Title: Quantum Field Theory I - Lecture 221017

Speakers: Gang Xu

Collection: Quantum Field Theory I (2022/2023)

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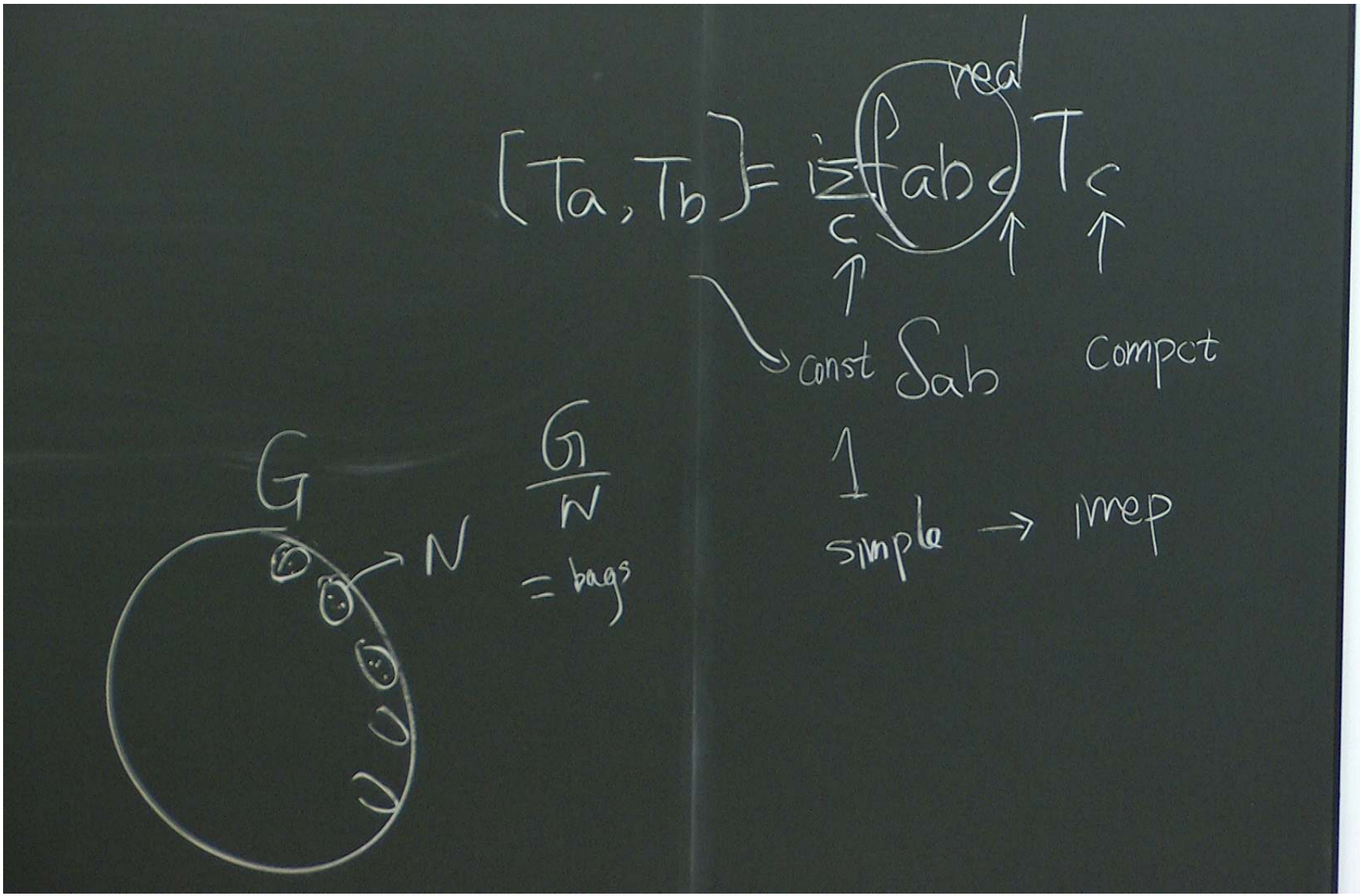
URL: https://pirsa.org/22100051


[Ta, Tb] = i Σ_c fabc Tc
real
const δab
compct
1
simple → irrep
G
N
G/N
= bags

generator

$$U = e^{i\epsilon_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu}J^{\mu\nu}}$$

$$i[J^{\mu\nu}, J^{\rho\sigma}] = \eta^{\nu\rho} J^{\mu\sigma} + 3 \text{ terms}$$

$$i[P^\mu, J^{\rho\sigma}] = \eta^{\mu\sigma} P^\sigma + 1 \text{ more}$$

$$[P^\mu, P^\nu] = 0$$

non-abelian

non-compact

not-direct product

$$T(\Lambda_1, a_1)\, T(\Lambda_2, a_2)$$

$$= T(\Lambda_1\Lambda_2, \Lambda_1 a_2 + a_1)$$

$= e^{i\epsilon_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu}J^{\mu\nu}}$

$[J^{\mu\nu}, J^{\rho\sigma}] = \eta^{\nu\rho} J^{\mu\sigma} + 3 \text{ terms}$

$[P^\mu, J^{\rho\sigma}] = \eta^{\mu\sigma} P^\sigma + 1 \text{ more}$

$[P^\mu, P^\nu] = 0$

non-abelian

non-compact

not-direct product

not-simple.

$g(N) = (N)g$

$T(\Lambda_1, a_1)\, T(\Lambda_2, a_2)$

$= T(\Lambda_1\Lambda_2, \Lambda_1 a_2 + a_1)$

$\mu P^\mu + \frac{i}{2} \omega_{\mu\nu} J^{\mu\nu}$

$\sigma] = \eta^{\nu\rho} J^{\mu\sigma}$ + 3 terms.

$J^{\mu\sigma}] = \eta^{\mu\sigma} P^\sigma$ + 1 more

$P^\mu, P^\nu] = 0$

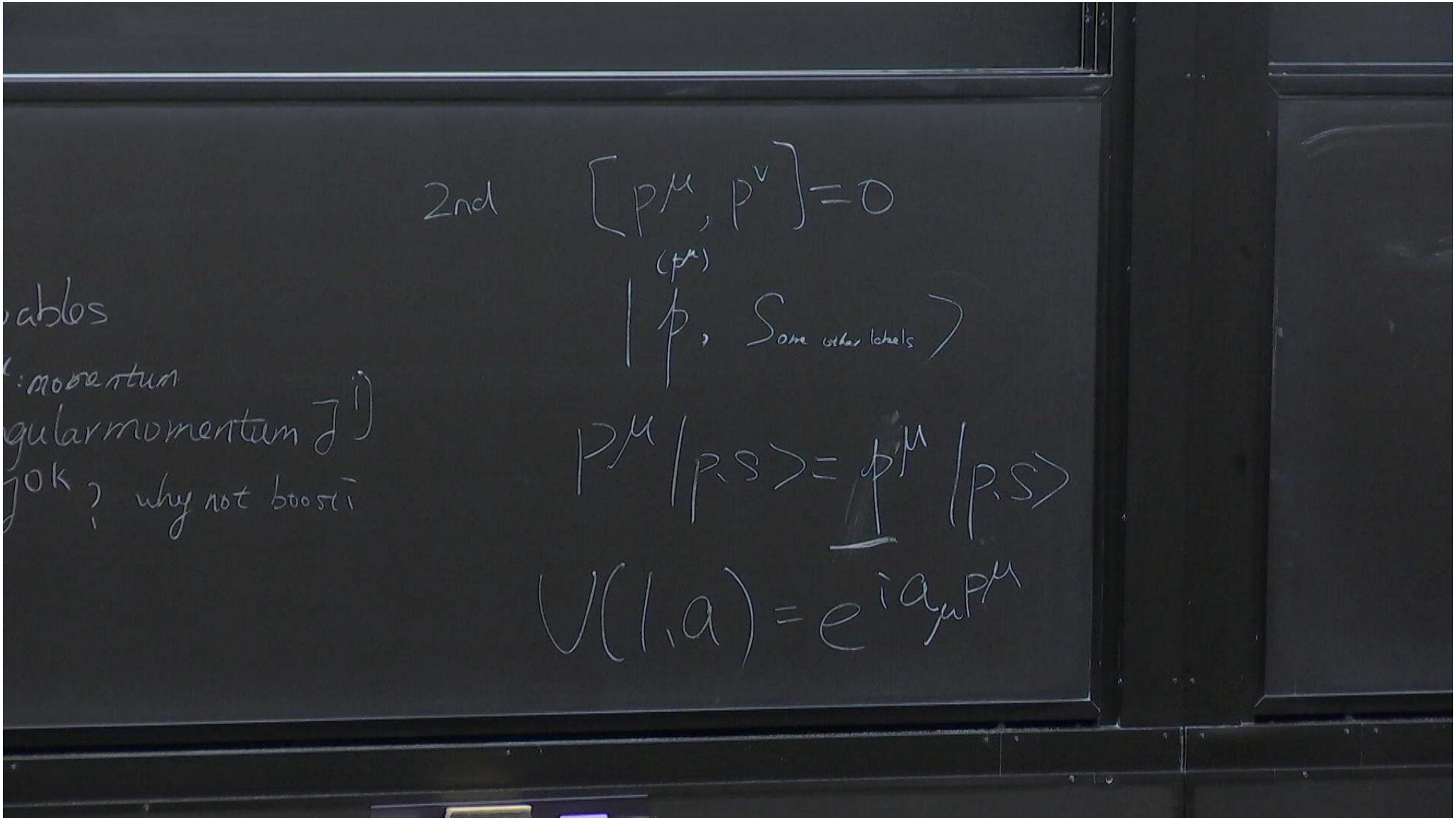
non-abelian
non-compact
not-direct product
not-simple.

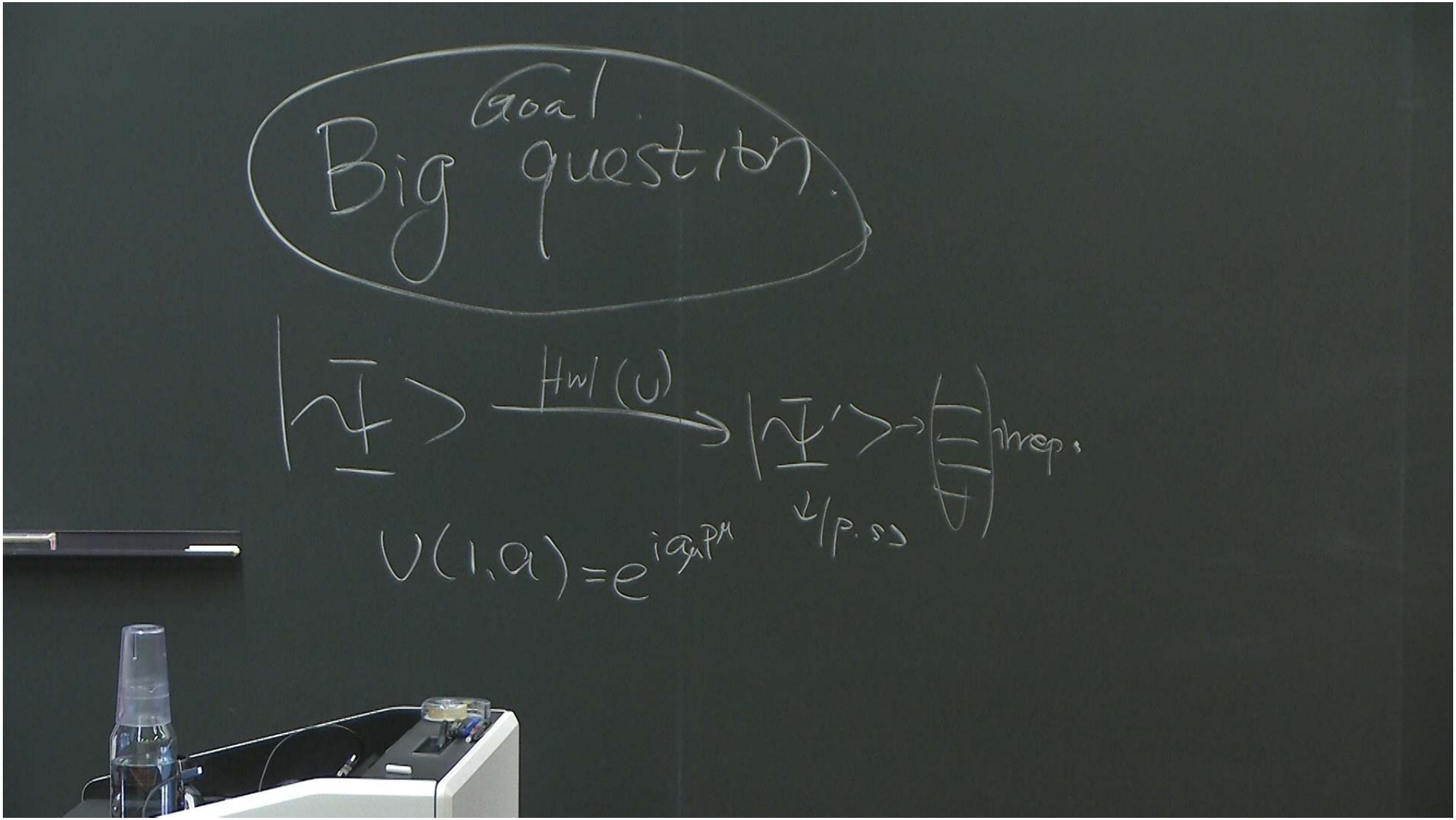
Oh crap!

