

Title: Berry phases, wormholes and factorization in AdS/CFT

Speakers: Johanna Erdmenger

Series: Quantum Fields and Strings

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Abstract: Within the AdS/CFT correspondence, the entanglement properties of the CFT are related to wormholes in the dual gravity theory. This gives rise to questions about the factorisation properties of the Hilbert spaces on both sides of the correspondence. We show how the Berry phase, a geometrical phase encoding information about topology, may be used to reveal the Hilbert space structure. Wormholes are characterized by a non-exact symplectic form that gives rise to the Berry phase. For wormholes connecting two spacelike regions in AdS<sub>3</sub> spacetimes, we find that the non-exactness is linked to a variable appearing in the phase space of the boundary CFTs. Mathematical concepts such as coadjoint orbits and geometric actions play an important role in this analysis. We classify Berry phases according to the type of dual bulk diffeomorphism involved, distinguishing between Virasoro, gauge and modular Berry phases.

In addition to its relevance for quantum gravity, the approach presented also suggests how to experimentally realize the Berry phase and its relation to entanglement in table-top experiments involving photons or electrons. This provides a new example for relations between very different branches of physics that follow from the AdS/CFT correspondence and its generalizations. Based on 2202.11717 and 2109.06190.

Zoom link: <https://pitp.zoom.us/j/96113910200?pwd=YXJnSWxiMHRIb21xdGFnNFM0cFFvUT09>

# Berry phase, factorization and wormholes in AdS/CFT

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# Overview

- Wormholes and factorization in AdS/CFT:
- Berry phase in quantum mechanics
- 2d CFTs and their gravity dual
- Relation to von Neumann algebras

# Berry phases for wormholes: Motivation

- Wormholes provide relation between entanglement and geometry in AdS/CFT  
van Raamsdonk; Maldacena, Susskind
- Concept of wormhole also present in simple quantum mechanics  
H. Verlinde
- Berry phase provides a geometrical understanding of how degrees of freedom entangle
- Factorization puzzle
- Also for AdS<sub>3</sub>/CFT<sub>2</sub>

## Part I based on

- Berry phase in quantum mechanics and wormholes

Nogueira, Banerjee, Dorband, J.E., Meyer, van den Brink  
arXiv:2109.06190, PRD

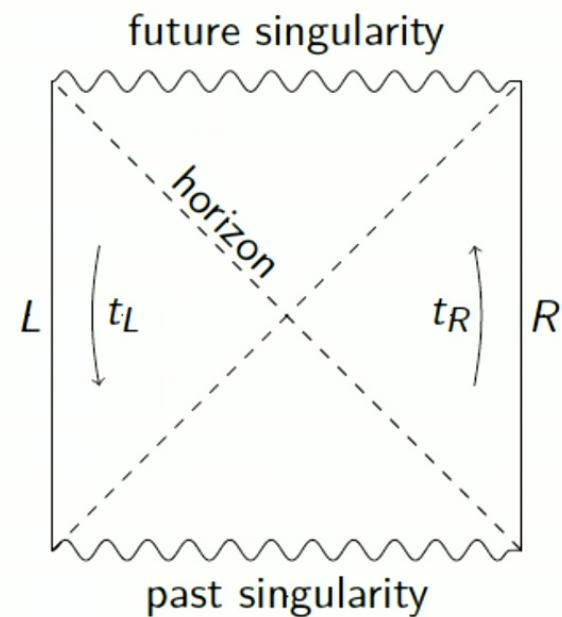
- Berry phase in AdS<sub>3</sub>/CFT<sub>2</sub>

Banerjee, Dorband, J.E., Meyer, Weigel arXiv:2202.11717 , JHEP



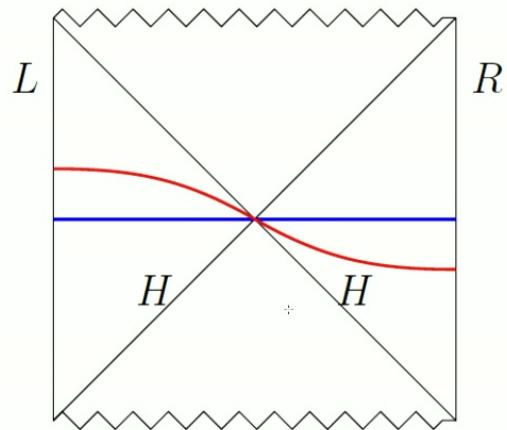
## Eternal AdS black hole

- Global coordinates (Kruskal)
- Non-traversable wormhole
- Singularity in time coordinate:  
Time-like Killing vector  
switches sign at horizon



