Title: A Celestial Matrix Model

Speakers: Charles Marteau

Collection: Quantum Gravity Around the Corner

Date: October 06, 2022 - 3:15 PM

URL: https://pirsa.org/22100061

Abstract: We construct a Hermitian random matrix model that provides a stable non-perturbative completion of Cangemi-Jackiw (CJ) gravity, a two-dimensional theory of black holes in asymptotically flat spacetimes. The matrix model reproduces, to all orders in the topological expansion, the Euclidean partition function of CJ gravity with an arbitrary number of boundaries. The non-perturbative completion enables the exact computation of observables in flat space quantum gravity which we use to explicitly characterize the Bondi Hamiltonian spectrum. We discuss the implications of our results for the flat space S-matrix and black holes.

A Celestial Matrix Model

Charles Marteau, University of British Columbia

Based on 2005.08999, 2103.13422 with V. Godet and 2208.05974, 2205.02240 with A. Kar, L. Lamprou and F. Rosso
QG around the corner conference @ PI
Oct. 6th 2022

The S-matrix & Flat Space Gravity

- Natural observable: *S-matrix*.
- t'Hooft 80's → Assume QFT valid near horizon → Black hole spectrum *continuous*.
- Differs radically from other quantum mechanical systems
- Probing black hole microstates calls for very non-perturbative gravitational effects.
- These effects are the subject of today's talk.

The S-matrix & Flat Space Gravity

- S-matrix in QFT: "understood", what about Quantum Gravity?
- $G_N \neq 0 \rightarrow$ backreaction, black holes, etc.
- Need for a notion of *asymptotic flatness*.
- Classically: studied by Bondi, Metzner and Sachs 60's.
- Quantization out of reach.
- Other approach: Celestial Holography.
- S-matrix $\leftrightarrow$ Correlators of a dual theory [Strominger & al.].
- *Non-perturbative definition* of the theory.

The S-matrix & Flat Space Gravity

- ***In this talk*:**
- 2d model of gravity, very simple phase space (but black holes, infinite-dim symmetry)
- S-matrix can be worked out
- Euclidean observables can be computed exactly
- Connection Lorentzian/Euclidean: interpretation in terms of asymptotic states
- Holographically dual to a matrix model
- Microscopic Bondi spectrum $\rightarrow$ signature of microstates
- Non-perturbative asymptotic Hilbert spaces/S-matrix?

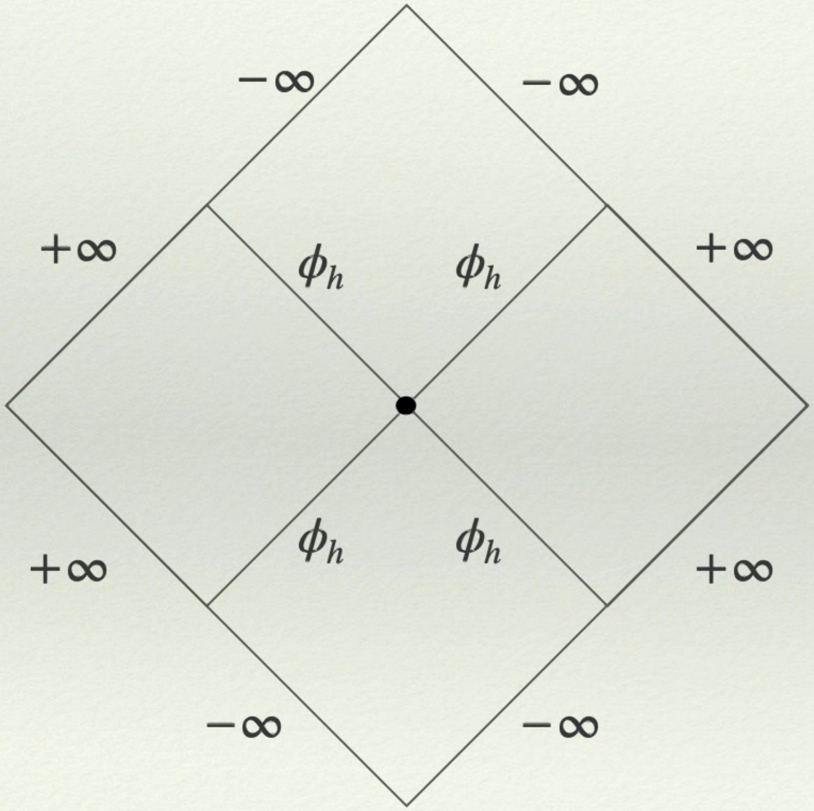
Flat Jackiw–Teitelboim Gravity

- Einstein-Hilbert: topological.
- Simplest dilaton gravity: $\int d^2x\sqrt{g}(\phi R - 2\Lambda)$.
- Related to CGHS without matter by Weyl rescaling.
- EOM: $R = 0, \quad \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi = g_{\mu\nu} \Lambda$.
- Solution on disk: $\phi(\tau, r) = \dfrac{\beta \Lambda r}{2\pi} + O(1)$.
- Dirichlet: $\phi \sim \gamma r \ \rightarrow$ temperature is fixed! $\beta = \dfrac{2\pi\gamma}{\Lambda}$, ($\gamma$ dimensionful)

