Title: Classical Physics - Lecture 221004

Speakers:

Collection: Classical Physics (2022/2023)

Date: October 04, 2022 - 9:00 AM

URL: https://pirsa.org/22100059

YESTERDAY: RELATIVISTIC FIELD THEORIES

$$S[\varphi] = \int_\Omega d^4x \, \mathcal{L}(\varphi, \partial_\mu \varphi)$$

$\delta\varphi$:

$$\boxed{\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = 0}$$

$T_c{}^\mu{}_\nu$

• $T_c^{\mu\nu}$

$$\boxed{T_c{}^{\mu}{}_{\nu} = \eta^{\mu}{}_{\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\partial_\nu \varphi}$$

(INVARIANCE W.R.T. SPACETIME TRANSL.)

$\boxed{\partial_\mu T_c^{\mu\nu} = 0}$ ... "4 CONSERVED CURRENTS"

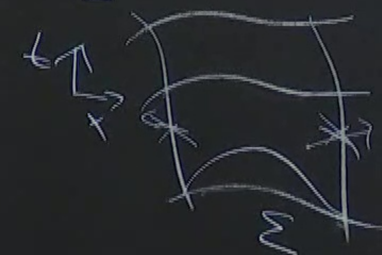
CHARGES: $P_\nu = \int_\Sigma T^{\mu}{}_{\nu}\, d\Sigma_\mu$

• $T_c^{\mu\nu}$ CAN BE UPGRADED TO A SYMMETRIC $T^{\mu\nu}$.

$$\boxed{T_c{}^{\mu}{}_{\nu} = \eta^{\mu}{}_{\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\partial_\nu \varphi}$$ (INVARIANCE W.R.T. SPACETIME TRANSL.)

CHARGES:

$$\boxed{\partial_\mu T_c^{\mu\nu} = 0}$$ ... "4 CONSERVED CURRENTS" $$P_\nu = \int_\Sigma T^{\mu}{}_{\nu}\, d\Sigma_\mu$$



$T_c^{\mu\nu}$ CAN BE UPGRADED TO A SYMMETRIC $T^{\mu\nu}$.

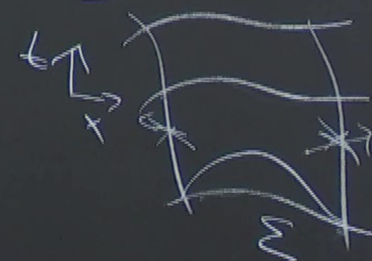
$$\boxed{T_c{}^{\mu}{}_{\nu} = \eta^{\mu}{}_{\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\,\partial_\nu \varphi}$$

(INVARIANCE W.R.T. SPACETIME TRANSL.)

$\boxed{\partial_\mu T_c^{\mu\nu} = 0}$ ... "4 CONSERVED CURRENTS"

CHARGES:

$$P_\nu = \int_\Sigma T^{\mu}{}_{\nu}\, d\Sigma_\mu$$



- $T_c^{\mu\nu}$ CAN BE UPGRADED TO A SYMMETRIC $T^{\mu\nu}$
- LORENTZ TRANSFORMATIONS → CONSERVED ANGULAR MOM TENSOR

$$\boxed{M^{\mu}{}_{\rho\sigma} = 2T^{\mu}{}_{[\rho}x_{\sigma]}} = T^{\mu}{}_{\rho}x_{\sigma} - T^{\mu}{}_{\sigma}x_{\rho}$$

SYMMETRIC $\underline{T^{\mu\nu}}$ (ABSORBED INTERNAL SPIN IN $T^{\mu\nu}$)

$$\partial_\mu M^{\mu}{}_{\rho\sigma} = 0 \ldots$$ 6 CURRENTS

6 CONSERVED CHARGES

$$L_{\rho\sigma} = \int_{\Sigma} M^{\mu}{}_{\rho\sigma}\, d\Sigma_\mu$$

$$\boxed{M^{\mu}{}_{[\rho\,x\,\sigma]}} = T^{\mu}{}_{\rho}\, x_{\sigma} - T^{\mu}{}_{\sigma}\, x_{\rho}$$

