

Title: Quantum Field Theory I - Lecture 221031

Speakers: Gang Xu

Collection: Quantum Field Theory I (2022/2023)

Date: October 31, 2022 - 10:45 AM

URL: <https://pirsa.org/22100057>

What?

4c3RWYpf

Year 2022

Clue of

new: Wu's observation  
(Crime scene)

What?  
4c3RWYof  
Year

clue of

new: Wu's observation  
(crime scene)

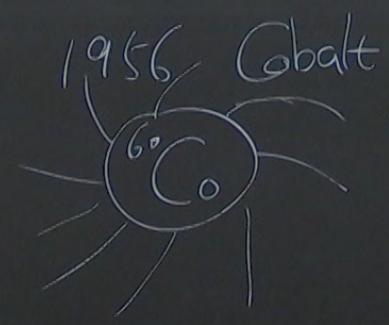
suspects:



Clue:  $\gamma$

raw: Wu's observation  
(crime scene)

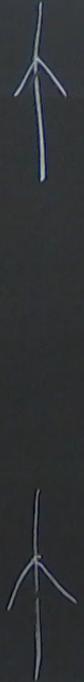
suspects:



6 Cobalt



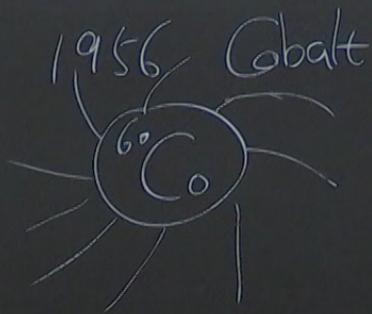
mirror



Clue:  $\gamma$

raw: Wu's observation  
(crime scene)

suspects: Dirac Spinor  
Weyl Spinor

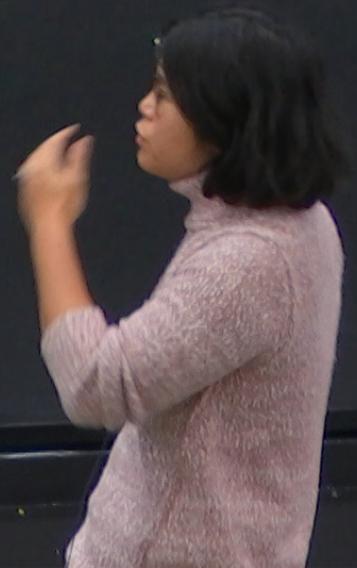


mirror

① choose coordinate system  
record  $\vec{p}$  spin direction

mirror " " " "

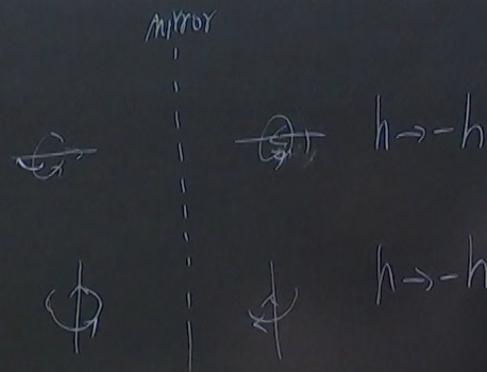
is  $\vec{p}$  same is helicity same?  
spin same



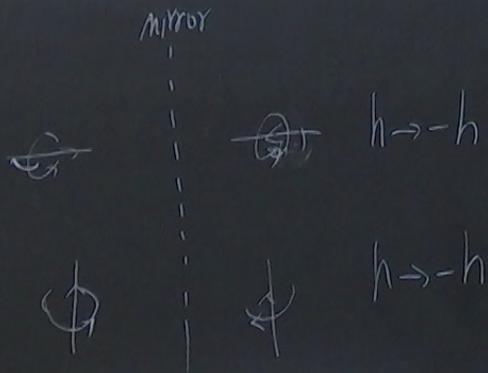
choose coordinate system  
record  $\vec{p}$  spin direction

mirror " " " "

is  $\vec{p}$  same is helicity same?  
spin same



Co



$h \rightarrow -h$

$h \rightarrow -h$

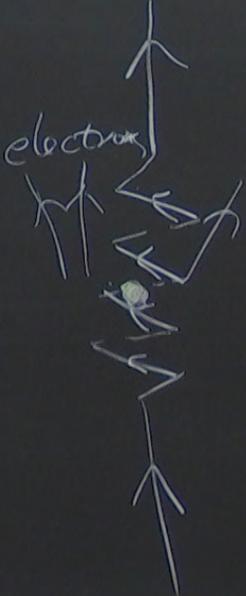
conclusion  
 right-handed Weyl  
 $\rightarrow$  left-handed Weyl