Cangemi–Jackiw Gravity

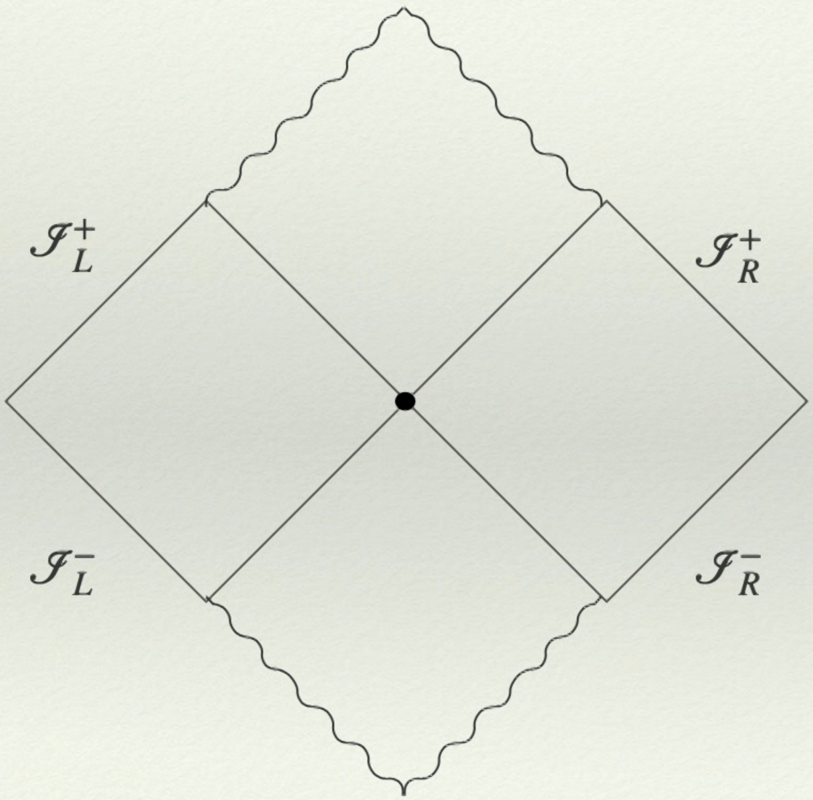
- Non-trivial $Z(\beta)$: make Λ dynamical.
- Integrate in a gauge field to ensure Λ constant on-shell.
- Action : $I_{CJ} = S_0\,\chi + \frac{1}{2}\int d^2x\sqrt{g}(\phi R - 2\Psi + 2\Psi F), \quad F = \star\, dA, \quad S_0 \sim 1/G_N\,.$
- EOM: $R = 0, \quad \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi = g_{\mu\nu}\Psi, \quad \Psi = cst, \quad F = 1.$
- Solution space: $ds^2 = dUdV, \quad \phi = \frac{\Lambda}{2}UV + \phi_h, \quad F = dU \wedge dV, \quad \Psi = \Lambda\,.$
- Parametrized by Λ and ϕ_h .

Cangemi–Jackiw Gravity



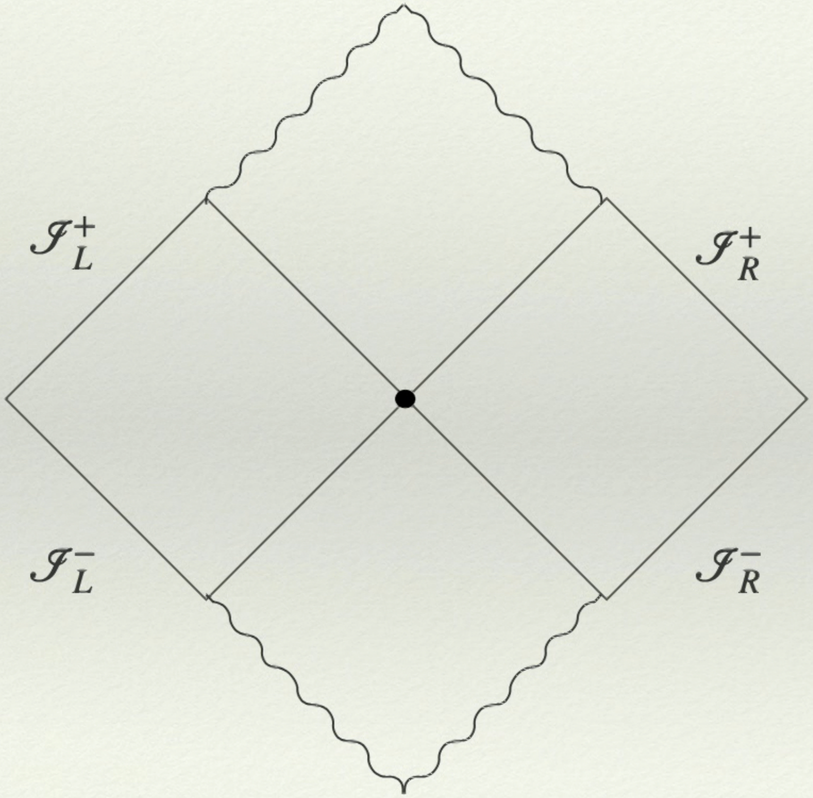
- ϕ is size of transverse sphere in higher-d.
- $\phi = \frac{\Lambda}{2} UV + \phi_h\,, \quad ds^2 = dUdV$