SYMMETRIC $\underline{T^{\mu\nu}}$ (ABSORBED INTERNAL SPIN IN $T^{\mu\nu}$)

0 ... 6 CURRENTS

$M^{\mu}{}_{\rho\sigma}\, d\Sigma_{\mu}$

3 ANGULAR MOMENTA

3 LINEAR MOMENTA (FOR CENTER OF MASS)

ANGULAR MOM TENSOR

"$0 = \frac{dL_{\rho\sigma}}{dt} = \{L_{\rho\sigma}, H\} + \frac{\partial L_{\rho\sigma}}{\partial t}$"

NEED THIS

x_ρ

SPIN)

ENTA

NTA (FOR CENTER OF MASS)

EXAMPLES:

- SCALAR FIELD

$$S = \int d^4x \left(-\frac{1}{2} \underbrace{(\partial_\mu \varphi)(\partial^\mu \varphi)}_{\eta^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi} - V(\varphi) \right)$$

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \\ & & & 1 \end{pmatrix}$$

...IPLES:

SCALAR FIELD

$$S = \int d^4x \left(-\frac{1}{2} \underbrace{(\partial_\mu \varphi)(\partial^\mu \varphi)}_{} - V(\varphi) \right)$$

$$\eta^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi$$

$$= \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \quad - \boxed{\partial_t \varphi \, \partial_t \varphi} \quad + \boxed{\partial_i \varphi \, \partial_i \varphi}$$

$$ds^2 = -\boxed{\eta_{\mu\nu}} dx^\mu dx^\nu$$

MPLES:

SCALAR FIELD

$$S = \int d^4x \, \Big(\underbrace{-\tfrac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) - V(\varphi)}_{\mathcal{L}} \Big)$$

$$-\frac{\partial V}{\partial \varphi} - \partial_\mu \left(- \partial^\mu \varphi \right) = 0$$

$$\boxed{\underbrace{\partial_\mu \partial^\mu}_{\Box} \varphi - \frac{\partial V}{\partial \varphi} = 0}$$

$$\boxed{\left(\partial^2 - m^2 \right) \varphi = 0}$$

KLEIN-GORDON EQ

- SIMPLEST EXAMPLE:

$$V = \frac{1}{2} m^2 \varphi^2$$

$$-\frac{\partial V}{\partial \varphi} - \partial_\mu\left(-\partial^\mu \varphi\right) = 0$$

$$\boxed{\underbrace{\partial_\mu \partial^\mu}_{\Box} \varphi - \frac{dV}{d\varphi} = 0}$$

$$\boxed{\left(\partial^2 - m^2\right)\varphi = 0}$$

KLEIN-GORDON EQ

- SIMPLEST EXAMPLE:

$$V = \frac{1}{2} m^2 \varphi^2$$

$$\boxed{T_C^{\mu\nu} = \eta^{\mu\nu}\mathcal{L} + \partial^\mu\varphi\,\partial^\nu\varphi}$$

• CLASSICAL ELECTRODYNAMICS

$$S[A^\mu, y^\nu] = -\frac{1}{16\pi}\underbrace{\int d^4x\, F_{\mu\nu}F^{\mu\nu}}_{\text{EM FIELD}} -$$

J^{μ} .. ELECTRON

$A_\mu J^\mu$

$$\ldots\nu - \underbrace{m\int d\lambda\,\sqrt{\eta_{\mu\nu}\frac{dy^\mu}{d\lambda}\frac{dy^\nu}{d\lambda}}}_{\text{ELECTRON(S)}} \;\underbrace{-\,e\int d\lambda\,\frac{dy^\mu}{d\lambda}A_\mu(y)}_{\text{(INTERACTION TERM)}}$$

$\eta_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}$

(S)

J^μ ... ELECTRON

$$-e \int d\lambda \frac{dy^\mu}{d\lambda} A_\mu(y)$$

(INTERACTION TERM)

$$\boxed{A_\mu J^\mu}$$

GAUGE INVARIANT ?

