Title: Classical Physics - Lecture 221004

Speakers:

Collection: Classical Physics (2022/2023)

Date: October 04, 2022 - 9:00 AM

URL: https://pirsa.org/22100059

YESTERDAY: RELATIVISTIC FIELD THEORIES

$$S[\varphi] = \int_\Omega d^4x \, \mathcal{L}(\varphi, \partial_\mu \varphi)$$

$\delta\varphi$:

$$\boxed{\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = 0}$$

$T_c^{\mu}{}_{\nu}$

$\cdot T_c^{\mu\nu}$

$$\boxed{T^{\mu}_{c\,\nu} = \eta^{\mu}_{\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\partial_\nu \varphi}$$ (INVARIANCE W.R.T. SPACETIME TRANSL.)

$$\boxed{\partial_\mu T^{\mu\nu}_{c} = 0}$$ ... "4 CONSERVED CURRENTS"

CHARGES: $P_\nu = \int_\Sigma T^{\mu}{}_{\nu}\, d\Sigma_\mu$

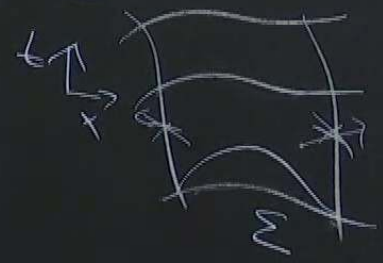
- $T^{\mu\nu}_{c}$ CAN BE UPGRADED TO A SYMMETRIC $T^{\mu\nu}$.

$$T_c^{\mu}{}_{\nu} = \eta^{\mu}{}_{\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\partial_\nu \varphi$$ (INVARIANCE W.R.T. SPACETIME TRANSL.)

$$\partial_\mu T_c^{\mu\nu} = 0$$ ... "4 CONSERVED CURRENTS"

CHARGES:

$$P_\nu = \int_\Sigma T^{\mu}{}_{\nu}\, d\Sigma_\mu$$



$T^{\mu\nu}_c$ CAN BE UPGRADED TO A SYMMETRIC $T^{\mu\nu}$.

$$T^{\mu}_{c\,\nu} = \eta^{\mu}_{\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\partial_\nu \varphi$$ (INVARIANCE W.R.T. SPACETIME TRANSL.)

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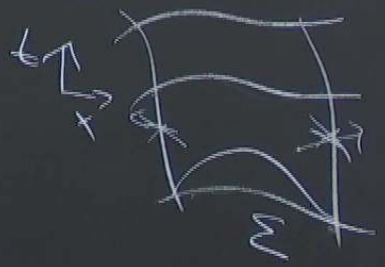
- $T^{\mu\nu}_{c}$ CAN BE UPGRADED TO A SYMMETRIC $T^{\mu\nu}$



- LORENTZ TRANSFORMATIONS → CONSERVED ANGULAR MOM TENSOR

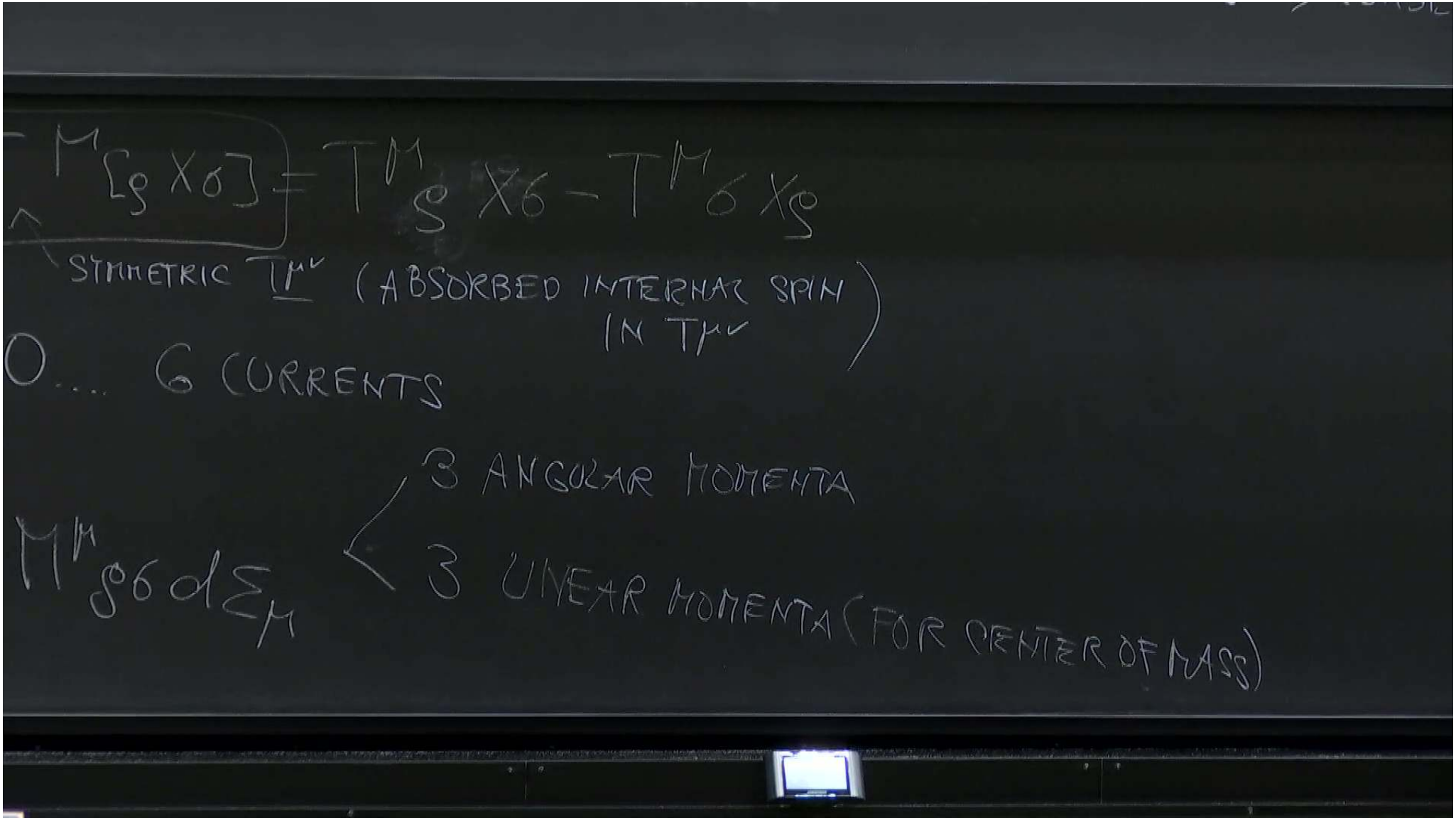
$$M^{\mu}{}_{\rho\sigma} = 2T^{\mu}{}_{[\rho}X_{\sigma]} = T^{\mu}{}_{\rho}X_{\sigma} - T^{\mu}{}_{\sigma}X_{\rho}$$

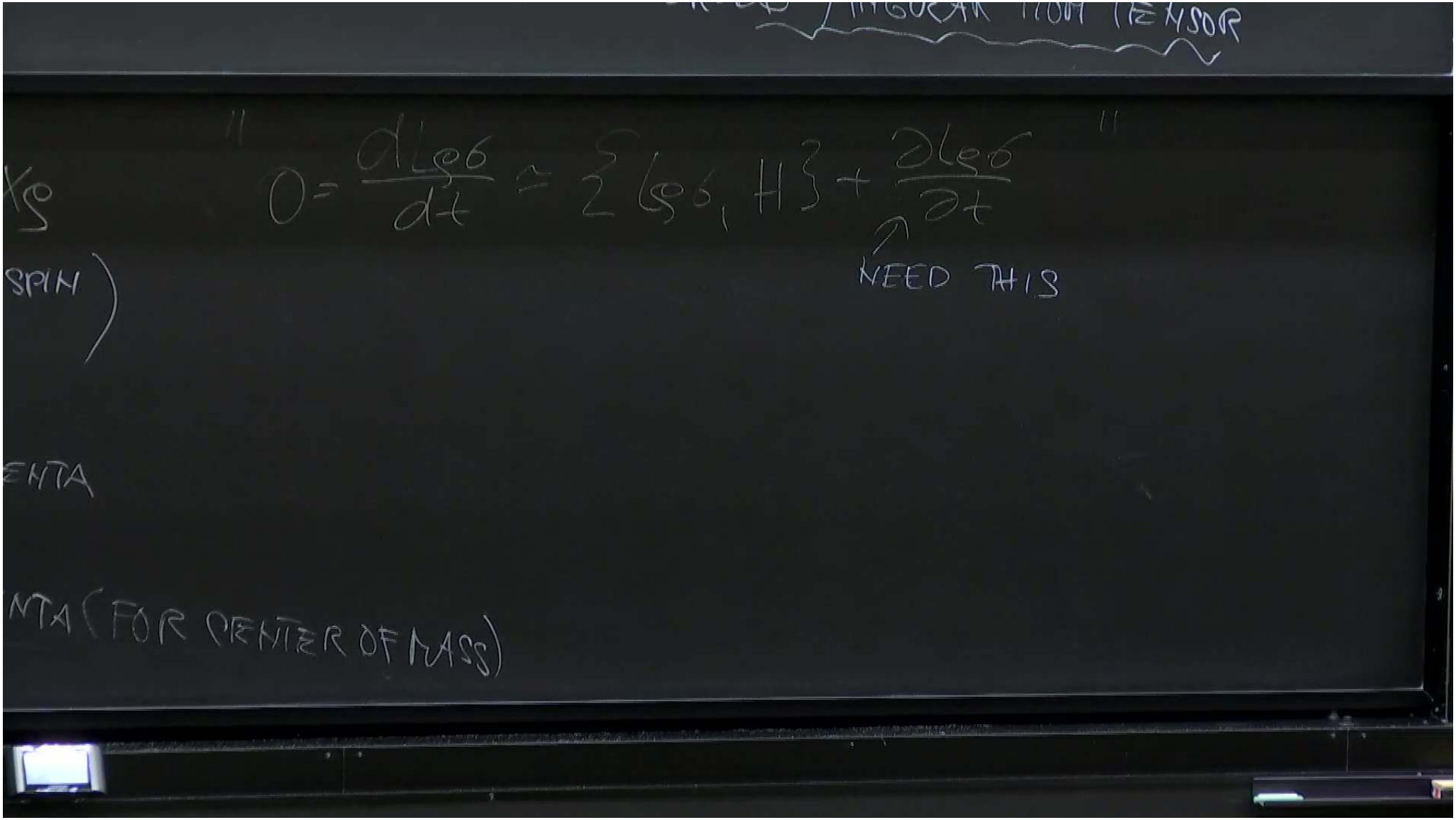
SYMMETRIC $T^{\mu\nu}$ (ABSORBED INTERNAL SPIN IN $T^{\mu\nu}$)

