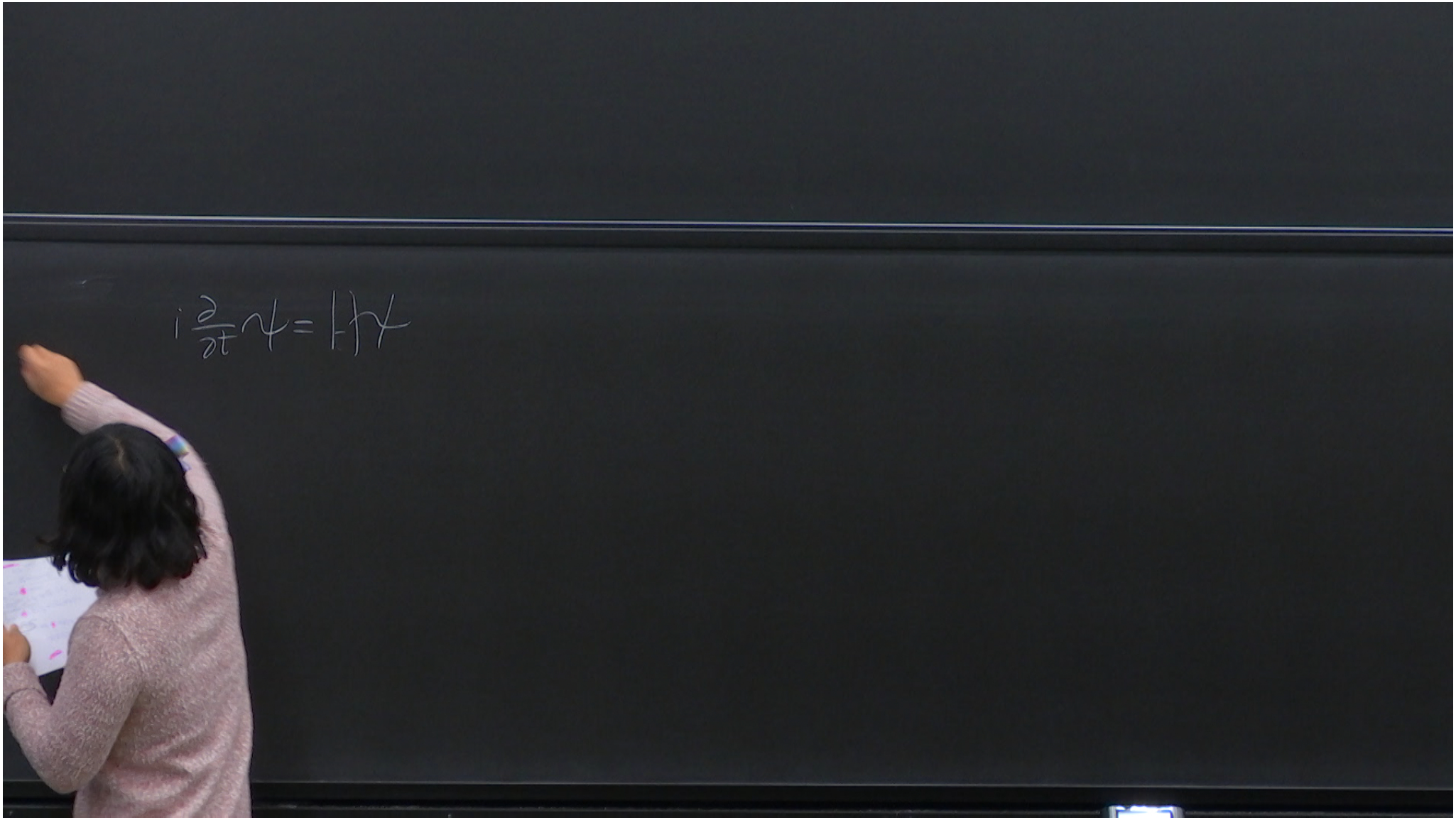
Title: Quantum Field Theory I - Lecture 221026

Speakers: Gang Xu

Collection: Quantum Field Theory I (2022/2023)

Date: October 26, 2022 - 10:45 AM

URL: https://pirsa.org/22100055


$i\frac{\partial}{\partial t}\psi = H\psi$

Gasel Hamiltonian

$$i\frac{\partial}{\partial t}\psi = H\psi$$

$$(i\gamma^{\mu}_{ab}\partial_{\mu} - m\mathbb{1}_{ab})\psi_b(x) = 0$$

Gasel Hamiltonian

$$i\frac{\partial}{\partial t}\psi = H\psi$$

$$(i\gamma^{\mu}_{ab}\partial_{\mu} - m\mathbb{1}_{ab})\psi_b(x) = 0$$

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi(x) = 0$$

Good Hamiltonian

$$i\frac{\partial}{\partial t}\psi = H\psi$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$$

$$(i\gamma^\mu_{ab}\partial_\mu - m\mathbb{1}_{ab})\psi_b(x) = 0$$

$$\boxed{(i\gamma^\mu\partial_\mu - m)\psi(x) = 0}$$

Good Hamiltonian

$$i\frac{\partial}{\partial t}\psi = H\psi$$

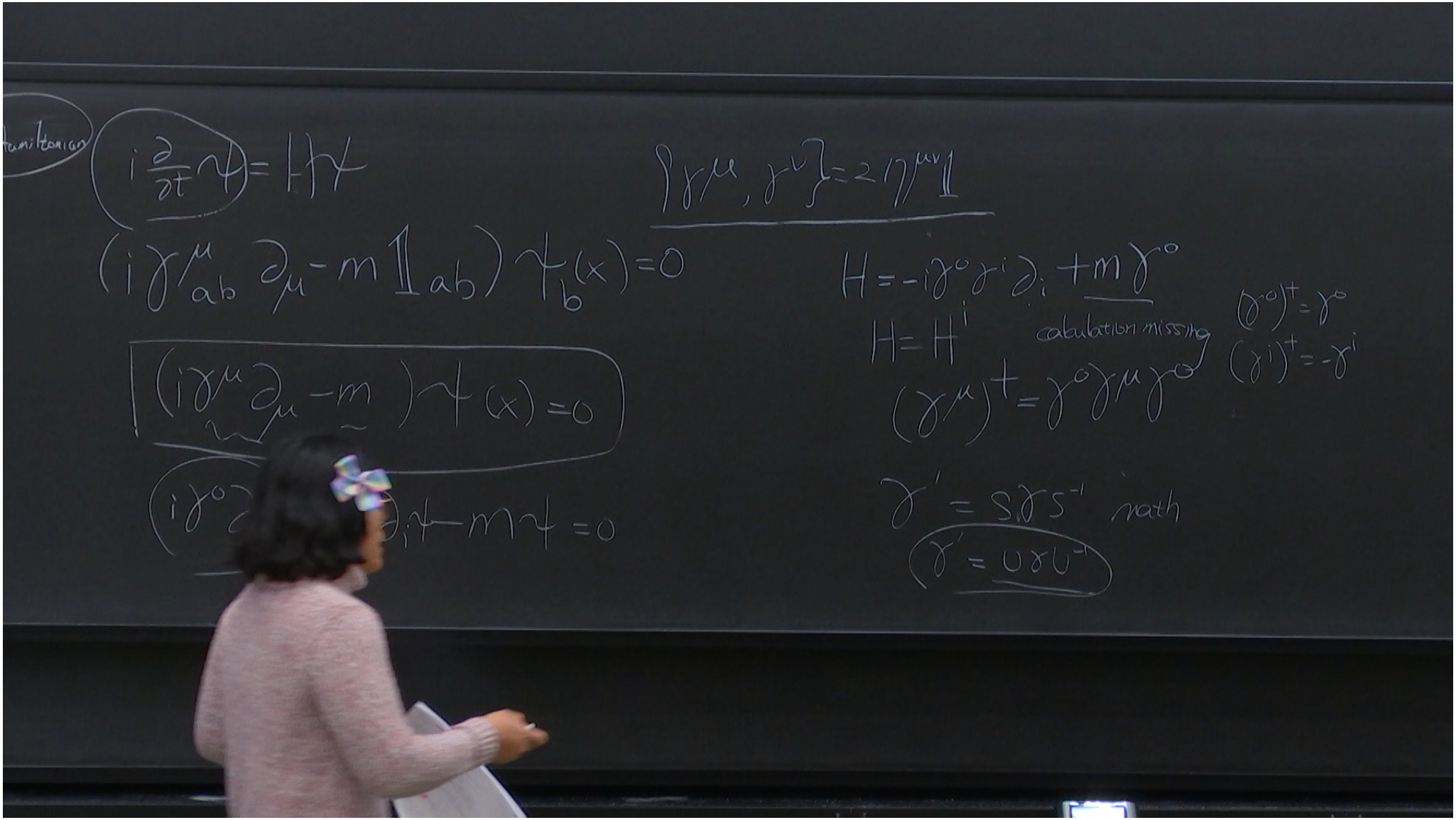
$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$$

$$(i\gamma^\mu_{ab}\partial_\mu - m\mathbb{1}_{ab})\psi_b(x) = 0$$

$$H = -i\gamma^0\gamma^i\partial_i + m\gamma^0$$

$$\boxed{(i\gamma^\mu\partial_\mu - m)\psi(x) = 0}$$

$$i\gamma^0\partial_0\psi + i\gamma^i\partial_i\psi - m\psi = 0$$


Hamiltonian
$i\frac{\partial}{\partial t}\psi = H\psi$
$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$
$(i\gamma^\mu_{ab}\partial_\mu - m\mathbb{1}_{ab})\psi_b(x) = 0$
$H = -i\gamma^0\gamma^i\partial_i + m\gamma^0$
$H = H^\dagger$
calculation missing
$(\gamma^0)^\dagger = \gamma^0$
$(\gamma^i)^\dagger = -\gamma^i$
$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0$
$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$
$i\gamma^0\partial_t\psi + i\gamma^i\partial_i\psi - m\psi = 0$
$\gamma' = S\gamma S^{-1}$ math
$\gamma' = U\gamma U^{-1}$