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$A_\mu J^\mu \to A_\mu J^\mu + J^\mu \partial_\mu \Lambda$$

$$\eta_{\mu\nu}\frac{dy^\mu}{d\lambda}\frac{dy^\nu}{d\lambda}$$

(S)

J^μ ... ELECTRON

$$-e\int d\lambda\,\frac{dy^\mu}{d\lambda}A_\mu(y)$$

INTERACTION TERM

$$\boxed{A_\mu J^\mu}$$

GAUGE INVARIANT ?

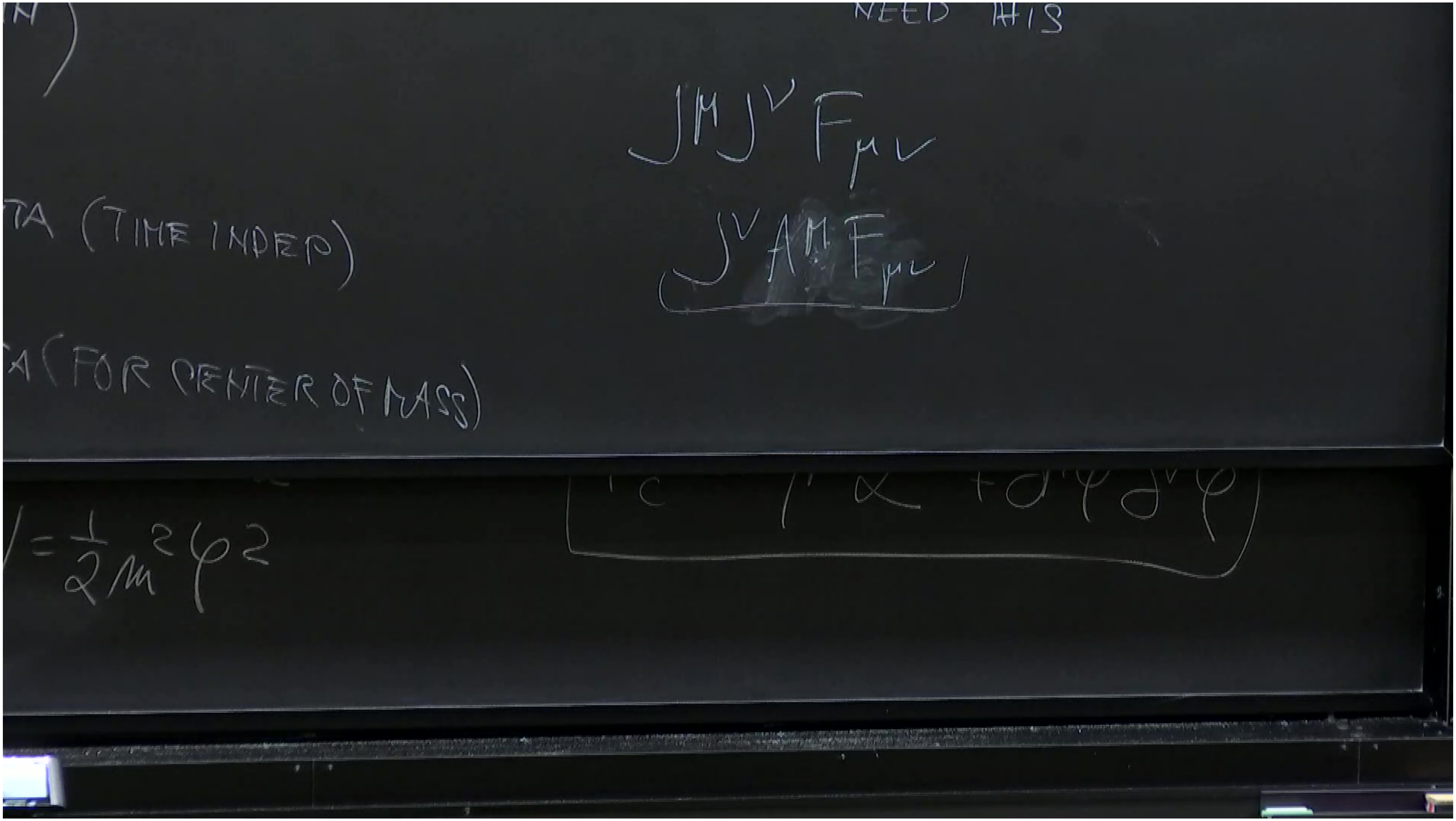
$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$A_\mu J^\mu \to A_\mu J^\mu + J^\mu \partial_\mu \Lambda$$

