Title: Quantum Field Theory I - Lecture 221011

Speakers: Gang Xu

Collection: Quantum Field Theory I (2022/2023)

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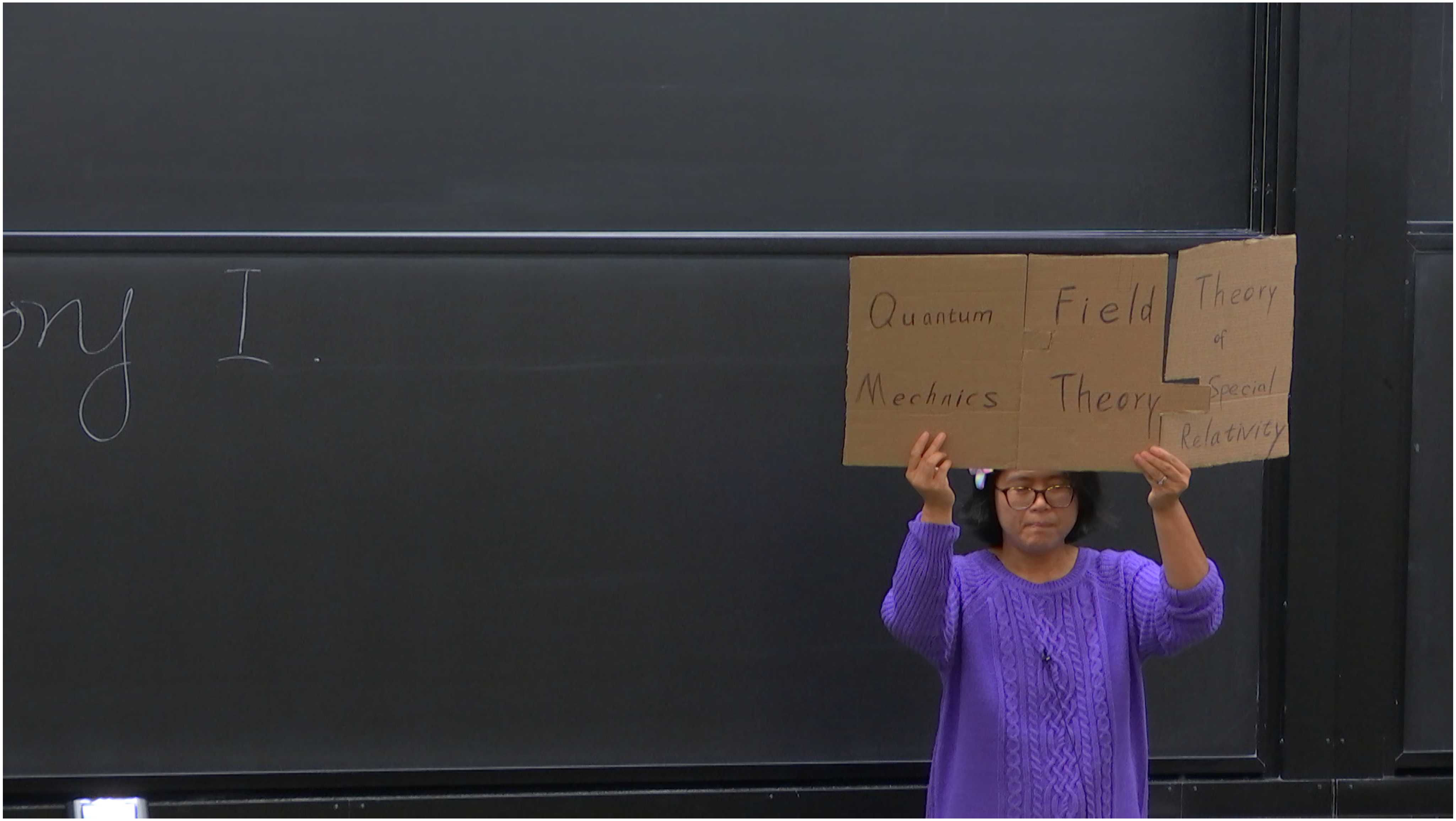
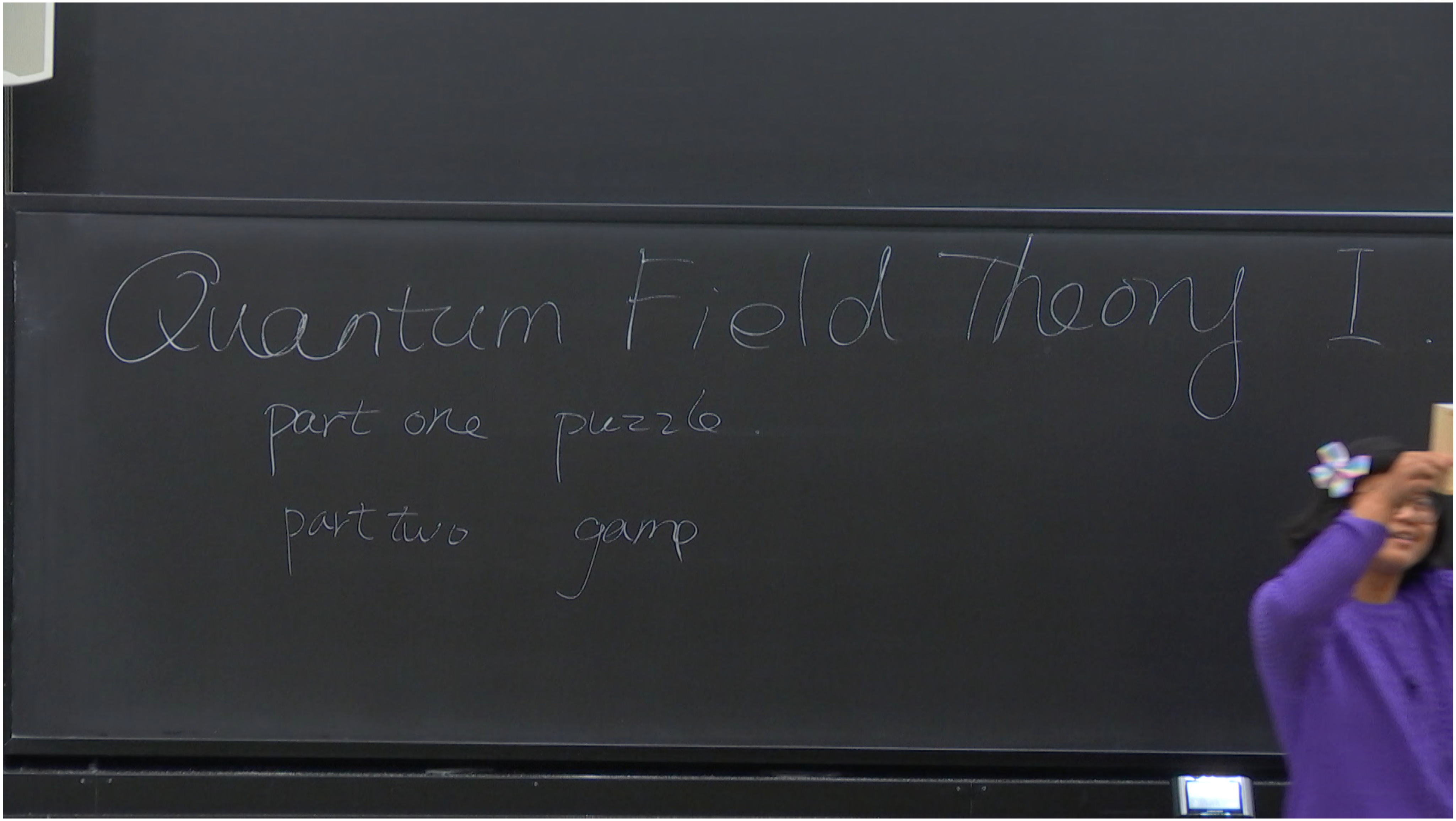
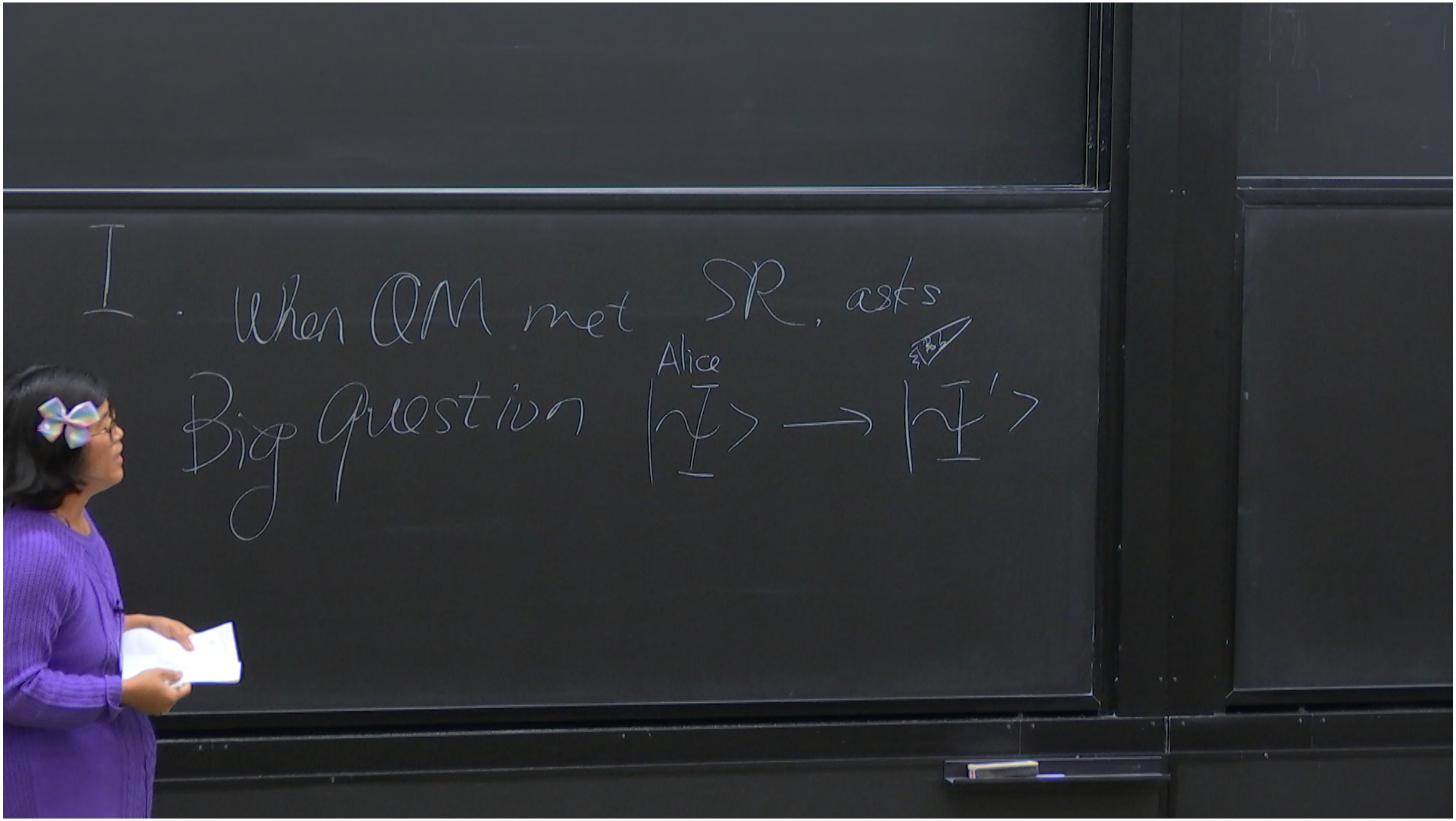
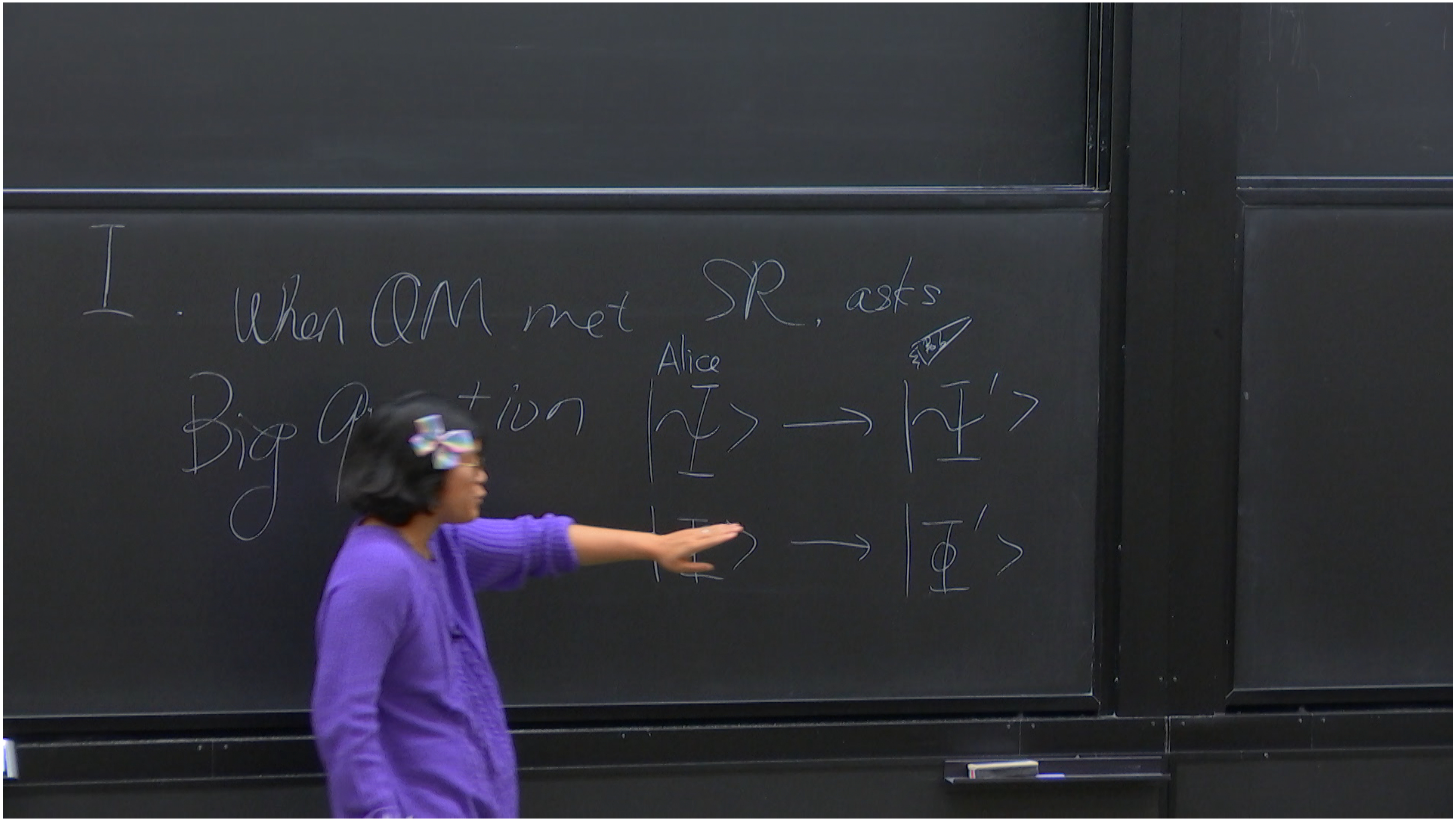
Quantum Field Theory I