Cangemi–Jackiw Gravity



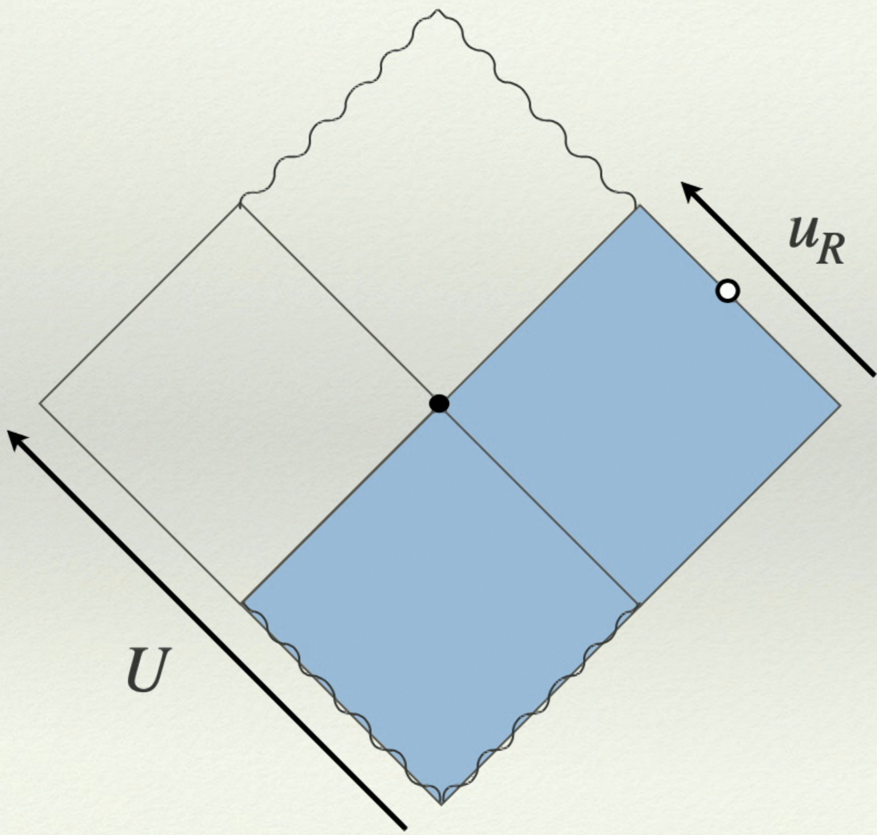
- ϕ is size of transverse sphere in higher-d.
- $\phi = \dfrac{\Lambda}{2} UV + \phi_h\,, \quad ds^2 = dUdV$
- Same causality structure as *eternal black hole* in higher-d.
- Two disjoint universes connected by Einstein-Rosen bridge.
- $\beta_H \sim \dfrac{1}{\Lambda}$ and $\phi_h =$ horizon value of dilaton.

Cangemi–Jackiw Gravity: Phase Space



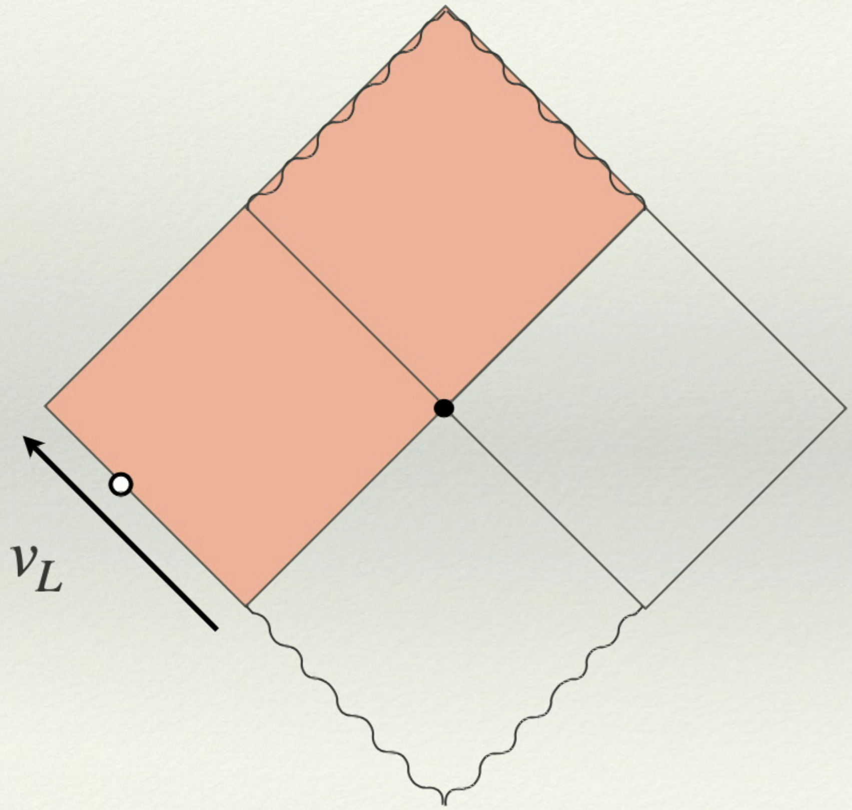
- #d.o.f.? Global $\rightarrow \Lambda, \phi_h$
- Local? Need for a boundary time

Cangemi–Jackiw Gravity: Phase Space



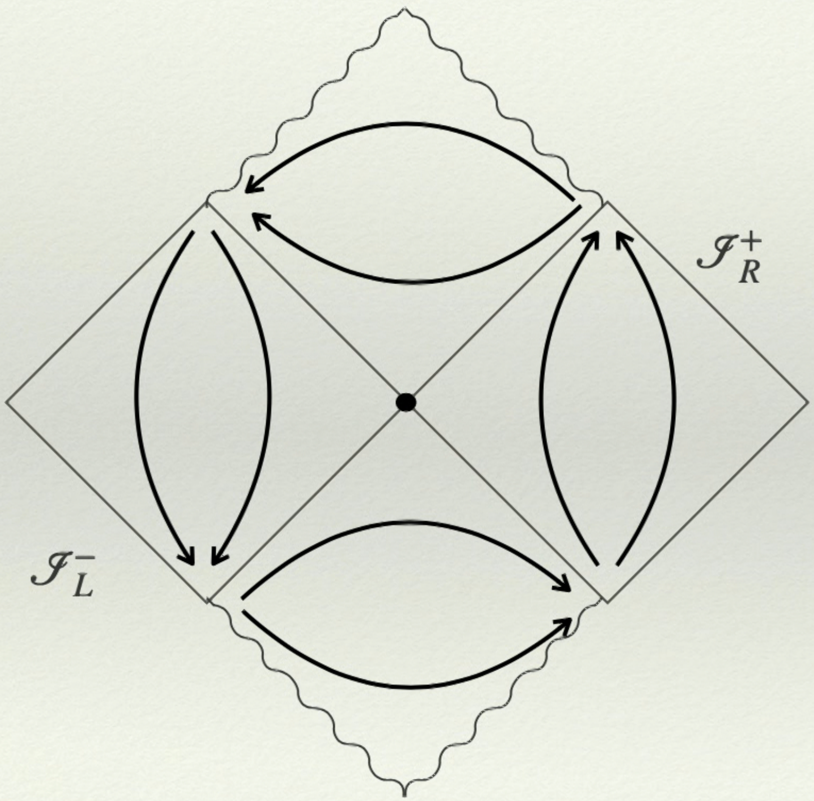
- #d.o.f.? Global $\rightarrow \Lambda, \phi_h$
- Local? Need for a boundary time u_R
- $ds_R^2 = -\frac{4\pi r_R}{\beta} du_R - 2du_R dr_R$
- $U \propto -exp\left[-\frac{\Lambda}{\gamma}\left(u_R - u_R^0\right)\right]$

