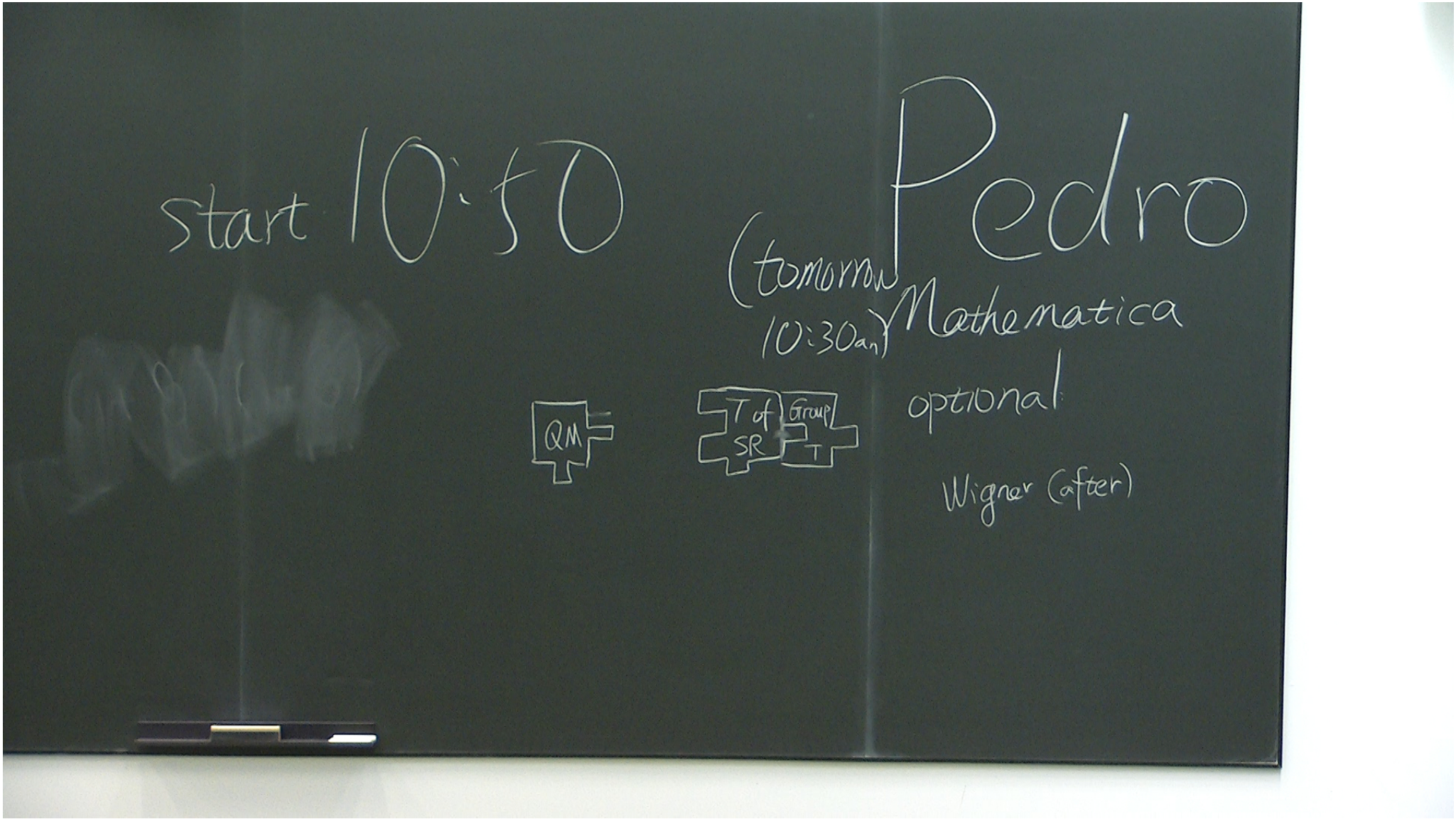
Title: Quantum Field Theory I - Lecture 221012

Speakers: Gang Xu

Collection: Quantum Field Theory I (2022/2023)

Date: October 12, 2022 - 10:45 AM

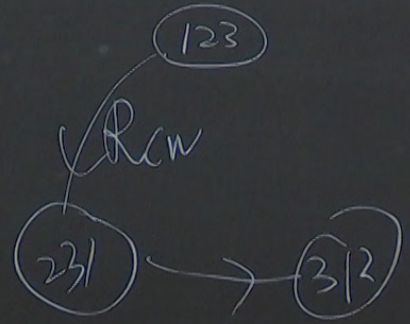
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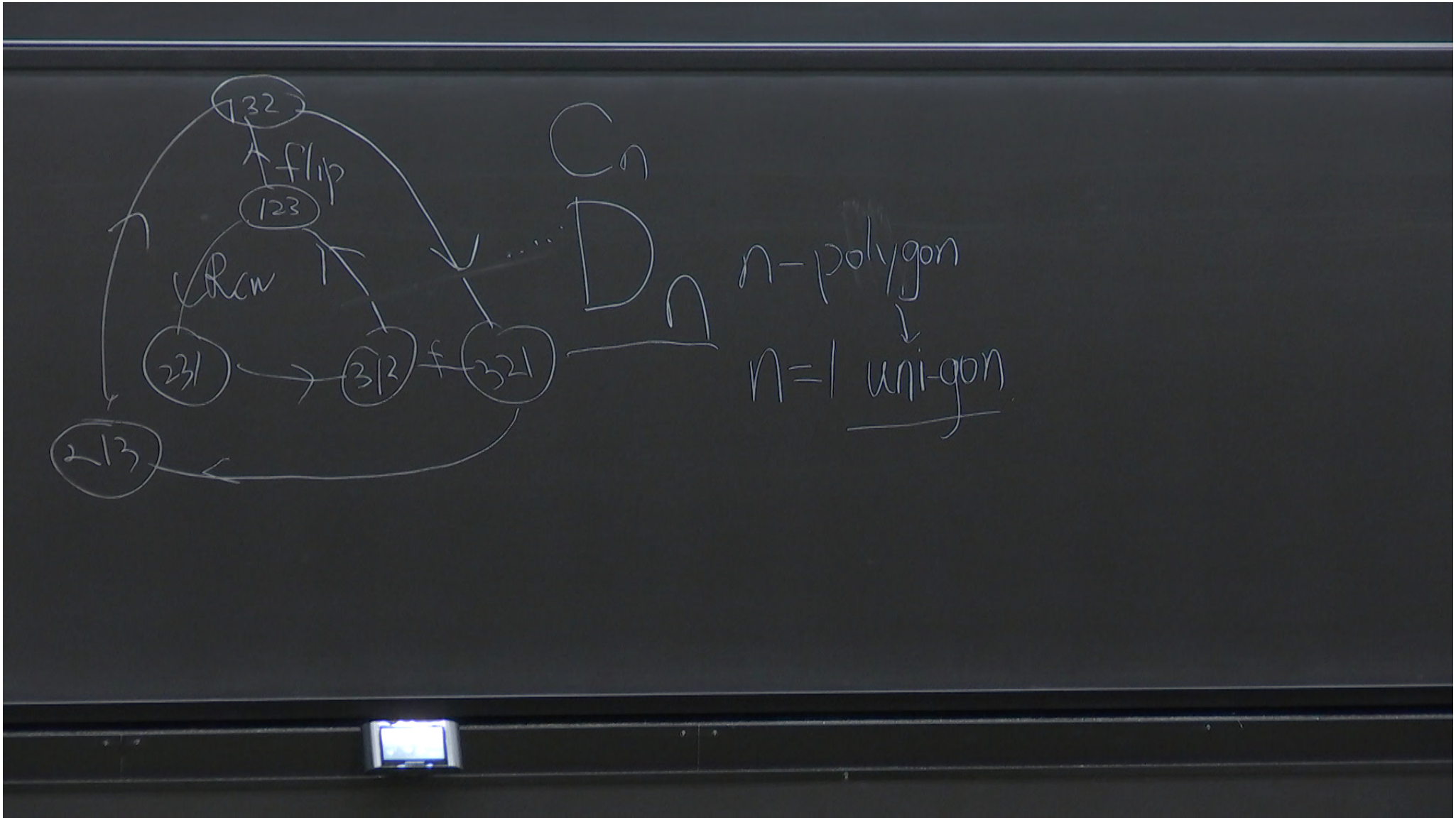

start 10:50
Pedro
(tomorrow 10:30am)
Mathematica
optional
QM
T of SR
Group
T
Wigner (after)

