Title: Quantum Field Theory I - Lecture 221028

Speakers: Gang Xu

Collection: Quantum Field Theory I (2022/2023)

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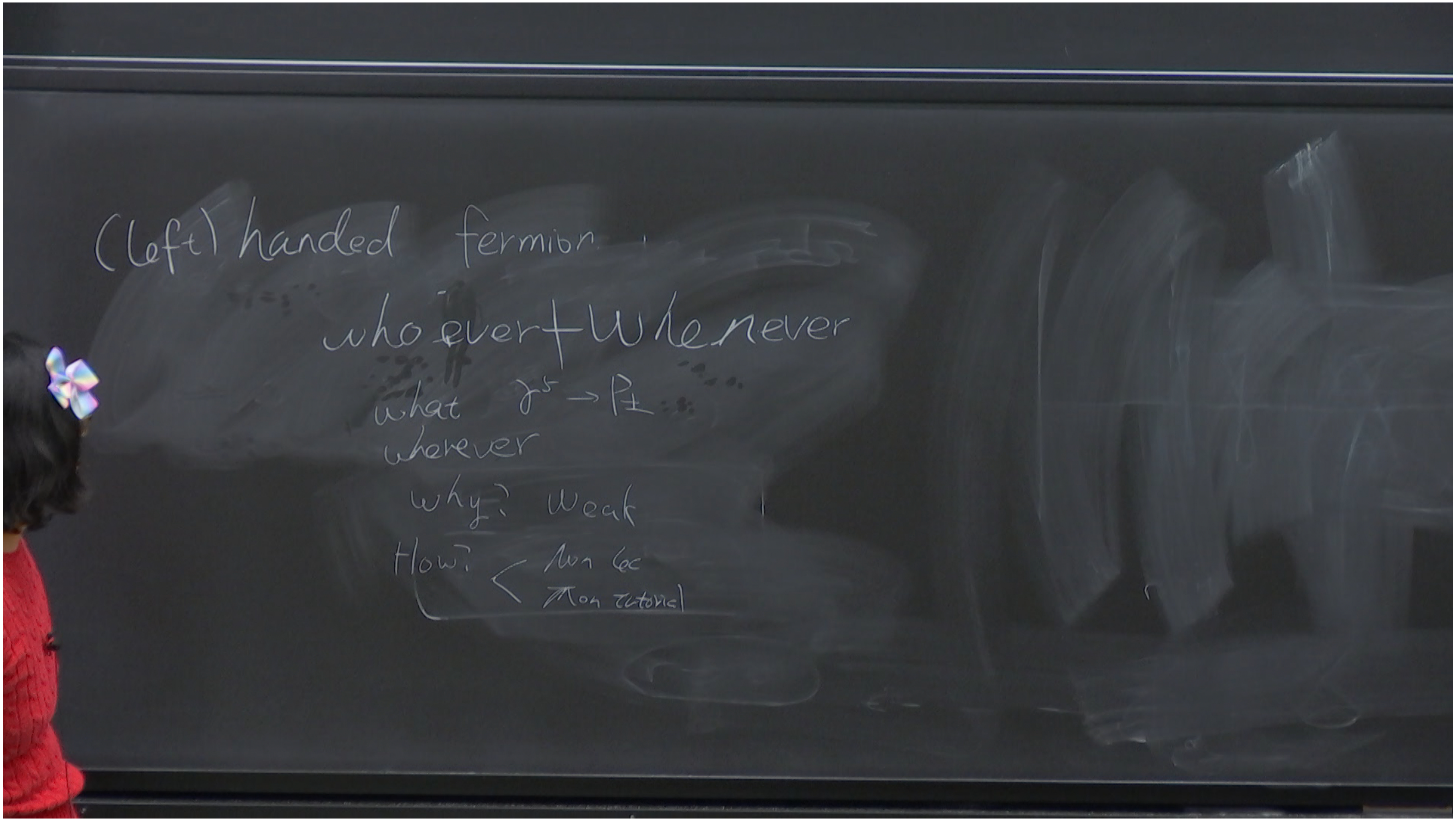

(left) handed fermion
who ever + Whenever
what $\gamma^5 \to P_L$
wherever
why? weak
How? non loc
non rational

(left) handed fermion

who ever + whenever

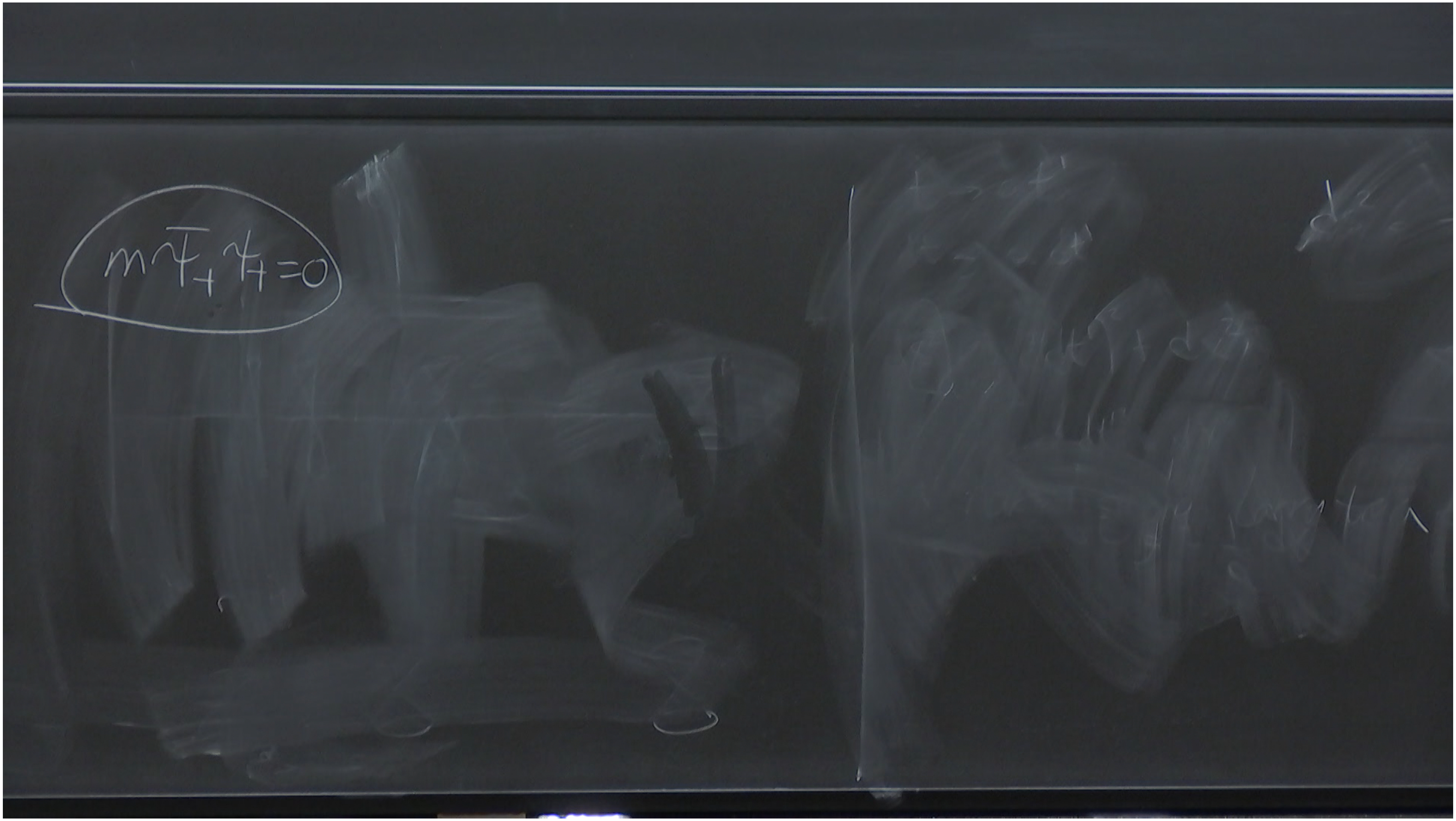
what $\gamma^5 \to P_\pm$

wherever

why? weak

How? < non loc / non tutorial

$m\bar{\psi}_+\psi_+$



$$m \bar{\psi}_+ \psi_+ = 0$$

$$m \bar{\psi} \psi \in m \bar{\psi}_+ \psi_-$$

$$\Downarrow$$

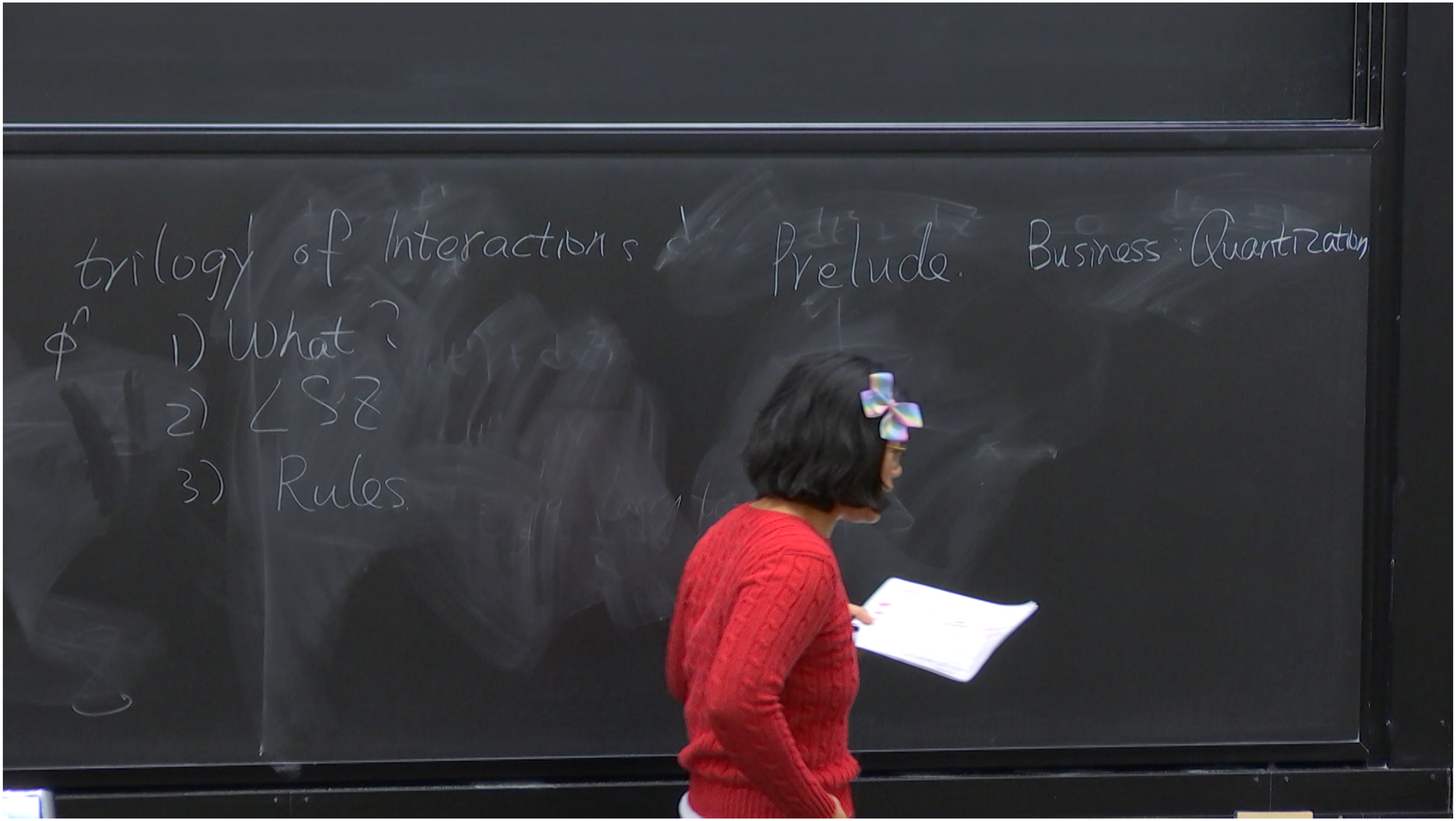
$$\lambda \bar{\psi}_+ H \psi_-$$

trilogy of Interactions

Prelude.

ϕ^n

1) What?
2) LSZ
3) Rules


trilogy of Interactions
Prelude.
Business: Quantization
1) What?
2) LSZ
3) Rules

trilogy of Interactions

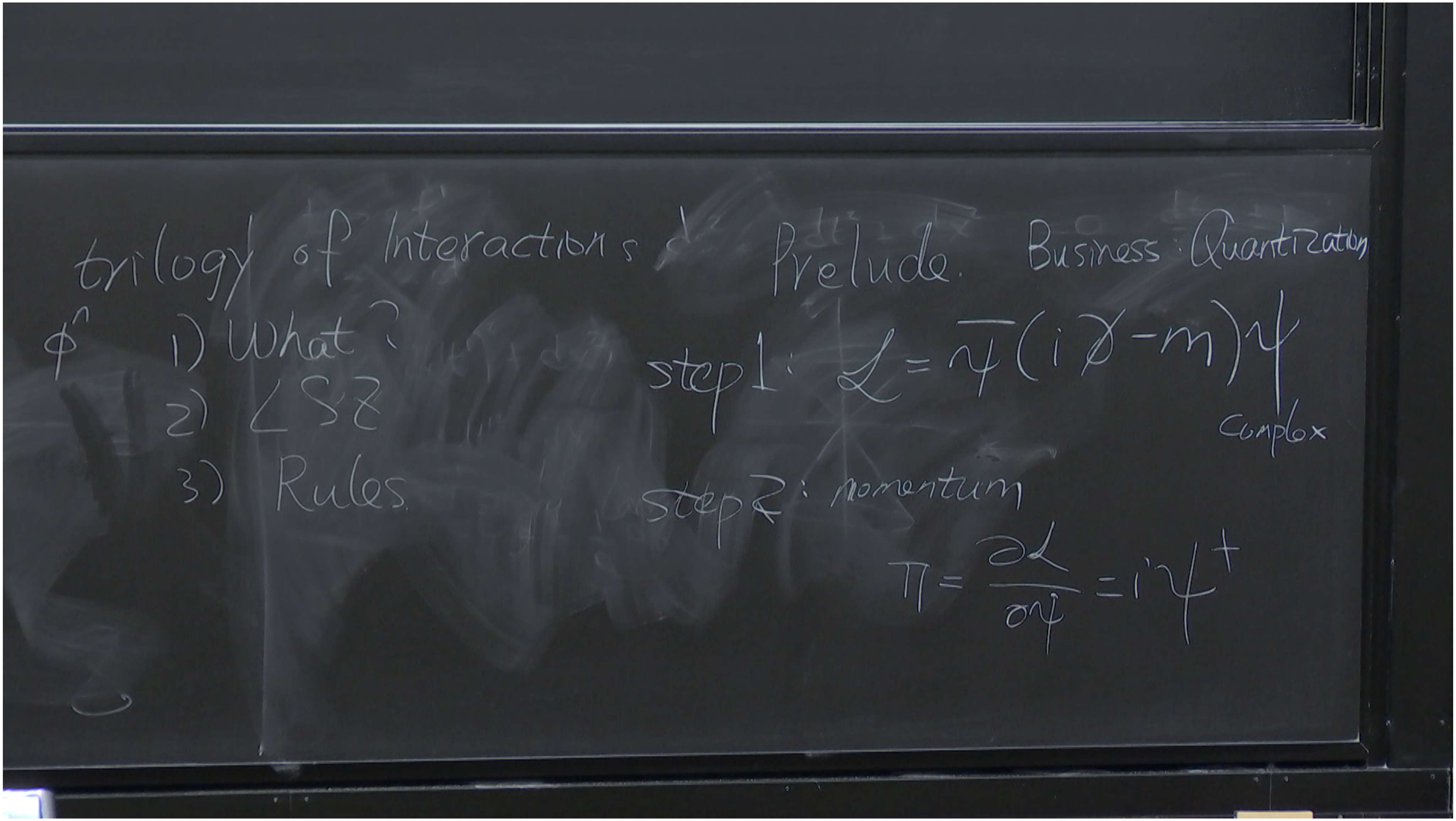
ϕ^n 1) What?