$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$

ab) $\psi_b(x) = 0$

$(\not{\partial} + m)\psi(x) = 0$

$\not{p} - m\psi = 0$

$H = -i\gamma^0\gamma^i\partial_i + m\gamma^0$

$H = H^\dagger$ calculation missing

$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$

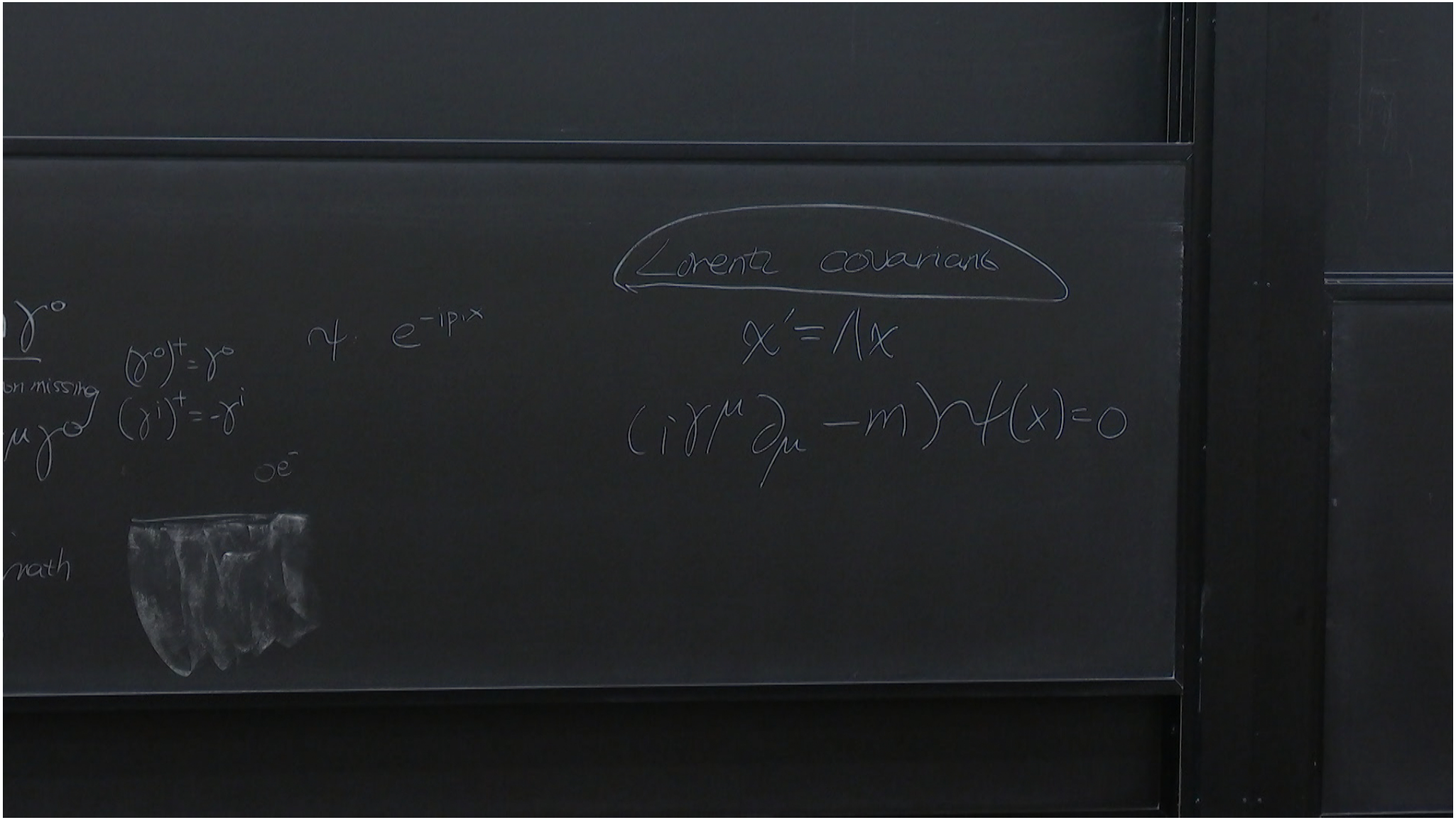
$(\gamma^0)^\dagger = \gamma^0$

$(\gamma^i)^\dagger = -\gamma^i$

$\psi \cdot e^{-ip\cdot x}$

$\gamma' = S\gamma S^{-1}$ math

$\gamma' = U\gamma U^{-1}$


Lorentz covariant
$x' = \Lambda x$
$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$
$(\gamma^0)^\dagger = \gamma^0$
$(\gamma^i)^\dagger = -\gamma^i$
$\psi \cdot e^{-ip\cdot x}$
γ^0
$\mu\gamma^0$
$0e^-$

Lorentz covariance

$x' = \Lambda x$

$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$

$\psi'(x') = D(\Lambda)\psi(x)$

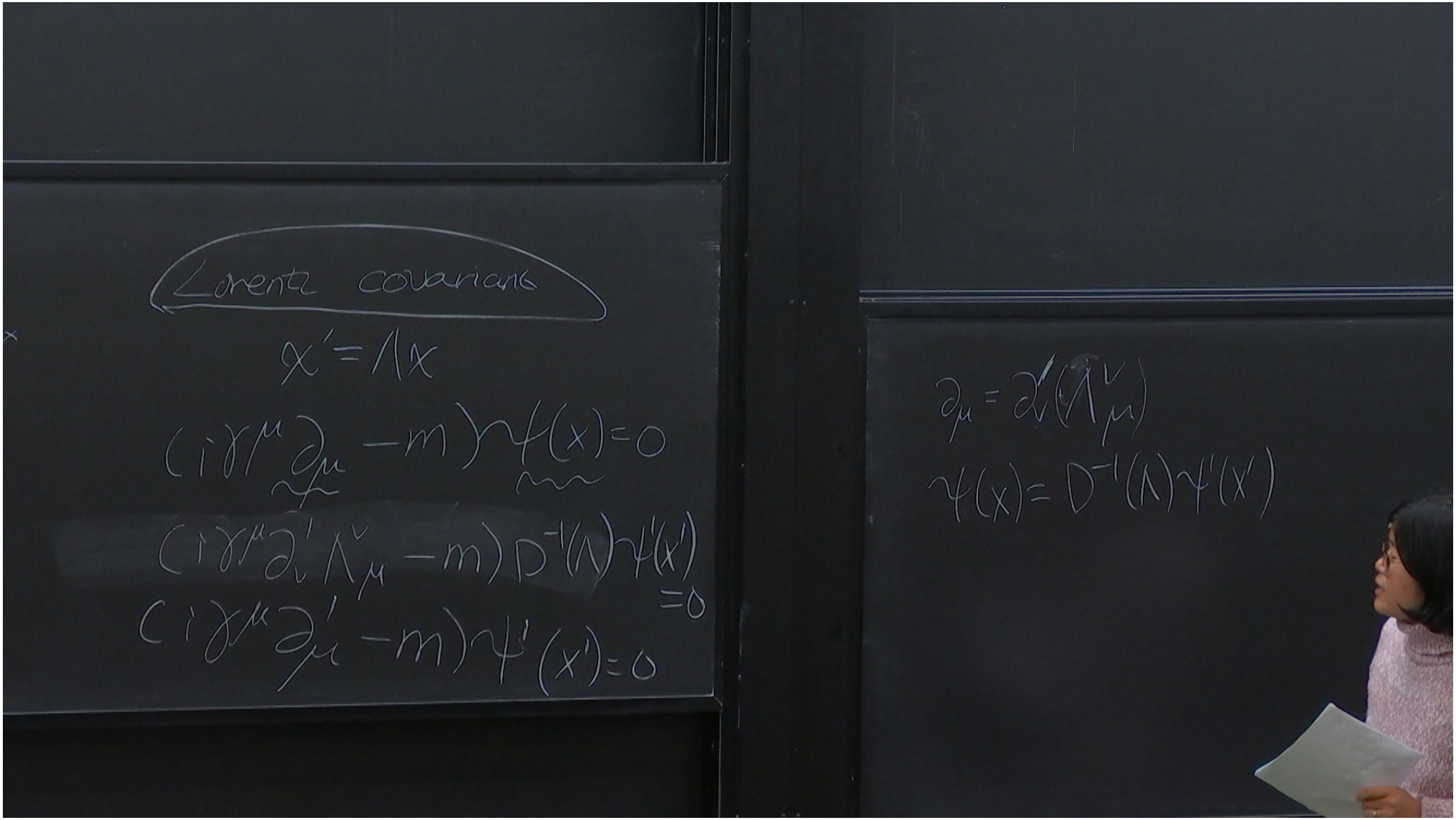
$(i\gamma^\mu \partial'_\mu - m)\psi'(x') = 0$