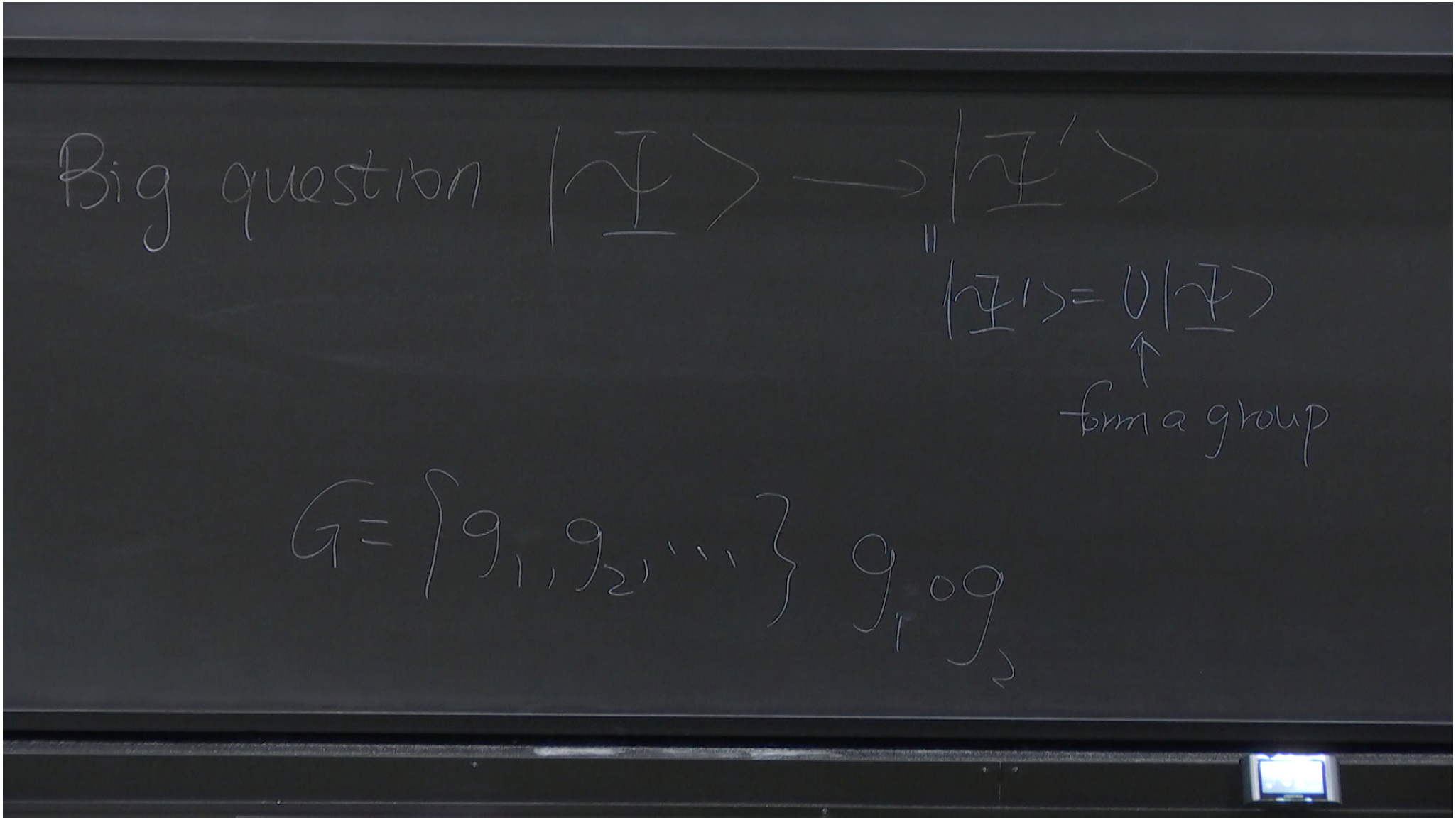
Coose end Cayley diagram

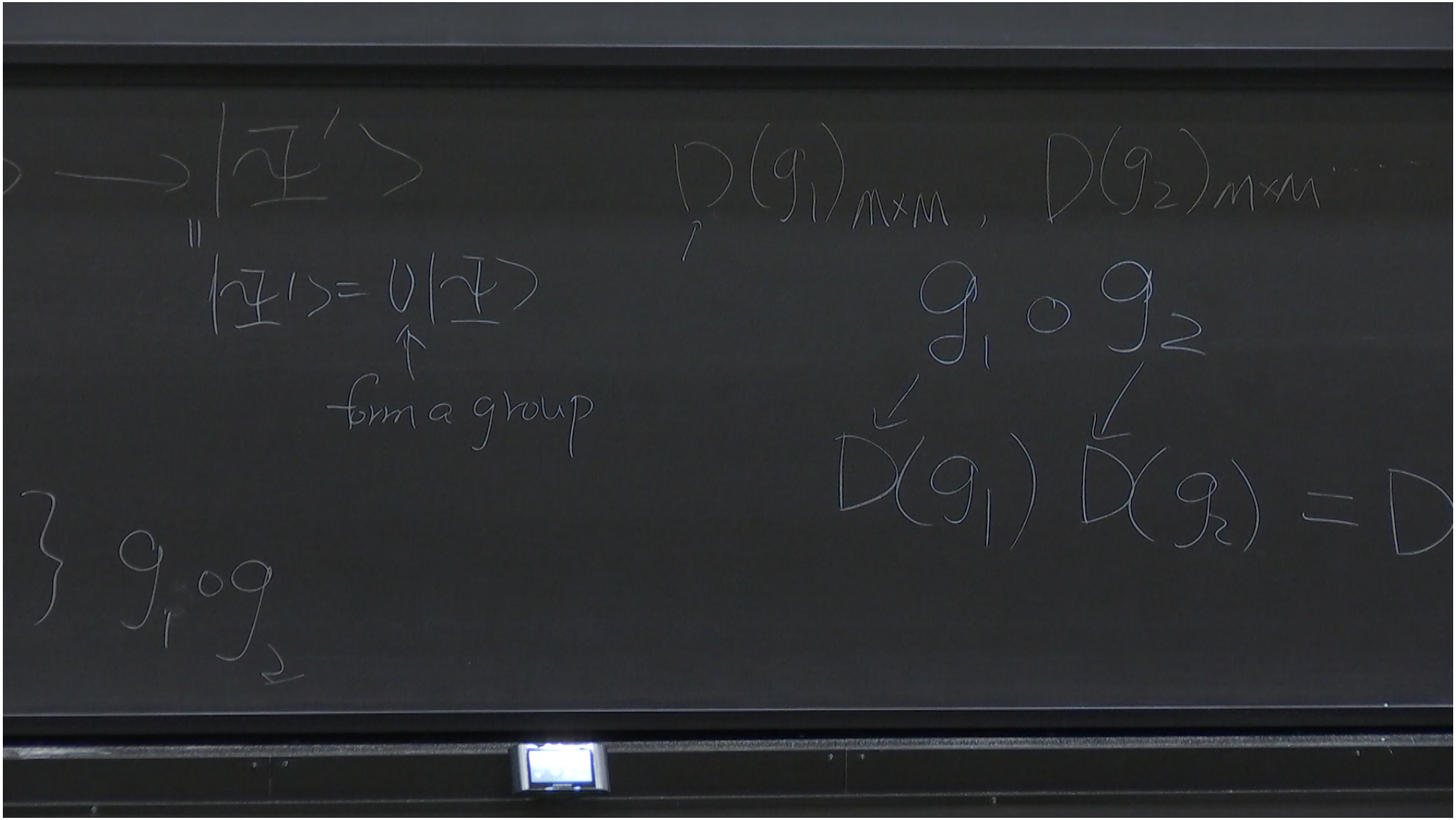
step 1. label mechanism

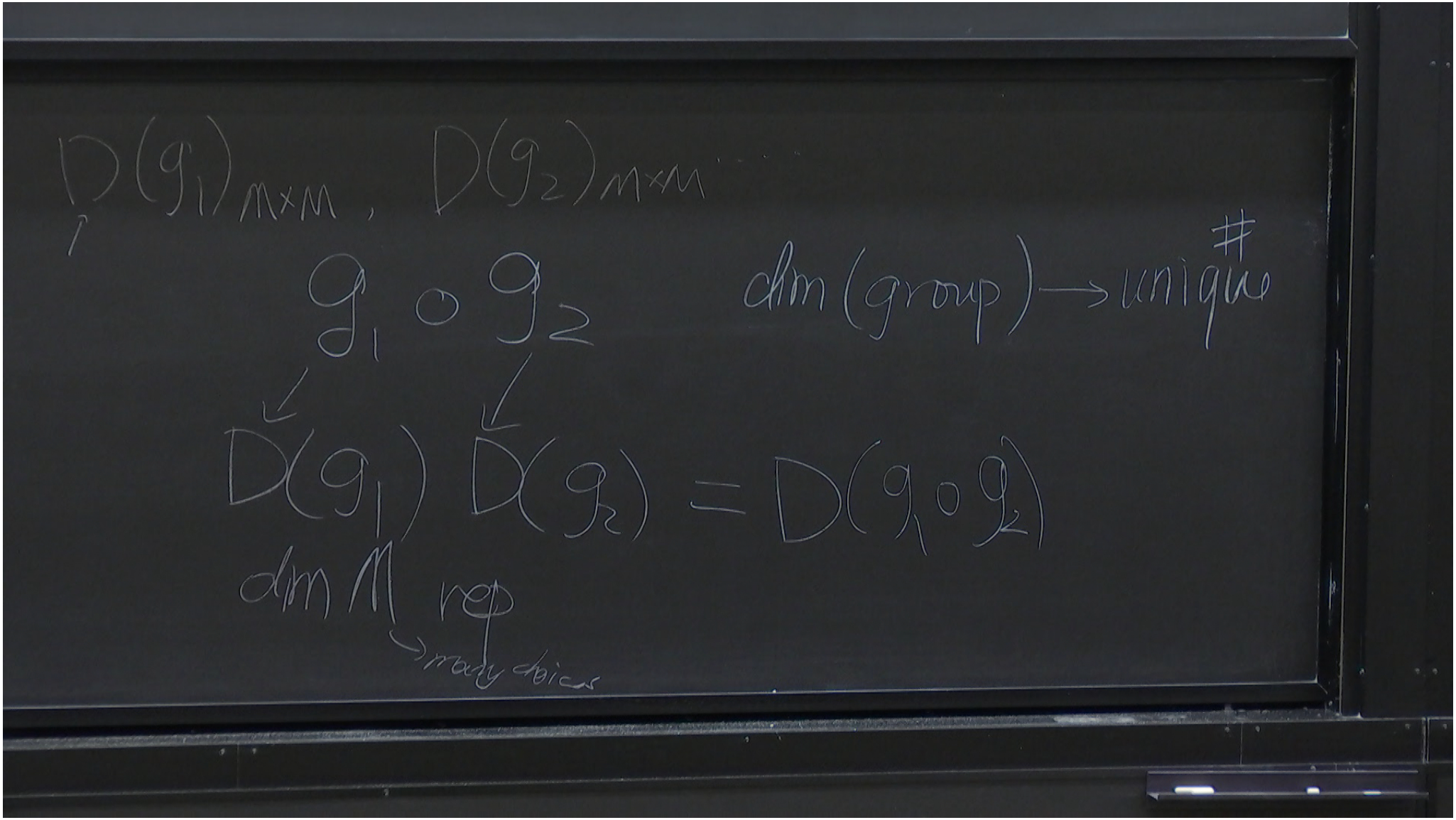
step 2: symmetry operations

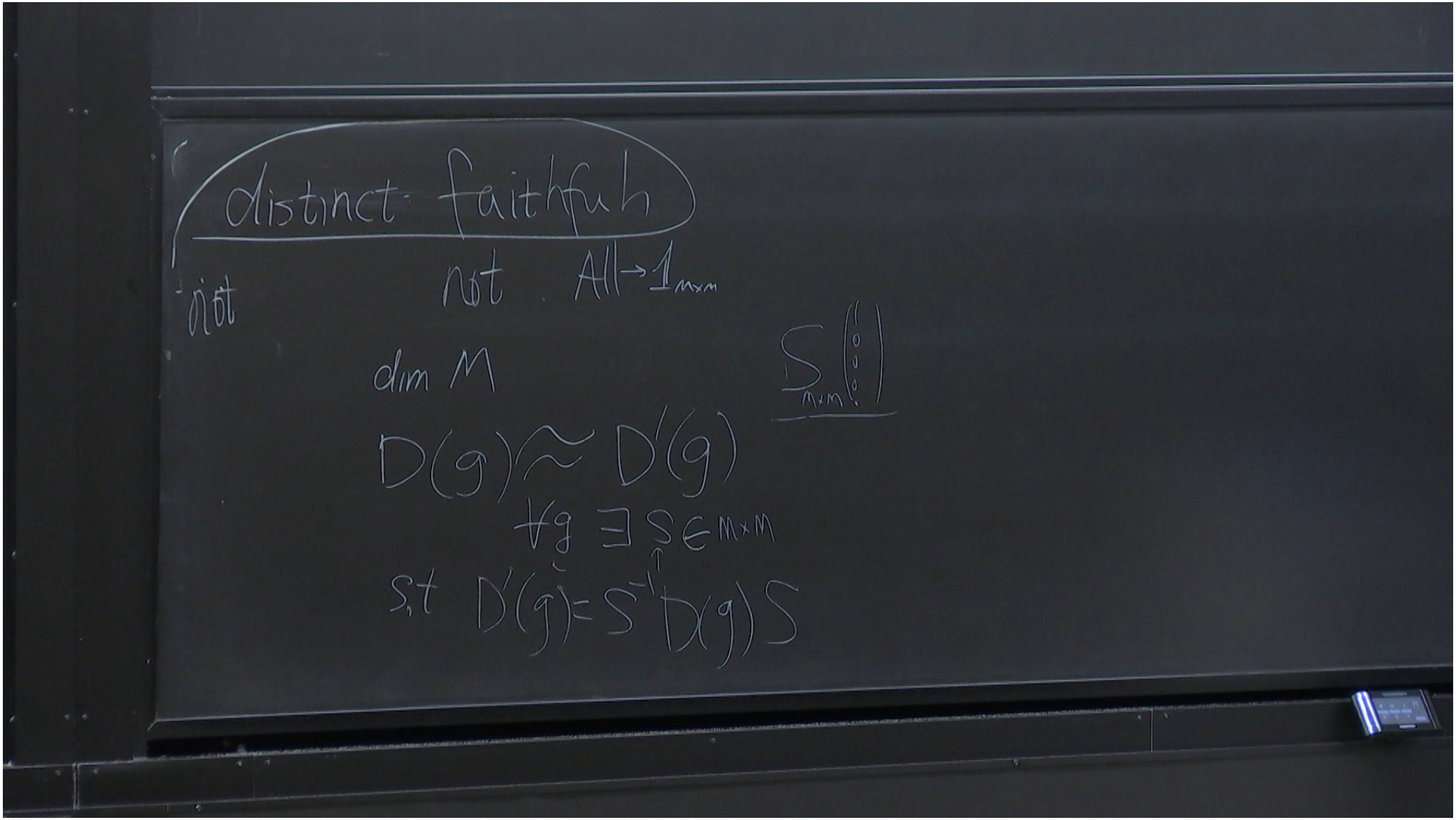


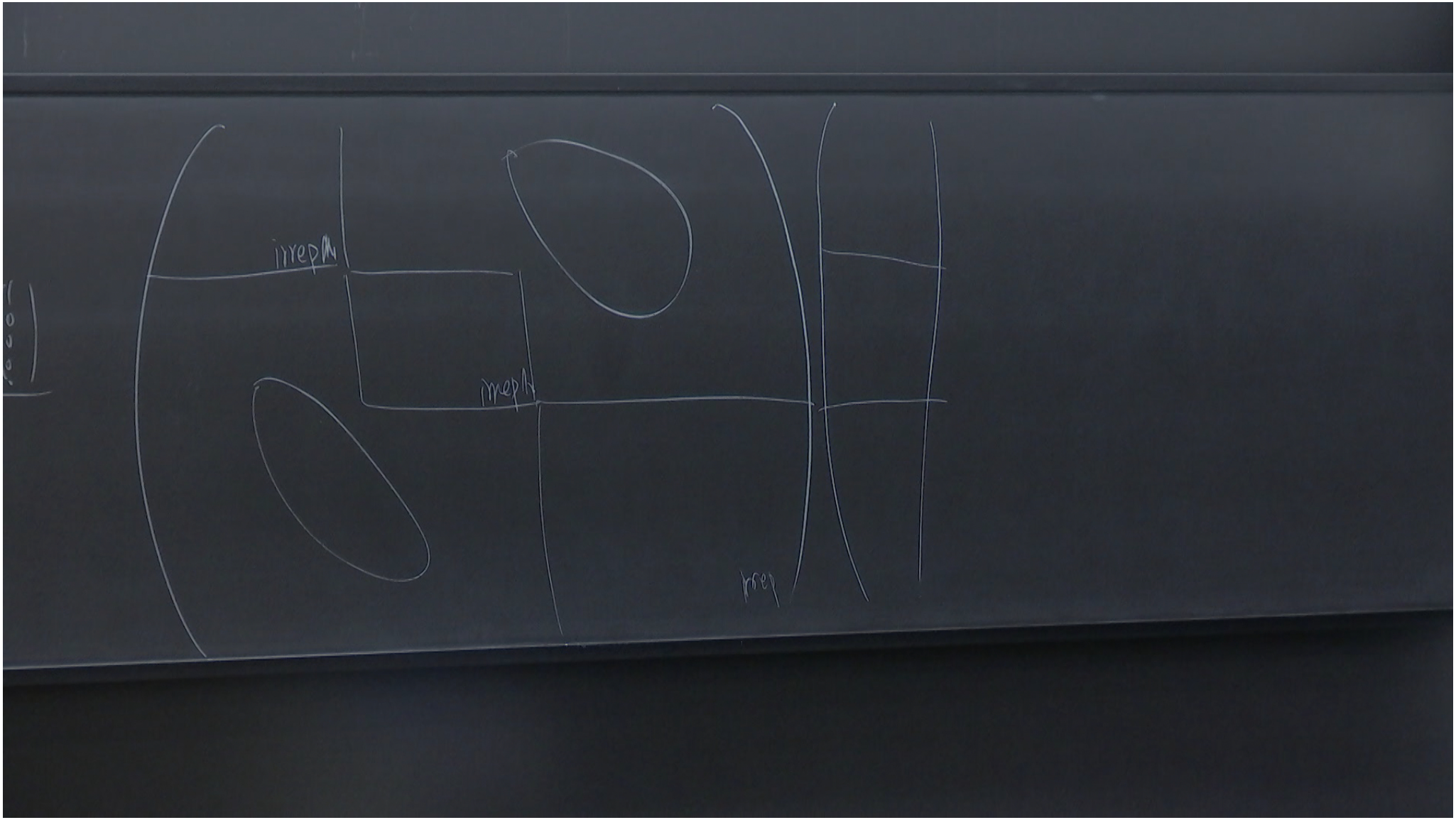

132
flip
123
Rcw
231
312
321
213
C_n
D_n
n-polygon
n=1 uni-gon


Big question $|\psi\rangle \longrightarrow |\psi'\rangle$
$|\psi'\rangle = U|\psi\rangle$
form a group
$G = \{g_1, g_2, \cdots\}$ $g_1 \circ g_2$


$|\psi'\rangle$
$|\psi'\rangle = U|\psi\rangle$
form a group
$\{g_1 \circ g_2$
$D(g_1)_{m\times m}$, $D(g_2)_{m\times m}$
$g_1 \circ g_2$
$D(g_1)\,D(g_2) = D$


D(g1)m×m . D(g2)m×m
g1 ∘ g2
dim(group) → unique #
D(g1) D(g2) = D(g1∘g2)
dim M rep
→ many choices


distinct: faithful
not
not All → $1_{m\times m}$
dim M
$S_{m\times m}\begin{pmatrix}1&&\\&0&\\&&\ddots\end{pmatrix}$
$D(g) \cong D'(g)$
$\forall g \; \exists S \in M\times M$
s.t $D'(g) = S^{-1} D(g) S$