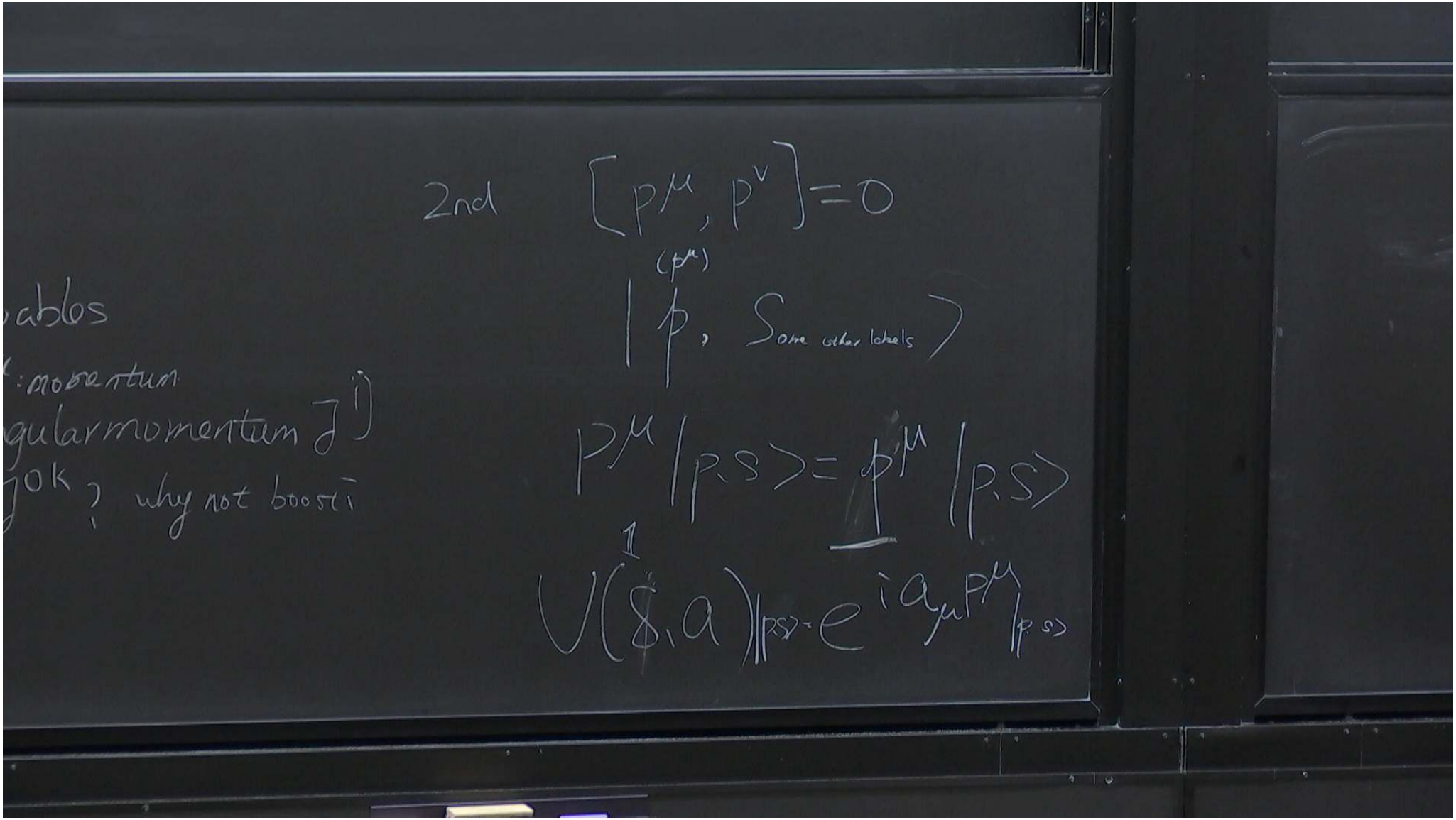
$g(\Lambda) = \Lambda g$

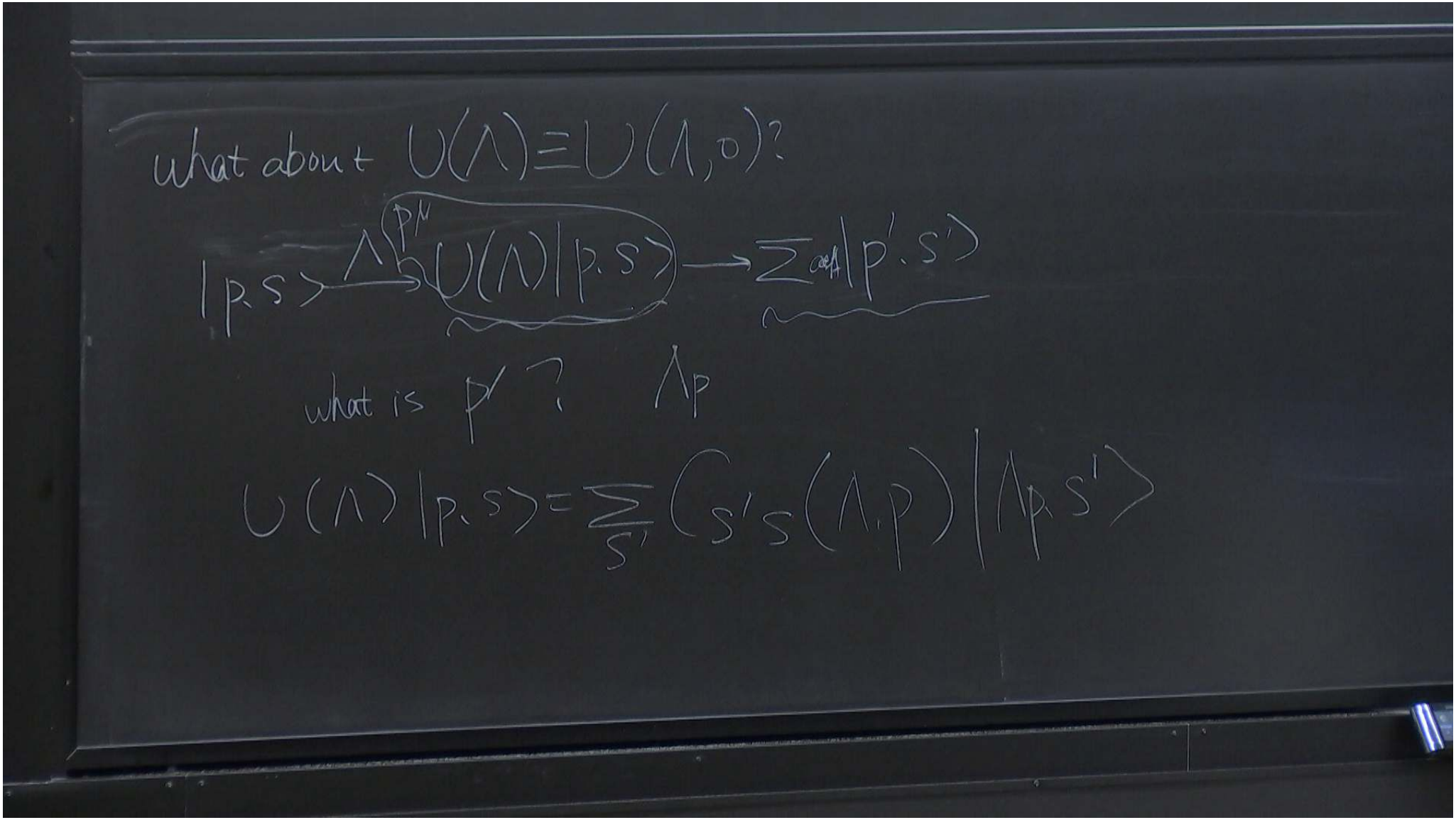
$T(\Lambda_1, a_1) T(\Lambda_2, a_2)$

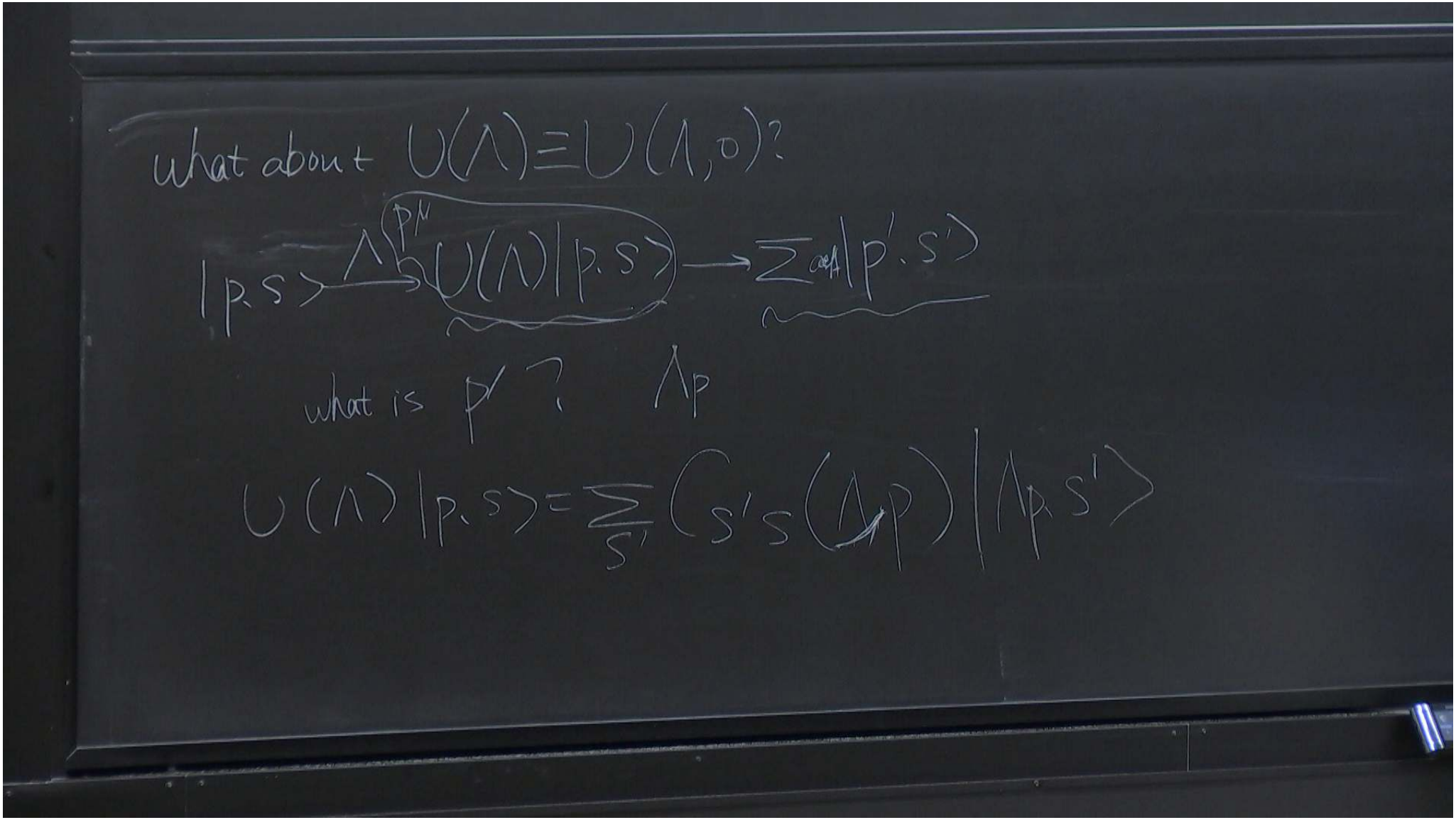
$= T(\Lambda_1 \Lambda_2, \Lambda_1 a_2 + a_1)$


2nd $[P^\mu, P^\nu] = 0$
$|p^\mu, \text{some other labels}\rangle$
$P^\mu |p, s\rangle = p^\mu |p, s\rangle$
$U(1, a) = e^{i a_\mu P^\mu}$
ables
: momentum
gular momentum J^i)
J^{0k} ; why not boosts


Goal
Big question.
|Ψ> Hw/(U) |Ψ'> → irrep.
|p.s>
U(1,a) = e^{i a_μ p^μ}


2nd $[P^\mu, P^\nu]=0$
$|p^\mu, \text{some other labels}\rangle$
$P^\mu|p,s\rangle = p^\mu|p,s\rangle$
$U(\$,a)|p,s\rangle = e^{ia_\mu P^\mu}|p,s\rangle$
ables
momentum
angular momentum J^{ij}
ok? why not boosts


what about $U(\Lambda) \equiv U(\Lambda, 0)$?
$|p,s\rangle \xrightarrow{\Lambda^{p^\mu}} U(\Lambda)|p,s\rangle \longrightarrow \sum_{\alpha\beta} |p',s'\rangle$
what is p'? Λp
$U(\Lambda)|p,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,|\Lambda p, s'\rangle$


