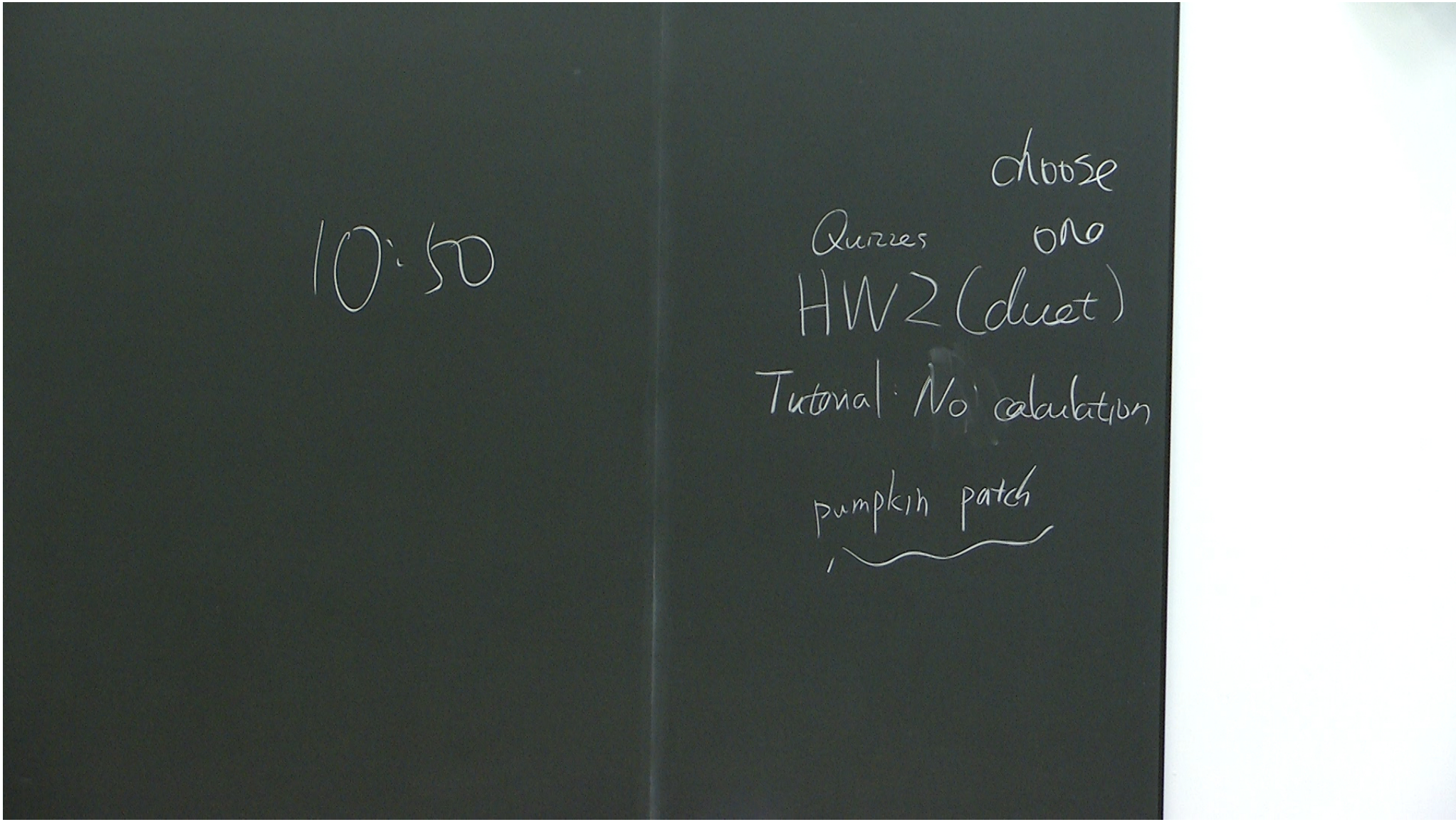
Title: Quantum Field Theory I - Lecture 221024

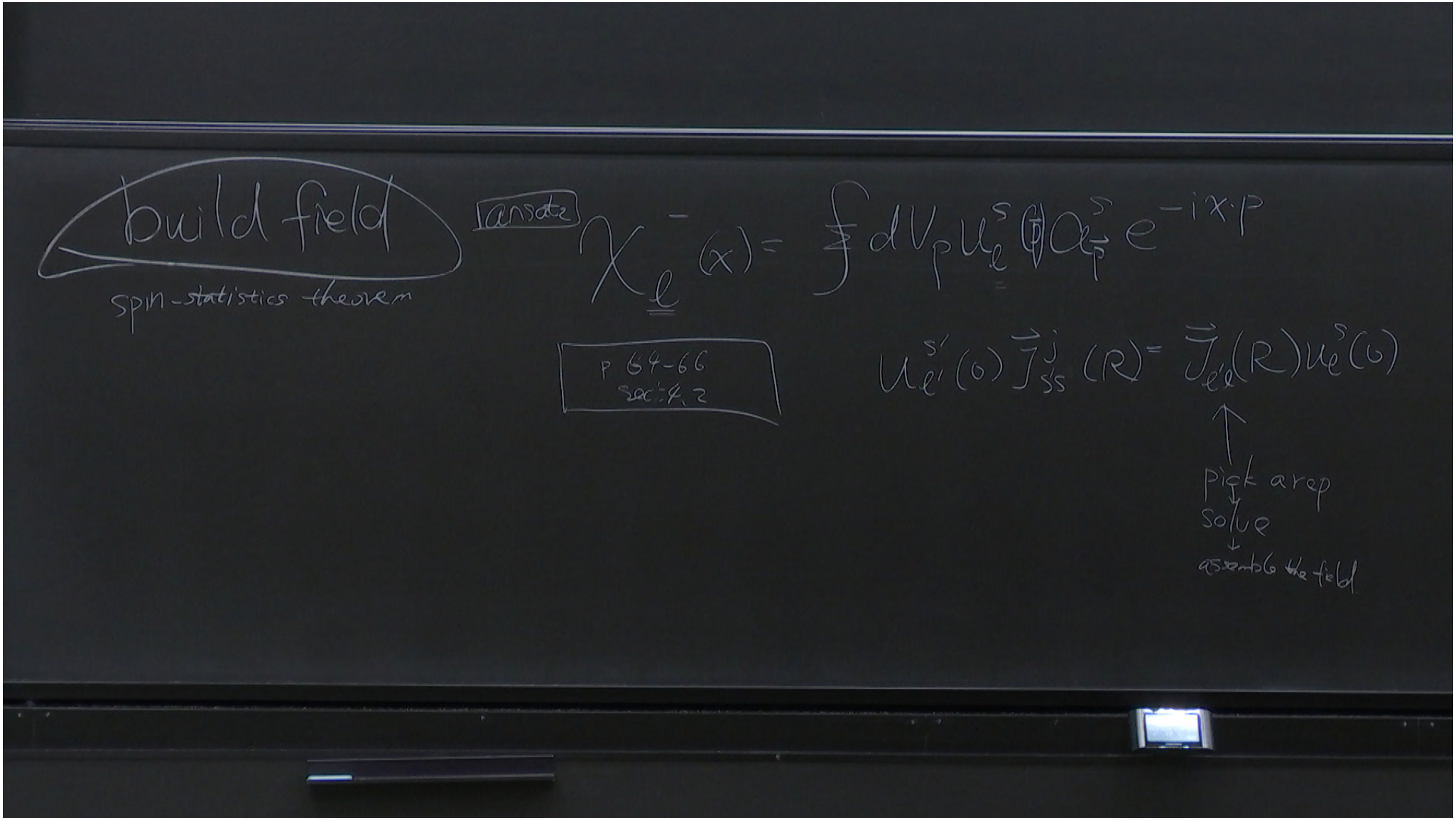
Speakers: Gang Xu

Collection: Quantum Field Theory I (2022/2023)

Date: October 24, 2022 - 10:45 AM

URL: https://pirsa.org/22100054


10:50
choose one
Quizzes
HW2 (due)
Tutorial: No calculation
pumpkin patch


build field
spin-statistics theorem
ansatz
$\chi_{\underline{\ell}}^{-}(x) = \int dV_p \, u_{\ell}^{s}(\vec{p}) \, a_{\vec{p}}^{s} \, e^{-ix\cdot p}$
p 64-66
sec: 4.2
$u_{\ell'}^{s'}(0) \, \vec{J}_{ss}^{j}(R) = \vec{J}_{\ell\ell}(R) \, u_{\ell}^{s}(0)$
pick a rep
solve
assemble the field

Ansatz

$$\chi_{\underline{\ell}}^{-}(x) = \int dV_p \, u_{\ell}^{s}(p) \, O_{\ell}^{s} \, e^{-ix\cdot p}$$

p 64-66
sec: 4.2

$$u_{\ell'}^{s'}(0) \, \vec{J}_{ss'}^{j}(R) = \vec{J}_{\ell'\ell}(R) \, u_{\ell}^{s}(0)$$