# $ER = EPR$

Van Raamsdonk 2010; Maldacena, Susskind 2013

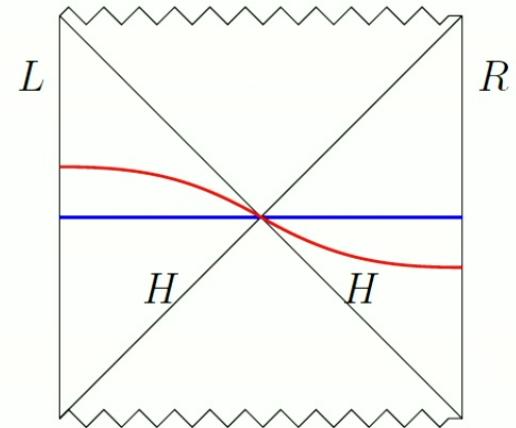


- Relation between entanglement and geometry
- EPR: Einstein-Podolsky-Rosen entanglement
- ER: Einstein-Rosen bridge (wormhole)
- Two entangled CFTs with EPR correlation are connected through a wormhole (ER bridge)

## Factorization puzzle

Maldacena+Maoz '13; Harlow '16

- The two CFTs have disjoint Hilbert spaces since there is no interaction between them,  $\mathcal{H}_L \otimes \mathcal{H}_R$
- The wormhole Hilbert space does not factorize
- Apparent contradiction?



# Wormholes in quantum mechanics

Verlinde 2021

$$Z(\beta) = \text{tr}(e^{-\beta H})$$

$$Z(D) = \int [dX] e^{\int_D \Omega - \oint_{\partial D} H dt}$$



generalized coordinates and momenta  $X^a$ , symplectic form  $\Omega = \frac{1}{2}\omega_{ab}dX^a \wedge dX^b$

Exact symplectic structure:  $\Omega = d\alpha$ ,  $\int_D \Omega = \oint_{\partial D} \alpha$

$$Z(\beta) = Z(D)$$

If symplectic structure is non-exact:

$$\langle Z(\beta)^n \rangle = Z(\Sigma_n)$$

$$\Sigma_n =$$



# Berry phases in wormholes and in quantum mechanics

- Berry phase for an interacting two-qubit system in quantum mechanics
- States with the same entanglement may have different Berry phase
- Mathematically described by a non-exact symplectic form:
- Non-factorization! Wormholes share this mathematical structure
- May be realised experimentally



# Berry Phase

- Time-dependent Schrödinger eq.
- Ground state
- Berry connection
- Berry phase

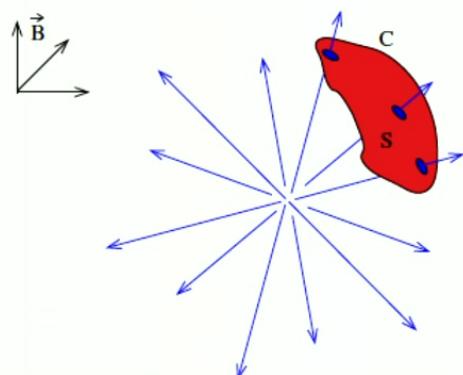
$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = H(\lambda(t))|\psi\rangle$$

$$|\psi(t)\rangle = U(t) |n(\lambda(t))\rangle$$

$$\mathcal{A}_i(\lambda) = -i\langle n | \frac{\partial}{\partial \lambda^i} | n \rangle$$

$$\dot{U} = -i\mathcal{A}_i \dot{\lambda}^i U$$

$$e^{i\gamma} = \exp \left( -i \oint_C \mathcal{A}_i(\lambda) d\lambda^i \right)$$



Review: Lectures by D. Tong

# Non-Factorization model in quantum mechanics

Nogueira, Banerjee, Dorband, J.E., Meyer, van den Brink '21

- Coupled spins in external magnetic field:  
Electronic Zeeman interaction in hydrogen atom 
$$H = JS_1 \cdot S_2 - 2\mu_B BS_{1z}$$
- Generalize second term to  $\propto B \cdot S_1$  and consider unitary transformations returning to the original Zeeman term:

$$U_1(\phi, \theta) = e^{-i\phi S_{1z}} e^{-i\theta S_{1y}} e^{-i\psi S_{1z}} \text{ for two patches} \begin{cases} \phi = \psi \\ \phi = -\psi \end{cases}$$