what about $U(\Lambda) \equiv U(\Lambda, 0)$?
$|p,s\rangle \xrightarrow{\Lambda^{p'}_{p}} U(\Lambda)|p,s\rangle \longrightarrow \sum_{\alpha\beta} |p',s'\rangle$
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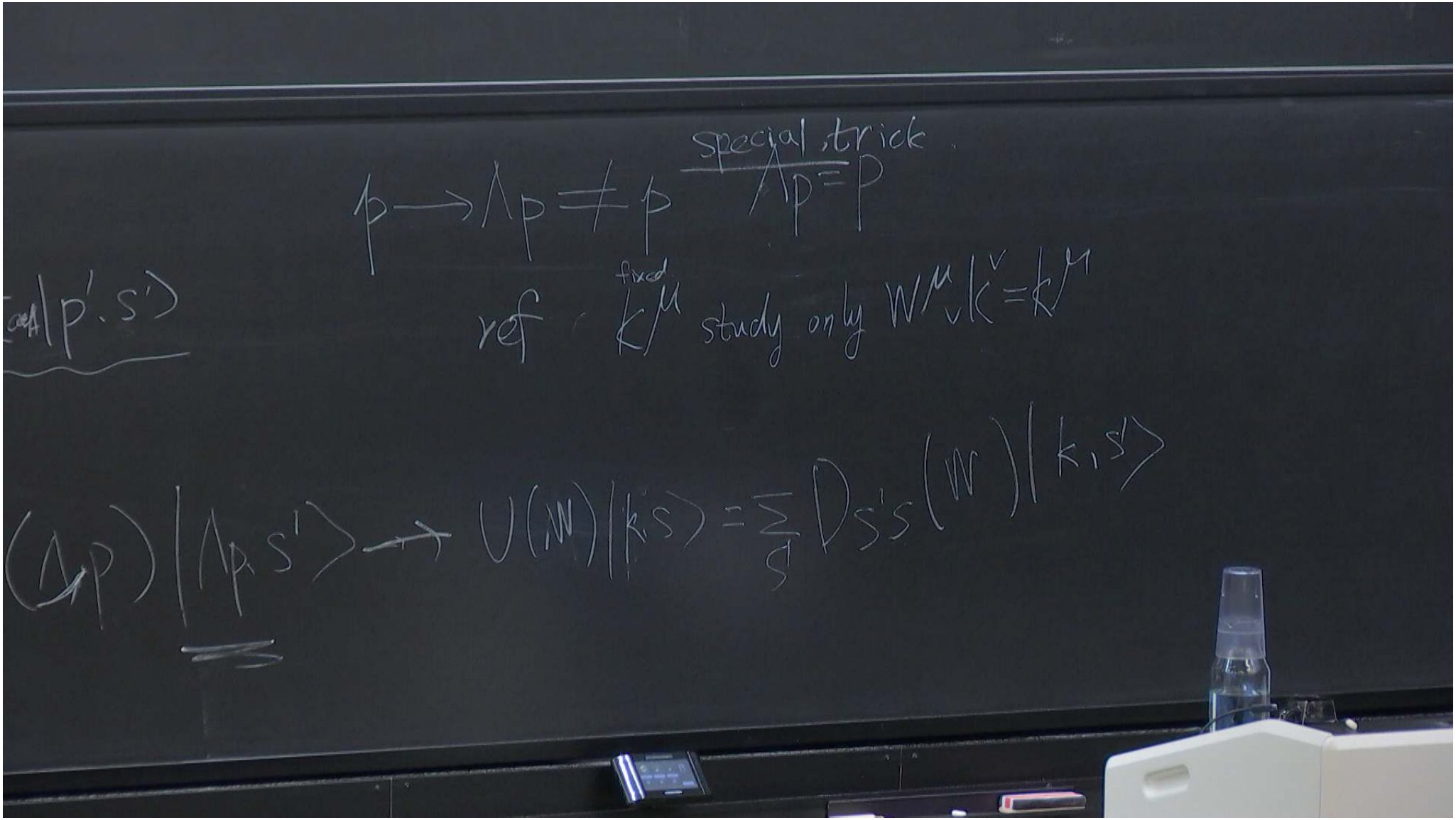
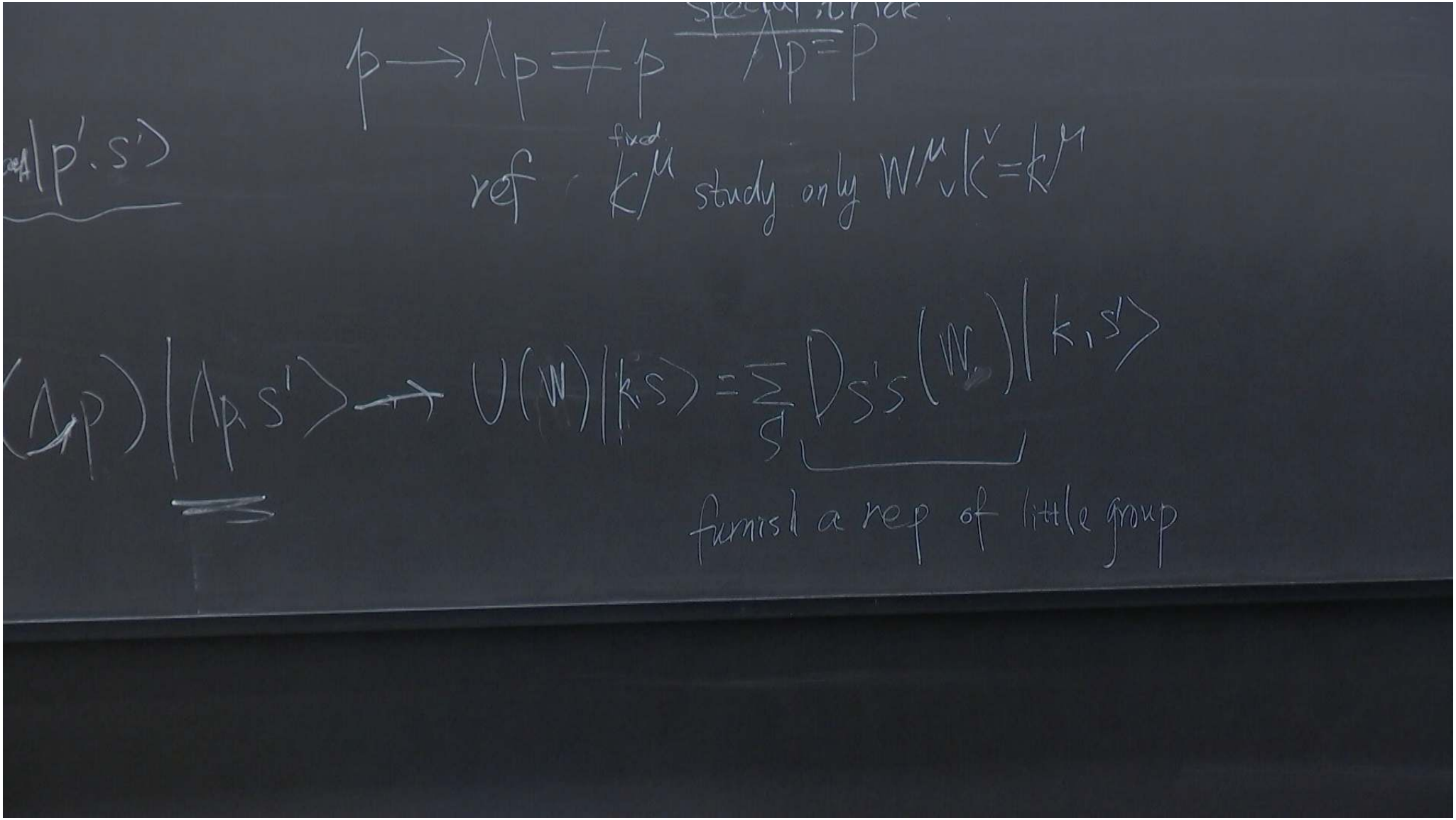
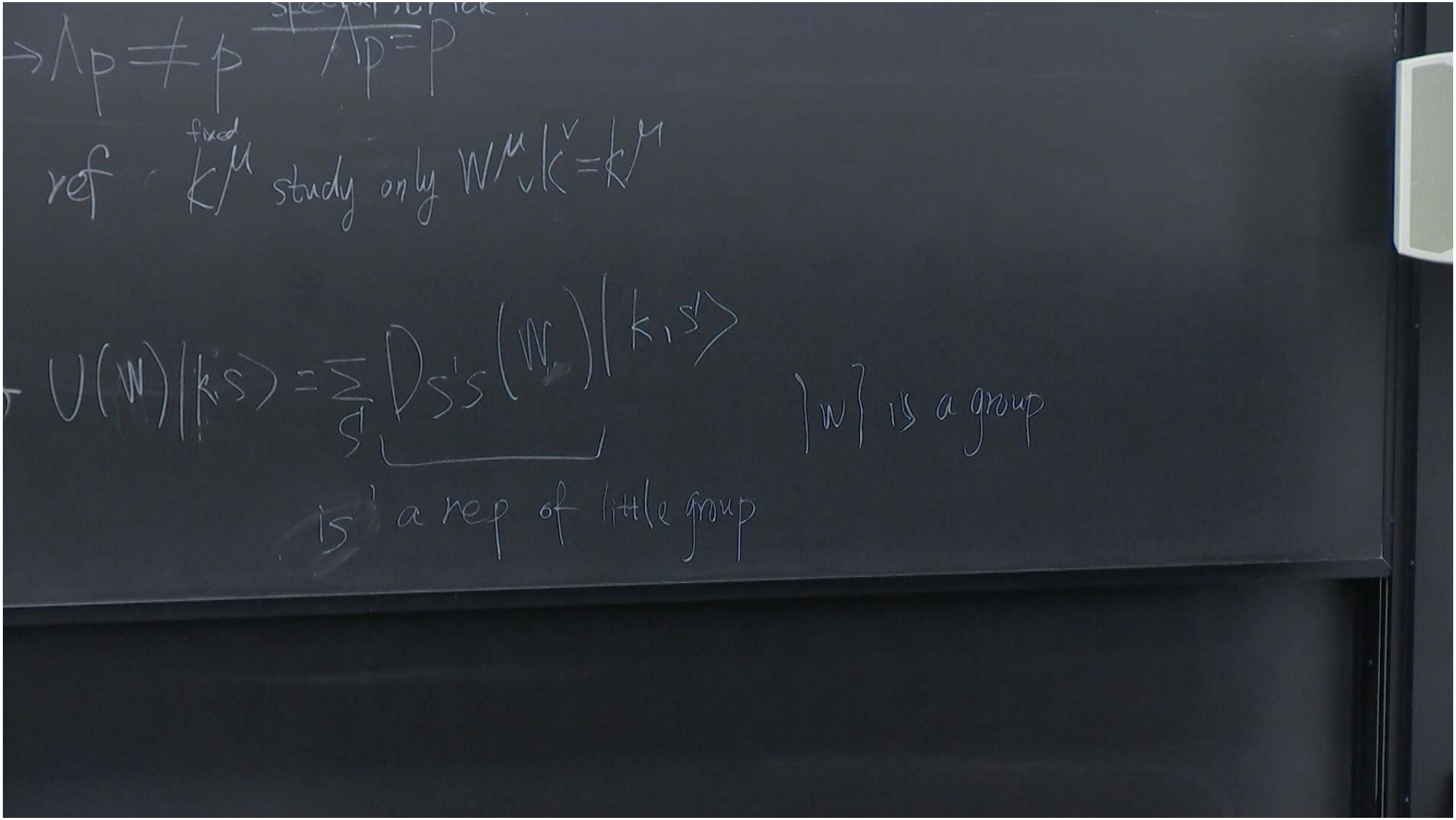
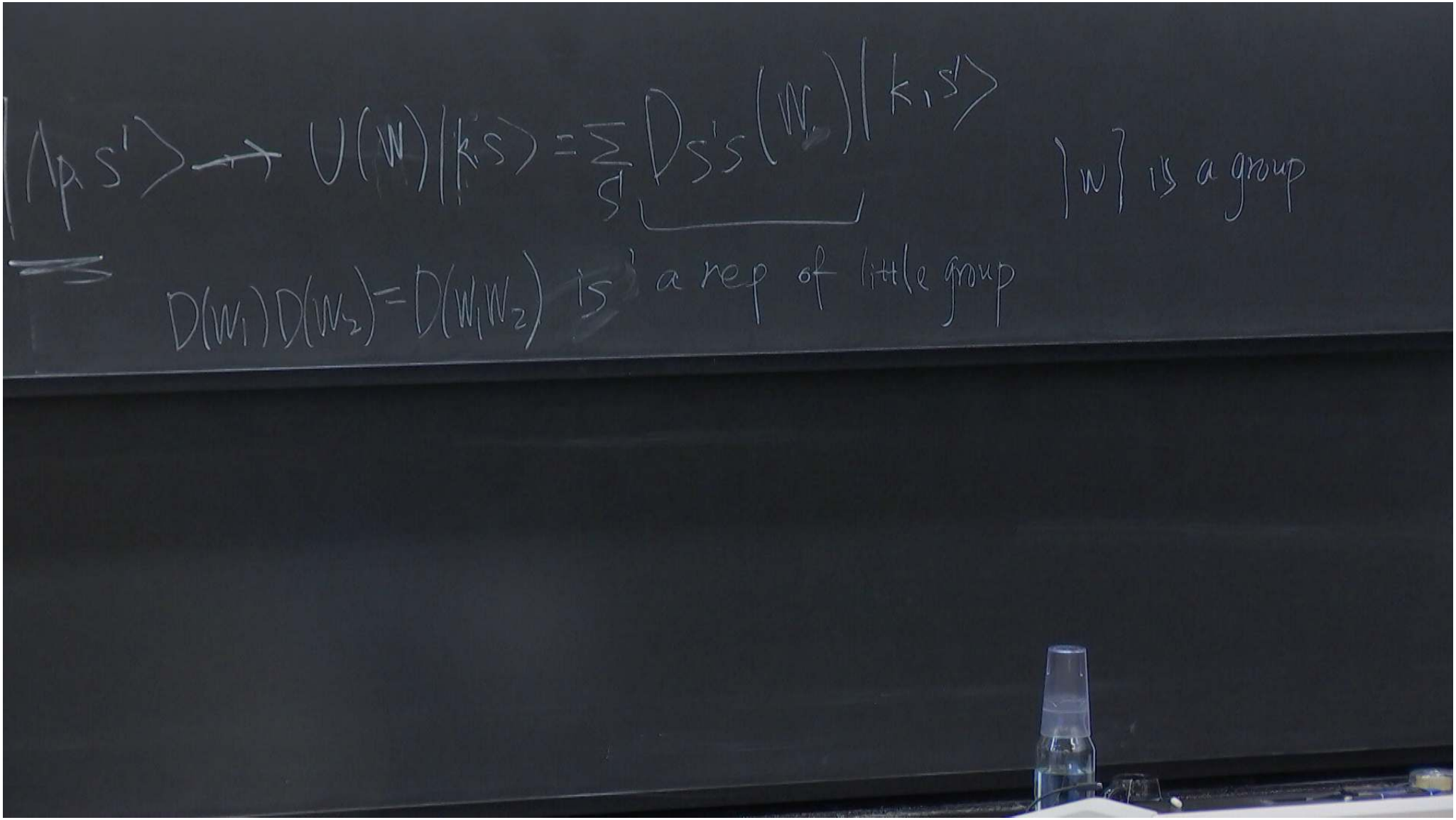
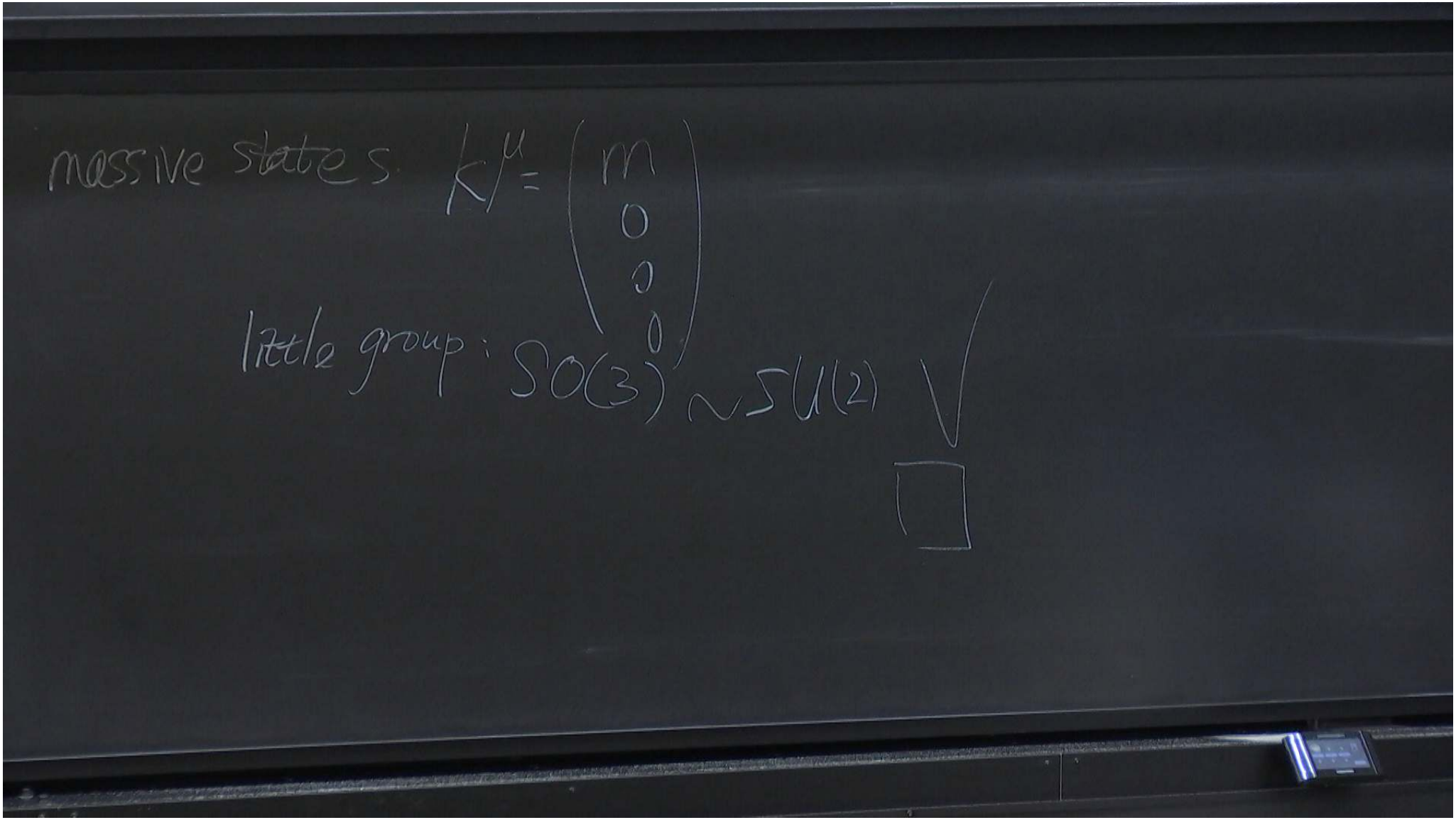
$U(\Lambda)|p,s\rangle = \sum_{s'} C_{s's}(\Lambda p)\,|\Lambda p, s'\rangle$