$SU(2)$

pick a rep
↓
solve
↓
assemble the field

$$\bar{\ell}(x) = \int dV_p \, u^{s}_{\ell}(p) \, a^{s\dagger}_{\vec{p}} \, e^{-ix\cdot p}$$

$$u^{s'}_{\ell'}(0) \, \vec{J}^{j}_{ss'}(R) = \vec{J}_{\ell'\ell}(R) \, u^{s}_{\ell}(0)$$

$su(2)$

pick a rep

solve

assemble the field

4-66
4.2

1st example. scalar

trivial → spin 0

$\Rightarrow [\phi(x), \phi(y)]$ assumption

$D(\lambda) = = = 1 \rightarrow J = 0 \Rightarrow j = 0$

$u^s_\ell(0) \xrightarrow[s=0]{\ell=1} u(0) \rightarrow 1$

$\phi^-(x) = \int dV_p \; a_{\vec{p}} e$

1st example: scalar

trivial → spin 0

$[\phi(x), \phi(y)]$ assumption

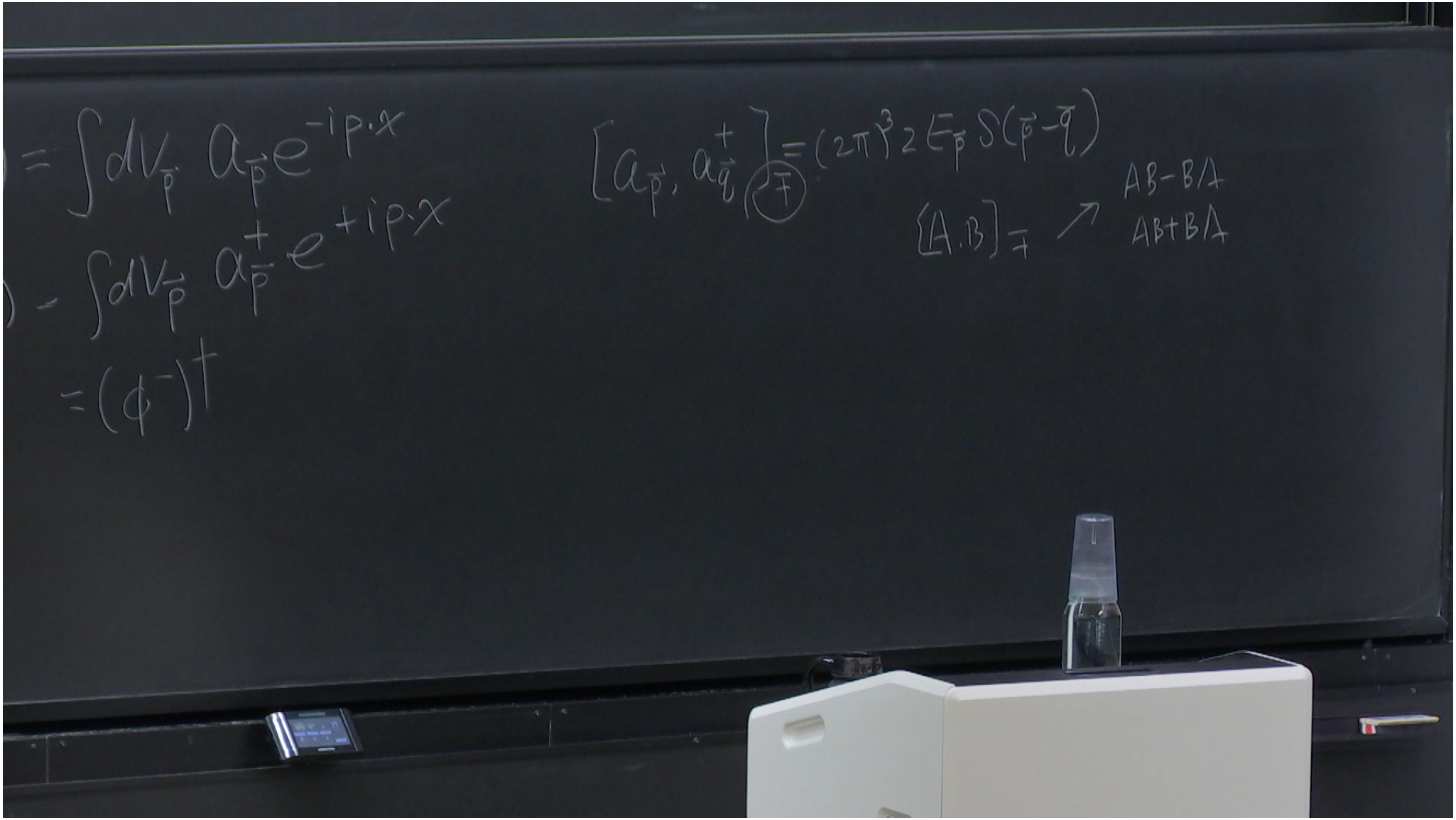
$D(\Lambda) = = = 1 \rightarrow J = 0 \Rightarrow j = 0$

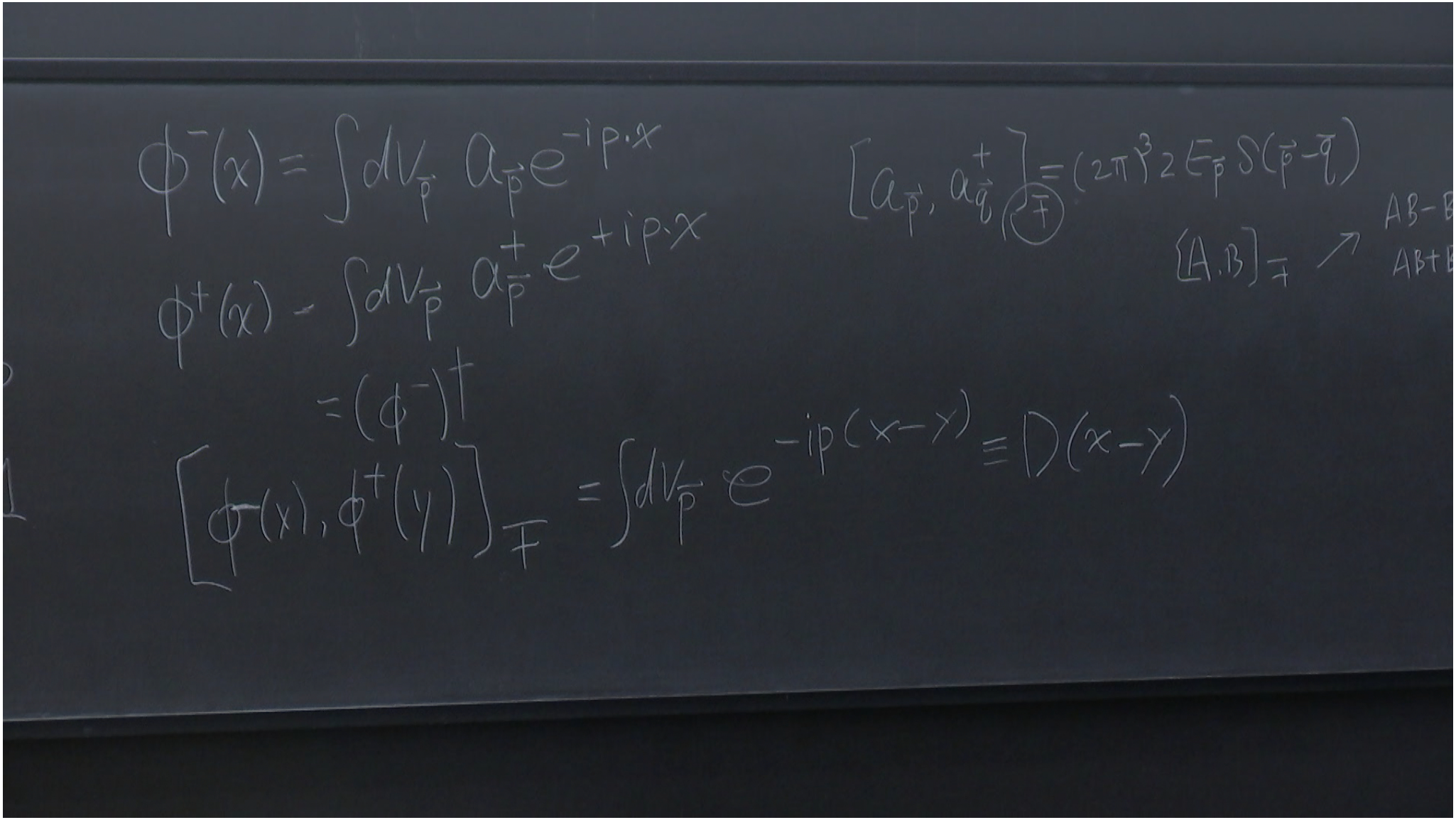
$U^S_\ell(0) \xrightarrow[S=0]{\ell=1} U(0) \rightarrow 1$

$$\phi^-(x) = \int dV_{\vec{p}}\, a_{\vec{p}} e^{-ip\cdot x}$$

$$\phi^+(x) = \int dV_{\vec{p}}\, a^+_{\vec{p}} e^{+ip\cdot x}$$

$$= (\phi^-)^\dagger$$


$= \int dV_{\vec{p}}\, a_{\vec{p}} e^{-ip\cdot x}$
$- \int dV_{\vec{p}}\, a^{+}_{\vec{p}} e^{+ip\cdot x}$
$= (\phi^{-})^{\dagger}$
$[a_{\vec{p}}, a^{+}_{\vec{q}}]_{\mp} = (2\pi)^3 2E_{\vec{p}}\, \delta(\vec{p}-\vec{q})$
$[A,B]_{\mp}$
$AB - BA$
$AB + BA$