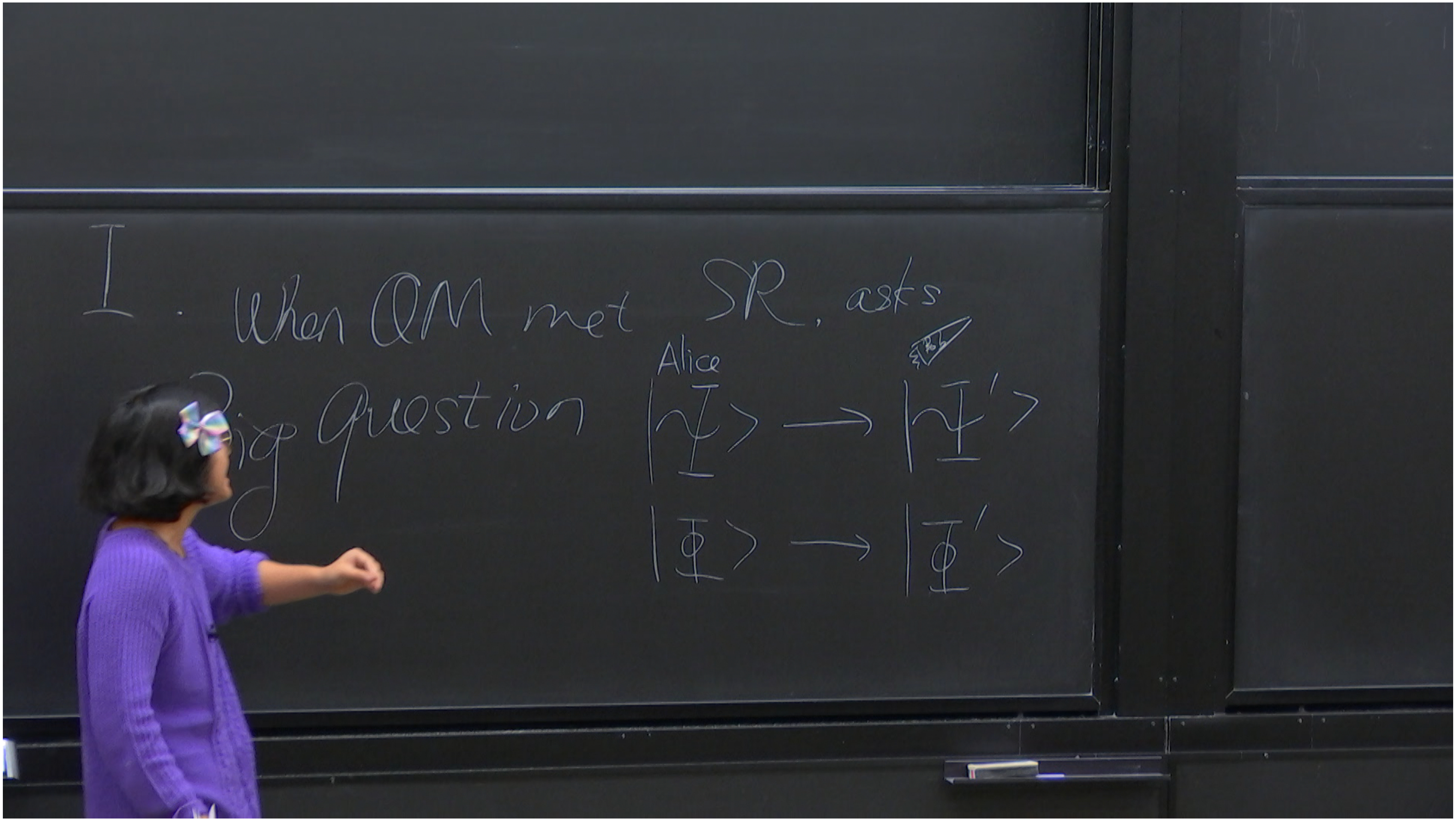

ory I.
Quantum Field Theory
Mechnics Theory
of
Special
Relativity

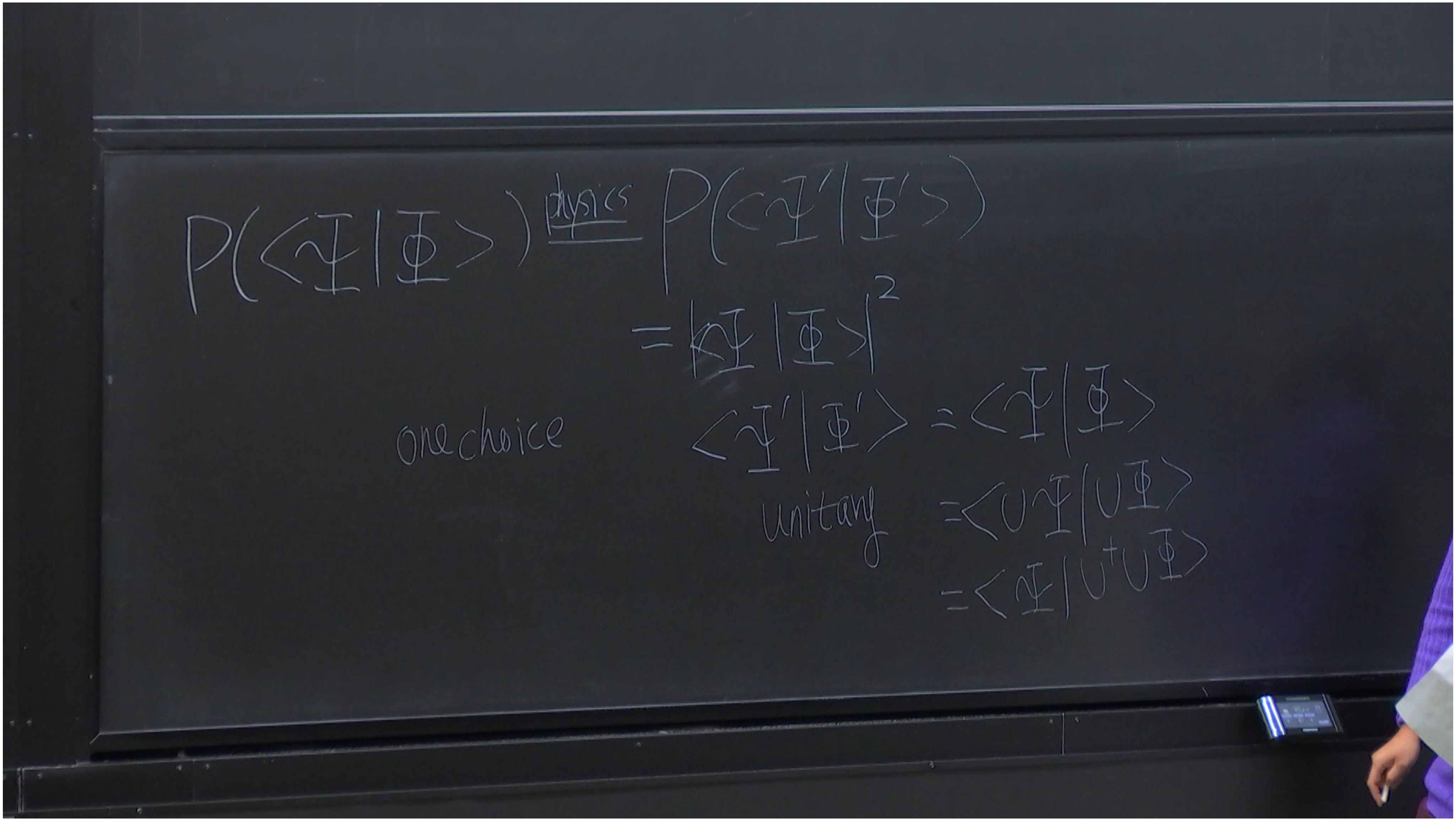

QM
T
SR


Quantum Field Theory I.
part one puzzle.
part two game


I. When QM met SR, asks
Alice
Big question |Ψ> → |Ψ'>


I. When QM met SR, asks
Alice
Big
tion $|\Psi\rangle \longrightarrow |\Psi'\rangle$
$|\Phi\rangle \longrightarrow |\Phi'\rangle$