special trick
$p \to \Lambda p \neq p$
$\Lambda p = p$
fixed
ref k^μ study only $W^\mu{}_\nu(k) = k^\mu$
$|p', s'\rangle$
$(\Lambda p)\,|\Lambda p, s'\rangle \to U(W)\,|k, s\rangle = \sum_{s'} D_{s's}(W)\,|k, s'\rangle$

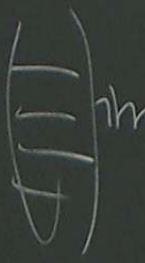

$p \to \Lambda p \neq p$
special little group
$\Lambda p = p$
$|p', s'\rangle$
ref k^μ fixed study only $W^\mu{}_\nu k^\nu = k^\mu$
$(\Lambda p)\,|\Lambda p, s'\rangle \to U(W)|k, s\rangle = \sum_{s'} D_{s's}(W)\,|k, s'\rangle$
furnish a rep of little group


$\Lambda p \neq p$
$\Lambda p = p$
ref: k^μ (fixed) study only $W^\mu{}_\nu k^\nu = k^\mu$
$U(W)|k,s\rangle = \sum_{s'} D_{s's}(W)\,|k,s'\rangle$
$\{W\}$ is a group
is a rep of little group


$|\Lambda p, s'\rangle \to U(W)|k,s\rangle = \sum_{s} D_{s's}(W)\,|k,s'\rangle$
$\{W\}$ is a group
$D(W_1)D(W_2) = D(W_1W_2)$ is a rep of little group


massive states: $k^\mu = (m, 0, 0, 0)$
little group: $SO(3) \sim SU(2)$

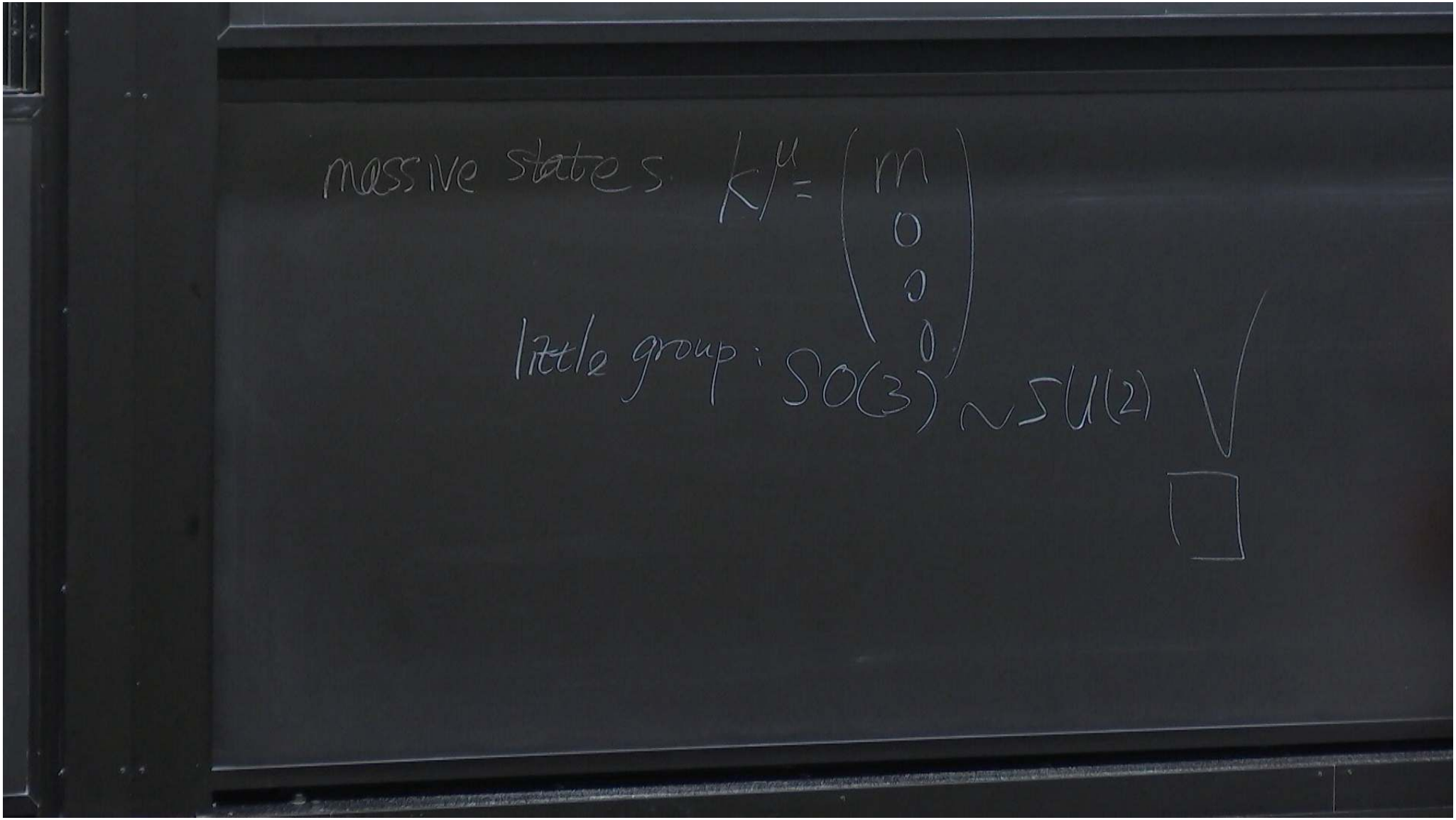
$$U(\Lambda)|p,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,|\Lambda p, s'\rangle$$

$\rangle \to$  irreps.

$p, s\rangle$

if choose k nicely

$\sim$ $W \sim SO(3) \sim SU(2)$ ✓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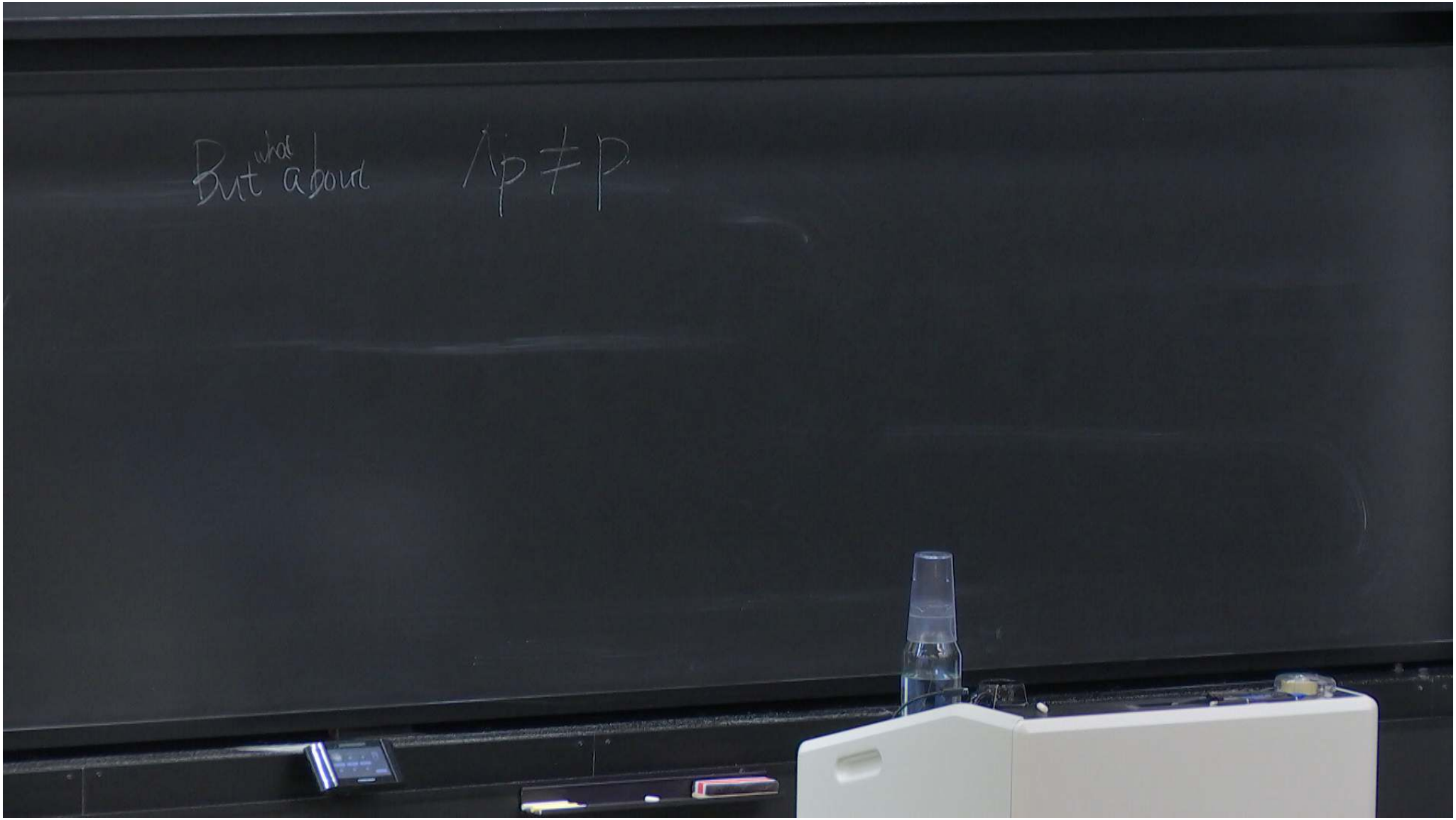
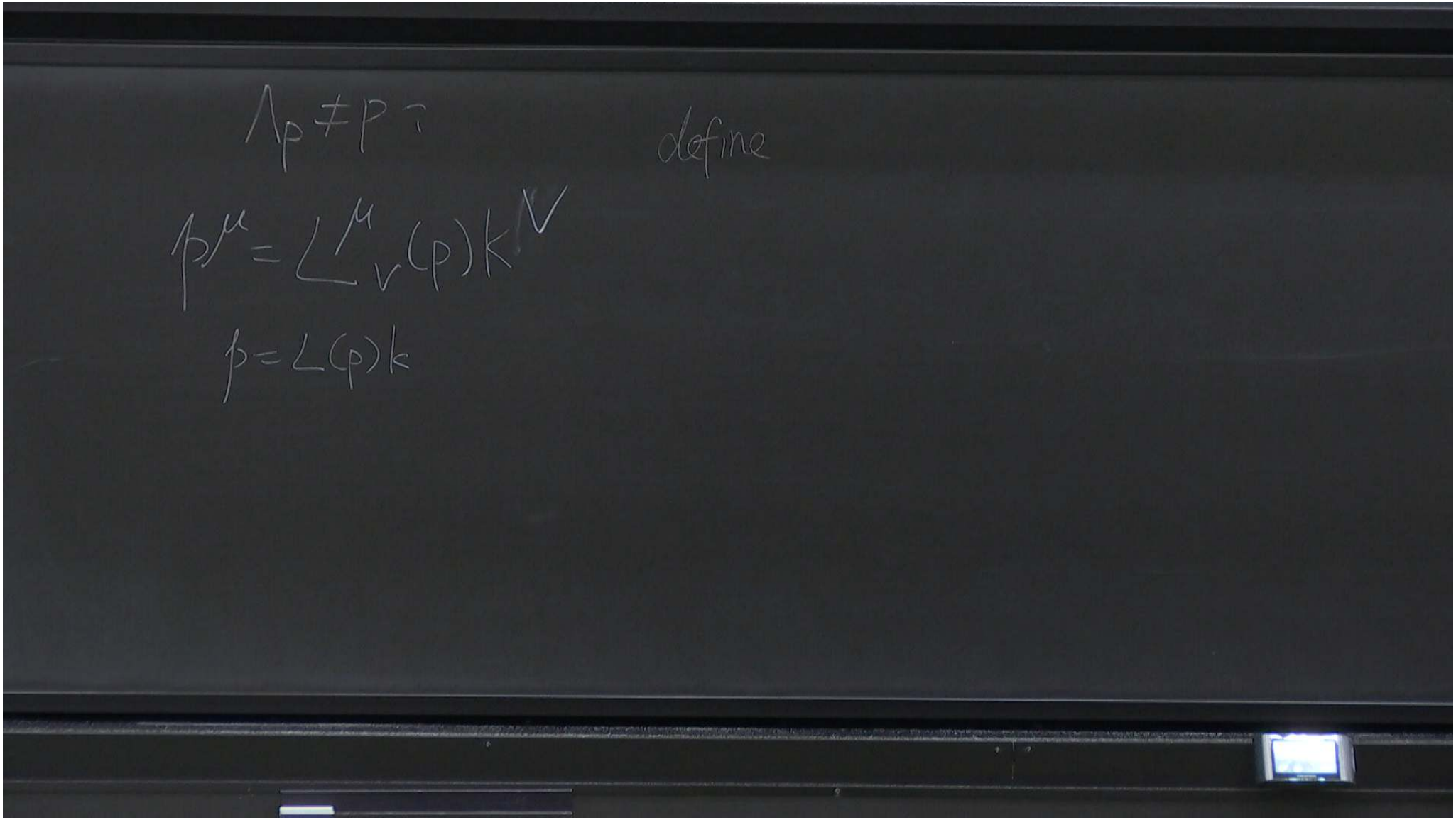