$\phi^-(x) = \int dV_{\vec{p}}\, a_{\vec{p}} e^{-ip\cdot x}$
$\phi^+(x) = \int dV_{\vec{p}}\, a^+_{\vec{p}} e^{+ip\cdot x}$
$= (\phi^-)^\dagger$
$[a_{\vec{p}}, a^+_{\vec{q}}]_{\mp} = (2\pi)^3 2E_{\vec{p}}\, \delta(\vec{p}-\vec{q})$
$[A,B]_{\mp}$
$[\phi^-(x), \phi^+(y)]_{\mp} = \int dV_{\vec{p}}\, e^{-ip(x-y)} \equiv D(x-y)$

$$\phi^-(x) = \int dV_{\vec{p}}\, a_{\vec{p}}\, e^{-ip\cdot x}$$

$$\phi^+(x) = \int dV_{\vec{p}}\, a^\dagger_{\vec{p}}\, e^{+ip\cdot x}$$

$$= (\phi^-)^\dagger$$

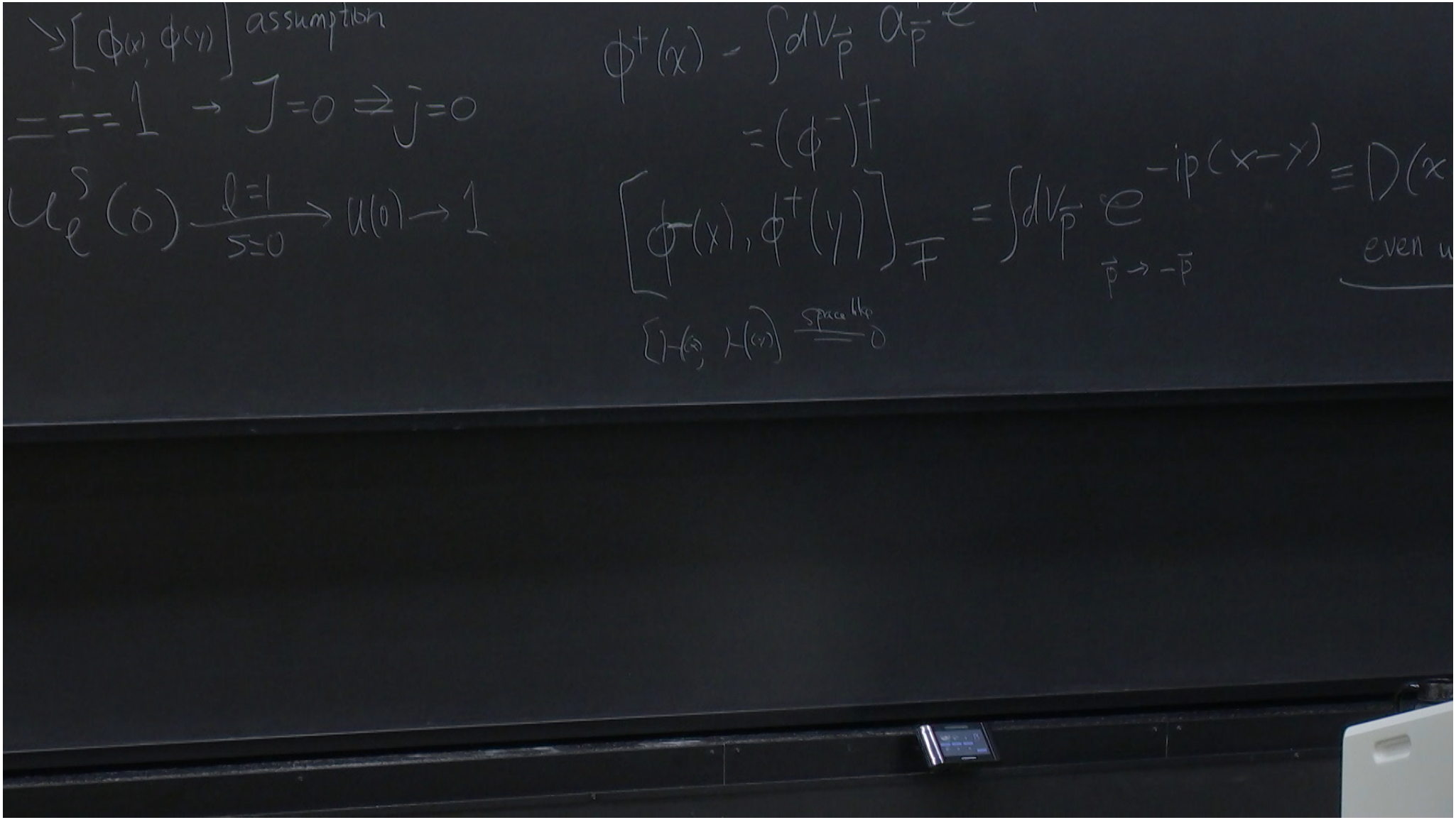
$$\left[a_{\vec{p}}, a^\dagger_{\vec{q}}\right]_{\mp} = (2\pi)^3 2E_{\vec{p}}\,\delta(\vec{p}-\vec{q})$$

$[A,B]_{\mp}$ → $AB - BA$, $AB + BA$

$$\left[\phi(x), \phi^\dagger(y)\right]_{\mp} = \int dV_{\vec{p}}\, e^{-ip(x-y)} \equiv D(x-y)$$

$\vec{p} \to -\vec{p}$

even when spacelike


assumption
$\phi^+(x) - \int dV_{\vec{p}}\, a_{\vec{p}}$
$= (\phi^-)^\dagger$
$\vec{p} \to -\vec{p}$
even

$$\phi(x) = k\phi^{-}(x) + \lambda\phi^{+}(x)$$

$$\left[\phi(x), \phi(y)\right]_{\mp}$$

$$= \left[k\phi^{-}(x) + \lambda\phi^{+}(x),\ k\phi^{-}(y) + \lambda\phi^{+}(y)\right]_{\mp}$$

$$=$$

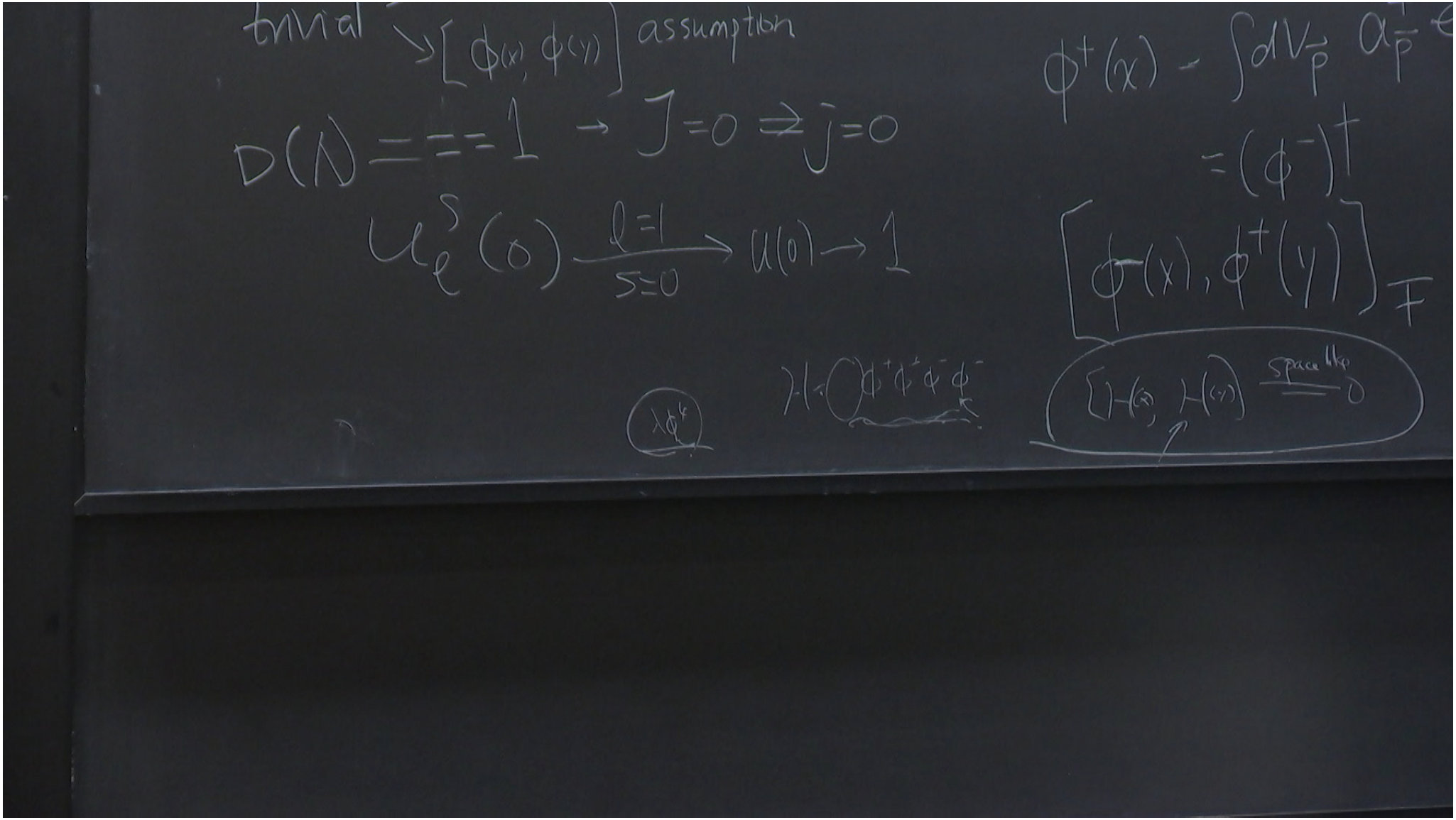
$$\phi(x) = k\phi^-(x) + \lambda\phi^+(x)$$