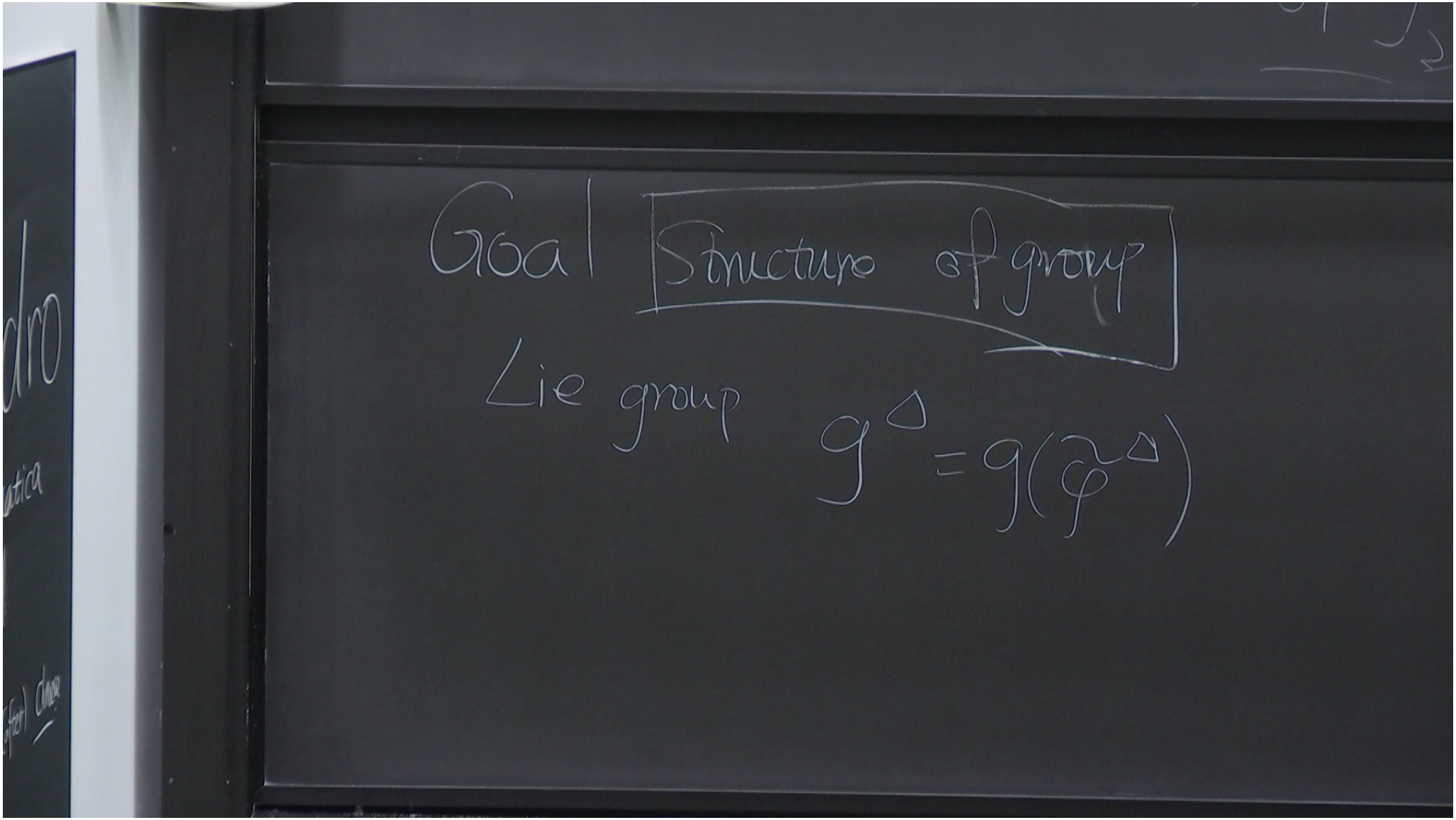

irrep
irrep
irrep

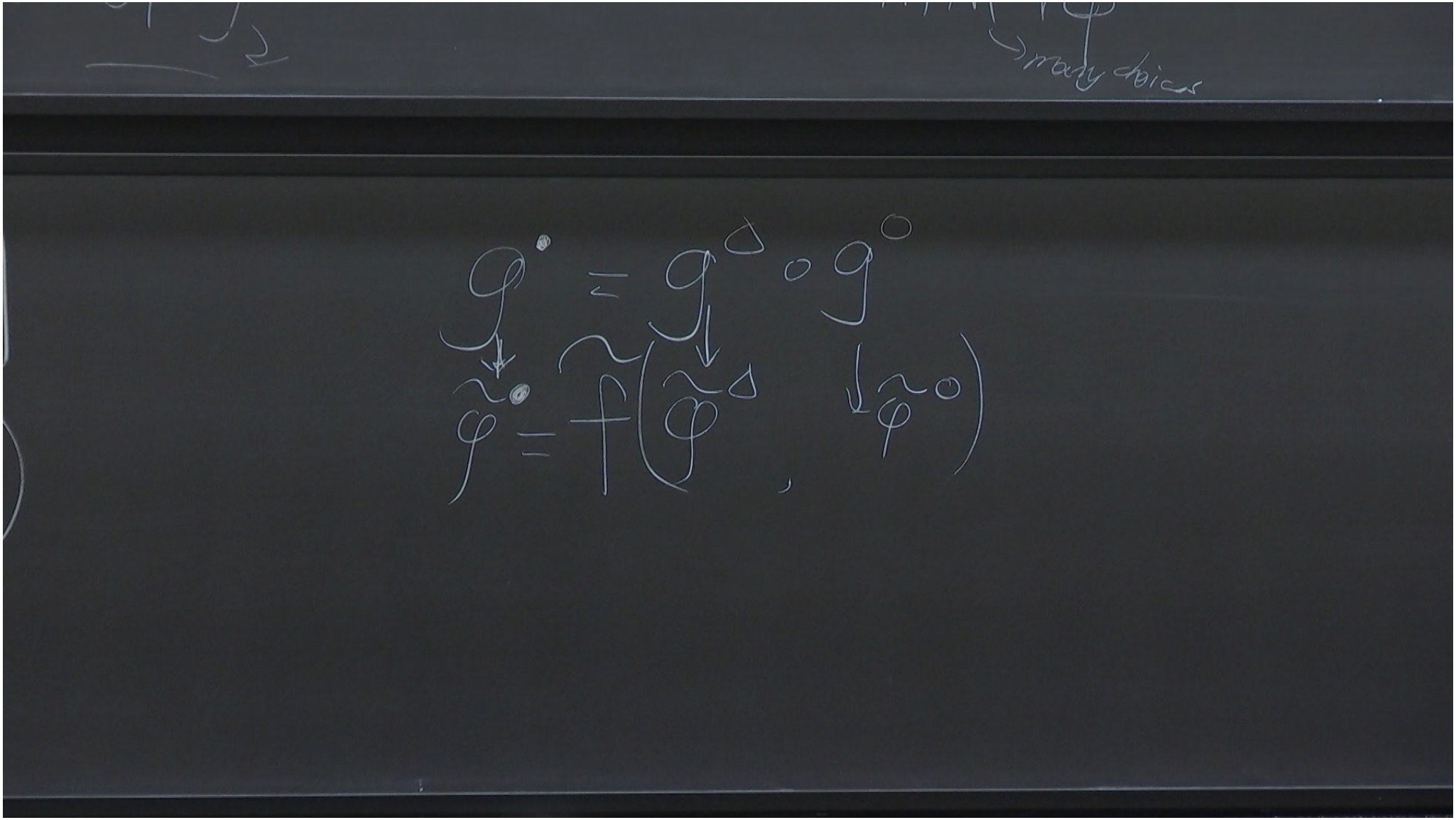
form a group

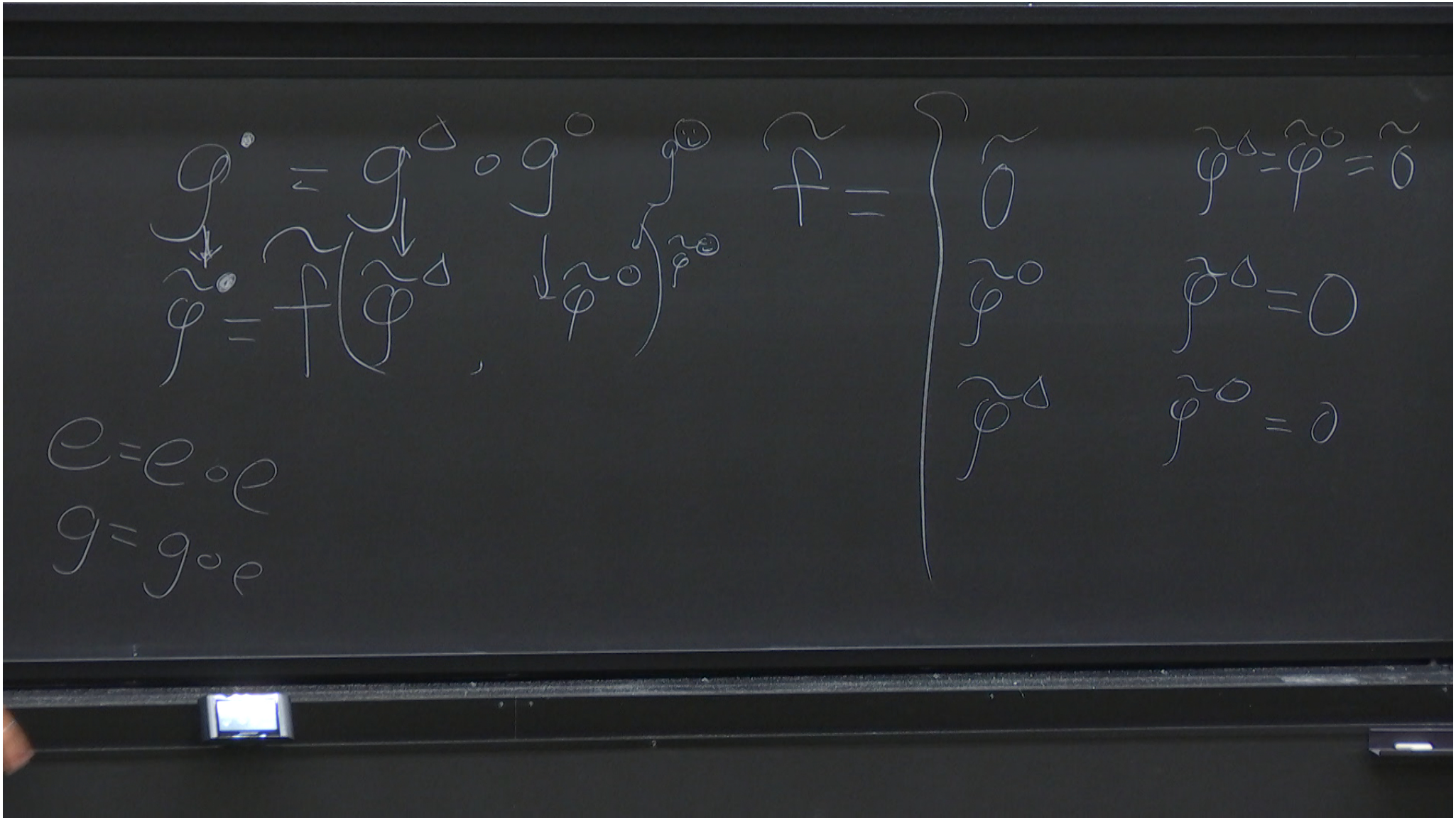
$G = \{g_1, g_2, \cdots\}$ $g_1 \circ g_2$