But what about $\Lambda p \neq p$


$\Lambda p \neq p$:
define
$p^\mu = \Lambda^\mu{}_\nu(p) k^\nu$
$p = \Lambda(p) k$

$\Lambda_p \neq p$:

$p^\mu = L^\mu{}_\nu(p) k^\nu$

$p = L(p) k$

define $|p, s\rangle \equiv N(p)\, U(L(p)) |k s\rangle$

define $|p,s\rangle \equiv N(p)\, U(L(p))\, |k,s\rangle$

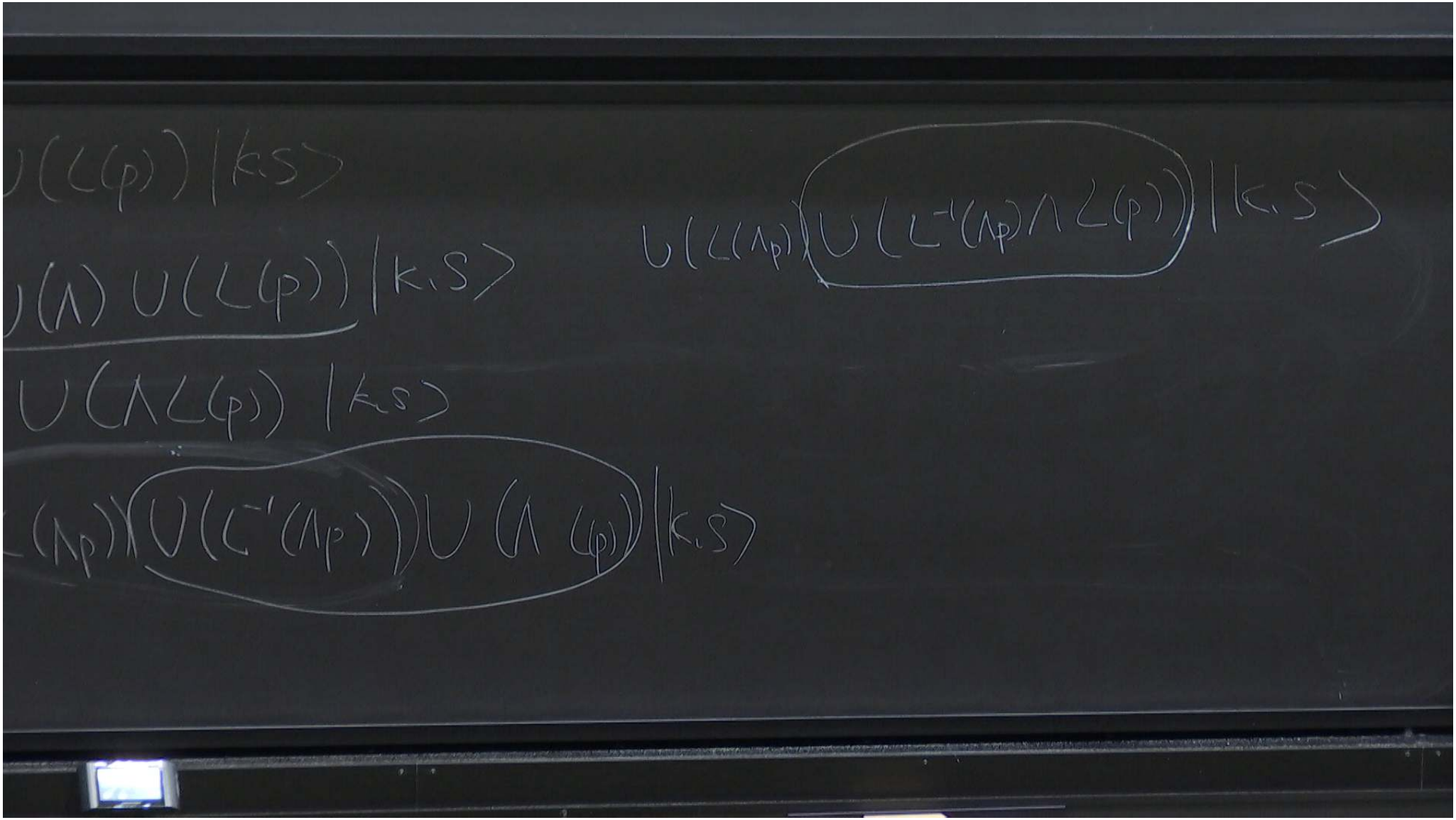
$U(\Lambda)\,|p,s\rangle \propto U(\Lambda)\, U(L(p))\, |k,s\rangle$

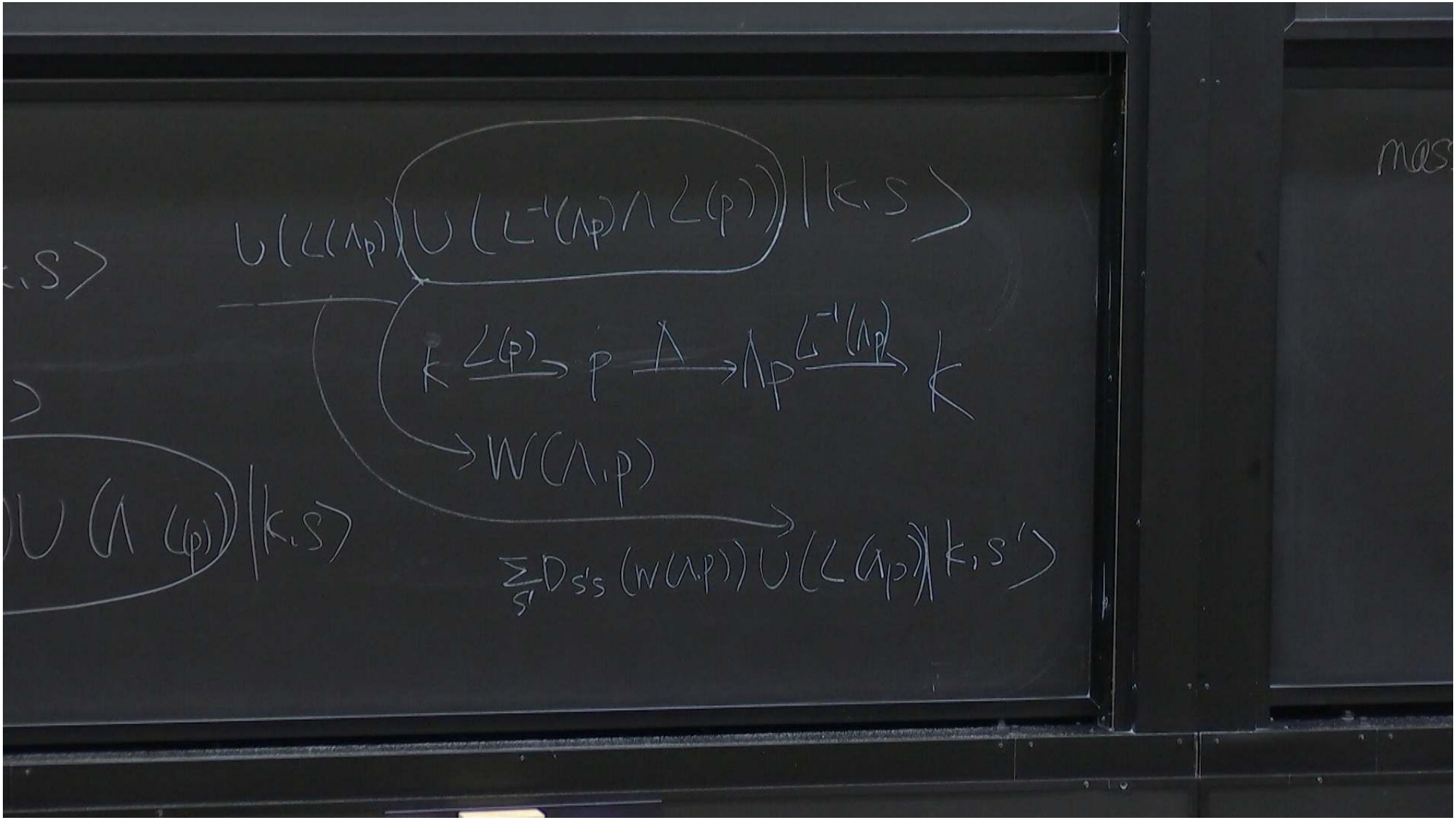
$)k^{\nu}$

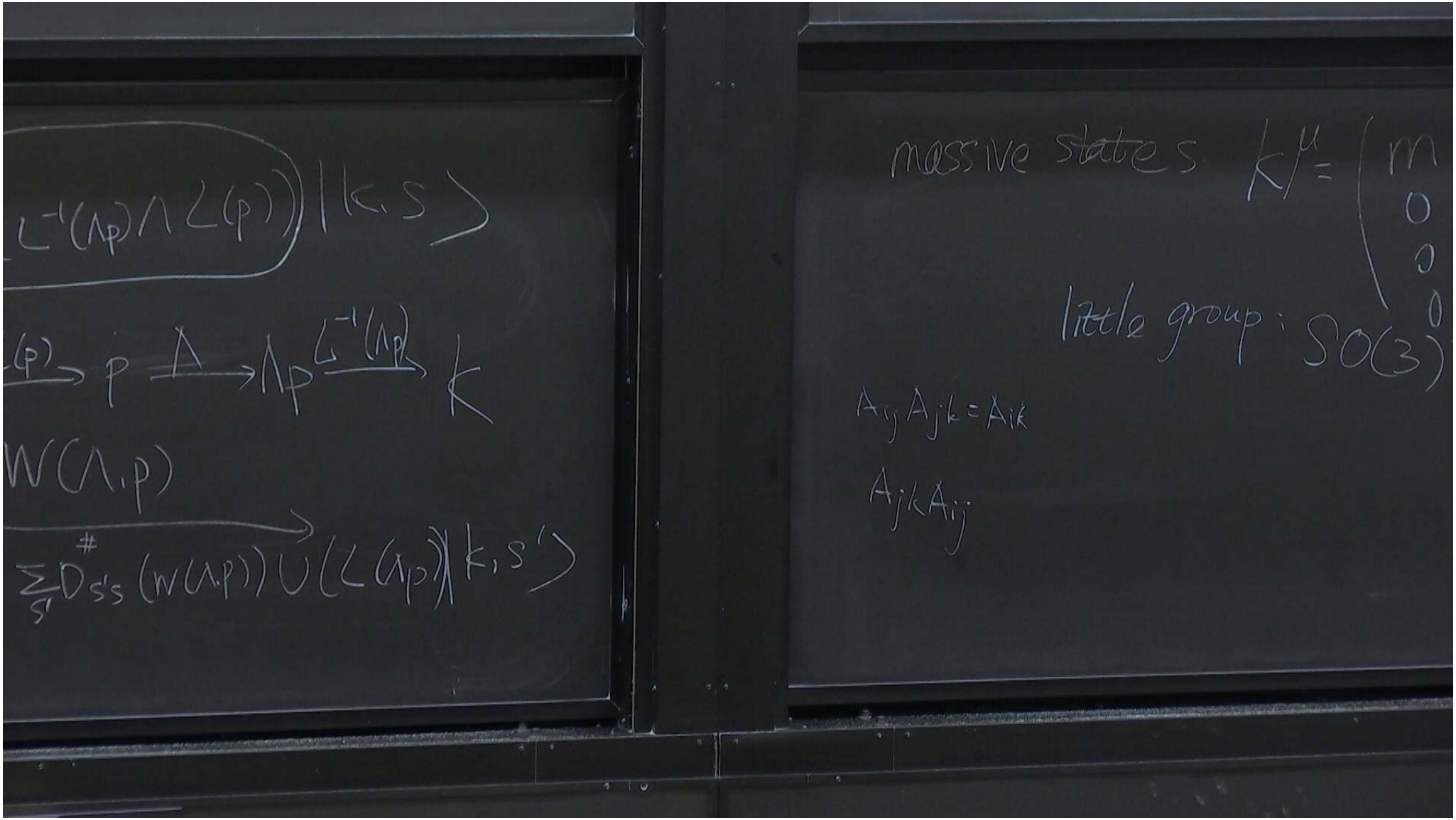
define $|p,s\rangle \equiv N(p)\, U(L(p))\, |k,s\rangle$