2) LSZ

3) Rules

Prelude. Business: Quantization

step 1: $\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$

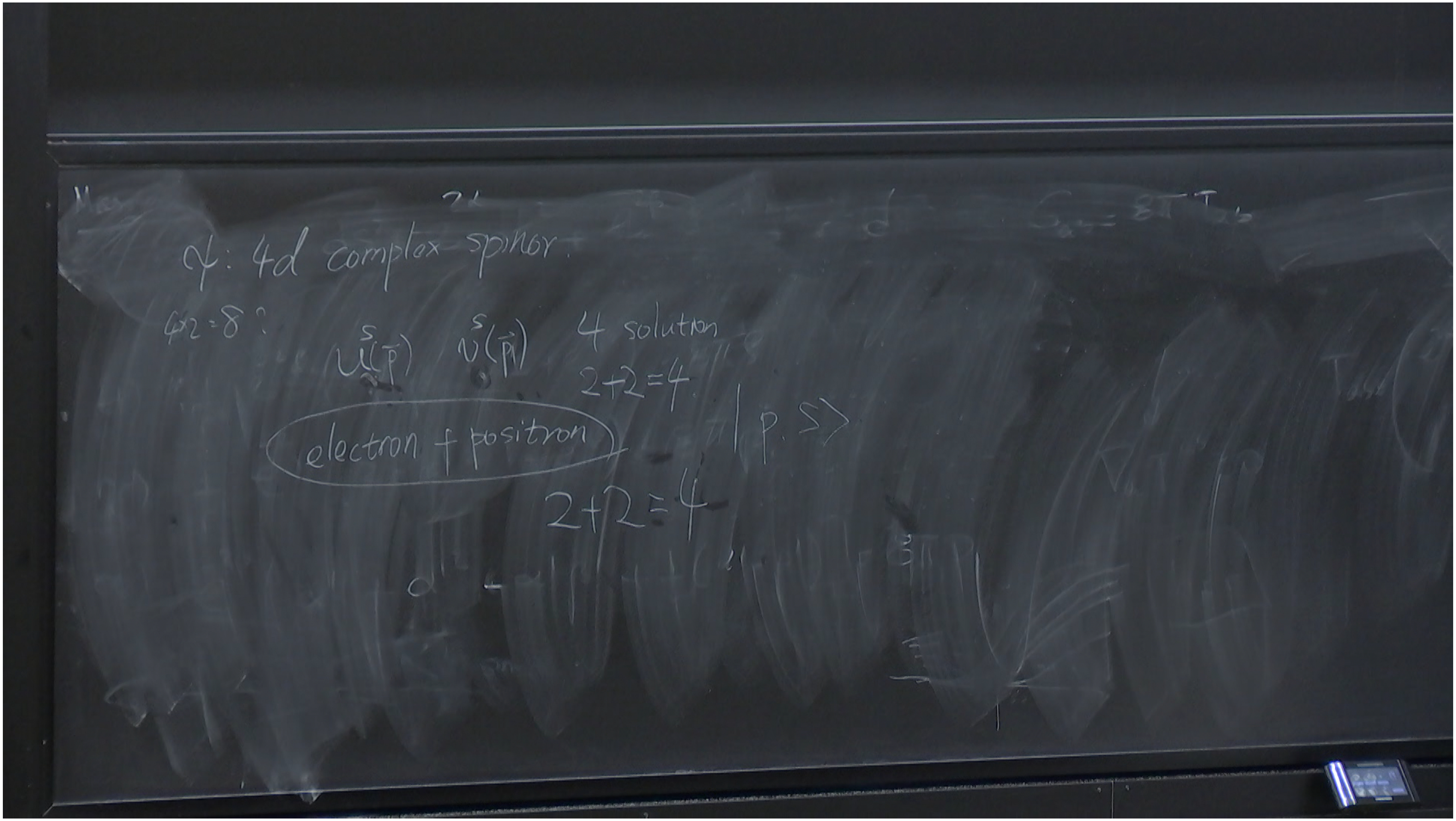

trilogy of Interactions
Prelude. Business: Quantization
ϕ^n 1) What?
2) LSZ
3) Rules
step1: $\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$
complex
step2: momentum
$\pi = \frac{\partial \mathcal{L}}{\partial \psi} = i\psi^{+}$

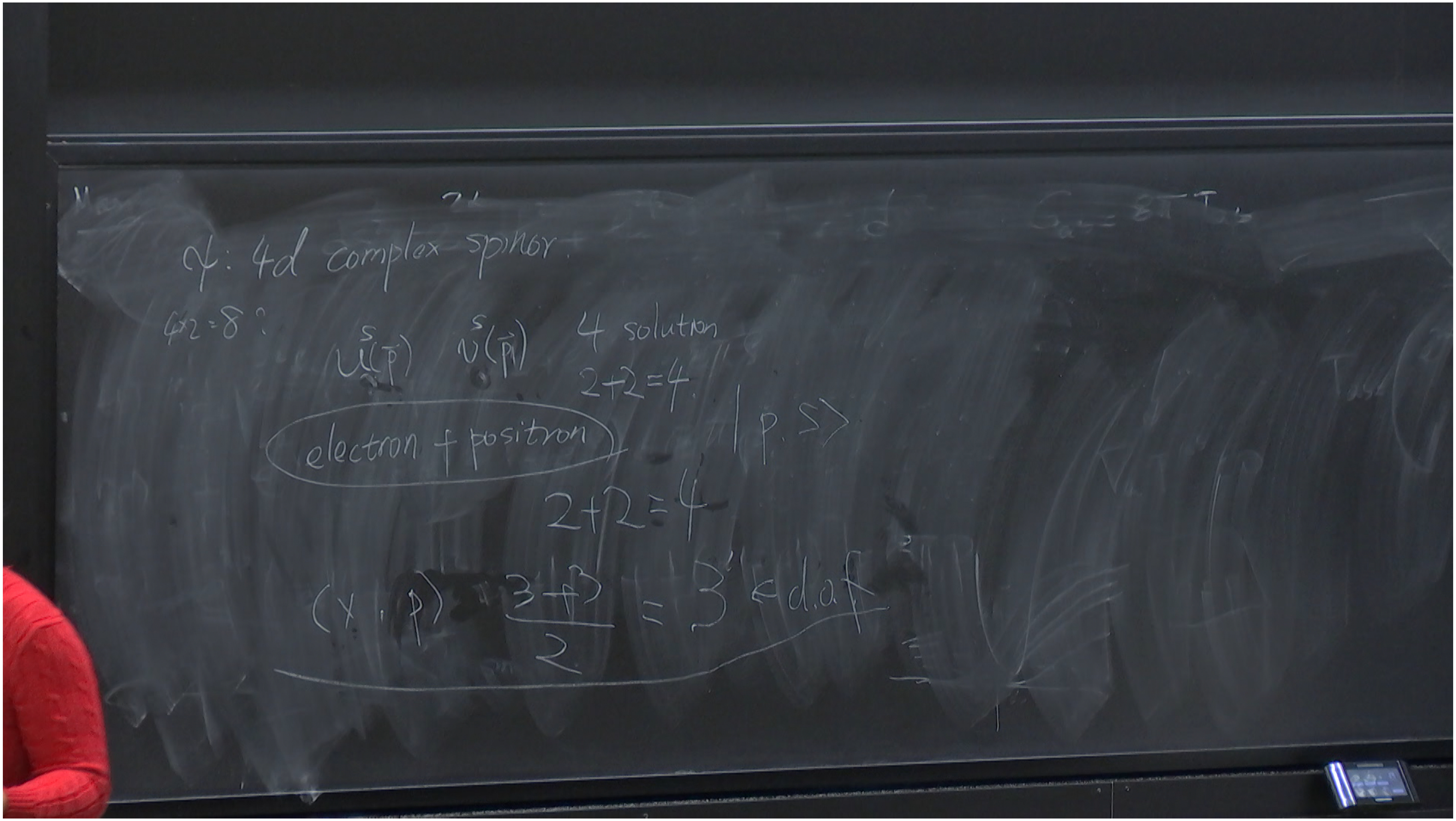
ψ: 4d complex spinor

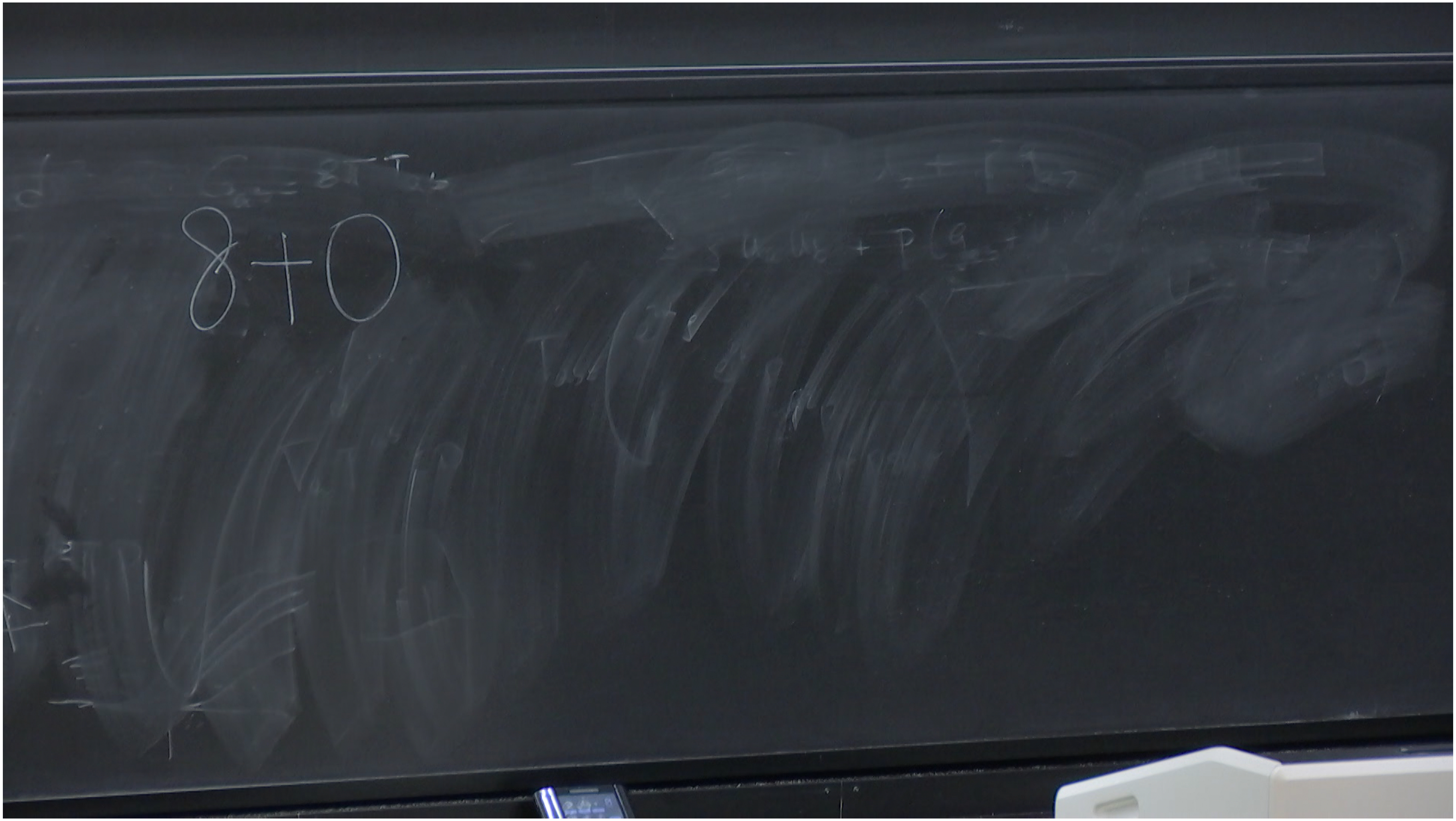
$4\times 2 = 8$?

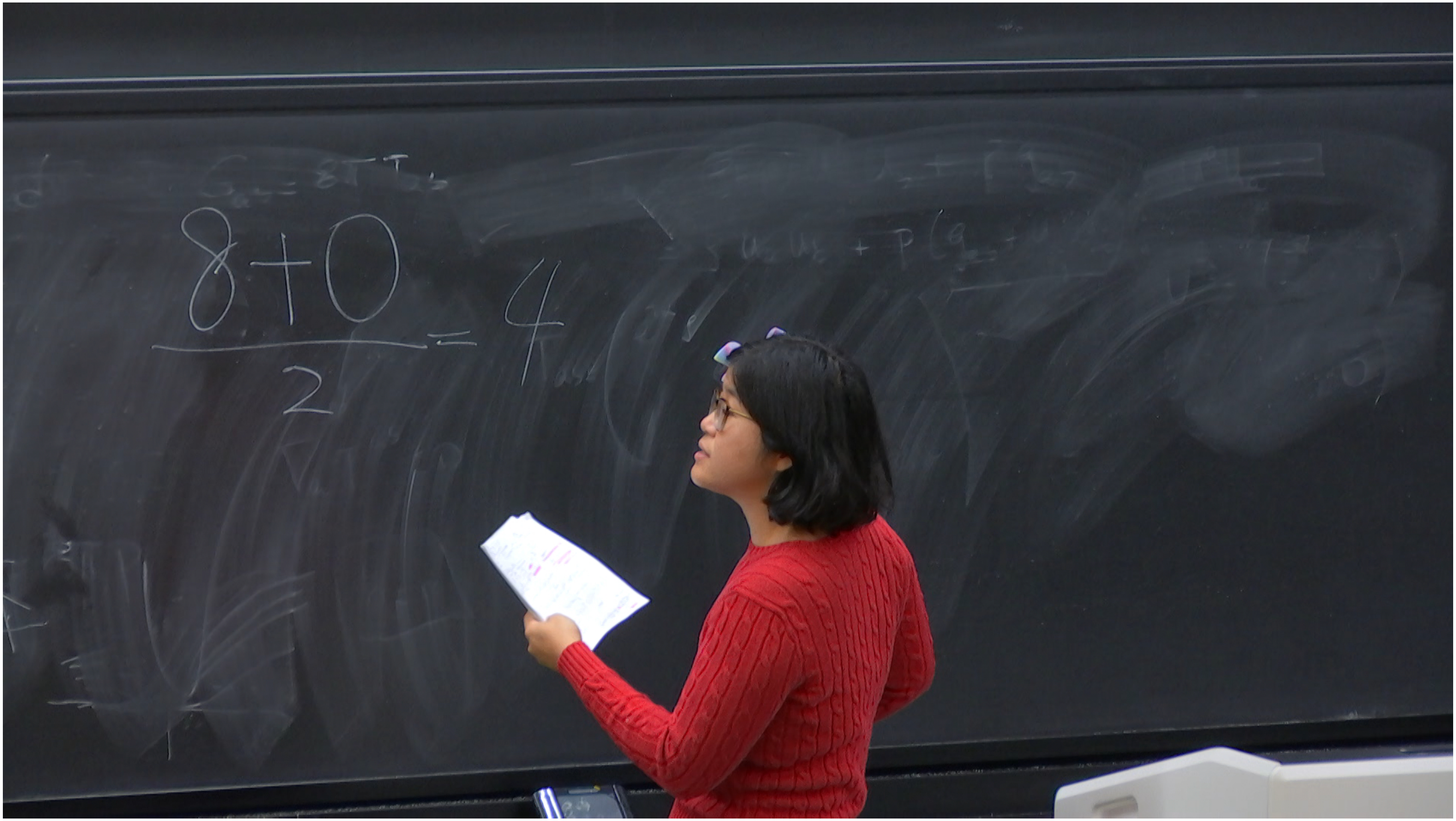
$\overset{s}{u}(\vec{p})$ $\quad$ $\overset{s}{v}(\vec{p})$ $\quad$ 4 solution

$2+2=4$


ψ: 4d complex spinor
4×2=8 :
u^s(p) v^s(p) 4 solution
2+2=4
electron + positron
|p, s>
2+2=4

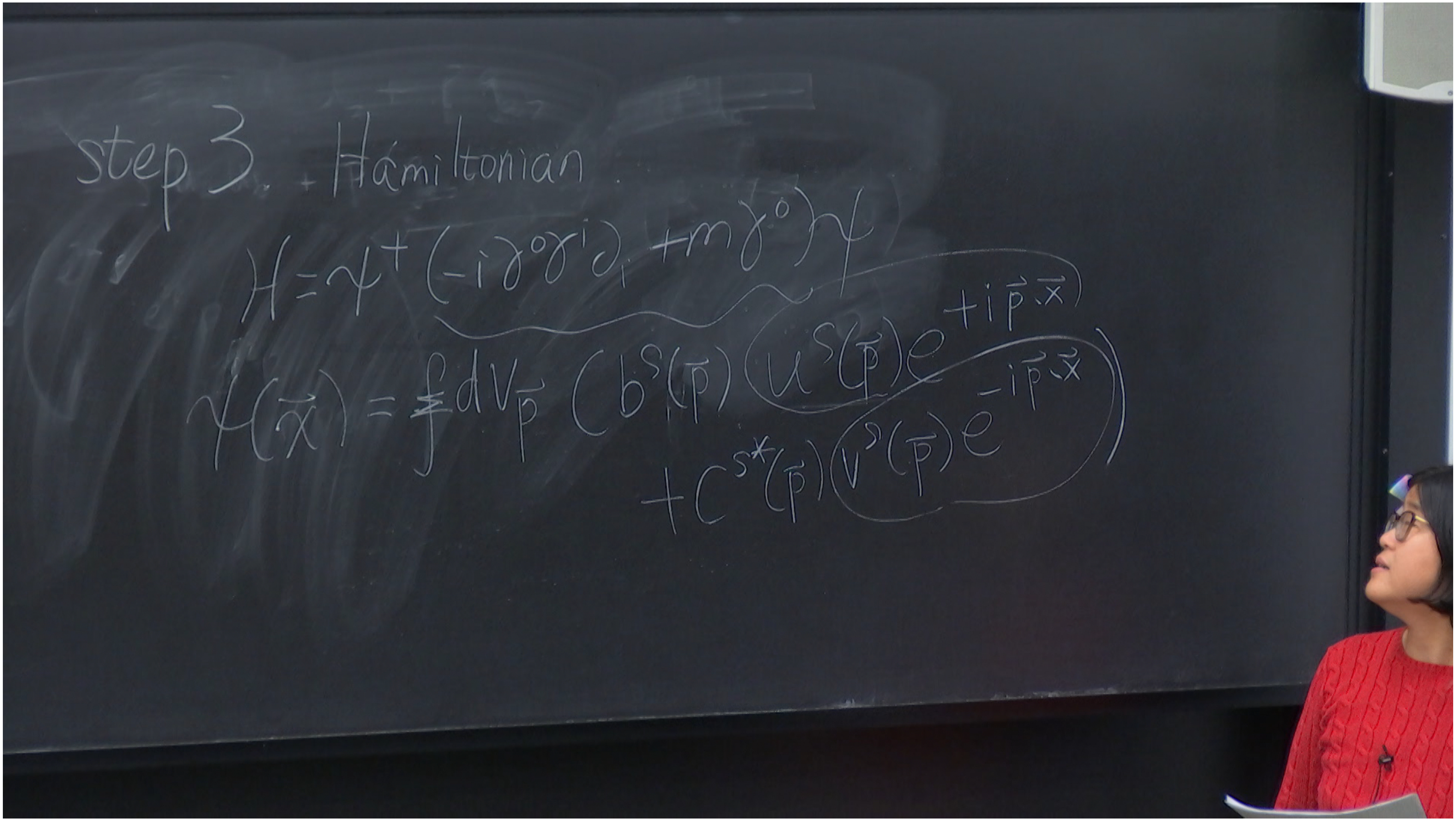

ψ: 4d complex spinor
4×2 = 8 :
u^s(p) v^s(p) 4 solution
2+2=4
|p, s>
electron + positron
2+2=4
(x, p) 3+3 / 2 = 3 d.o.f

