Title: Gong Show

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Abstract: Speaker Order:
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Ward Identities for near horizon symmetries

-Ankit Aggarwal
(Based on w.i.p. with N. Gaddam)

"Quantum Gravity around the corner" @ PITP

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Introduction and Summary

- Soft theorems are the ward identities for asymptotic symmetries in asymptotically flat spacetime.
  Asymptotic Symmetries ↔ Soft Theorems

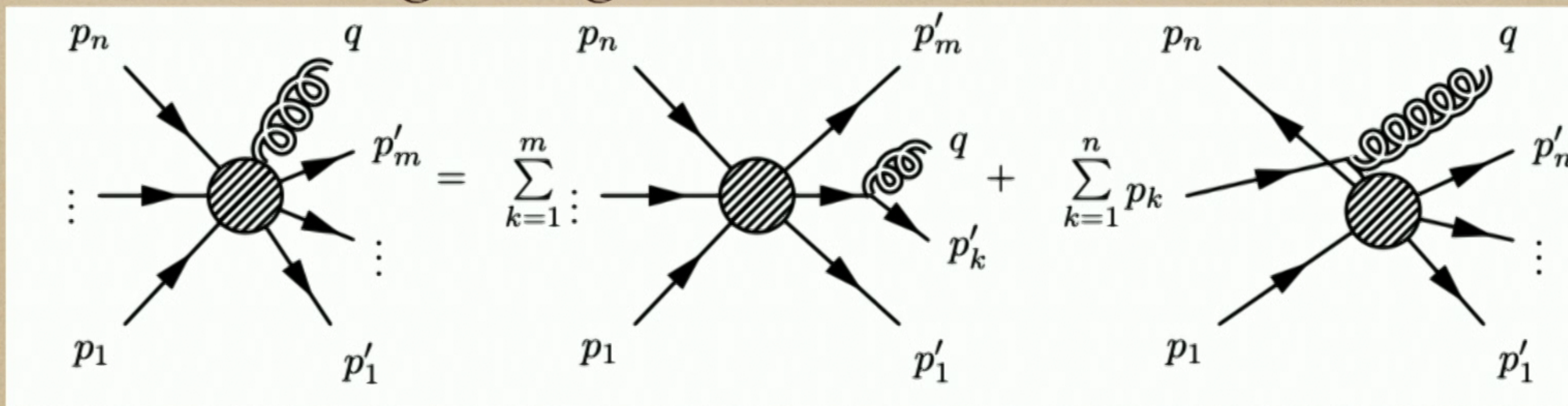
- An interesting boundary in the presence of blackholes is the horizon of the blackhole- near horizon symmetries.

- What are the ward identities associated with near-horizon symmetries?
- We explore this question in the context of Schwarzschild blackhole.
- We prove an effective soft graviton theorem for scattering processes near the blackhole horizon.
- We claim that this is the desired ward identity for near horizon symmetries.

Soft graviton theorem from BMS in angular momentum basis

- Consider Einstein Hilbert action + minimally coupled scalar field; with BMS boundary conditions.
- One can compute the (gravitational + matter) charge, Q_{BMS}, associated to supertranslations.
- Assuming BMS is a symmetry of quantum gravity S-matrix implies $\langle in | [Q_{BMS}, S] | out \rangle = 0$.

- When in (out) states are momentum eigenstates, we get well-known Weinberg's soft graviton theorem with factorisation.



Weinberg's soft theorem in momentum basis

$$\lim_{q\to 0} \mathscr{M}_{\mu\nu}(q, p'_1, \cdots) = \sqrt{8\pi G}\left[\sum_{k=1}^{m}\frac{p'_{k\mu}p'_{k\nu}}{p'_k\cdot q} - \sum_{k=1}^{n}\frac{p_{k\mu}p_{k\nu}}{p_k\cdot q}\right]\mathscr{M}(p'_1, \cdots)\,.$$

