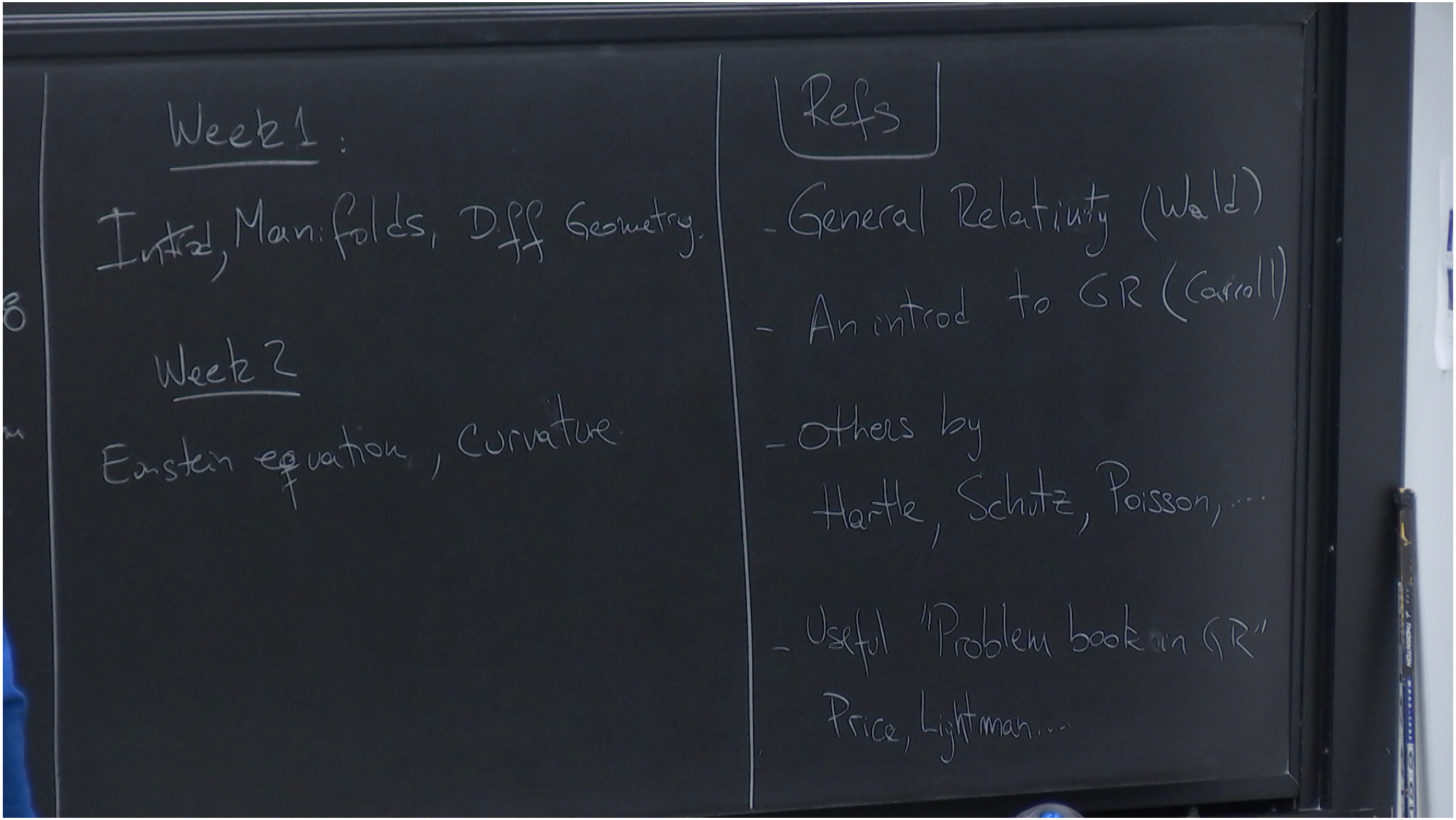
Title: Relativity - Lecture 221011

Speakers:

Collection: Relativity (2022/2023)

Date: October 11, 2022 - 9:00 AM

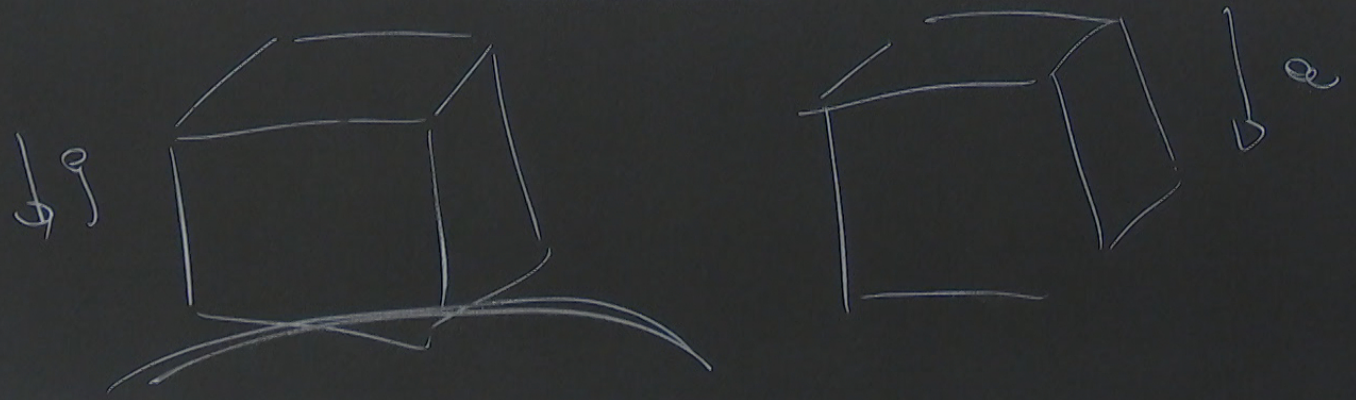
URL: https://pirsa.org/22100075


Week 1 :
Intro, Manifolds, Diff Geometry.
Week 2
Einstein equation, Curvature
Refs
- General Relativity (Wald)
- An introd to GR (Carroll)
- others by
Hartle, Schutz, Poisson, ...
- Useful "Problem book in GR"
Price, Lightman...

Newton's 2nd law $F = m_i a$

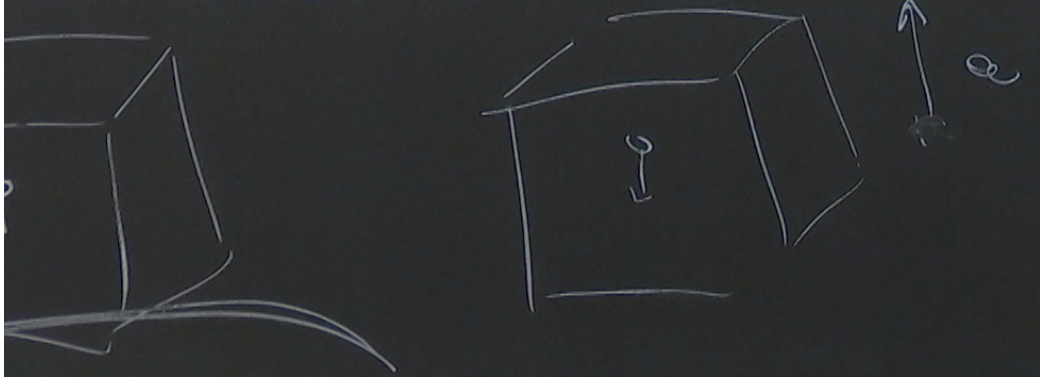
Newton's law for gravity $F_g = -m_g \nabla\phi$

$\Big\}$ $m_g < m_i$ $\quad a =$



$F = m_i a$ } $m_g < m_i$ $a = -\nabla\phi$ Weak equivalence

$F_g = -m_g \nabla\phi$



$a = -D\phi$

Weak equivalence principle

Einstein equiv. pple

- (Strong) → Locally all interactions are as in special relativity (except gravity)
- (including gravity)

m_i $\quad a = -D\phi$ — Weak equivalence principle

Einstein equiv. ppl. → Locally all interactions are as in special relativity (except gravity)