I. When QM met SR, asks
Alice
Big question
$|\Psi\rangle \longrightarrow |\Psi'\rangle$
$|\Phi\rangle \longrightarrow |\Phi'\rangle$


$P(\langle\tilde{\Psi}|\Phi\rangle) \overset{\text{physics}}{=} P(\langle\tilde{\Psi}'|\Phi'\rangle)$
$= |\langle\Psi|\Phi\rangle|^2$
one choice
$\langle\tilde{\Psi}'|\Phi'\rangle = \langle\tilde{\Psi}|\Phi\rangle$
unitary
$= \langle U\tilde{\Psi}|U\Phi\rangle$
$= \langle\tilde{\Psi}|U^{\dagger}U\Phi\rangle$

$$P(\langle \Psi | \Phi \rangle) \xrightarrow{\text{physics}} P(\langle \Psi' | \Phi' \rangle)$$

$$= |\langle \Psi | \Phi \rangle|^2$$

one choice $\quad \langle \Psi' | \Phi' \rangle = \langle \Psi | \Phi \rangle$

unitary $\quad = \langle U\Psi | U\Phi \rangle$

$$= \langle \Psi | \underbrace{U^\dagger U}_{\mathbb{1}} \Phi \rangle$$

Bob Alice

$$|\Psi'\rangle = U_1|\Psi\rangle$$

Caro Bob

$$|\Psi''\rangle = U_2|\Psi'\rangle$$

Alice

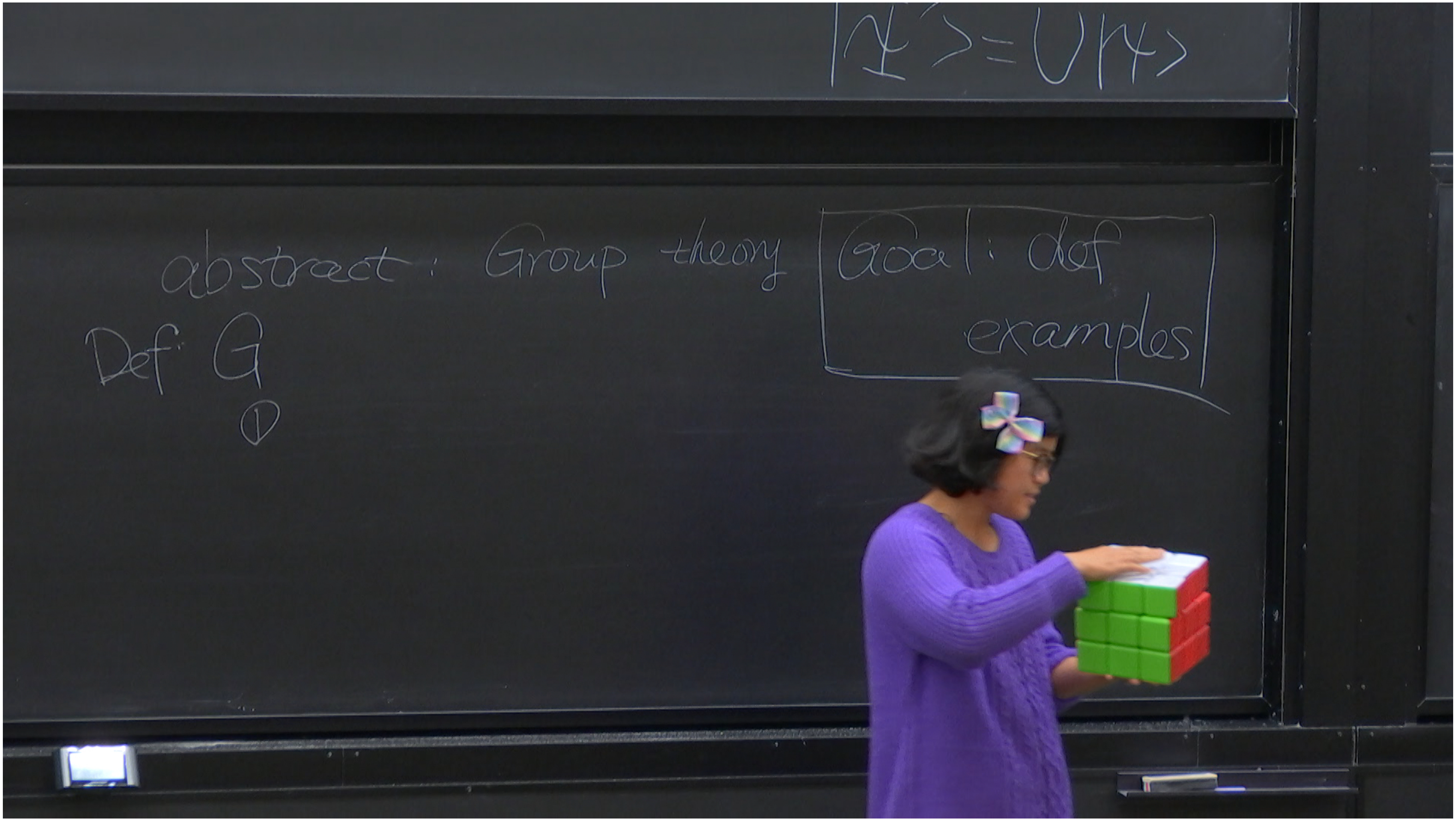
$$= U_2 U_1|\Psi\rangle$$

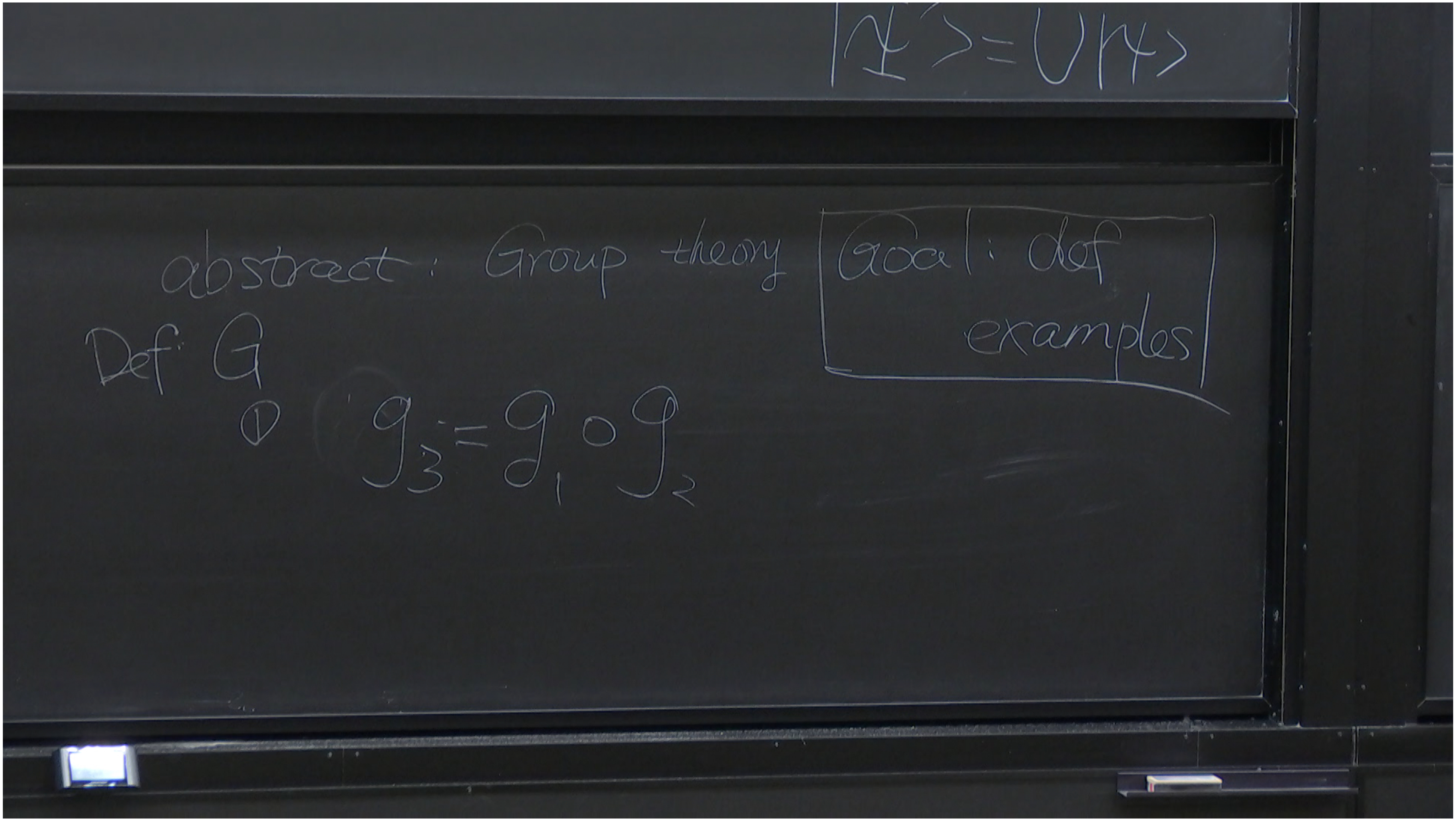
Alice

$$= U_3|\Psi\rangle$$

$$U_3 = U_1 U_2$$

symmetry


|Ψ'> = U|Ψ>
abstract: Group theory
Goal: def
examples
Def: G
①



$|\Psi'\rangle = U|\Psi\rangle$
abstract: Group theory
Goal: def
examples
Def: G
① $g_3 = g_1 \circ g_2$