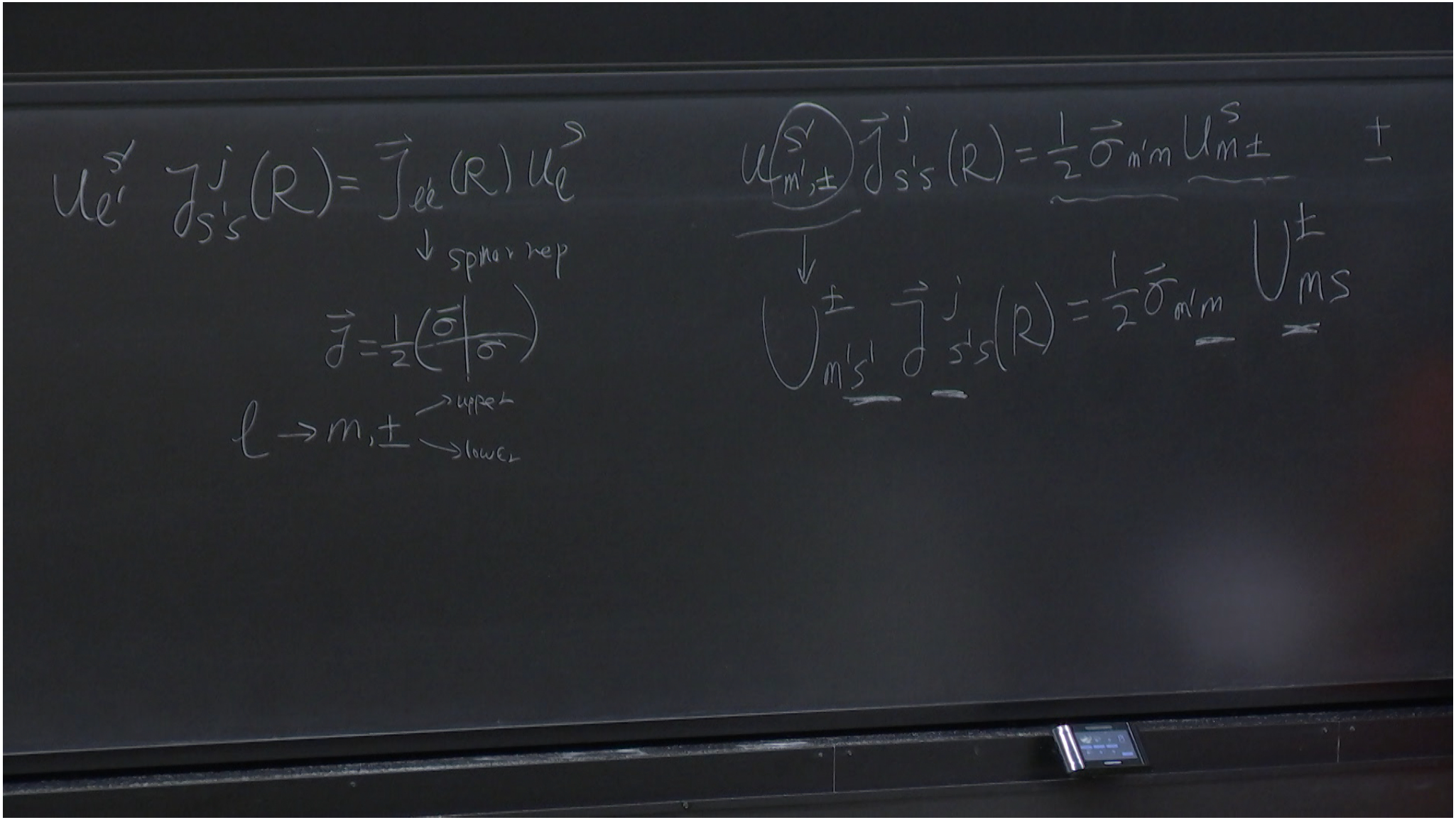
$$[\phi(x), \phi(y)]_\mp$$

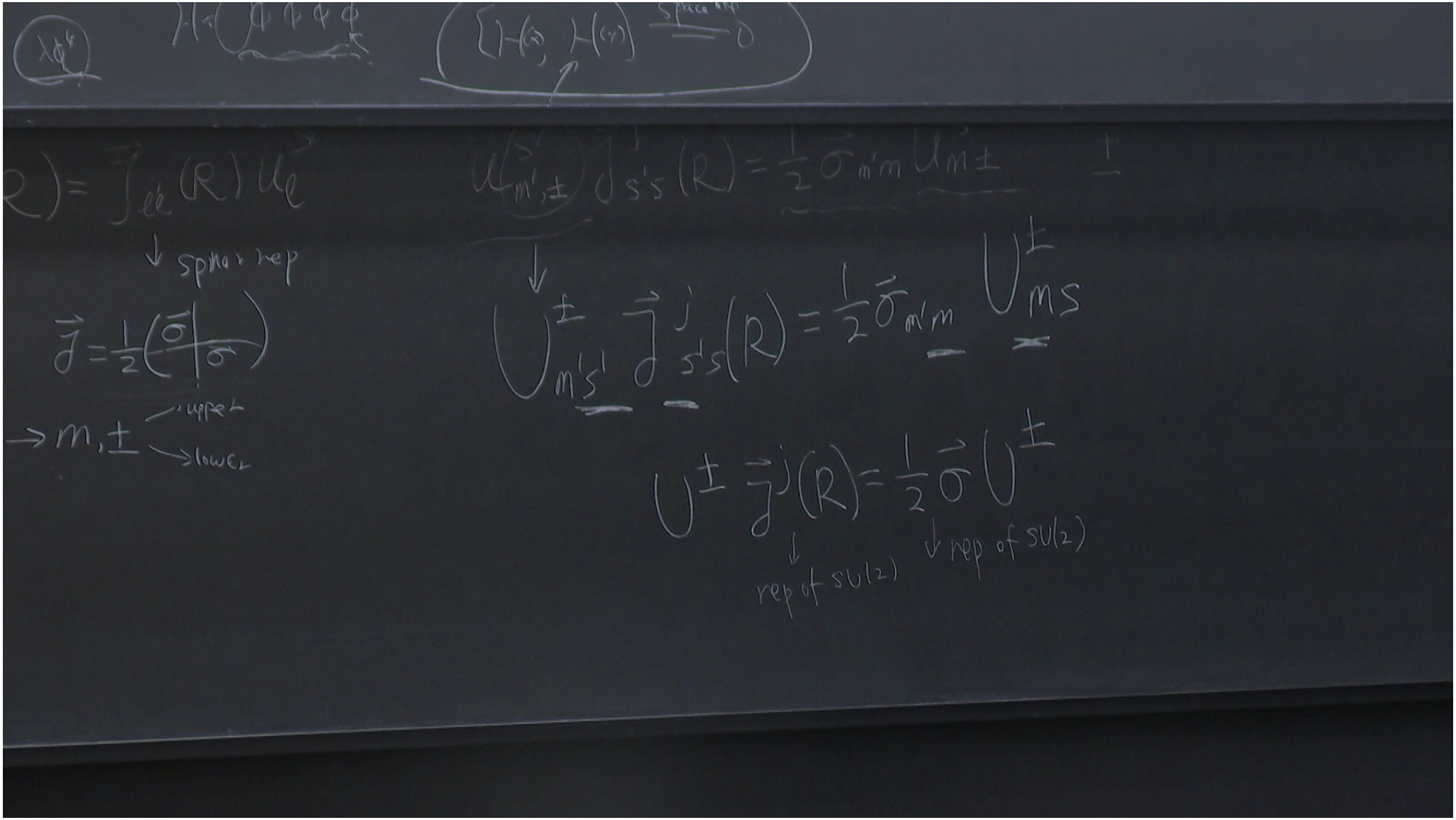
$$= [k\phi^-(x) + \lambda\phi^+(x),\ k\phi^-(y) + \lambda\phi^+(y)]_\mp$$