- Transformation of second spin is free to choose:

$$U_2(\phi, \theta) = U_1(\lambda\phi, \lambda\theta), \quad \lambda \in [0, 1] \qquad \qquad U = U_1 \otimes U_2$$

# Ground state and entanglement entropy

- Ground state ( $|s,m\rangle$  singlet and triplet states):

$$|\psi_0\rangle = -\sin\left(\frac{\alpha}{2}\right)|1,0\rangle + \cos\left(\frac{\alpha}{2}\right)|0,0\rangle \quad \tan \alpha = 2\mu_B B/J$$

- Entanglement entropy not changed when applying the unitary transformation that acts on each spin individually:

$$S_{\text{EE}}^i(\text{tr}_j(|\psi_0\rangle\langle\psi_0|)) = S_{\text{EE}}^i(\text{tr}_j(U|\psi_0\rangle\langle\psi_0|U^\dagger)) \quad \text{where } i, j \in \{1, 2\} \text{ and } i \neq j.$$

$$S_{\text{EE}} = -\ln\left(\frac{\cos(\alpha)}{2}\right) - \operatorname{arctanh}(\sin(\alpha))\sin(\alpha)$$

# Berry phase

- **Maurer-Cartan form:** Connection on a group manifold  $M$  defined for any group element  $\sigma$

- $A_{\text{MC}} = \sigma^{-1} d\sigma$

- **Berry connection:** Ground state expectation value of the **Maurer-Cartan form**

$$\begin{aligned} A_B(\lambda) &= i \langle \psi_0 | A_{\text{MC}} | \psi_0 \rangle = i \langle \psi_0 | (U^\dagger dU) | \psi_0 \rangle \\ &= \frac{\sin \alpha}{2} \{(1 - \cos \theta) - \lambda [1 - \cos(\lambda \theta)]\} d\varphi \end{aligned}$$

- **Berry curvature:**  $F_B(\lambda) = i \langle \psi_0 | \omega_{\text{KK}} | \psi_0 \rangle = \frac{\sin \alpha}{2} (\sin \theta - \lambda^2 \sin(\lambda \theta))$

$$\omega_{\text{KK}} = dA_{\text{MC}}$$

- **Berry phase:**  $\Phi_B = \int_0^{2\pi} \int_0^\pi F_B(\lambda) = \pi \sin \alpha \{2 - \lambda [1 - \cos(\lambda \pi)]\}$

Kirillov-Kostant  
symplectic form

# Berry Phase

- Time-dependent Schrödinger eq.
- Ground state
- Berry connection
- Berry phase

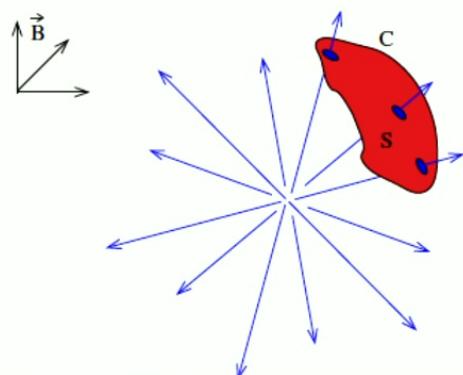
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Review: Lectures by D. Tong

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# Berry phase

- Berry phase:  $\Phi_B = \int_0^{2\pi} \int_0^\pi F_B(\lambda) = \pi \sin \alpha \{2 - \lambda[1 - \cos(\lambda\pi)]\}$

non-trivial as long as  $\lambda \neq 1$ , i.e.  $U_2 \neq U_1$

Non-trivial since Kirillov-Kostant form is not globally exact  
(form proportional to volume form of  $S^2$ )

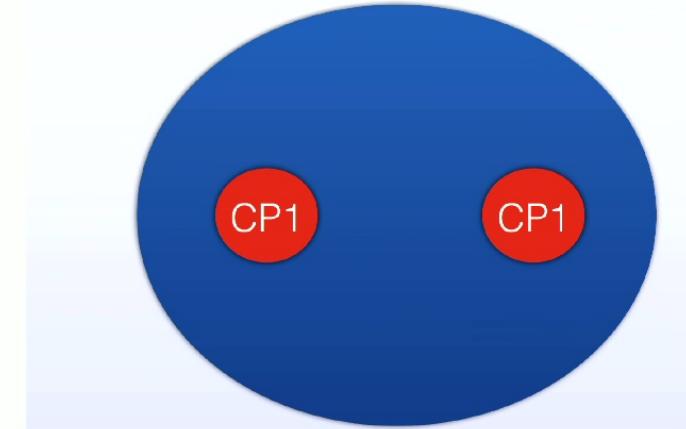
two states  $|\psi_0\rangle$  and  $U|\psi_0\rangle$

have the same entanglement entropy but different Berry phase



# State spaces

- Projective Hilbert space  $\text{CP}3 = \text{SU}(4)/\text{U}(3)$
- $\text{CP}1 \times \text{CP}1$ : Two single-qubit subspaces of  $\text{CP}3$