$\chi_{pf}$   
 2022

clue:  $\chi$   
 new: Wu's observation  
 (crime scene)

suspects: Dirac Spinor  $\rightarrow \gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^5$   
 Weyl Spinor  $\begin{pmatrix} \chi \\ \chi \end{pmatrix} \rightarrow \chi$

1956 Cobalt  

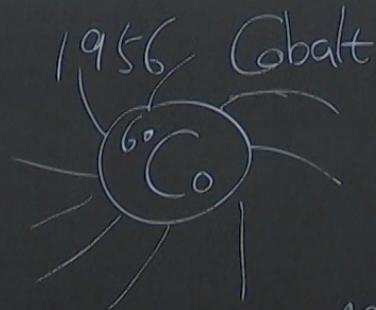

electrons  


mirror

Ypf  
2022

Clue:  $\gamma$

new: Wu's observation  
(crime scene)



suspects: Dirac Spinor  
Weyl Spinor

new suspects  
 $\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^5$

$\begin{pmatrix} 4 \\ + \end{pmatrix} \gamma \rightarrow \gamma$   
 $\begin{pmatrix} 4 \\ - \end{pmatrix}$  flipped.

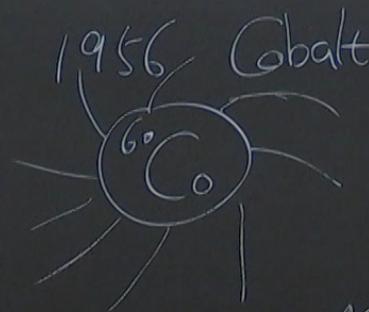


2022

Clue:  $\gamma$

new: Wu's observation  
(crime scene)

suspects: Dirac Spinor  
Weyl Spinor

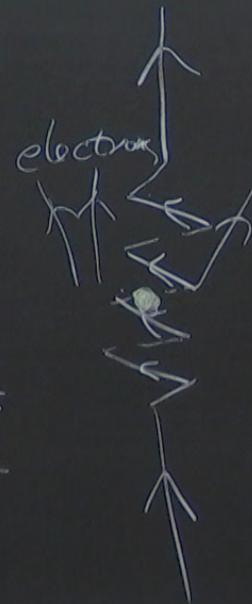


$$\begin{pmatrix} \sigma^1 \\ \sigma^1 \\ 1 \end{pmatrix}$$

new suspects

$$\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^5$$

$$\begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \rightarrow \begin{pmatrix} 4 \\ 4 \end{pmatrix} \text{ flipped.}$$



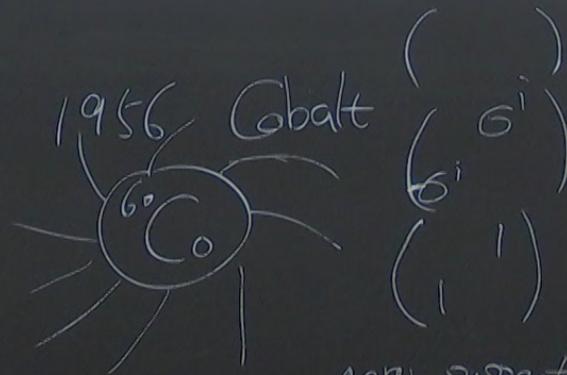
mirror

Ypf  
2022

Clue:  $\gamma$

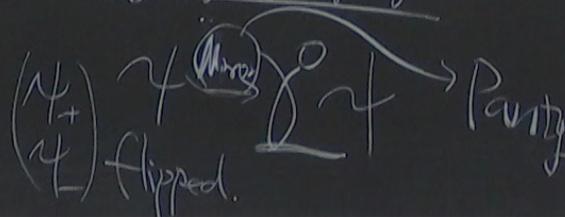
raw: Wu's observation  
(crime scene)

suspects: Dirac Spinor  
Weyl Spinor



new suspects

$\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^5$



mirror

Weyl Spinor

$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$  flipped.