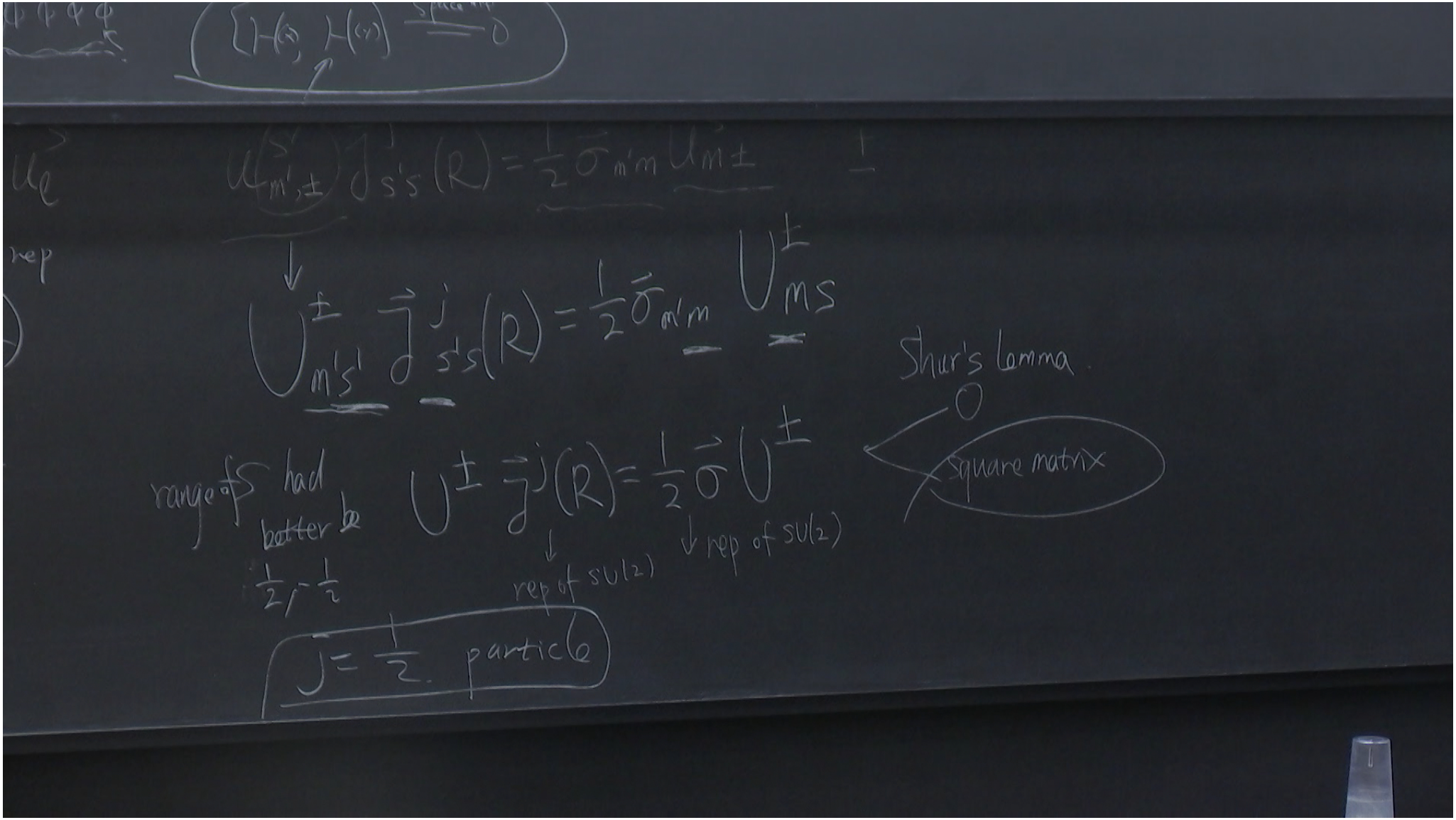
$$= k\lambda\underbrace{[\phi^-(x), \phi^+(y)]_\mp}_{k\lambda D(x-y)} + k\lambda[\phi^+(x), \phi^-(y)]_\mp$$