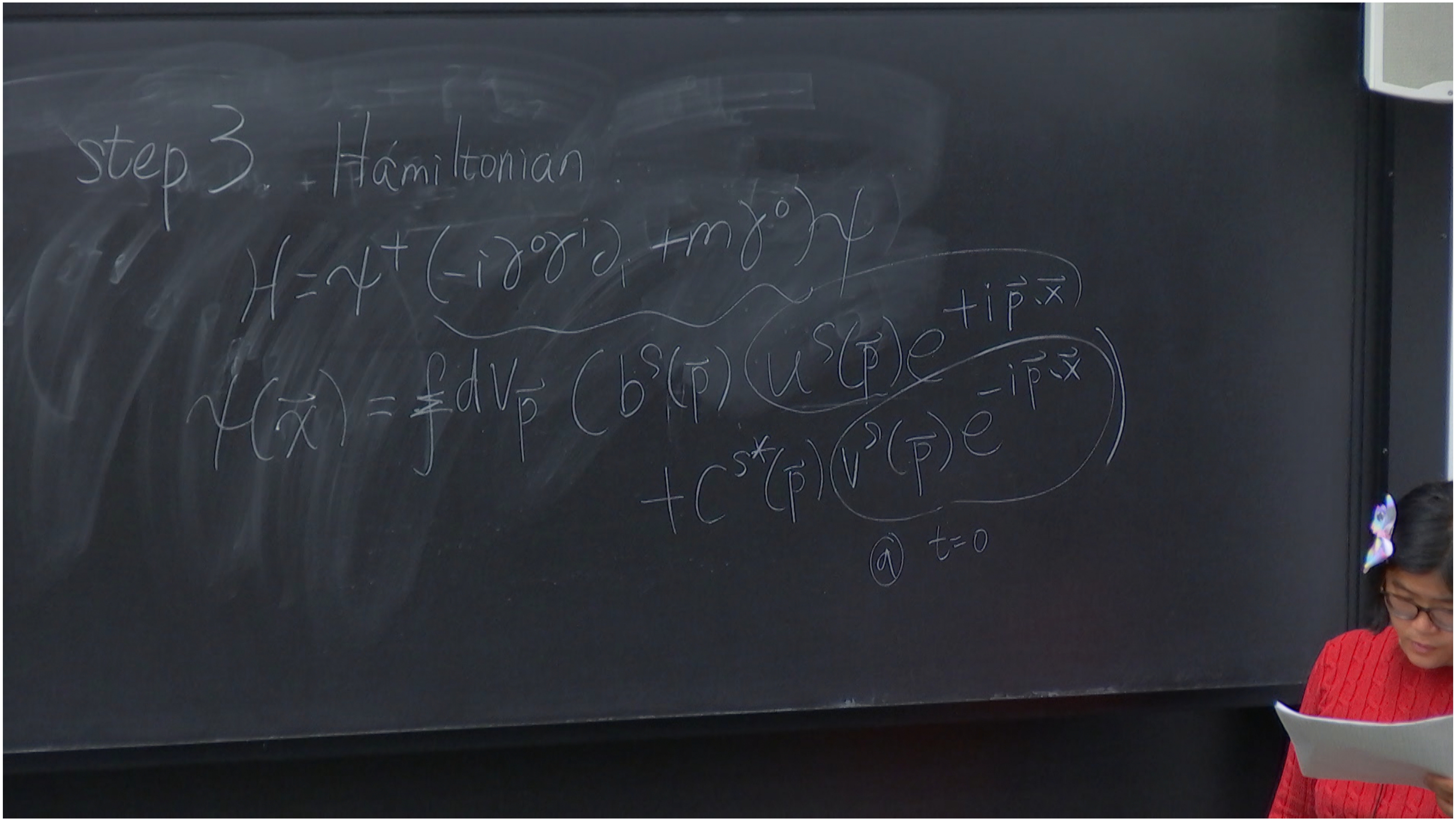

8+0


$\frac{8+0}{2} = 4$

step 3. Hamiltonian.

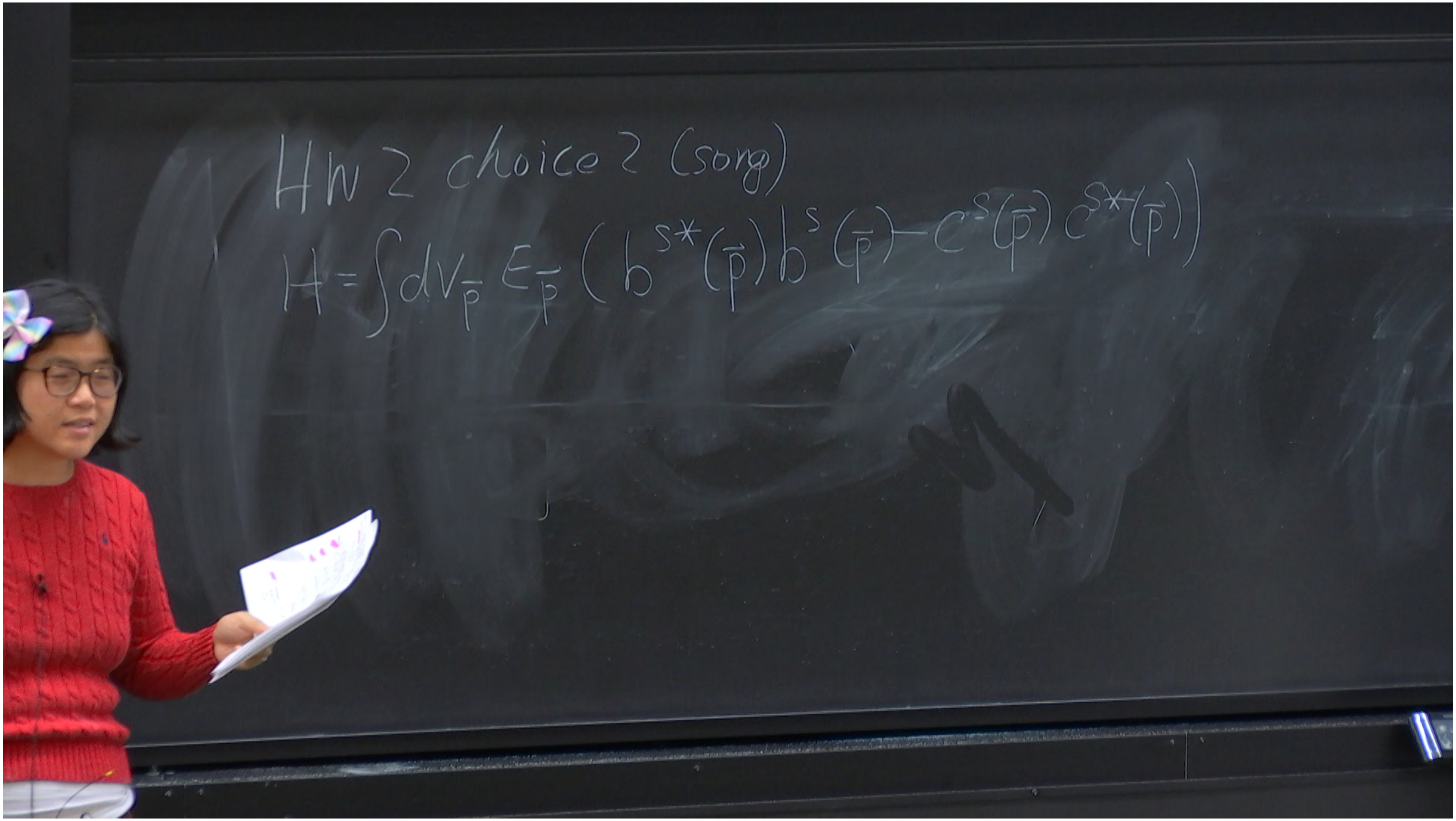
$$H = \psi^{\dagger}(-i\gamma^{0}\gamma^{i}\partial_{i} + m\gamma^{0})\psi$$


step 3. Hamiltonian
$H = \psi^\dagger(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$
$\psi(\vec{x}) = \sum\!\!\!\!\!\!\int dV_{\vec{p}}\, \big(b^s(\vec{p})\, u^s(\vec{p})\, e^{+i\vec{p}\cdot\vec{x}}$
$+ c^{s*}(\vec{p})\, v^s(\vec{p})\, e^{-i\vec{p}\cdot\vec{x}} \big)$


step 3. Hamiltonian
$H = \psi^{\dagger}(-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi$
$\psi(\vec{x}) = \int dV_{\vec{p}} \left(b^s(\vec{p})\, u^s(\vec{p})\, e^{+i\vec{p}\cdot\vec{x}} + c^{s*}(\vec{p})\, v^s(\vec{p})\, e^{-i\vec{p}\cdot\vec{x}} \right)$
@ $t=0$

HW 2 choice 2 (song)

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{s*}(\vec{p})\, b^{s}(\vec{p}) - c^{s}(\vec{p})\, c^{s*}(\vec{p}) \right)$$


HW 2 choice 2 (song)
$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{s*}(\vec{p})\, b^{s}(\vec{p}) - c^{s}(\vec{p})\, c^{s*}(\vec{p}) \right)$

HW 2 choice 2 (song)

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{s*}(\vec{p})\, b^{s}(\vec{p}) - c^{s}(\vec{p})\, c^{s*}(\vec{p}) \right)$$

$\downarrow$ "promote"

step 4

HW 2 choice 2 (song)

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{s*}(\vec{p})\, b^{s}(\vec{p}) - c^{s}(\vec{p})\, c^{s*}(\vec{p}) \right)$$

step 4

$$\Big\downarrow \text{"promote"}$$

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{s+}_{\vec{p}}\, b^{s}_{\vec{p}} - c^{s}_{\vec{p}}\, c^{s+}_{\vec{p}} \right)$$

ste

HW 2 choice 2 (sorry)

step 5:

$$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{s*}(\vec{p})\, b^{s}(\vec{p}) - c^{s}(\vec{p})\, c^{s*}(\vec{p}) \right)$$

step 4

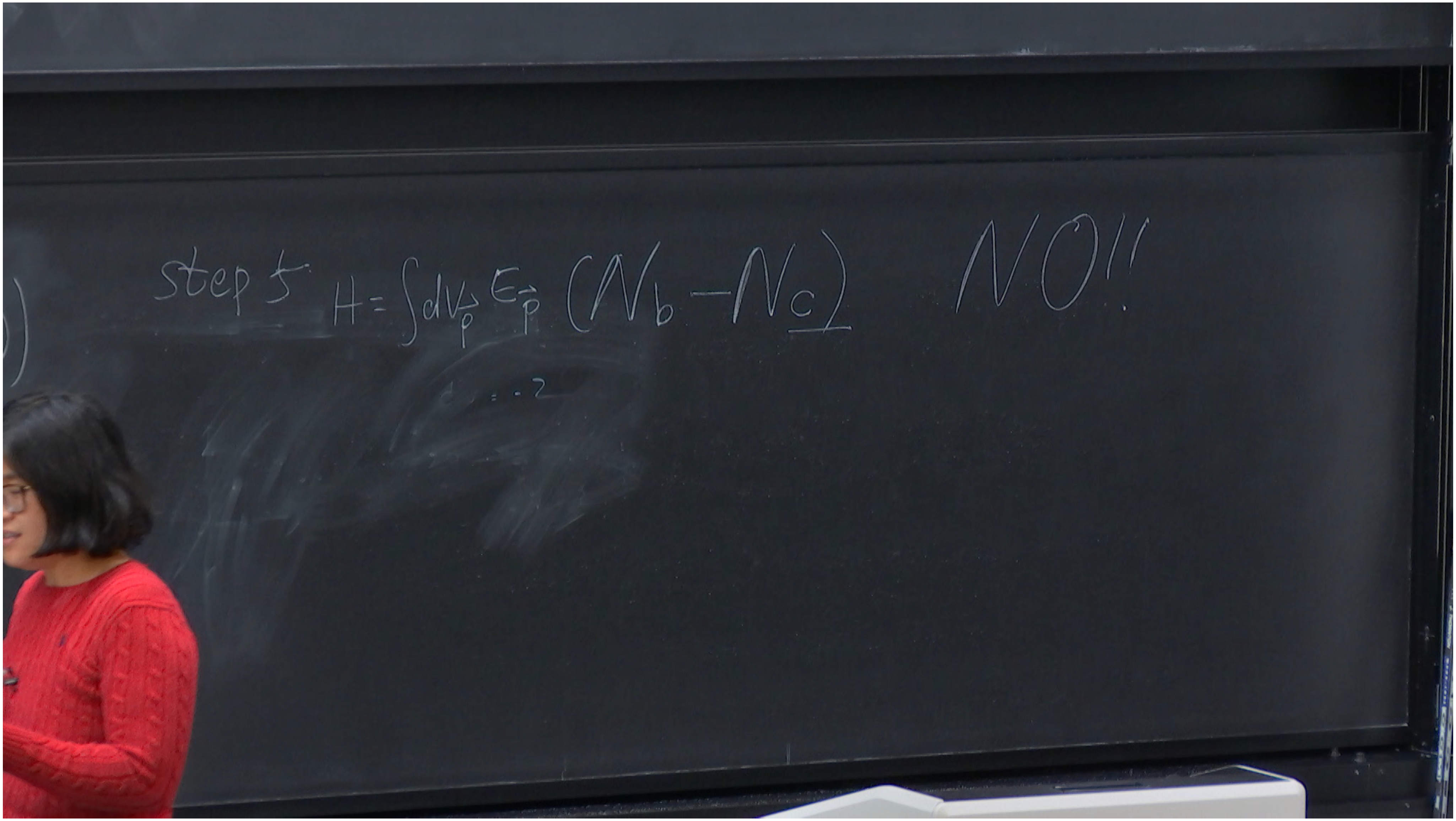
$$\Big\downarrow \text{"promote"}$$

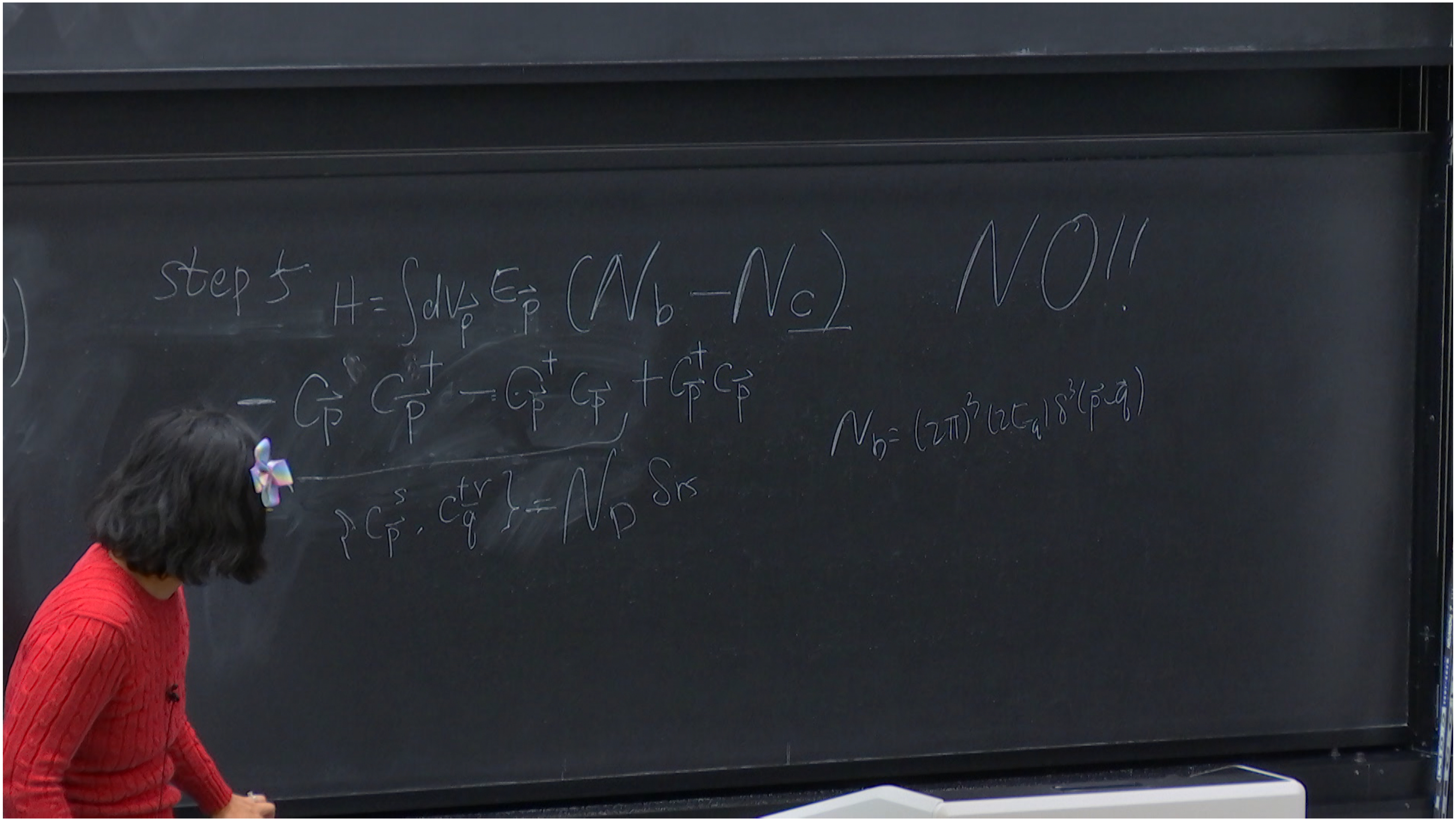
$$H = \int dV_{\vec{p}}\, E_{\vec{p}} \left(b^{s\dagger}_{\vec{p}}\, b^{s}_{\vec{p}} - \underline{c^{s}_{\vec{p}}\, c^{s\dagger}_{\vec{p}}} \right)$$