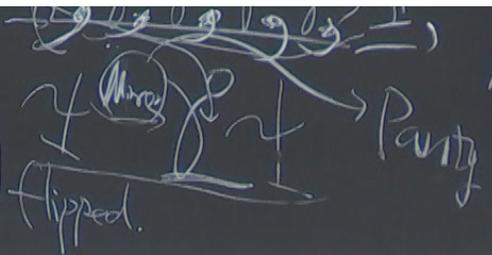
$$\psi(t, x) \xrightarrow{P} \beta \psi(t, -x)$$

$\beta = \gamma_0$

Weyl Spinor

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

flipped.



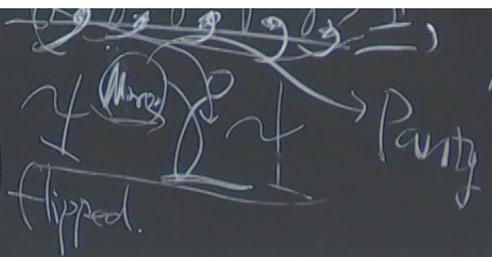
$$P \psi(t, x) P^{-1} = \beta \psi(t, -x)$$

$$P_t = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Weyl Spinor

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

flipped.



$$P \psi(t, x) P^{-1} = \beta \psi(t, -x)$$

$$U^{-1}(A) H(x) U^{-1}(A) = H(Ax + b)$$

$$P_t = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Weyl Spinor

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

flipped.

Parity  $\uparrow$

$$\psi^T = \beta \psi(t, -x)$$

$$P_t \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}$$

Weyl Spinor

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

flipped.

Parity



$$P^T = \beta \psi(t, -x)$$

transformation

$$P_t =$$

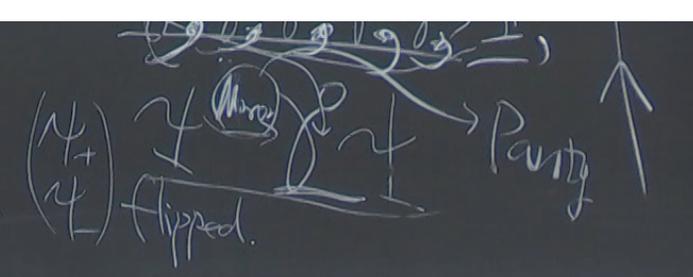
$$\begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}$$

$$U^{-1}(x) = f(ax+b)$$

Weyl Spinor



$$\psi(t, \vec{x}) \stackrel{P}{=} \beta \psi(t, -\vec{x})$$

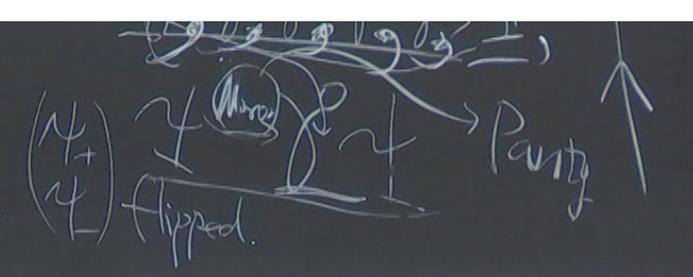
transformation

$$P_t = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$U^{-1}(\Lambda) H(\alpha) U^{-1}(\Lambda) = H(\Lambda \alpha)$$

Weyl Spinor



$$\psi(t, \vec{x}) P^\dagger = \beta \psi(t, -\vec{x})$$

$\beta = \gamma_0$

transformation

$$P_t = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

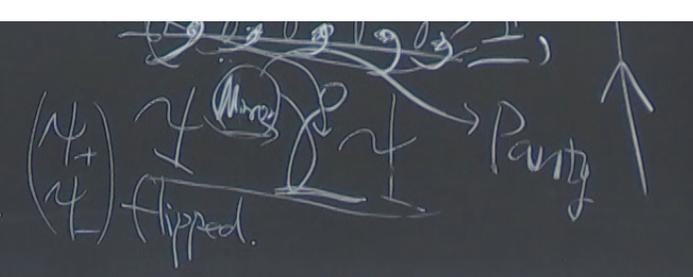
$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$U(\Lambda) H(x) U^{-1}(\Lambda) = H(\Lambda x)$$

$\downarrow$  scalar

$$U(\Lambda) V(x) U^{-1}(\Lambda) = V(\Lambda x)$$

Weyl Spinor



$$\psi(t, \vec{x}) \rightarrow \beta \psi(t, -\vec{x})$$

$\beta \rightarrow$  activity

transformation

$$P_t = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} t \\ -x \\ -y \\ -z \end{pmatrix} \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