$$|\Psi'\rangle = U|\Psi\rangle$$

abstract: Group theory

$\boxed{\text{Goal: def, examples}}$

Def: G

① $$g_3 = g_1 \circ g_2$$

if $g_1 \in G$, $g_2 \in G$, then $g_3 \in G$

$|\Psi'\rangle = U|\Psi\rangle$

abstract: Group theory

Goal: def
examples

Def: G

① closed $g_3 = g_1 \circ g_2$

if $g_1 \in G$, $g_2 \in G$, then $g_3 \in G$

② e: identity exists

$|\Psi'\rangle = U|\psi\rangle$

abstract: Group theory

$\boxed{\text{Goal: def, examples}}$

Def: G

closed

① $g_3 = g_1 \circ g_2$

if $g_1 \in G$, $g_2 \in G$, then $g_3 \in G$

② e: identity exists

③ inverse exists

$|\Psi'\rangle = U|\Psi\rangle$

abstract: Group theory

$\boxed{\text{Goal: def examples}}$

Def: G

① closed

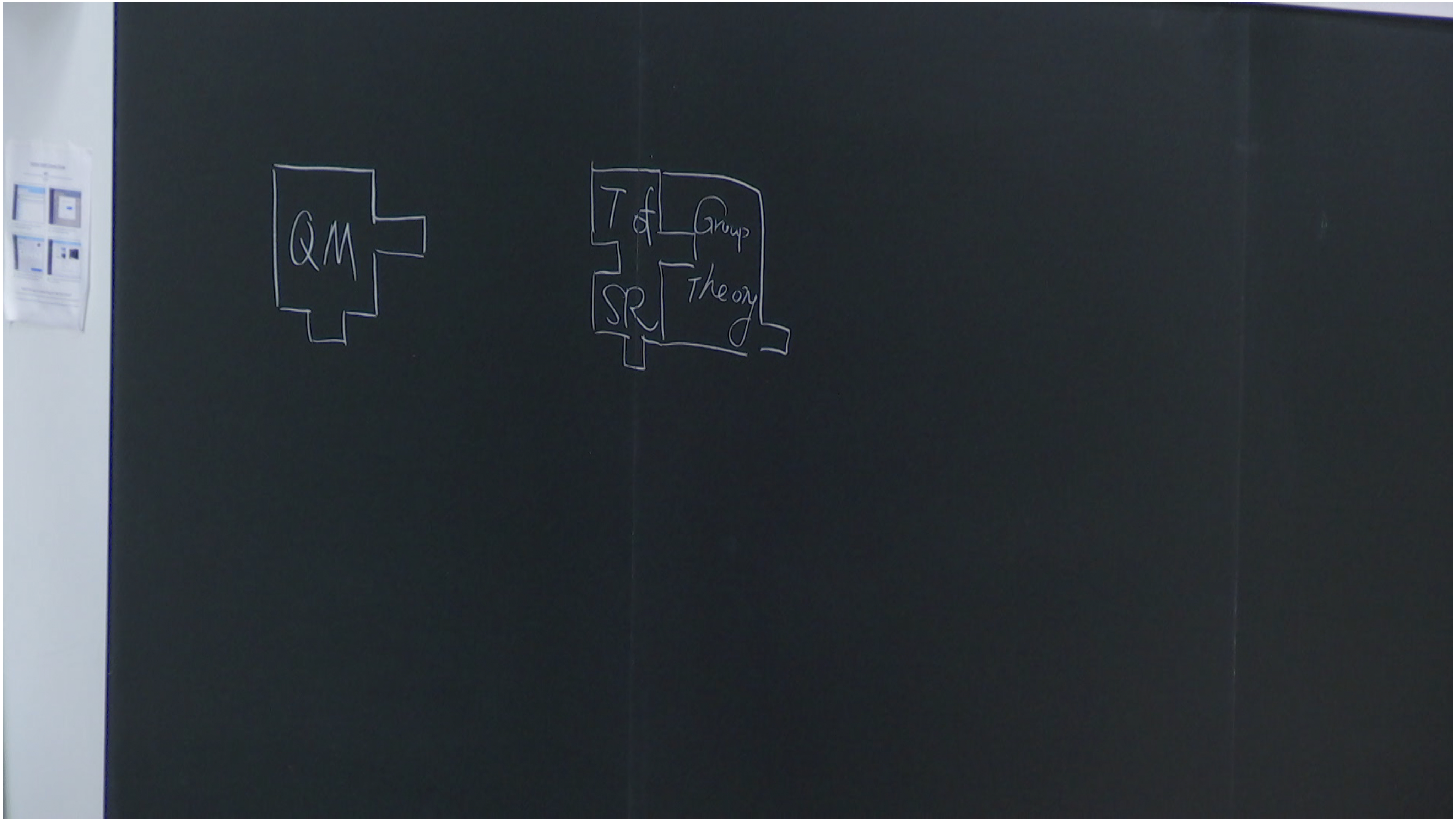
$$g_3 = g_1 \circ g_2$$

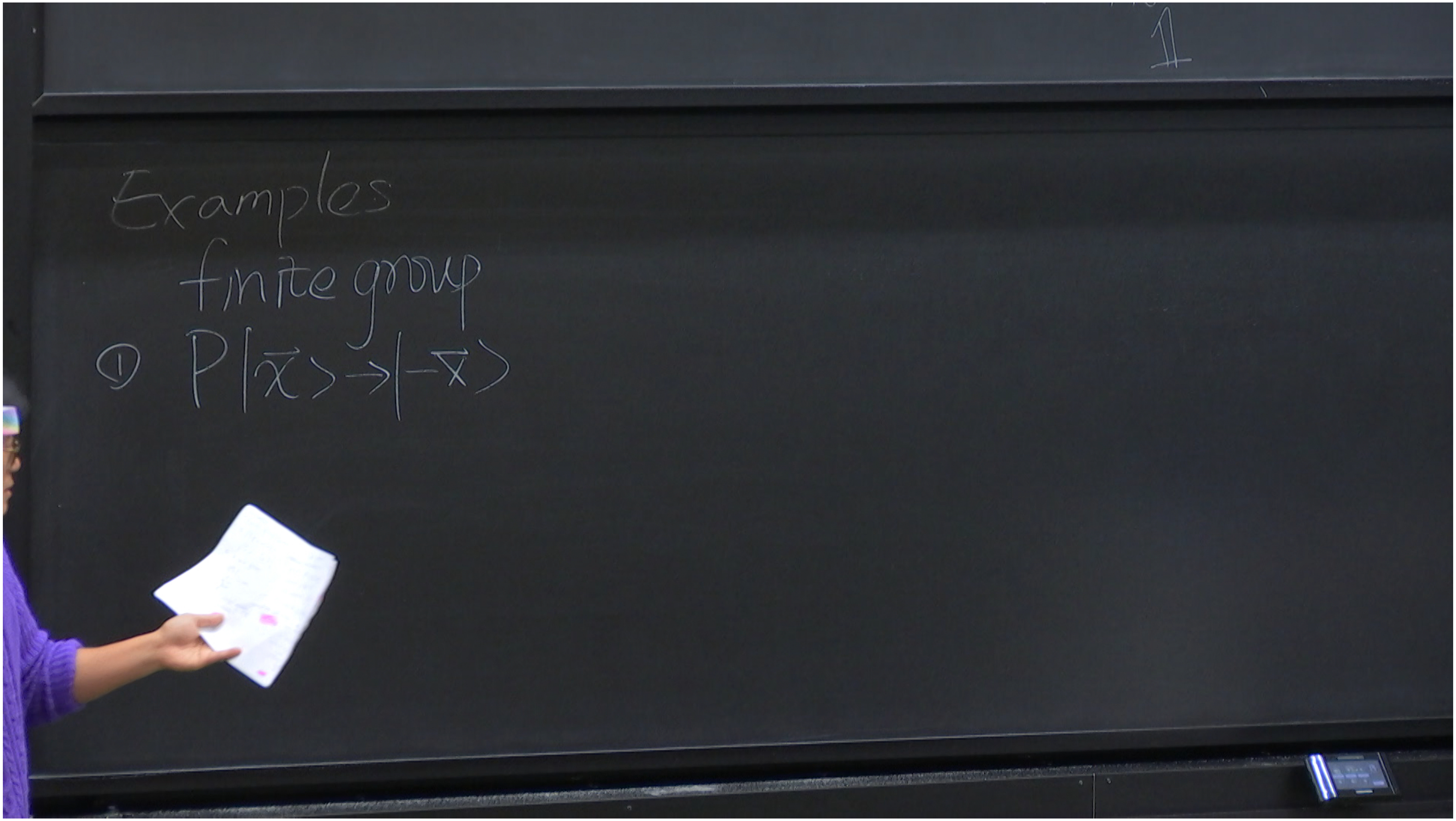
if $g_1 \in G$, $g_2 \in G$, then $g_3 \in G$