**No entanglement:** ground state lies in  $\text{CP}1 \times \text{CP}1$   
Describes local (one-sided) properties

**Maximal entanglement:**

States in non-diagonally embedded submanifold of  $\text{CP}3$   
Maximally entangled orbit is Lagrangian submanifold

$$\frac{\text{SU}(2)}{\text{U}(1)} \times \frac{\text{SU}(2)}{\text{U}(1)} \rightarrow \frac{\text{SU}(2) \times \text{SU}(2)}{\text{U}(1)}$$

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# Quantum tomography

Rainer Blatt and David Wineland, “Entangled states of trapped atomic ions,” [Nature 453, 1008–1015 \(2008\)](#).

A quantum state is reconstructed using measurements on an ensemble of identical quantum states

## Density matrix

For  $|\Psi_\phi\rangle = (|00\rangle + \exp(i\phi)|11\rangle)/\sqrt{2}$  , the measurement of the product of Pauli matrices  $\sigma_x \otimes \sigma_x$  allows to reconstruct the phase

$$\langle \Psi_\phi | \sigma_x \otimes \sigma_x | \Psi_\phi \rangle = \cos \phi$$



# Quantum tomography and Berry phase: Proposals for experiments

- Simultaneous measurement of Berry phase and entanglement involves interference between original and rotated state for an ensemble of identical quantum states
- Hydrogen atom in magnetic field
- Multiple-spin quits accessible in liquid-state NMR, [Ryan et al 0808.3973](#)
- Quantum dots coupled to optical cavity, [A Imamoglu et al, quant-ph/9904096](#)
- Superconducting quantum circuits
  - [M. Steffen et al Science 313 \(2006\), C. Song et al 1703.10302](#)
- Quantum simulators (teleportation)
  - [Bhattacharyya, Joshi, Sundar 2111.11945](#)



### III. Berry phases in AdS<sub>3</sub>/CFT<sub>2</sub>

Banerjee, Dorband, J.E., Meyer, Weigel arXiv:2202.11717

# AdS<sub>3</sub>/CFT<sub>2</sub>

Virasoro group:  $\widehat{Diff}(S^1)$  group elements  $(f(\phi), \alpha)$

generator:  $T(\phi) = \sum_n L_n e^{in\phi} \quad [L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}m(m^2 - 1)\delta_{m+n,0}$

Highest weight-state  $|h\rangle$ :

CFT vacuum  $|0\rangle$ : invariant under  $L_{-1}, L_0, L_1$ , SL(2,R) symmetry

General  $|h\rangle$  with  $h > 0$  invariant under  $L_0$ , U(1) symmetry

Stabilizer group

Left- and right-moving sector:

$$|0\rangle \quad \frac{\widehat{Diff}(S^1)}{SL(2,R)} \times \frac{\widehat{Diff}(S^1)}{SL(2,R)}$$

$$|h\rangle \quad \frac{\widehat{Diff}(S^1)}{U(1)} \times \frac{\widehat{Diff}(S^1)}{U(1)}$$

# AdS<sub>3</sub>/CFT<sub>2</sub>

In 3d, the bulk spacetime is completely fixed by

- Boundary metric:  $g_{ij}^{(0)}$
- Expectation value  $\langle h | T(\phi) | h \rangle$

$$ds^2 = \frac{dr^2}{r^2} + \left( \frac{1}{r^2} g_{ij}^{(0)} + g_{ij}^{(2)} + r^4 g_{ij}^{(4)} \right) dx^i dx^j$$

$$g_{ij}^{(2)} = -\frac{1}{2} R^{(0)} g_{ij}^{(0)} - \frac{6}{c} \langle h | T_{ij} | h \rangle \quad \text{and} \quad g_{ij}^{(4)} = \frac{1}{4} \left( g^{(2)} (g^{(0)})^{-1} g^{(2)} \right)_{ij}$$