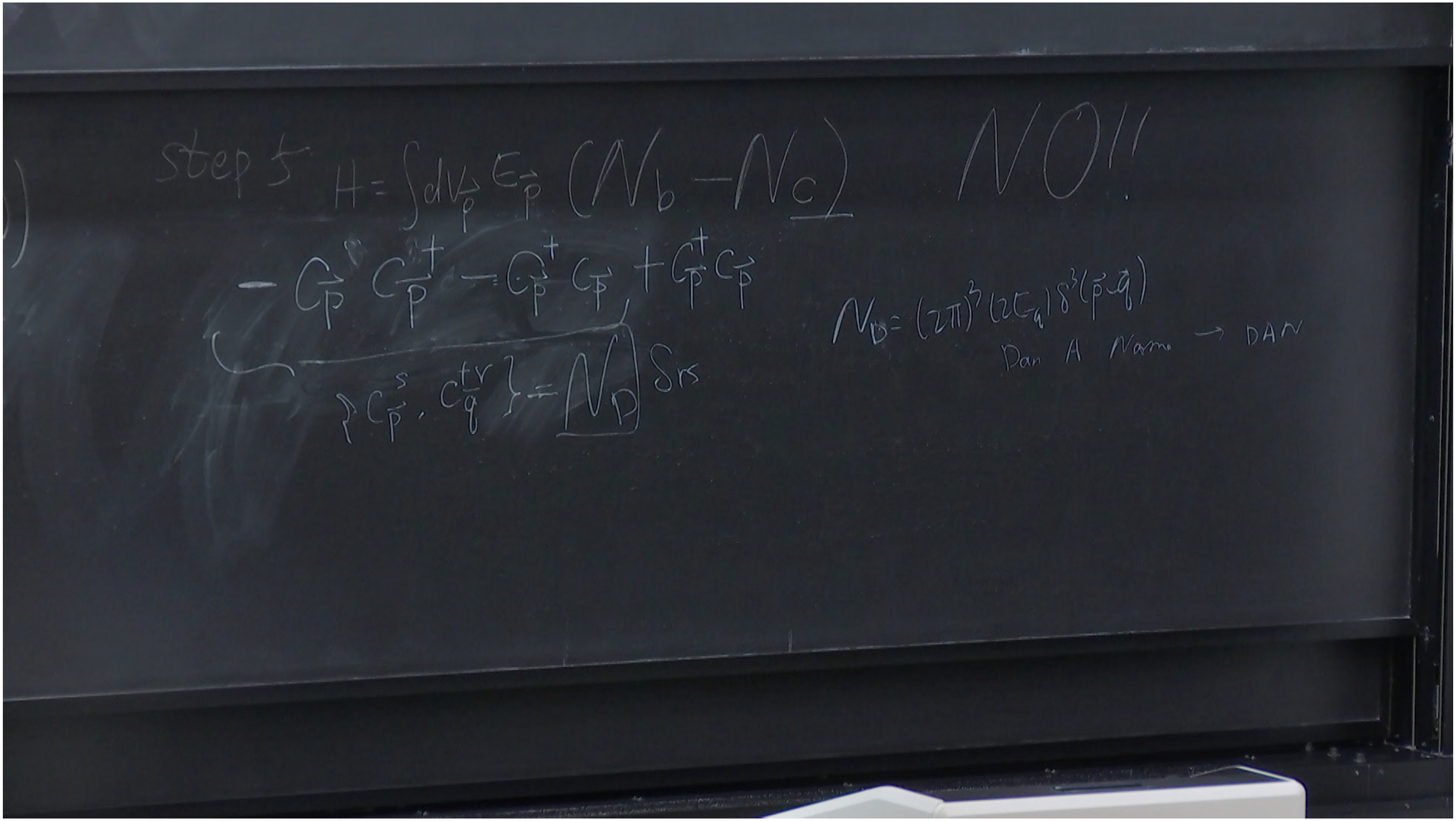
$$\downarrow$$

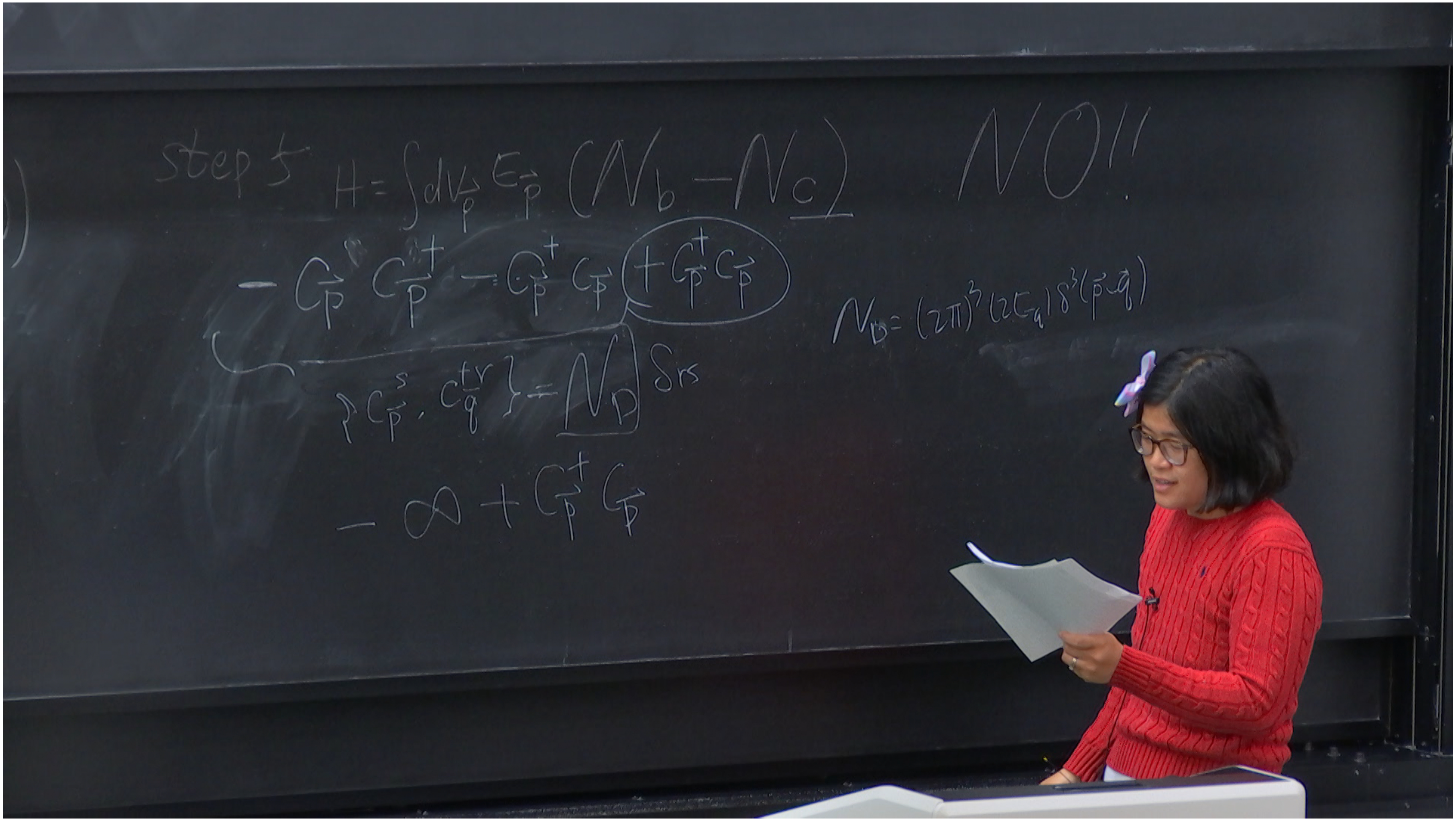
$$c^{s\dagger}_{\vec{p}}\, c^{s}_{\vec{p}}$$

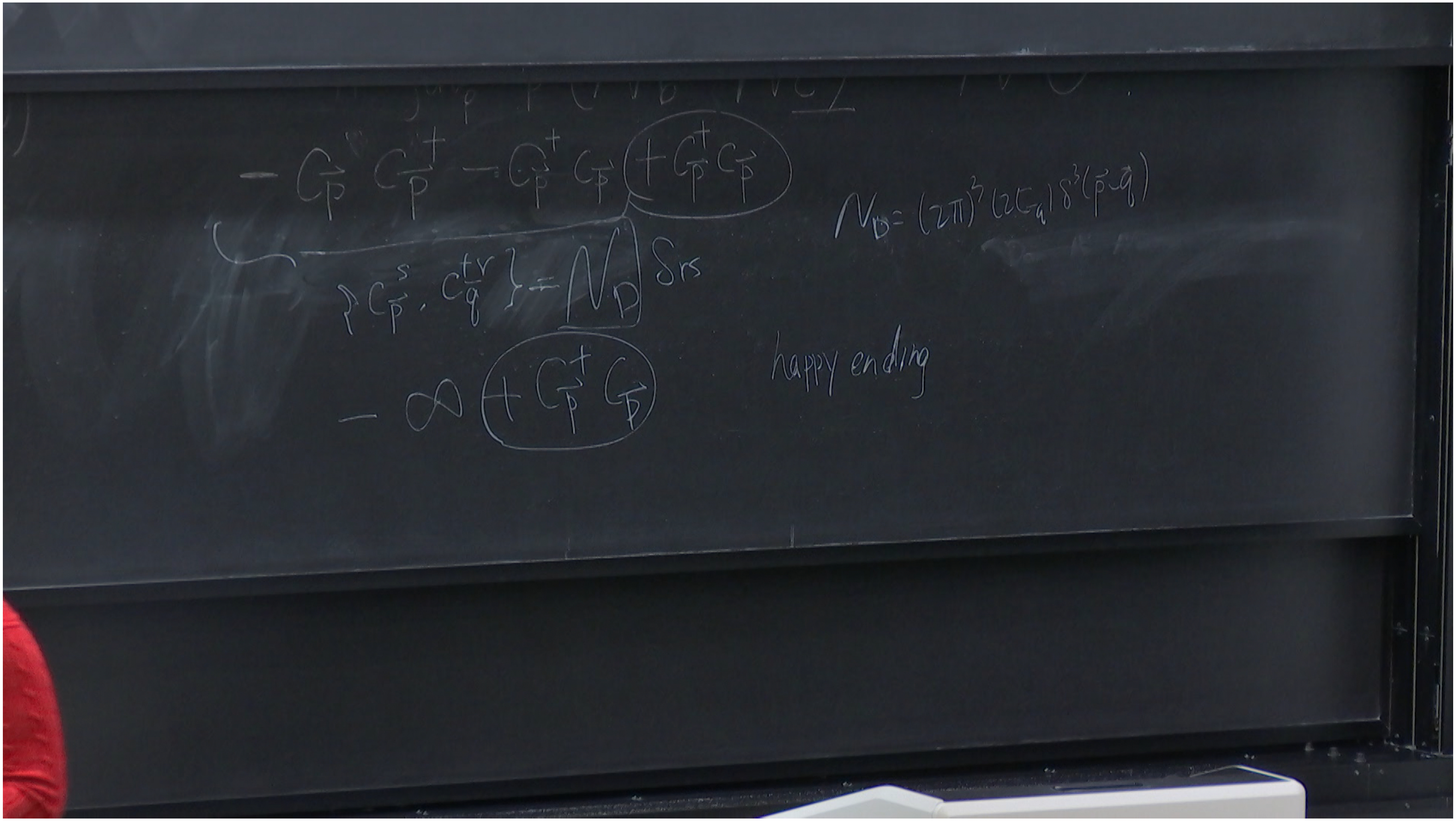
step 5: $H = \int dV_{\vec{p}}\, E_{\vec{p}}\, (N_b - N_c)$

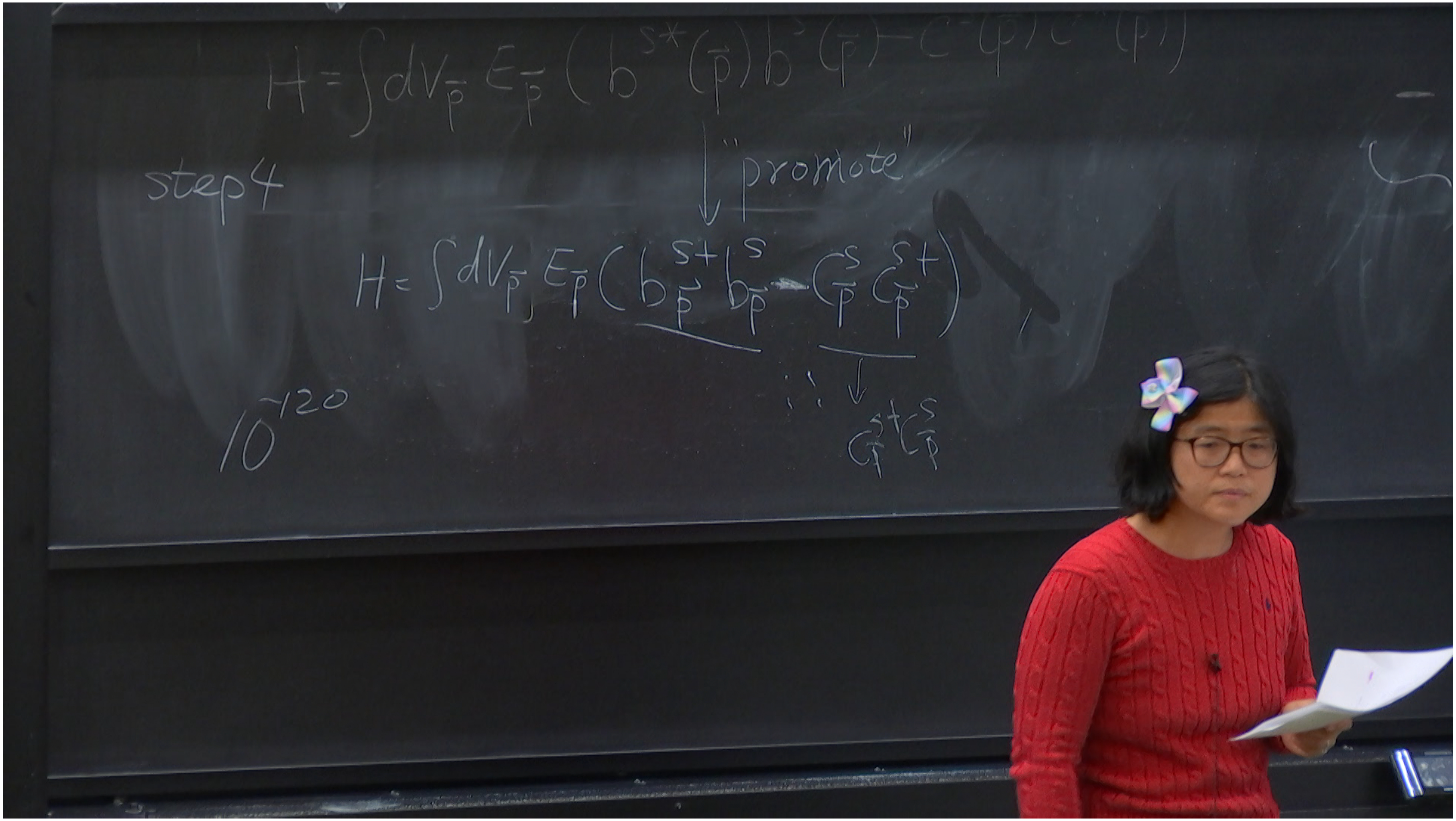

step 5 $H = \int dV_{\vec{p}}\, \epsilon_{\vec{p}} (N_b - N_c)$
NO!!

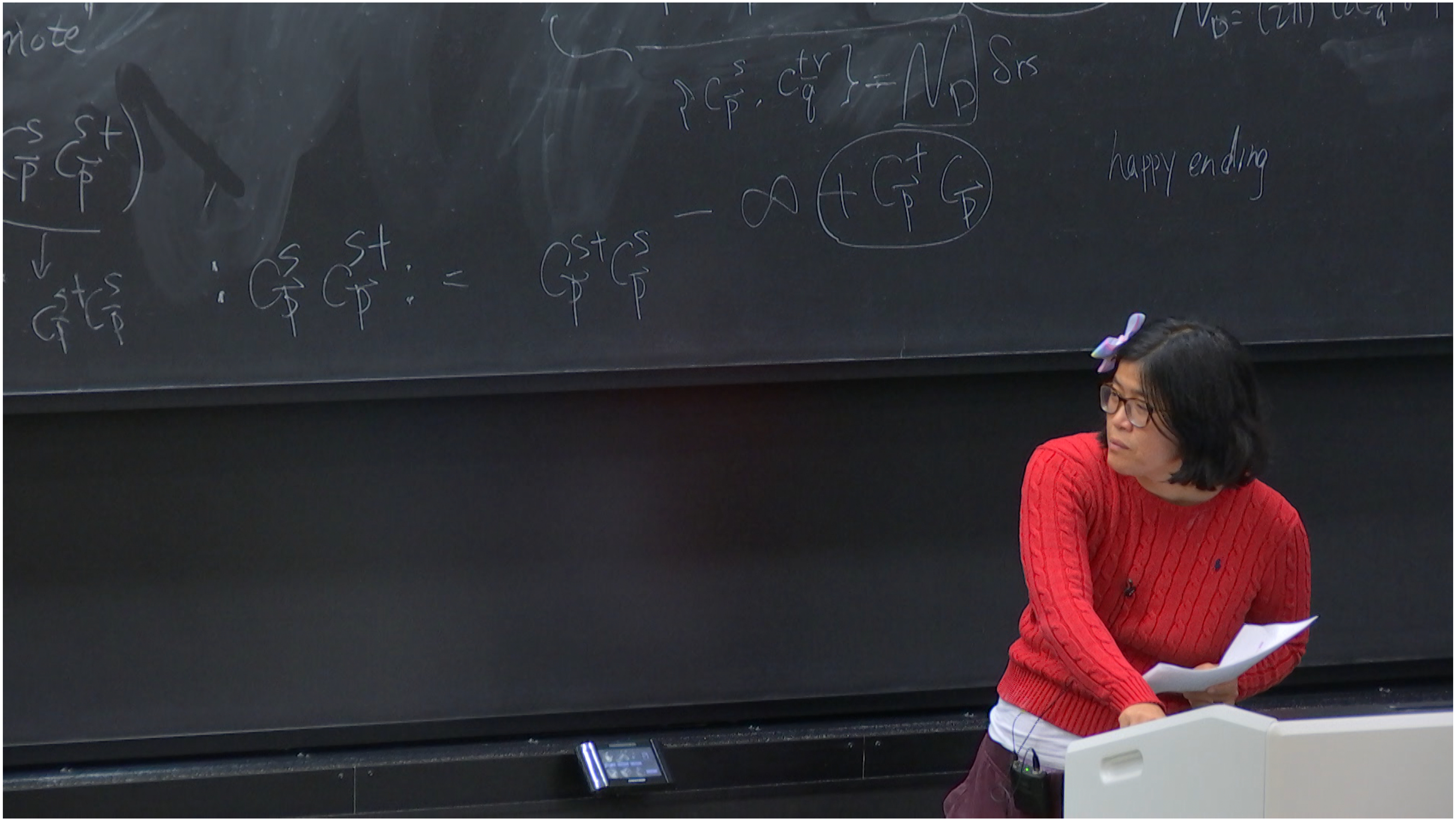

step 5 $H = \int dV_{\vec{p}}\, E_{\vec{p}} (N_b - N_c)$
NO!!
$- c_{\vec{p}}\, c^{\dagger}_{\vec{p}} - c^{\dagger}_{\vec{p}}\, c_{\vec{p}} + c^{\dagger}_{\vec{p}}\, c_{\vec{p}}$
$N_b = (2\pi)^3 (2E_q)\, \delta^3(\vec{p}-\vec{q})$
$\{c^{s}_{\vec{p}}, c^{\dagger r}_{\vec{q}}\} = N_D\, \delta_{rs}$