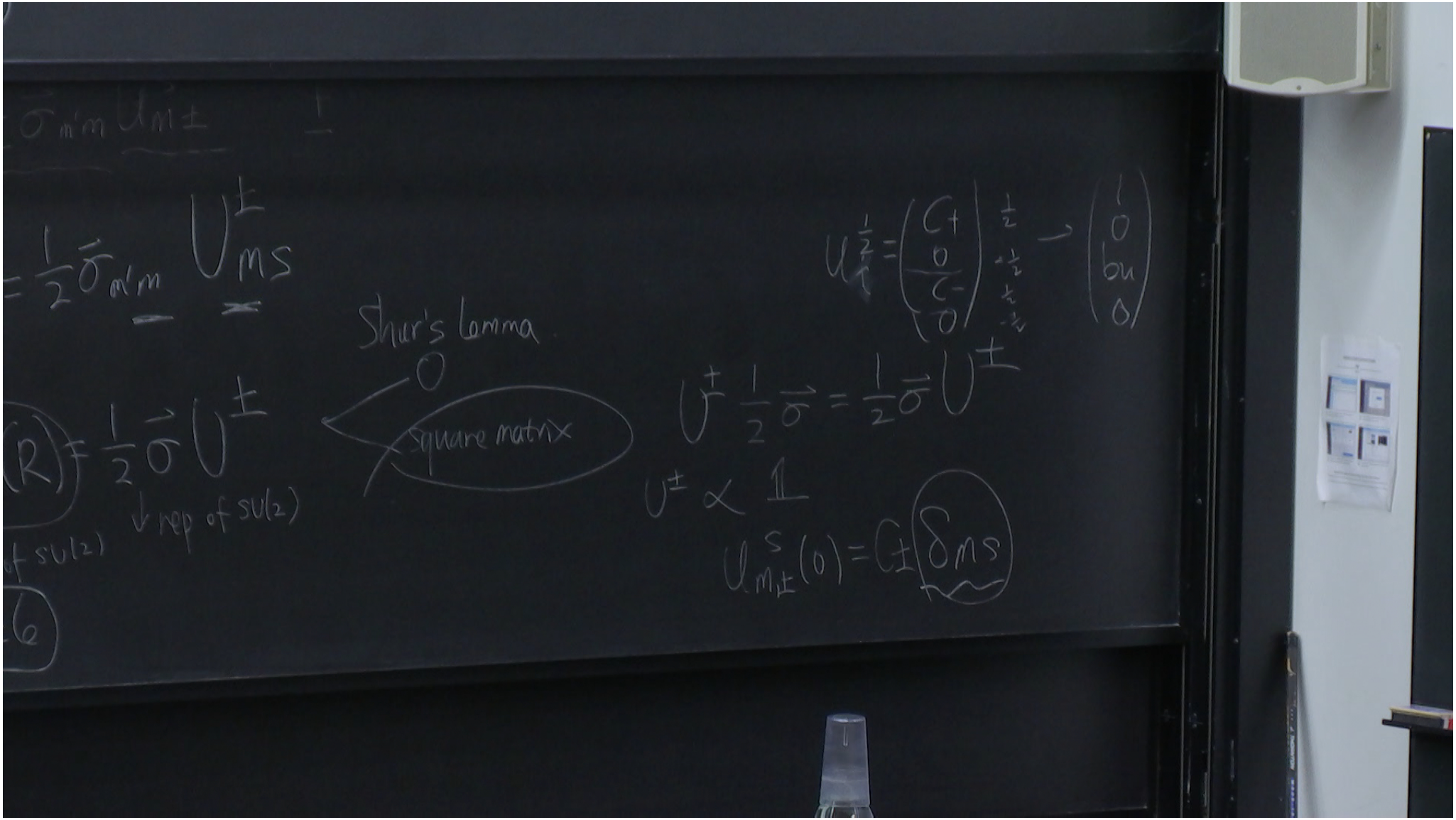
$$\mp k\lambda D(y-x)$$

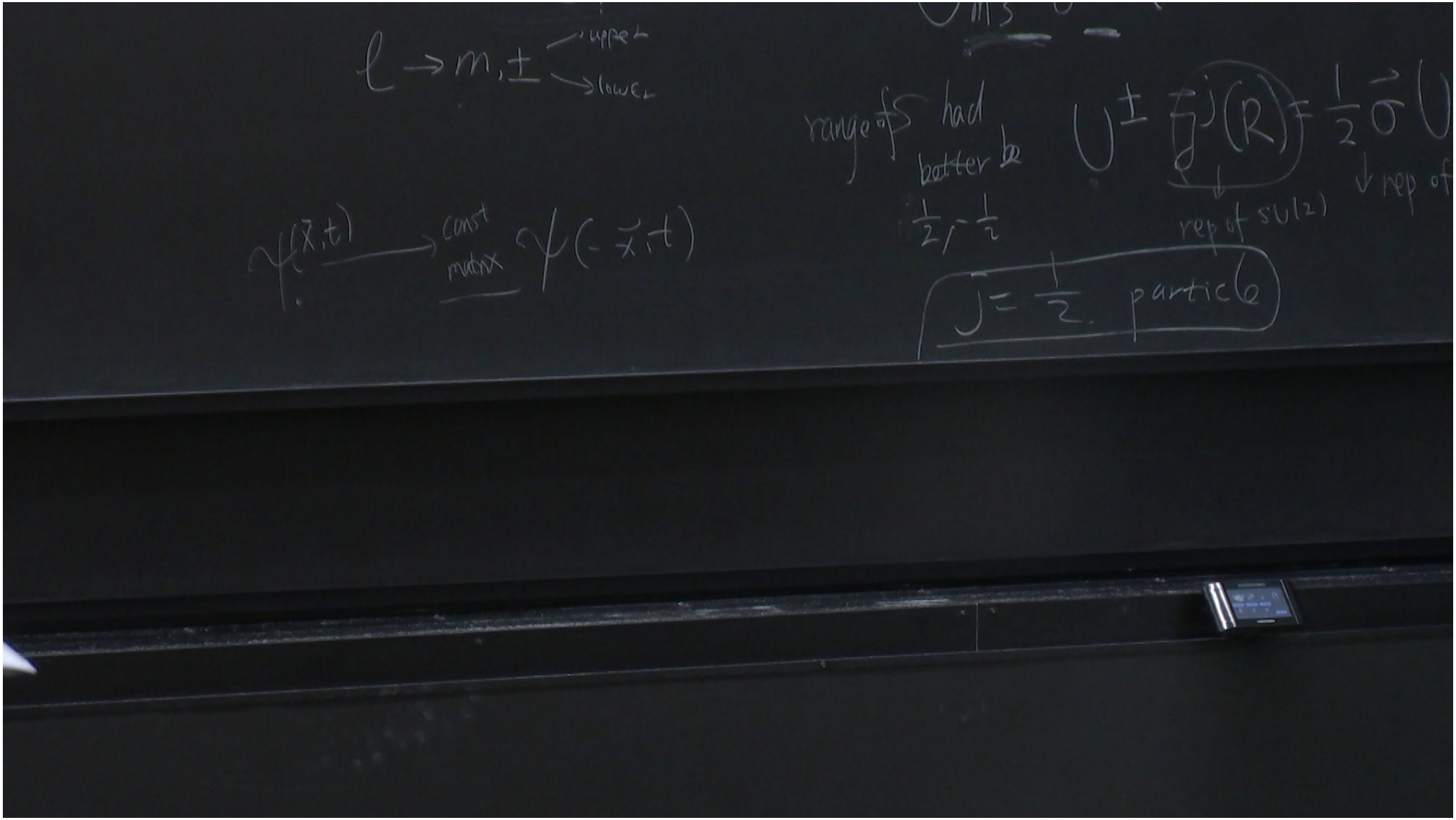

trivial
$[\phi(x), \phi(y)]$ assumption
$D(\lambda) = = = 1 \rightarrow J = 0 \Rightarrow \tilde{J} = 0$
$u_\ell^s(0) \xrightarrow[s=0]{\ell=1} u(0) \rightarrow 1$
$\phi^+(x) - \int d\tilde{V}_p \, a_p^+ e$
$= (\phi^-)^\dagger$
$[\phi^-(x), \phi^+(y)]_F$
$\lambda(\phi^+\phi^+\phi^-\phi^-)$
$\lambda\phi^4$
$[\lambda(x), \lambda(y)] \overset{\text{space like}}{=} 0$

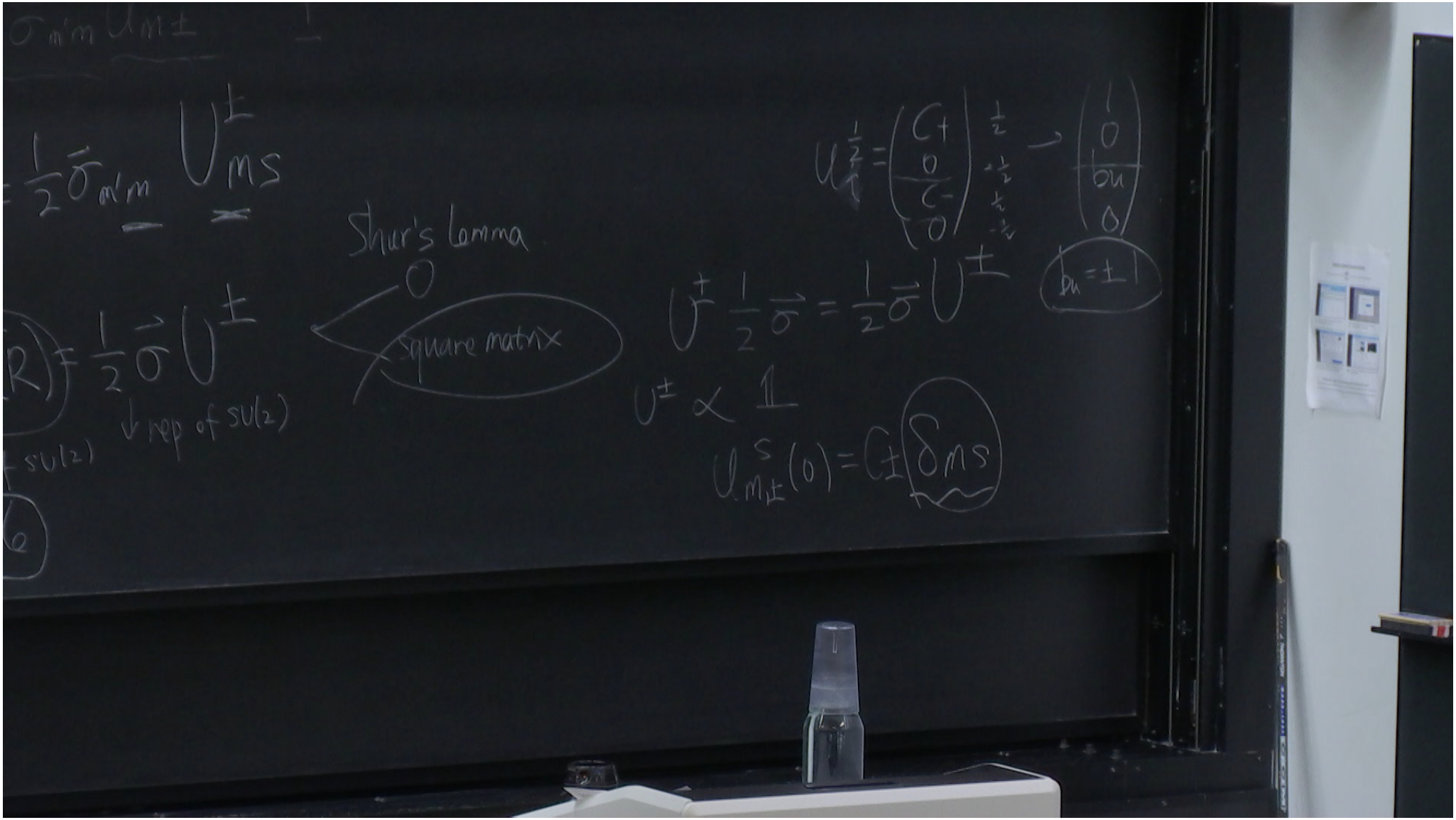

$U^{s'}_{\ell i}\ \mathcal{J}^{j}_{s's}(R) = \vec{\mathcal{J}}_{\ell\ell}(R)\, U^{s}_{\ell}$
↓ spinor rep
$\vec{J} = \frac{1}{2}\begin{pmatrix}\vec{\sigma} & \\ & \vec{\sigma}\end{pmatrix}$
$\ell \to m, \pm$ → upper → lower
$U^{s'}_{m',\pm}\ \vec{\mathcal{J}}^{j}_{s's}(R) = \frac{1}{2}\vec{\sigma}_{m'm}\, U^{s}_{m\pm}$ $\pm$
$U^{\pm}_{m's'}\ \vec{J}^{j}_{s's}(R) = \frac{1}{2}\vec{\sigma}_{m'm}\, U^{\pm}_{ms}$