(Strong) → (including gravity)

a

$\exists$ Special trajectories. Universal (do not depend on m)

m_i $a = -D\phi$ Weak equivalence principle

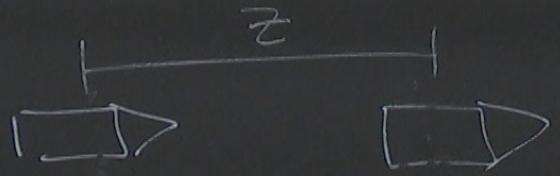
Einstein equiv. pple → Locally all interactions are as in special relativity (except gravity)

(Strong) → (including gravity)

a

$\exists$ Special trajectories. Universal (do not depend on m)

SR mass isn't special $E = mc^2$



t_0 $\vec{n}$

t_1 $\vec{n}$

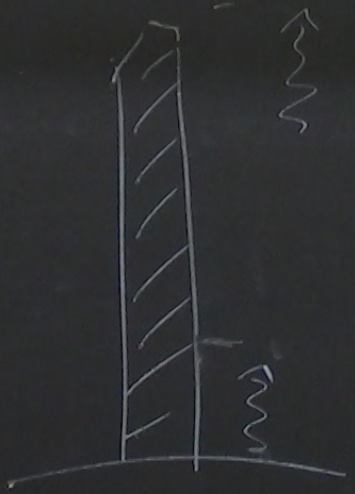
λ_0 λ_1

v v_1

$v = a \cdot \Delta t$

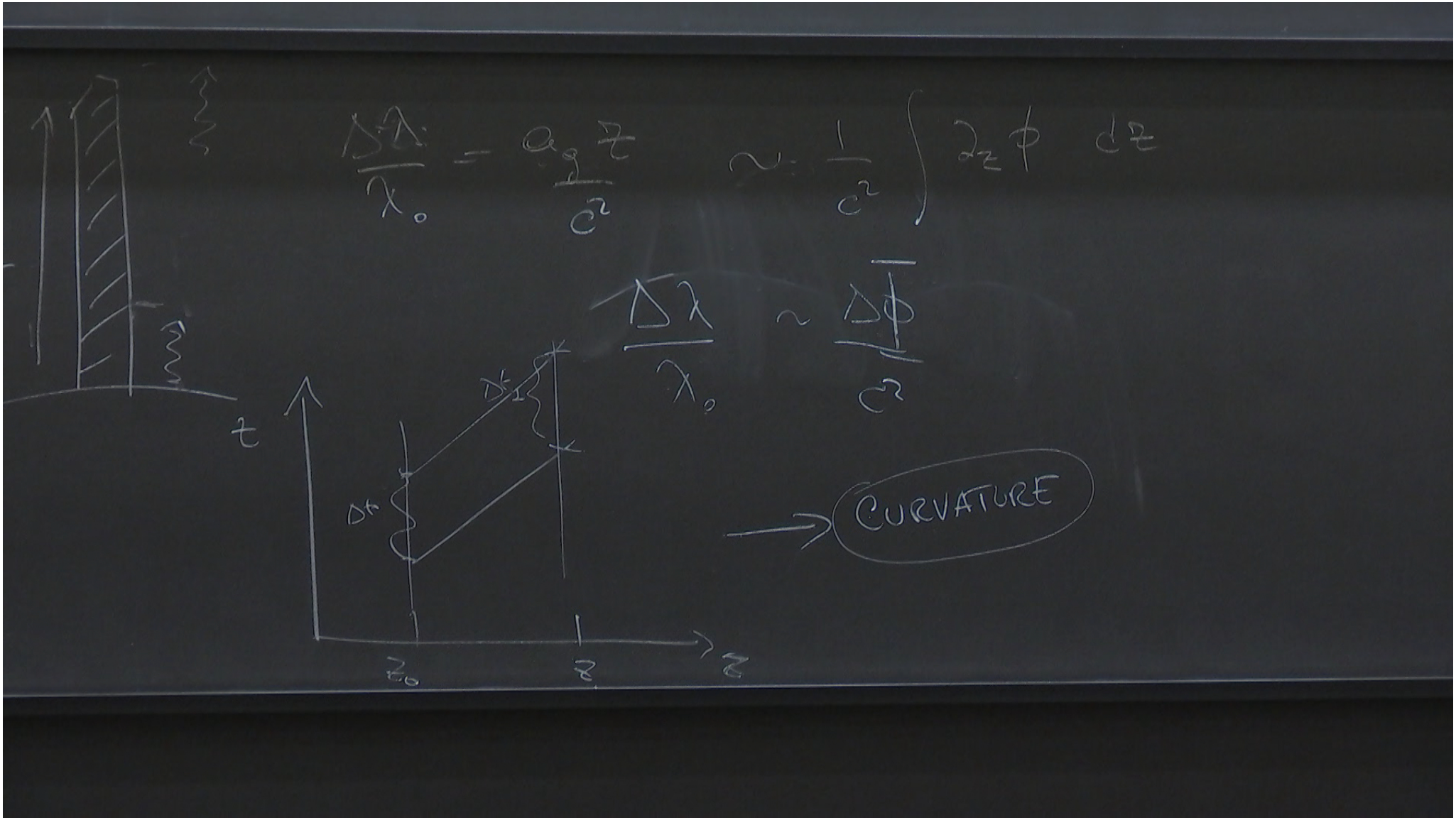
$= a \frac{z}{c}$

$\frac{\Delta\lambda}{\lambda_0} = \frac{\Delta v}{c} = \frac{a z}{c^2}$



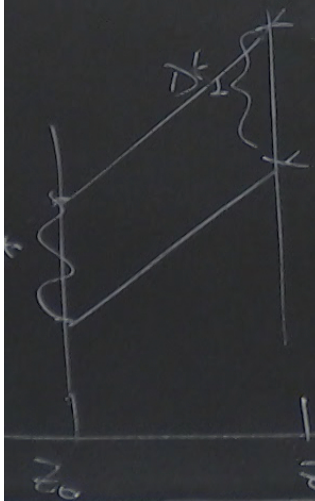
$$\frac{\Delta\lambda}{\lambda_0} = \frac{a_g z}{c^2} \sim \frac{1}{c^2}\int \partial_z \phi \, dz$$

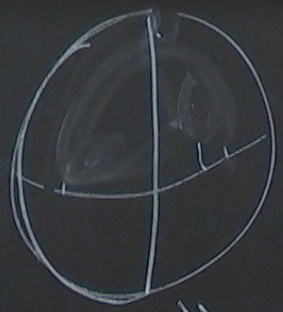
$$\frac{\Delta\lambda}{\lambda_0} \sim \frac{\Delta\phi}{c^2}$$


$\frac{\Delta\lambda}{\lambda_0} = \frac{a_g z}{c^2} \simeq \frac{1}{c^2}\int z_2 \phi \, dz$
$\frac{\Delta\lambda}{\lambda_0} \simeq \frac{\Delta\phi}{c^2}$
t
Δt_1
Δt_0
z_0
z
z
CURVATURE

$\frac{\Delta\lambda}{\lambda_0} = a_g \frac{1}{c^2} z \sim \frac{1}{c^2} \int \partial_z \phi \, dz$