$$\partial_\mu (J^\mu \Lambda) - (\partial_\mu J^\mu)\Lambda$$

$\downarrow$

BOUNDARY


NEED THIS
$J^\mu J^\nu F_{\mu\nu}$
$J^\nu A^\mu F_{\mu\nu}$
TA (TIME INDEP)
A (FOR CENTER OF MASS)
$V = \frac{1}{2} m^2 \varphi^2$
$+\partial^\mu \varphi \partial^\nu \varphi$

$$S[A^\mu, y^\nu] = -\frac{1}{16\pi}\underbrace{\int d^4x\, F_{\mu\nu}F^{\mu\nu}}_{\text{EM FIELD}} - \underbrace{m\int d\lambda \sqrt{\eta_{\mu\nu}\frac{dy^\mu}{d\lambda}\frac{dy^\nu}{d\lambda}}}_{\text{ELECTRON(S)}}$$

δA^μ: $\partial_\mu F^{\mu\nu} = -4\pi J^\nu$ ✓, $F_{(\mu\nu,\sigma)} = 0$ ✓ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

ALREADY ASSUMED!

δy^ν: $\frac{dp^\mu}{d\tau} = e F^{\mu\nu} u_\nu$ ✓

• CONSERVATION OF CHARGE IN MAXWELL'S EQS.

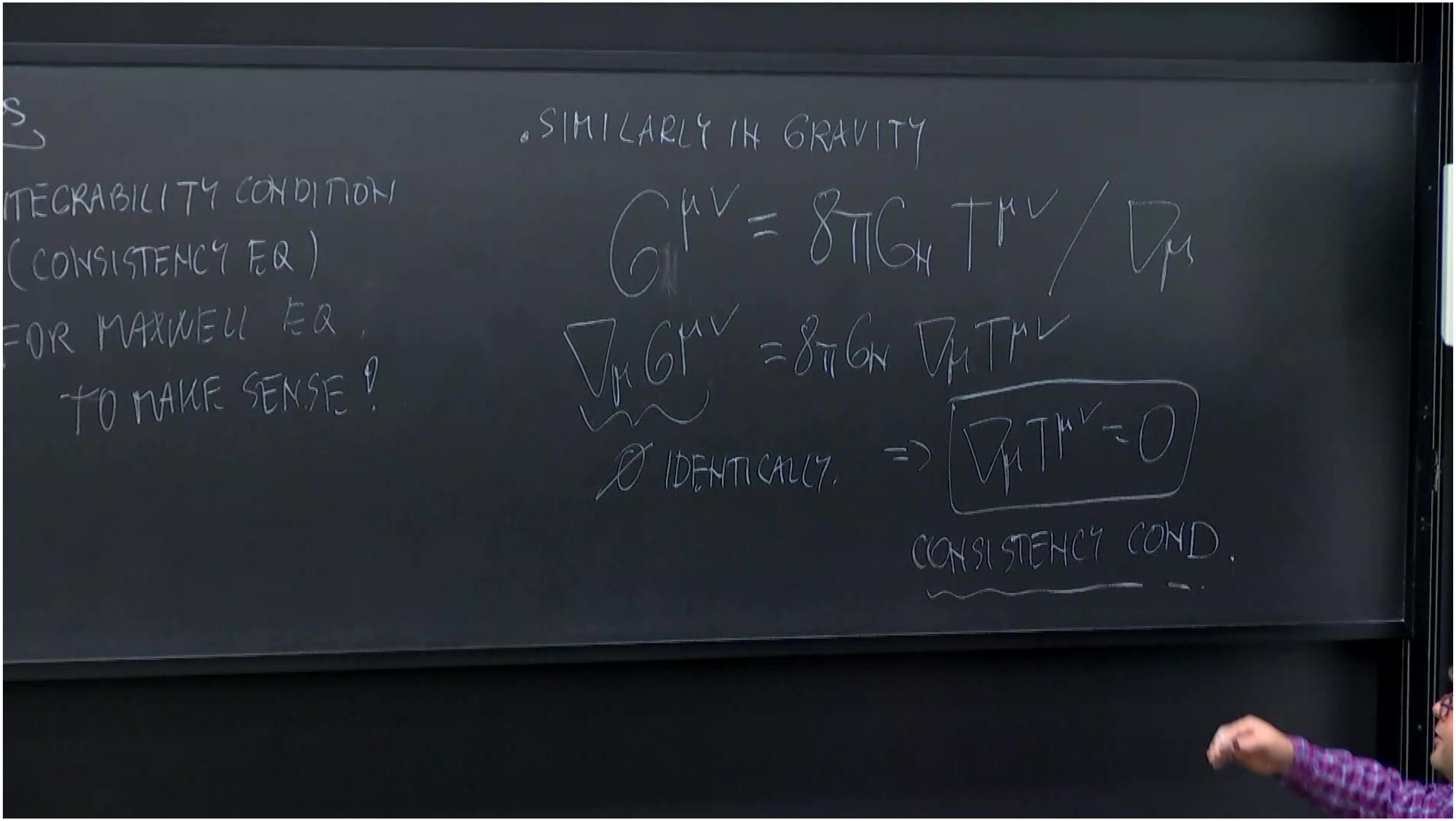
$$\partial_\mu F^{\mu\nu} = -4\pi J^\nu \quad / \, \partial_\nu$$