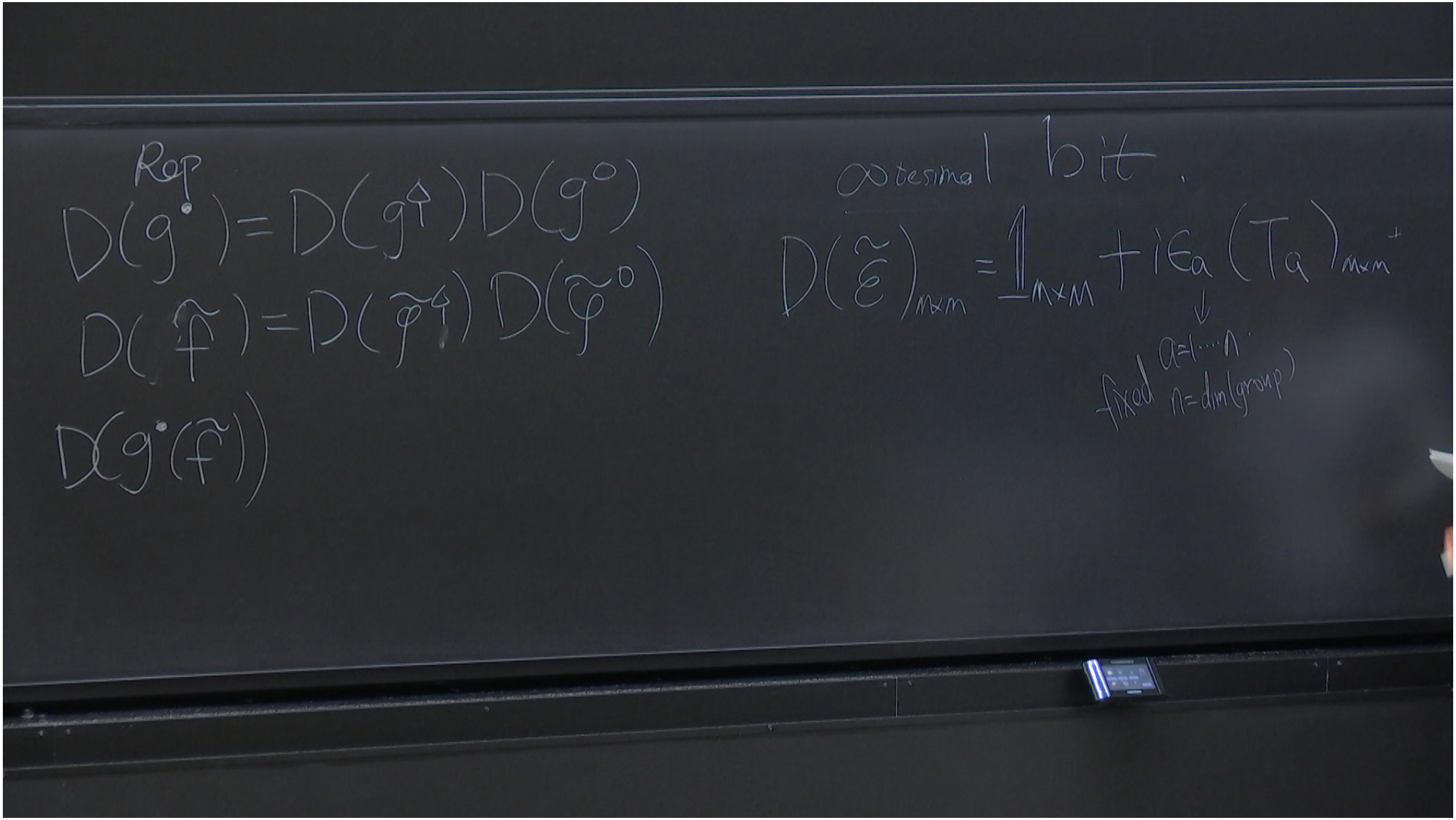
Goal **Structure of group**

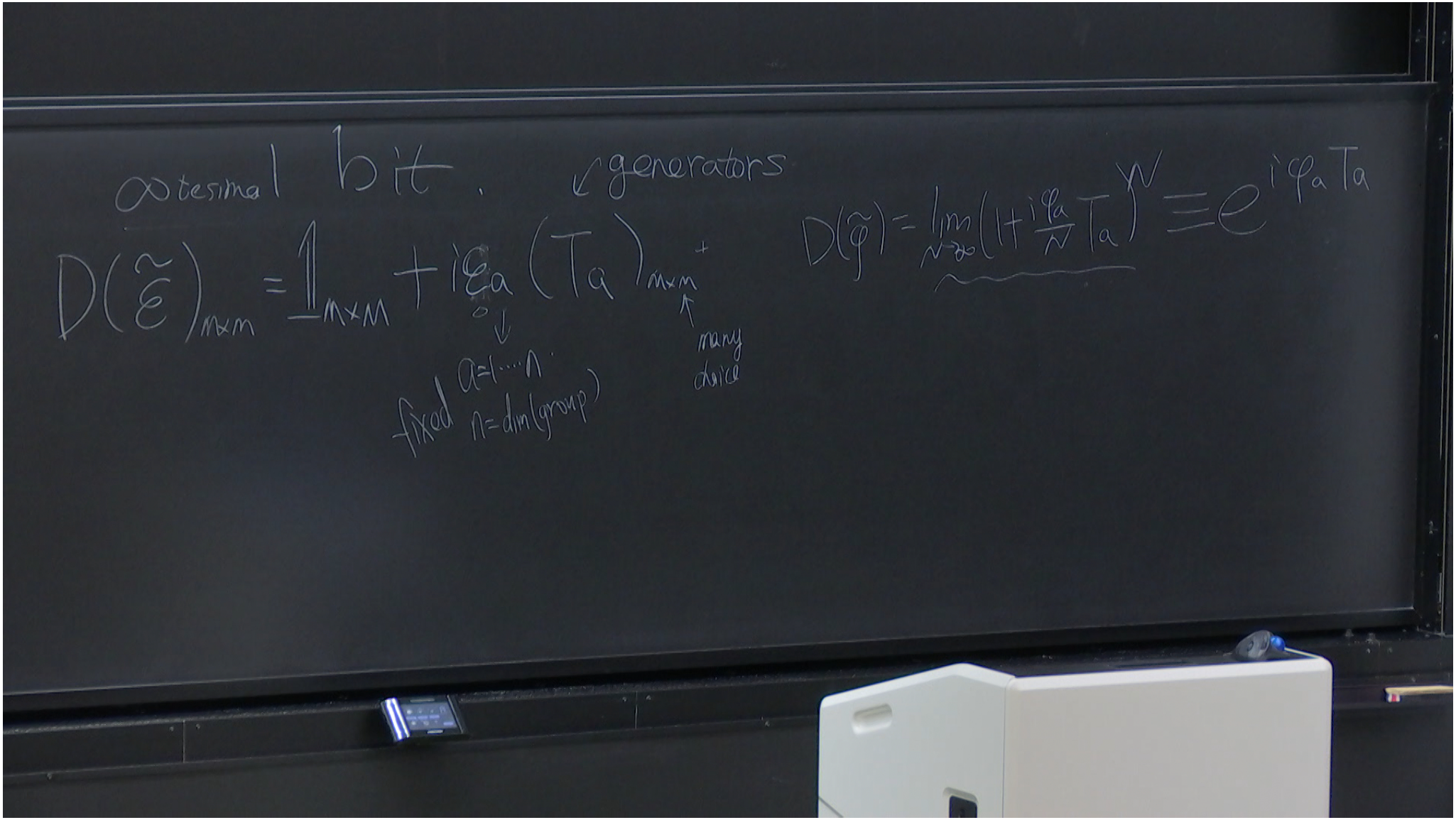
Lie group

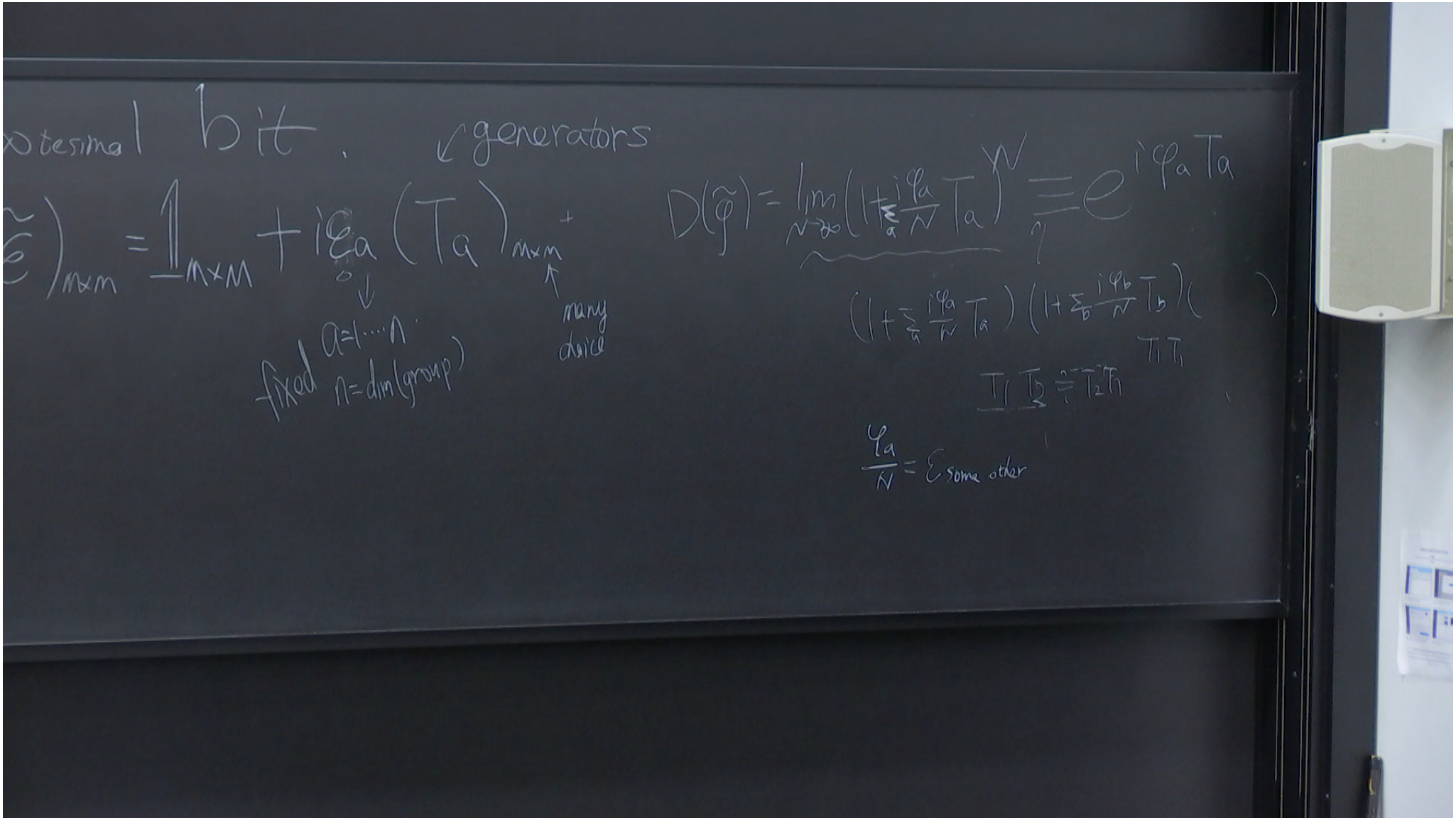

Goal Structure of group
Lie group
$g^\Delta = g(\tilde{\varphi}^\Delta)$

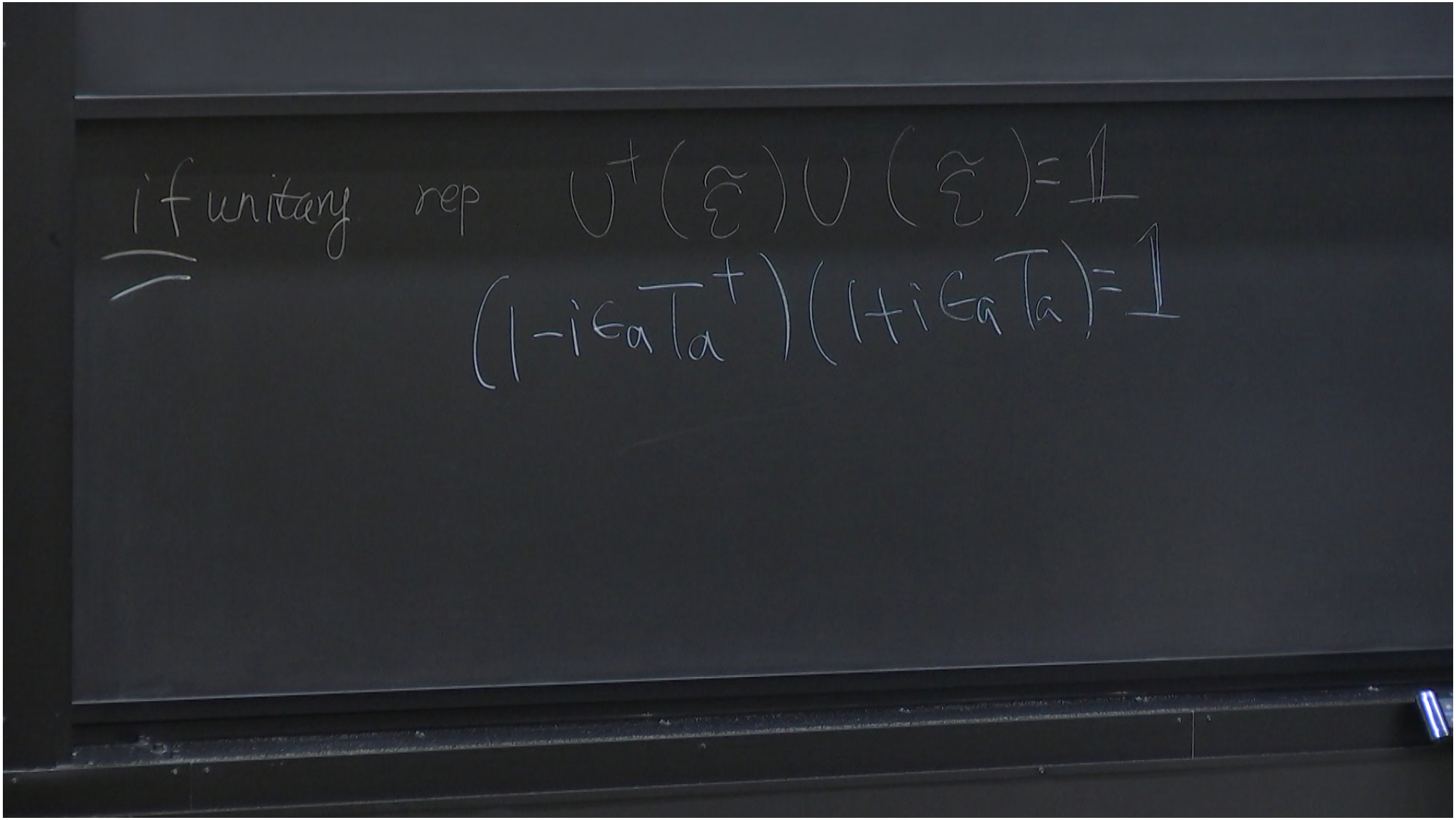

many choices
$g = g^\circ \circ g^\circ$
$\tilde{g} = \tilde{f}(\tilde{g}^\circ, \tilde{g}^\circ)$