Symmetries:  $\langle h | T(\phi) | h \rangle$  invariant under  $SL(2,R)/U(1)$ , becomes Killing symmetry of the bulk

Example: CFT in vacuum  $\langle 0 | T(\phi) | 0 \rangle = -\frac{c}{24} SL(2,R)$  symmetry

Dual to empty AdS  $SL(2,R)$  Killing symmetry

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# Coadjoint orbits

A Virasoro group element  $g$  acts on a gate  $X$  by the *adjoint transformation*

$$\text{Ad}_g(X) = \frac{d}{dt} (g \cdot e^{tX} \cdot g^{-1})|_{t=0}$$

and on a state  $v$  by the *coadjoint transformation*

$$\langle \text{Ad}_g^*(v), X \rangle = \langle v, \text{Ad}_{g^{-1}}(X) \rangle$$

*Coadjoint orbit:* set of states  $v$  reachable through coadjoint transformations on fixed state  $v_0$ ,

$$O_{v_0} = \{v = \text{Ad}_g^*(v_0) \mid g \in G\}$$

for Virasoro group: coadjoint orbit = Verma module



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Coadjoint orbits

- Coadjoint orbit has symplectic form

$$\omega = d\alpha = -d\langle v, \theta_g \rangle \text{ where } \theta_g = \frac{d}{ds} [g^{-1}(t) \cdot g(s)] \Big|_{s=t}$$

- Geometric action on coadjoint orbits

$$S_{\text{geo}} = \int \alpha = - \int dt \langle \text{Ad}_{g(t)}^* v_0, \theta_{g(t)} \rangle$$

Coadjoint orbit for Virasoro group

$$\mathcal{O}_{b_0} = \{b = \text{Ad}_{(f,\alpha)}^* b_0 \mid f \in \text{Diff}(S^1)\},$$
$$\text{Ad}_{(f,\alpha)}^* b_0 = f'^2 b_0 - \frac{c}{24\pi} \{f, \phi\}$$

# Virasoro Berry phase

Oblak 1703.06142

generated by diffeomorphisms that change the CFT state

Berry phase for group manifold  $A = i\langle \phi(t) | d | \phi(t) \rangle = i\langle \phi | \mathfrak{u}(\theta) | \phi \rangle$ ,  $\mathfrak{u}(\theta) = U_g^\dagger d U_g$

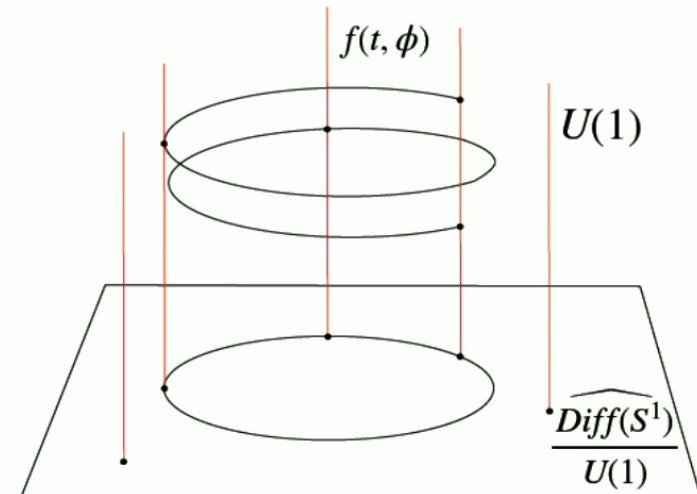
with central extension  $A = \langle h | \mathfrak{u}(\theta) | h \rangle + c \langle h | \mathfrak{u}(m_\theta) | h \rangle$

$(\theta, m_\theta)$ : Maurer-Cartan form

Connection gives rise to Berry phase as before

Virasoro Berry phases probe the geometry of a particular coadjoint orbit

For states outside the coadjoint orbit, the stabilizer group leads to a phase



# Virasoro Berry phase

For holographic CFTs: Note that stress tensor is element of dual Lie algebra

$$2\pi b = 2\pi \text{Ad}_{(f,\alpha)}^* b_0 = \langle h | \tilde{T}(\phi) | h \rangle = f'^2 \langle h | T(\tilde{\phi}) | h \rangle - \frac{c}{12} \{f, \phi\},$$

$$b_0 = \frac{1}{2\pi} \langle h | T(\tilde{\phi}) | h \rangle = \frac{1}{2\pi} \left( h - \frac{c}{24} \right)$$

