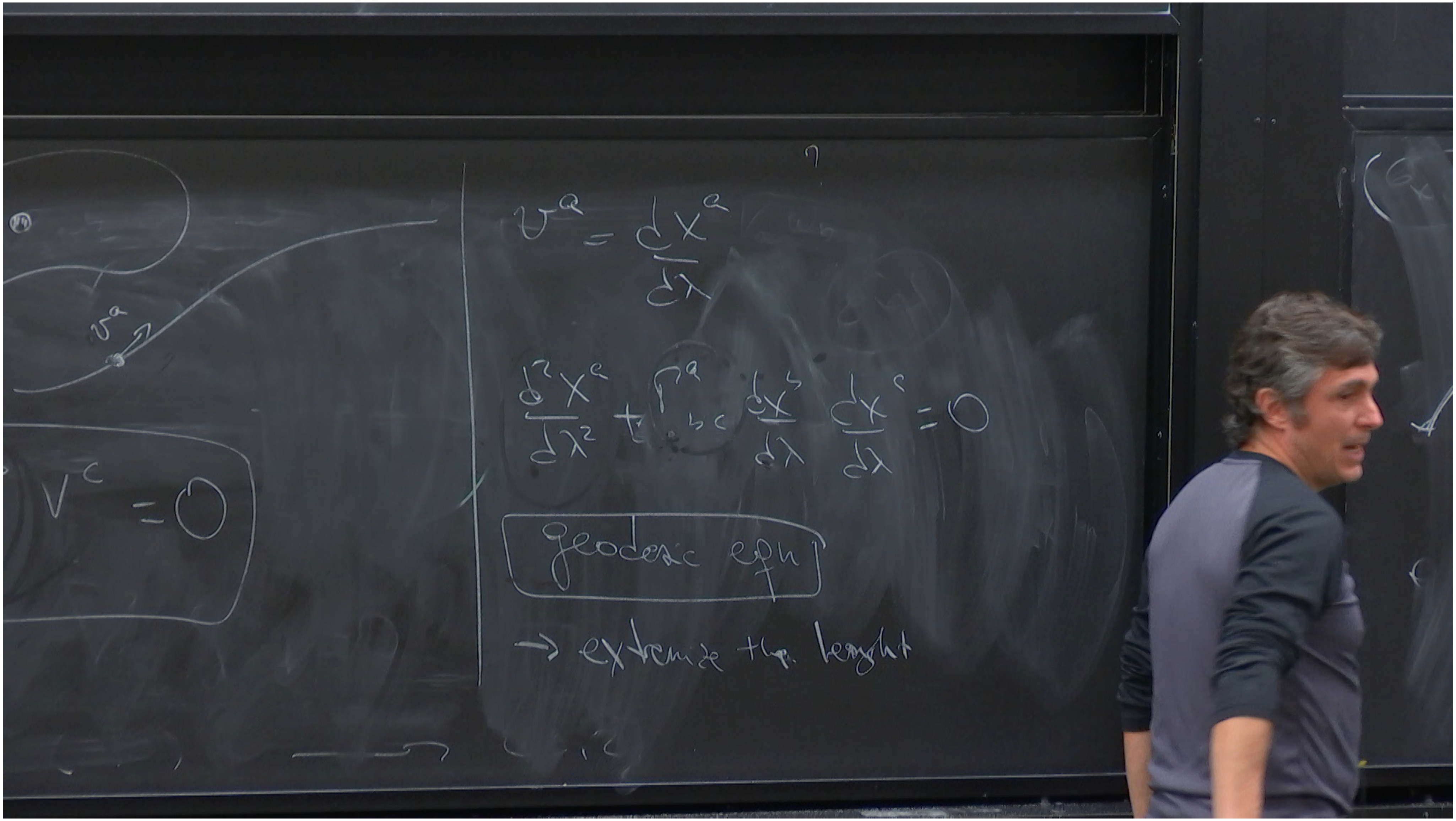
Trivial obs

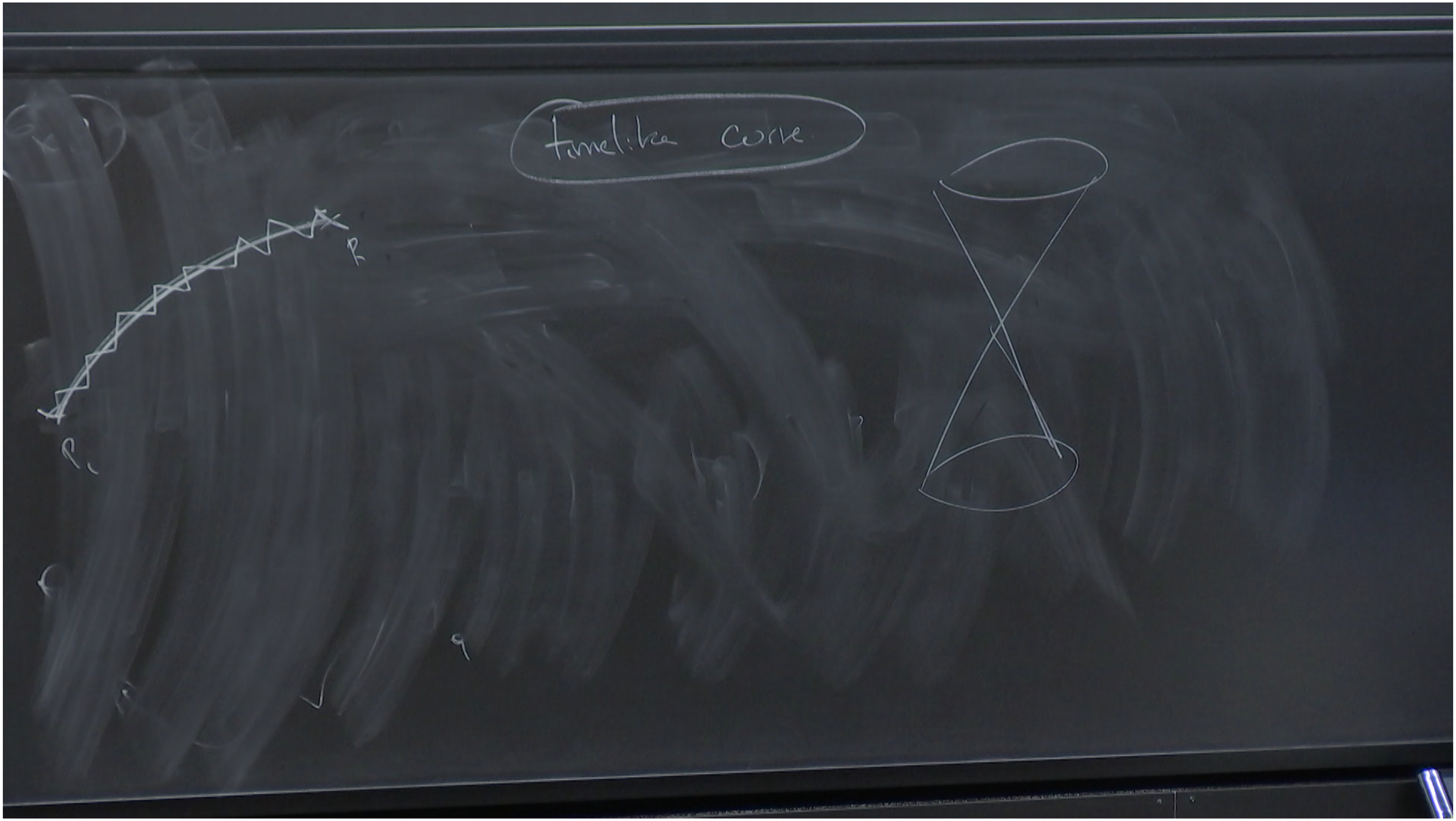
metric is parallel

$\longrightarrow$ transported


$v^a = \frac{dx^a}{d\lambda}$
$\frac{d^2x^a}{d\lambda^2} + \Gamma^a{}_{bc}\frac{dx^b}{d\lambda}\frac{dx^c}{d\lambda} = 0$
geodesic eqn


$\frac{dx^c}{d\lambda} \frac{\partial}{\partial x^c} v^a$
$\frac{dx^b}{d\lambda}$
v^a
$= \frac{dx^c}{d\lambda} \nabla_c v^a$
$\frac{d}{d\lambda} v^a + \Gamma^a{}_{bc} \frac{dx^b}{d\lambda} v^c = 0$
metric is parallel
transported