$\frac{\Delta\lambda}{\lambda_0} \sim \frac{\Delta\phi}{c^2}$





"π" $= \frac{\text{CIRC}}{\text{DIAMETER}} = \frac{2\pi R}{\pi R}$

$= 2$

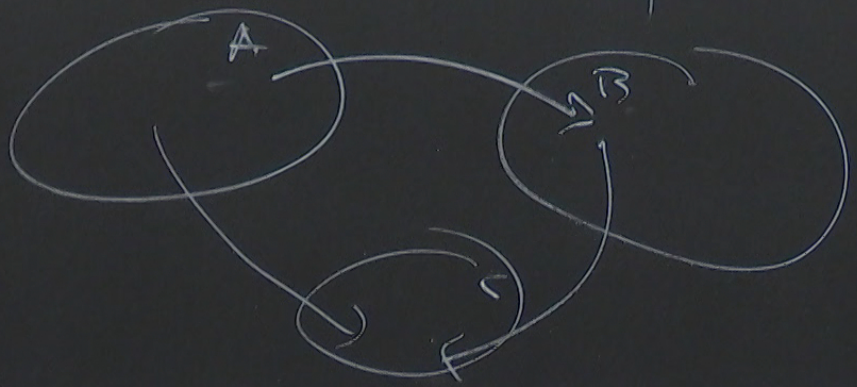
→ CURVATURE

$\mathbb{R}^n \to (x^1, \ldots x^n)$

Manifold "=" Space consists a collection of "patches" that look locally like $\mathbb{R}^n$ and they are smoothly sewn together

- Map $\phi : M \longrightarrow N$
- compositions of maps. $\phi : A \to B$ $\xi : A \longrightarrow C$

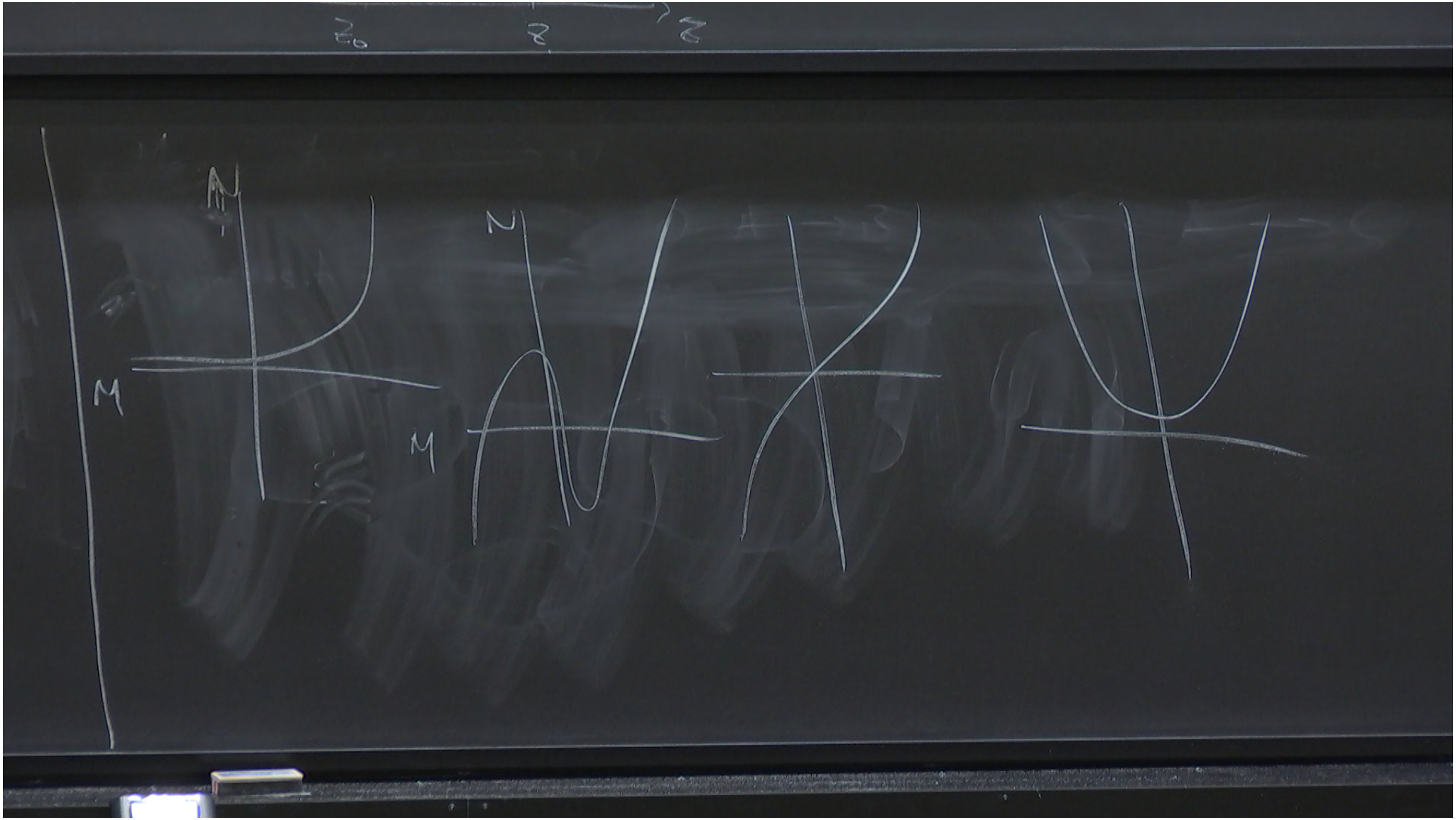
$\psi : B \to C$



$\xi : \psi \circ \phi$

Map can be (injective) 1-to-1 if each element of N has at most one of M.

" " Surjective (onto) if each element of N has at most one element of M

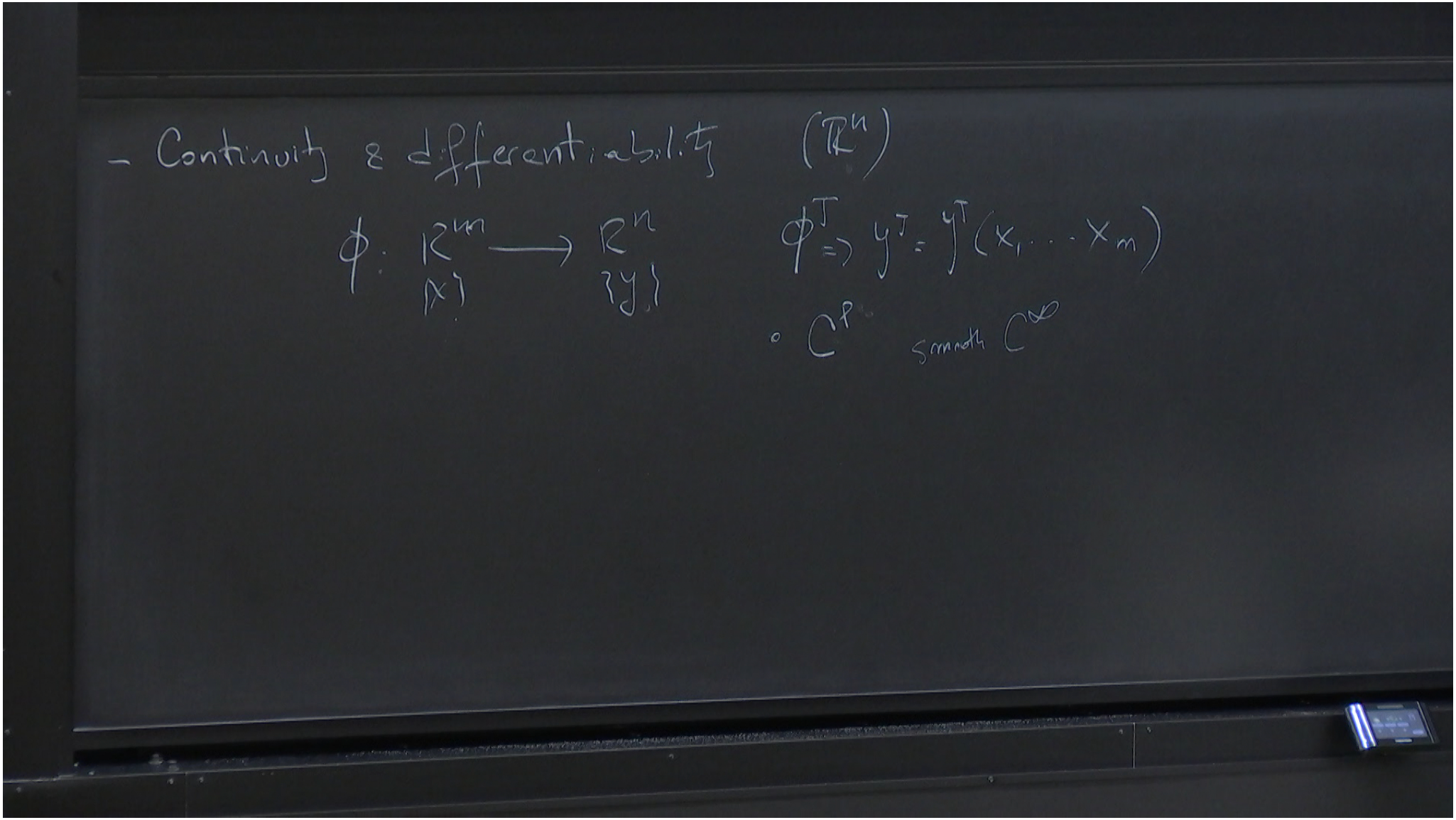


Map can be 1-to-1 (injective) if each element of N has at most one of M.