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Dual bulk geometries associated to Virasoro coadjoint orbits: [Banados geometries](#)

[On-shell solutions to AdS3 gravity with Brown-Henneaux boundary conditions](#)

$$ds^2 = \ell^2 \frac{dr^2}{r^2} - \left( rdx^+ - \ell^2 \frac{L_-(x^-) dx^-}{r} \right) \left( rdx^- - \ell^2 \frac{L_+(x^+) dx^+}{r} \right)$$

$$L(x^\pm) = \frac{6}{c} \langle h | \tilde{T} | h \rangle = \frac{12\pi}{c} b \text{ and } x^\pm = t \pm \phi$$



Berry phase for wormholes in AdS3      Henneaux, Merbis, Ranjbar arXiv:1912.09465

Two boundary CFTs for eternal AdS black hole

Chern-Simons action

$$S_{[CS]} = \frac{k}{2\pi} \int dt dr d\varphi (A_\varphi \partial_t A_r + A_t F_{r\varphi})$$

Abelian case:  $A_\varphi = \partial_\varphi \mu + k_0$

$\Phi, \Psi$  Boundary values of  $\mu$

Holonomy  $k_0 = \frac{1}{2\pi} \oint d\varphi A_\varphi$

Boundary action for  $\Phi, \Psi, k_0$

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## Boundary action

$$S = \frac{k}{4\pi} \left( \int dt \left[ \oint d\varphi (\partial_\varphi \Phi \partial_t \Phi) - H_\Phi \right] + \int dt \left[ - \oint d\varphi (\partial_\varphi \Psi \partial_t \Psi) - H_\Psi \right] + 2 \int dt \left[ \oint d\varphi k_0 (\partial_t \Phi - \partial_t \Psi) - H_0 \right] \right),$$

where

$$H_\Phi = \int d\varphi (\partial_\varphi \Phi)^2,$$

$$H_\Psi = \int d\varphi (\partial_\varphi \Psi)^2,$$

$$H_0 = 2\pi (k_0)^2.$$



## Conjugate momentum and symplectic form on phase space

Conjugate momentum for holonomy  $\Pi_0 = -\frac{k}{2\pi} \oint d\varphi (\Phi - \Psi) = -\frac{k}{2\pi} \oint d\varphi \left( \int_{r_1}^{r_2} dr A_r \right)$

Symplectic form on boundary phase space  $x = (\Phi, \Psi, \Pi_0, \Pi_\Phi, \Pi_\Psi, k_0)$

$$\omega = d\Pi_\Phi \wedge d\Phi + d\Pi_\Psi \wedge d\Psi + dk_0 \wedge d\Pi_0$$

Non-exact due to contribution of holonomy!

$$\frac{\widehat{LG}}{G} \times \frac{\widehat{LG}}{G} \rightarrow \frac{\check{LG} \times \widehat{LG}}{G}$$



## Berry phases for wormholes in AdS<sub>3</sub>

A similar analysis may be performed for non-abelian  $SL(2,R) \times SL(2,R)$  symmetry

### Virasoro Berry phase

Symmetry transformations change states in the CFTs

Symplectic form on phase space can be mapped to symplectic form on Virasoro group manifold with coupling between both CFTs

Banerjee, Dorband, J.E., Meyer, Weigel arXiv:2202.11717

$$\frac{\text{Diff}(S^1)}{S^1} \times \frac{\text{Diff}(S^1)}{S^1} \rightarrow \frac{\text{Diff}(S^1) \times \text{Diff}(S^1)}{S^1}$$

Symplectic form non-exact



## Relation to geometric action

Reparametrize chiral boson

$$\Phi = k_0(f(t, \varphi) - \varphi) - \ln(-k_0 f'(t, \varphi))$$

$$\Psi = k_0(\varphi - g(t, \varphi)) - \ln(k_0 g'(t, \varphi))$$

Coupled Virasoro Berry phase  $b_0 = \frac{k}{8\pi} k_0(t)^2$

$$b_0 = \frac{1}{2\pi} \langle h | T(\phi) | h \rangle \text{ couples both Berry phases}$$

Action becomes

$$S[k_0, \Phi, \Psi] = S_{\text{geo}}^-[f, b_0] - S_{\text{geo}}^+[g, b_0]$$

$$S_{\text{geo}}^\pm[h, b_0] = \int dt d\sigma \left( b_0 h' \partial_\pm h + \frac{k}{8\pi} \frac{h'' \partial_\pm h'}{(h')^2} \right)$$