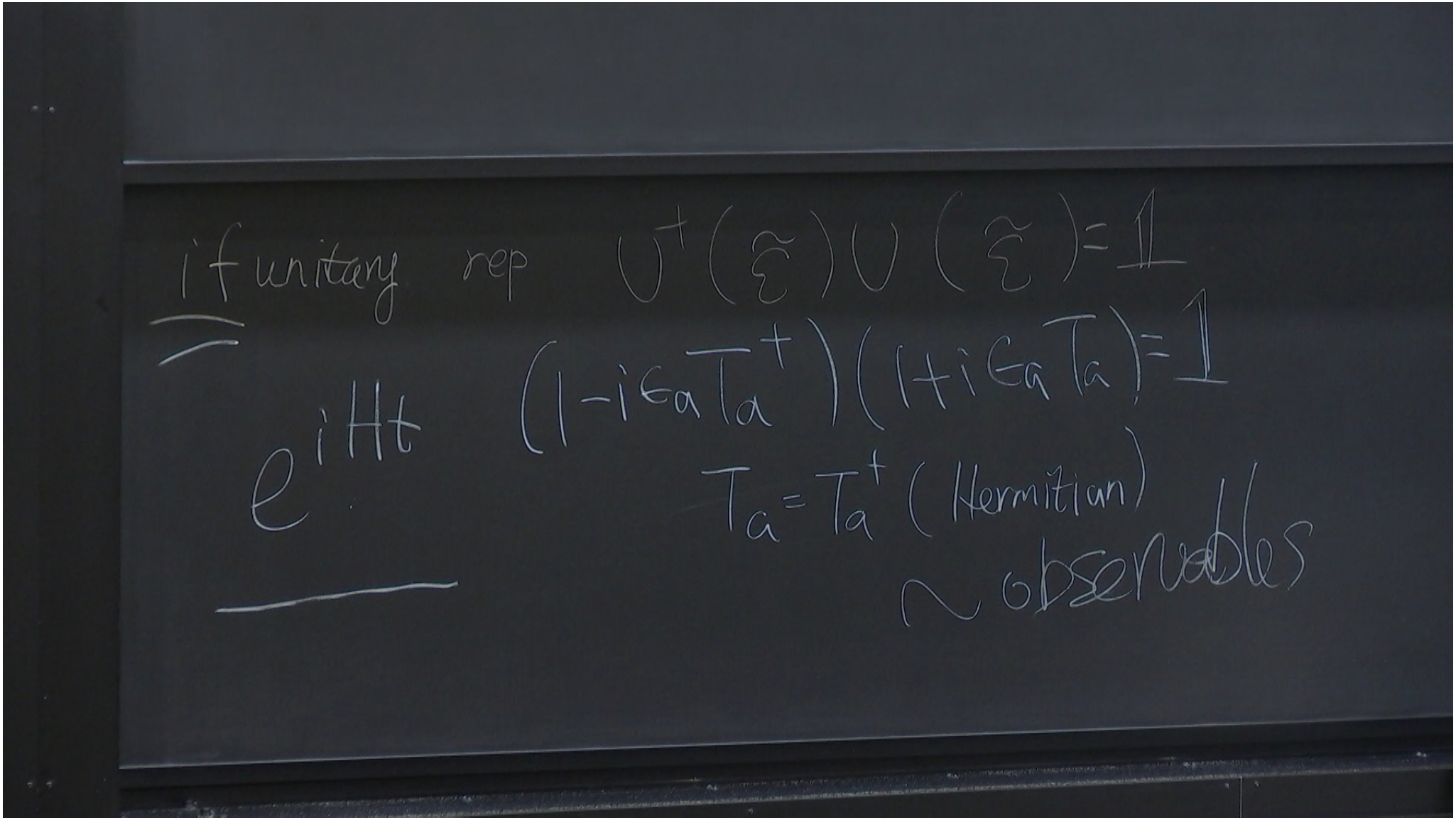

$g = g^{\Delta} \circ g^{\circ}$
$\tilde{\varphi} = \tilde{f}(\tilde{\varphi}^{\Delta}, \tilde{\varphi}^{\circ})$
$\tilde{f} =$
$\tilde{0}$ $\tilde{\varphi}^{\Delta} = \tilde{\varphi}^{\circ} = \tilde{0}$
$\tilde{\varphi}^{\circ}$ $\varphi^{\Delta} = 0$
$\tilde{\varphi}^{\Delta}$ $\tilde{\varphi}^{\circ} = 0$
$e = e \circ e$
$g = g \circ e$