- When in (out) states are angular momentum eigenstates, we get

$$\lim_{\omega_q \to 0^+} \mathcal{M}\left(\{\omega_q, \ell_q, m_q\}, \{E_1, \ell_1, m_1\}, \cdots\right) = \frac{\sqrt{8\pi G}}{\omega_q} \sum_{\ell', m'} \sum_k \frac{1}{\ell_q(\ell_q+1)}$$

$$\Big[C(\ell_q, m_q;\ \ell_k^{in}, m_k^{in};\ \ell', m') E_k^{in} \mathcal{M}\left(\{E_1, \ell_1, m_1\}, \cdots, \{E_k^{in}, \ell', m'\}, \cdots\right)$$

$$- in \leftrightarrow out \Big]$$

where $C(\ell_1, m_1;\ \ell_2, m_2;\ \ell_3, m_3) = \int d\Omega Y_{\ell_1, m_1} Y_{\ell_2, m_2} Y_{\ell_3, m_3}$

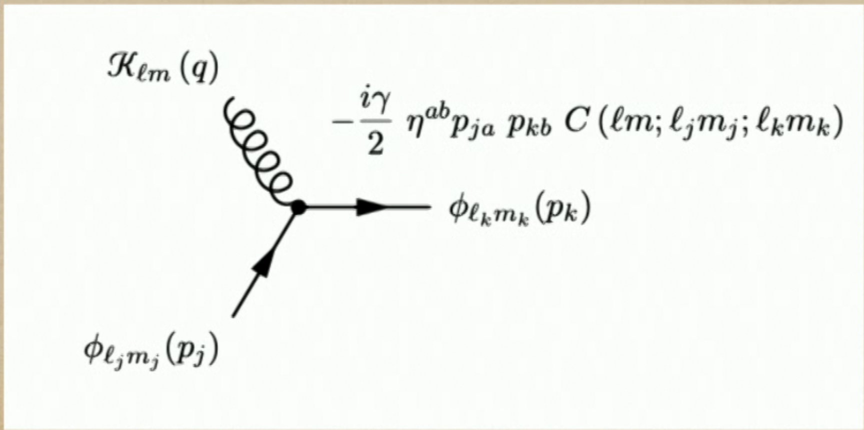
Near horizon symmetries

- Diffeomorphisms that act non-trivially on horizon.
- Donnay, Giribet, Gonzalez, Pino (DGGP) showed that near horizon symmetries of a non-extremal blackhole are two copies of supertranslations + Virasoro.

Near horizon scattering

- We consider EH action + minimally coupled scalar field.
- We study the metric perturbations around Schwarzschild background.
- Spatial translation symmetry is absent.
- Exploiting the rotational symmetry, reduce the problem to 2d problem with infinitely many Kaluza-Klein fields.

- The relevant radiative mode turns out to be a transverse scalar $\mathscr{K}_{\ell,m}$ with mass $\mu = \frac{1}{R_s}\sqrt{\ell(\ell+1)+1}$.
- Scalar field $\phi_{\ell,m}$ also behaves in the same way.
- One can then derive Feynman rules near the horizon.



Interaction vertex

Near-horizon soft limit

- Let the longitudinal 2-momentum of $\mathcal{K}_{\ell,m}$ be $q = \left[q_x, \frac{\mu}{q_x}\right]$.
- Soft limit: Dimensionless momenta, $\hat{q}_x := \dfrac{q_x}{M_{pl}} \to 0$ and dimensionless blackhole mass, $\hat{M} = \dfrac{M_{BH}}{M_{pl}} \to \infty$, keeping $\hat{M}\hat{q}_x$ fixed.
- $\hat{q} = \dfrac{q}{M_{pl}} \to 0$ in this limit.

Near horizon soft theorem

- In the near-horizon soft limit, one can derive the near-horizon soft theorem using the Feynman rules of the 2d theory
- $$\lim_{\hat{q}\to 0^+} \mathcal{M}\left(\{q, \ell_q, m_q\}, \{E_1, \ell_1, m_1\}, \cdots\right) = \frac{\sqrt{8\pi G}}{8GM} \sum_{\ell', m'} \sum_k$$
$$\Bigg[C(\ell_q, m_q;\, \ell_k^{in}, m_k^{in};\, \ell', m') \frac{\mu_k^2}{p_k^{in}\cdot q} \mathcal{M}\left(\{E_1, \ell_1, m_1\}, \cdots, \{p_k^{in}, \ell', m'\}, \cdots\right)$$
$$- in \leftrightarrow out \Bigg]$$
- Can also get it from the DGGP charge : $\langle in | [Q_{DGGP}, S] | out\rangle = 0.$

Conslusions and outlook

- Soft theorems for near horizon scattering.
- Corespondence between NH symmetries and NH soft theorems.
- Ward identities of other near horizon symmetries? For other blackholes?
- Ward identity without the near horizon soft limit?
- Connection to 'soft hair' proposal?
- Something similar for the $\Lambda - BMS$?