Henneaux et al  
Jensen et al

Asymptotic dynamics of BTZ black hole is described by coupled Virasoro Berry phase

Both sides are coupled by the holonomy and connected by a radial Wilson line

Symmetry group is enhanced boundary no longer factorizes

$$\frac{\text{Diff}(S^1)}{S^1_{32}} \times \frac{\text{Diff}(S^1)}{S^1} \rightarrow$$

$$\frac{\text{Diff}(S^1) \times \text{Diff}(S^1)}{S^1}$$

## Gauge Berry phase

Proper diffeomorphisms correspond to gauge symmetries at the boundary  
yield Killing charges and satisfy  $\delta_\xi g_{\mu\nu} = 0$

Brown, Henneaux 1986  
Compère; Mao, Seraj, Sheikh-Jabbari 2015

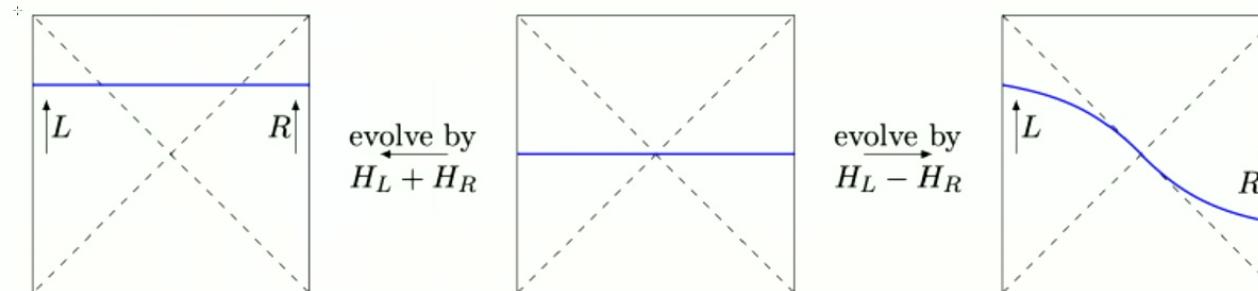
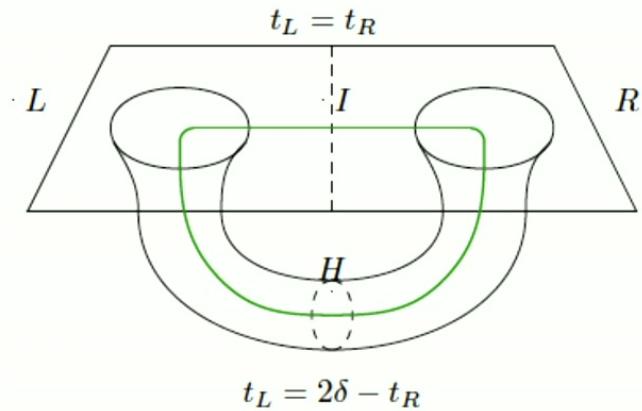
In presence of an eternal black hole, these charges may only be defined  
locally near the two boundaries



## Gauge Berry phase

No global Killing vector in the presence of a wormhole

Leads to non-exact symplectic form



$$|\text{TFD}_\alpha\rangle = e^{-i(H_L + H_R)\delta} |\text{TFD}\rangle$$

Symmetry:  
does not transform state

# Wormhole Berry Phase

Non-zero Berry connection for diffeomorphism

$$A_\delta = i \langle \text{TFD}_\alpha | \partial_\delta | \text{TFD}_\alpha \rangle = \frac{2}{Z} \sum_n E_n e^{-\beta E_n}$$

$$u_0(\delta) = e^{-i(H_L + H_R)\delta}$$

$$|\text{TFD}_\alpha\rangle = \tilde{u}_0(\delta) |\text{TFD}_{\alpha=0}\rangle$$

Vanishing Berry connection for diffeomorphism

$$A_\delta = i \langle \text{TFD}_{\alpha=0} | u_1^\dagger \partial_\delta u_1 | \text{TFD}_{\alpha=0} \rangle = 0$$

$$u_1(\delta) = e^{-i(H_L - H_R)\delta}$$



# Example: Wormhole Berry phase for JT gravity

- Theory of 2d gravity with a scalar field (dilaton)  
$$R + 2 = 0 \quad \& \quad (\nabla_\mu \nabla_\nu - g_{\mu\nu}) \Phi = 0$$
- Boundary conditions:  $\gamma_{tt}|_{\partial M} = r_c^2$  &  $\Phi|_{\partial M} = \phi_b r_c$
- Diffeomorphisms given by time translations with