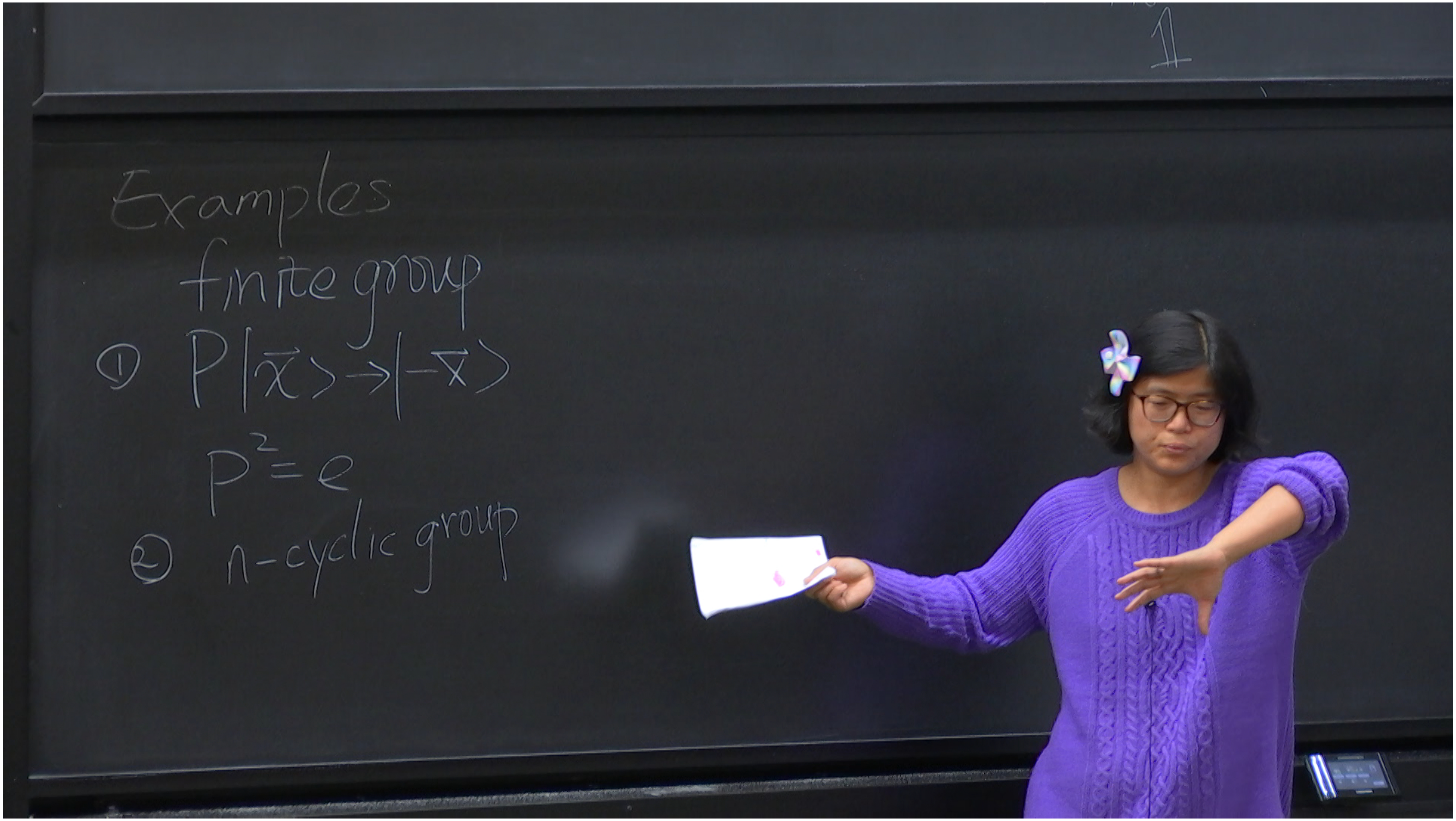
② e: identity exists

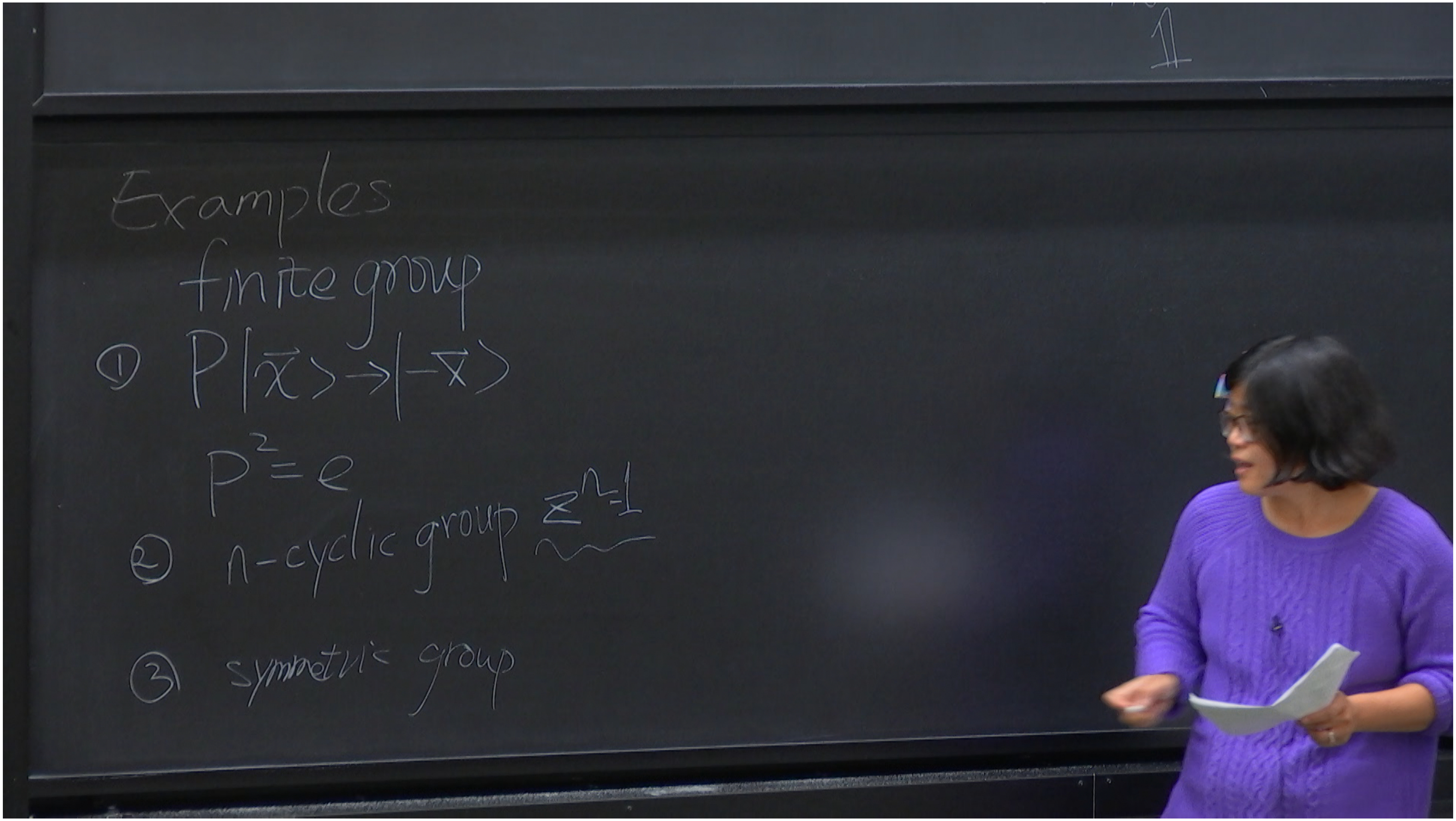
③ inverse exists

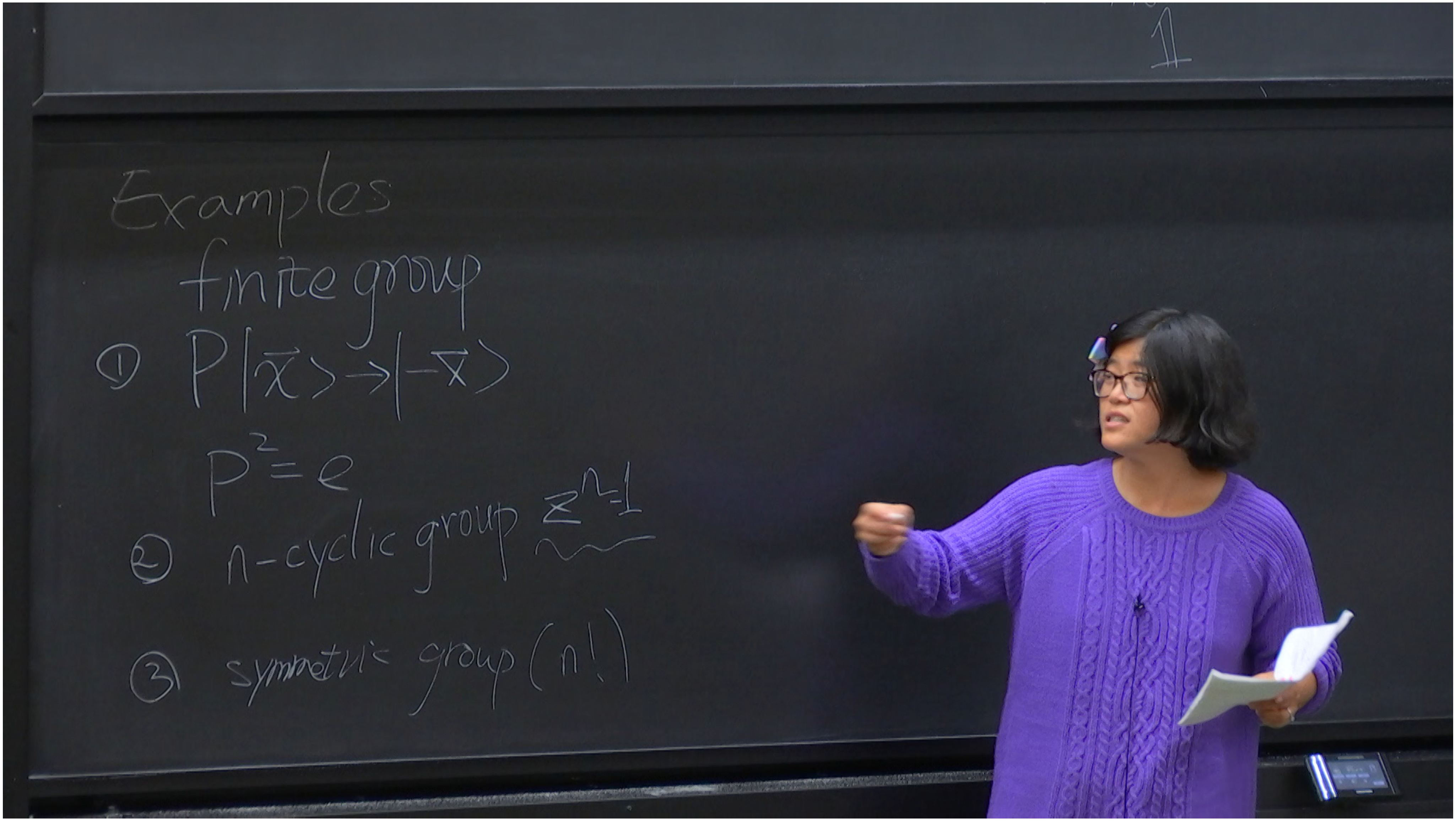
④ associativity

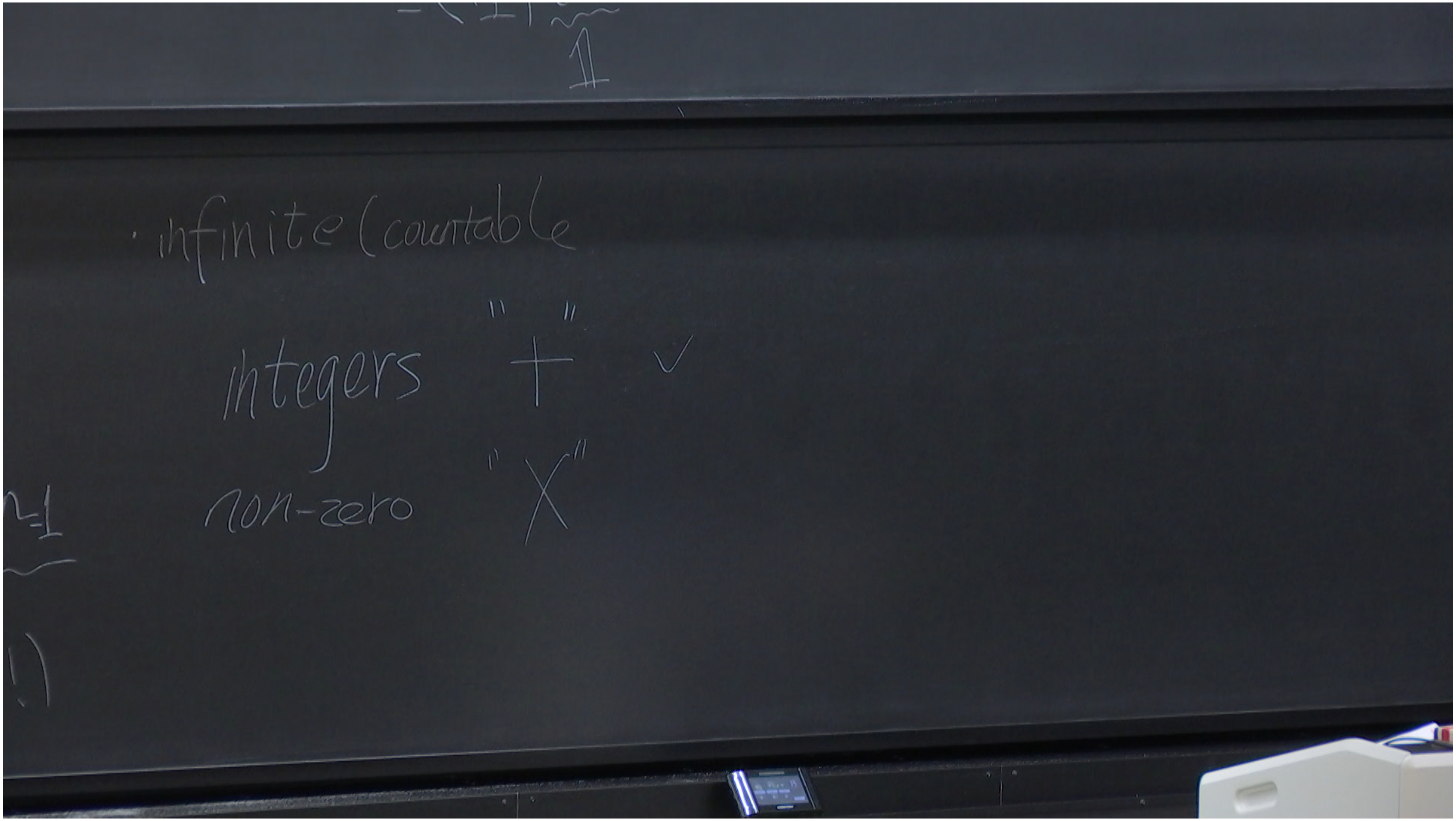

QM
T of
Group
SR
Theory


Examples
finite group
① $P|\vec{x}\rangle \to |-\vec{x}\rangle$


Examples
finite group
① P|x⃗> → |−x⃗>
P² = e
② n-cyclic group


Examples
finite group
① P|x⃗> → |−x⃗>
P² = e
② n-cyclic group zⁿ=1
③ symmetric group


Examples
finite group
① $P|\vec{x}\rangle \to |-\vec{x}\rangle$
$P^2 = e$
② n-cyclic group $z^n = 1$
③ symmetric group (n!)