Holographic Entanglement Entropy and Mutual Information in Deformed Field Theories at Finite Temperature

Monireh Ahmadpour

Department of Physics, University of Tehran

Based on the work by H. Ebrahim, M. Ahmadpour
"Entanglement Entropy in Presence of $T\bar{T}$",
to appear on arXiv[2210...].

Conference on Quantum Gravity Around the Corner
Perimeter Institute, October 2022

Outline

1. Motivation
   - $T\bar{T}$ deformation of CFT
   - Holographic Entanglement Entropy (HEE)
   - Holographic Mutual Information (HMI)
2. Our Results
   - HEE Main Results
   - HMI Main Results

$T\bar{T}$ deformation represents a geometric cutoff at finite distance $z = zc$ in the bulk.

[McGough, Mezei, Verlinde 2018], [Taylor 2018], [Hartman, Kruthof, Shaghoulian, Tajdini 2019]

- Removes the asymptotic region of AdS.
- Places the QFT on the new boundary of AdS.

$$\frac{dS_{QFT}}{d\lambda} = \int d^d x \sqrt{\gamma}\, X(x) \,, \quad \lambda \to 0 \,, \tag{1}$$

$$\lambda = \left(\frac{8\pi G_N^{d+1}\, R}{2d} \right) \left(\frac{z_c}{R^2} \right)^d \,. \tag{2}$$

$$ds^2 = \frac{R^2}{z^2} \left(-\left(1 - \left(\frac{z}{z_H}\right)^d\right) dt^2 + dx_{d-1}^2 + \frac{dz^2}{\left(1 - \left(\frac{z}{z_H}\right)^d\right)} \right) \tag{3}$$

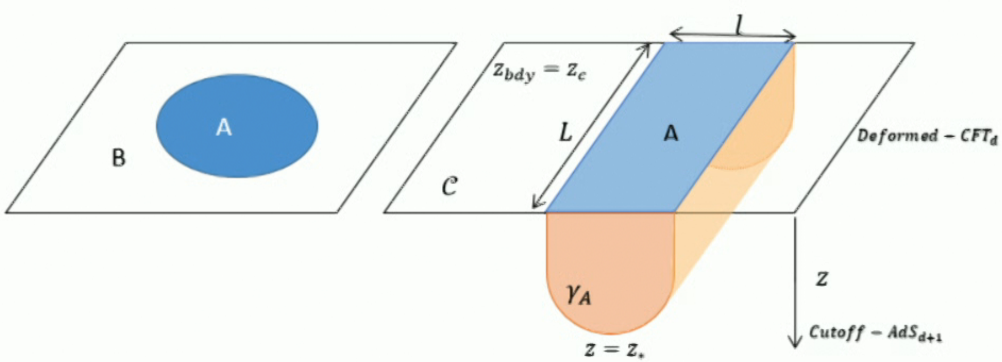
Entanglement Entropy

A measure of entanglement in a given pure quantum state $|\psi\rangle$.

Ryu-Takayanagi (RT) Prescription [Ryu-Takayanagi 2006]

$$S_A = -tr\left(\rho_A \ln \rho_A\right) = min\left[\frac{Area(\gamma_A)}{4G_N^{(d+1)}}\right] \quad ; \quad \partial\gamma_A = \partial A \qquad (4)$$

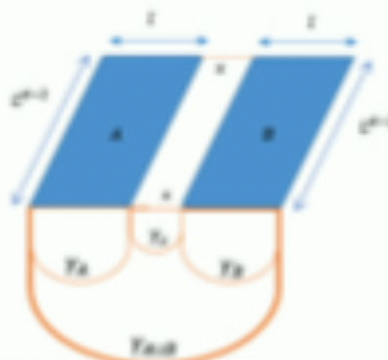
Where $Area(\gamma_A)$ is a minimal codimension-2 surface.