$$\underbrace{\partial_\nu \partial_\mu F^{\mu\nu}}_{0 \text{ IDENTICALLY}} = -4\pi \, \partial_\nu J^\nu$$

$$\Rightarrow \boxed{\partial_\nu J^\nu = 0}$$

BETTER BE TRUE.

INTEGRABILITY CONDITION
(CONSISTENCY EQ)
FOR MAXWELL EQ.
TO MAKE SENSE!


ITEGRABILITY CONDITION
(CONSISTENCY EQ)
FOR MAXWELL EQ
TO MAKE SENSE!
. SIMILARLY IN GRAVITY
$G^{\mu\nu} = 8\pi G_N T^{\mu\nu} \,/\, \nabla_\mu$
$\underbrace{\nabla_\mu G^{\mu\nu}} = 8\pi G_N \nabla_\mu T^{\mu\nu}$
$\emptyset$ IDENTICALLY. $\Rightarrow$ $\boxed{\nabla_\mu T^{\mu\nu} = 0}$
CONSISTENCY COND.

- GRAVITY:

BASIC FIELD IS THE METRIC $g_{\mu\nu}$

$$\mathcal{L} = \mathcal{L}(g, \cancel{\partial g}) \quad ?$$

DOES NOT EXIST!

$\mathcal{L} = \mathcal{L}(g, \partial g, \partial\partial g)$