Cangemi–Jackiw Gravity: Phase Space



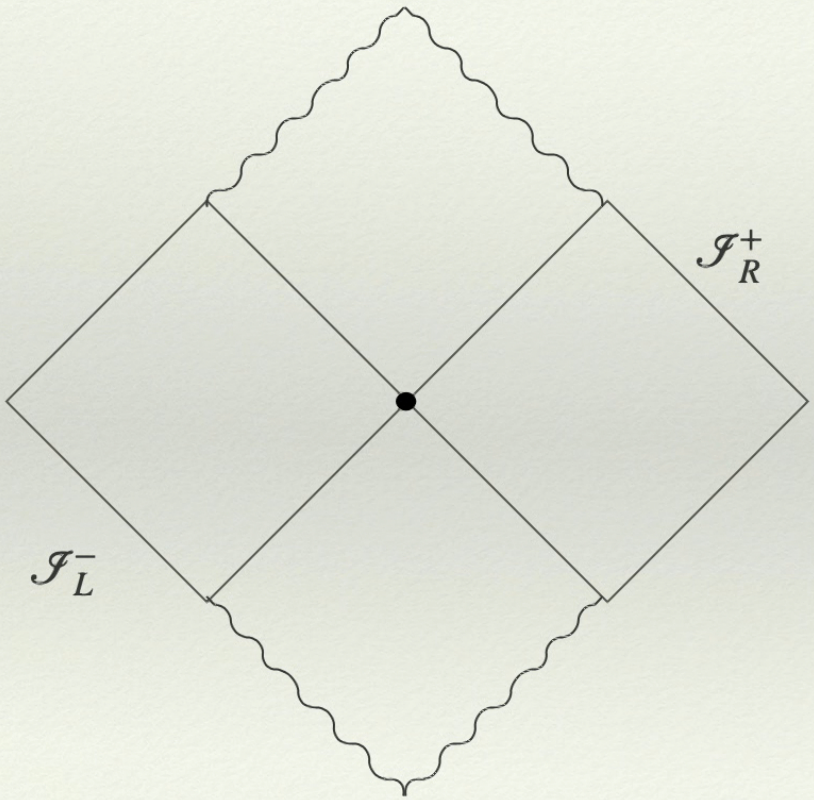
- #d.o.f.? Global $\rightarrow \Lambda, \phi_h$
- Local? Need for a boundary time v_L
- $ds_L^2 = -\frac{4\pi r_L}{\beta} dv_L + 2 dv_L dr_L$
- $U \propto exp\left[\frac{\Lambda}{\gamma}\left(v_L - v_L^0\right)\right]$

Cangemi–Jackiw Gravity: Phase Space



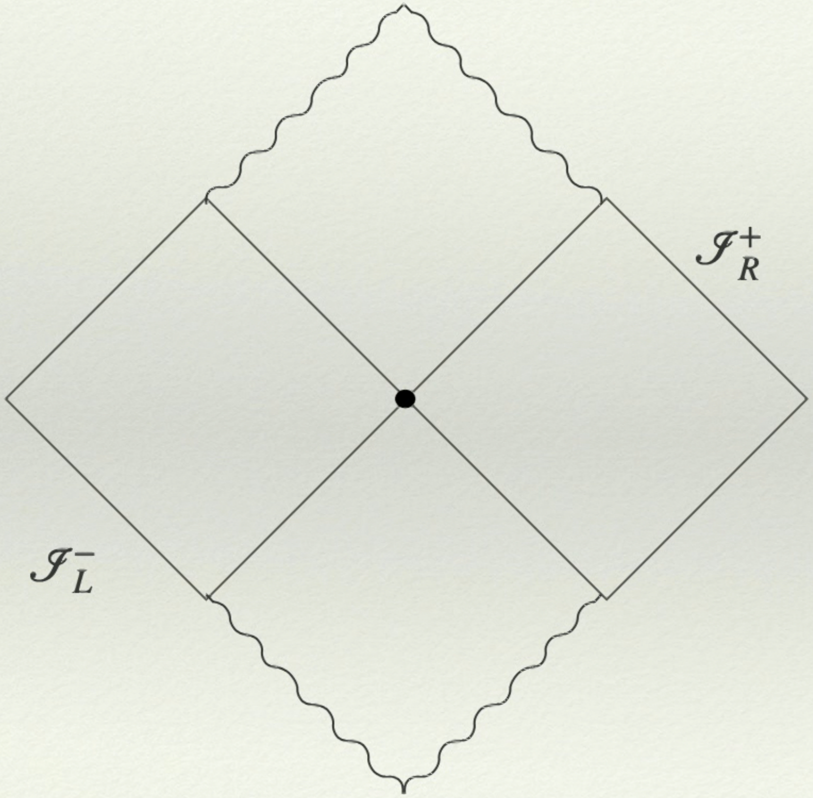
- #d.o.f.? Global $\rightarrow \Lambda, \phi_h$
- Local? Boost symmetry
- 1 physical local d.o.f : $\dfrac{u_R^0 + v_L^0}{2} \equiv u_{LR}$