" " Surjective (onto) if each element of N has at most one element of M

M: domain of ϕ

N: image of ϕ


- Continuity & differentiability ($\mathbb{R}^n$)
$\phi: \mathbb{R}^m \longrightarrow \mathbb{R}^n$
$\{x_i\}$ $\{y_j\}$
$\phi^j \Rightarrow y^j = y^j(x_1 \cdots x_m)$
C^p smooth C^∞

Map can be (injective) 1-to-1 if each element of N has at most one of. M.

Surjective (onto) if each element of N has at least one element of M

M: domain of ϕ

N: image of ϕ

ϕ^{-1}

M

- Continuity & differentiability $(\mathbb{R}^n)$

$\phi: \mathbb{R}^m \longrightarrow \mathbb{R}^n$
$\{x\}$ $\{y\}$

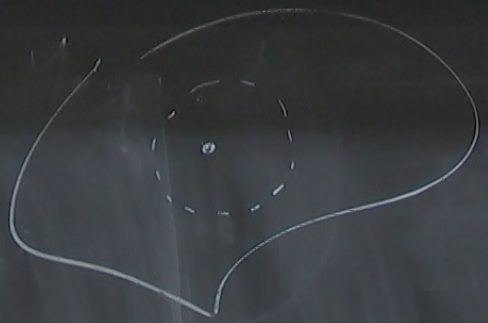
$\phi^j \Rightarrow y^j = y^j(x_1 \dots x_m)$

- C^p smooth C^∞

- M & N are diffeomorphic

if ϕ is C^∞; with a ϕ^{-1} C^∞

all $x \in \mathbb{R}^n$ $|x-y| < r$



system") of $U \subset M$, $\phi : U \longrightarrow \mathbb{R}^n$

onto, invertible

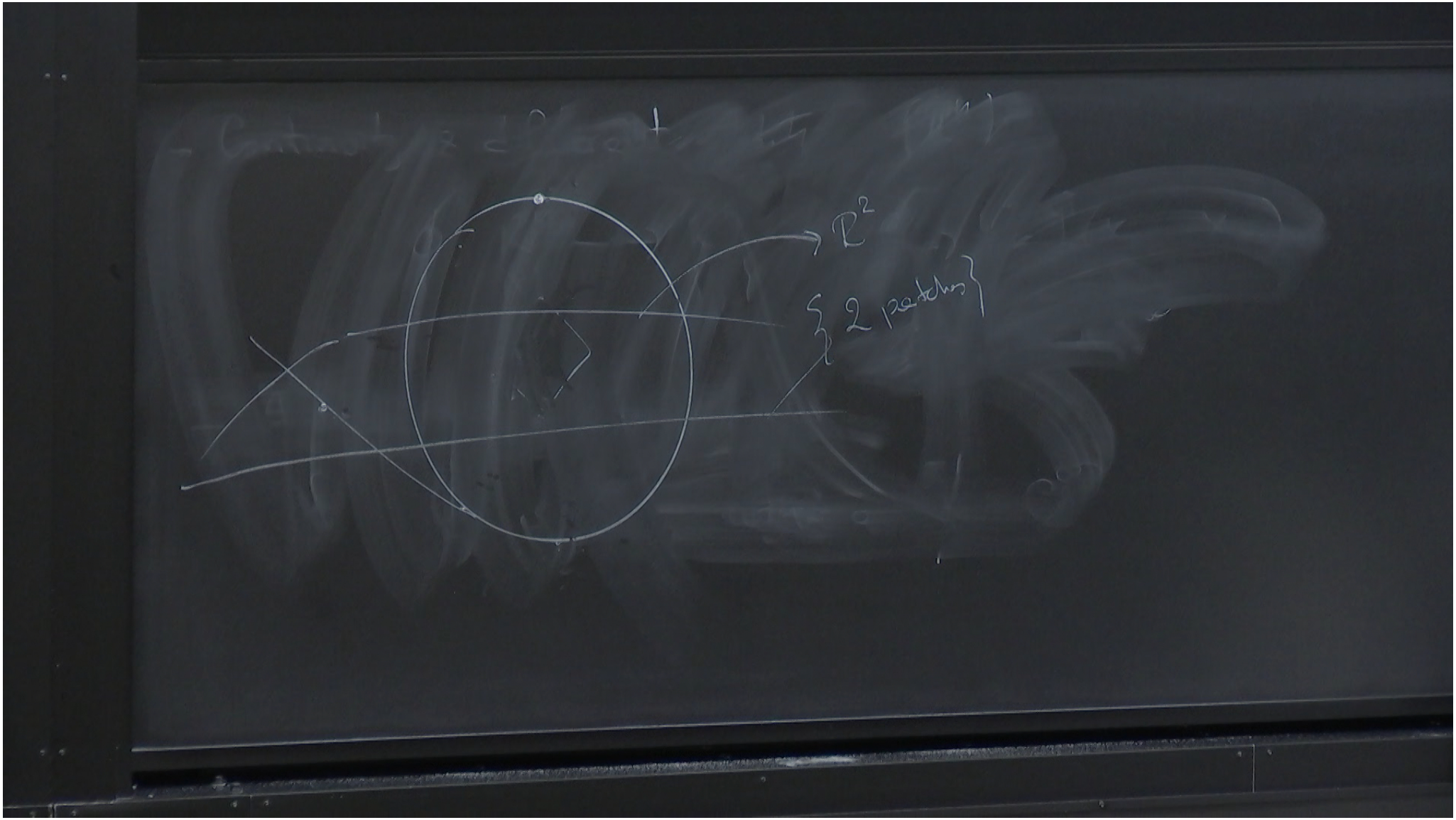


① Open ball in $\mathbb{R}^n$, B_r, $r>0$, all $x \in \mathbb{R}^n$ $|x-y|<r$

② Open set $\bigcup B_r$.

③ A <u>chart</u> ("coordinate system") of $U \subset M$, $\phi: U \longrightarrow \mathbb{R}^n$