Rep
$D(g^{\uparrow}) = D(g^{\uparrow}) D(g^{\circ})$
$D(\tilde{f}) = D(\tilde{f}^{\uparrow}) D(\tilde{f}^{\circ})$
$D(g^{\circ}(\tilde{f}))$
infinitesimal bit.
$D(\tilde{\varepsilon})_{n\times m} = \mathbb{1}_{n\times M} + i\epsilon_a (T_a)_{n\times m}$
fixed
$a = 1 \dots n$
$n = \dim(\text{group})$


infinitesimal bit.
generators
$D(\tilde{\varepsilon})_{n\times n} = \mathbb{1}_{n\times M} + i\varepsilon_a (T_a)_{n\times n}$
fixed $a=1\dots n$
$n=\dim(\text{group})$
many choice
$D(\tilde{\varphi}) = \lim_{N\to\infty}\left(1+i\frac{\varphi_a}{N}T_a\right)^N \equiv e^{i\varphi_a T_a}$


tesimal bit.
generators
$\tilde{e})_{n\times n} = \mathbb{1}_{n\times n} + i\varepsilon_a (T_a)_{n\times n}$
fixed
$a=1\cdots n$
$n=\dim(\text{group})$
many choice
$D(\tilde{\varphi}) = \lim_{N\to\infty}\left(1+\sum_a \frac{i\varphi_a}{N} T_a\right)^N \equiv e^{i\varphi_a T_a}$
$\left(1+\sum_a \frac{i\varphi_a}{N} T_a\right)\left(1+\sum_b \frac{i\varphi_b}{N} T_b\right)($ $)$
$T_1 T_1$
$T_1 T_2 \neq T_2 T_1$
$\frac{\varphi_a}{N} = \varepsilon$ some other


if unitary rep $U^\dagger(\epsilon)U(\epsilon)=\mathbb{1}$
$(1-i\epsilon_a T_a^\dagger)(1+i\epsilon_a T_a)=1$


if unitary rep $U^\dagger(\xi)U(\xi)=\mathbb{1}$
e^{iHt} $(1-i\epsilon_a T_a^\dagger)(1+i\epsilon_a T_a)=1$
$T_a=T_a^\dagger$ (Hermitian)
~ observables

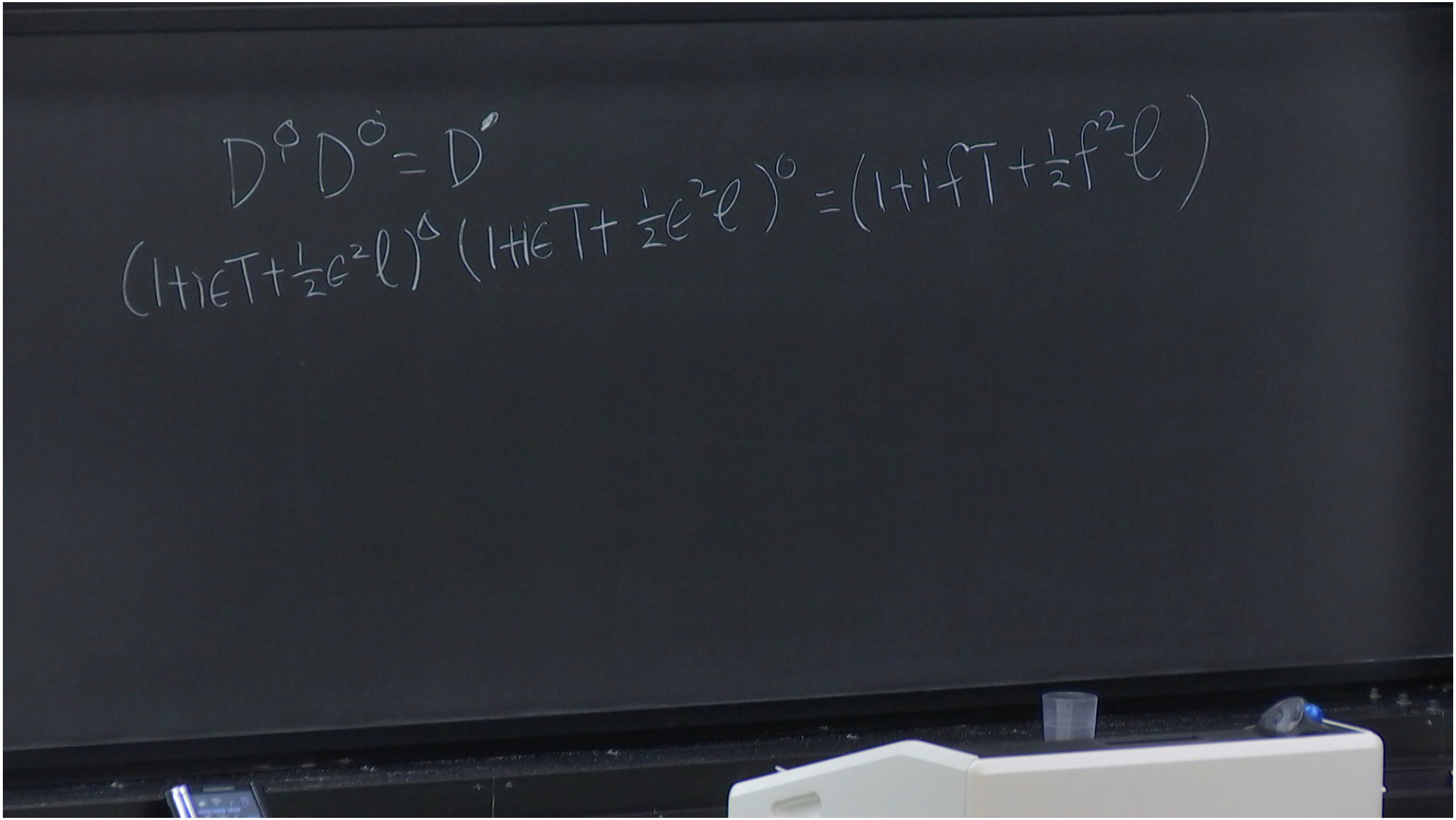
$$D(\tilde{\varepsilon}^{\uparrow})\,D(\tilde{\varepsilon}^{0}) = D(f)$$

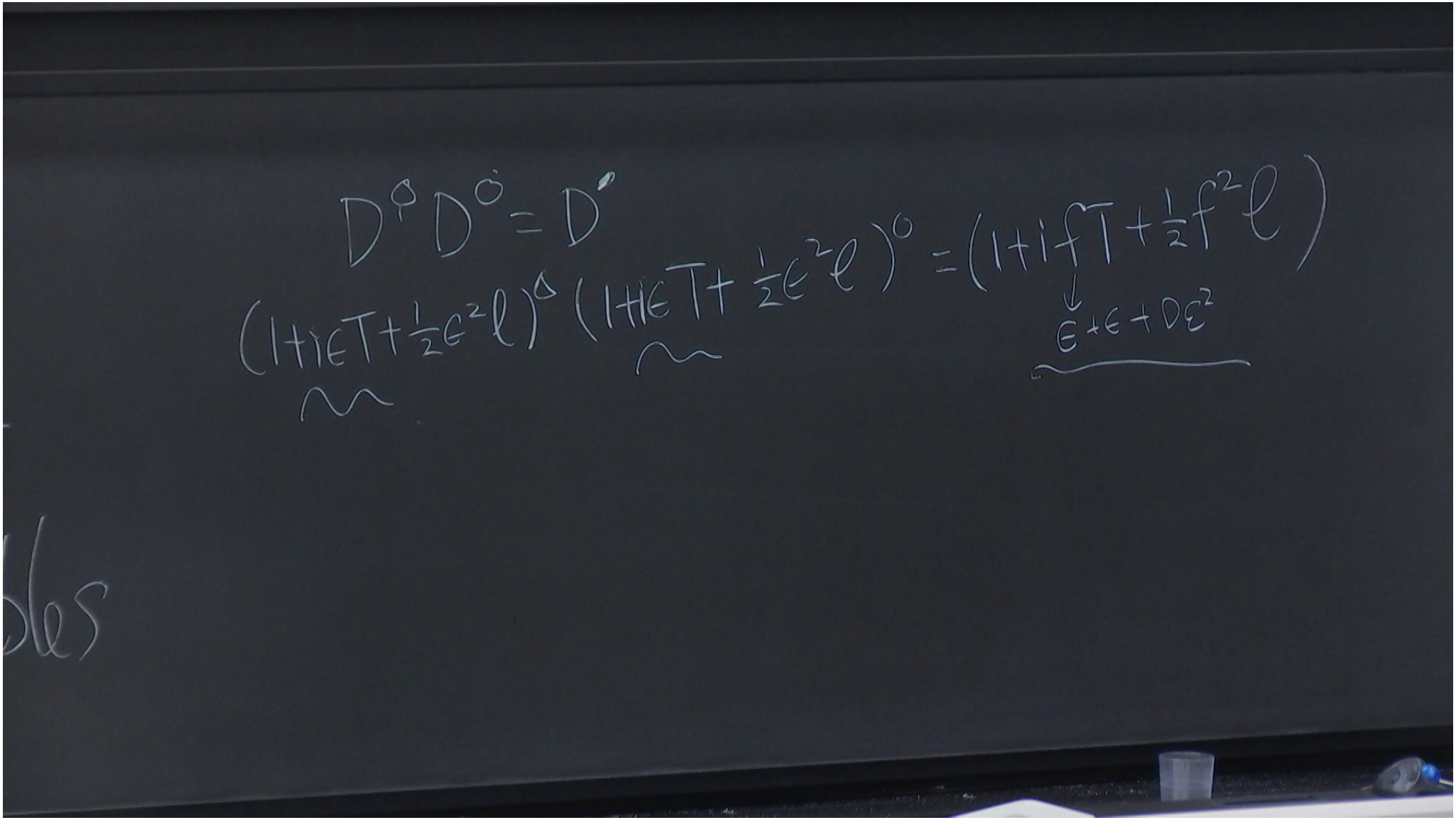
order #2 $\quad D(\varepsilon) = \mathbb{1} + i\varepsilon_a T_a + \frac{1}{2}\varepsilon_a\varepsilon_b\, l_{ab}$ symmetric