Cangemi–Jackiw Gravity: Phase Space



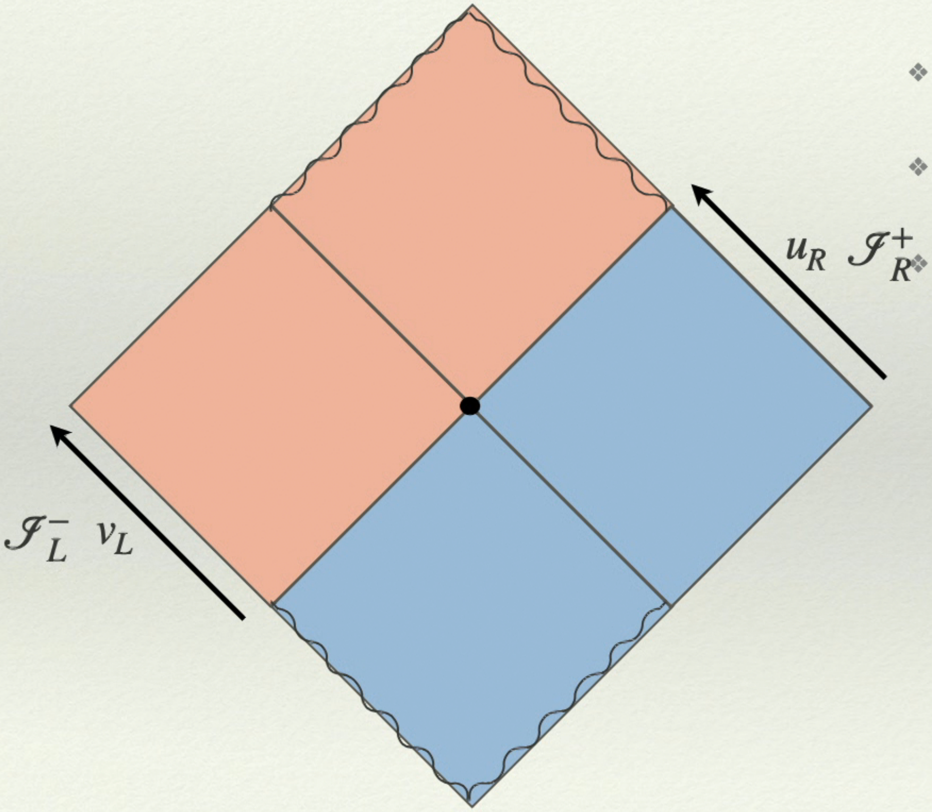
- #d.o.f.? Global $\to \Lambda, \phi_h$
- Local? Boost symmetry
- 1 physical local d.o.f : $\dfrac{u_R^0 + v_L^0}{2} \equiv u_{LR}$
- Gauge field: $A_R = r_R du_R + dg_R$
- Wilson line $\displaystyle\int_L^R A = g_R - g_L \equiv g_{LR}$
- 4d phase space: $\Lambda, \phi_h, u_{LR}, g_{LR}$
- Symplectic form?

Cangemi–Jackiw Gravity: Phase Space



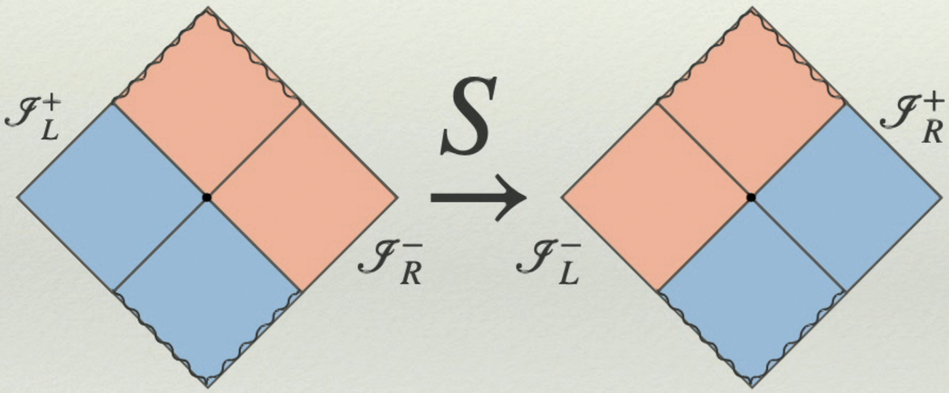
- #d.o.f.? Global $\rightarrow \Lambda, \phi_h$
- Local? Boost symmetry
- 1 physical local d.o.f : $\dfrac{u^0_R + v^0_L}{2} \equiv u_{LR}$
- Gauge field: $A_R = r_R du_R + dg_R$
- Wilson line $\displaystyle\int_L^R A = g_R - g_L \equiv g_{LR}$
- 4d phase space: $\Lambda, \phi_h, u_{LR}, g_{LR}$
- $\omega = \delta u_{LR} \wedge \delta E + \delta g_{LR} \wedge \delta\Lambda, \quad E \equiv 2\gamma^{-1}\Lambda\,\phi_h$

Cangemi–Jackiw Gravity: Classical S-Matrix



- $\omega = \delta u_{LR} \wedge \delta E + \delta g_{LR} \wedge \delta\Lambda, \quad E \equiv 2\gamma^{-1}\Lambda\,\phi_h$
- Phase space associated to $\mathscr{I}_L^- \cup \mathscr{I}_R^+$
- $\rightarrow \omega^{-+} = \delta u_{LR}^{-+} \wedge \delta E + \delta g_{LR}^{-+} \wedge \delta\Lambda$

Cangemi–Jackiw Gravity: Classical S-Matrix



- $\omega = \delta u^0_{LR} \wedge \delta E + \delta g_{LR} \wedge \delta \Lambda, \quad E \equiv 2\gamma^{-1}\Lambda\,\phi_h$
- Phase space associated to $\mathscr{I}_L^- \cup \mathscr{I}_R^+ \rightarrow \omega^{-+}$
- Second copy associated to $\mathscr{I}_L^+ \cup \mathscr{I}_R^- \rightarrow \omega^{+-}$
- Classical S-matrix = symplectomorphism
- $S^*(\omega^{+-}) = \omega^{-+}$
- Corresponds to a matching of ingoing and outgoing Wilson lines

$$S : (u^{+-}_{LR}, g^{+-}_{LR}, E, \Lambda) \rightarrow (u^{-+}_{LR}, g^{-+}_{LR}, E, \Lambda)$$