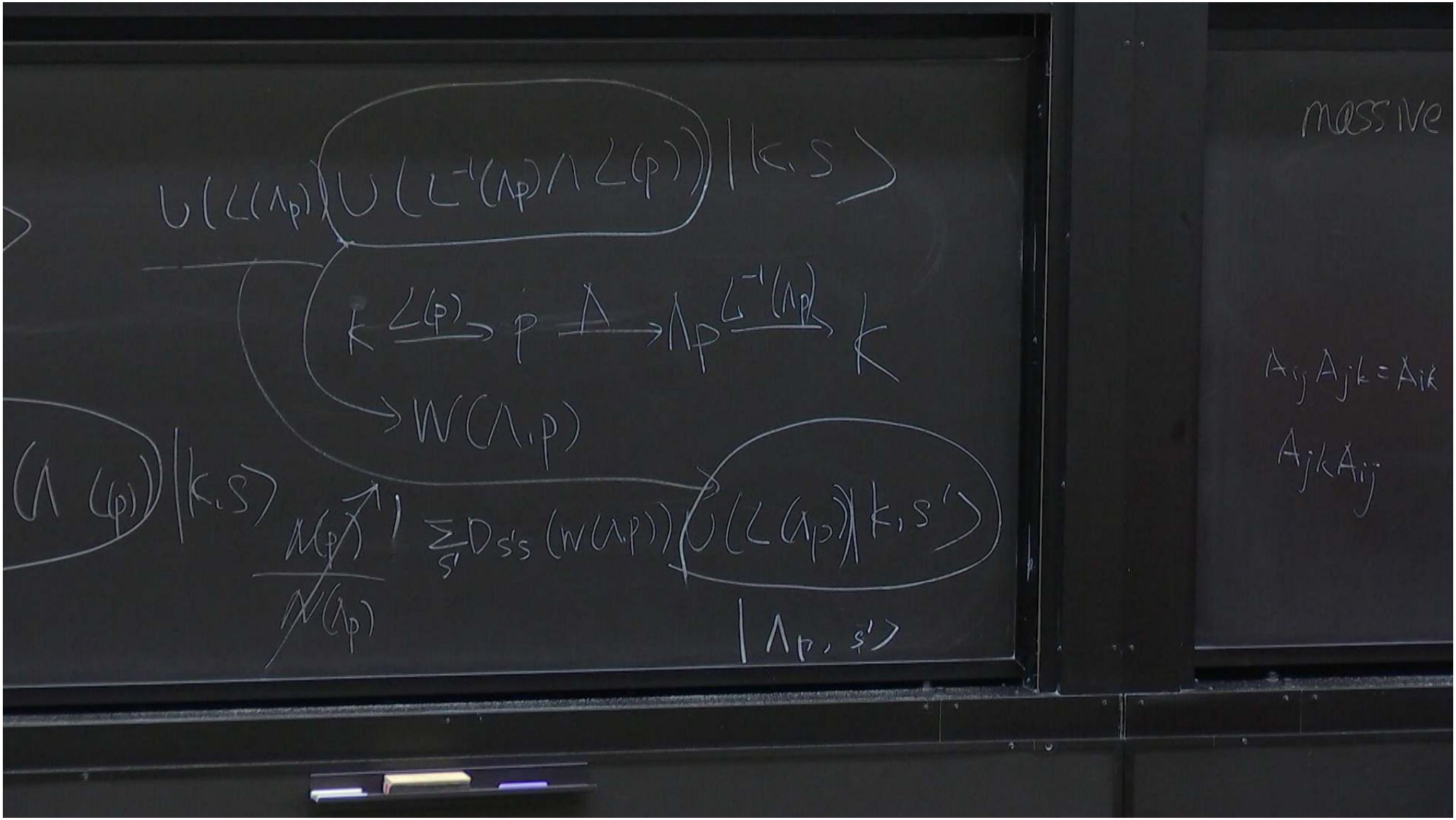
$U(\Lambda)\,|p,s\rangle \propto U(\Lambda)\, U(L(p))\, |k,s\rangle$

$=$

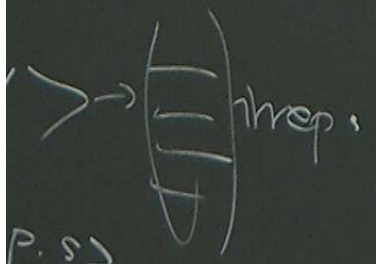

U(L(p)) |k,s⟩
U(Λ) U(L(p)) |k,s⟩
U(ΛL(p)) |k,s⟩
U(Λp) U(L⁻¹(Λp)) U(Λ) |k,s⟩
U(L(Λp)) U(L⁻¹(Λp)∧L(p)) |k,s⟩


$U(L(\Lambda p))\,U(L^{-1}(\Lambda p)\Lambda L(p))\,|k,s\rangle$
$k \xrightarrow{L(p)} p \xrightarrow{\Lambda} \Lambda p \xrightarrow{L^{-1}(\Lambda p)} k$
$W(\Lambda,p)$
$\sum_{s'} D_{s's}(W(\Lambda,p))\,U(L(\Lambda p))\,|k,s'\rangle$


massive states: $k^\mu = (m, 0, 0, 0)$
little group: $SO(3)$
$A_{ij}A_{jk} = A_{ik}$
$A_{jk}A_{ij}$
$\Lambda p \xrightarrow{L^{-1}(\Lambda p)} k$
$W(\Lambda, p)$
$\sum_{s'} D^{\#}_{s's}(W(\Lambda, p))\, U(L(\Lambda p))|k, s'\rangle$


massive
$A_{ij}A_{jk} = A_{ik}$
$A_{jk}A_{ij}$
$k \xrightarrow{L(p)} p \xrightarrow{\Lambda} \Lambda p \xrightarrow{L^{-1}(\Lambda p)} k$
$W(\Lambda, p)$
$\sum_{s'} D_{s's}(W(\Lambda,p))$
$|\Lambda p, s'\rangle$

$$U(\Lambda)|p,s\rangle = \sum_{s'} C_{s's}(\Lambda,p)\,|\Lambda p, s'\rangle$$



$|p,s\rangle$

if choose k nicely

$\sim$ $W \sim SO(3) \sim SU(2)$ ✓

P $\quad p = \angle k$ ← solved

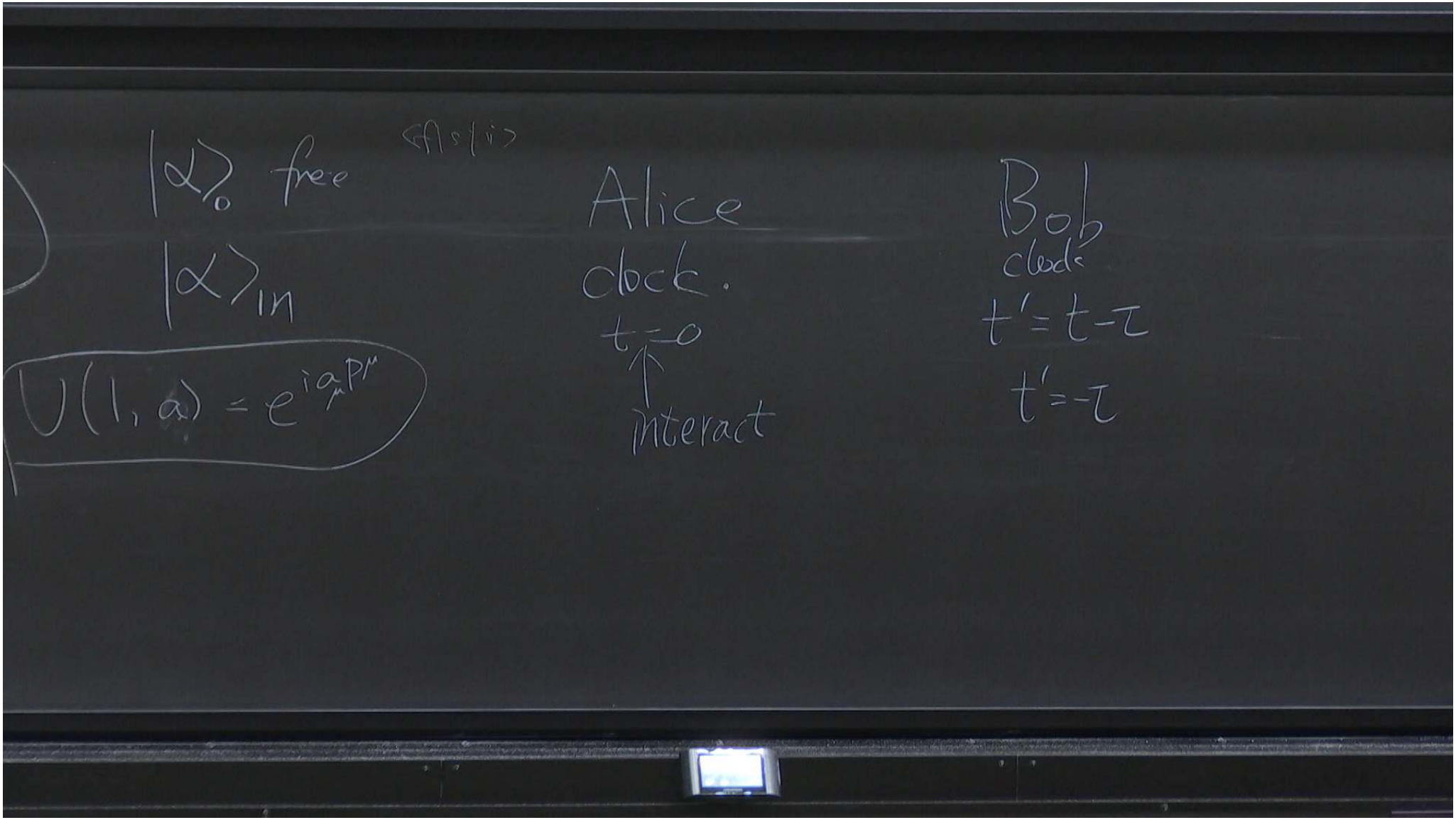
$U(\Lambda)|p,s\rangle \to \sum_{s'} |\Lambda p, s'\rangle$

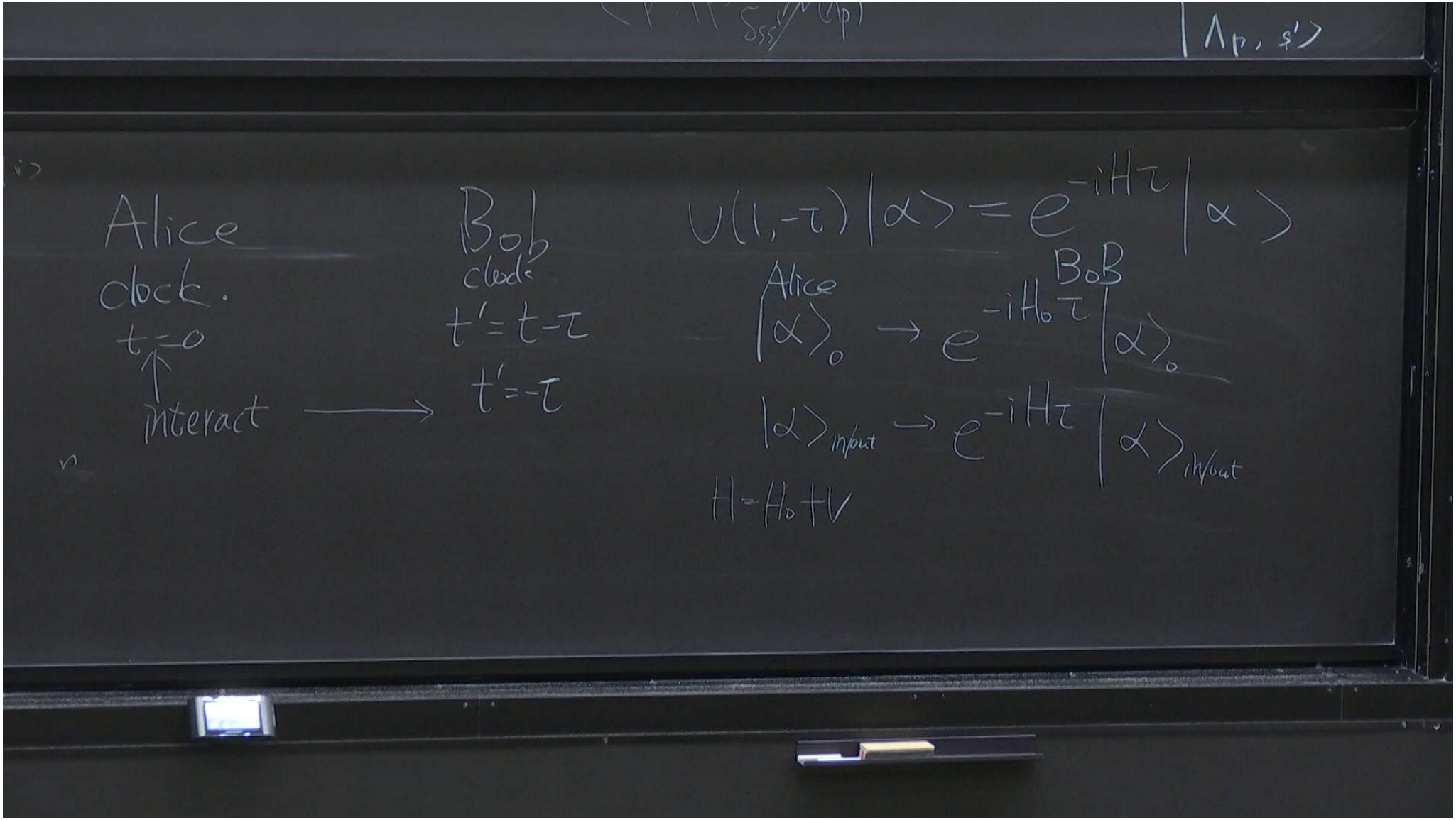
Goal $S_{\beta\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in}$