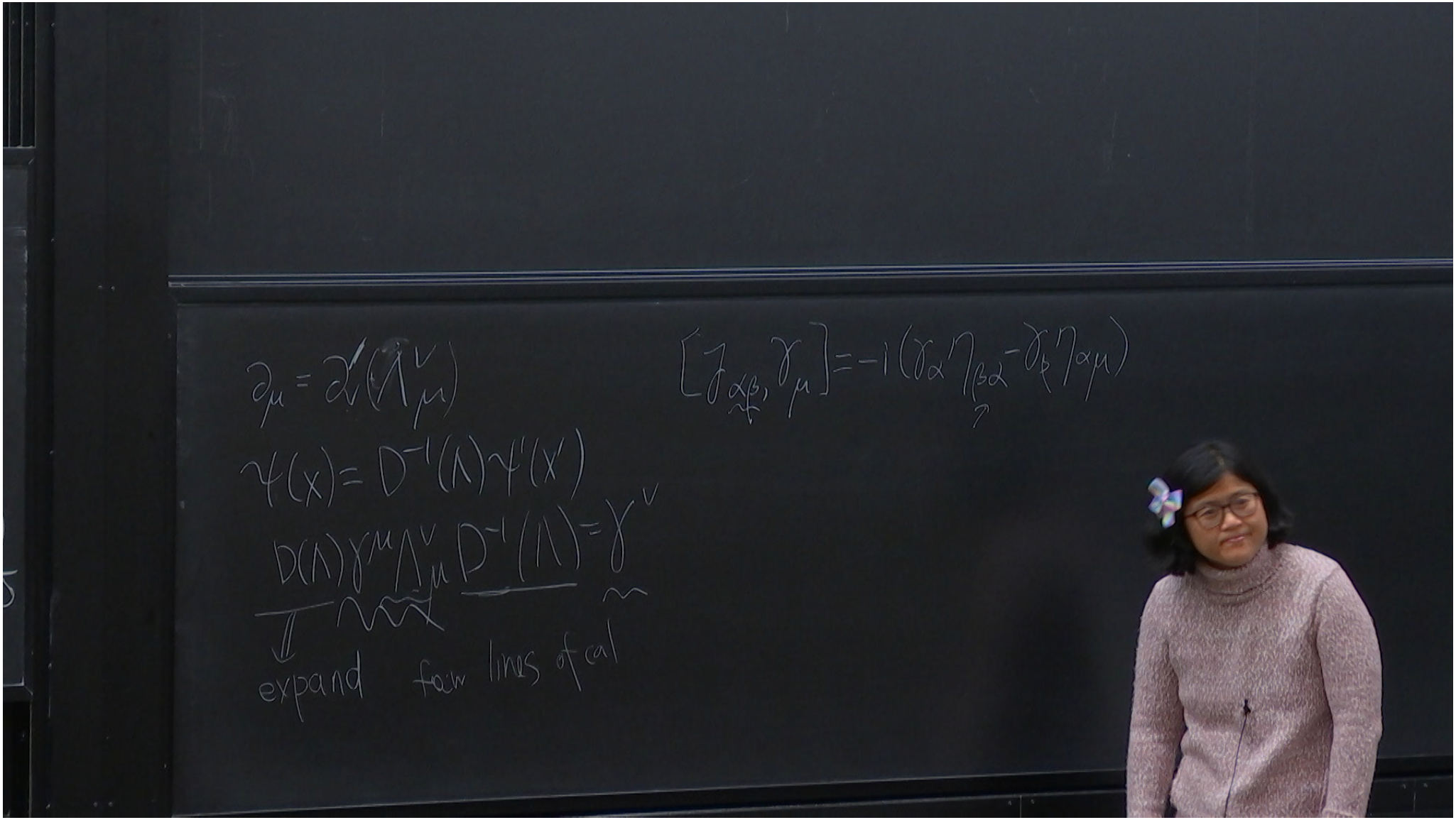
$(\gamma^0)^\dagger = \gamma^0$

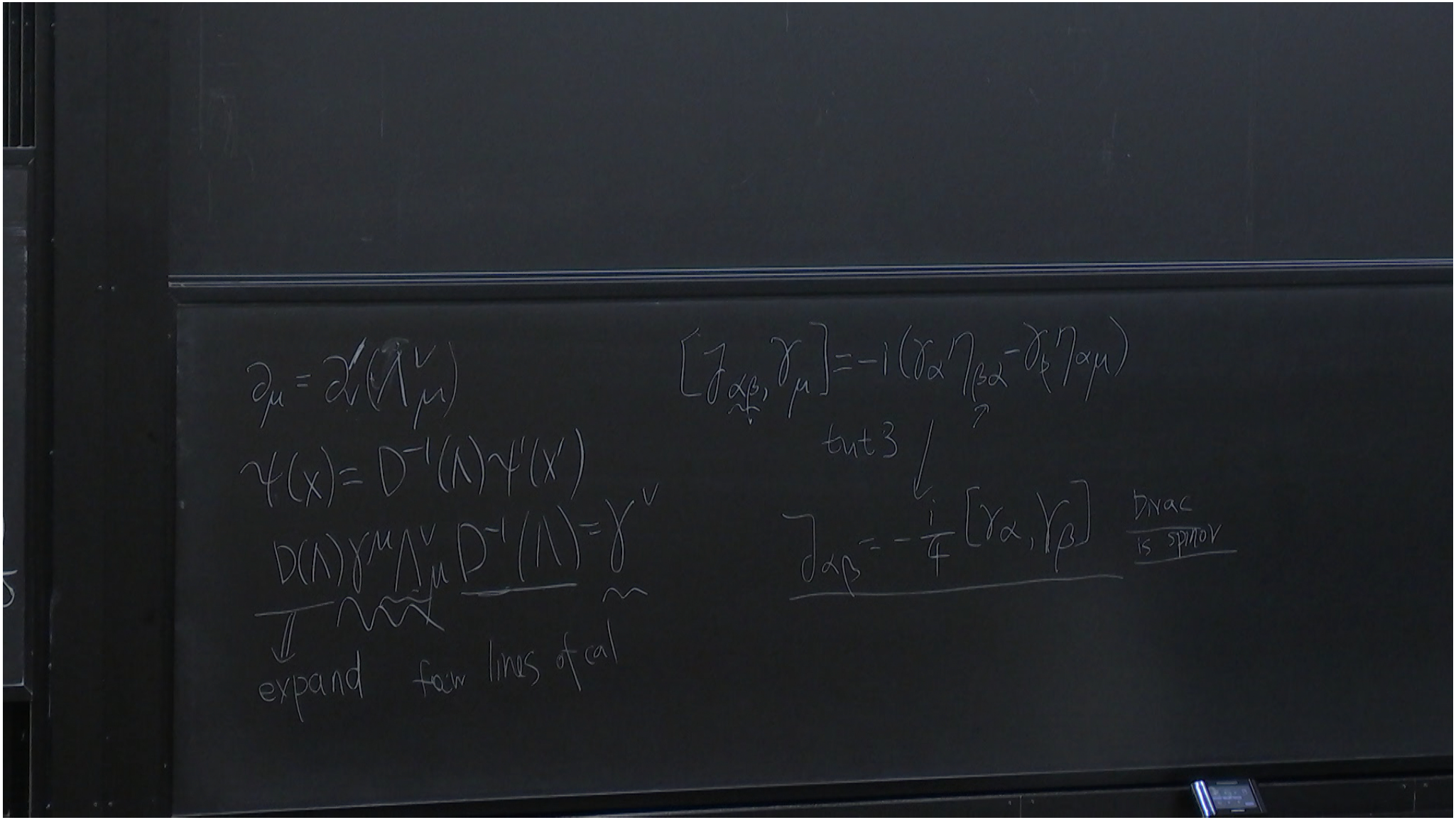
$(\gamma^i)^\dagger = -\gamma^i$

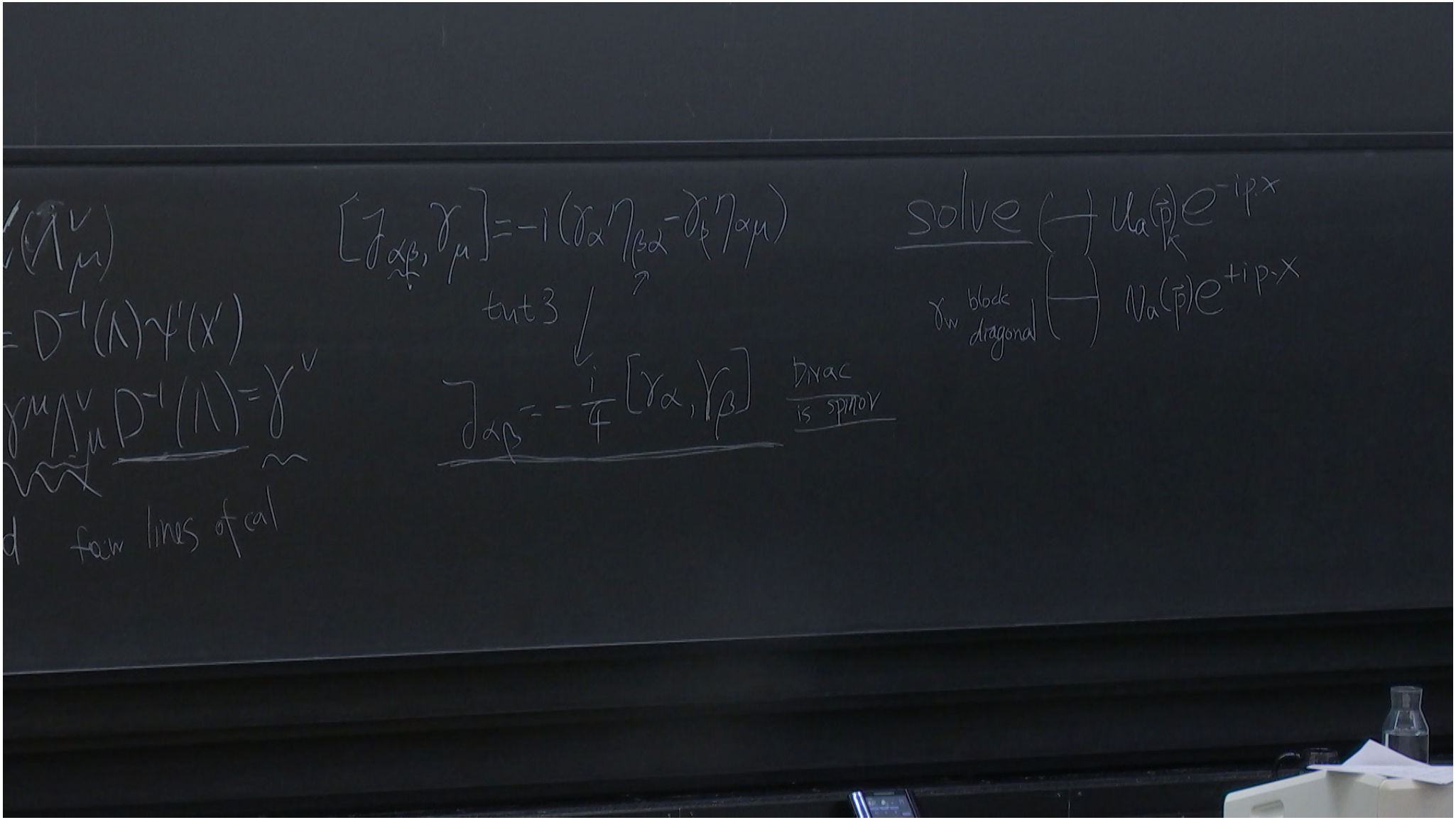
$\psi \cdot e^{-ip\cdot x}$

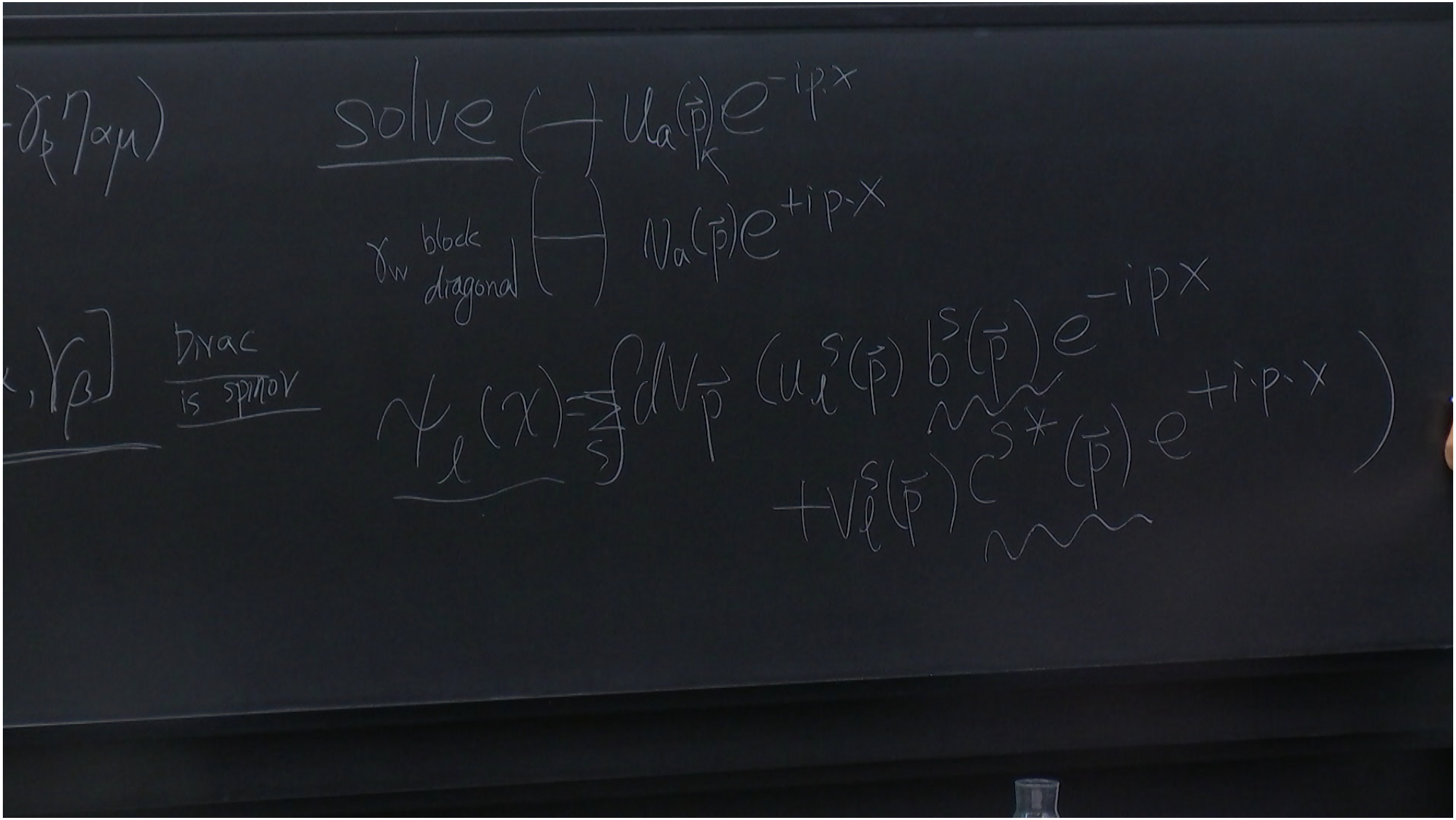
$0e^-$

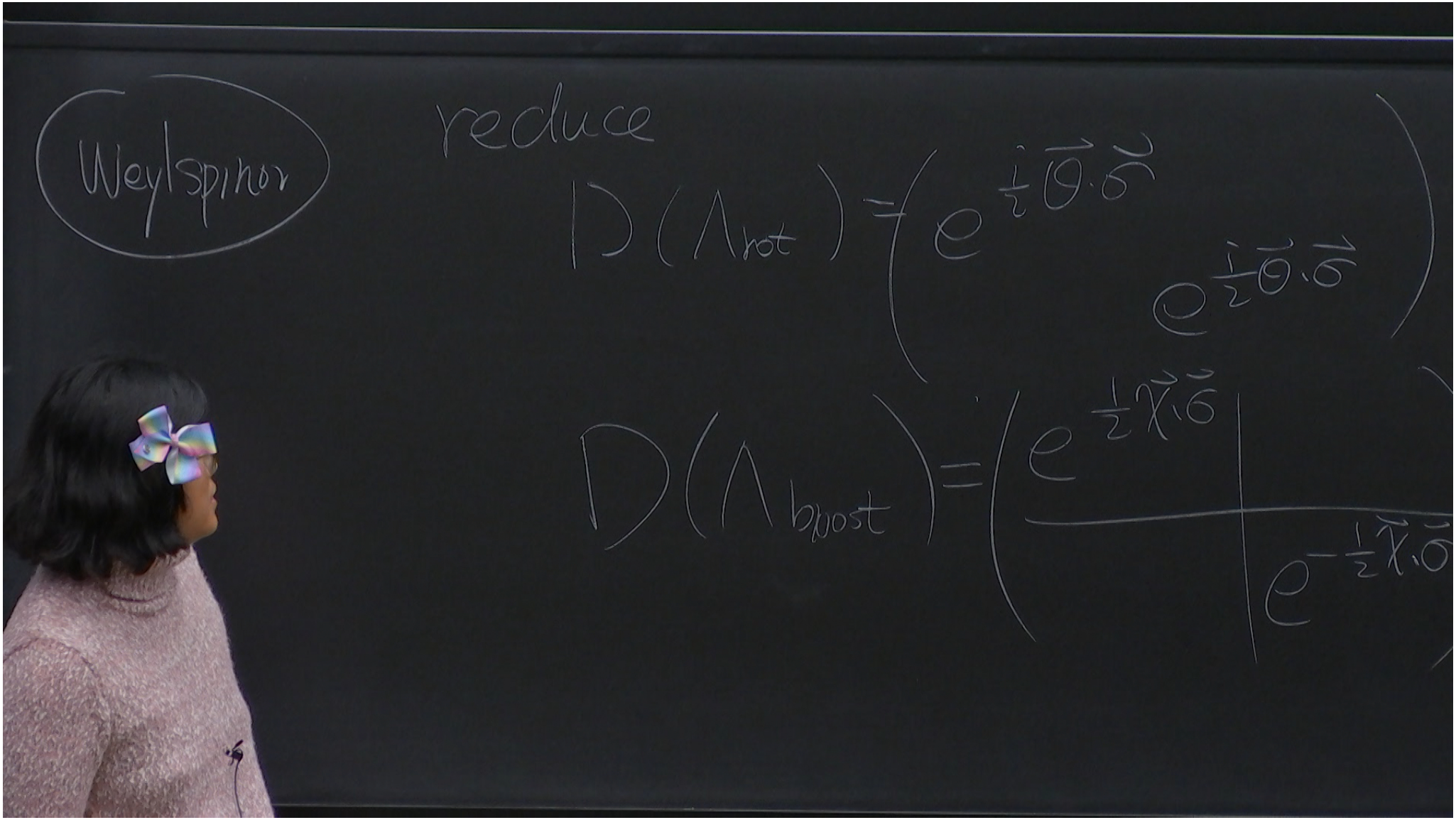

Lorentz covariance
$x' = \Lambda x$
$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$
$(i\gamma^\mu \partial'_\nu \Lambda^\nu_\mu - m) D^{-1}(\Lambda)\psi'(x') = 0$
$(i\gamma^\mu \partial'_\mu - m)\psi'(x') = 0$
$\partial_\mu = \partial'_\nu (\Lambda^\nu_\mu)$
$\psi(x) = D^{-1}(\Lambda)\psi'(x')$


$\partial_\mu = \partial'_\nu (\Lambda^\nu{}_\mu)$
$[J_{\alpha\beta}, \gamma_\mu] = -i(\gamma_\alpha \eta_{\beta\mu} - \gamma_\beta \eta_{\alpha\mu})$
$\psi(x) = D^{-1}(\Lambda)\psi'(x')$
$D(\Lambda)\gamma^\mu \Lambda^\nu{}_\mu D^{-1}(\Lambda) = \gamma^\nu$
expand for infinitesimal


$\partial_\mu = \partial'(\Lambda^\nu{}_\mu)$
$\psi(x) = D^{-1}(\Lambda)\psi'(x')$
$D(\Lambda)\gamma^\mu\Lambda^\nu{}_\mu D^{-1}(\Lambda) = \gamma^\nu$
expand for lines of cal
$[J_{\alpha\beta}, \gamma_\mu] = -i(\gamma_\alpha \eta_{\beta\mu} - \gamma_\beta \eta_{\alpha\mu})$
tut 3
$J_{\alpha\beta} = -\frac{i}{4}[\gamma_\alpha, \gamma_\beta]$
Dirac is spinor