Cangemi–Jackiw Gravity: Classical S-Matrix

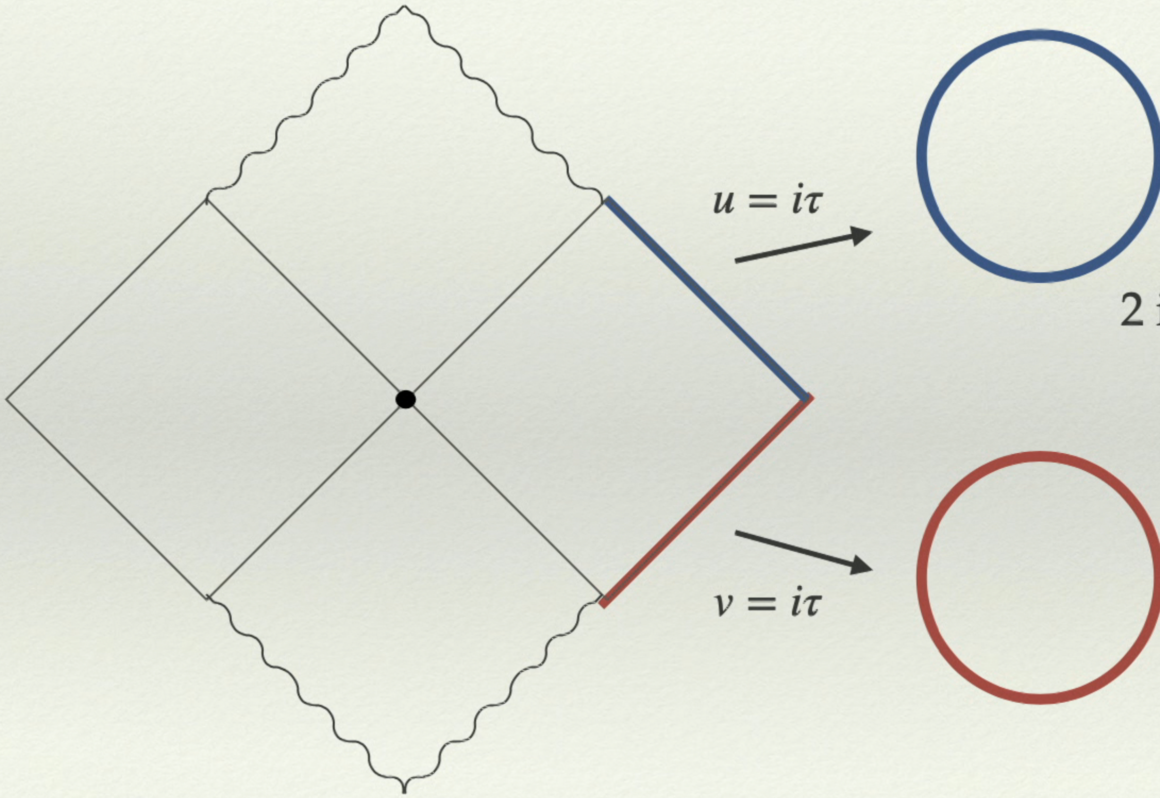
- Most general S-matrix? We define $x = (u_{LR}, g_{LR})$ and $p = (E, \Lambda)$,

  $\exists K(p_1^{-+}, p_2^{-+})$ such that $\quad S : p_i^{+-} \to p_i^{-+}$
  $$x_i^{+-} \to x_i^{-+} + \partial_{p_i^{-+}} K$$

- Quantum mechanically $\quad p_i^{+-} = p_i^{-+}, \quad \partial_{p_i^{+-}} = \partial_{p_i^{-+}} + i\partial_{p_i^{-+}} K$
- S-matrix is nothing but a **trivial** diagonal phase redefinition

$$|p_1, p_2\rangle^{-+} = \exp[-iK(p_1, p_2)]\,|p_1, p_2\rangle^{+-}$$

Euclidean Path Integral



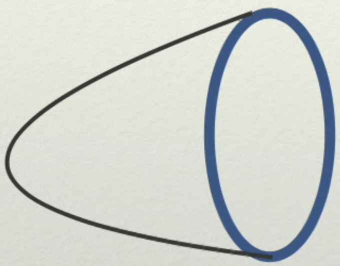
$$ds^2 = \frac{4\pi r}{\beta} d\tau^2 - 2i d\tau dr$$

2 inequivalent analytic continuation
2 inequivalent disks
$\tau \sim \tau + \beta$