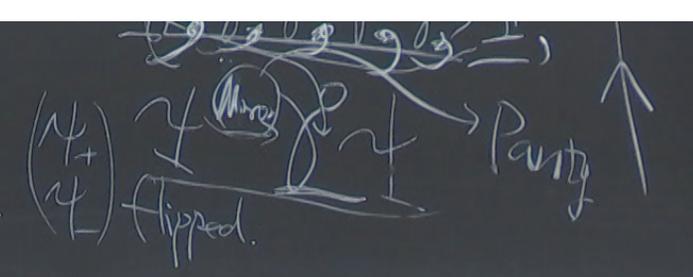
$$U(\Lambda) H(\alpha) U^{-1}(\Lambda) = H(\Lambda \alpha)$$

$\downarrow$  scalar  $\uparrow$

$$U(\Lambda) V_{\alpha}^{\mu} U^{-1}(\Lambda) = \Lambda^{\mu}_{\nu} V_{\alpha}^{\nu}$$

$$P^2 = 1$$

Weyl Spinor



$$\psi(t, \vec{x}) \rightarrow \beta \psi(t, -\vec{x})$$

$\beta \rightarrow$  activity

transformation

$$P_t = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ -x \\ -y \\ -z \end{pmatrix} \begin{pmatrix} & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$U^{-1}(\Lambda) H(\alpha) U^{-1}(\Lambda) = H(\Lambda \alpha)$$

↓ scalar

$$U(\Lambda) \psi(x) U^{-1}(\Lambda) \rightarrow \psi(\Lambda x)$$

$$P^2 = 1 \quad P = P^{-1}$$

List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \gamma^\mu \psi$

$\bar{\psi} \gamma^5 \psi$

$\bar{\psi} \gamma^5 \gamma^\mu \psi$

$\bar{\psi} \gamma^5 \gamma^\mu \gamma^5 \psi$

$\bar{\psi} \gamma^5 \gamma^\mu \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^5 \gamma^\mu \gamma^5 \gamma^5 \gamma^5 \psi$



List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \not{\partial} \psi$

$\mu=0, 1, 2, 3$   $\bar{\psi} \not{\partial}^\mu \psi$

$\bar{\psi} \not{\partial}^\mu \not{\partial}^\nu \psi$

$\bar{\psi} \not{\partial}^\mu \not{\partial}^\nu \not{\partial}^\rho \psi$

$\bar{\psi} \not{\partial}^\mu \not{\partial}^\nu \not{\partial}^\rho \not{\partial}^\sigma \psi$

$\bar{\psi} \not{\partial}^\mu \not{\partial}^\nu \not{\partial}^\rho \not{\partial}^\sigma \not{\partial}^\tau \psi$



List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \not{\partial} \psi$

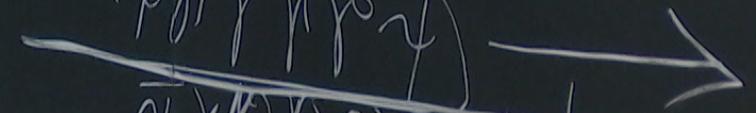
$\mu=0, 1, 2, 3$   $\bar{\psi} \not{\partial}^\mu \psi$

$\bar{\psi} \not{\partial}^2 \psi$

$\bar{\psi} \not{\partial}^3 \psi$

~~$\bar{\psi} \not{\partial}^4 \psi$~~

~~$\bar{\psi} \not{\partial}^5 \psi$~~



List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \gamma^\mu \partial_\mu \psi$

$\mu=0,1,2,3$   $\bar{\psi} \gamma^\mu \psi$

~~$\bar{\psi} \gamma^\mu \psi$~~

~~$\bar{\psi} \gamma^\mu \psi$~~

~~$\bar{\psi} \gamma^\mu \psi$~~

~~$\bar{\psi} \gamma^\mu \psi$~~

$\bar{\psi} \gamma^5 \psi$

$$(\gamma^0)^2 = 1$$

$$(\gamma^i)^2 = -1$$

List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \gamma^\mu \partial_\mu \psi$

$\mu=0,1,2,3$   $\bar{\psi} \gamma^\mu \gamma^\nu \psi$

~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \psi$~~

~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \psi$~~

~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau \psi$~~

~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau \gamma^\delta \psi$~~

$[\gamma^\mu, \gamma^\nu]$

$\bar{\psi} \gamma^\mu \psi$

$\bar{\psi} \gamma^5 \gamma^\mu \psi$

$\bar{\psi} \gamma^5 \psi$

$$(\gamma^0)^2 = 1$$

$$(\gamma^i)^2 = -1$$

double

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix} \quad \text{mass term} \quad \underline{\underline{F \quad F}}$$