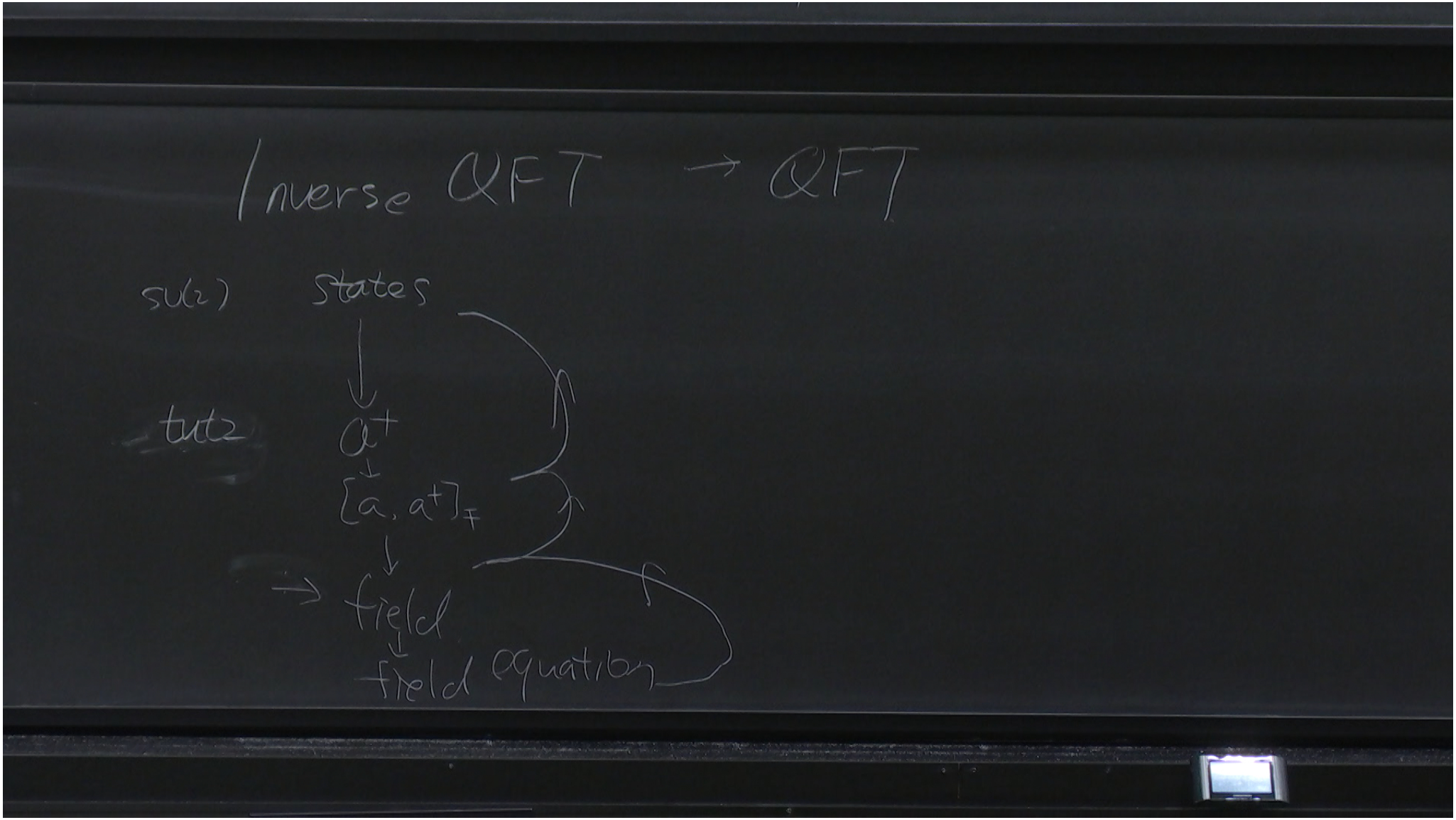

spin rep
upper
lower
rep of SU(2)
rep of SU(2)

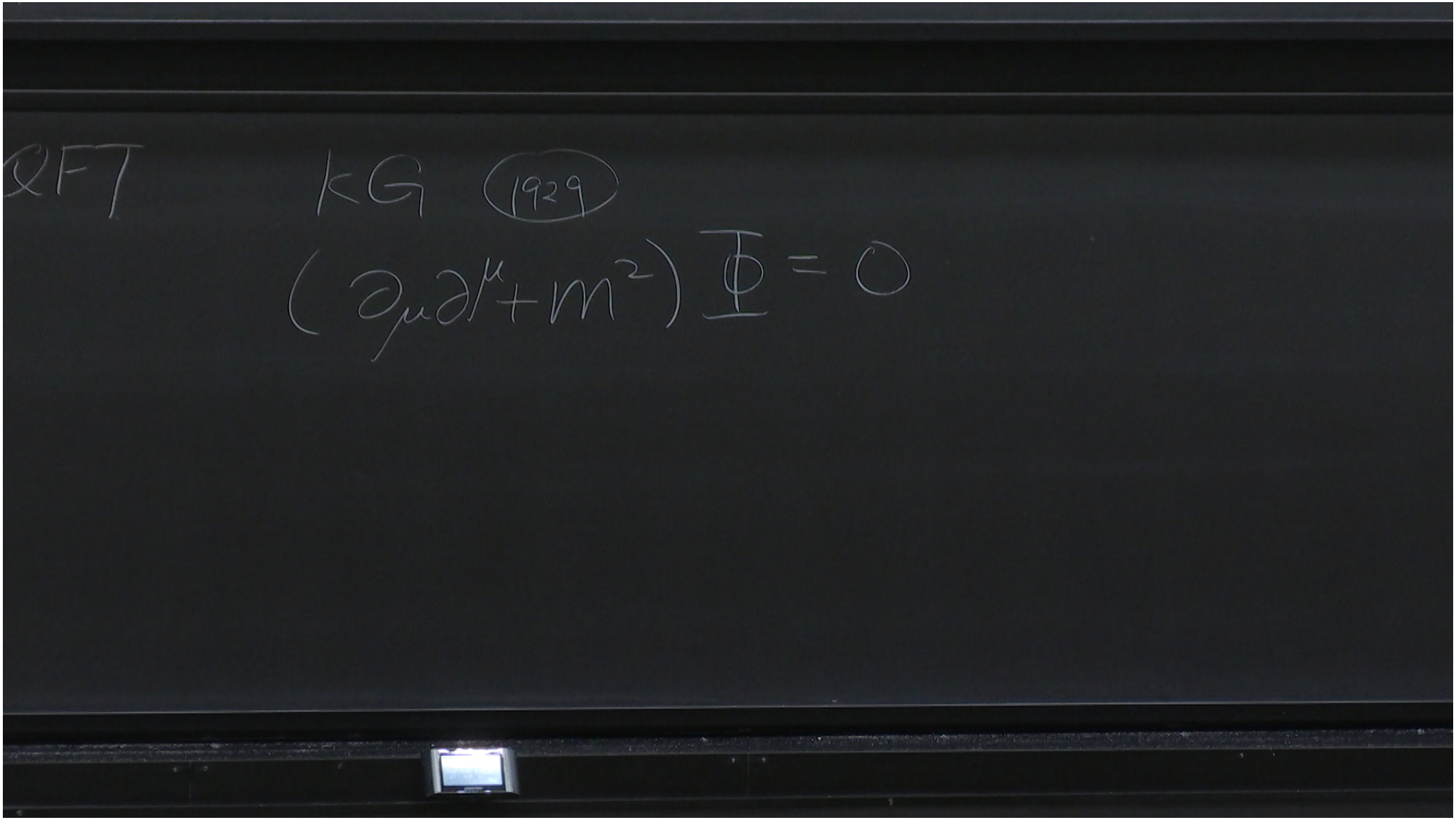

Shur's lemma.
square matrix
rep of SU(2)
rep of SU(2)
J = 1/2 particle


Shur's lemma
square matrix
rep of SU(2)


l → m, ±
upper
lower
range of s had better be
1/2, -1/2
rep of SU(2)
J = 1/2 particle


Schur's lemma
square matrix
rep of SU(2)
SU(2)


Inverse QFT → QFT
SU(2)
States
tut2
a†
[a, a†]_T
→ field
field equation


QFT
KG
1929
$(\partial_\mu \partial^\mu + m^2)\,\Phi = 0$

KG (1929)