$$ds^2 = \frac{4\pi r}{\beta} d\tau^2 + 2i d\tau dr$$

Euclidean Path Integral

Euclidean black hole in higher-d



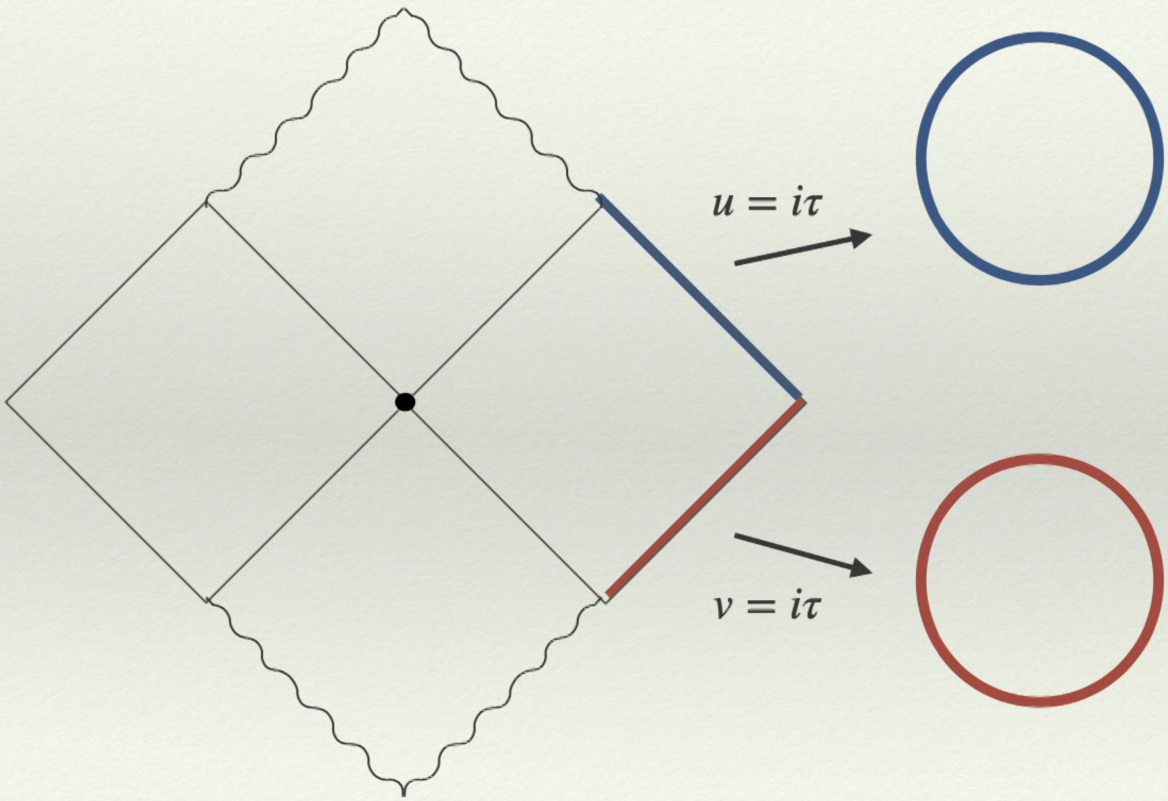
× ~~S^d~~

In 2d it's a disk

Disk = saddle point of Euclidean path integral

$$Z_{grav}(\beta) = \int D\Phi \, e^{-I_{CJ}} \sim e^{-I_{disk}}$$

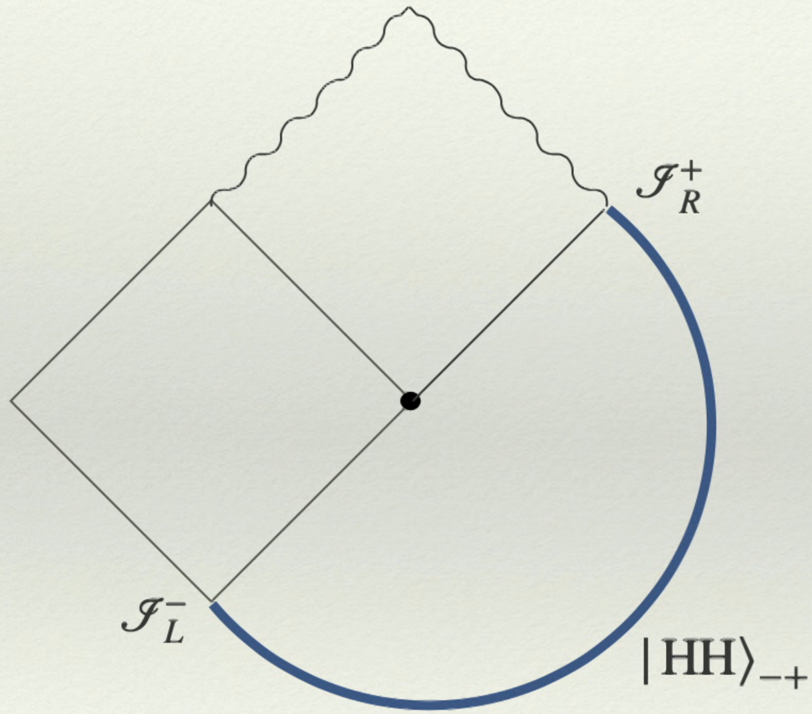
Euclidean Path Integral



$$Z^{+}_{grav}(\beta) = \mathrm{Tr}\, e^{-\beta H^{+}}$$

$$Z^{-}_{grav}(\beta) = \mathrm{Tr}\, e^{-\beta H^{-}}$$

Euclidean Path Integral



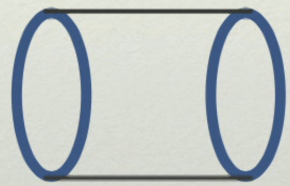
$$|\mathrm{HH}\rangle_{-+} \in \mathscr{H}_{\mathscr{I}_L^- \cup \mathscr{I}_R^+}$$

Euclidean Path Integral

- Simplicity of the model: $Z^{\pm}_{grav}(\beta)$ can be computed exactly.
- Need to define off-shell configurations:
- $\tau \to f(\tau), \quad A \to A + dg(\tau), \quad (f, g) \in \text{BMS}_2 \simeq$ Warped Virasoro.
- Off-shell metric: $2(P(\tau)r + T(\tau))d\tau^2 + 2id\tau dr$, where $\tau \sim \tau + \beta, \quad P(\tau) = \dfrac{2\pi}{\beta}f' - \dfrac{f''}{f'}, \quad T = \ldots$
- Becomes boundary path integral $\quad Z^{\pm}_{grav}(\beta) = \displaystyle\int DfDg\, e^{-I_\partial[f,g]}$.
- Where $I_\partial(f, g)$ breaks $\text{BMS}_2 \to ISO(1,1) \times \mathbb{R}$