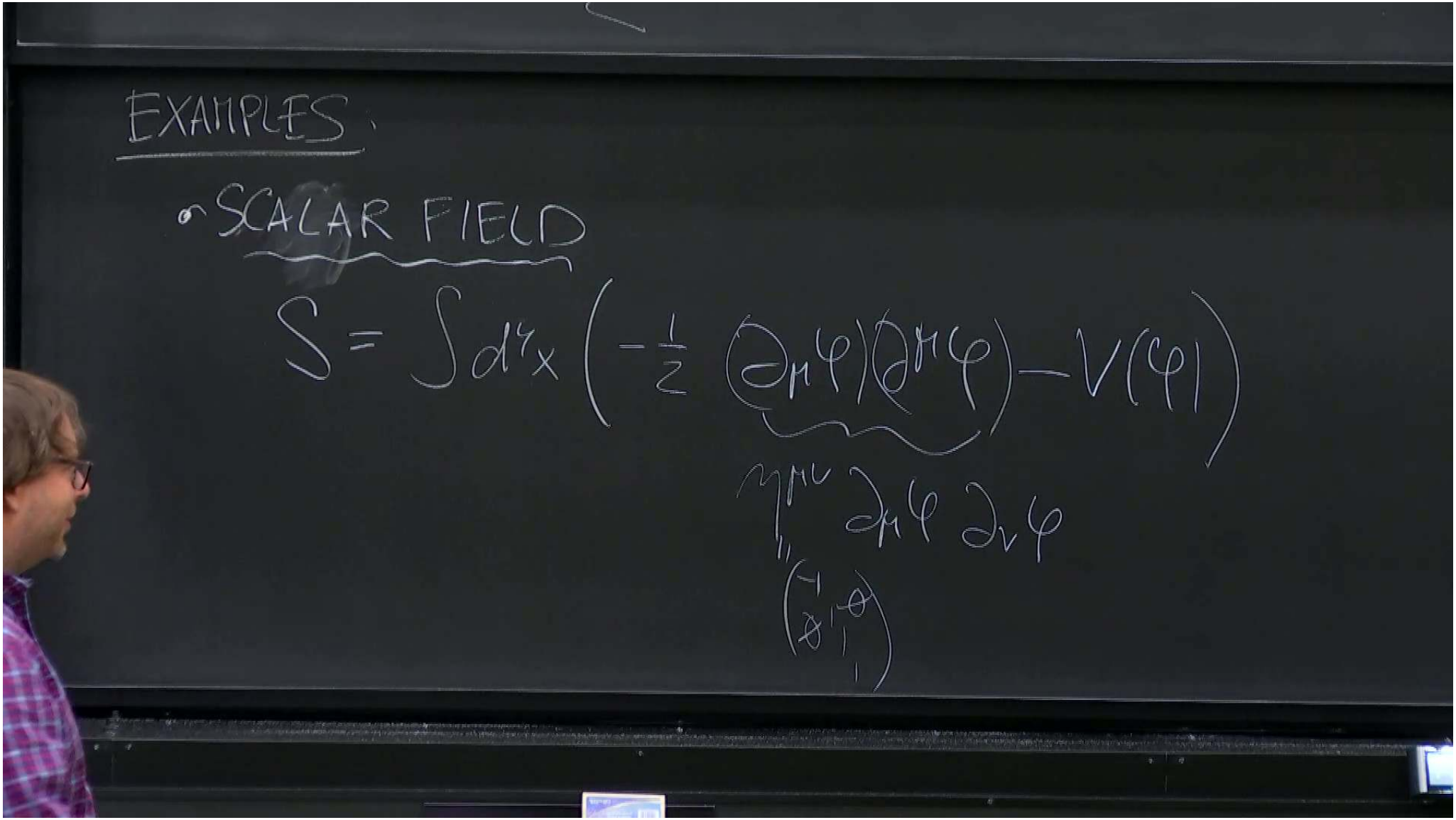
$$\partial_{\mu} M^{\mu}{}_{\rho\sigma} = 0 \ldots$$ 6 CURRENTS