· infinite (countable
integers "+" ✓
non-zero "×"

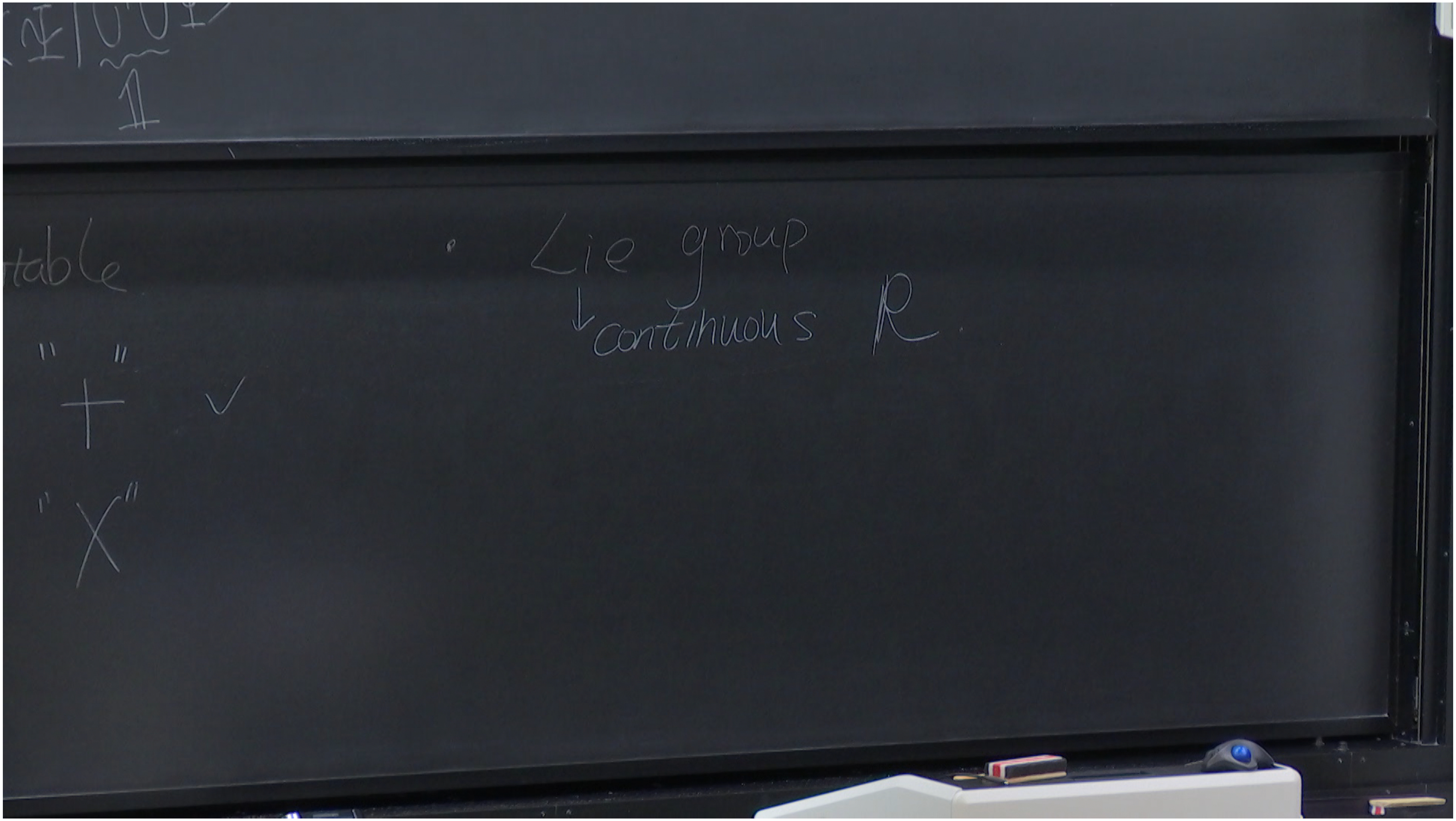


table
• Lie group
continuous R
"+"
"X"

ntable

" + " ✓

" X "

- Lie group

↳ continuous $\mathbb{R}$.

$g(\tilde{\varphi})$ $\tilde{\varphi} = (\varphi_1, \varphi_2, \ldots)$

choose $g(\tilde{\varphi} = 0) = e$

2D rotation $g(\theta)$

ntable

" + " ✓

" X "

• Lie group

↓ continuous R.

$g(\tilde{\varphi})$ $\tilde{\varphi} = (\varphi_1, \varphi_2, \cdots)$

choose $g(\tilde{\varphi} = 0) = e$

1 parameter: 2D rotation $g(\theta)$

ntable

"+" ✓

"X"

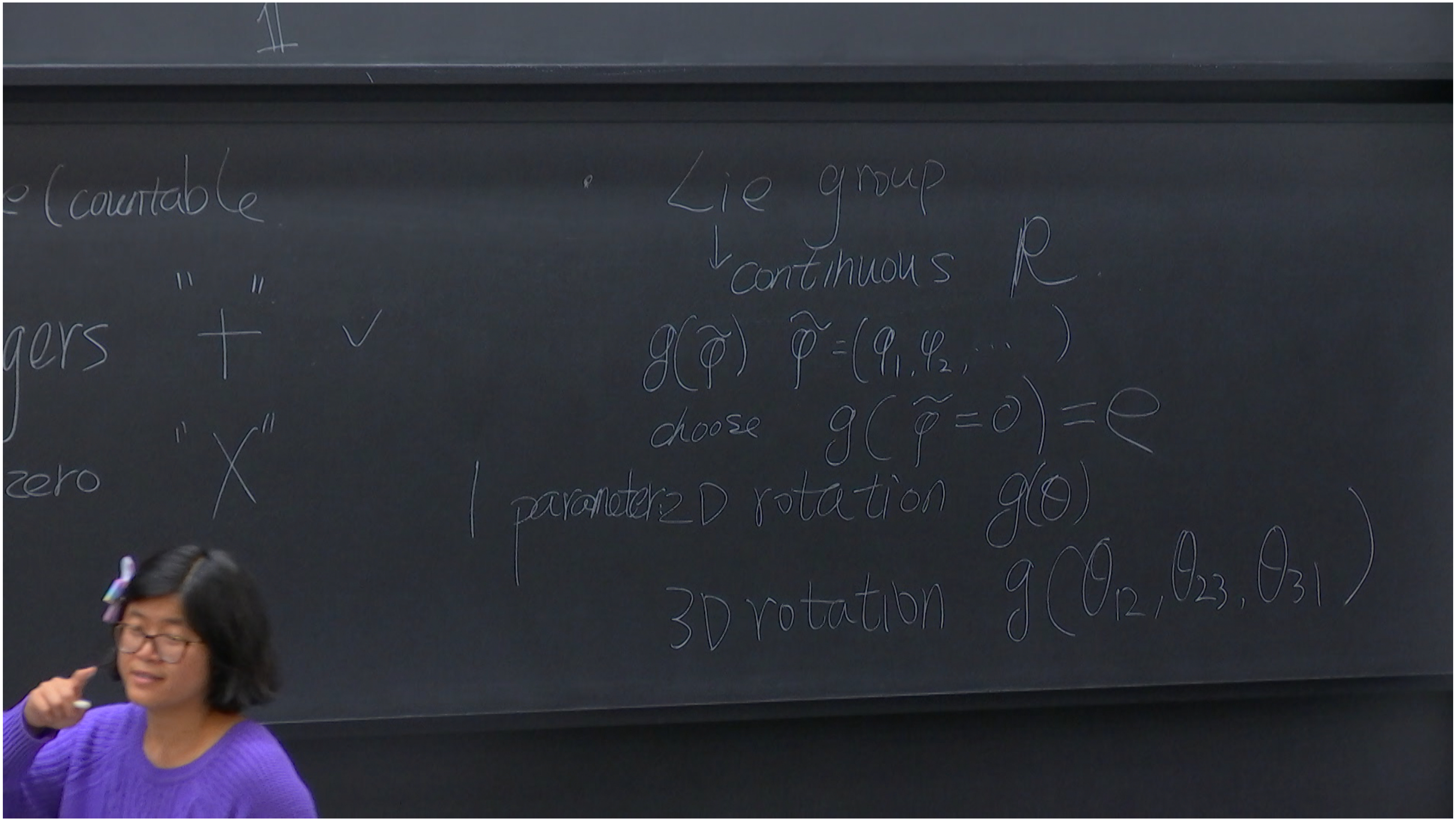
• Lie group

↓ continuous $\mathbb{R}$.

$g(\tilde{\varphi})$ $\tilde{\varphi} = (\varphi_1, \varphi_2, \dots)$

choose $g(\tilde{\varphi} = 0) = e$

1 parameter: 2D rotation $g(\theta)$


e (countable
Lie group
continuous R
gers
" + "
zero
" X "
$g(\tilde{\varphi})$ $\tilde{\varphi} = (\varphi_1, \varphi_2, \dots)$
choose $g(\tilde{\varphi} = 0) = e$
1 parameterizeD rotation $g(\theta)$
3D rotation $g(\theta_{12}, \theta_{23}, \theta_{31})$