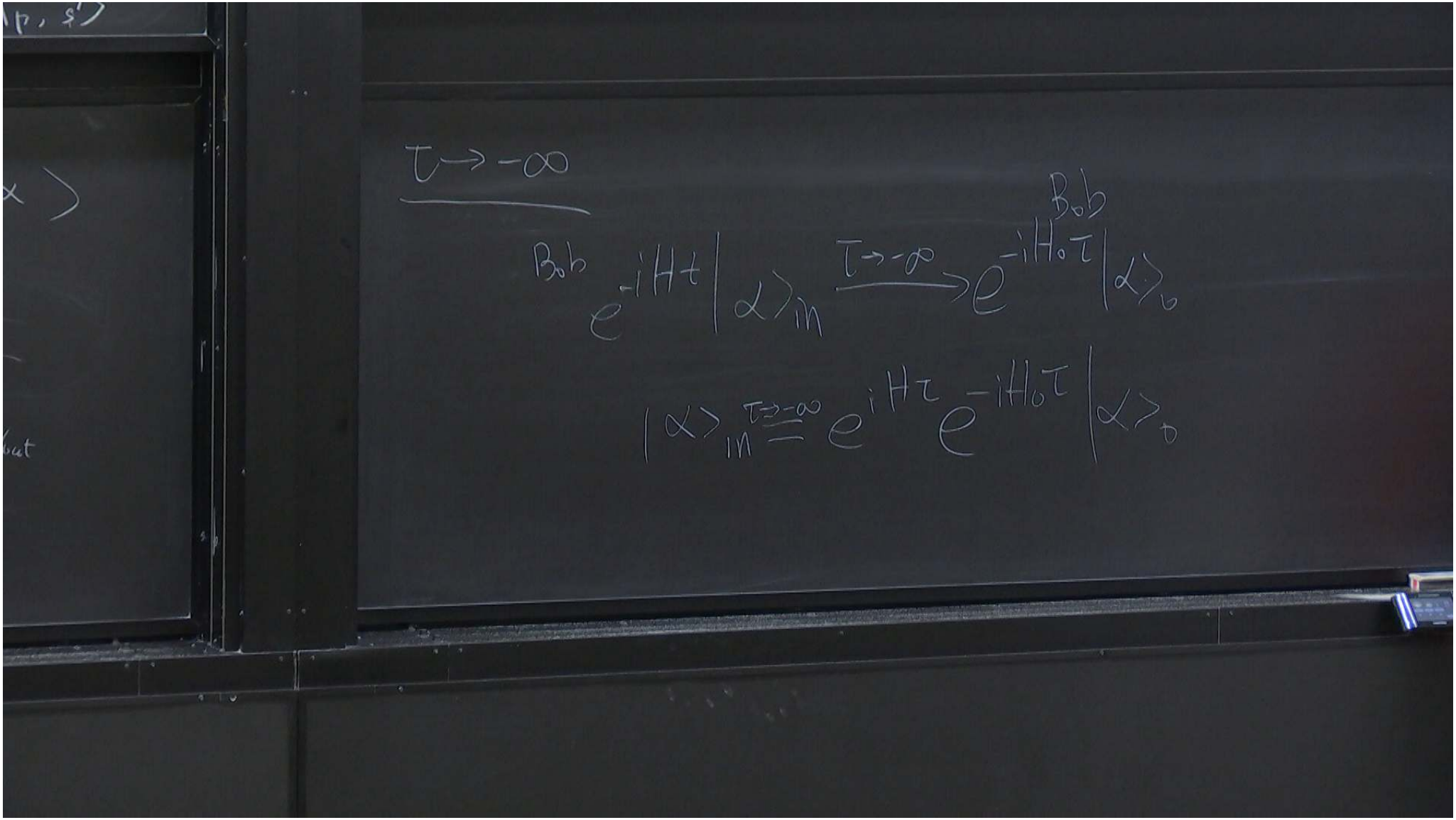
$|\alpha\rangle_0$ free $\quad {}_0\langle\beta|\alpha\rangle_0$

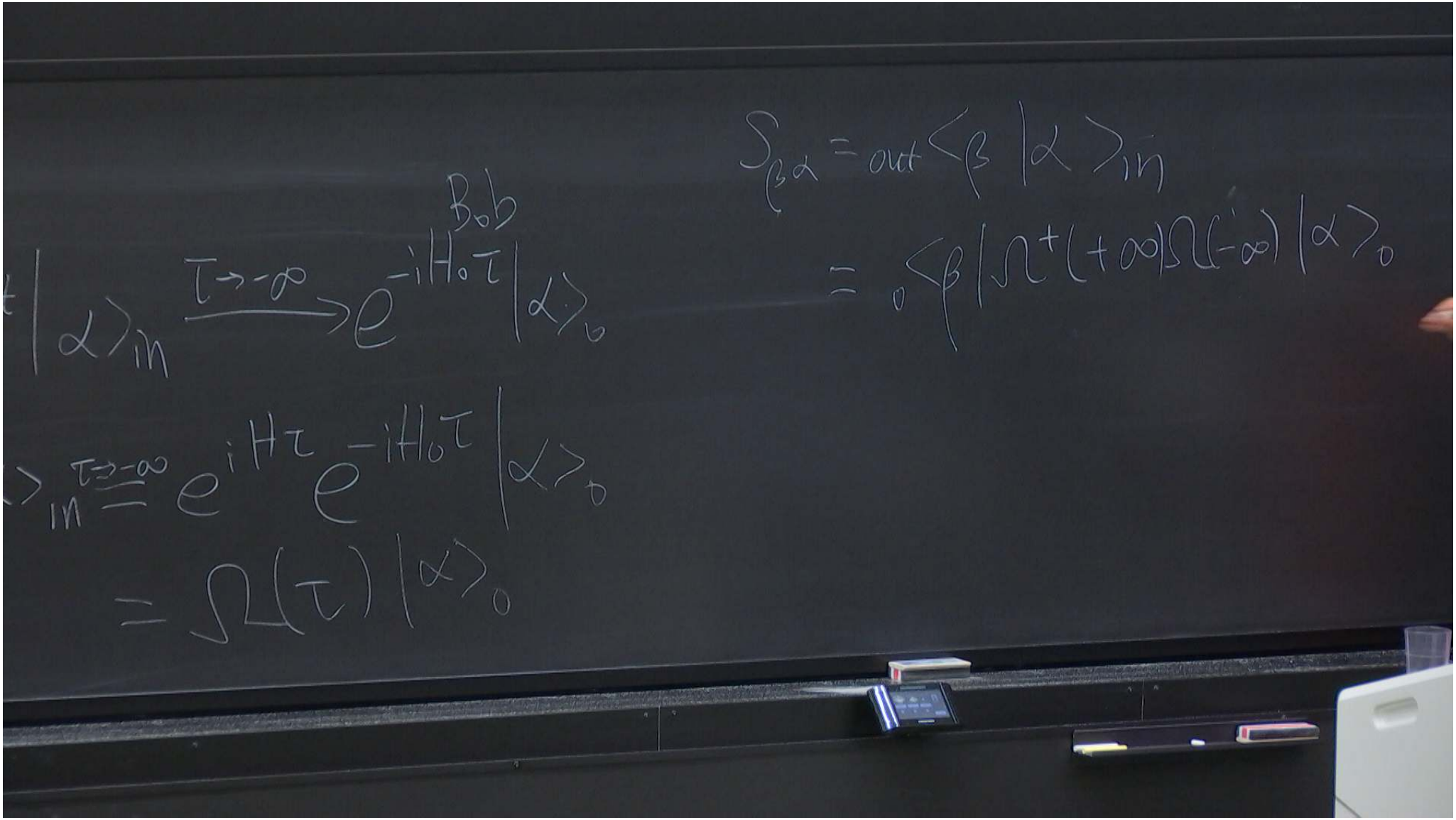
$|\alpha\rangle_{in}$

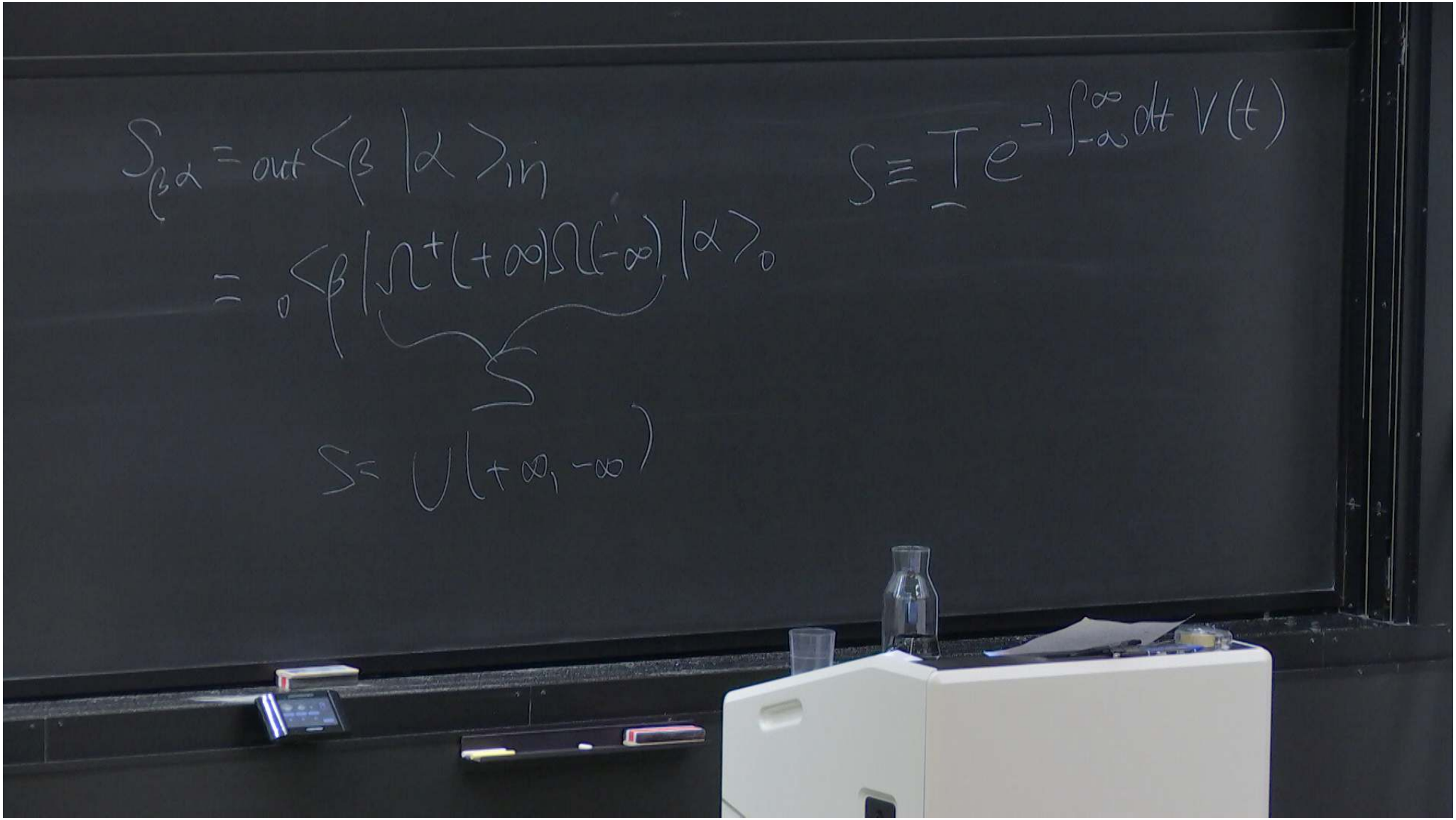
$U(1, a) = e^{ia_\mu P^\mu}$

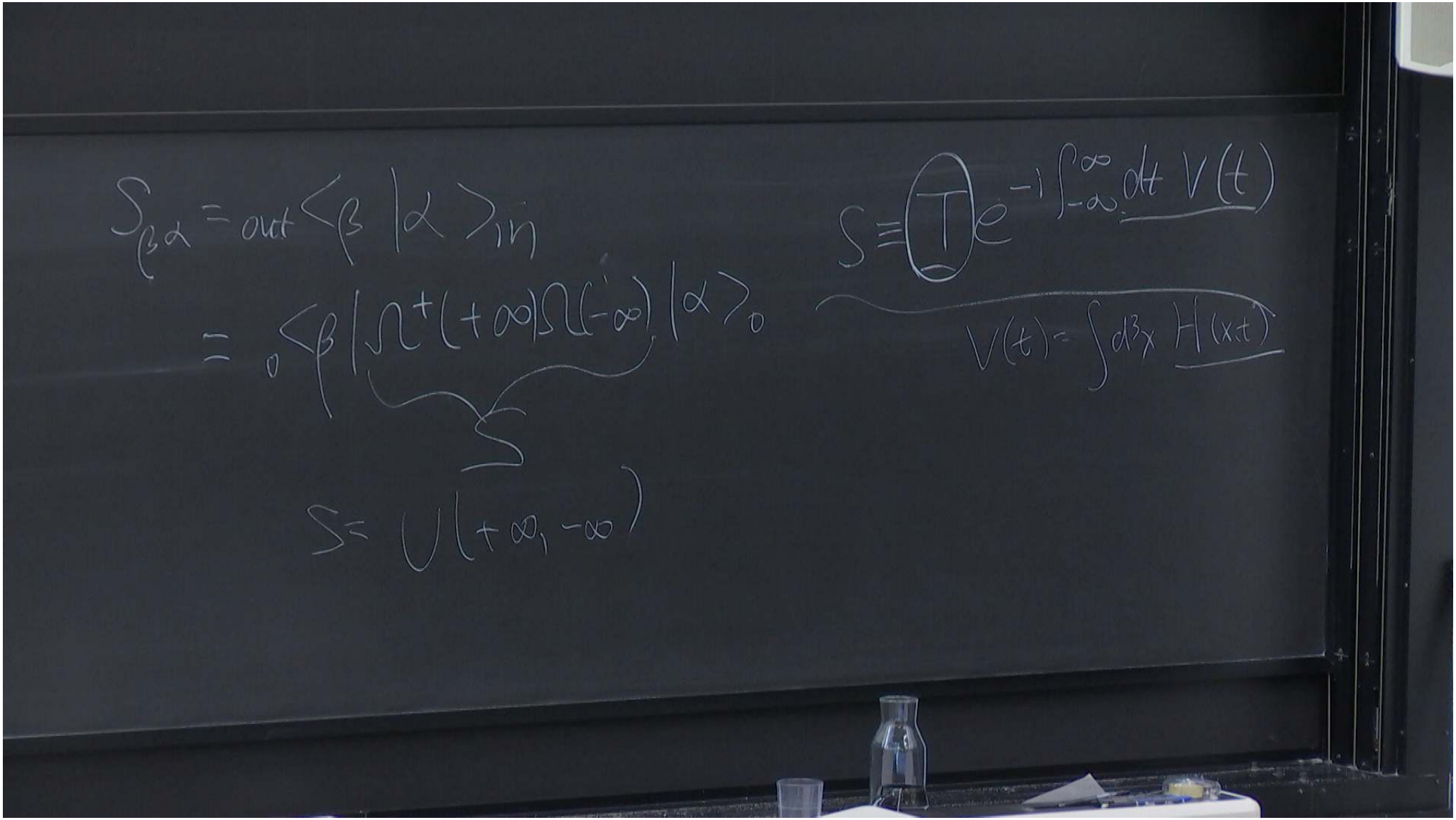

$|\alpha\rangle_0$ free
$\langle f|i\rangle$
$|\alpha\rangle_{in}$
$U(1, a) = e^{ia_\mu P^\mu}$
Alice
clock.
$t = 0$
interact
Bob
clock
$t' = t - \tau$
$t' = -\tau$