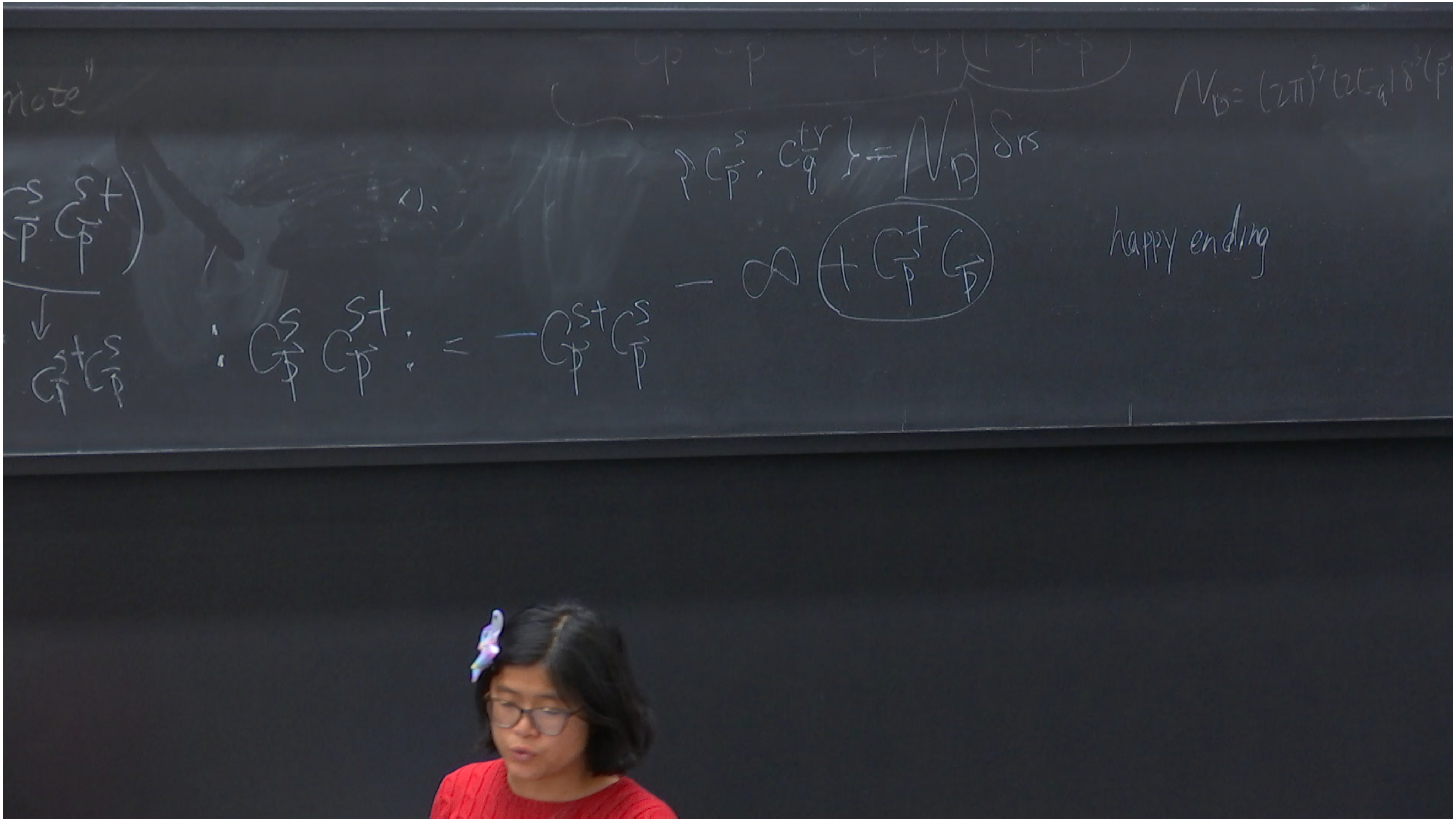

step 5 $H = \int dV_{\vec{p}} \, E_{\vec{p}} \, (N_b - N_c)$
NO!!
$- C_{\vec{p}} \, C^{\dagger}_{\vec{p}} \to C^{\dagger}_{\vec{p}} \, C_{\vec{p}} + C^{\dagger}_{\vec{p}} \, C_{\vec{p}}$
$\{ C^{s}_{\vec{p}} , C^{\dagger r}_{\vec{q}} \} = N_D \, \delta_{rs}$
$N_D = (2\pi)^3 (2E_q) \delta^3(\vec{p} - \vec{q})$
Dan A Norma → DAN

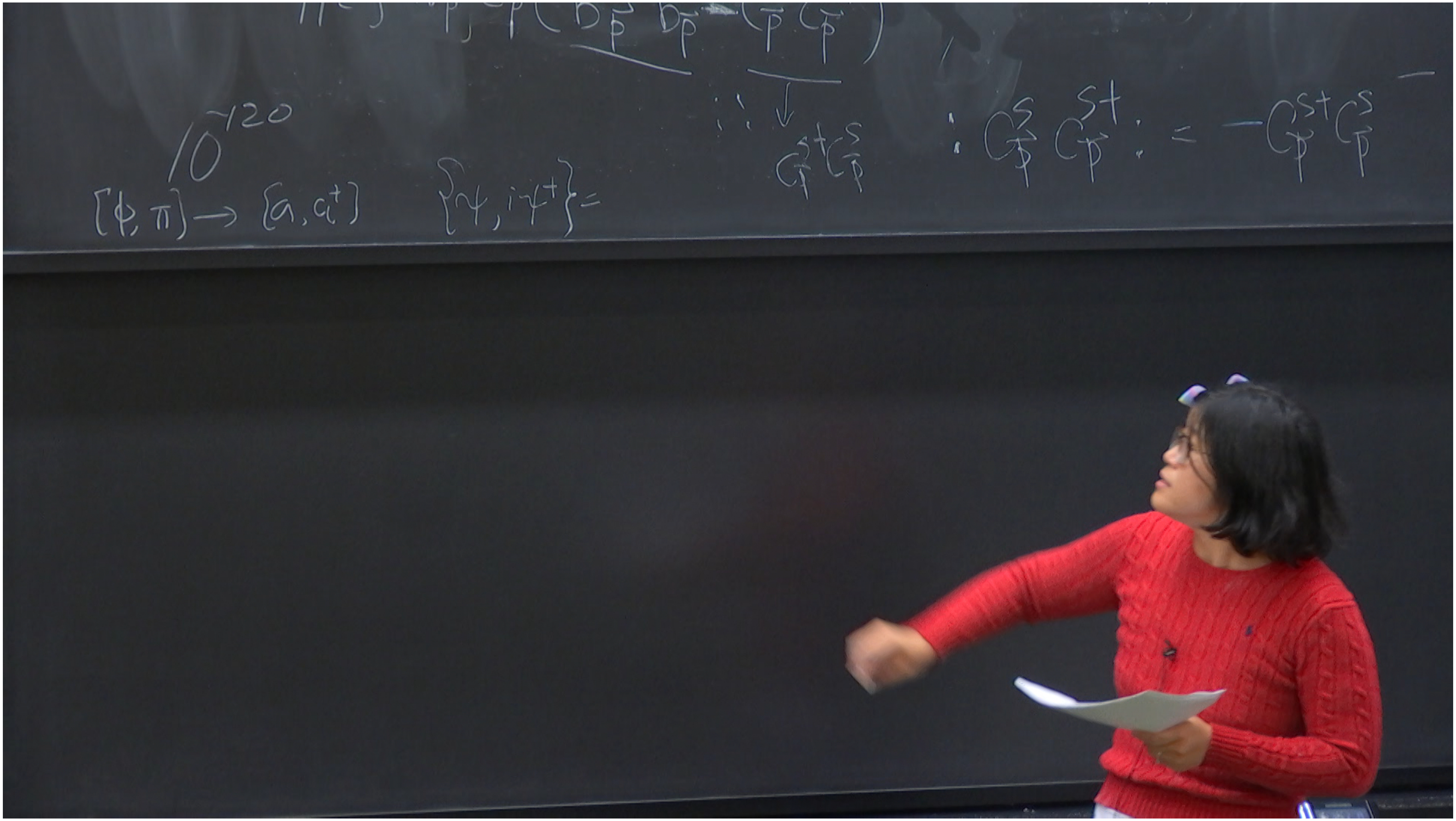

step 5
$H = \int dV_{\vec p}\, E_{\vec p} (N_b - N_c)$
NO!!
$N_D = (2\pi)^3 (2E_q) \delta^3(\vec p - \vec q)$
$\{c^s_{\vec p}, c^{\dagger r}_{\vec q}\} = N_D \delta_{rs}$
$-\infty + c^\dagger_{\vec p} c_{\vec p}$


$- c_{\vec p} c^{\dagger}_{\vec p} - c^{\dagger}_{\vec p} c_{\vec p} + c^{\dagger}_{\vec p} c_{\vec p}$
$\{ c^{s}_{\vec p}, c^{\dagger r}_{\vec q} \} = N_D \, \delta_{rs}$
$N_D = (2\pi)^3 (2E_q) \delta^3(\vec p - \vec q)$
$- \infty + c^{\dagger}_{\vec p} c_{\vec p}$
happy ending

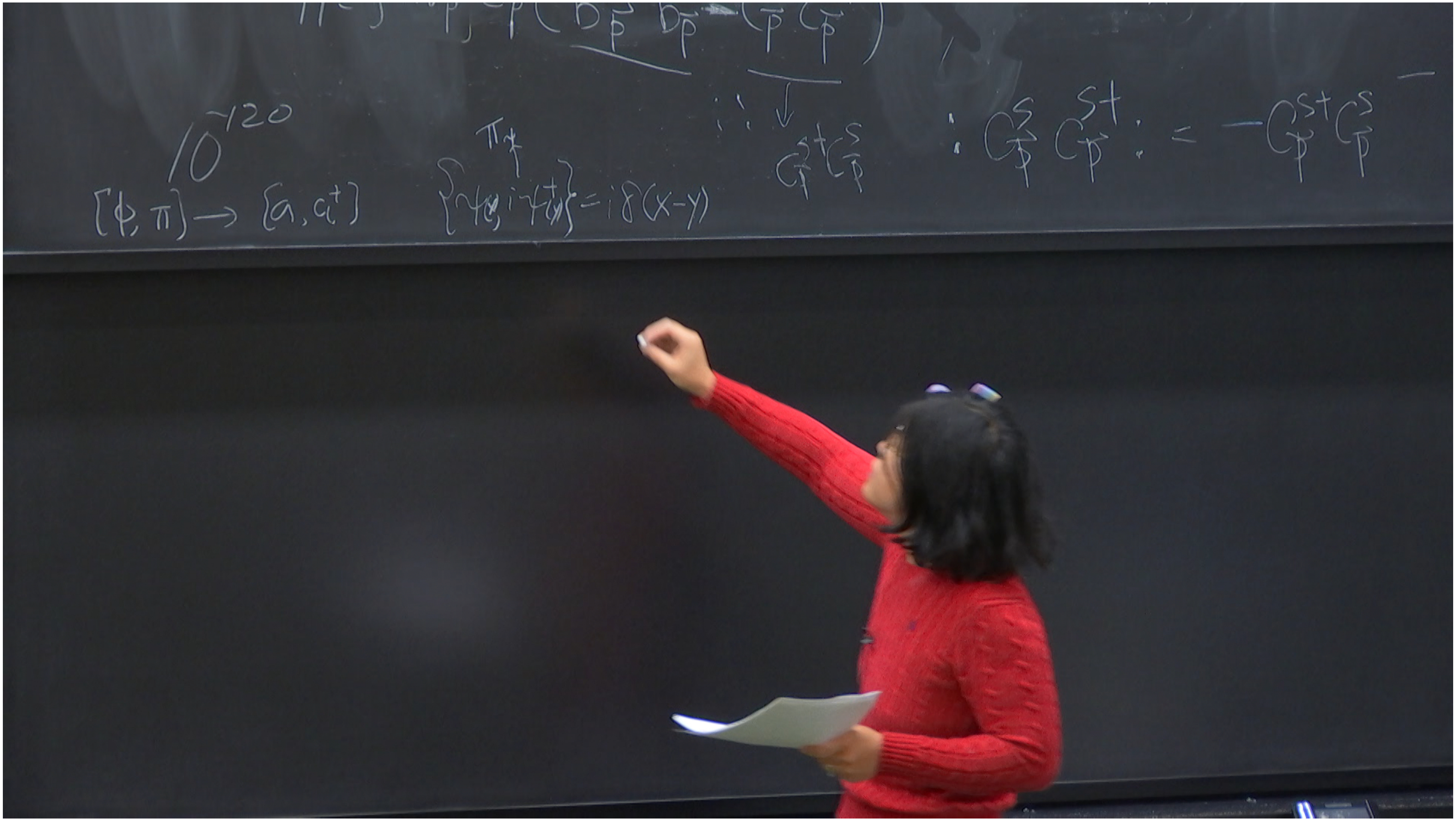

H = ∫dV_p E_p (b^{s*}(p) b^s(p) − c^s(p) c^{s*}(p))
step4
"promote"
H = ∫dV_p E_p (b^{s+}_p b^s_p − c^s_p c^{s+}_p)
c^{s+}_p c^s_p
10^{-120}