onto, invertible

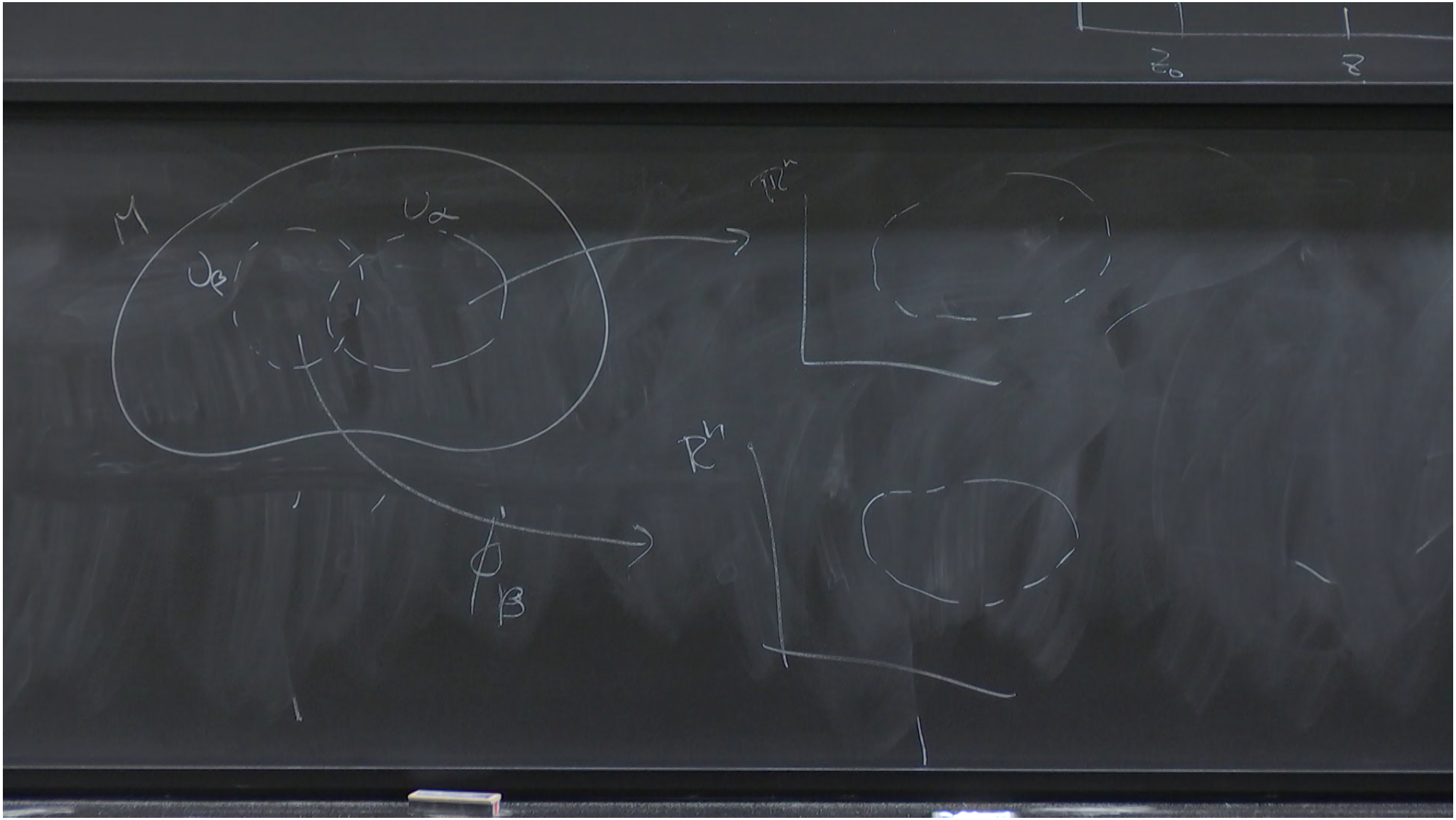

R^2
2 rotations

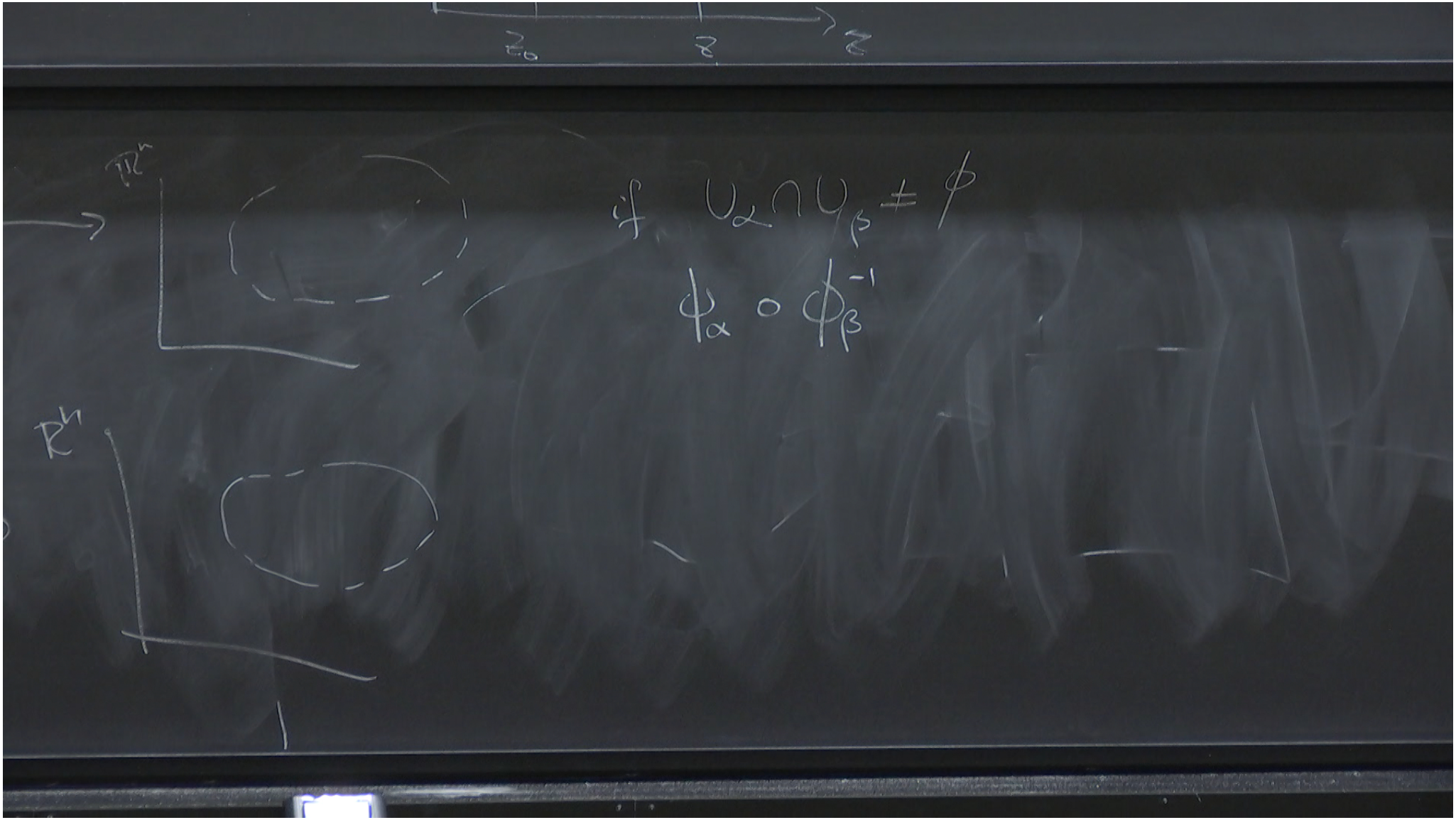
- An Atlas : an indexed collection of charts. $\{(U_\alpha, \phi_\alpha)\}$

that satisfies

i) $\bigcup U_\alpha = M$

ii) charts are smoothly sewn together.


M
U_α
U_β
$\mathbb{R}^4$
$\mathbb{R}^4$
ϕ_β


z_0
z
z
$\mathbb{R}^n$
$\mathbb{R}^4$
if $U_\alpha \cap U_\beta \neq \emptyset$
$\psi_\alpha \circ \phi_\beta^{-1}$

if $U_\alpha \cap U_\beta \neq \emptyset$

$\phi_\alpha \circ \phi_\beta^{-1}$ takes a point in

$\phi_\beta(U_\alpha \cap U_\beta) \subset \mathbb{R}^n$ onto

an open set of $\phi_\alpha(U_\alpha \cap U_\beta) \subset \mathbb{R}^n$.