A measure of correlations for two disjoint intervals.

$$I(A:B) = S_A + S_B - S_{A\cup B}\,, \quad \text{[Casini,Huerta 2007]} \qquad (5)$$

- $I(A:B) \geq 0$.
- It is finite and Scheme-independent.
- there are two choices for the $S_{A\cup B}$:
  - $x \gg \ell$ So $I(A:B) = 0$.
  - $x \ll \ell$, $I(A:B) = 2S(\ell) - S(x) - S(2\ell + x)$.
- Finite temperature case: dimensionless parameter is Tx .



Motivating Question:

How measures of quantum entanglement are affected by this deformation?

Expansion Parameters:

Field theory parameters: (T,λ,ℓ).
Dimensionless expansion parameters: $T\ell$ and $\tilde{\lambda}/\ell$. $\tilde{\lambda} \equiv \lambda^{1/d}$
On the gravity side expansion parameters are: $\left(\frac{z_c}{z_*}\right)$ and $\left(\frac{z_*}{z_H}\right)$.
Where z_* is turning point of the minimal surface in the bulk.

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Area and Length of entangling surface

$$A = 2R^{d-1}\left(\frac{L}{z_*}\right)^{d-2}\sum_{k=0}^{\infty}\frac{\Gamma(k+\frac{1}{2})}{\Gamma(k+1)}\left(\frac{z_*}{z_H}\right)^{kd}\left[\frac{\Gamma\left(\frac{d(k-1)+2}{2(d-1)}\right)}{2(d-1)\Gamma\left(\frac{1+kd}{2(d-1)}\right)}\right.$$
$$\left. - \frac{1}{\sqrt{\pi}(d(k-1)+2)}\left(\frac{z_c}{z_*}\right)^{d(k-1)+2} {}_2F_1\left(\frac{1}{2},\frac{d(k-1)+2}{2(d-1)};\frac{d(k+1)}{2(d-1)},\left(\frac{z_c}{z_*}\right)^{2(d-1)}\right)\right]. \quad (6)$$

$$\frac{\ell}{2} = z_*\sum_{k=0}^{\infty}\frac{\Gamma(k+\frac{1}{2})}{\Gamma(k+1)}\left(\frac{z_*}{z_H}\right)^{kd}\left[\frac{\Gamma\left(\frac{d(k+1)}{2(d-1)}\right)}{(1+kd)\Gamma\left(\frac{1+kd}{2(d-1)}\right)}\right.$$
$$\left. - \frac{1}{\sqrt{\pi}d(k+1)}\left(\frac{z_c}{z_*}\right)^{d(k+1)} {}_2F_1\left(\frac{1}{2},\frac{d(k+1)}{2(d-1)};\frac{d(3+k)-2}{2(d-1)},\left(\frac{z_c}{z_*}\right)^{2(d-1)}\right)\right]. \quad (7)$$

We consider three different thermal limits at finite cutoff. Our analytic calculations are similar to the methods of [Fischler,Kundu 2013].

- $z_H \gg z_* \gg z_c$
- $z_H \gg z_* \simeq z_c$
- $z_H \simeq z_* \gg z_c$

$z_H \gg z_* \gg z_c$

$$
\begin{aligned}
S_A = \frac{2R^{d-1}}{4G_N^{(d+1)}} \Bigg\{ & \left(\frac{L}{z_c}\right)^{d-2} \left[\frac{1}{d-2} - \frac{1}{4}\left(\frac{z_c}{\ell}\right)^{d} \left(\frac{\ell}{z_H}\right)^{d} - \frac{3}{8\,(d+2)} \left(\frac{z_c}{\ell}\right)^{2d} \left(\frac{\ell}{z_H}\right)^{2d} \right] \\
& + \left(\frac{L}{\ell}\right)^{d-2} \Bigg[a_1 + a_2 \left(\frac{z_c}{\ell}\right)^{d} - a_3 \left(\frac{z_c}{\ell}\right)^{2d} + \left(a_4 + a_5 \left(\frac{z_c}{\ell}\right)^{d} + a_6 \left(\frac{z_c}{\ell}\right)^{2d} \right) \left(\frac{\ell}{z_H}\right)^{d} \\
& + \left(a_7 + a_8 \left(\frac{z_c}{\ell}\right)^{d} + a_9 \left(\frac{z_c}{\ell}\right)^{2d} \right) \left(\frac{\ell}{z_H}\right)^{2d} \Bigg] \Bigg\} . \qquad (8)
\end{aligned}
$$

Where coefficients a_i depend on the dimension of the spacetime.