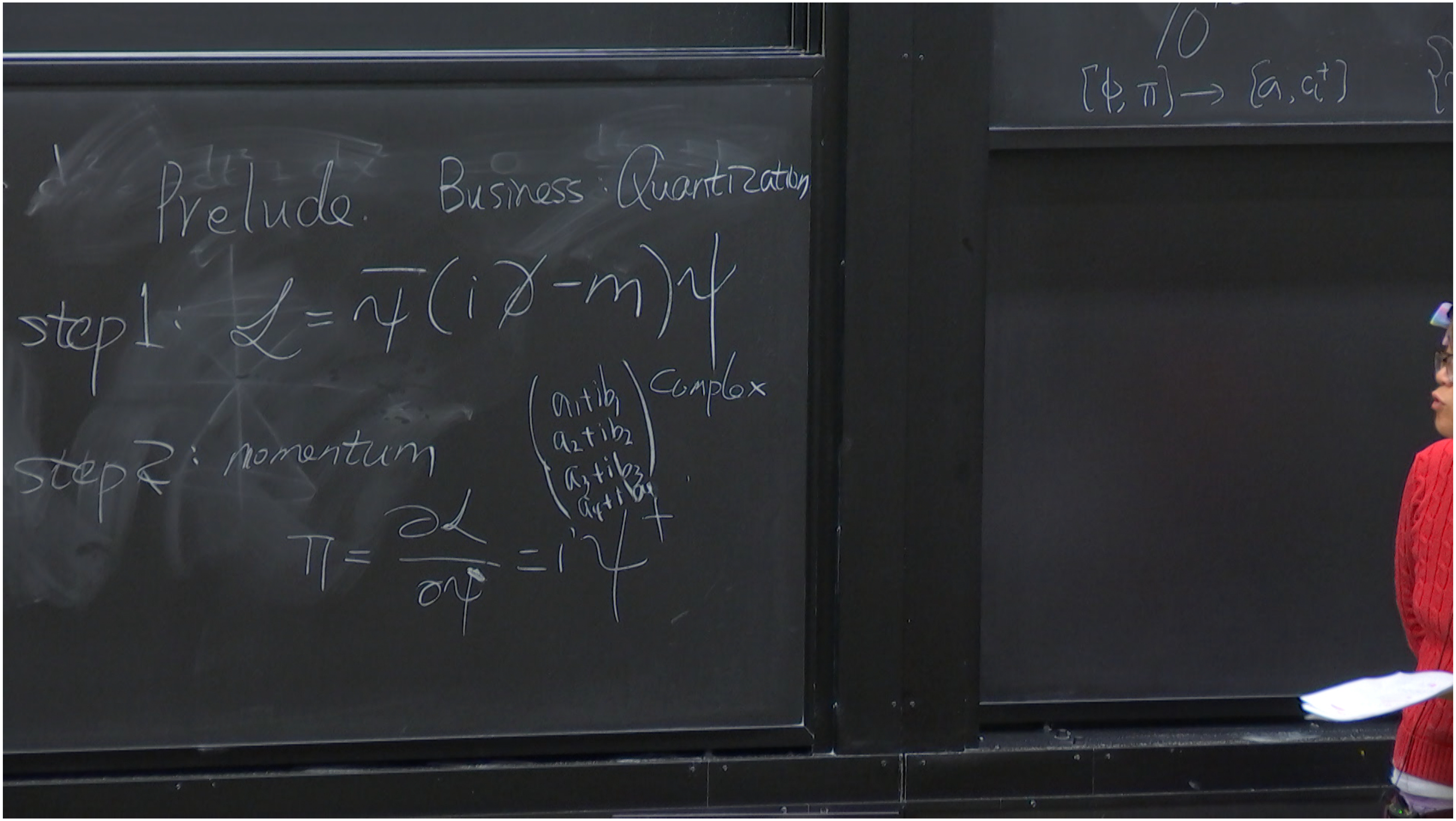

note
happy ending

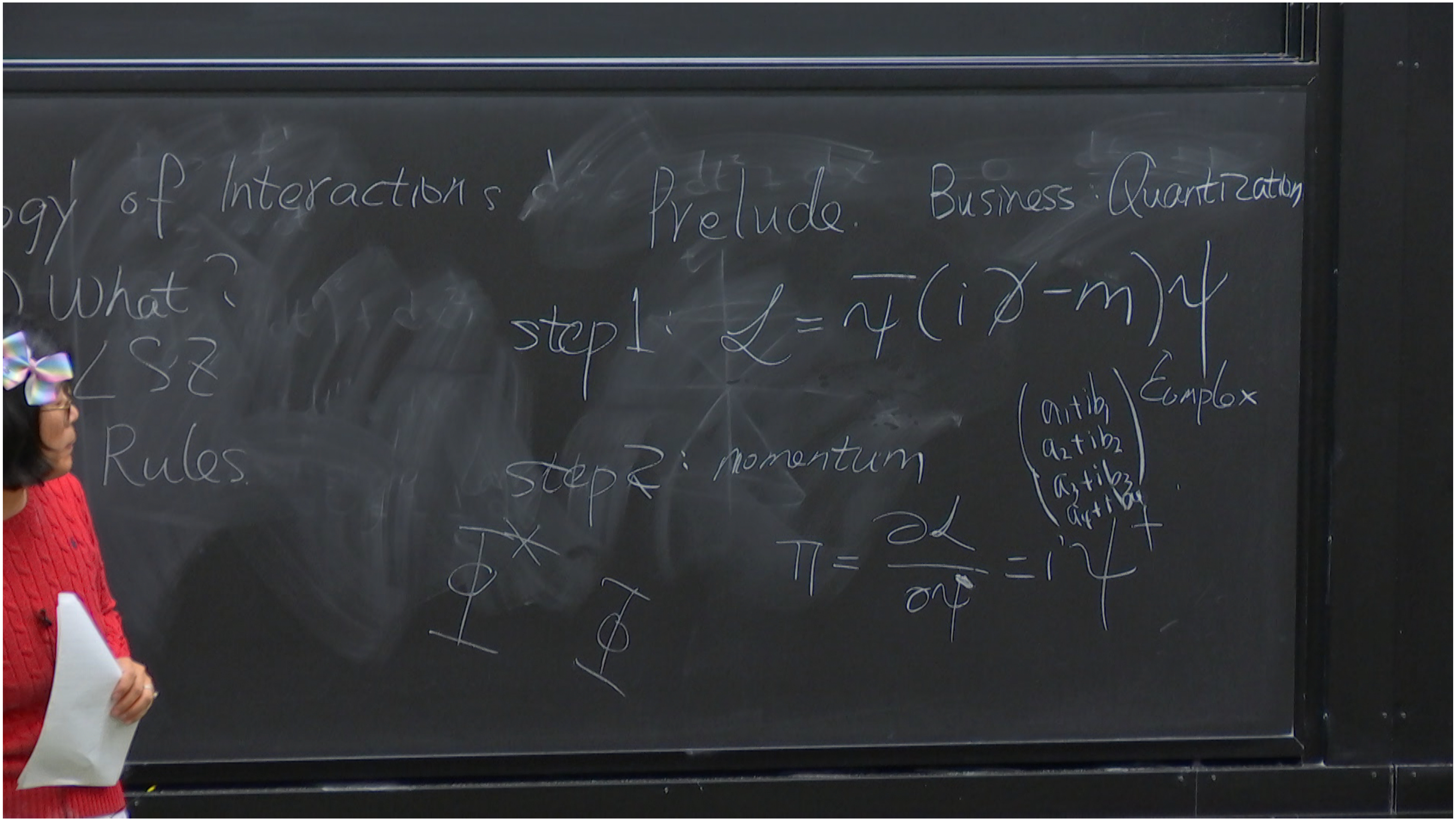

happy ending

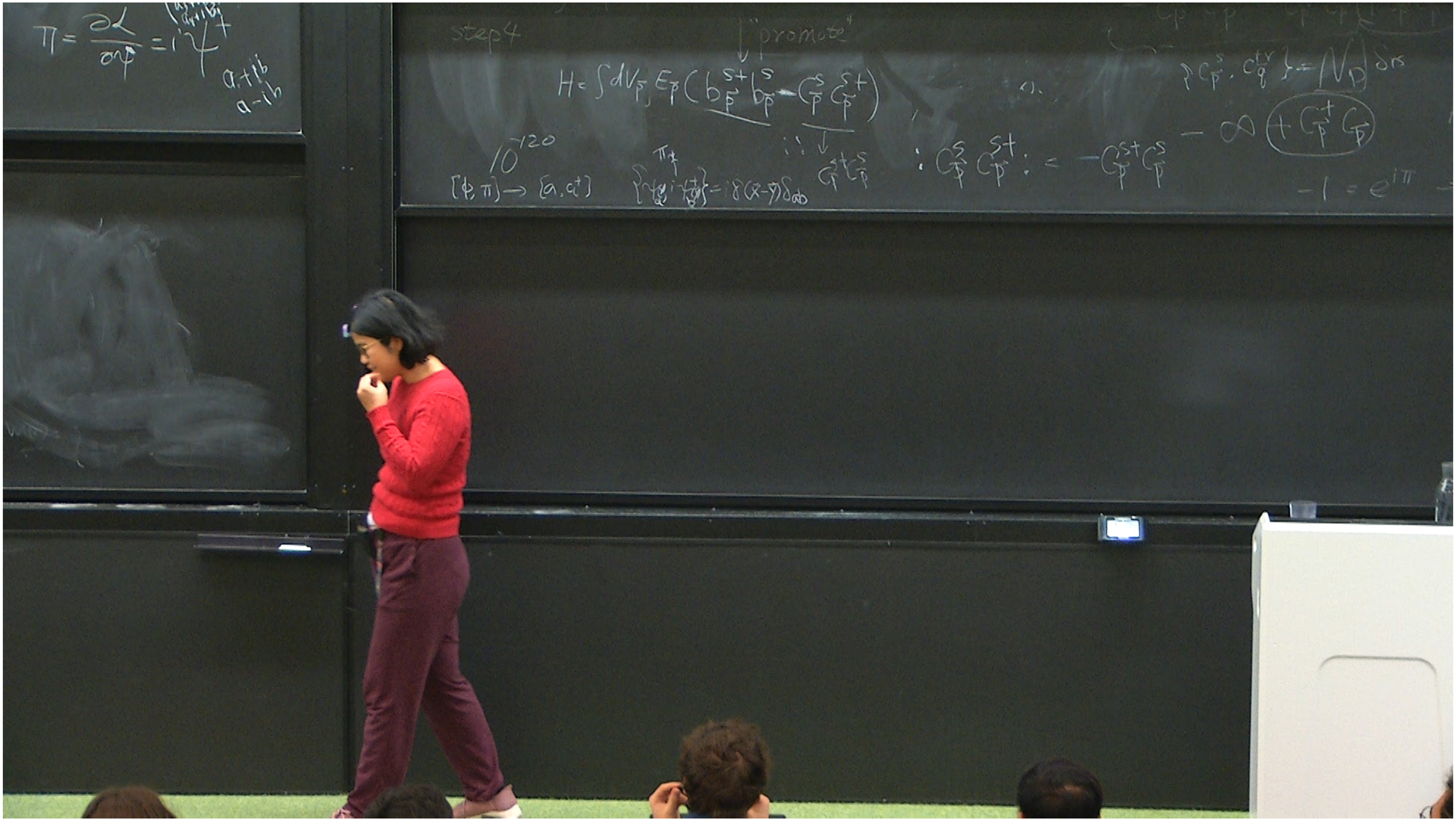


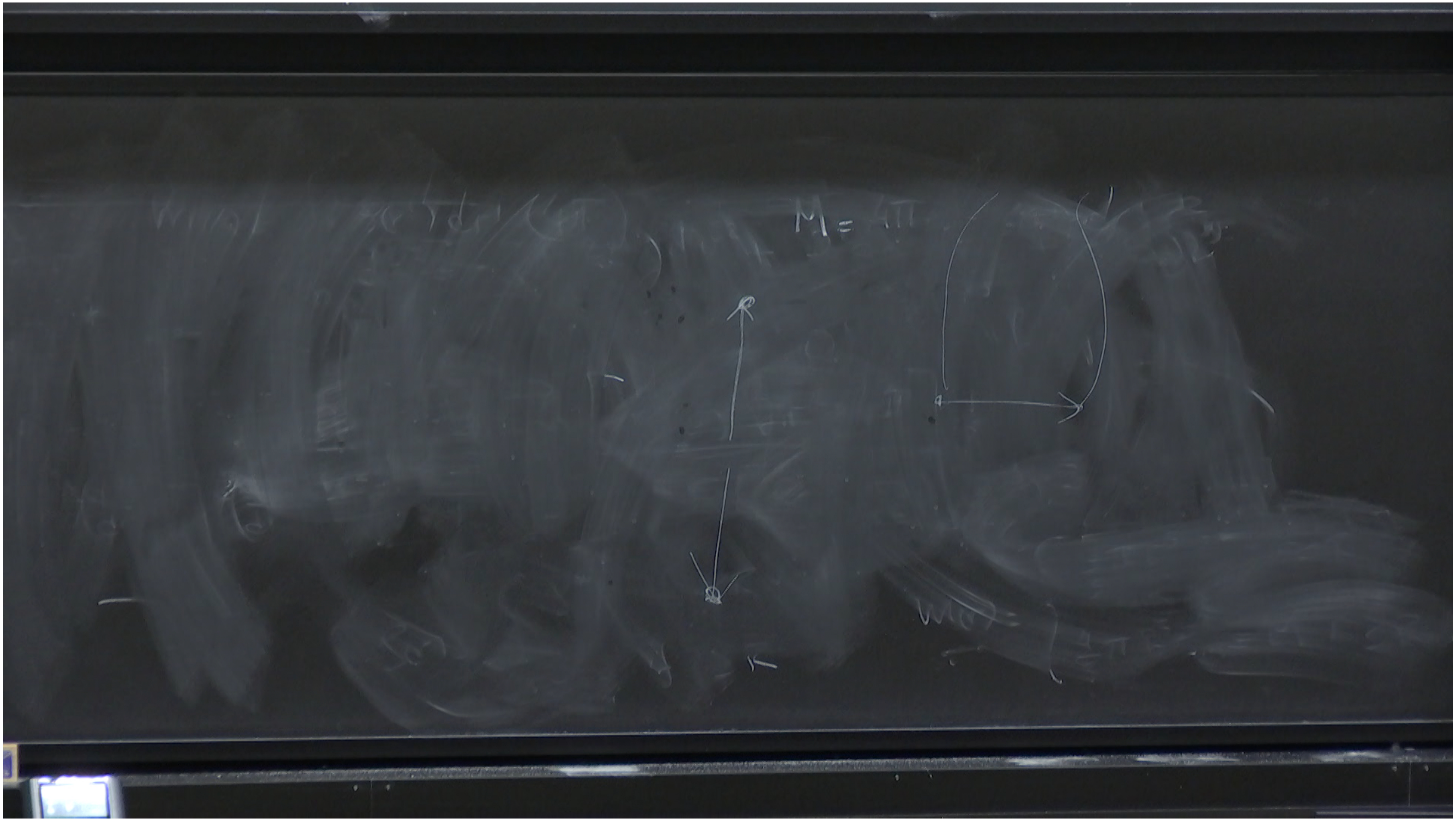
10^-120
[φ, π] → {a, a†}
{ψ, iψ†} =
C^s† C^s
: C^s C^s† : = − C^s† C^s


10^{-120}
$[\phi, \pi] \rightarrow \{a, a^\dagger\}$
π_ψ
$\{\psi(x), i\psi^\dagger(y)\} = i\delta(x-y)$
$C_p^{s\dagger} C_p^{s}$
$:C_p^{s} C_p^{s\dagger}: = -C_p^{s\dagger} C_p^{s}$


[φ, π] → {a, a†}
Prelude
Business: Quantization
step 1: $\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$
complex
$\begin{pmatrix} a_1+ib_1 \\ a_2+ib_2 \\ a_3+ib_3 \\ a_4+ib_4 \end{pmatrix}$
step 2: momentum
$\pi = \frac{\partial \mathcal{L}}{\partial \psi} = i\psi^\dagger$


ogy of Interactions
Prelude.
Business: Quantization
What?
LSZ
Rules
step 1: $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$
Complex
a_1+ib_1
a_2+ib_2
a_3+ib_3
a_4+ib_4
step 2: momentum
$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = i\psi^\dagger$


step4
"promote"