$(\partial_\mu \partial^\mu + m^2)\,\Phi = 0$

$a^2 + b^2$

$(-ia - b)(ia - b)$

$a = \partial_\mu \quad b = m$

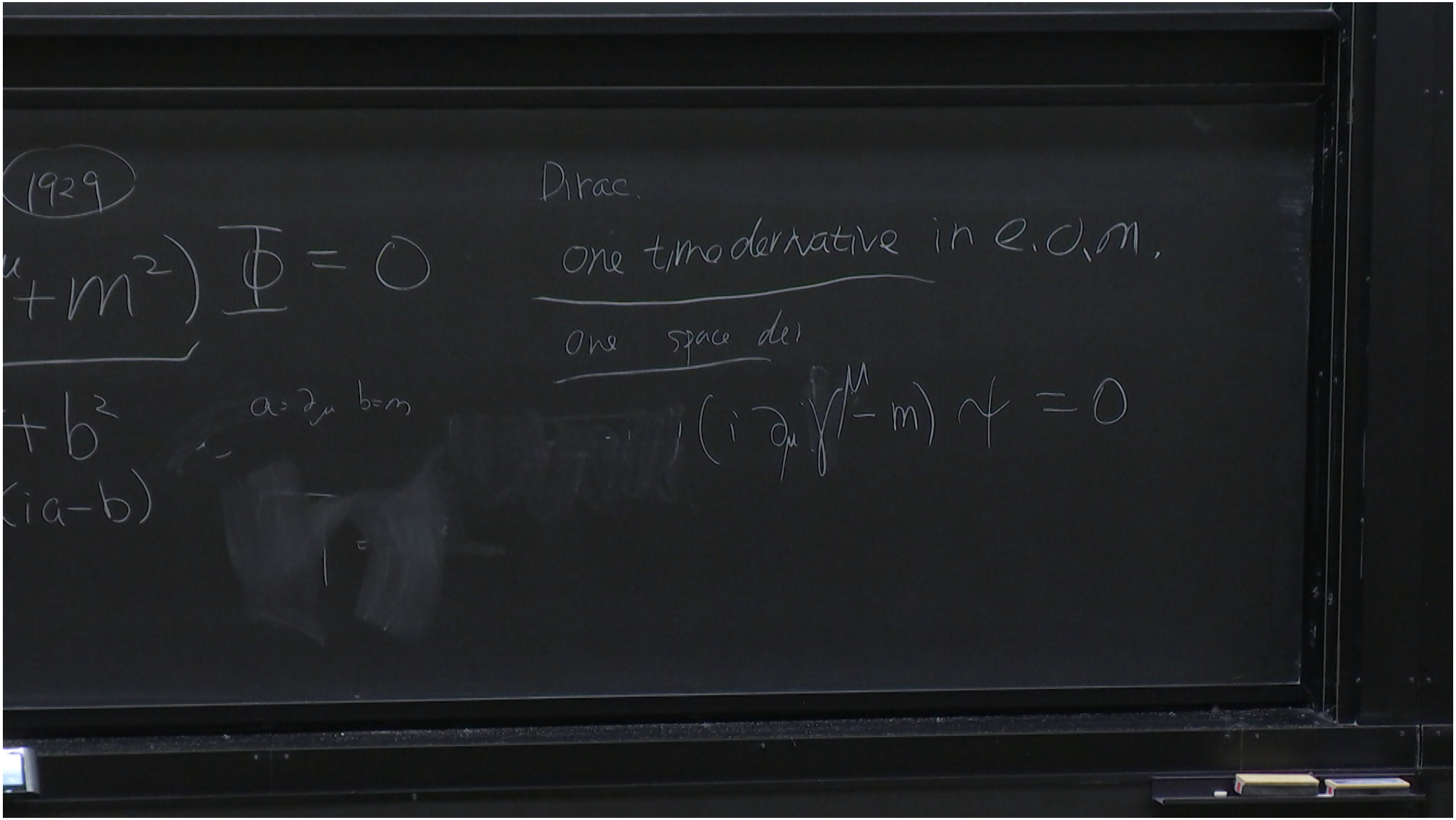
$\sqrt{|a|^2} = \sqrt{aa^*}$

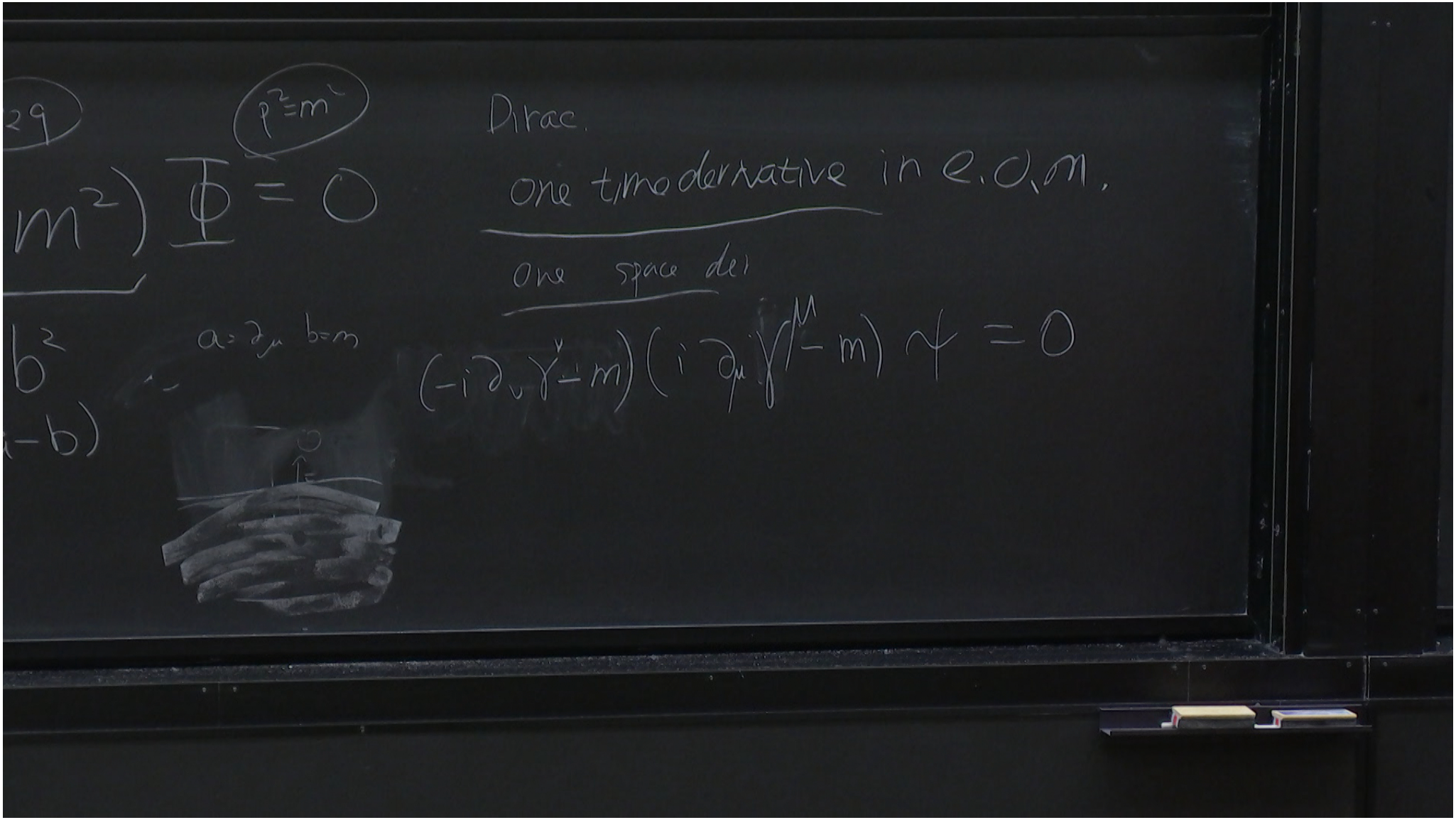
Dirac.

one time derivative in e.o.m.

one space del

$(i\partial_\mu - m)\,\psi$


1929
Dirac.
one t/m derivative in e.o.m.
one space der
$+m^2)\,\Phi = 0$
$+b^2$
$a=\partial_\mu \quad b=m$
$(ia-b)$
$(i\partial_\mu \gamma^\mu - m)\,\psi = 0$


29
$p^2=m^2$
$m^2)\,\Phi = 0$
Dirac.
one time derivative in e.o.m.
one space del
$a=\partial_\mu \quad b=m$
b^2
$-b)$
$(-i\partial_\nu\gamma^\nu - m)(i\partial_\mu\gamma^\mu - m)\,\psi = 0$

$p^2 = m^2$

$\Phi = 0$

$a = \partial_\mu \quad b = m$

Dirac.

one time derivative in e.o.m.

one space del

$$(-i\partial_\nu \gamma^\nu - m)(i\partial_\mu \gamma^\mu - m)\psi = 0$$

$$(\partial_\nu \gamma^\nu \partial_\mu \gamma^\mu + m^2)\psi = 0$$