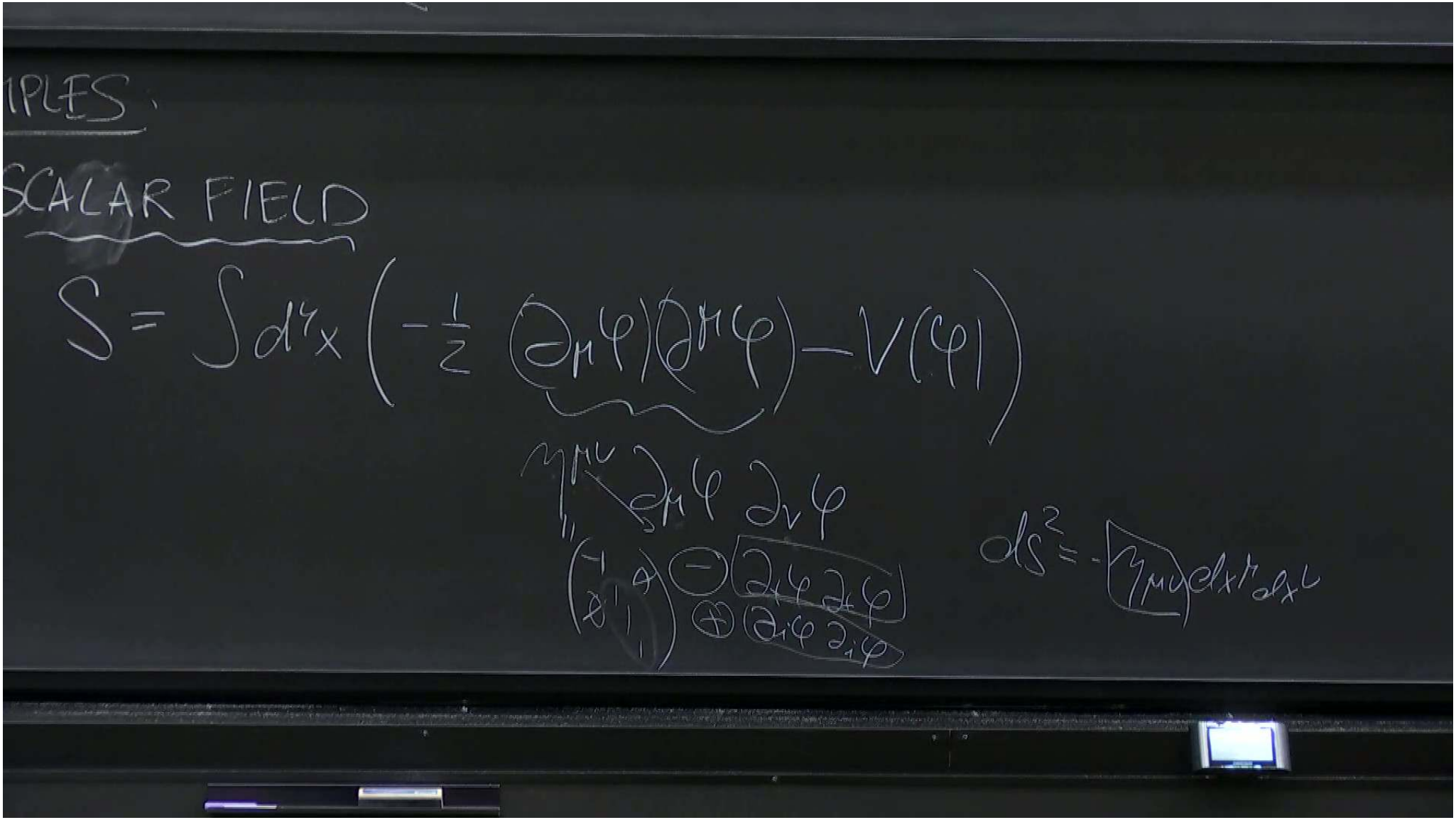
6 CONSERVED CHARGES

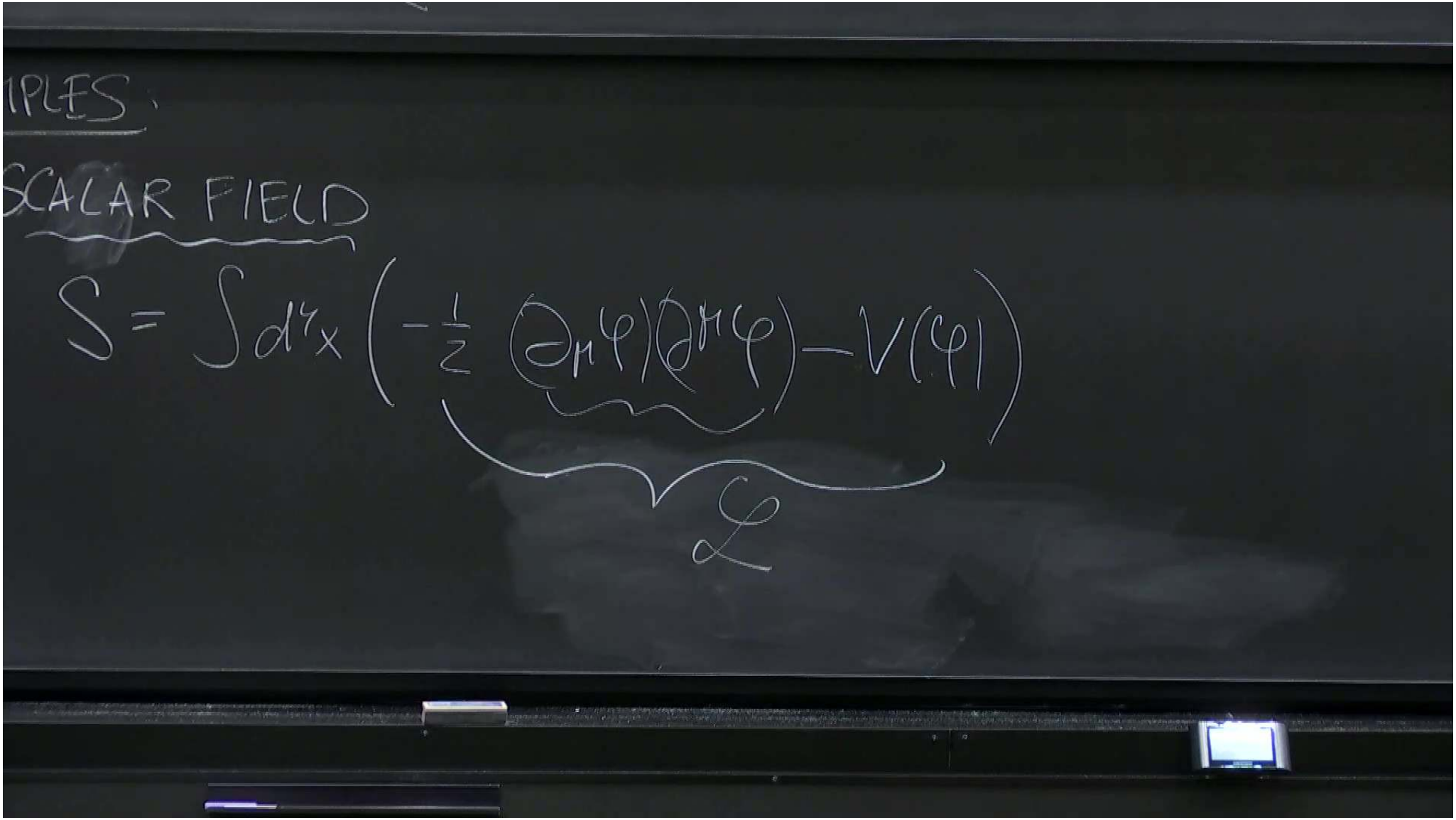
$$L_{\rho\sigma} = \int_{\Sigma} M^{\mu}{}_{\rho\sigma} d\Sigma_{\mu}$$


$M^{\mu}{}_{[\rho}\,x_{\sigma]} = T^{\mu}{}_{\rho}\,x_{\sigma} - T^{\mu}{}_{\sigma}\,x_{\rho}$
SYMMETRIC $T^{\mu\nu}$ (ABSORBED INTERNAL SPIN IN $T^{\mu\nu}$)
0 ... 6 CURRENTS
3 ANGULAR MOMENTA
3 LINEAR MOMENTA (FOR CENTER OF MASS)
$M^{\mu}{}_{\rho\sigma}\,d\Sigma_{\mu}$


" $0 = \frac{d L_{SO}}{dt} = 2 L_{SO} \cdot I^3 + \frac{2 L_{SO}}{2t}$ "
NEED THIS


EXAMPLES:
SCALAR FIELD
$S = \int d^4x \left(-\frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) - V(\varphi) \right)$
$\eta^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi$
$(-1, 1, 1, 1)$


IPLES:
SCALAR FIELD
$S = \int d^4x \left(-\frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) - V(\varphi) \right)$
$\eta^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi$
$(-1, +1, +1, +1)$
$ds^2 = -\eta_{\mu\nu} dx^\mu dx^\nu$


IPLES:
SCALAR FIELD
$S = \int d^4x \left(-\frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) - V(\varphi) \right)$
$\mathcal{L}$

$$-\frac{\partial V}{\partial \varphi} - \partial_\mu(-\partial^\mu \varphi) = 0 \qquad \boxed{(\partial^2 - m^2)\varphi = 0}$$

$$\boxed{\partial_\mu \partial^\mu \varphi - \frac{dV}{d\varphi} = 0} \qquad \boxed{\Box}$$

KLEIN-GORDON EQ

- SIMPLEST EXAMPLE:

$$V = \frac{1}{2} m^2 \varphi^2$$

$$-\frac{\partial V}{\partial \varphi} - \partial_\mu\left(-\partial^\mu \varphi\right) = 0 \qquad \boxed{\left(\partial^2 - m^2\right)\varphi = 0}$$

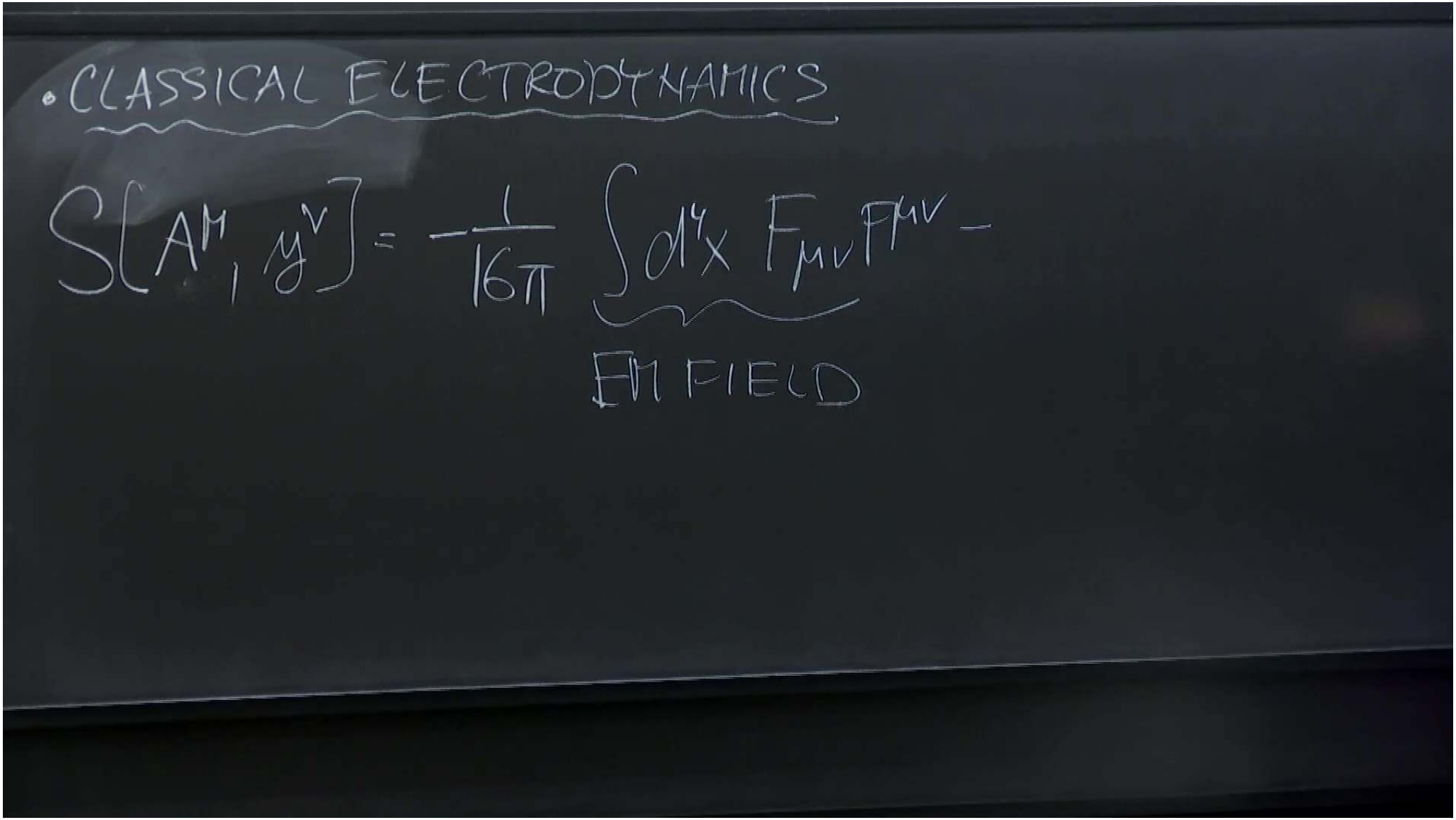
$$\boxed{\partial_\mu \partial^\mu \varphi - \frac{dV}{d\varphi} = 0}$$