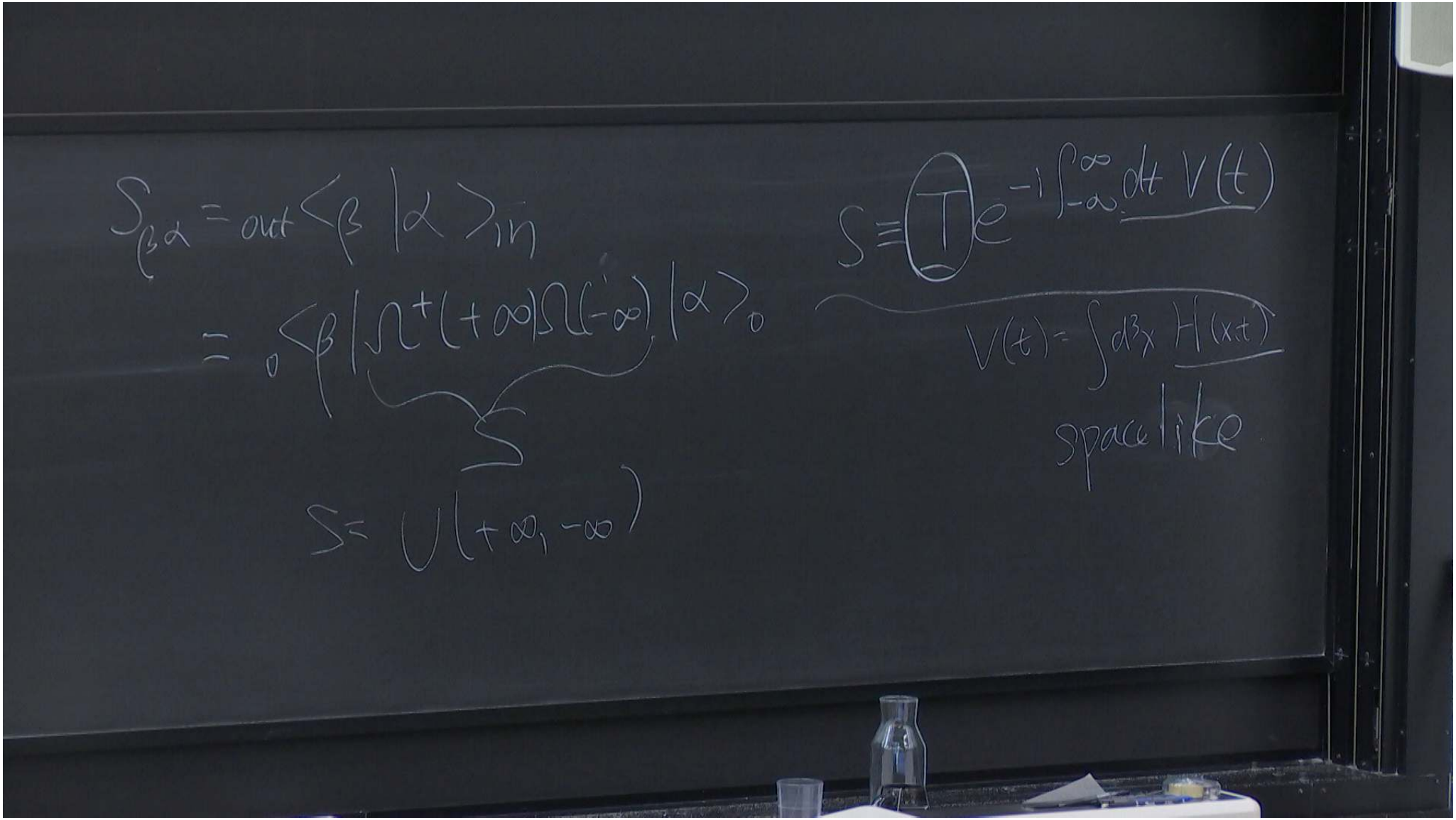

Alice
clock.
t=0
interact
Bob
clock
t'=t-τ
t'=-τ
U(1,-τ)|α⟩ = e^{-iHτ}|α⟩
Alice
Bob
|α⟩_0 → e^{-iH_0τ}|α⟩_0
|α⟩_input → e^{-iHτ}|α⟩_input
H=H_0+V

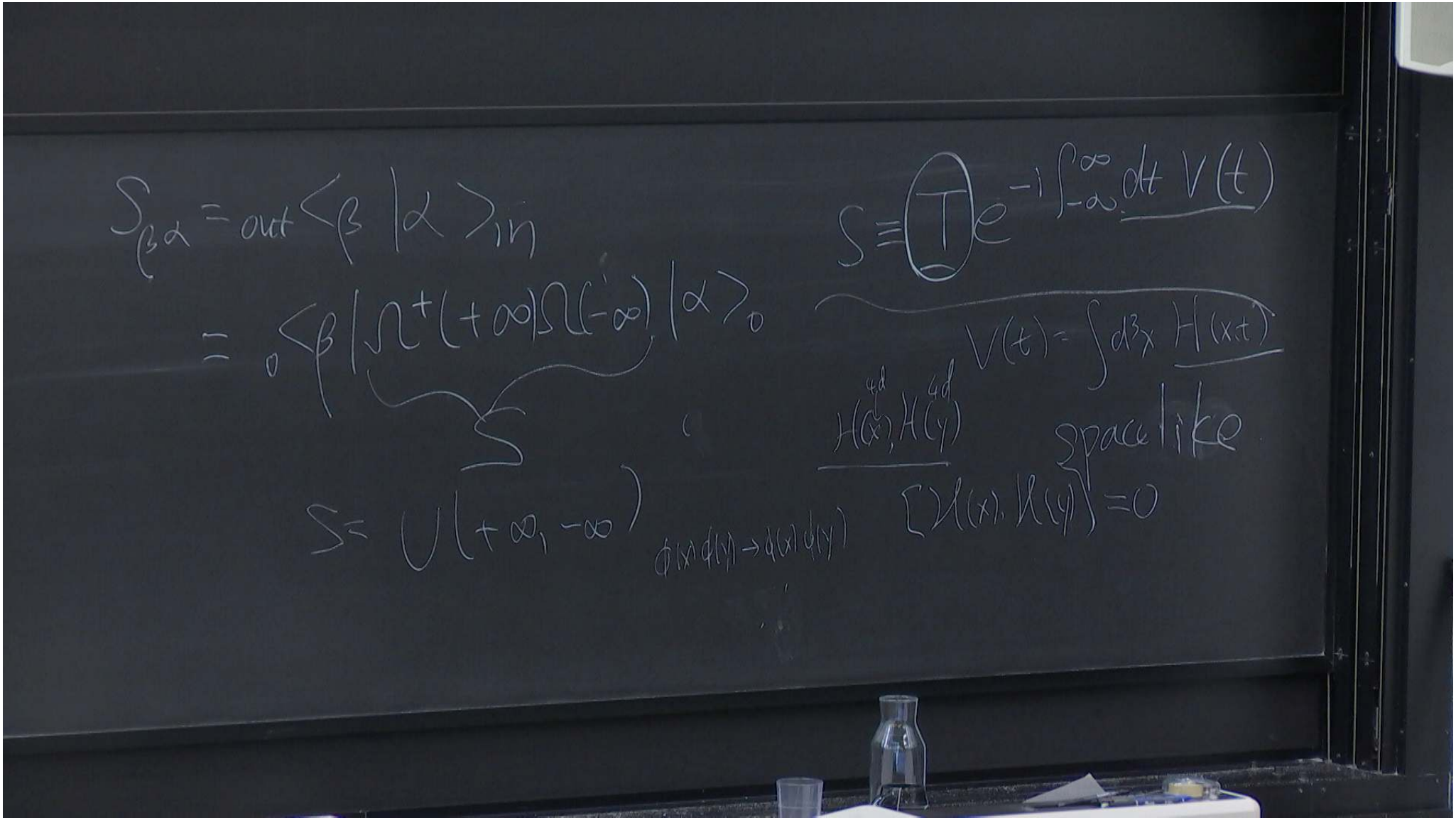

$\tau \to -\infty$
Bob
$e^{-iHt}|\alpha\rangle_{in} \xrightarrow{\tau\to-\infty} e^{-iH_0^{Bob}\tau}|\alpha\rangle_0$
$|\alpha\rangle_{in} \overset{\tau\to-\infty}{=} e^{iH\tau}e^{-iH_0\tau}|\alpha\rangle_0$

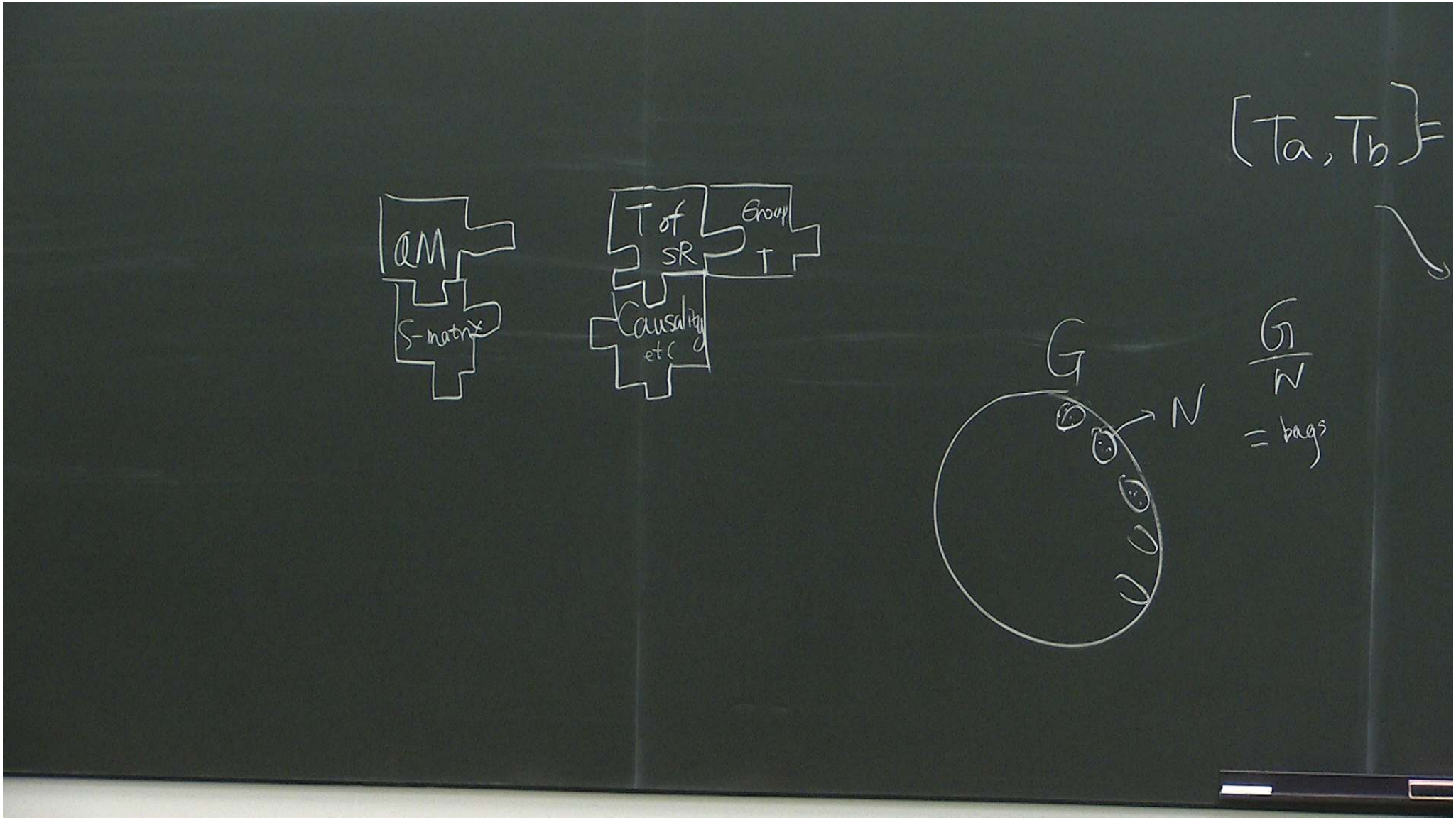

$S_{\beta\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in}$
$= {}_0\langle\beta|\Omega^\dagger(+\infty)\Omega(-\infty)|\alpha\rangle_0$
$|\alpha\rangle_{in} \xrightarrow{\tau\to-\infty} e^{-iH_0\tau}|\alpha\rangle_0$
B.b
$|\alpha\rangle_{in} \overset{\tau\to-\infty}{=} e^{iH\tau}e^{-iH_0\tau}|\alpha\rangle_0$
$= \Omega(\tau)|\alpha\rangle_0$


$S_{\beta\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in}$
$= {}_0\langle\beta|\Omega^+(+\infty)\Omega(-\infty)|\alpha\rangle_0$
$S = U(+\infty, -\infty)$
$S \equiv \underline{T} e^{-i\int_{-\infty}^{\infty} dt\, V(t)}$


$S_{\beta\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in}$
$= {}_0\langle\beta|\Omega^\dagger(+\infty)\Omega(-\infty)|\alpha\rangle_0$
$S = U(+\infty, -\infty)$
$S \equiv T e^{-i\int_{-\infty}^{\infty} dt\, V(t)}$
$V(t) = \int d^3x\, H(x,t)$


$S_{\beta\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in}$
$= {}_0\langle\beta|\Omega^\dagger(+\infty)\Omega(-\infty)|\alpha\rangle_0$
$S = U(+\infty, -\infty)$
$S \equiv T e^{-i\int_{-\infty}^{\infty} dt\, V(t)}$
$V(t) = \int d^3x\, H(x,t)$
spacelike


$S_{\beta\alpha} = {}_{out}\langle\beta|\alpha\rangle_{in}$
$= {}_0\langle\beta|\Omega^\dagger(+\infty)\Omega(-\infty)|\alpha\rangle_0$
$S = U(+\infty, -\infty)$
$S \equiv T\, e^{-i\int_{-\infty}^{\infty} dt\, V(t)}$
$V(t) = \int d^3x\, H(x,t)$
$H(x), H(y)$ 4d 4d
spacelike
$\phi(x)\phi(y) \to \phi(x)\phi(y)$
$[H(x), H(y)] = 0$


[Ta, Tb] =
QM
S-matrix
T of SR
Causality etc
Group
T
G
N
G/N
= bags