M = 4π

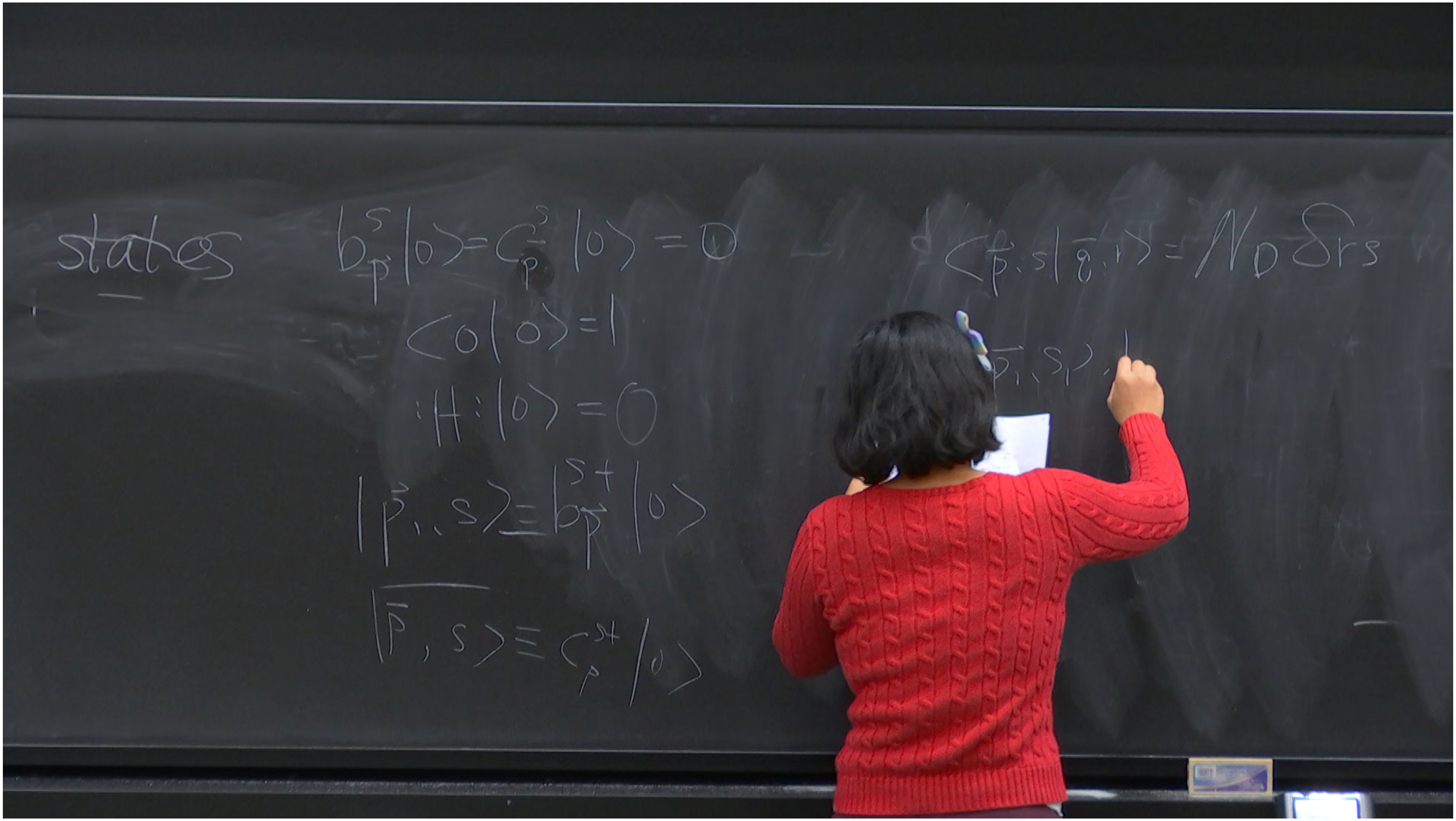

happy ending
2d
anyon

states

$$b^s_p|0\rangle = c^s_p|0\rangle = 0$$

$$\langle 0|0\rangle = 1$$

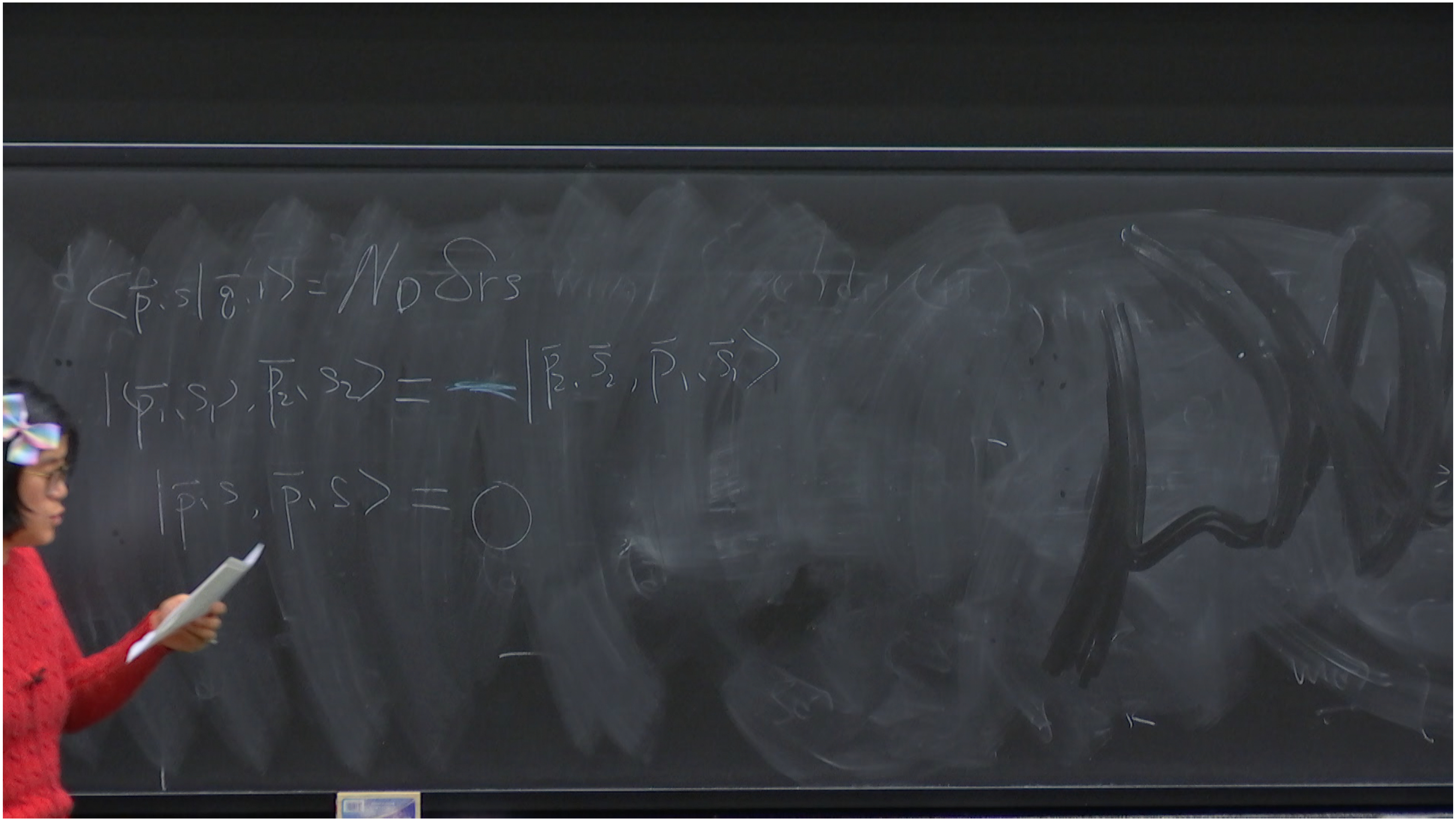
$$:H:|0\rangle =$$

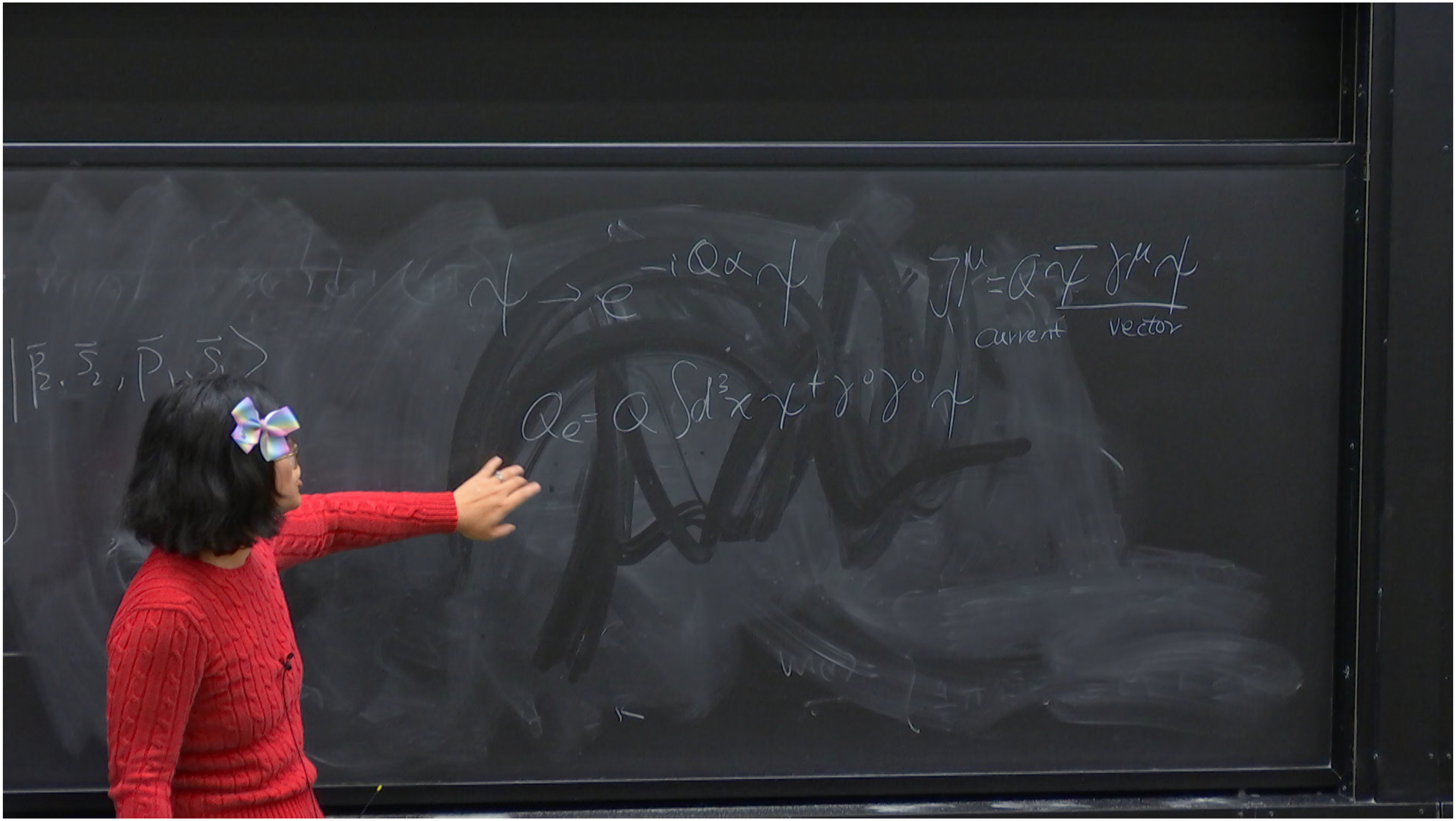

states
$b_{\vec{p}}^{s}|0\rangle = c_{\vec{p}}^{s}|0\rangle = 0$
$\langle 0|0\rangle = 1$
$:H:|0\rangle = 0$
$|\vec{p}, s\rangle \equiv b_{\vec{p}}^{s\dagger}|0\rangle$
$|\bar{\vec{p}}, s\rangle \equiv c_{\vec{p}}^{s\dagger}|0\rangle$
$\langle \vec{p}, s|\vec{q}, r\rangle = N_D \delta_{rs}$
$|\vec{p}_1, s_1\rangle,$

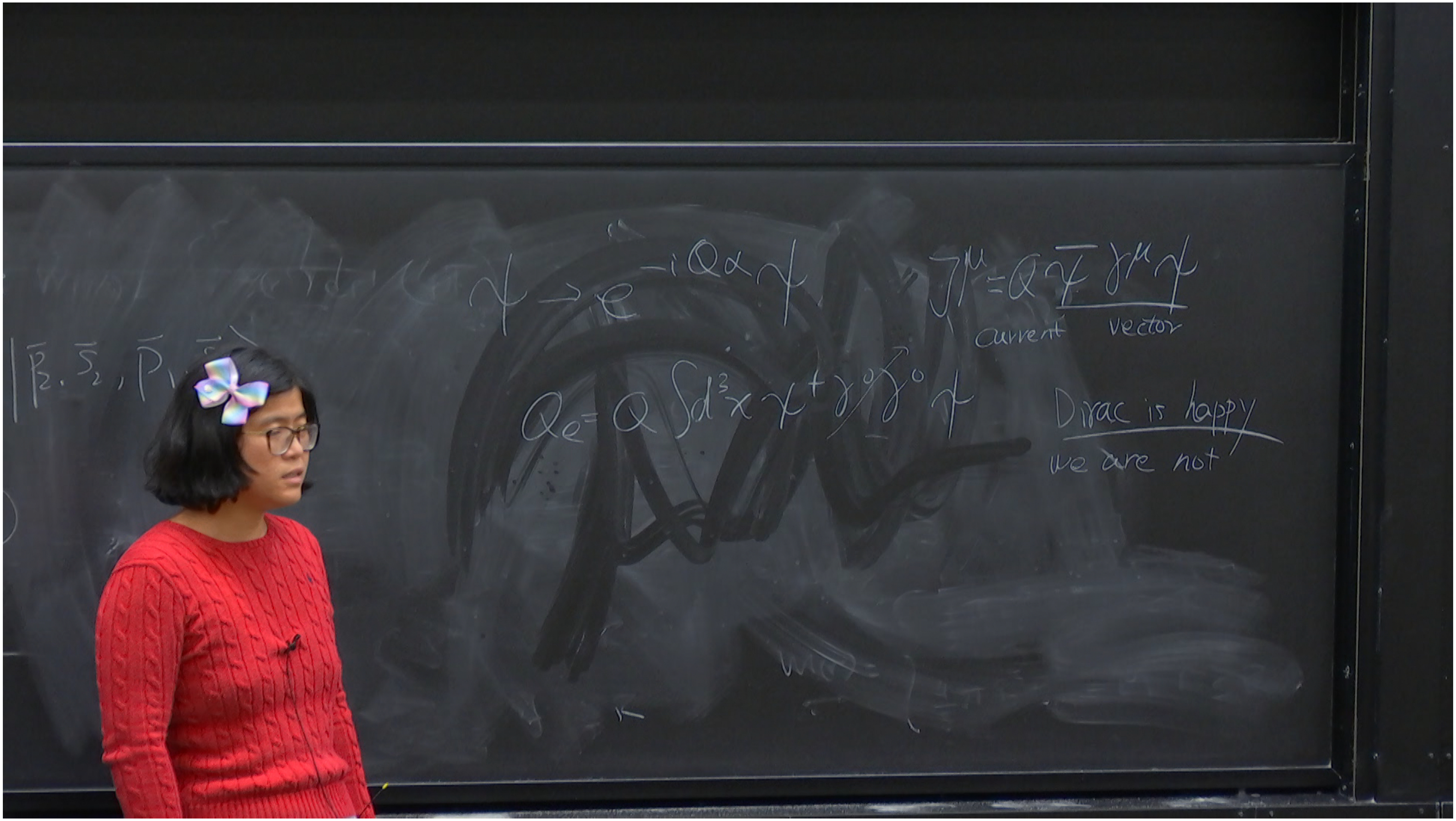
$$\langle \vec{p}, s | \vec{q}, r \rangle = N_D \delta_{rs}$$

$$|(\vec{p}_1, s_1), \vec{p}_2, s_2\rangle = - |\vec{p}_2, \vec{s}_2, \vec{p}_1, \vec{s}_1\rangle$$

$$|\vec{p}, s, \vec{p}, s\rangle =$$


$\langle \vec{p}, s | \vec{q}, r \rangle = N_D \delta_{rs}$
$|\vec{p}_1, s_1\rangle, \vec{p}_2, s_2\rangle = - |\vec{p}_2, \vec{s}_2, \vec{p}_1, \vec{s}_1\rangle$
$|\vec{p}, s, \vec{p}, s\rangle = 0$


ψ → e^{-iQα} ψ
j^μ = Q ψ̄ γ^μ ψ
current vector
Q_e = Q ∫d³x ψ† γ⁰ γ⁰ ψ
|p₂, s₂, p₁, s₁⟩


$\chi \to e^{-iQ\alpha}\chi$
$J^\mu = Q\bar{\chi}\gamma^\mu\chi$
current vector
$Q_e = Q\int d^3x\, \chi^\dagger \gamma^0\gamma^0 \chi$
Dirac is happy
we are not

$|\vec{p}_2, \bar{s}_2, \vec{p}_1, \bar{s}_1\rangle$

$\psi \to e^{-iQ\alpha}\psi$

$J^\mu = Q\underline{\bar{\psi}\gamma^\mu\psi}$

current vector

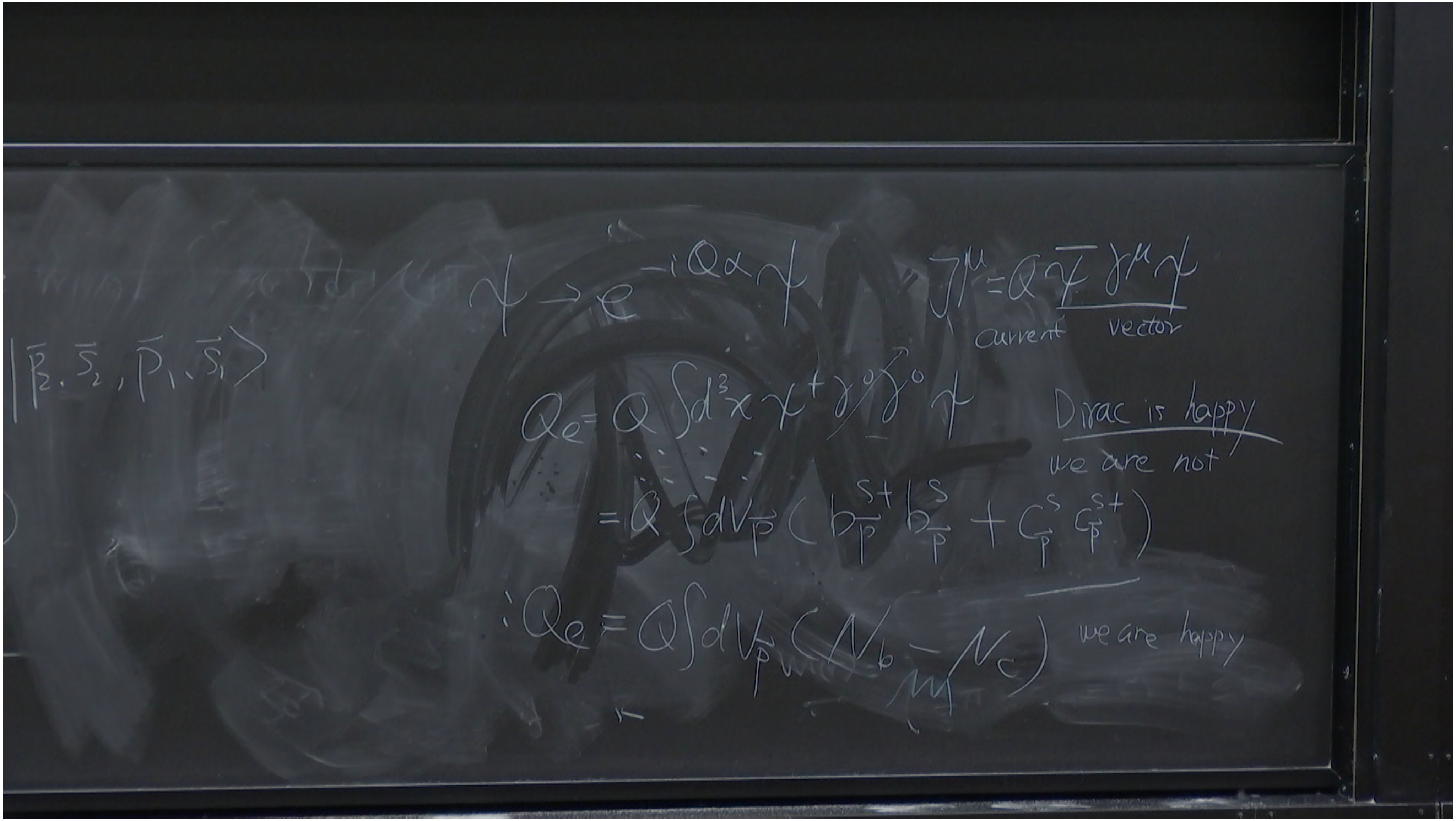
$Q_e = Q\int d^3x\, \psi^\dagger \gamma^0 \gamma^0 \psi$

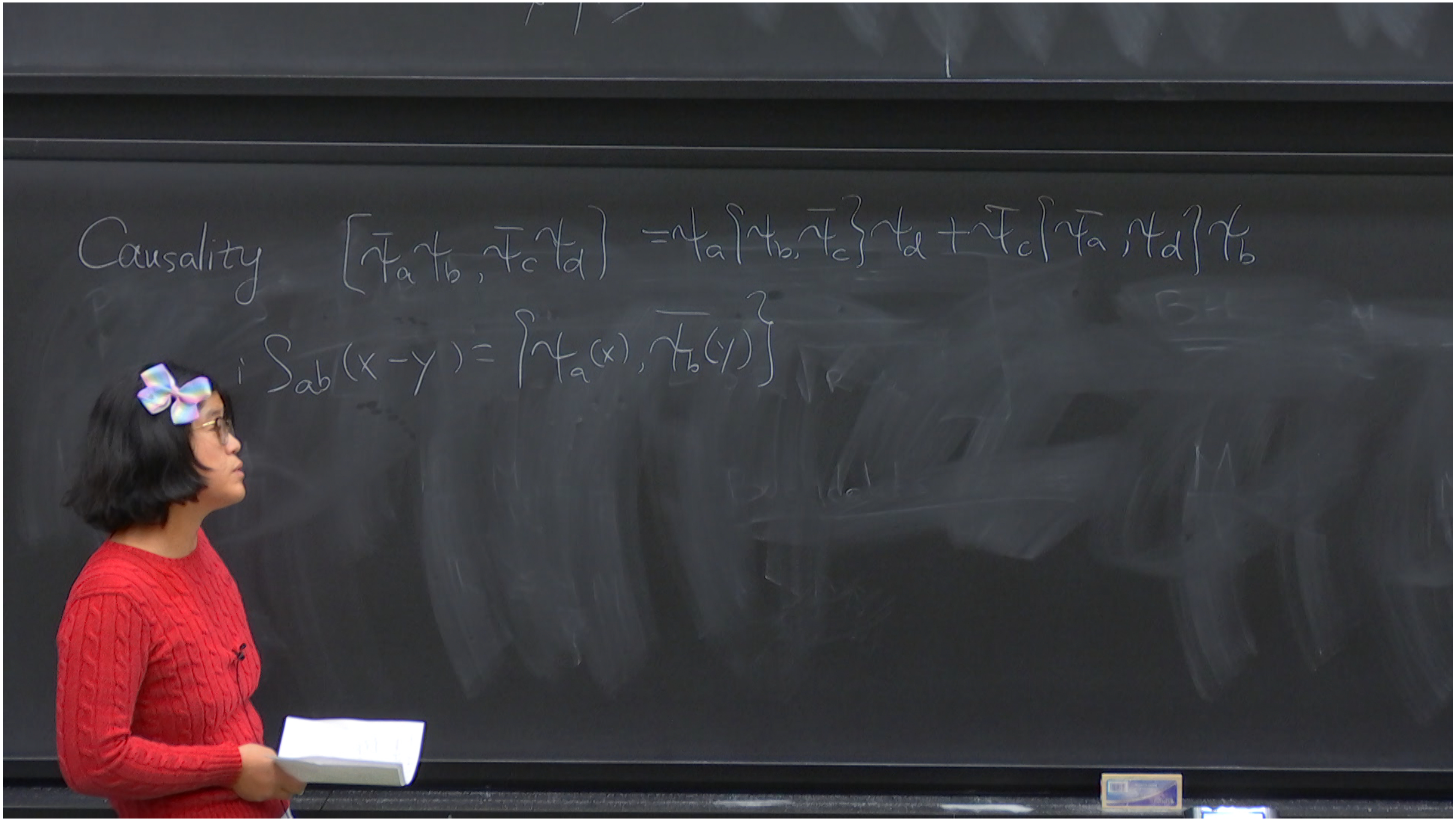
Dirac is happy

we are not

$= Q\int dV_{\vec{p}} \left(b^{s\dagger}_{\vec{p}} b^{s}_{\vec{p}} + c^{s}_{\vec{p}} c^{s\dagger}_{\vec{p}} \right)$