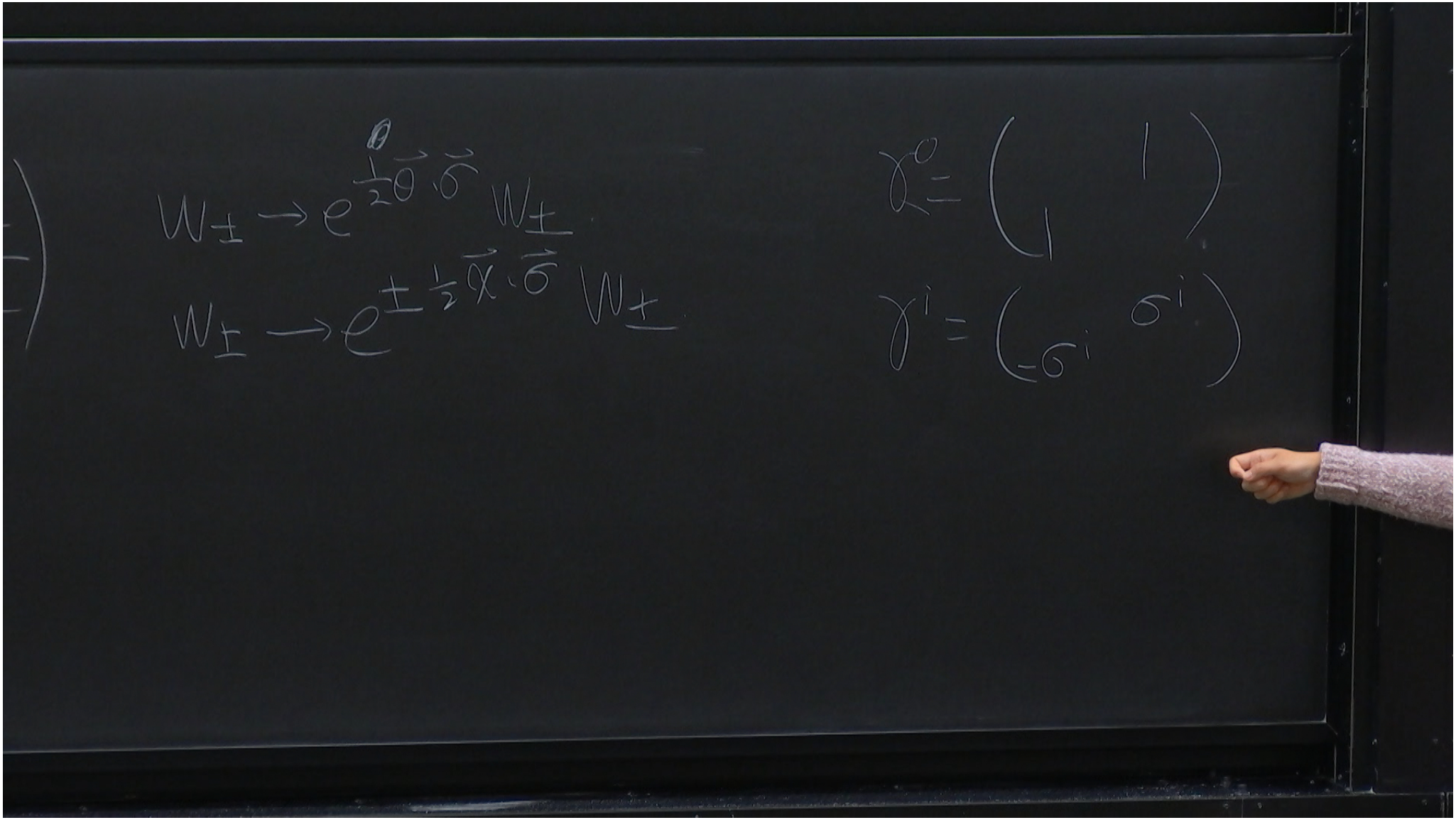

$[J_{\alpha\beta}, \gamma_\mu] = -i(\gamma_\alpha \eta_{\beta\mu} - \gamma_\beta \eta_{\alpha\mu})$
tut 3
$J_{\alpha\beta} = -\frac{i}{4}[\gamma_\alpha, \gamma_\beta]$
Dirac is spinor
solve
γ_w block diagonal
$u_a(\vec{p})e^{-ip\cdot x}$
$v_a(\vec{p})e^{+ip\cdot x}$
$D^{-1}(\Lambda)\gamma'(x')$
$\gamma^\mu \Lambda^\nu_\mu D^{-1}(\Lambda) = \gamma^\nu$
four lines of cal

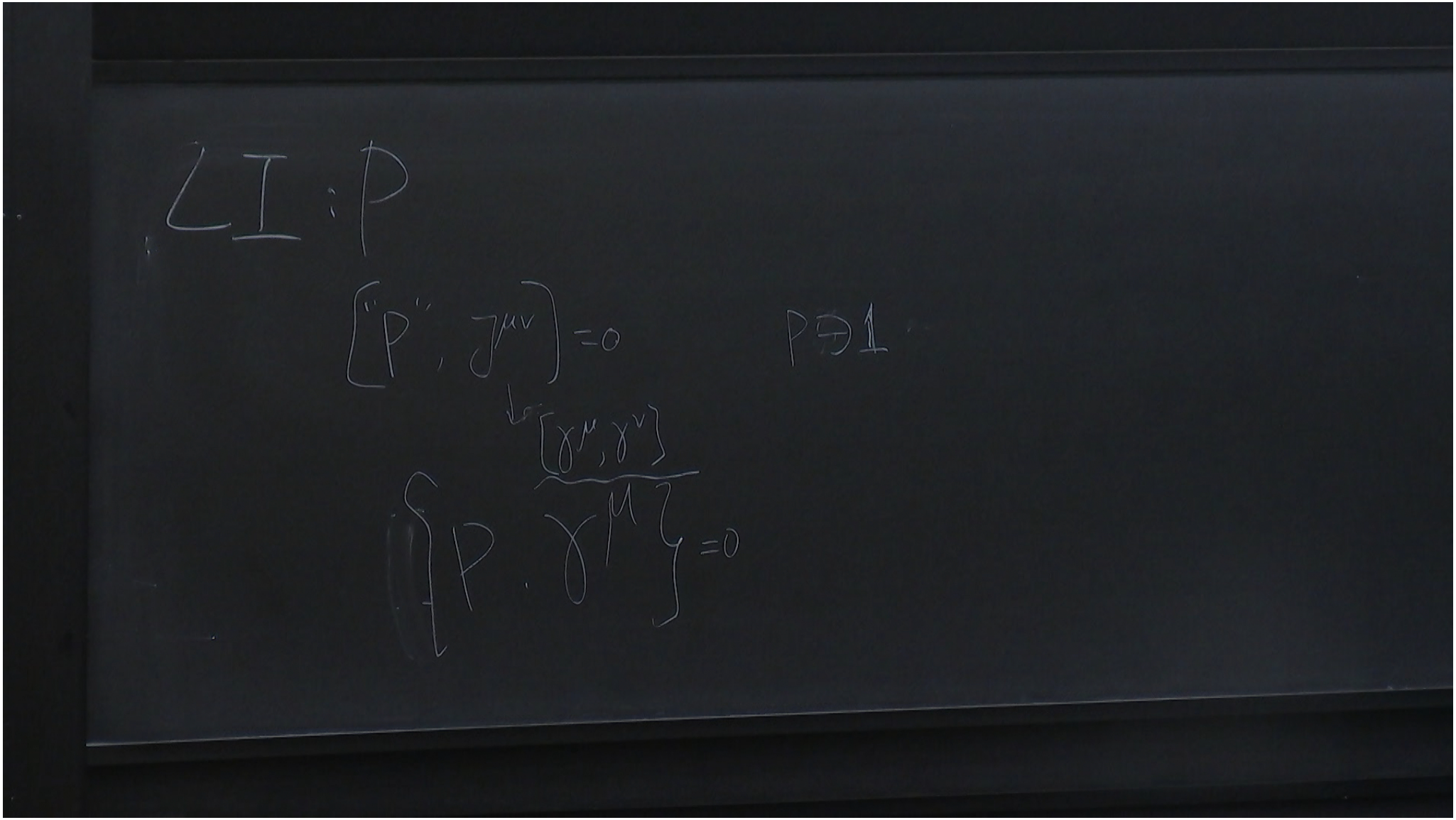

solve
block diagonal
Dirac
is spinor

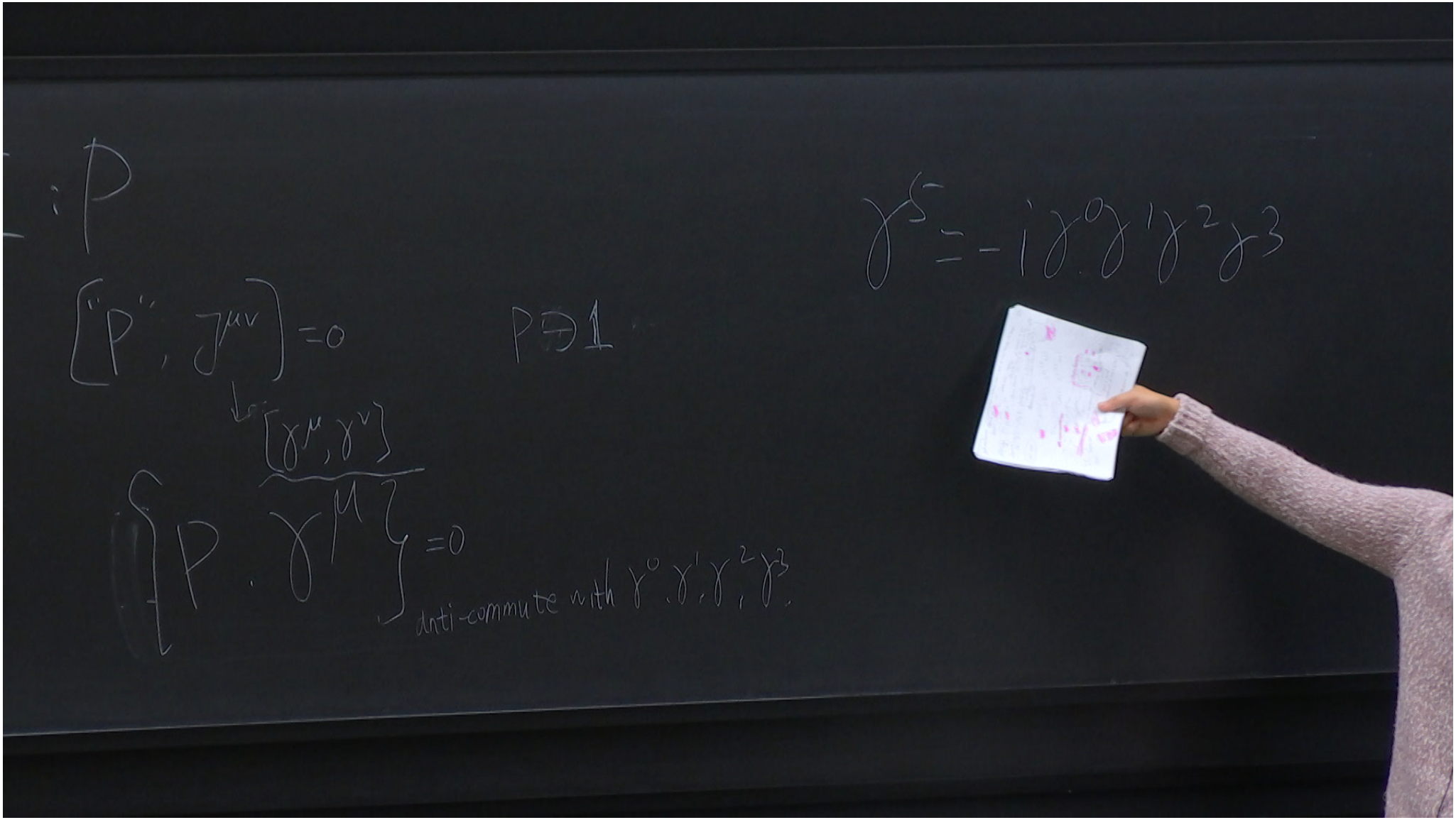

Weylspinor
reduce
$D(\Lambda_{rot}) = \begin{pmatrix} e^{\frac{i}{2}\vec{\theta}\cdot\vec{\sigma}} & \\ & e^{\frac{i}{2}\vec{\theta}\cdot\vec{\sigma}} \end{pmatrix}$
$D(\Lambda_{boost}) = \begin{pmatrix} e^{\frac{1}{2}\vec{\chi}\cdot\vec{\sigma}} & \\ & e^{-\frac{1}{2}\vec{\chi}\cdot\vec{\sigma}} \end{pmatrix}$