Examples

finite group

① $P|\vec{x}\rangle \to |-\vec{x}\rangle$

$P^2 = e$

② n-cyclic group $z^n = 1$ ~ C_n

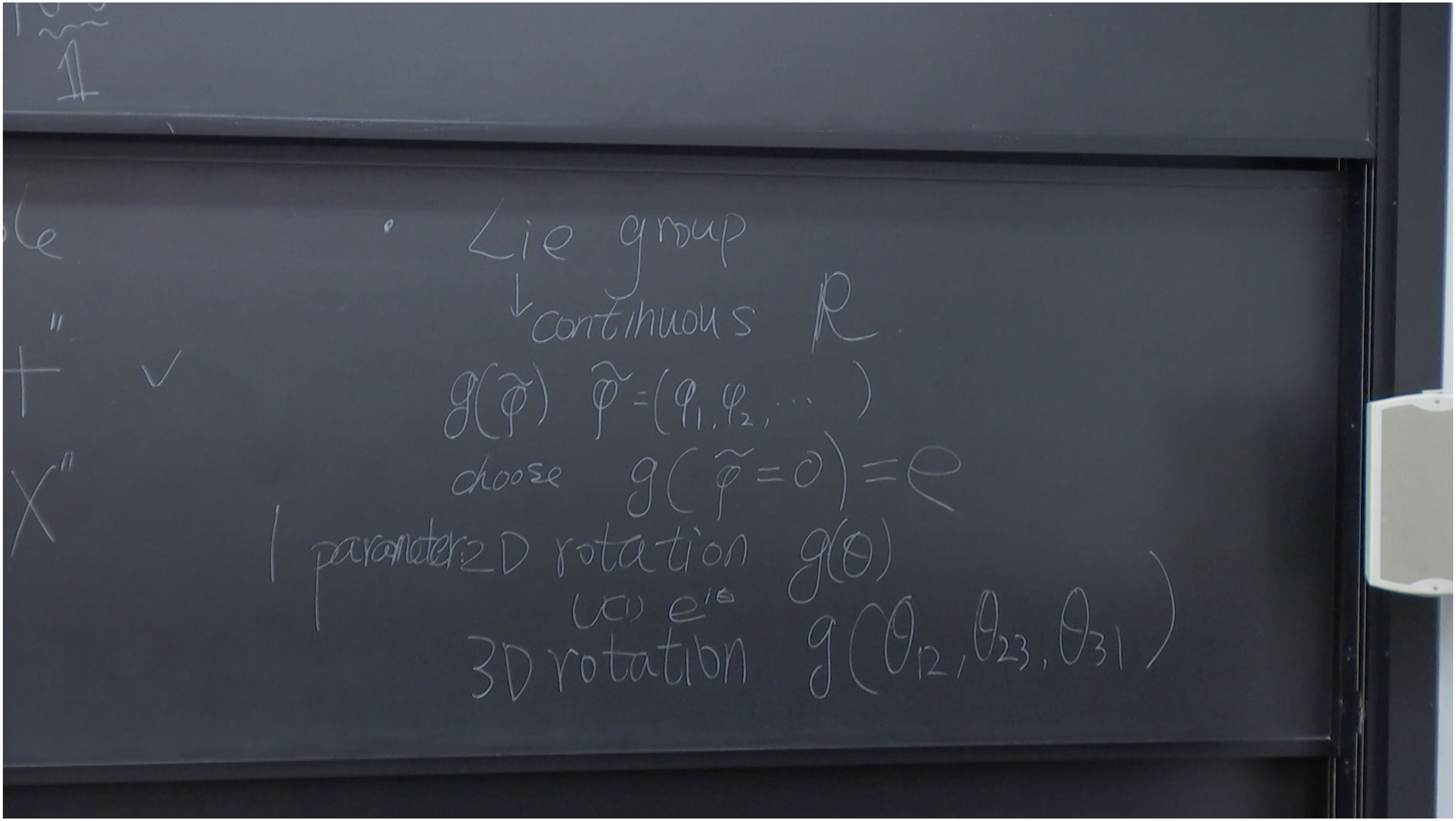
③ symmetric group $(n!)$

· infinite (countable

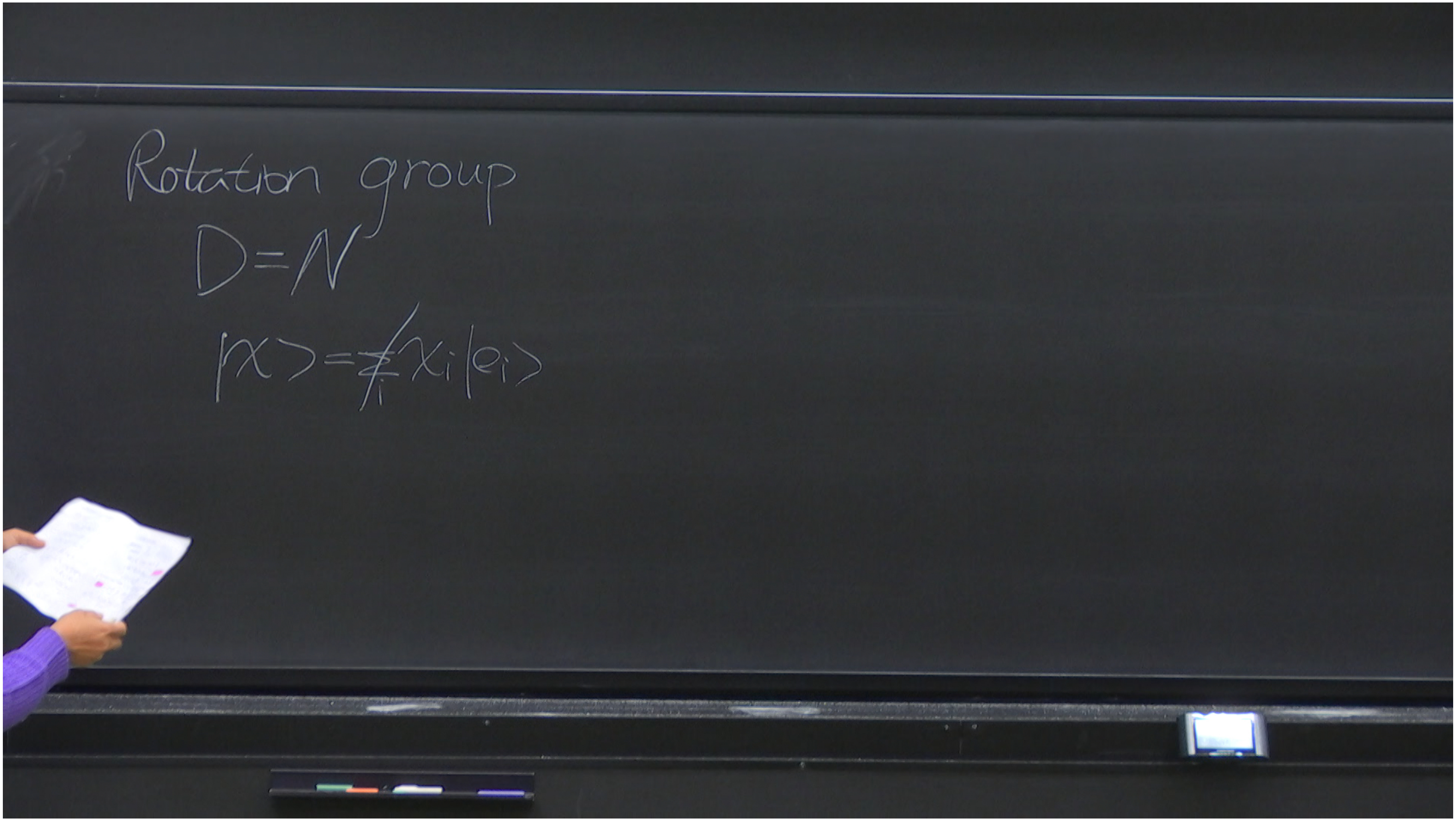
integers "+" ✓

non-zero "×"

1 paramete



Lie group
continuous R
$g(\tilde{\varphi})$ $\tilde{\varphi}=(\varphi_1,\varphi_2,\cdots)$
choose $g(\tilde{\varphi}=0)=e$
1 parameter 2D rotation $g(\theta)$
U(1) $e^{i\theta}$
3D rotation $g(\theta_{12},\theta_{23},\theta_{31})$


Rotation group
$D=N$
$|x\rangle = \sum_i x_i |e_i\rangle$

Rotation group

$D = N$

$|x\rangle = \sum_i x_i |e_i\rangle$

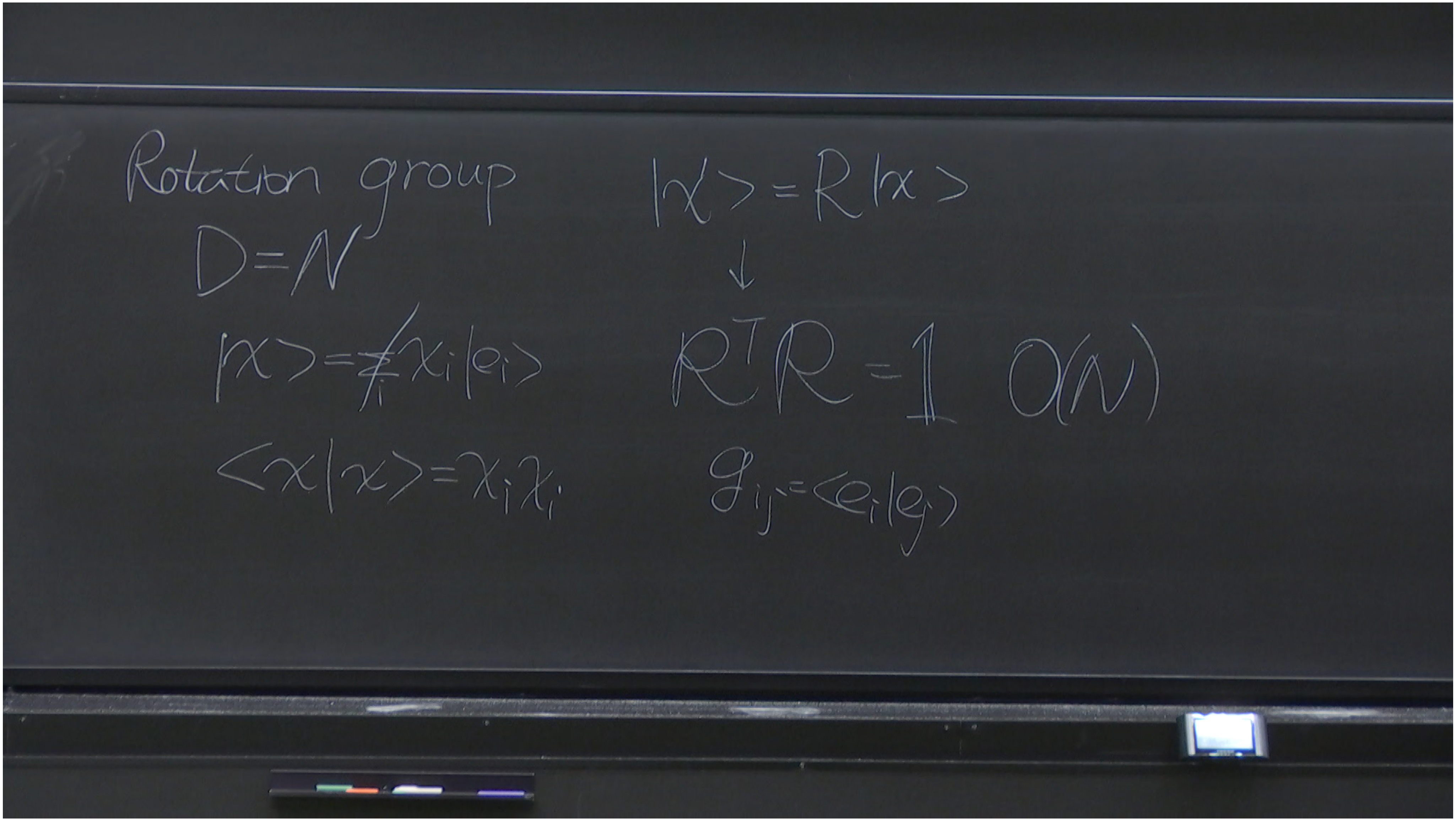
$\langle x|x\rangle = x_i x_i$

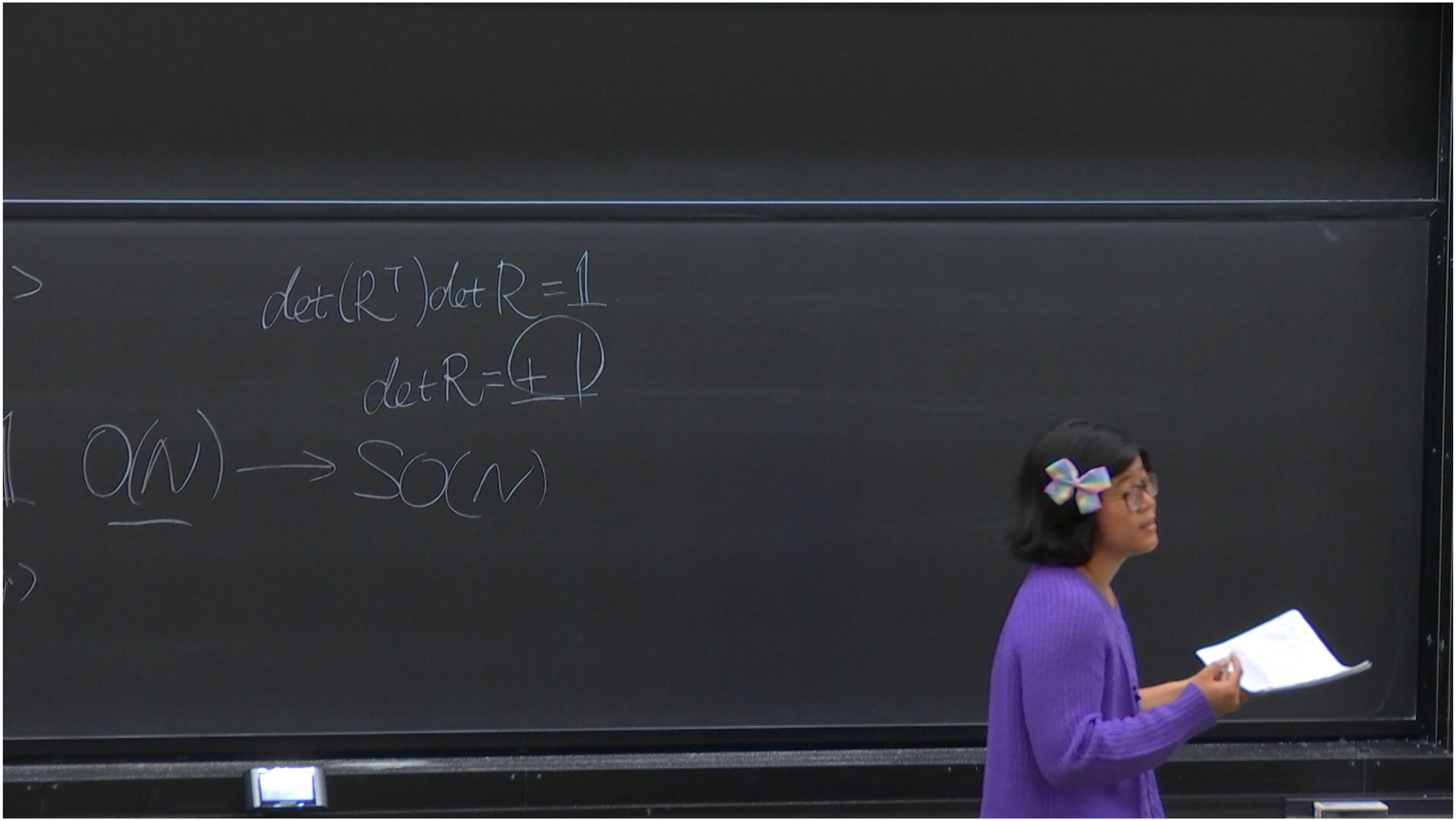
$|x'\rangle = R|x\rangle$

$\downarrow$

$R^T R = \mathbb{1}$

$g_{ij} = \langle e_i|e_j\rangle$


Rotation group
$D=N$
$|x\rangle = \sum_i x_i |e_i\rangle$
$\langle x|x\rangle = x_i x_i$
$|x'\rangle = R|x\rangle$
$\downarrow$
$R^T R = 1$
$O(N)$
$g_{ij} = \langle e_i|e_j\rangle$