$v^a = \frac{dx^a}{d\lambda}$
$\frac{d^2x^a}{d\lambda^2} + \Gamma^a_{bc} \frac{dx^b}{d\lambda} \frac{dx^c}{d\lambda} = 0$
geodesic eqn
→ extremize the lenght
v^a
$V^c = 0$


timelike curve
p
q

$x(\alpha)$

$$\frac{d^2 x^a}{d\alpha^2} + \Gamma^a{}_{bc}\frac{dx^b}{d\alpha}\frac{dx^c}{d\alpha} = f(\alpha)\frac{dx^a}{d\alpha}$$

$$f(\alpha) = -\left(\frac{d^2\alpha}{d\lambda^2}\right)\Big/\left(\frac{d\alpha}{d\lambda}\right)^2$$

Week 1 office hour 2:30

$$f(\alpha)\,\frac{dx^a}{d\alpha}$$

$$U^a = \frac{dx^a}{d\alpha}$$

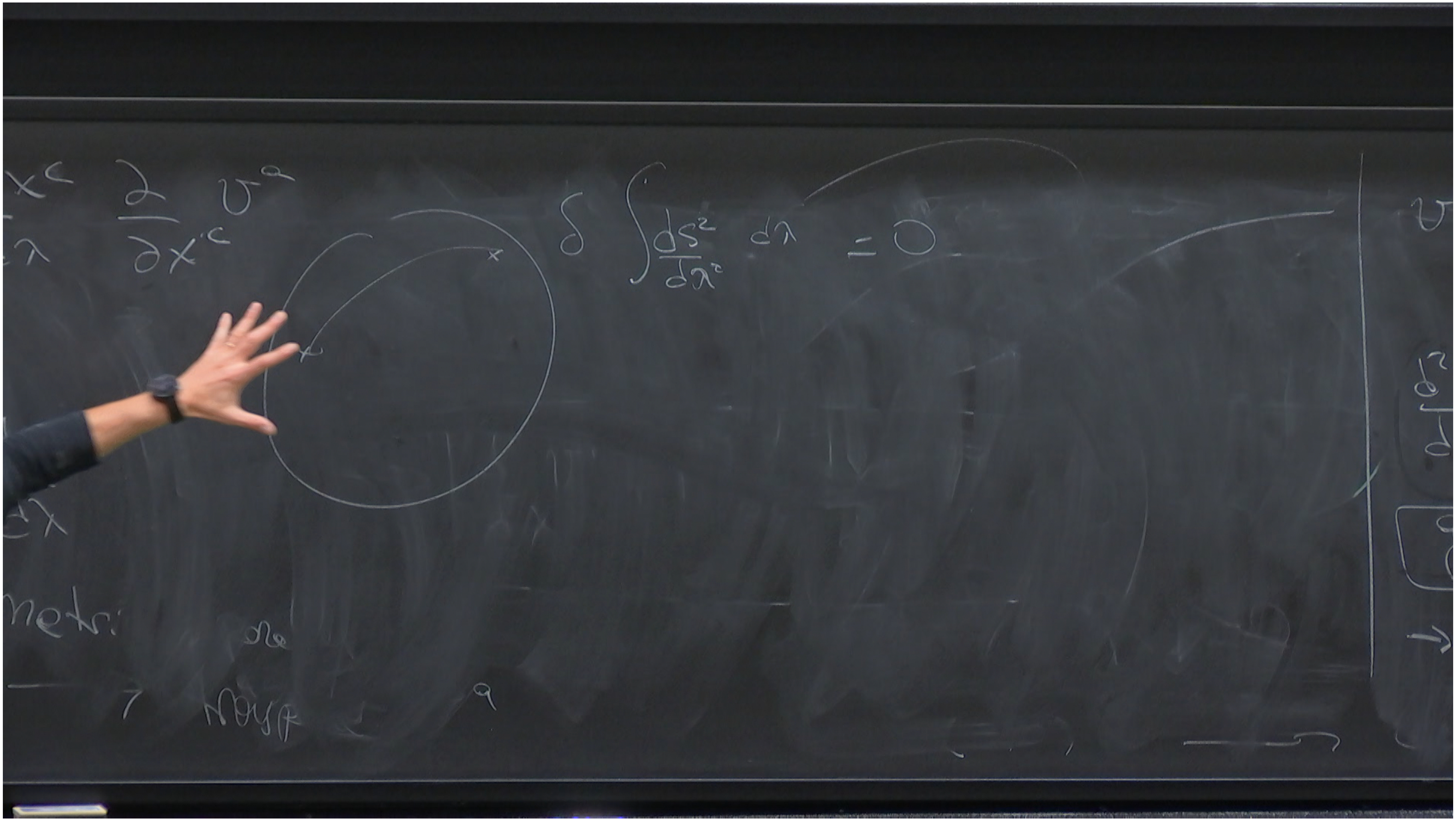
Week 1

$$\frac{d^2x^a}{d\alpha^2} + \Gamma^a_{bc}\frac{dx^b}{d\alpha}\frac{dx^c}{d\alpha} = f(\alpha)\frac{dx^a}{d\alpha}$$