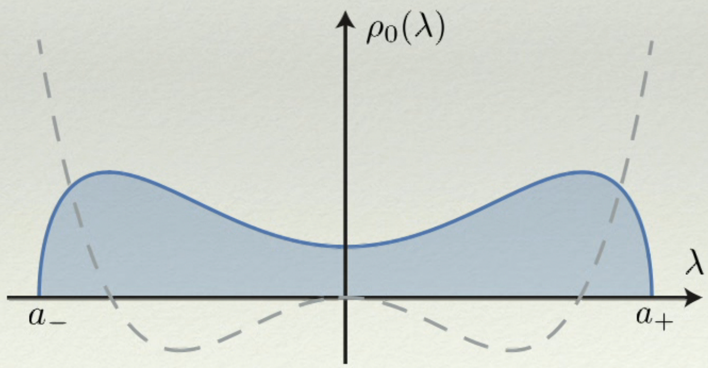
Euclidean Path Integral

Euclidean Path Integral

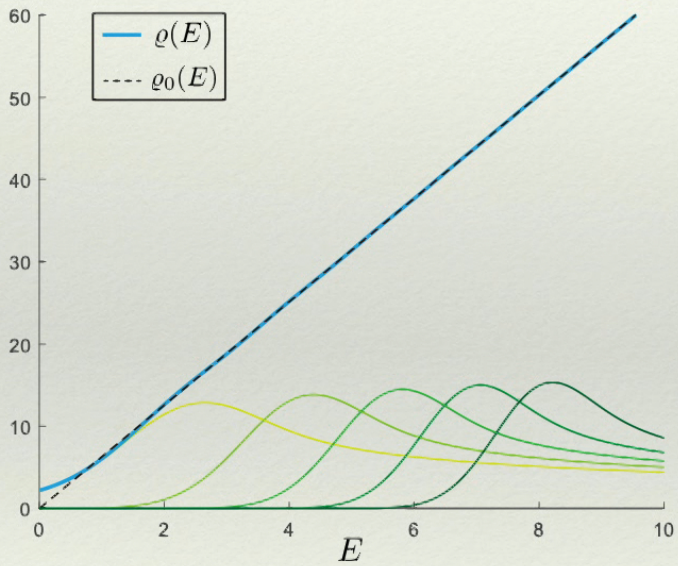
- Result: $Z^{\pm}(\beta) = \dfrac{e^{S_0}}{\beta^2} \quad \leftrightarrow \quad \rho^{\pm}(E) = e^{S_0}E \sim O(e^{1/G_N}).$
- Non-perturbative in G_N but *perturbative* in 1/#microstates $\sim e^{-S_0}$
- Multiple boundaries? Only cylinder: $Z^{\pm}(\beta_1, \beta_2) = \dfrac{1}{\beta_1 + \beta_2} =$ β_1  β_2
- $Z^{\pm}(\beta_1, \ldots, \beta_n) = 0, \quad n > 2$

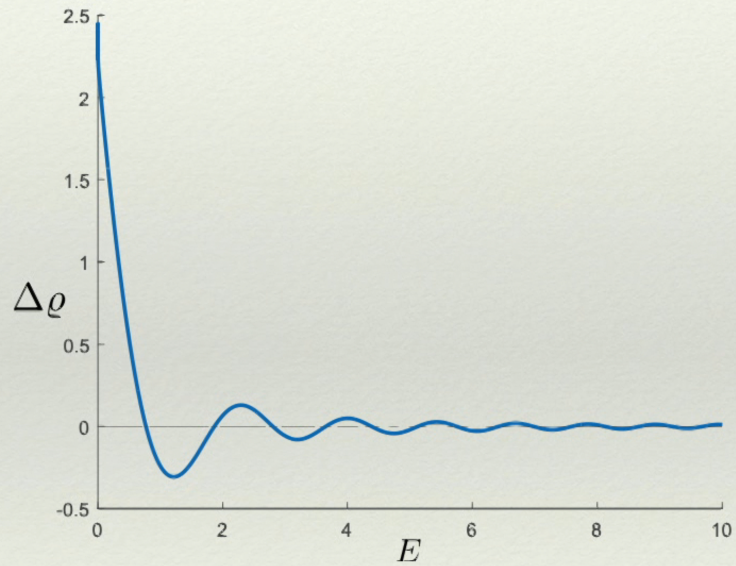
Celestial Matrix Model

- Our proposal: non-perturbative effects captured by *Matrix Model*.
- Replace $Z^{\pm}_{grav}(\beta) \to \langle \mathbb{O}^{\pm}(\beta) \rangle_{\rm MM}$, with $\langle \, .. \rangle_{\rm MM} = \int dM e^{-N{\rm Tr}V(M)}$, square Hermitian matrices
- $V(M) = -M^2 + \frac{1}{4}M^4$ (critical)
- $\mathbb{O}^{\pm}(\beta) = {\rm Tr} e^{-\beta H^{\pm}}, \quad H^{\pm} = M^2 \otimes \mathbb{I} + \mathbb{I} \otimes \hat{p}.$
- Double scaling limit $a_{\pm} \to \infty$
- $\langle \mathbb{O}^{\pm}(\beta) \rangle_{\rm MM} = \frac{e^{S_0}}{\beta^2} + O\left(e^{-e^{S_0}}\right).$



Celestial Matrix Model





Frequency of oscillations $\sim e^{-S_0}$

Conclusion

- 2d model of *two sided eternal black holes*.
- Can be canonically quantized to obtain asymptotic Hilbert spaces and S-matrix.
- Pure gravity S-matrix is trivial, need to include matter.
- Euclidean formulation of thermal trace of *Bondi Hamiltonian*.
- Holographic duality with Matrix Model.
- Allows for computation of the non-perturbative spectrum of Bondi Hamiltonian, *black hole microstates*.

Conclusion

- 2d model of *two sided eternal black holes*.
- Can be canonically quantized to obtain asymptotic Hilbert spaces and S-matrix.
- Pure gravity S-matrix is trivial, need to include matter.
- Euclidean formulation of thermal trace of *Bondi Hamiltonian*.
- Holographic duality with Matrix Model.
- Allows for computation of the non-perturbative spectrum of Bondi Hamiltonian, *black hole microstates*.
- *In the paper*: proposal for how to use Matrix Model to construct non-perturbative asymptotic Hilbert spaces and S-matrix elements.

Thank you!