det(R^T) det R = 1
det R = ±1
O(N) → SO(N)

$$\det(R^T)\det R = \mathbb{1}$$

$$\det R = \pm 1$$

$$O(N) \longrightarrow SO(N)$$

dim: group

$$\binom{N}{2} = \frac{N(N-1)}{2}$$

⑦ $SO(1,3)$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

③ symmetric group ($n!$)

unitary group dim

$U^\dagger U = \mathbb{1}$ $U(N)$

$\downarrow$ $\det U = \underline{1}$

$SU(N)$

② n-cyclic group $\sim C_n$

③ symmetric group $(n!)$

unitary group

$U^\dagger U = 1 \quad U(N)$

$(U^\dagger U)^\dagger = U^\dagger U$

$\det U = 1$

$SU(N)$

dim

$2N^2 -$

② n-cyclic group $\sim C_n$

③ symmetric group $(n!)$

unitary group

$U^\dagger U = 1$ $\quad U(N)$

$(U^\dagger U)^\dagger = U^\dagger U$ $\quad$ $\det U = 1$ $\quad SU(N)$

dim

$$2N^2 - \underbrace{\left(\frac{N(N-1)}{2}\times 2 + N\right)}_{\text{independent equation}} = N^2$$

② n-cyclic group $\mathbb{Z}_n$ ~ C_n par

③ symmetric group $(n!)$

unitary group

$U^\dagger U = 1$ $\quad U(N)$

$(U^\dagger U)^\dagger = U^\dagger U$

$\det U = 1$

$SU(N)$

$\det U = e^{i\theta}$

dim

$$2N^2 - \underbrace{\left(\frac{N(N-1)}{2}\times 2 + N\right)}_{\text{independent equation}} = N^2$$

$SU(N)$??

$N^2 - 1$

② n-cyclic group ~ C_n par

③ symmetric group ($n!$)

unitary group dim

$U^\dagger U = 1$ $U(N)$ $2N^2 - \left(\frac{N(N-1)}{2} \times 2 + N\right) = N^2$

independent equation

$(U^\dagger U)^\dagger = U^\dagger U$ det $U = 1$

$SU(N)$

$SU(N)$??

$N^2 - 1$

det $U = e^{i\theta}$

② n-cyclic group $\sim C_n$

③ symmetric group $(n!)$

unitary group

dim

$U^\dagger U = 1$ $\quad U(N) \quad 2N^2 - \left(\frac{N(N-1)}{2} \times 2 + N\right) = N^2$

independent equation

$(U^\dagger U)^\dagger = U^\dagger U$

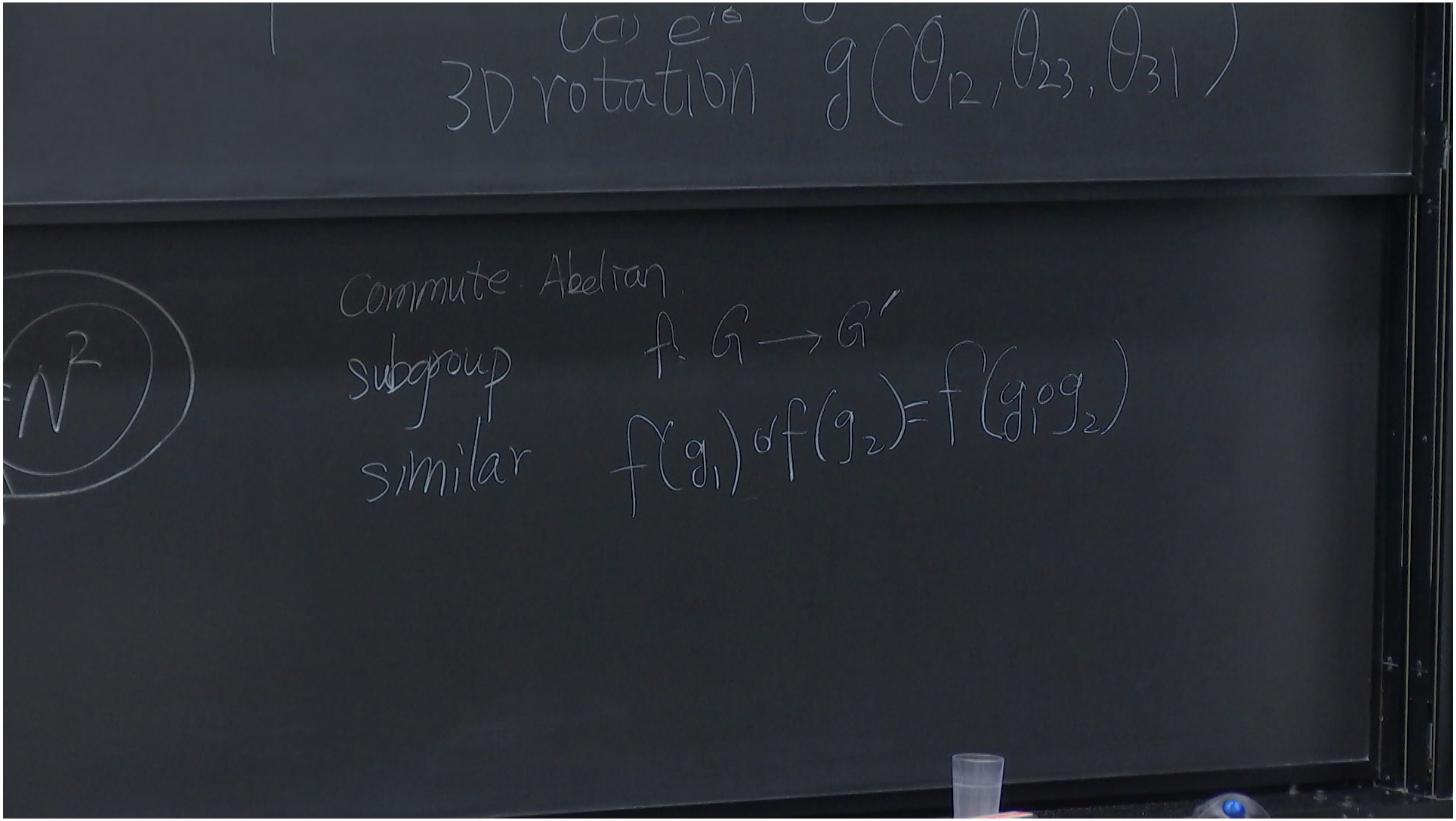
$\det U = 1$

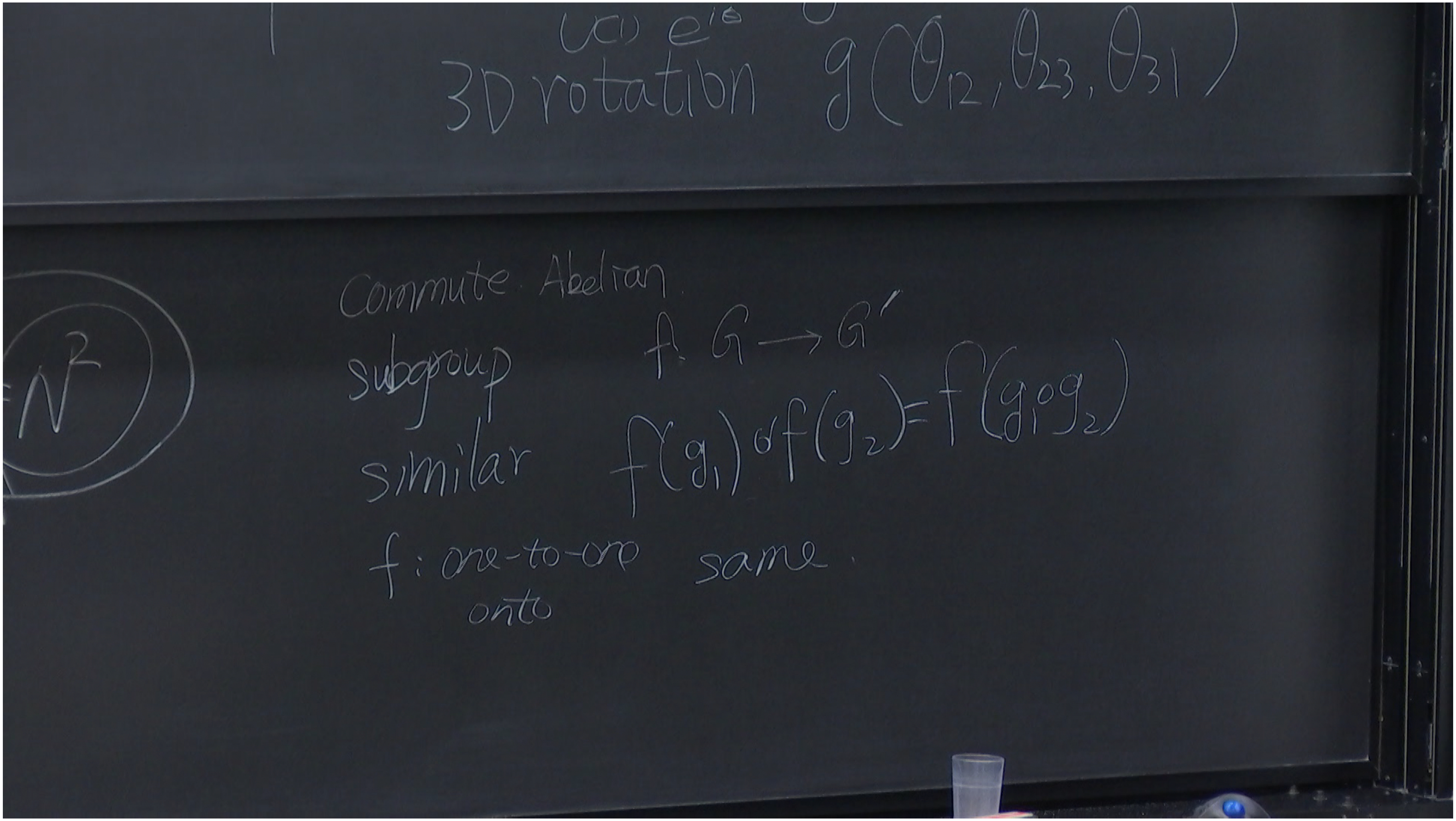
$SU(N)$

$SU(N)$: ?

$N^2 - 1$

$\det U = e^{i\theta}$


U(1) e^iθ
3D rotation g(θ12, θ23, θ31)
N2
Commute Abelian
subgroup
f: G → G'
similar
f(g1)∘f(g2) = f(g1∘g2)


3D rotation $g(\theta_{12}, \theta_{23}, \theta_{31})$
Commute. Abelian.
subgroup
$f: G \to G'$
similar
$f(g_1) \circ f(g_2) = f(g_1 \circ g_2)$
f: one-to-one same.
onto

dim: group (real)

$$\binom{N}{2} = \frac{N(N-1)}{2}$$

7 $SO(1,3)$ $\mathbb{R}^{1,3}$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$