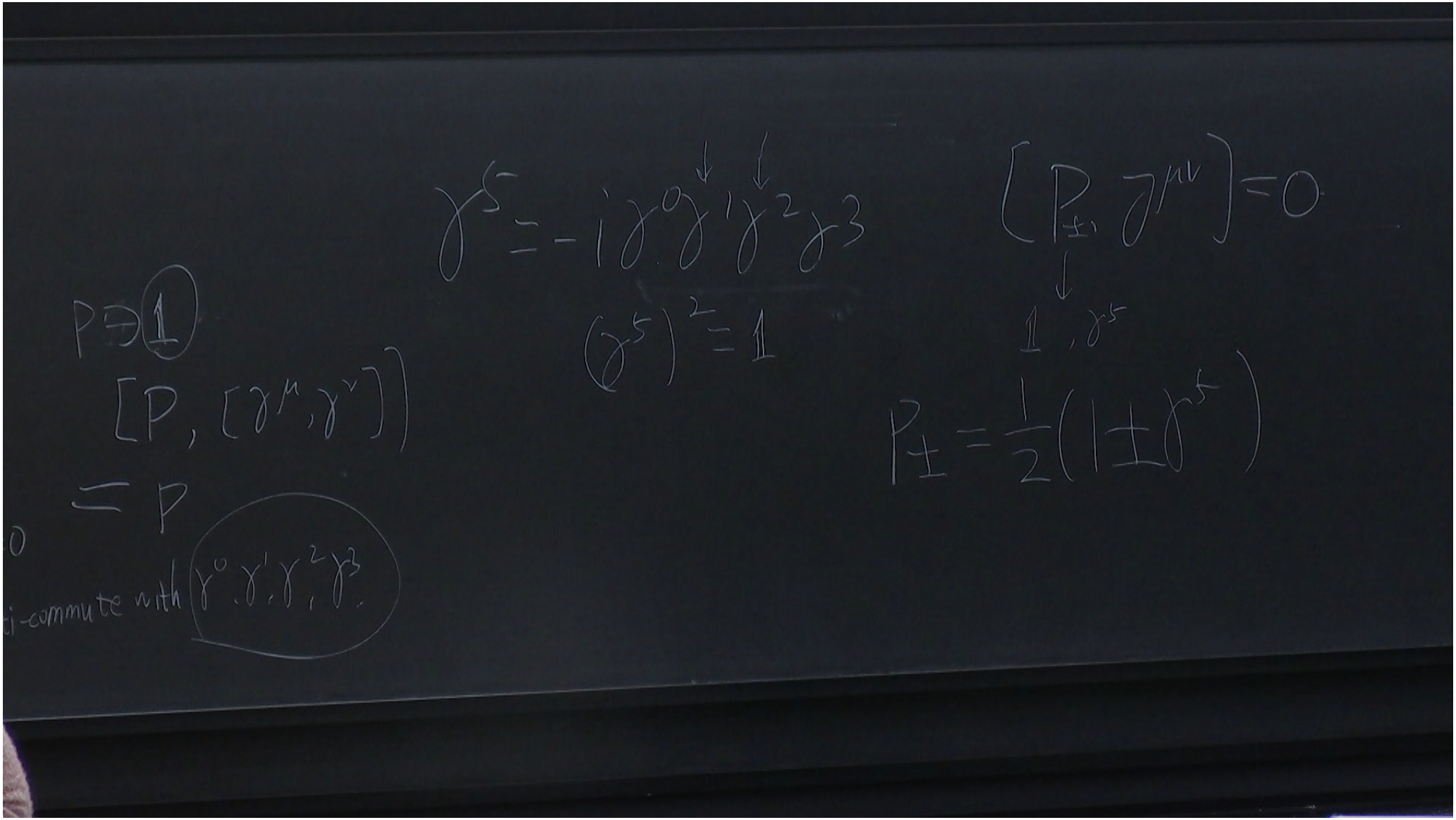
$$\psi = \begin{pmatrix} w_+ \\ w_- \end{pmatrix} \qquad w_\pm \to e^{\pm \frac{i}{2}\vec{\theta}\cdot\vec{\sigma}} w_\pm$$

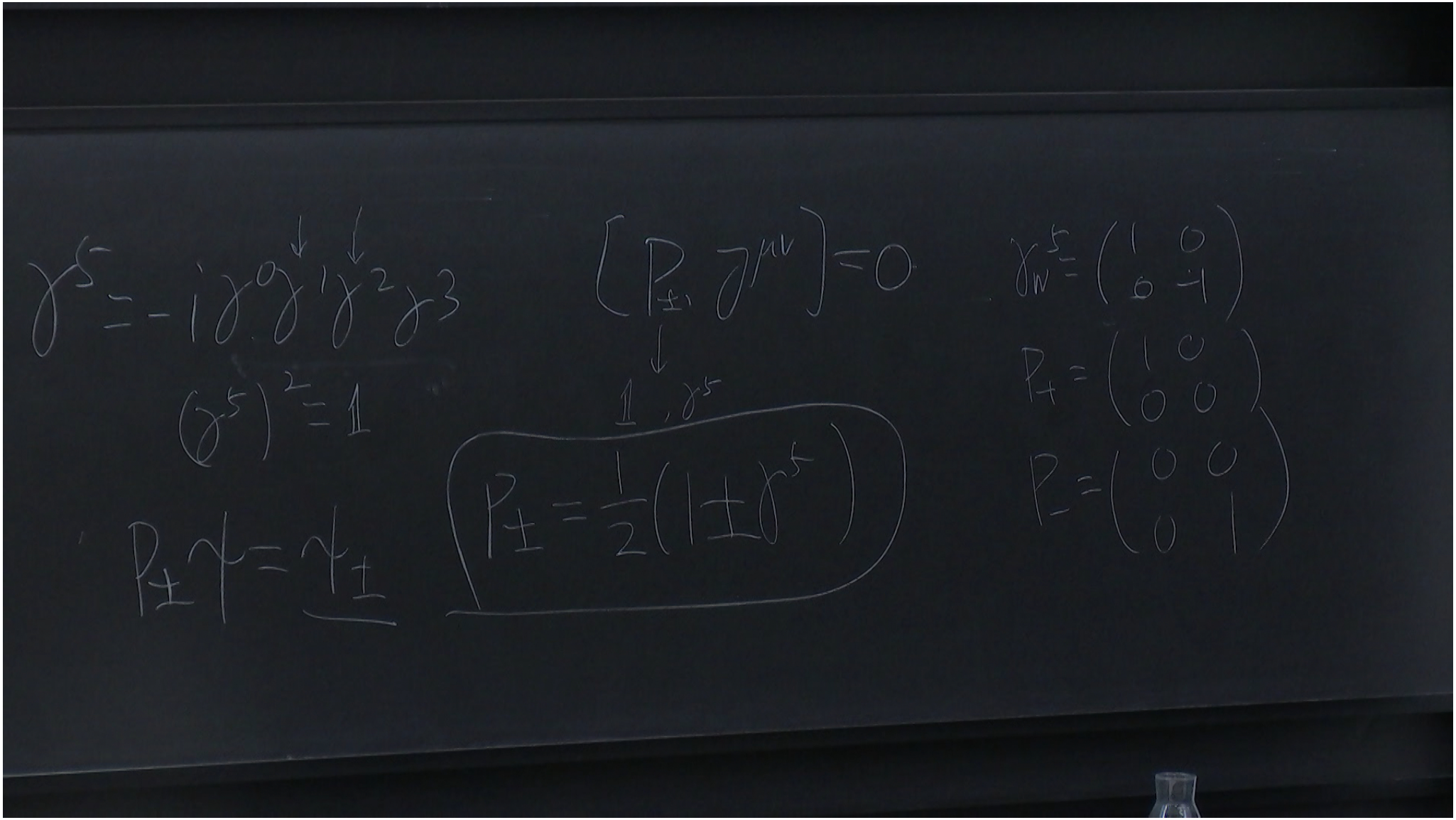
$$w_\pm \to e^{\pm \frac{1}{2}\vec{\alpha}\cdot\vec{\sigma}} w_\pm$$