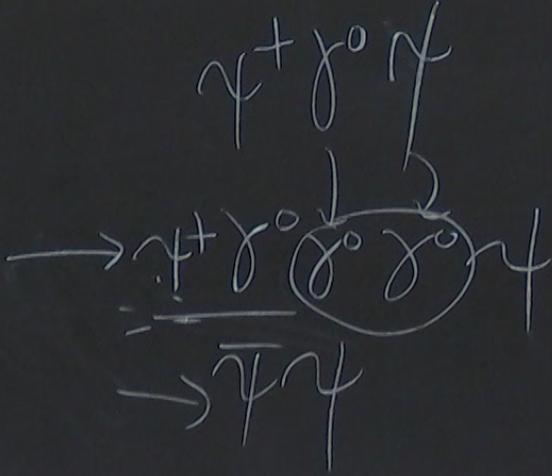
$$\psi^+ \rightarrow \psi^+(t, \vec{x}) \gamma^0$$

$$\begin{array}{c} \psi^+ \gamma^0 \psi \\ \rightarrow \psi^+ \gamma^0 \gamma^0 \gamma^0 \psi \end{array}$$

double

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix} \quad \text{mass term} \quad \underline{\underline{F \quad F}}$$

$$\psi^+ \rightarrow \psi^+(t, \vec{x}) \gamma^0$$



double

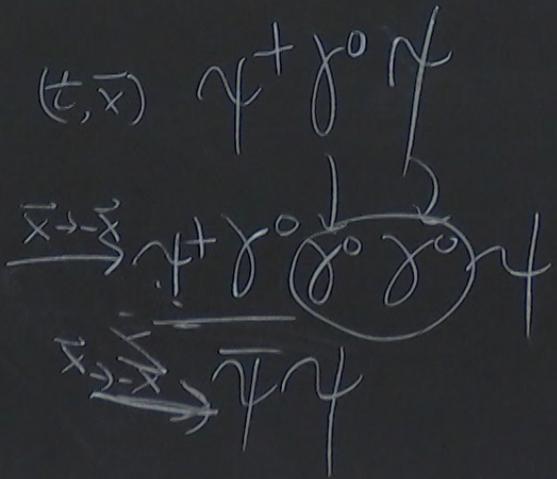
$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}$$

mass term

$$\overline{\psi} \psi$$

$$\overline{\psi} \gamma^5 \psi$$

$$\psi^+ \rightarrow \psi^+(t, -\vec{x}) \gamma^0$$





List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \gamma^\mu \partial_\mu \psi$

0, 1, 2, 3  $\bar{\psi} \gamma^\mu \gamma^\nu \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \psi$

~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \psi$~~   
 ~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \gamma^5 \psi$~~   
 ~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \gamma^5 \gamma^5 \psi$~~   
 ~~$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$~~

$[\gamma^\mu, \gamma^\nu]$

$\bar{\psi} \gamma^{\mu\nu} \psi$

$\bar{\psi} \gamma^5 \gamma^\mu \psi$

$\bar{\psi} \gamma^5 \psi$

Weak theory

$(\gamma^0)^2 = 1$

responsible  $(\gamma^i)^2 = -1$

List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \gamma^\mu \psi$  →

0, 1, 2, 3  $\bar{\psi} \gamma^{\mu\nu} \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^{\mu\nu} \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$[\gamma^\mu, \gamma^\nu]$

$F_{\mu\nu} \bar{\psi} \gamma^{\mu\nu} \psi \leftarrow \text{Mina}$

$\bar{\psi} \gamma^5 \gamma^\mu \psi$

Weak theory

$\bar{\psi} \gamma^5 \psi$

scalar

$(\gamma^0)^2 = 1$

responsible  $(\gamma^i)^2 = -1$

-1  
+1  
+1  
+1

axial vector

List of suspects

mass  $\bar{\psi} \psi$

kinetic  $\bar{\psi} \gamma^\mu \psi \rightarrow$

$\mu=0,1,2,3$   $\bar{\psi} \gamma^\mu \gamma^\nu \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \psi$