$$H_L = H_R = \Phi_H^2 / \phi_B$$

Harlow, Jafferis '18



## Example: Wormhole Berry phase for JT gravity

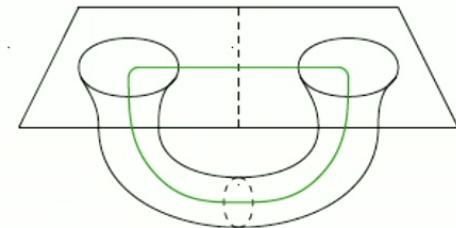
- Berry connection evaluates to:

$$A_\delta = 2\Phi_H^2/\phi_b$$

$$\alpha = -2E\delta \quad 2\pi \text{ periodic}$$

Resulting Berry phase with winding number interpretation:

$$\Phi_B^{JT} = \oint A_\delta d\delta = 2\frac{\Phi_H^2}{\phi_b} \int_0^{\frac{\pi}{E}} d\delta = 2\pi$$



# Relation to von Neumann algebras

Liu, Leutheusser; Witten; Chandrasekaran, Penington, Witten

Concept of algebraic QFT

for classifying operator algebras w. r. t. entanglement properties

Type I - density matrix and trace (as in quantum mechanics)

Type II - trace prescription, but does not act irreducibly

Type III - no trace prescription (eg. free QFTs)

Symmetry generated by  $H_L - H_R$  corresponds to outer automorphism  
that generates type II from type III von Neumann algebra



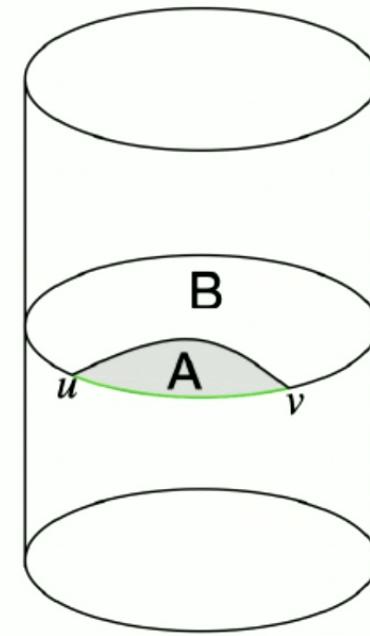
## Modular Berry phase

B. Czech

- Interval  $[u, v]$  on constant time slice in CFT with reduced density metric  $\rho_A$  dual bulk region bounded by RT geodesic

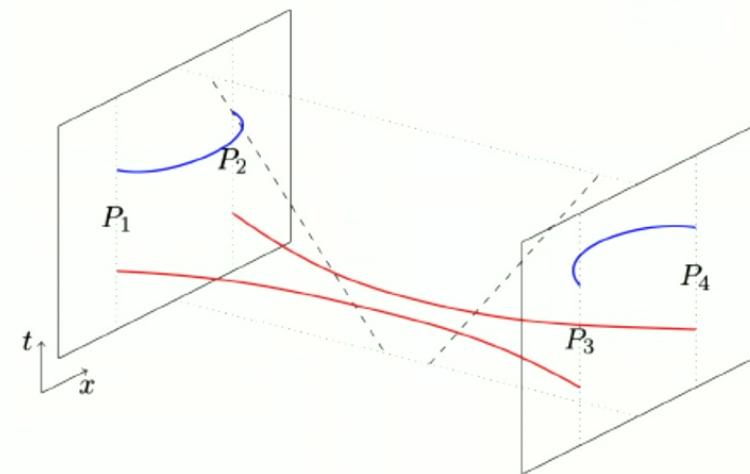
$$S(u, v) = \frac{l(u, v)}{4G_N}$$

- Modular Hamiltonian:  $H_{\text{mod}} = -\log(\rho_A)$  generates abstract time evolution with respect to modular time parameter
- Choice of modular time parameter is gauge symmetry for each interval
- Parallel transport of interval around closed loop leads to Berry phase



## Modular Berry phase: Two-sided case

Transition in entanglement entropy (Hartman et al)



Change in Berry curvature

## Conclusions and outlook

- Non-Factorization of wormhole Hilbert space due to non-exact symplectic form results in non-zero Berry phase
- Mathematical structure also present in quantum mechanics and in CFT
- Relation between entanglement and geometry
- New possibilities for experimental study
- Relation to von Neumann algebras