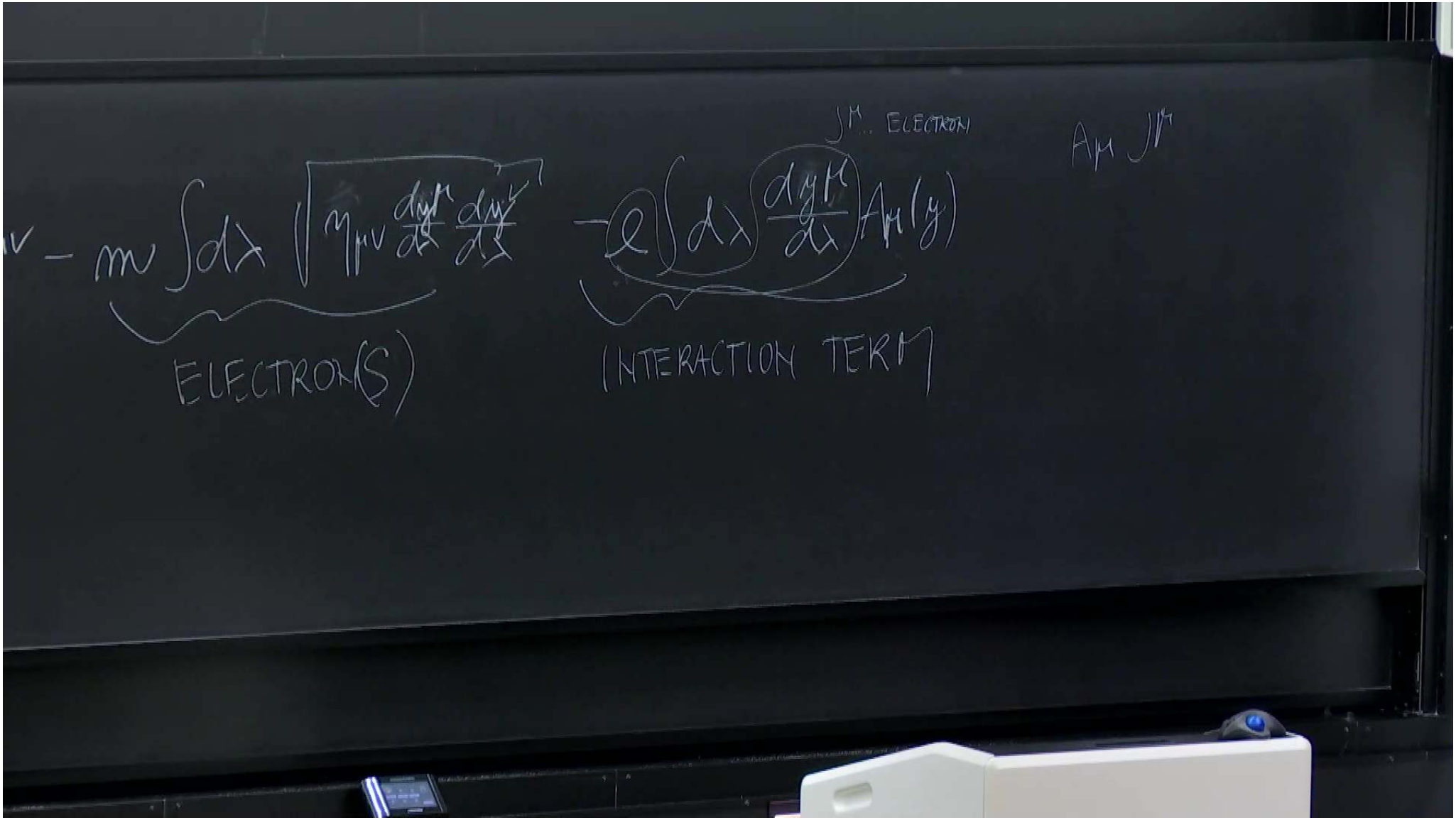
KLEIN-GORDON EQ

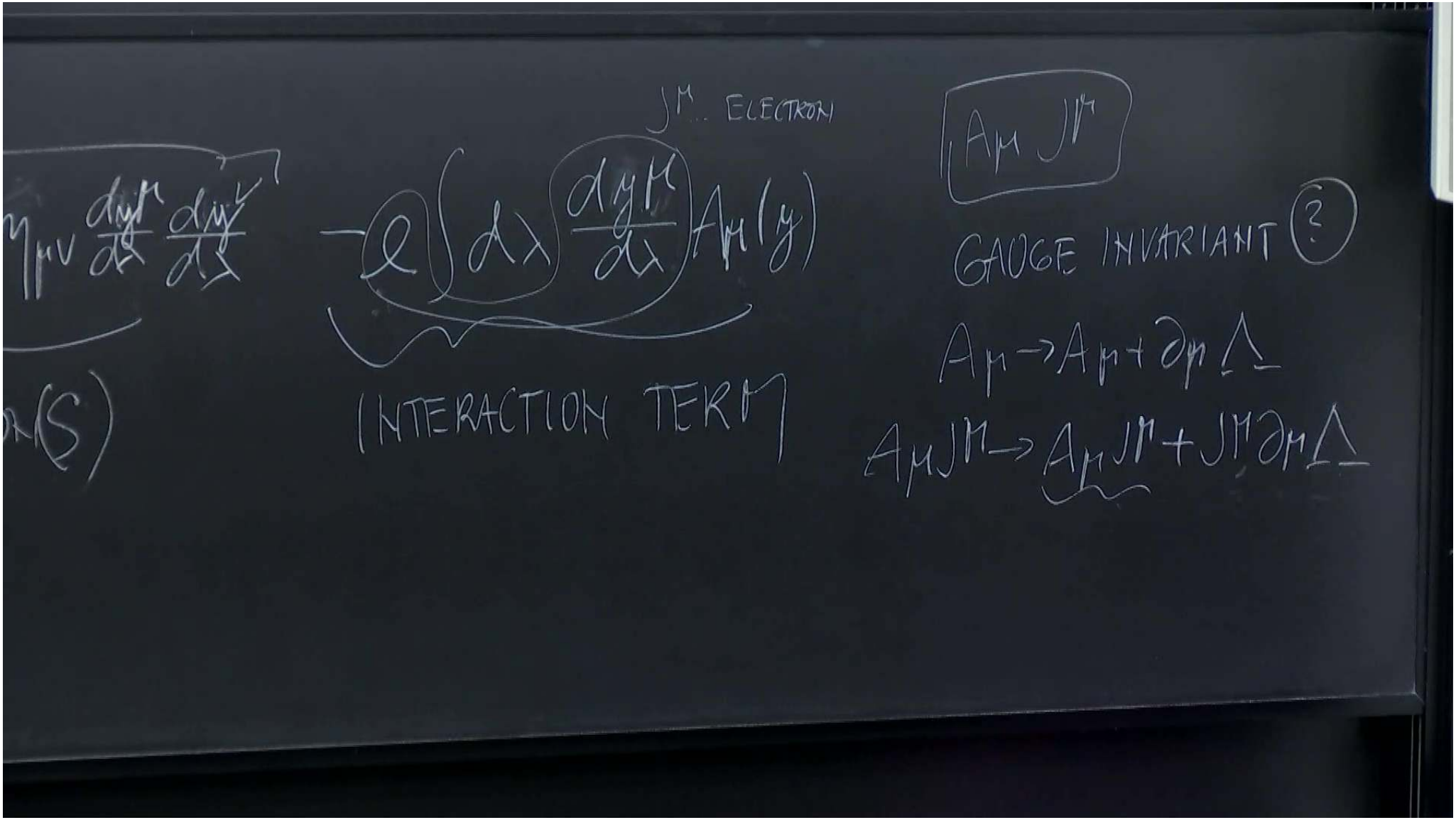
- SIMPLEST EXAMPLE:

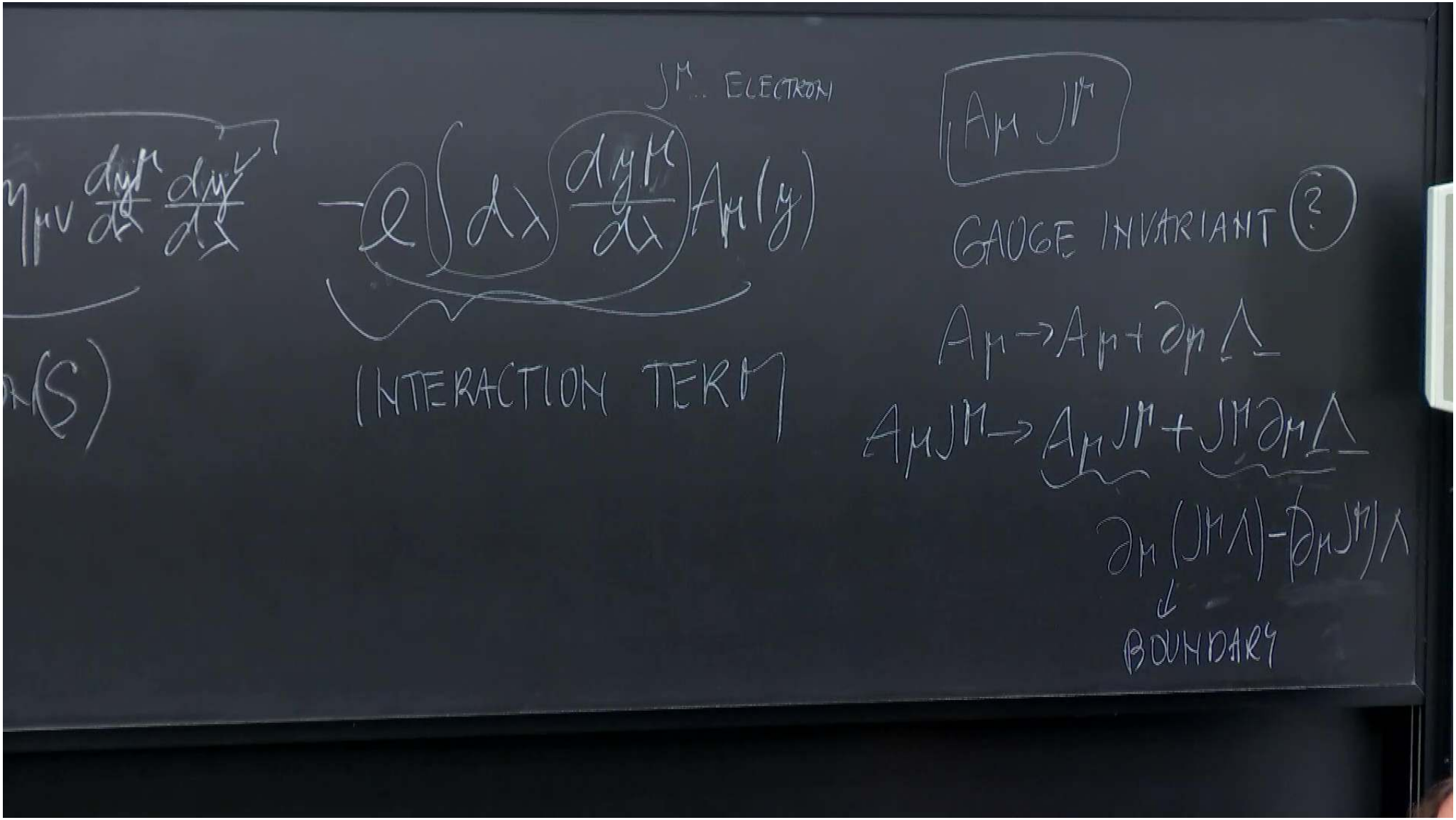
$$V = \frac{1}{2} m^2 \varphi^2$$

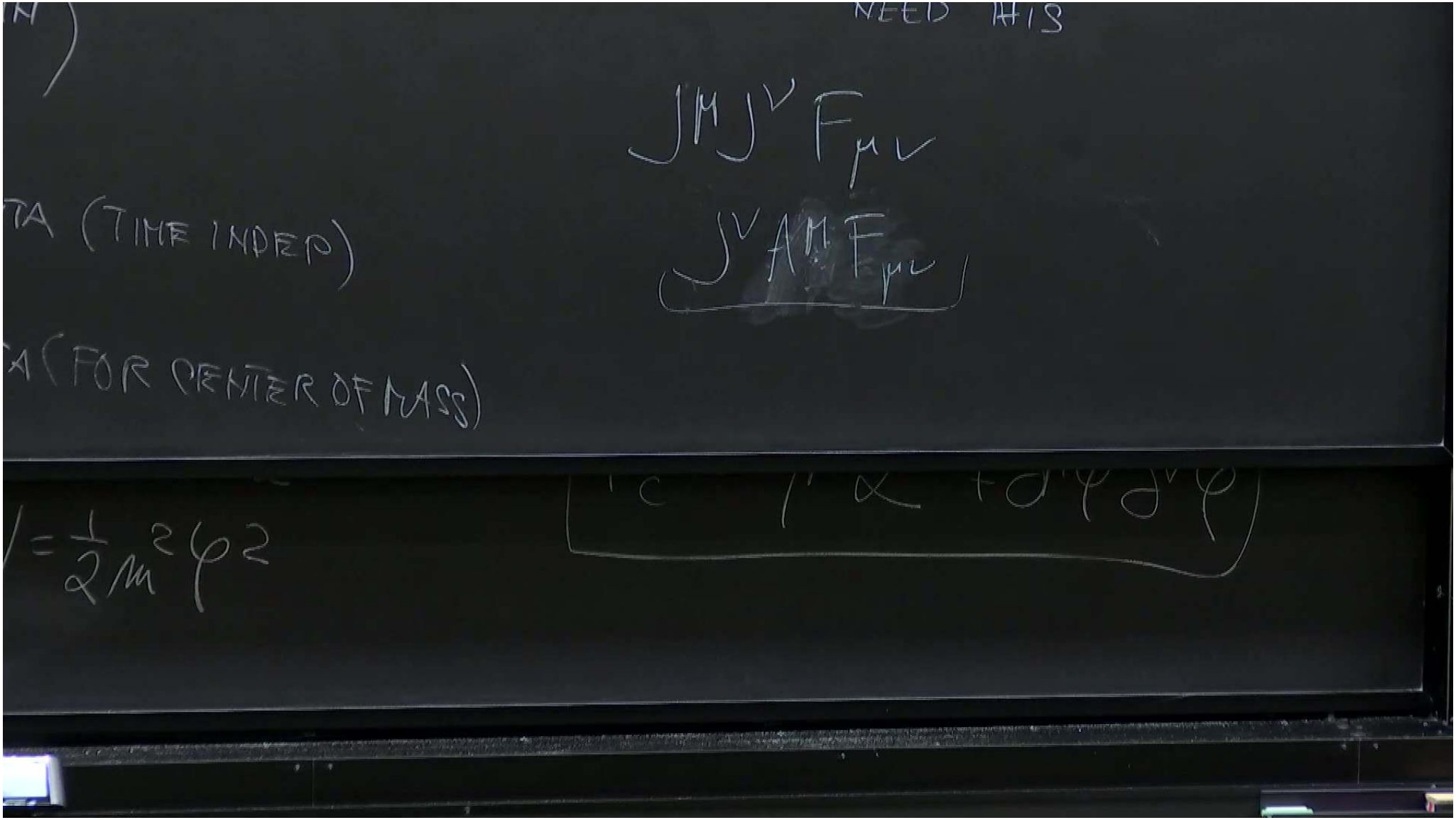
$$\boxed{T^{\mu\nu}_C = \eta^{\mu\nu}\mathcal{L} + \partial^\mu\varphi\,\partial^\nu\varphi}$$


• CLASSICAL ELECTRODYNAMICS
$S[A^\mu, y^\nu] = -\frac{1}{16\pi}\int d^4x\, F_{\mu\nu}F^{\mu\nu} -$
EM FIELD


J^μ ELECTRON
$A_\mu J^\mu$
$-m\int d\lambda \sqrt{\eta_{\mu\nu}\frac{dy^\mu}{d\lambda}\frac{dy^\nu}{d\lambda}}$
$-e\int d\lambda \frac{dy^\mu}{d\lambda} A_\mu(y)$
ELECTRON(S)
INTERACTION TERM