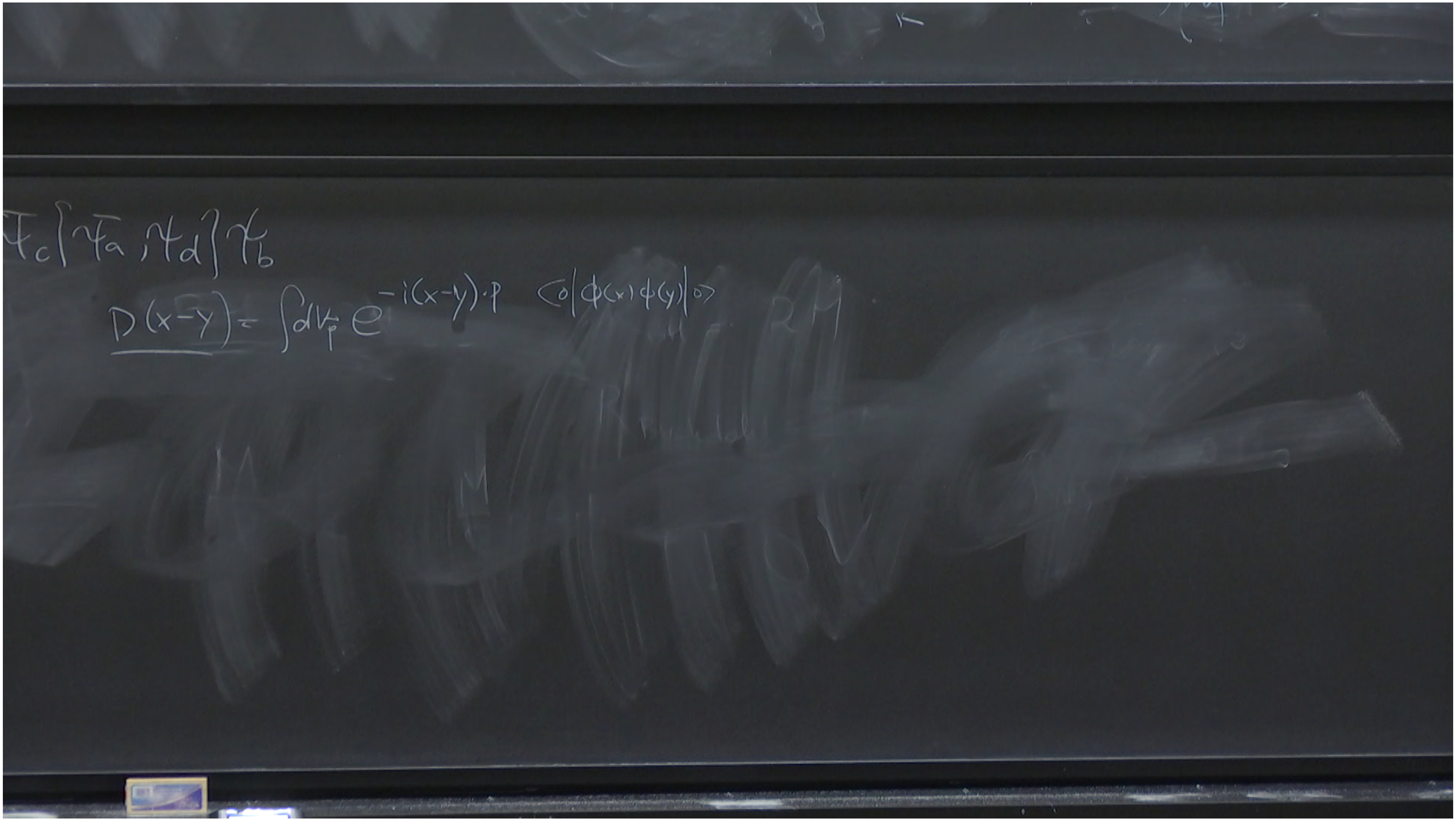
$\therefore Q_e = Q\int dV_{\vec{p}} \left(N_b - N_c \right)$

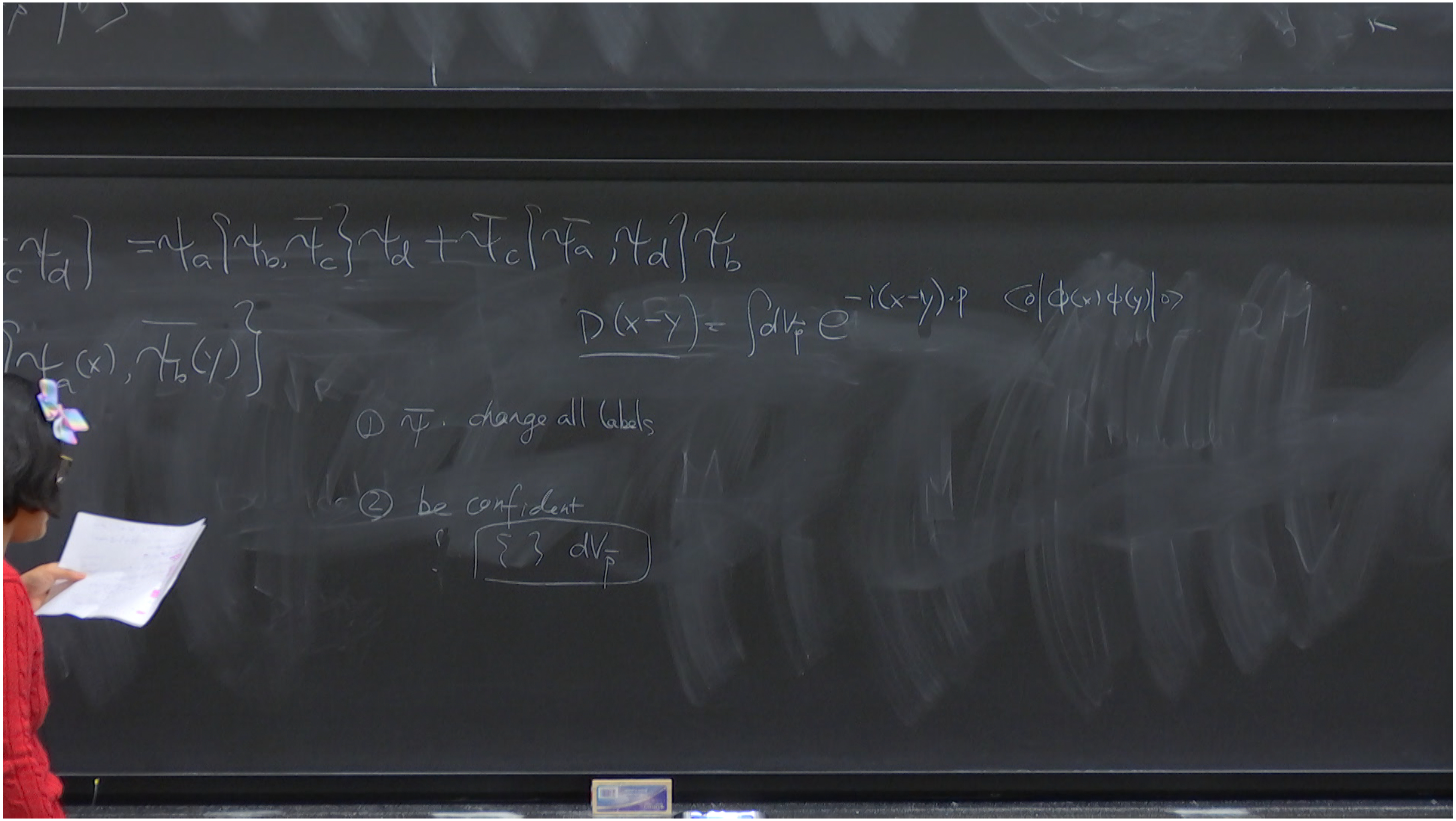

$\psi \to e^{-iQ\alpha}\psi$
$j^\mu = Q\bar{\psi}\gamma^\mu\psi$
current
vector
$|\vec{p}_2, \vec{s}_2, \vec{p}_1, \vec{s}_1\rangle$
$Q_e = Q\int d^3x\, \psi^\dagger \gamma^0\gamma^0 \psi$
Dirac is happy
we are not
$= Q\int dV_{\vec{p}} \left(b^{s\dagger}_{\vec{p}} b^{s}_{\vec{p}} + c^{s}_{\vec{p}} c^{s\dagger}_{\vec{p}} \right)$
$:Q_e: = Q\int dV_{\vec{p}} \left(N_b - N_c \right)$
we are happy

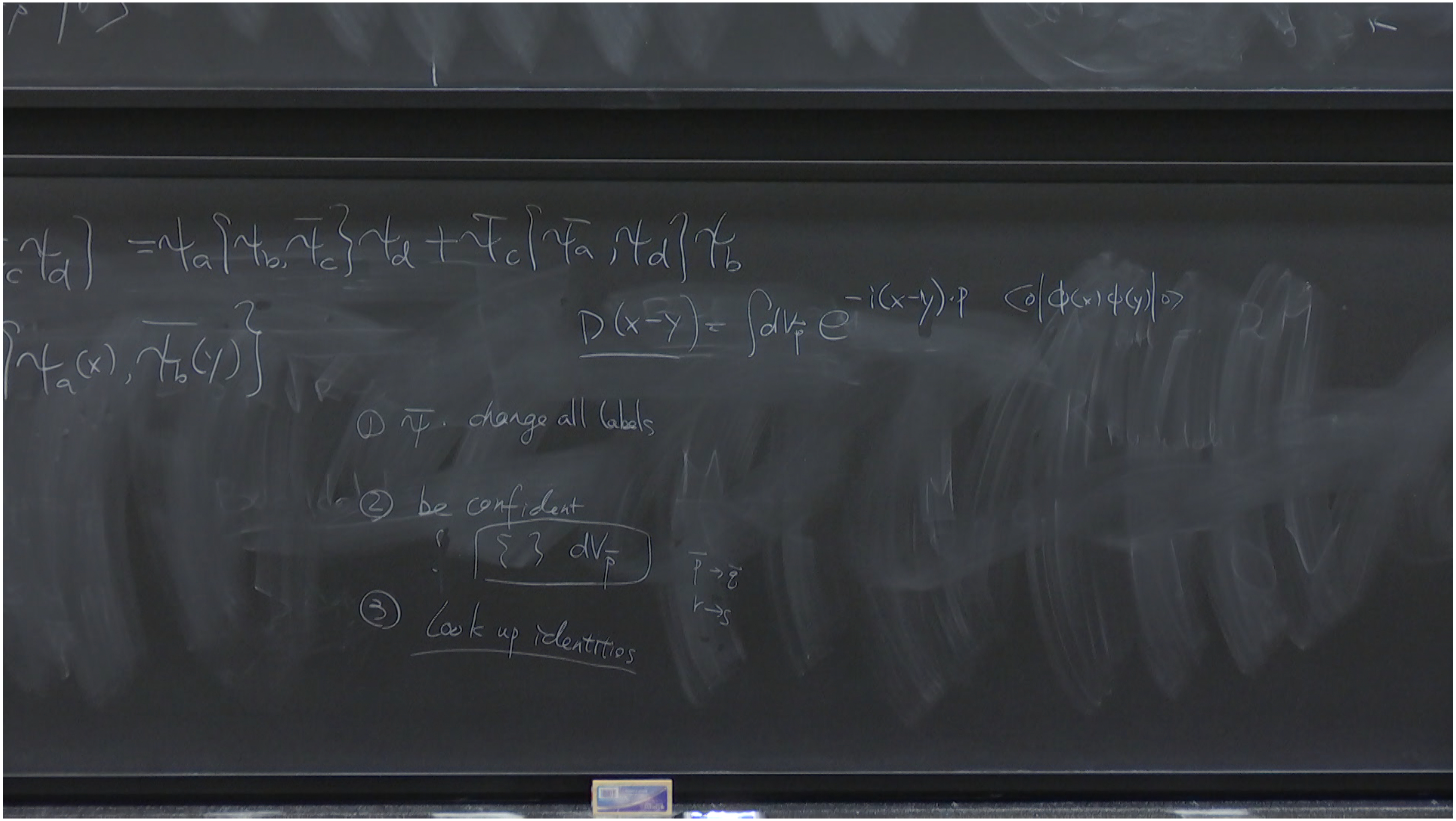

Causality $[\bar{\psi}_a \psi_b, \bar{\psi}_c \psi_d] = \psi_a \{\psi_b, \bar{\psi}_c\} \psi_d + \bar{\psi}_c \{\bar{\psi}_a, \psi_d\} \psi_b$
$i S_{ab}(x-y) = \{\psi_a(x), \bar{\psi}_b(y)\}$

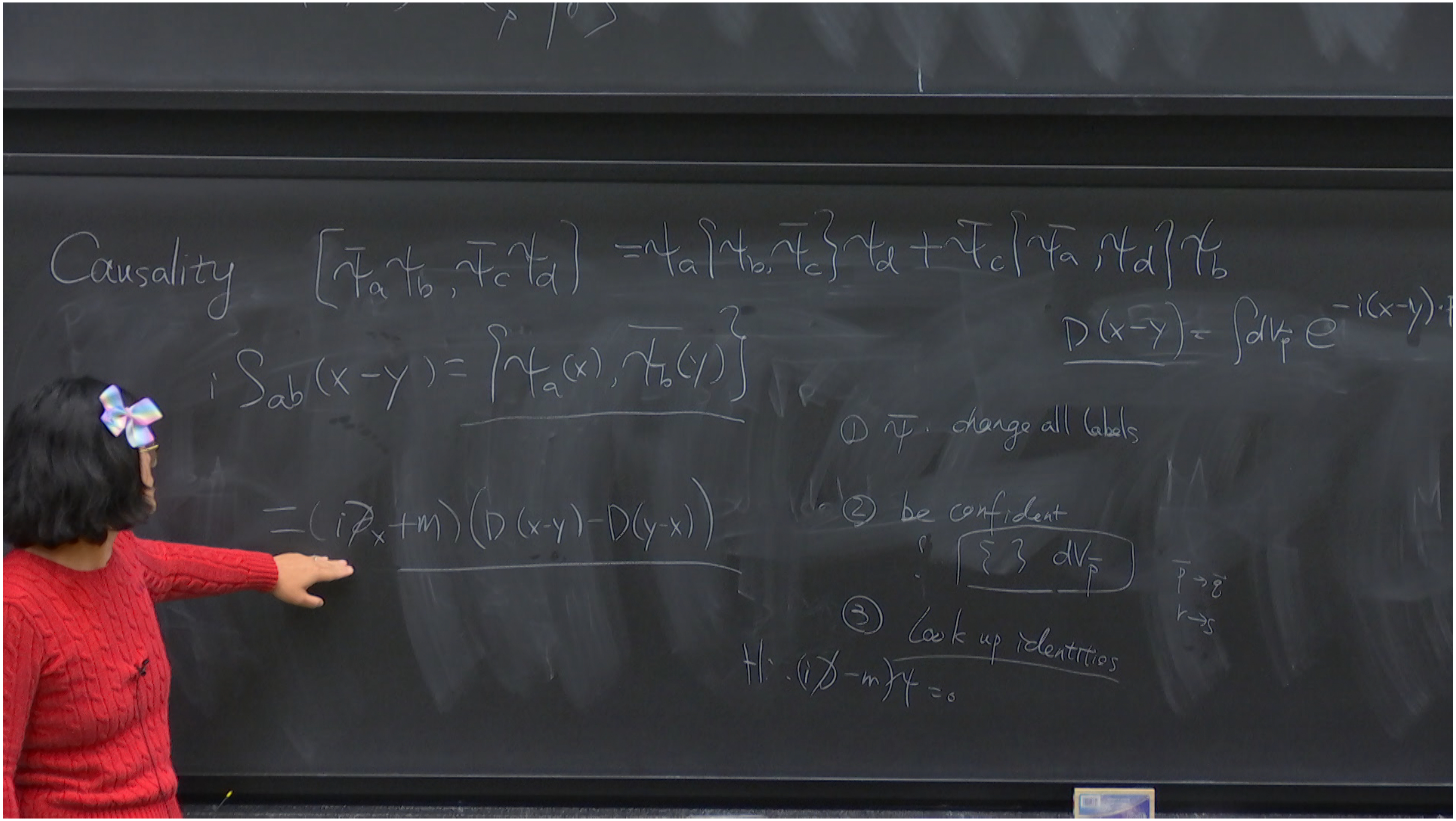
$$\bar{\psi}_c \{ \bar{\psi}_a, \psi_d \} \psi_b$$

$$\underline{D(x-y)} = \int dV_{\vec{p}}\, e^{-i(x-y)\cdot p}$$


ψ̄c {ψ̄a , ψd} ψb
D(x−y) = ∫dVp e^{−i(x−y)·p}   <0|φ(x)φ(y)|0>


$= \psi_a \{\psi_b, \bar{\psi}_c\} \psi_d + \bar{\psi}_c \{\psi_a, \psi_d\} \psi_b$
$\{\psi_a(x), \bar{\psi}_b(y)\}$
$D(x-y) = \int dV_{\vec{p}}\, e^{-i(x-y)\cdot p}$
$\langle 0 | \phi(x) \phi(y) | 0 \rangle$
① $\bar{\psi}$: change all labels
② be confident
$\{\ \}\ dV_{\vec{p}}$


D(x−y) = ∫dV_p e^{−i(x−y)·p}
⟨0|φ(x)φ(y)|0⟩
① ψ̄ : change all labels
② be confident
∫{ } dV_p
p → q
r → s
③ Look up identities


Causality
change all labels
be confident
Look up identities

Causality $[\bar\psi_a\psi_b, \bar\psi_c\psi_d] = \bar\psi_a\{\psi_b, \bar\psi_c\}\psi_d + \bar\psi_c\{\bar\psi_a, \psi_d\}\psi_b$

$D(x-y) = \int dV_p\, e^{-i(x-y)}$

$i S_{ab}(x-y) = \{\psi_a(x), \bar\psi_b(y)\}$

$u\bar{u}$

$\to (\not p + m)$ $= (i\not\partial_x + m)\big(D(x-y) - D(y-x)\big)$

① $\bar\psi$: change all labels

② be confident

$\boxed{\{\ \}\ dV_p}$

$p \to q$

$r \to s$

③ Look up identities

H: $(i\not\partial - m)\psi = 0$