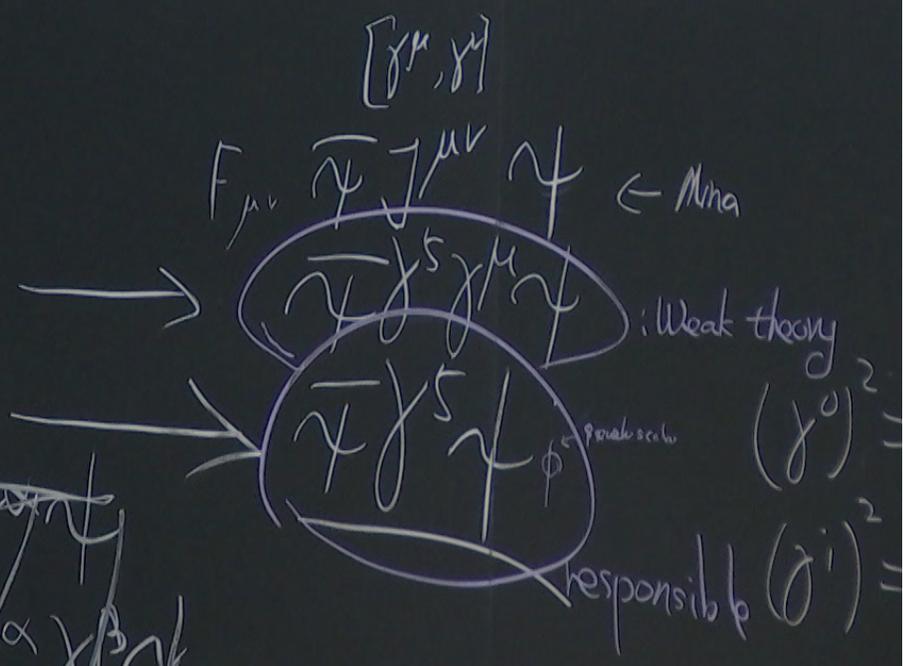
$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$\bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 \gamma^5 \gamma^5 \gamma^5 \gamma^5 \psi$

$$\begin{array}{r} (1+1)^4 \\ \hline 1 \\ 4 \\ 6 \\ 4 \\ 1 \\ \hline 16 \\ \hline = 2^4 = 4^2 \end{array}$$





a particular  $\overline{F} \cdot \overline{A}_\mu$

a particular  $\bar{\psi} \gamma^\mu \psi A_\mu$

$$A_\mu \rightarrow \partial_\mu \alpha$$

apple: nice global symmetry

as



apple: nice global symmetry  $\xrightarrow{\text{broken}}$  apple  
label: stem, smooth skin, ~~crispy~~  $\rightarrow$  gauge symmetry



apple: nice global symmetry  $\xrightarrow{\text{broken}}$  apple  
pineapple label: stem, smooth skin, ~~crispy~~  $\rightarrow$  gauge symmetry

photon: LC  $A_\mu$   
massless  $+ A_\mu \rightarrow \partial_\mu \phi \rightarrow$  gauge symmetry  
 $\downarrow$   
massive

apple: nice global symmetry  $\xrightarrow{\text{broken}}$  apple  
pineapple label: stem, smooth skin, ~~crispy~~  $\rightarrow$  gauge symmetry

photon: LC  $A_\mu$   
massless  $+ A_\mu \rightarrow \partial_\mu \phi \rightarrow$  gauge symmetry  
W boson massive

a particular  $\psi \gamma^\mu \psi A_\mu$

$A_\mu \rightarrow \partial_\mu \alpha + A_\mu$

$\int d^4x \bar{\psi} \gamma^\mu (\partial_\mu \psi) \psi$

apple: nice global symmetry broken a

pineapple label: stem smooth skin crispy → g

photon: LC  $A_\mu$

massless  $+ A_\mu \rightarrow \partial_\mu \alpha \rightarrow$  gauge symmetry

W boson massive



$$\bar{\psi} \psi$$

$$\int \bar{\psi} \gamma^\mu (\partial_\mu \psi) \psi$$

$$\psi \rightarrow \psi e^{-iQ\alpha(x)}$$

photon: LC  $A_\mu$   
 massless  
 $+ A_\mu \rightarrow \partial_\mu \alpha(x) \rightarrow g$   
 W boson  
 massive

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + \underline{Q\bar{\psi}A_{\mu}\gamma^{\mu}\psi}$$

$$D_{\mu} \equiv \partial_{\mu} - iQA_{\mu}$$

$$= \bar{\psi}(i\not{D} - m)\psi$$