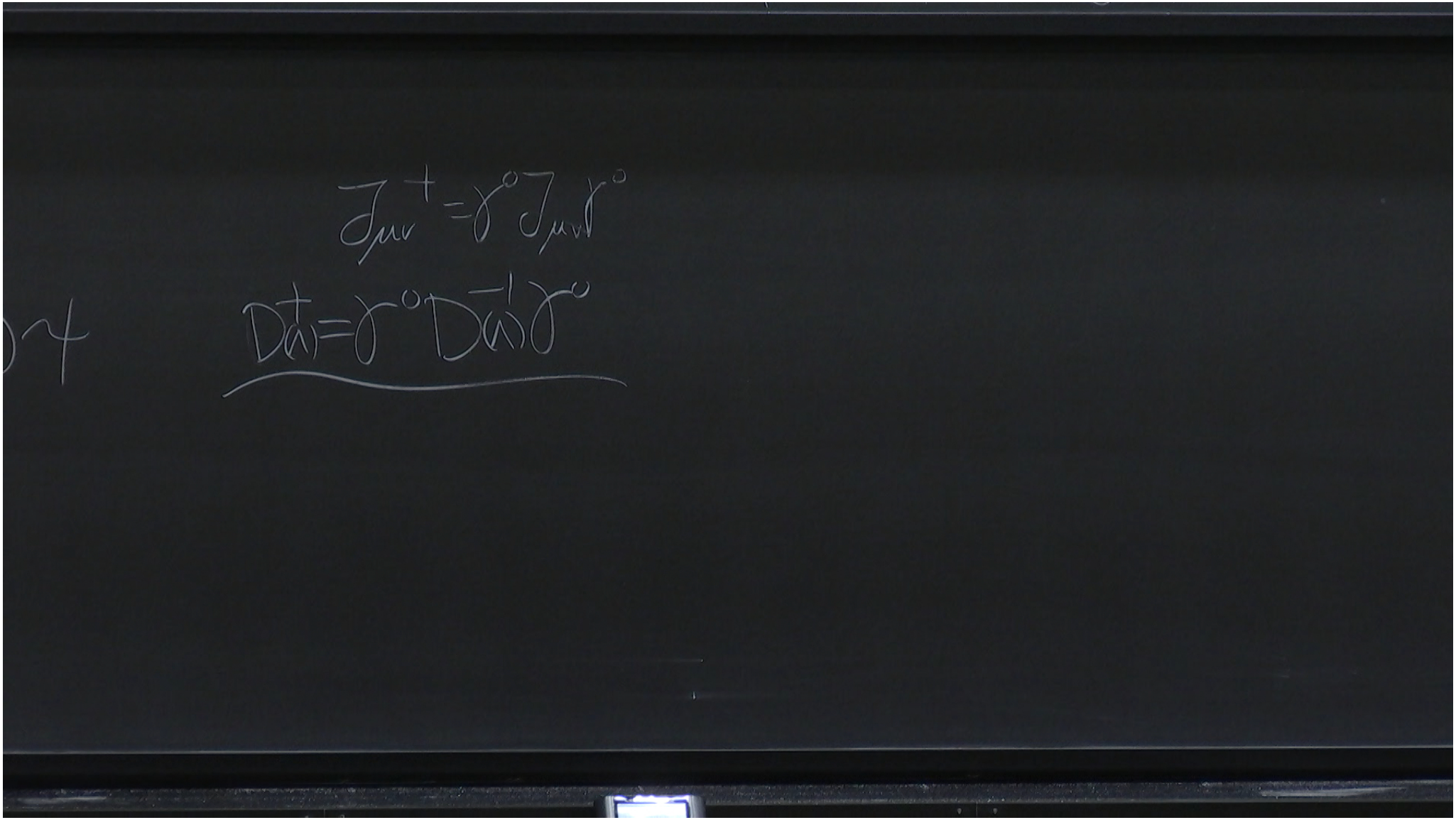

$W_\pm \to e^{\pm \frac{1}{2}\vec{\theta}\cdot\vec{\sigma}} W_\pm$
$W_\pm \to e^{\pm \frac{1}{2}\vec{\chi}\cdot\vec{\sigma}} W_\pm$
$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\gamma^i = \begin{pmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$

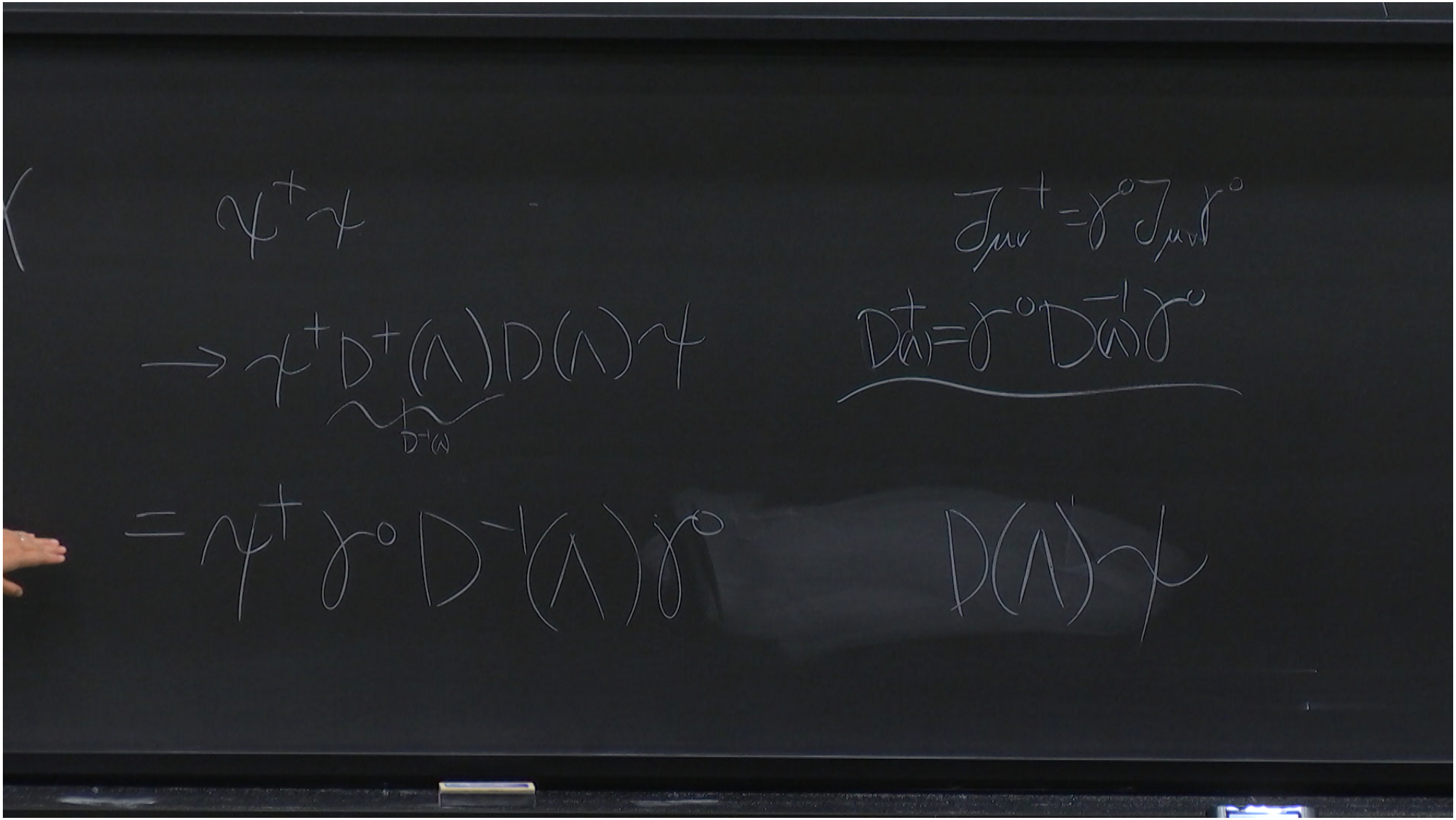

LI : P
[P , J^μν] = 0
P ∄ 1
[γ^μ , γ^ν]
{P , γ^μ} = 0

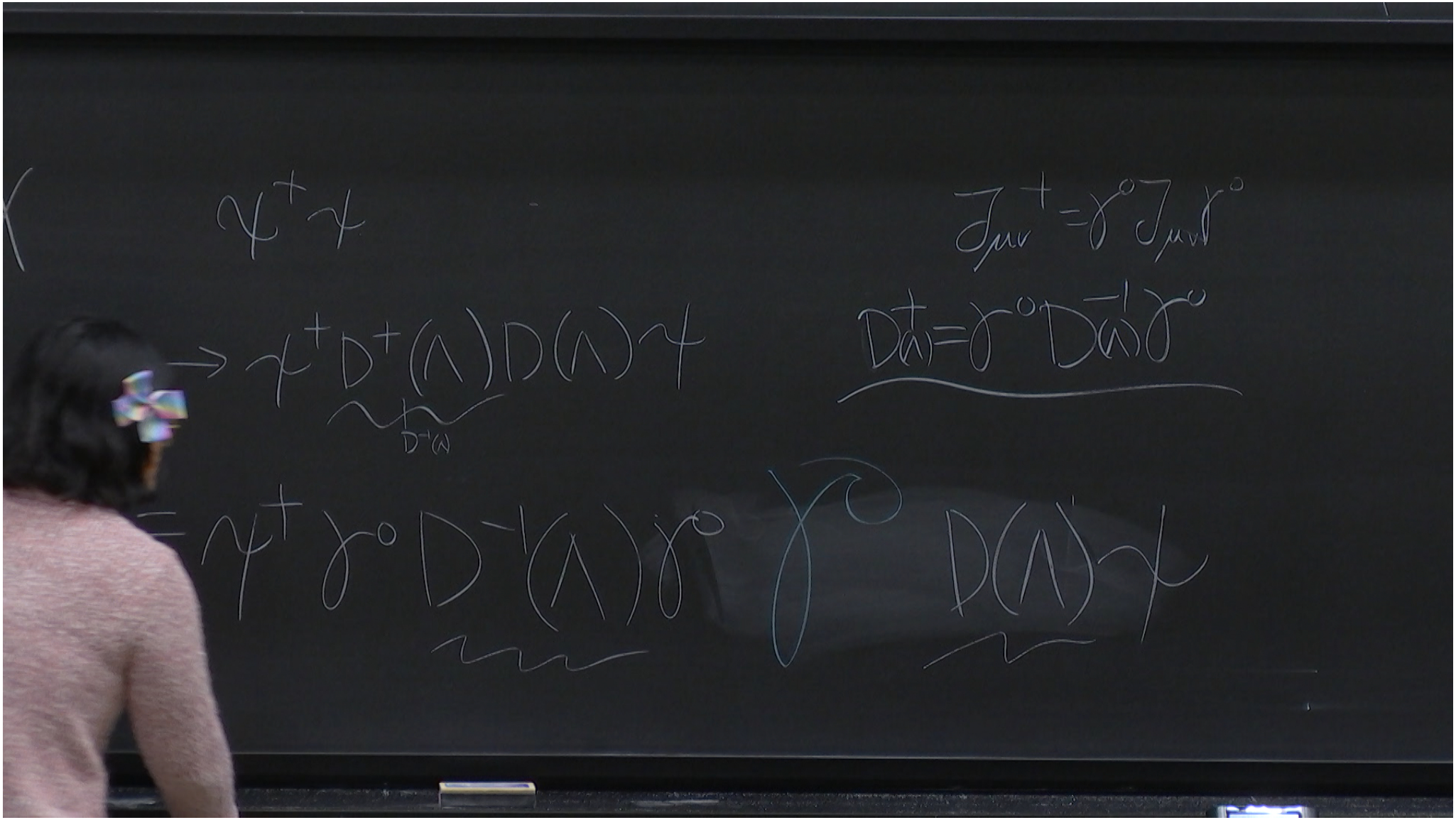

$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$
$[P, J^{\mu\nu}] = 0$
$P \neq 1$
$[\gamma^\mu, \gamma^\nu]$
$\{P, \gamma^\mu\} = 0$
anti-commute with $\gamma^0, \gamma^1, \gamma^2, \gamma^3$