HE METRIC $g_{\mu\nu}$

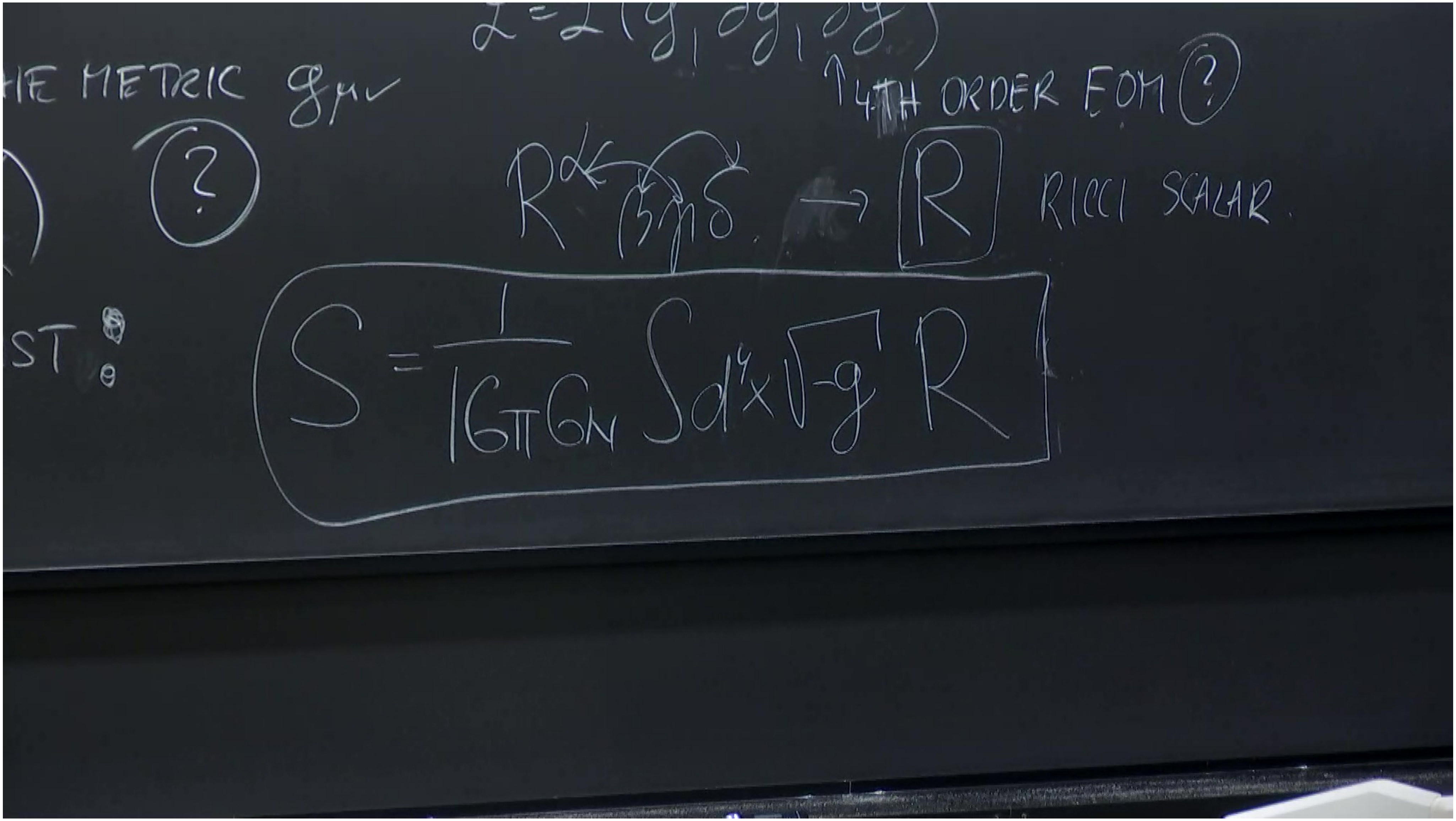
↑ 4TH ORDER EOM ?

?

$R^{\alpha}{}_{\beta\gamma\delta} \longrightarrow \boxed{R}$ RICCI SCALAR.

ST.

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}$$


$\mathcal{L} = \mathcal{L}(g, \partial g, \partial\partial g)$
HE METRIC $g_{\mu\nu}$
4TH ORDER EOM ?
?
$R^{\alpha}{}_{\beta\gamma\delta} \to R$
RICCI SCALAR.
ST
$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,R$

$\mathcal{L} = \mathcal{L}(g, \partial g, \partial^2 g)$

↑ 4TH ORDER EOM ?

$R^{\alpha}{}_{\beta\gamma\delta} \longrightarrow \boxed{R}$ RICCI SCALAR

$T^{\alpha\beta} g_{\alpha\beta} = T$

$$\boxed{\frac{1}{6\pi G_N} \int d^4x \sqrt{-g}\, R}$$