J^μ ELECTRON
$A_\mu J^\mu$
$-e\int d\lambda \frac{dy^\mu}{d\lambda} A_\mu(y)$
INTERACTION TERM
GAUGE INVARIANT ?
$A_\mu \to A_\mu + \partial_\mu \Lambda$
$A_\mu J^\mu \to A_\mu J^\mu + J^\mu \partial_\mu \Lambda$


J^μ ... ELECTRON
$A_\mu J^\mu$
GAUGE INVARIANT ?
$A_\mu \to A_\mu + \partial_\mu \Lambda$
$A_\mu J^\mu \to A_\mu J^\mu + J^\mu \partial_\mu \Lambda$
$\partial_\mu (J^\mu \Lambda) - \partial_\mu J^\mu \Lambda$
BOUNDARY
INTERACTION TERM


NEED THIS
$J^\mu J^\nu F_{\mu\nu}$
$J^\nu A^\mu F_{\mu\nu}$
TA (TIME INDEP)
A (FOR CENTER OF MASS)
$= \frac{1}{2} m^2 \varphi^2$

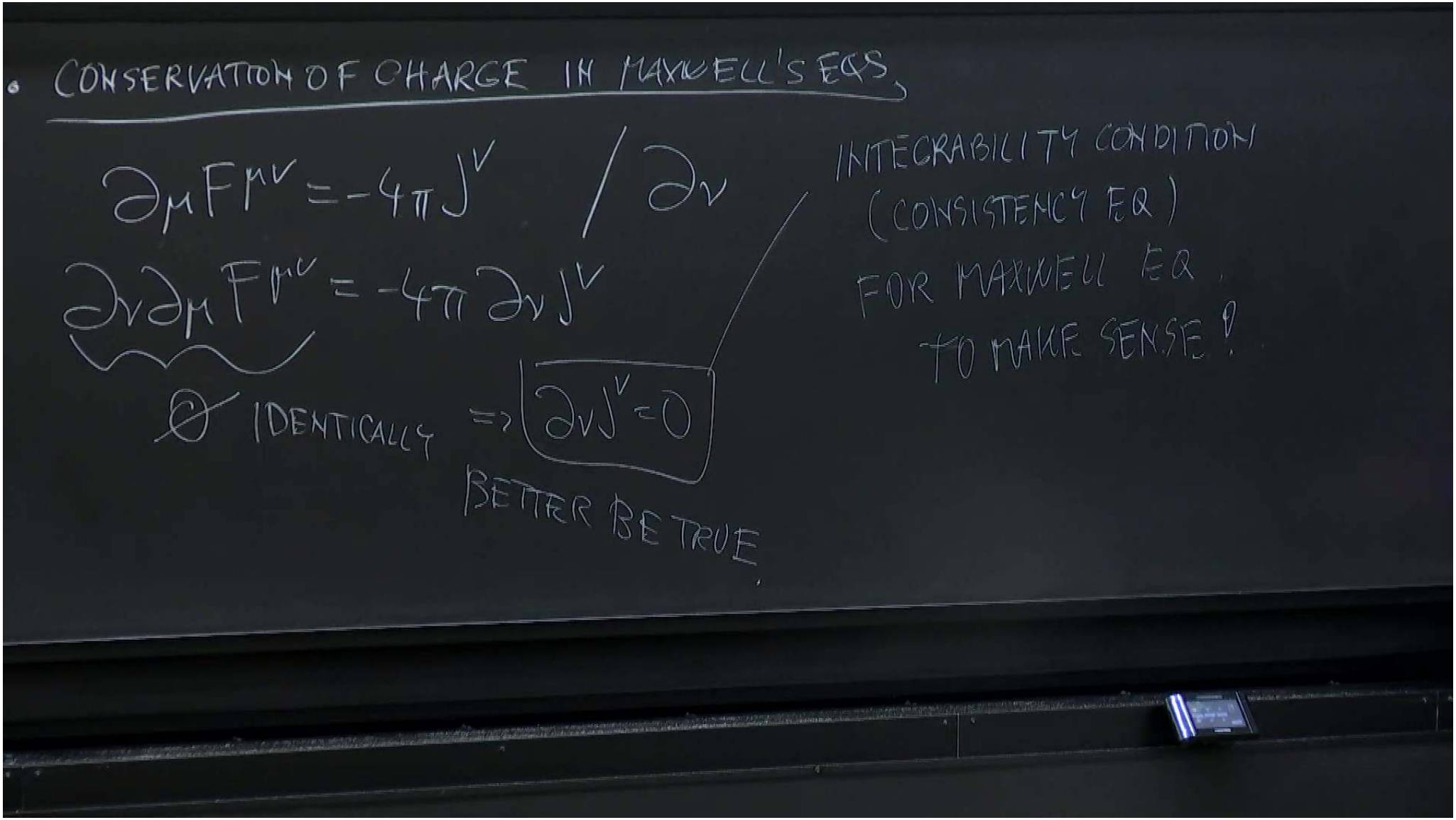
$$S[A^\mu, y^\nu] = -\frac{1}{16\pi}\int d^4x\, F_{\mu\nu}F^{\mu\nu} - m\int d\lambda \sqrt{\eta_{\mu\nu}\frac{dy^\mu}{d\lambda}\frac{dy^\nu}{d\lambda}}$$

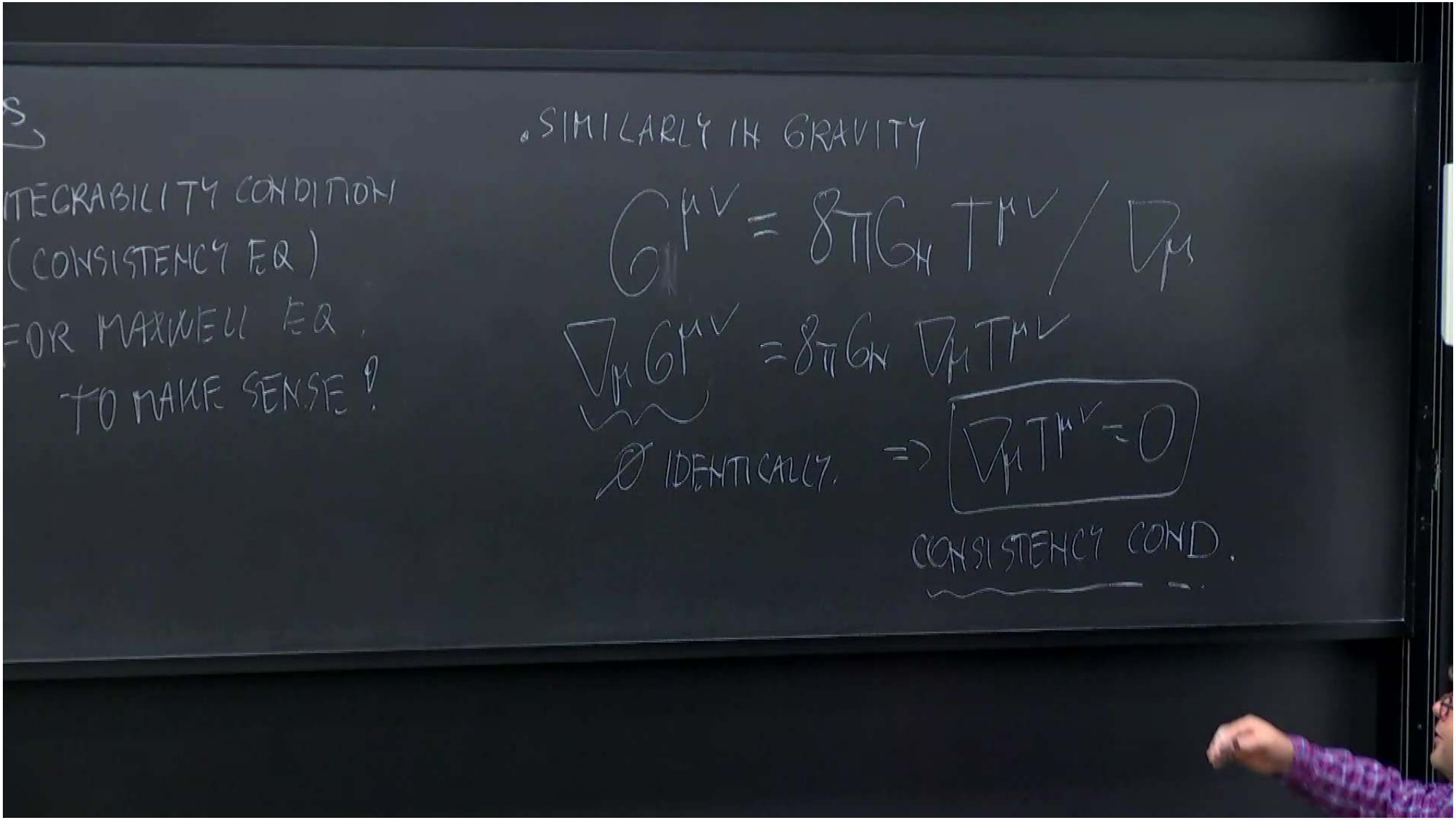
EM FIELD — ELECTRON(S)