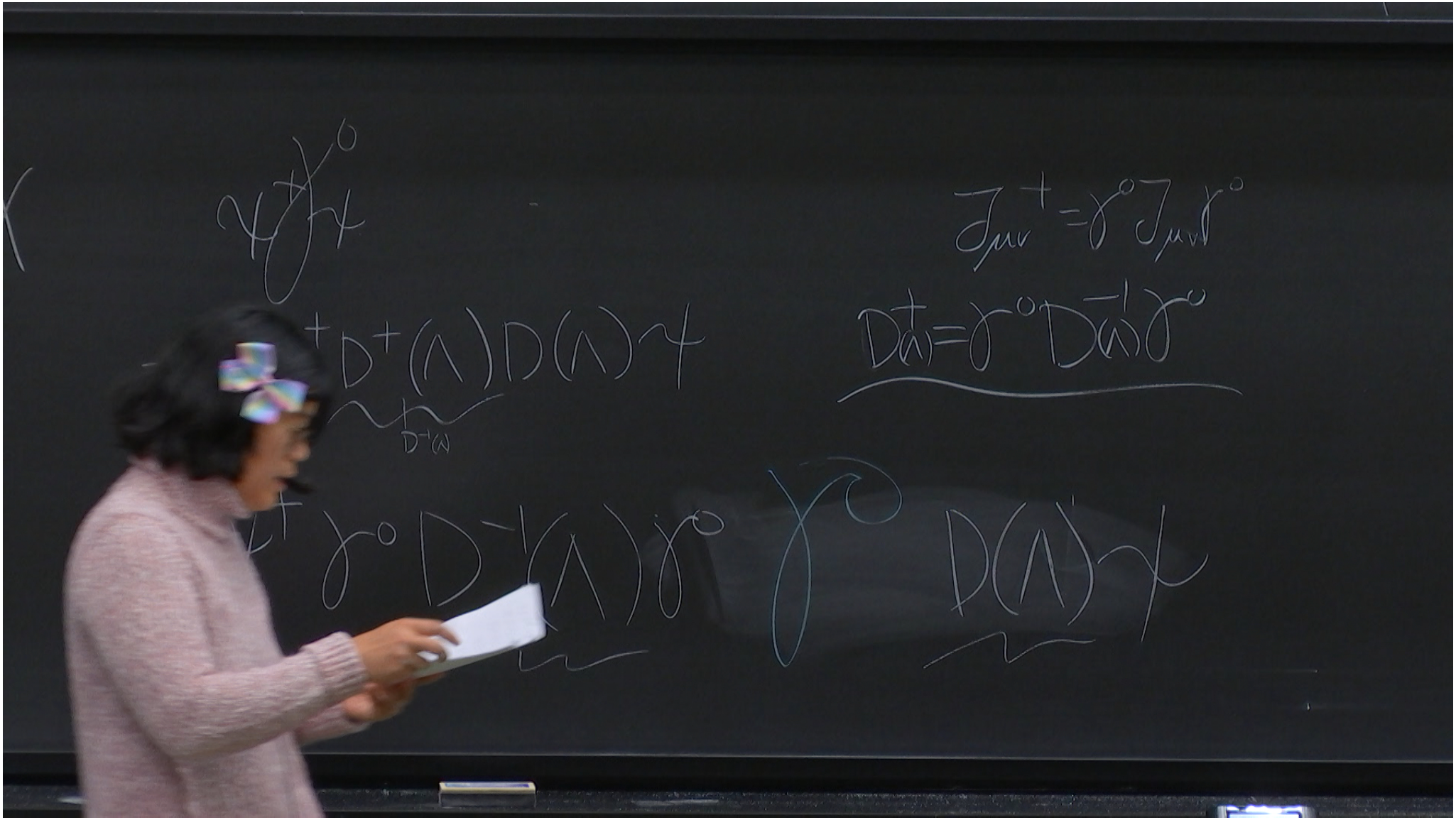

$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$
$(\gamma^5)^2 = 1$
$[P_\pm, \gamma^\mu] = 0$
$1, \gamma^5$
$P_\pm = \frac{1}{2}(1 \pm \gamma^5)$
$P = 1$
$[P, [\gamma^\mu, \gamma^\nu]]$
$= P$
$\gamma^0, \gamma^1, \gamma^2, \gamma^3$

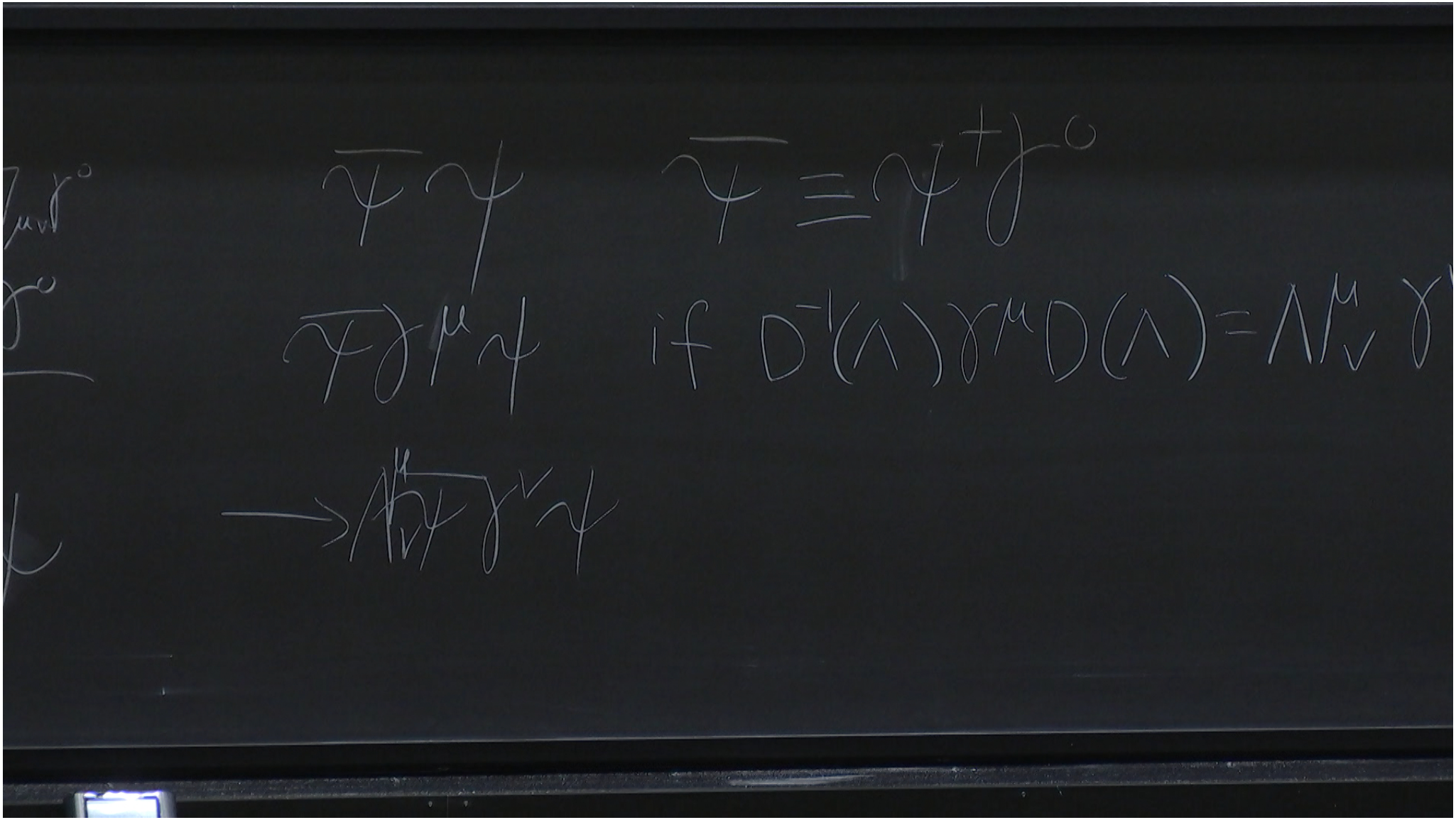

$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$
$(\gamma^5)^2 = \mathbb{1}$
$[P_\pm, \gamma^\mu] = 0$
$\mathbb{1}, \gamma^5$
$P_\pm = \frac{1}{2}(1 \pm \gamma^5)$
$P_\pm \psi = \psi_\pm$
$\gamma_W^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$


$J_{\mu\nu}^{\dagger} = \gamma^0 J_{\mu\nu} \gamma^0$
$D(\Lambda)^{\dagger} = \gamma^0 D(\Lambda)^{-1} \gamma^0$


$\chi^\dagger \gamma$
$\rightarrow \chi^\dagger D^\dagger(\Lambda) D(\Lambda) \gamma$
$D^{-1}(\Lambda)$
$= \chi^\dagger \gamma^0 D^{-1}(\Lambda) \gamma^0$
$J_{\mu\nu}{}^\dagger = \gamma^0 J_{\mu\nu} \gamma^0$
$D^\dagger(\Lambda) = \gamma^0 D^{-1}(\Lambda) \gamma^0$
$D(\Lambda) \chi$


$\chi^\dagger \chi$
$J_{\mu\nu}^\dagger = \gamma^0 J_{\mu\nu} \gamma^0$
$\chi^\dagger D^\dagger(\Lambda) D(\Lambda) \chi$
$D^\dagger(\Lambda)$
$D(\Lambda) = \gamma^0 D(\Lambda)^{-1} \gamma^0$
$= \chi^\dagger \gamma^0 D^{-1}(\Lambda) \gamma^0 \gamma^0 D(\Lambda) \chi$


$\gamma^0 \chi$
$J_{\mu\nu}^{\dagger} = \gamma^0 J_{\mu\nu} \gamma^0$
$D^{\dagger}(\Lambda) D(\Lambda) \chi$
$D^{-1}(\Lambda)$
$D^{\dagger}(\Lambda) = \gamma^0 D^{-1}(\Lambda) \gamma^0$
$\gamma^0 D^{-1}(\Lambda) \gamma^0$
$D(\Lambda) \chi$


$\bar{\psi}\psi$
$\bar{\psi} \equiv \psi^\dagger \gamma^0$
$\bar{\psi}\gamma^\mu\psi$
if $D^{-1}(\Lambda)\gamma^\mu D(\Lambda) = \Lambda^\mu_\nu \gamma$


$\mathcal{L} = \bar{\psi}(i\gamma - m)\psi$
$(i\gamma^\mu_{ab}\partial_\mu - m\mathbb{1}_{ab})\psi_b(x) = 0$
$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0$
$\rightarrow \psi^\dagger D^\dagger(\Lambda) D(\Lambda)\psi$
$= \psi^\dagger \gamma^0 D^{-1}(\Lambda)\gamma^0$