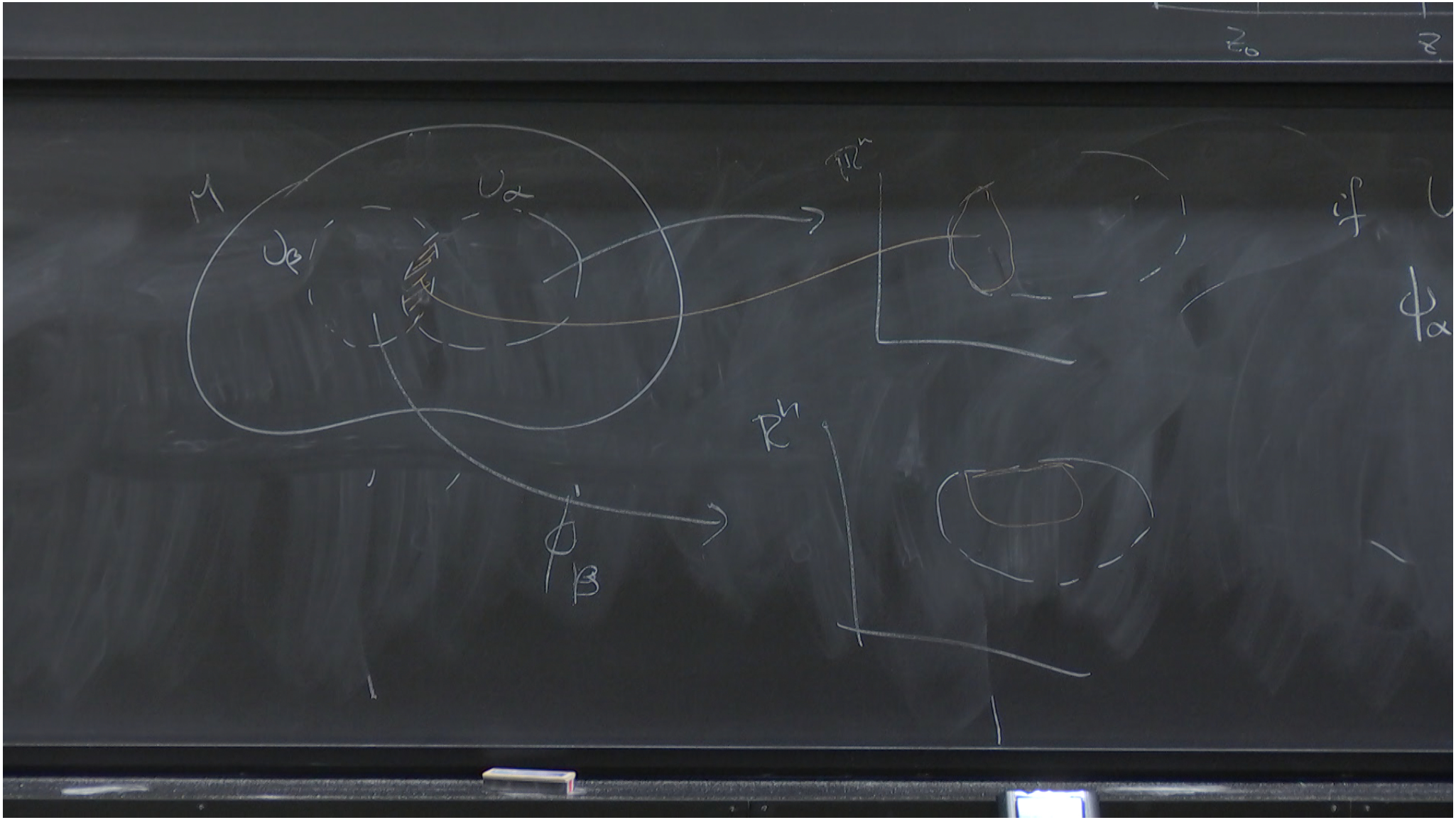
- An Atlas : an indexed collection of charts. $\{(U_\alpha, \phi_\alpha$