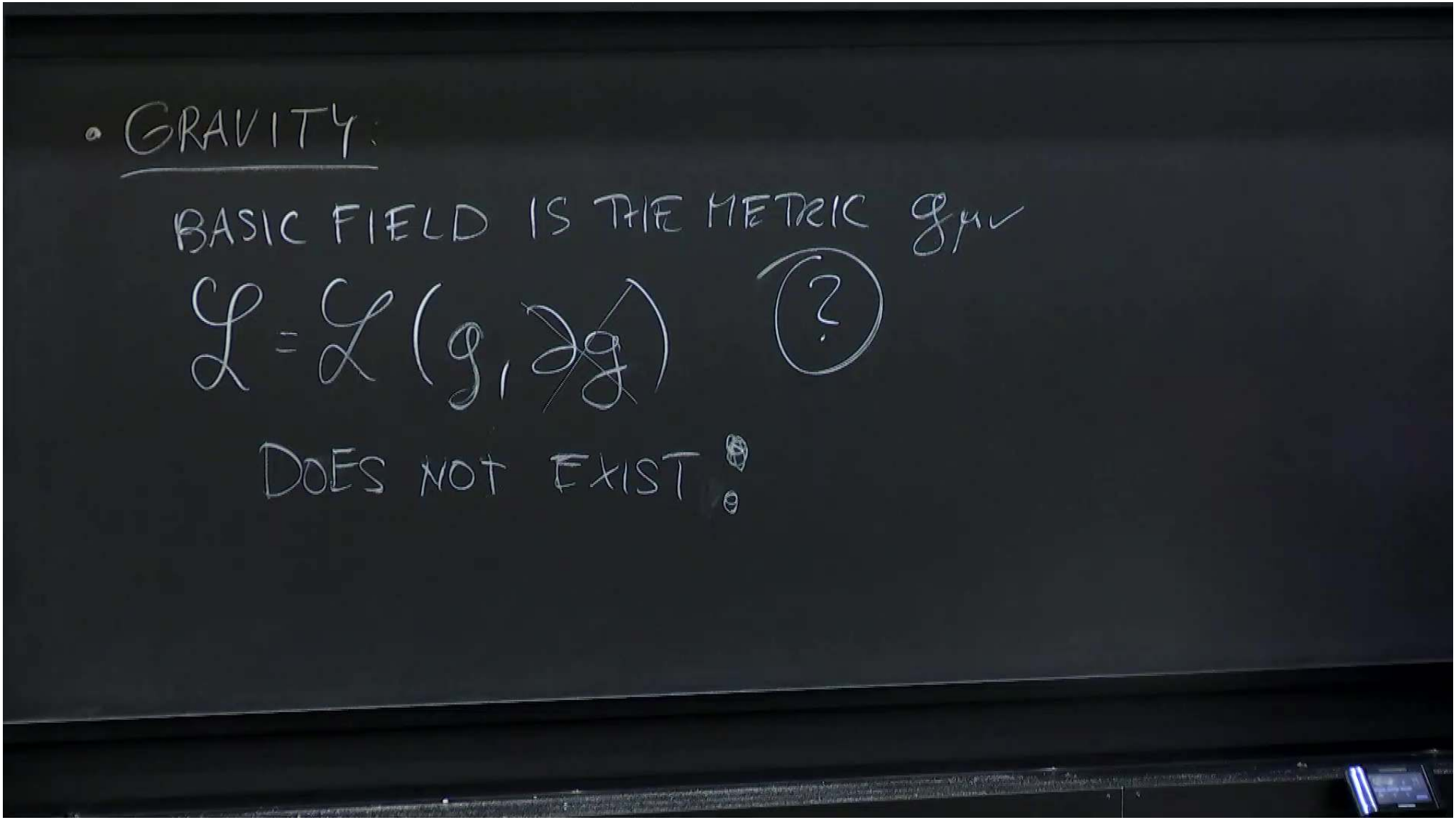
δA^μ: $\partial_\mu F^{\mu\nu} = -4\pi J^\nu$, $F(\mu\nu, S) = \emptyset$ ✓ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

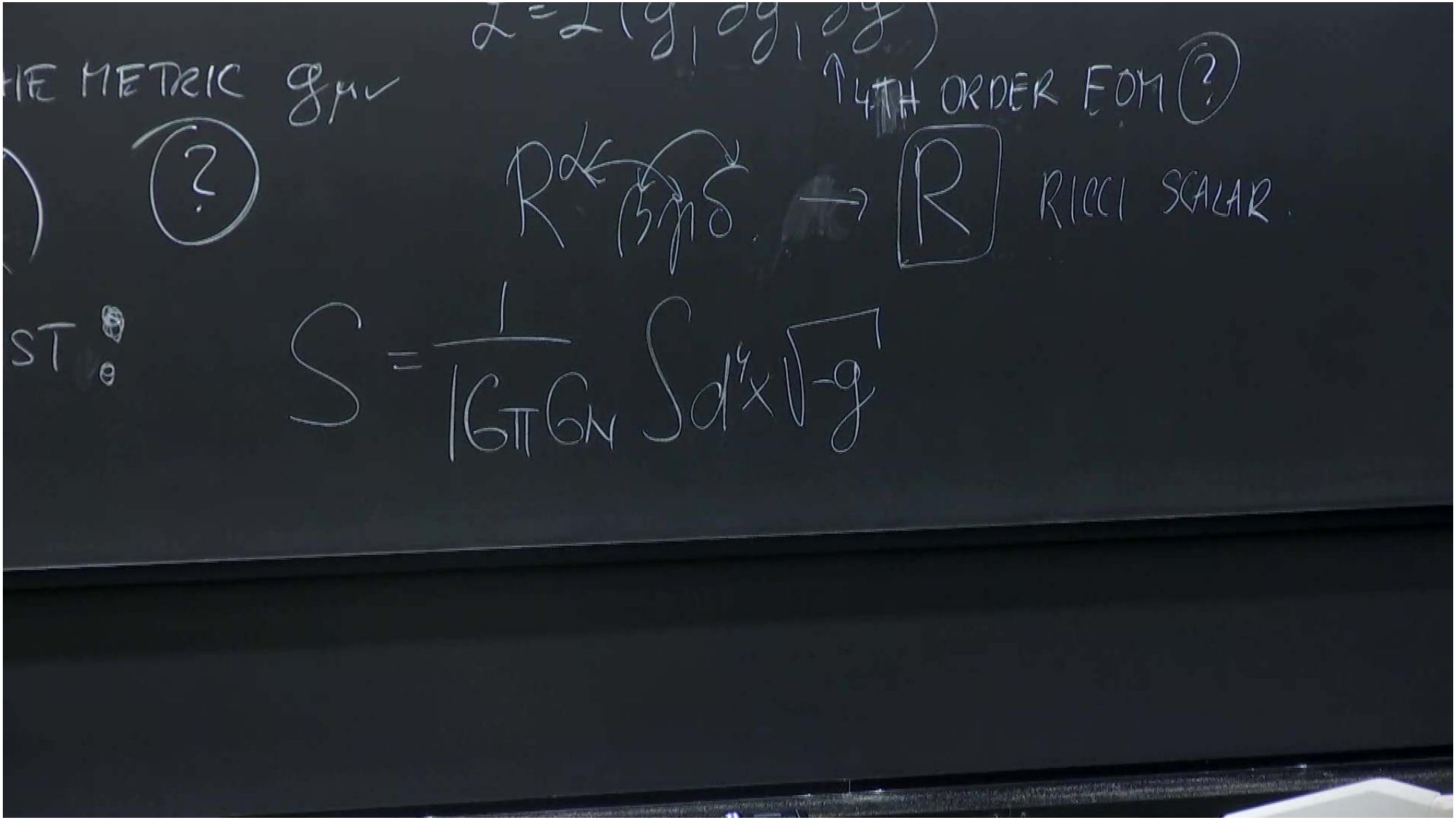
δy^ν: $\frac{dp^\mu}{d\tau} = eF^{\mu\nu}u_\nu$ ✓