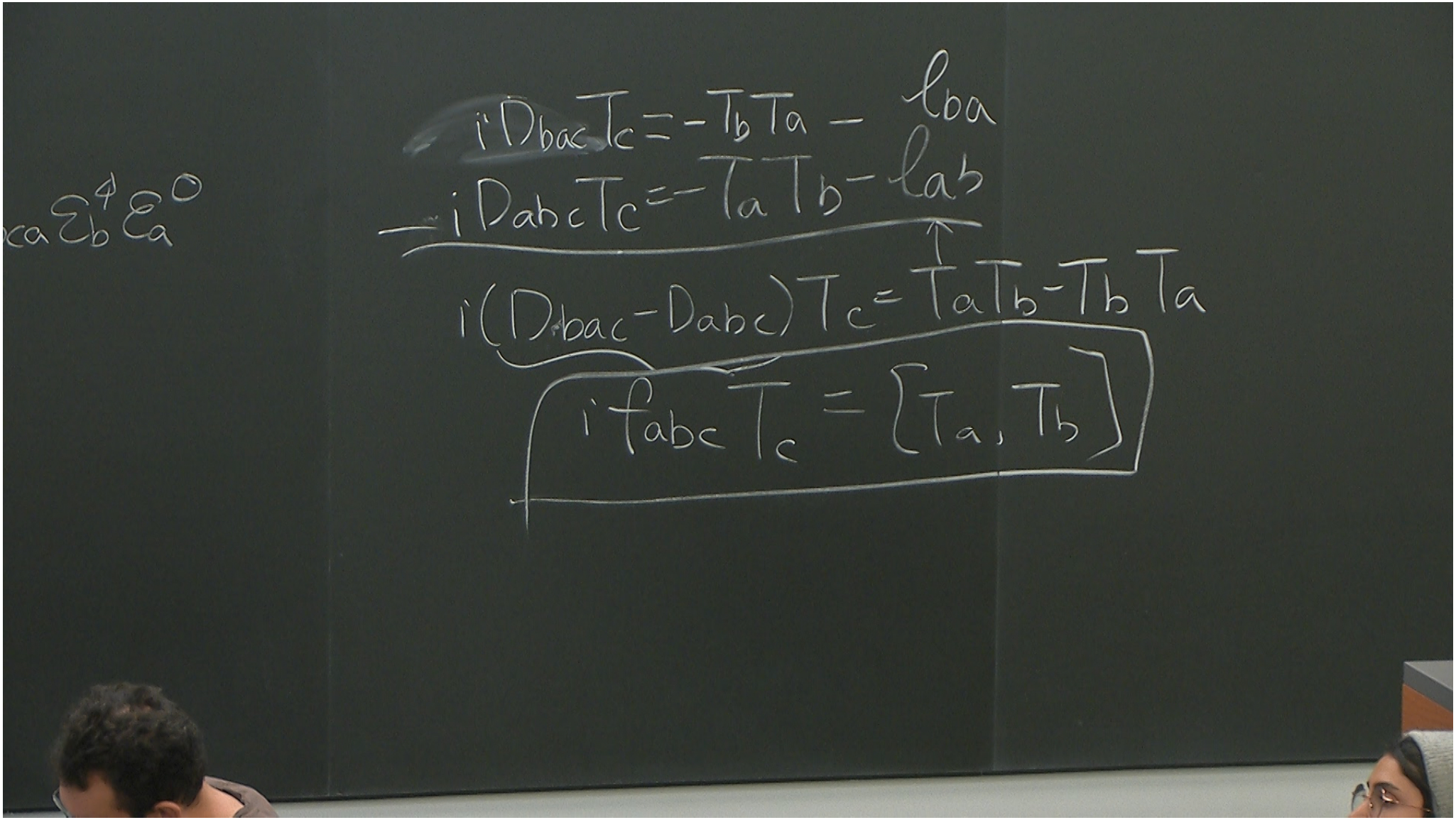
$$f_a(\tilde{\varepsilon}^{\uparrow}, \tilde{\varepsilon}^{0}) = \overset{\delta_{ba}}{B_{ba}}\,\varepsilon_b^{\uparrow} + \overset{\delta_{ba}}{\zeta_{ba}}\,\varepsilon_b^{0}$$

$$+ D_{bca}\,\varepsilon_b^{\uparrow}\varepsilon_c^{0} + E\!\!\!/_{bca}\,\varepsilon_b^{\uparrow}\varepsilon_c^{\uparrow} + F\!\!\!/_{bca}\,\varepsilon_b^{0}\varepsilon_c^{0}$$



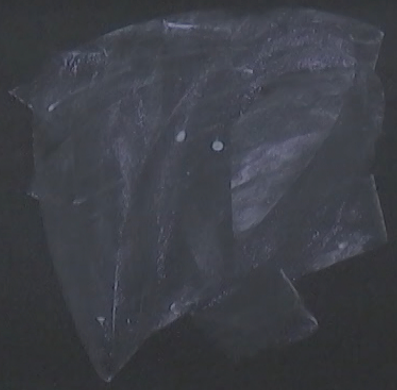

$D^0 D^0 = D^0$
$(1+i\epsilon T+\frac{1}{2}\epsilon^2\ell)^0 (1+i\epsilon T+\frac{1}{2}\epsilon^2\ell)^0 = (1+ifT+\frac{1}{2}f^2\ell)$


$D^{\alpha} D^{\circ} = D^{\circ}$
$(1+i\epsilon T+\frac{1}{2}\epsilon^2 \ell)^{\alpha} (1+i\epsilon T+\frac{1}{2}\epsilon^2 \ell)^{\circ} = (1+ifT+\frac{1}{2}f^2 \ell)$
$\epsilon+\epsilon+O\epsilon^2$


$i D_{bac} T_c = -T_b T_a - l_{ba}$
$-i D_{abc} T_c = -T_a T_b - l_{ab}$
$i(D_{bac} - D_{abc}) T_c = T_a T_b - T_b T_a$
$i f_{abc} T_c = [T_a, T_b]$

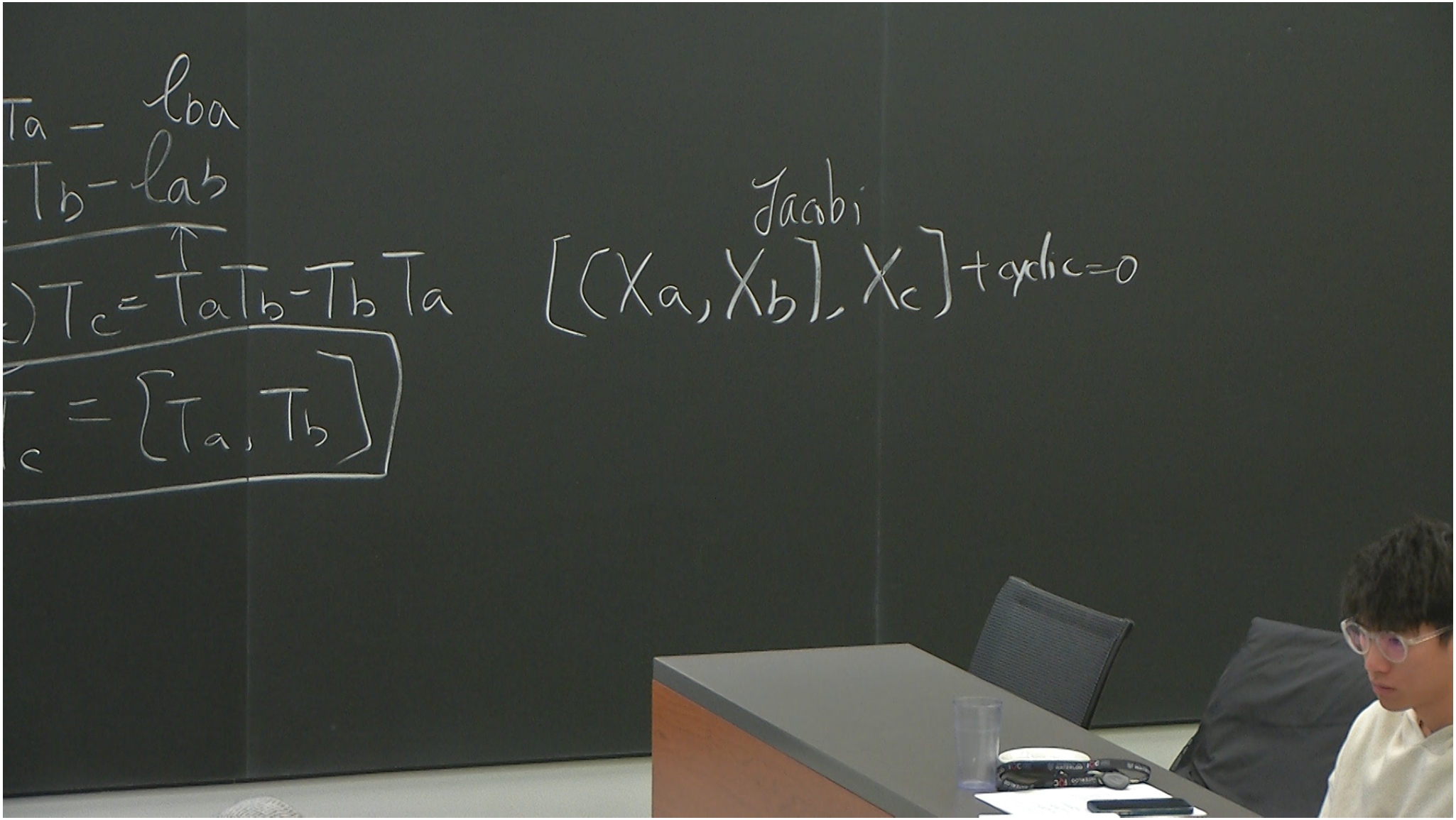
Goal: Structure constant of group

Lie group $g^{\Delta} = g(\tilde{\varphi}^{\Delta})$



$e = e$

$g = g$


Ta − lba
Tb − lab
)Tc = TaTb − TbTa
= [Ta, Tb]
Jacobi
[(Xa, Xb], Xc] + cyclic = 0