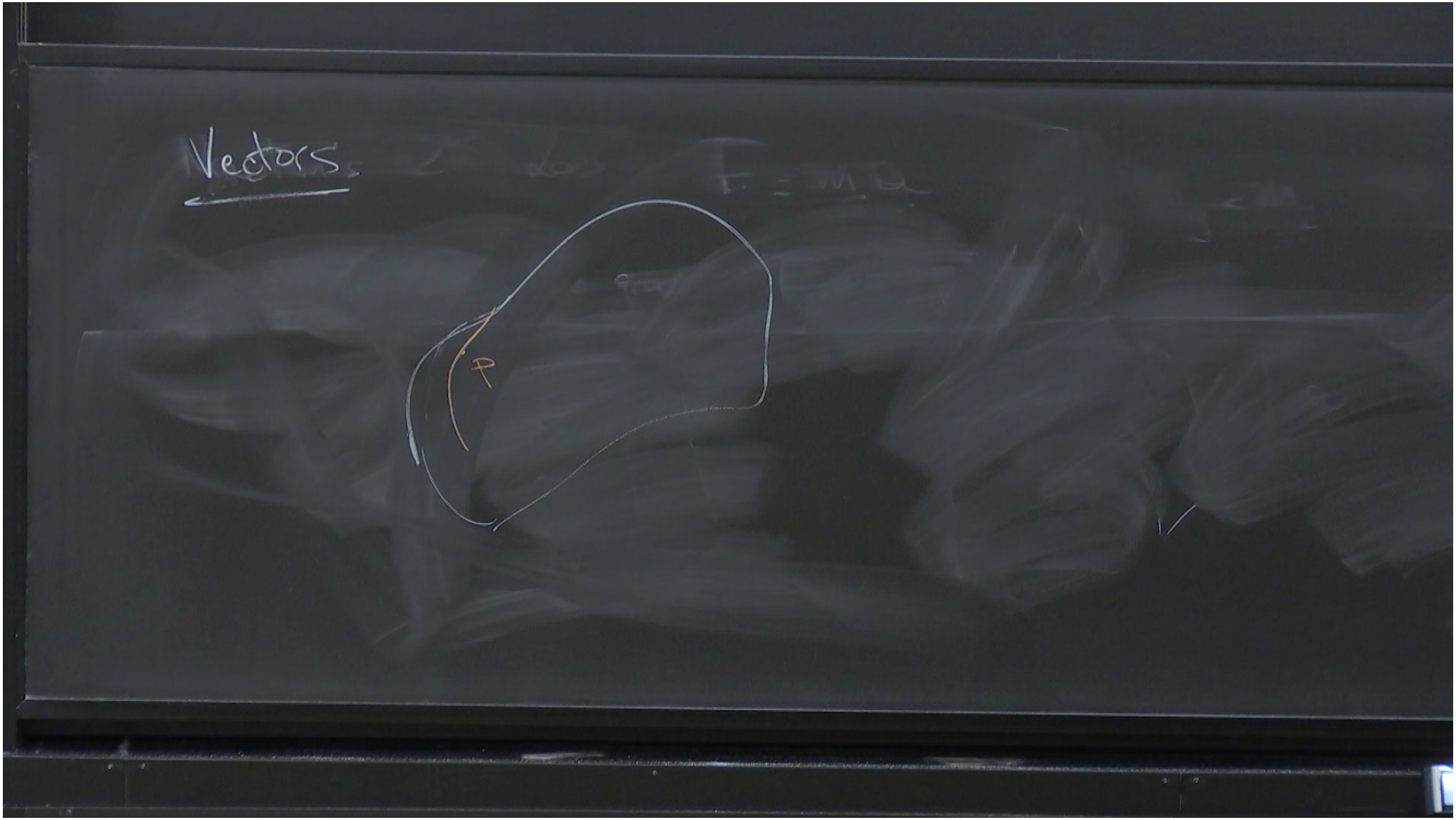
that satisfies

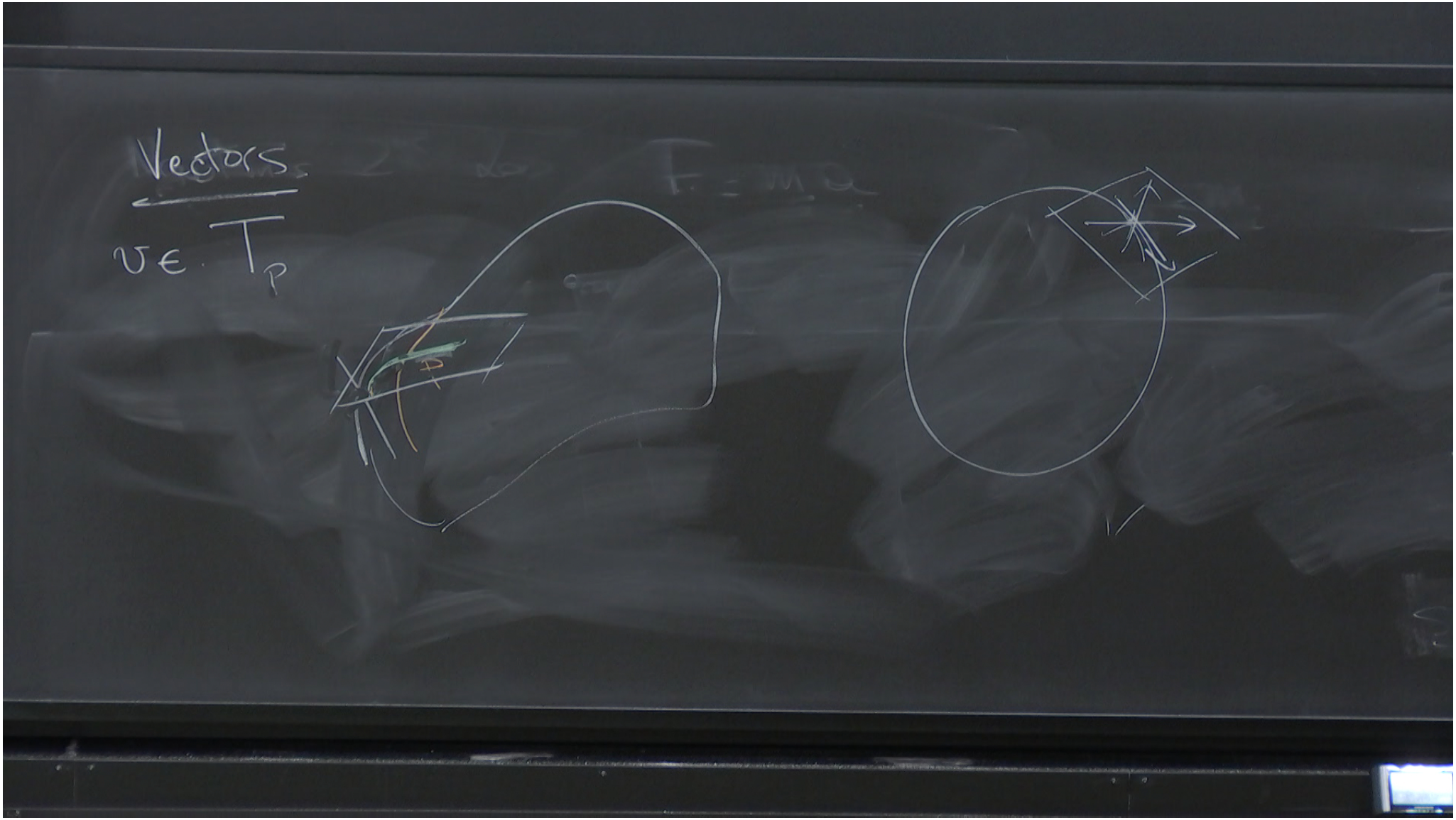
i) $\bigcup U_\alpha = M$

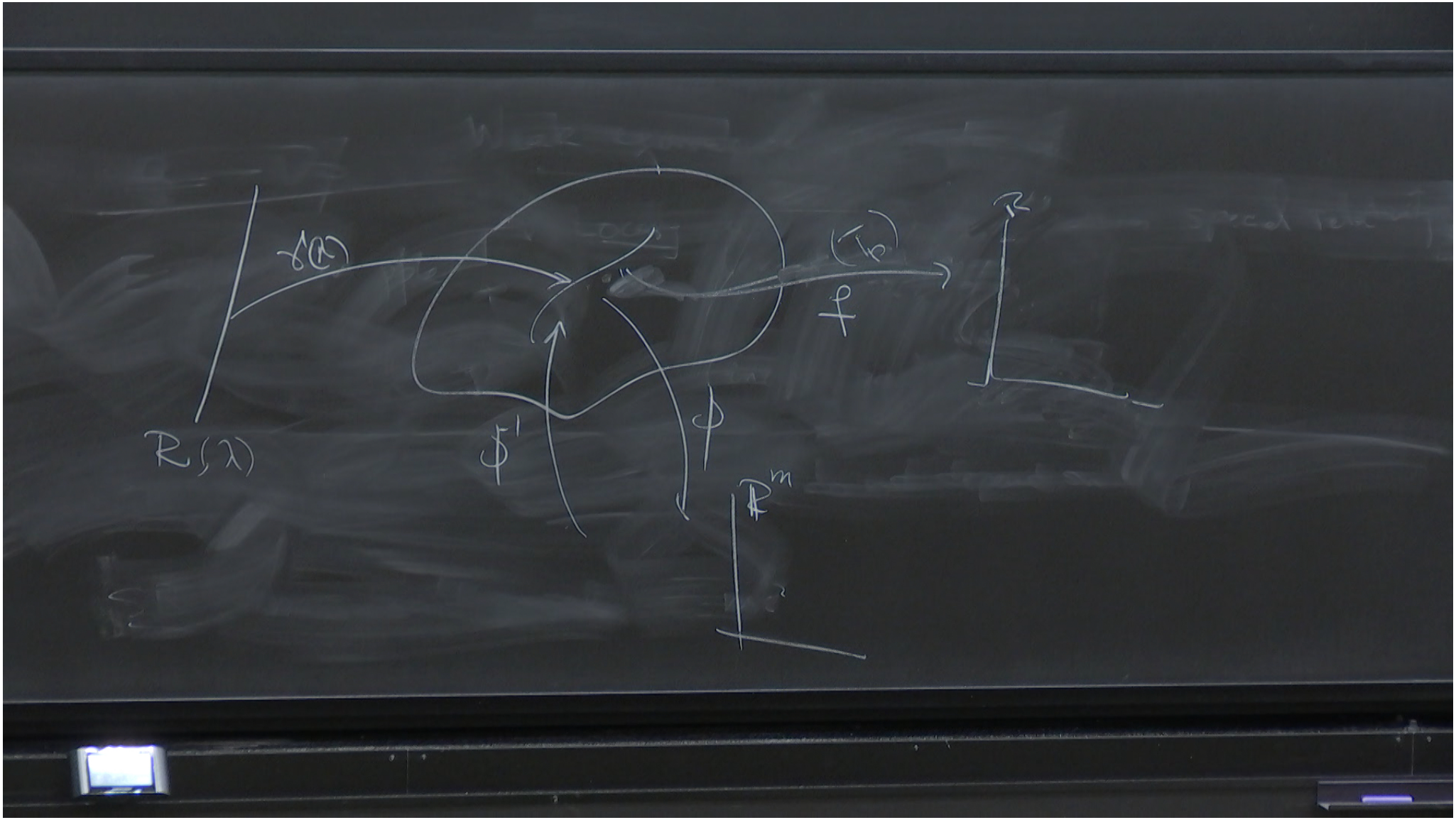
ii) charts are smoothly sewn together.