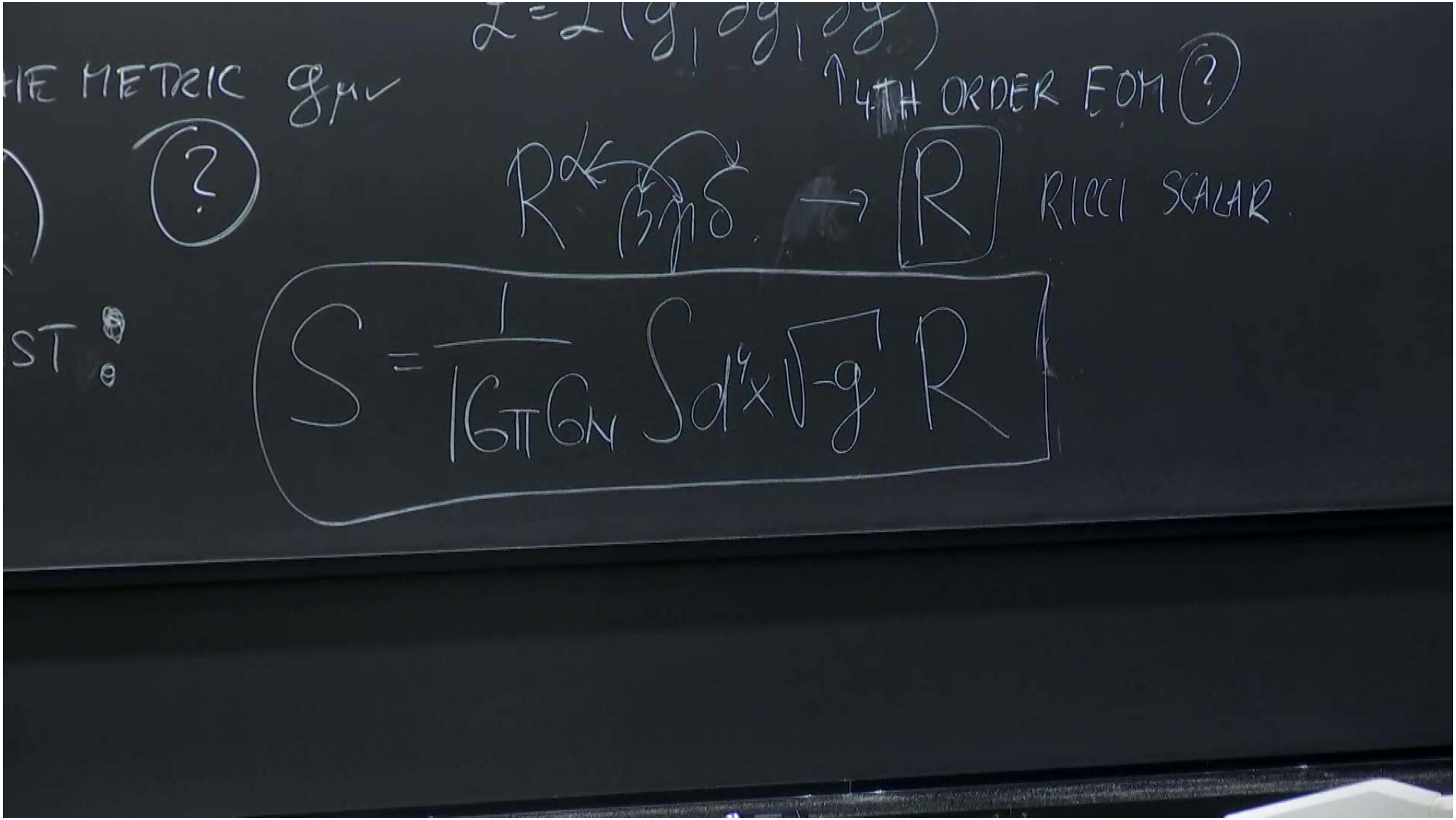
ALREADY ASSUMED!

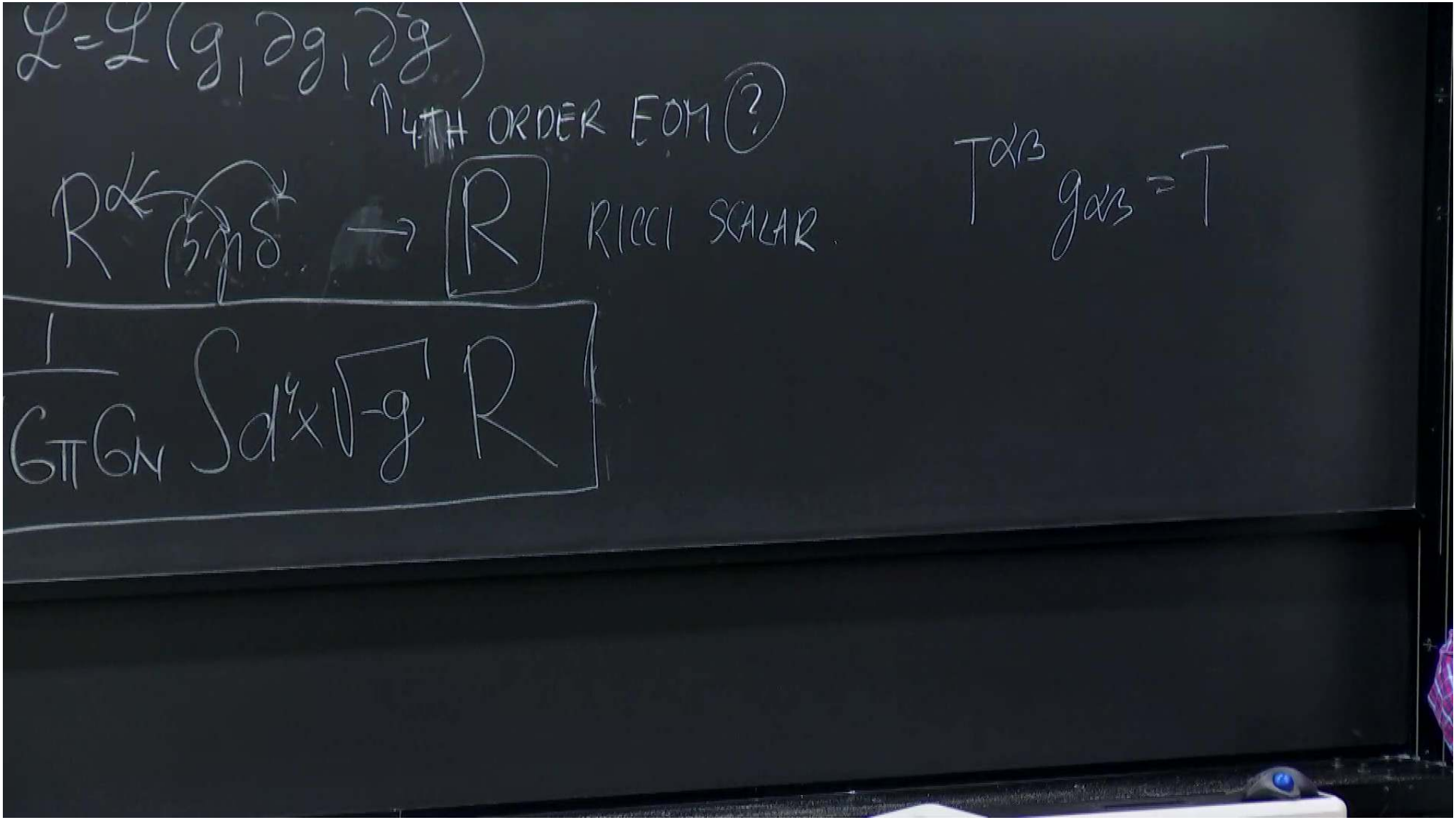

CONSERVATION OF CHARGE IN MAXWELL'S EQS.
$\partial_\mu F^{\mu\nu} = -4\pi J^\nu$ / ∂_ν
$\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$
0 IDENTICALLY $\Rightarrow$ $\partial_\nu J^\nu = 0$
BETTER BE TRUE.
INTEGRABILITY CONDITION
(CONSISTENCY EQ)
FOR MAXWELL EQ.
TO MAKE SENSE!


S
ITEGRABILITY CONDITION
(CONSISTENCY EQ)
FOR MAXWELL EQ.
TO MAKE SENSE!
. SIMILARLY IN GRAVITY
$G^{\mu\nu} = 8\pi G_N T^{\mu\nu} / \nabla_\mu$
$\nabla_\mu G^{\mu\nu} = 8\pi G_N \nabla_\mu T^{\mu\nu}$
IDENTICALLY. $\Rightarrow$ $\nabla_\mu T^{\mu\nu} = 0$
CONSISTENCY COND.


• GRAVITY:
BASIC FIELD IS THE METRIC $g_{\mu\nu}$
$\mathcal{L} = \mathcal{L}(g, \partial g)$
?
DOES NOT EXIST!


HE METRIC $g_{\mu\nu}$
?
$\mathcal{L} = \mathcal{L}(g, \partial g, \partial g)$
↑ 4TH ORDER EOM ?
$R^{\alpha}{}_{\beta\gamma\delta} \rightarrow R$
RICCI SCALAR.
ST
$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}$


HE METRIC $g_{\mu\nu}$
?
$\mathcal{L} = \mathcal{L}(g, \partial g, \partial g)$
↑ 4TH ORDER EOM ?
$R^{\alpha}{}_{\beta\gamma\delta} \rightarrow R$ RICCI SCALAR.
$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, R$


$\mathcal{L} = \mathcal{L}(g, \partial g, \partial^2 g)$
↑ 4TH ORDER EOM ?
$R^{\alpha\beta}{}_{\gamma\delta} \rightarrow R$ RICCI SCALAR
$T^{\alpha\beta} g_{\alpha\beta} = T$
$\frac{1}{G_\Pi G_N} \int d^4x \sqrt{-g} R$