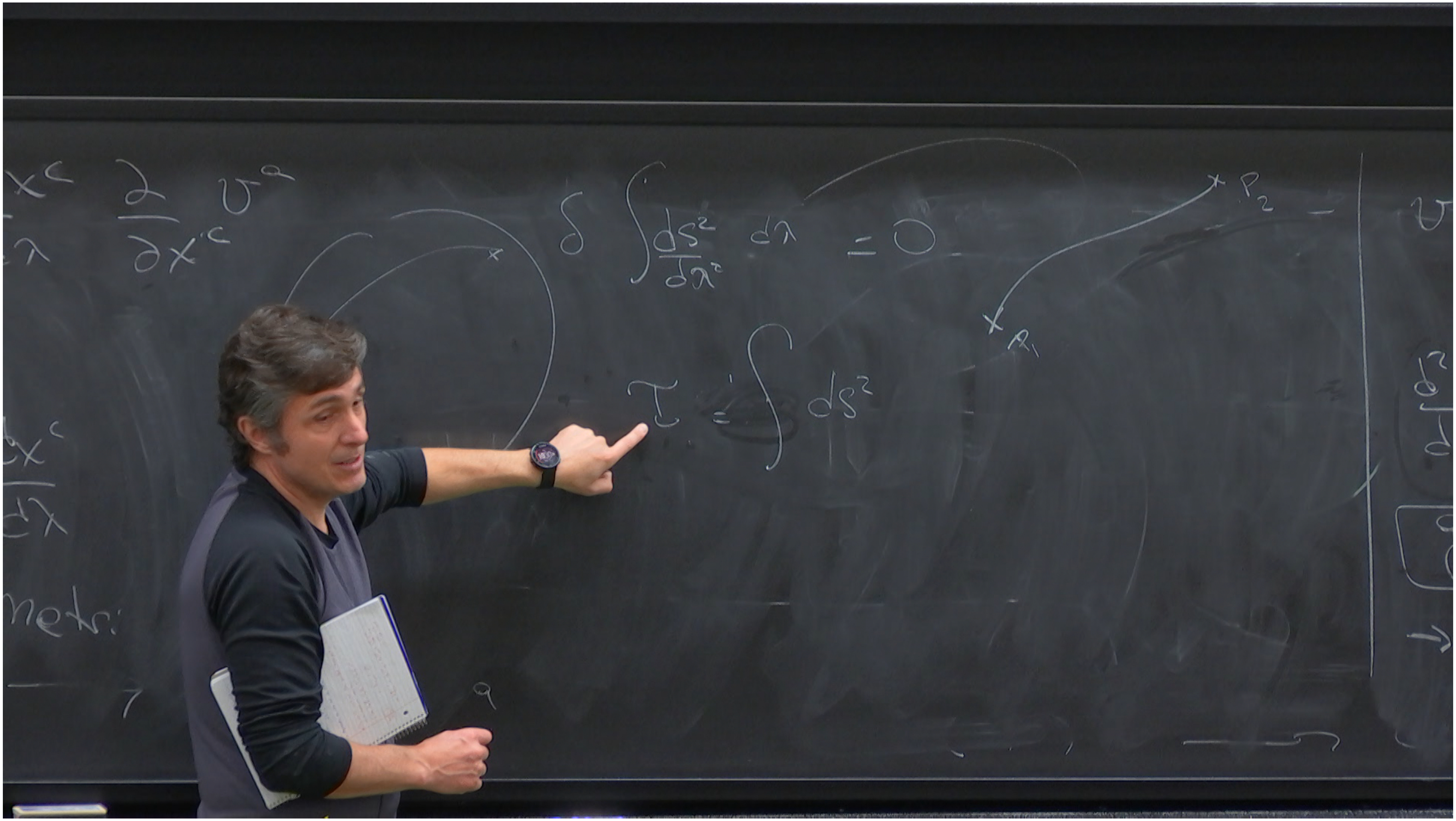
$$U^a = \frac{dx^a}{d\alpha}$$

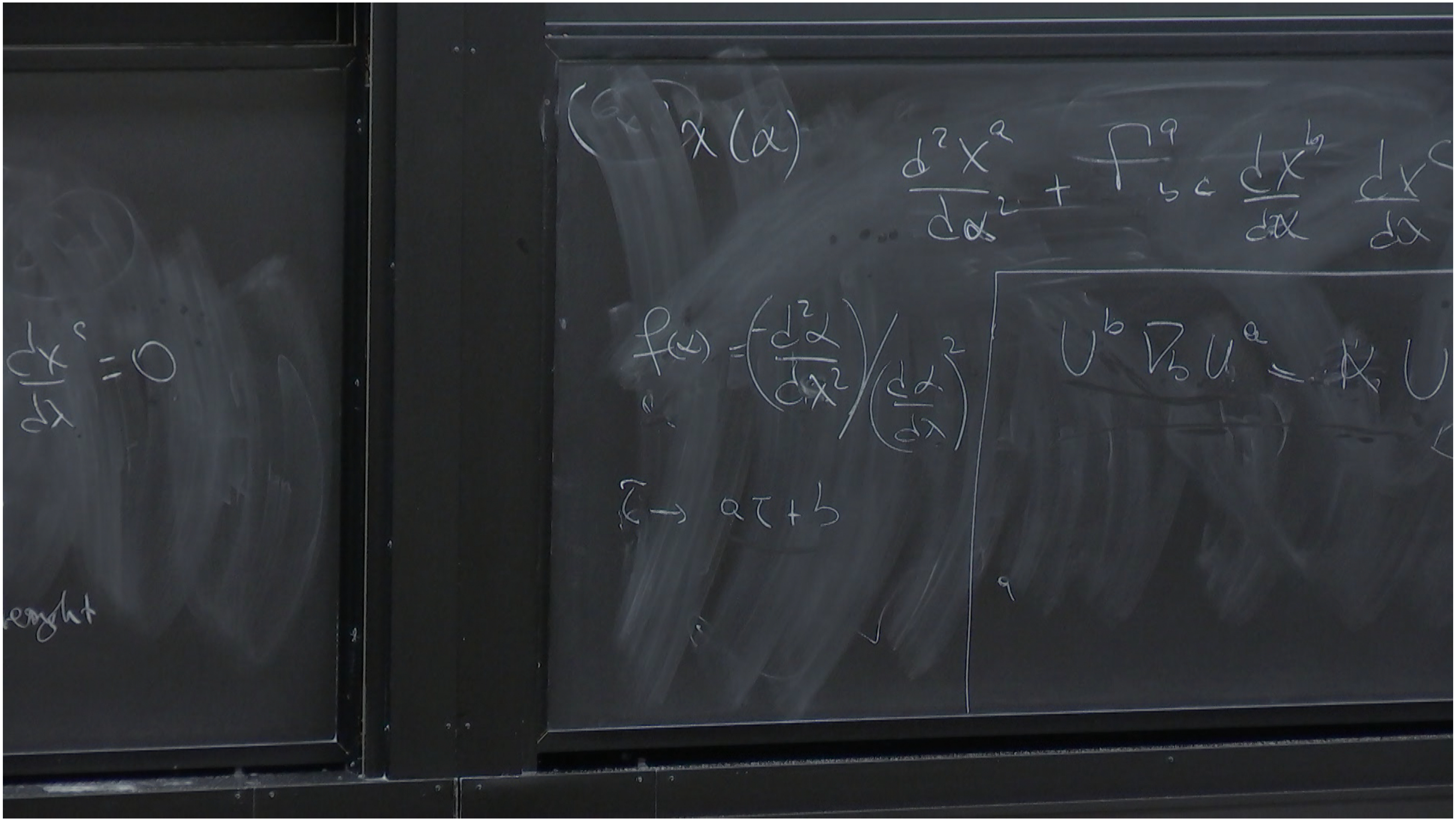
$\boxed{Ex\ 3}$

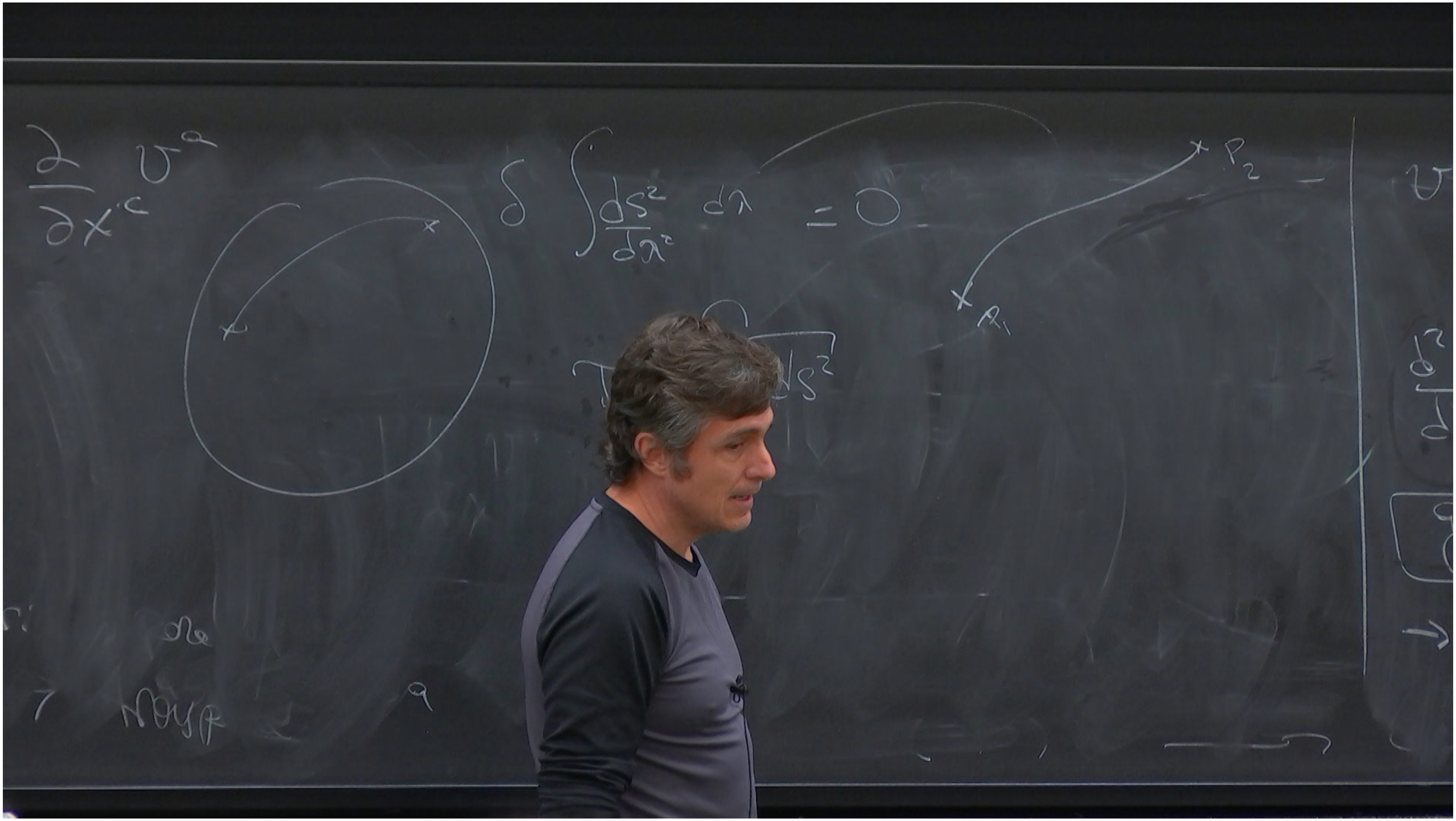