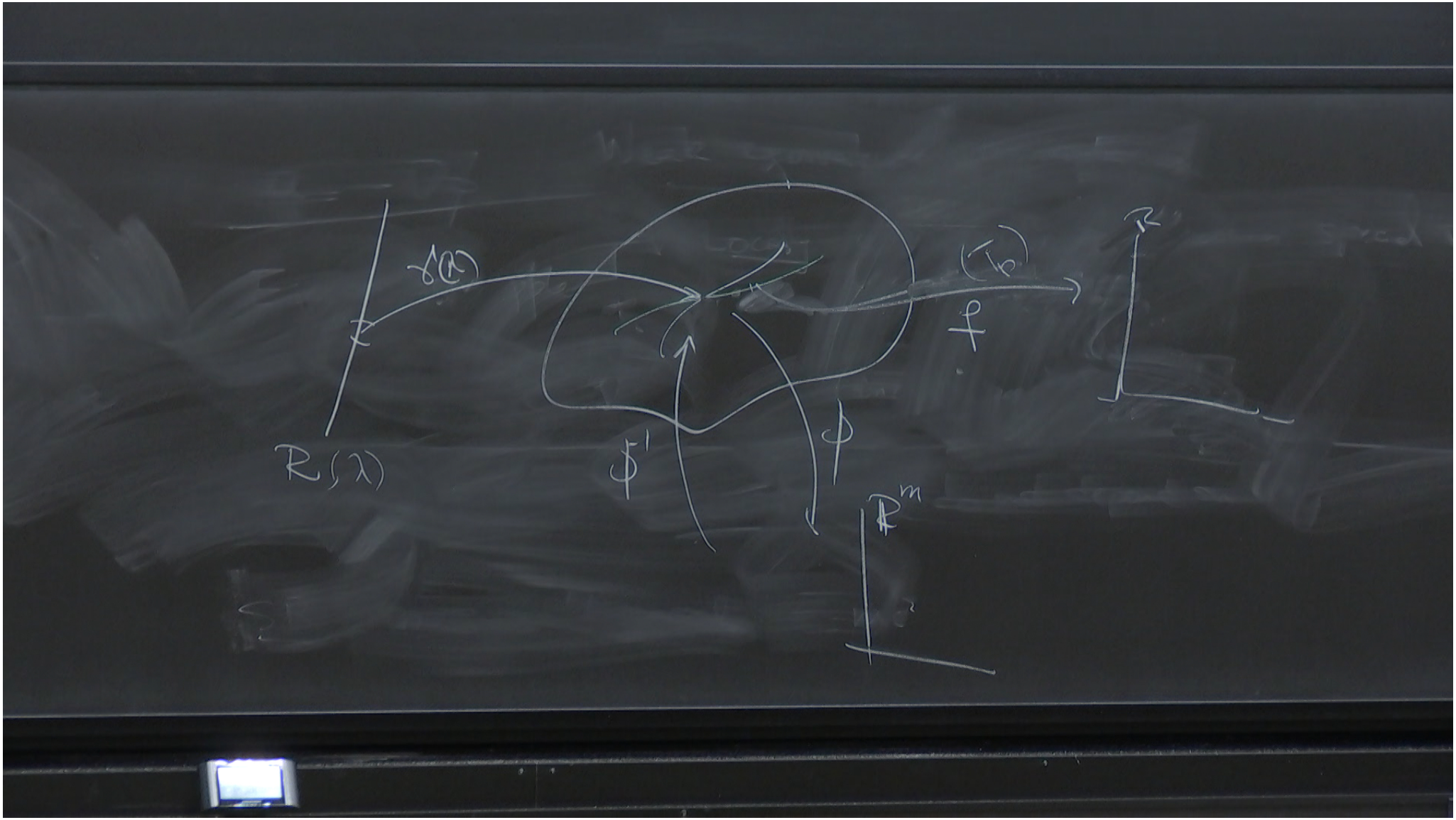
iii) all maps are C^∞.

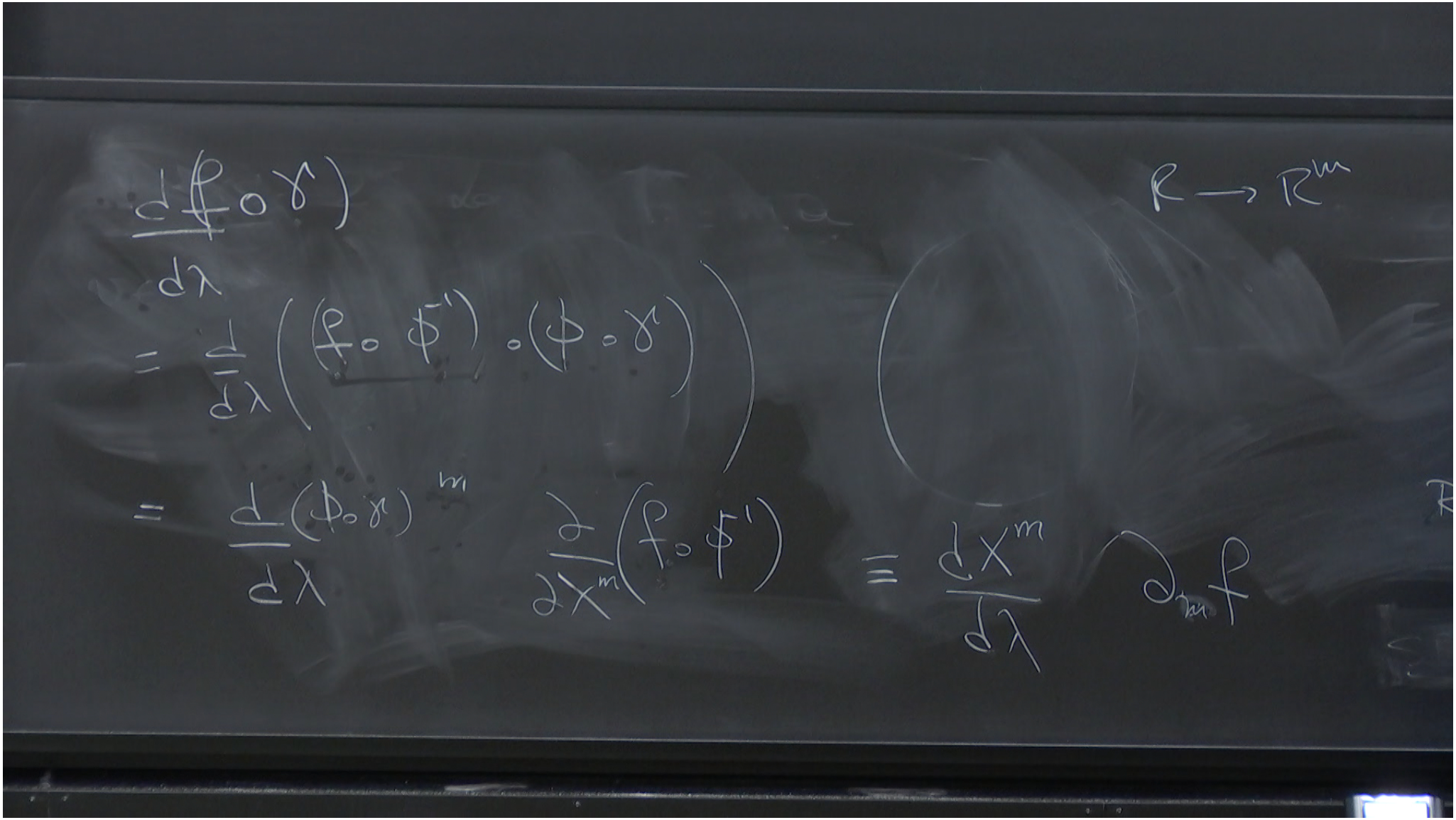

M
U_α
U_β
$\mathbb{R}^n$
ϕ_β
$\mathbb{R}^n$
ϕ_α


Vectors
F = ma
F


Vectors
$v \in T_p$
p


$\gamma(\alpha)$
(T_b)
f
$\mathbb{R}$
$\mathbb{R}(\lambda)$
Φ'
Φ
$\mathbb{R}^m$


γ(t)
R(λ)
φ'
φ
f
R
R^m


$\frac{d(f\circ\gamma)}{d\lambda}$
$= \frac{d}{d\lambda}\left((f\circ\phi^{-1})\circ(\phi\circ\gamma)\right)$
$= \frac{d}{d\lambda}(\phi\circ\gamma)^m \frac{\partial}{\partial x^m}(f\circ\phi^{-1}) \equiv \frac{dx^m}{d\lambda}\partial_m f$
$\mathbb{R} \to \mathbb{R}^m$

$R \to R^m$

$\circ(\phi \circ \gamma)$

$\frac{\partial}{\partial x^m}(f \circ \phi^{-1}) \equiv \frac{dx^m}{d\lambda} \partial_m f$