$$U^b \nabla_b U^a = \kappa U^a$$

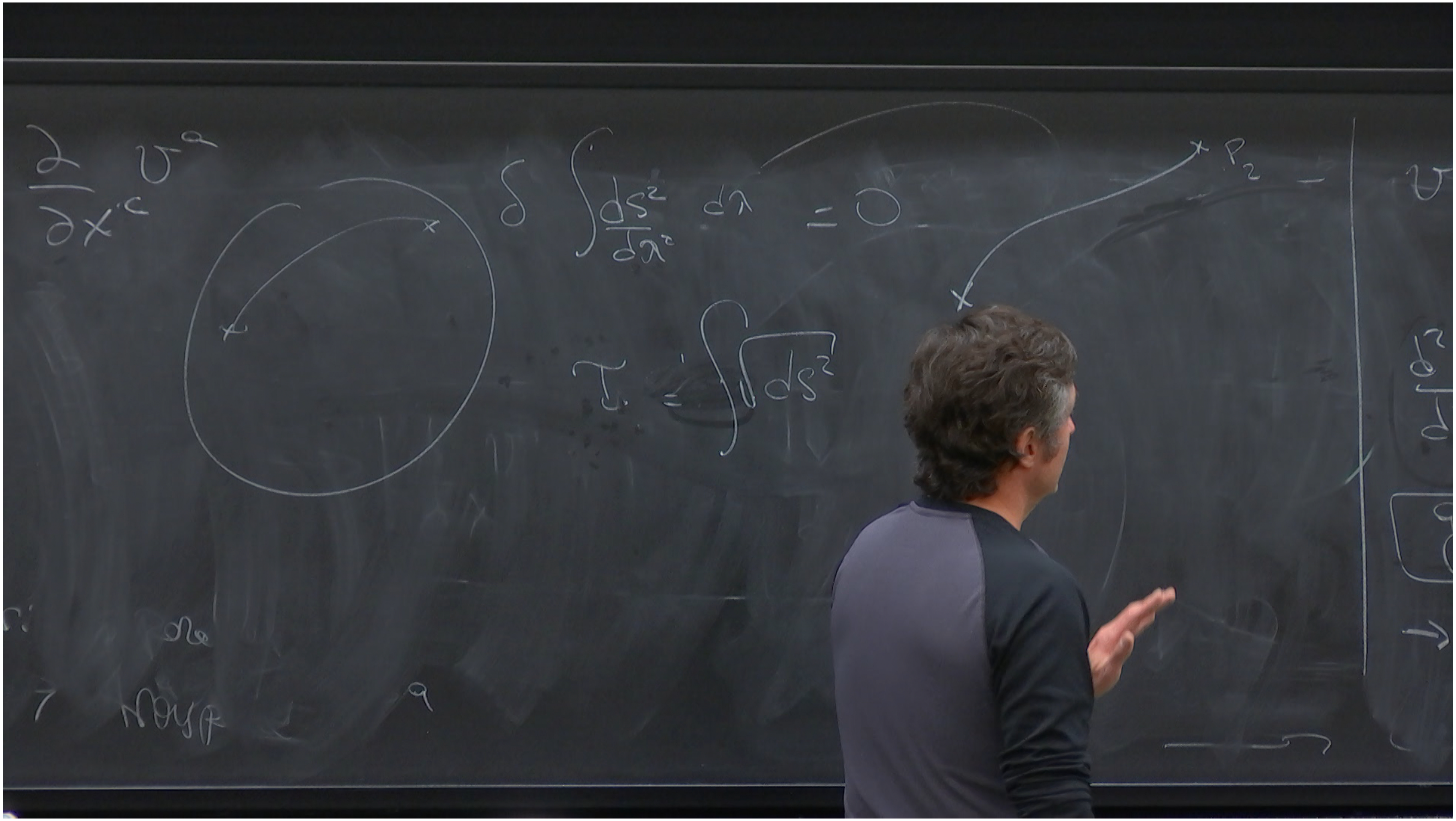


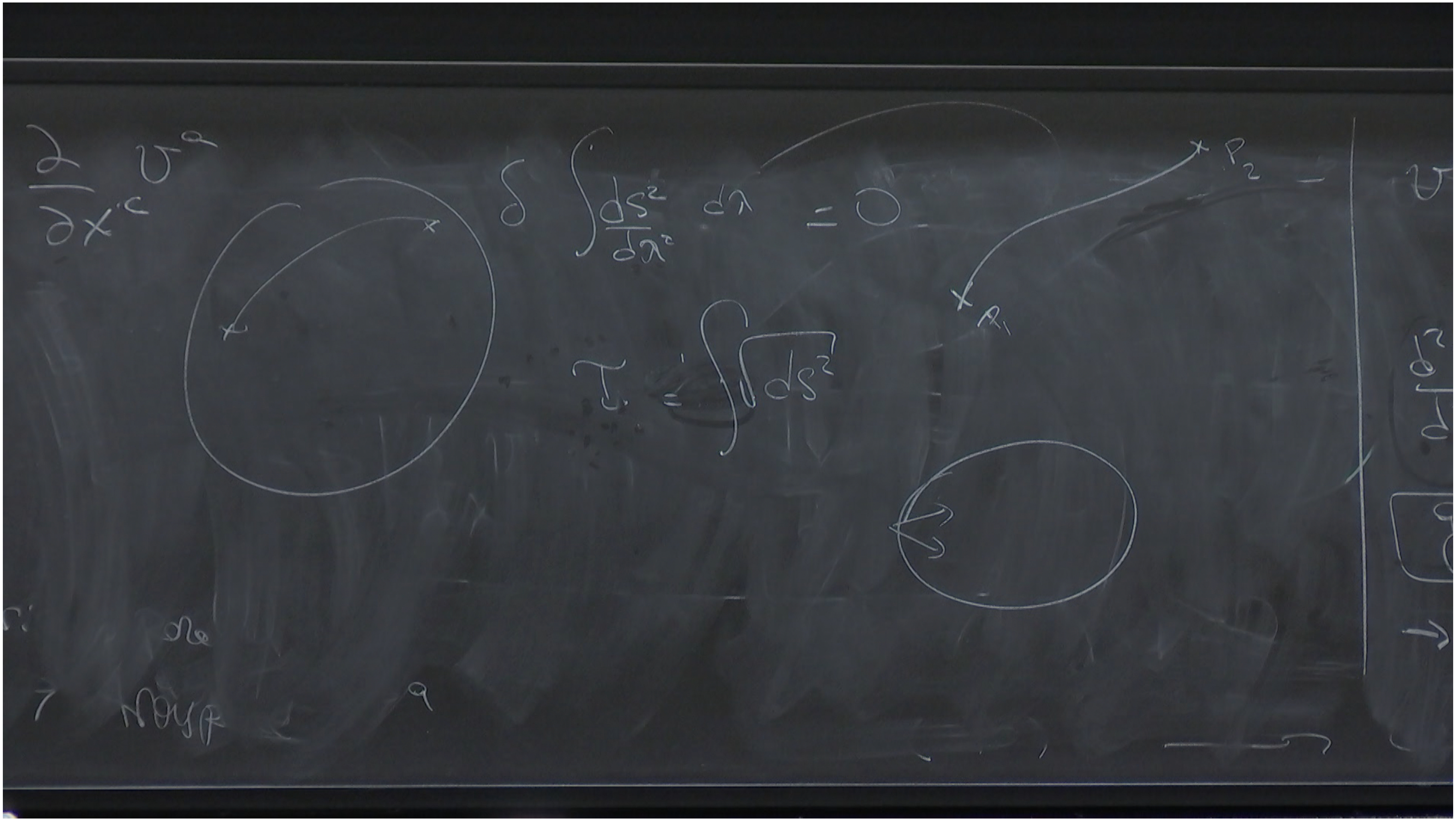
$\frac{\partial}{\partial x^c} v^a$
$\delta \int \frac{ds^2}{d\lambda^2} d\lambda = 0$
metri


δ ∫ ds²/dλ² dλ = 0
P2
P1
I = ∫ ds²
metri


x(α)
d²x^a/dα² + Γ^a_bc (dx^b/dα)(dx^c/dα)
dx^c/dλ = 0
f(λ) = -(d²α/dλ²)/(dα/dλ)²
τ → aτ + b
U^b ∇_b U^a =


$\frac{\partial}{\partial x^c} v^a$
$\delta \int \frac{ds^2}{d\lambda^2} \, d\lambda = 0$
P_2
P_1


∂/∂x^c v^a
δ ∫ ds²/dλ² dλ = 0
P₂
τ = ∫ √ds²


$\frac{\partial}{\partial x^c} v^a$
$\delta \int \frac{ds^2}{d\lambda^2} d\lambda = 0$
P_2
P_1
$\tau = \int \sqrt{ds^2}$