- Leading term shows Area Law behaviour.
- All corrections are finite and do not cause divergent pieces.

$z_H \gg z_* \simeq z_c$

$$S_A = \frac{R^{d-1}}{4G_N^{(d+1)}} \left(\frac{L}{z_c}\right)^{d-2} \left[\left(\frac{\ell}{z_c}\right) - \frac{(d-1)^2}{24}\left(1 - \left(\frac{z_c}{z_H}\right)^d\right)\left(\frac{\ell}{z_c}\right)^3\right] \quad (9)$$

- HEE is a decreasing function of z_c.
- Leading term shows volume law behaviour.
- This special limit just appears in the finite cutoff AdS_{d+1}.

$z_H \simeq z_* \gg z_c$

$$S_A = \frac{2R^{d-1}}{4G_N^{(d+1)}}\left(\frac{L}{z_H}\right)^{d-2}\left\{\frac{\ell}{2z_H} + \frac{1}{2}S_{high} - \sqrt{\frac{d-1}{2d}}\,\epsilon - \frac{1}{4}\left(\frac{z_c}{z_H}\right)^2 + \frac{1}{d-2}\left(\frac{z_c}{z_H}\right)^{2-d} + \frac{1}{2d}\left(\frac{z_c}{z_H}\right)^{d} - \frac{3}{8(d+2)}\left(\frac{z_c}{z_H}\right)^{d+2} + \frac{1}{8d}\left(\frac{z_c}{z_H}\right)^{2d}\right\}. \tag{10}$$

where

$$S_{high} = \sum_{k=0}^{\infty}\frac{\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(\frac{d(k+1)}{2(d-1)}\right)}{\Gamma\left(k+1\right)\Gamma\left(\frac{kd+1}{2(d-1)}\right)}\frac{d-1}{(kd+1)(d(k-1)+2)} \tag{11}$$

and

$$\epsilon = \epsilon_d\ \exp\left[-\sqrt{\frac{d(d-1)}{2}}\left\{\frac{\ell}{z_H} + \sum_{k=0}^{\infty}\frac{2}{\sqrt{\pi}\ d(k+1)}\frac{\Gamma(k+\frac{1}{2})}{\Gamma(k+1)}\left(\frac{z_c}{z_H}\right)^{d(k+1)}\ {}_2F_1\left(\frac{1}{2},\frac{d(k+1)}{2(d-1)};\frac{d(k+3)-2}{2(d-1)},\left(\frac{z_c}{z_*}\right)^{2(d-1)}\right)\right\}\right] \tag{12}$$

$$\epsilon_d = \frac{1}{d}\exp\left[\sqrt{\frac{d(d-1)}{2}}\left\{2c_0 + \sum_{k=1}^{\infty}\left(\frac{\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(\frac{d(k+1)}{2(d-1)}\right)}{\Gamma\left(k+1\right)\Gamma\left(\frac{kd+1}{2(d-1)}\right)}\frac{2}{kd+1} - \sqrt{\frac{2}{d(d-1)}}\frac{1}{k}\right)\right\}\right] \tag{13}$$

- Leading term shows volume law behaviour.
- Corrections appear as series with expansion parameter $\left(\frac{z_c}{z_H}\right)^n$.
- Isolated divergences are in ϵ_d relation.

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$(z_H \gg \ell, x) \wedge (x \ll z_c \ll \ell)$

$$I(A:B) = \frac{2R^{d-1}}{4G_N^{(d+1)}} \left(\frac{L}{z_c}\right)^{d-2} \Bigg\{ \frac{1}{d-2} - \frac{1}{4}\left(\frac{z_c}{z_H}\right)^d - \frac{3}{8(d+2)}\left(\frac{z_c}{z_H}\right)^{2d}$$
$$- \left(\frac{x}{2z_c}\right) + \frac{(d-1)^2}{6}\left(1 - \left(\frac{z_c}{z_H}\right)^d\right)\left(\frac{x}{2z_c}\right)^3$$
$$+ \left(a_1 + a_5\left(\frac{z_c}{z_H}\right)^d + a_9\left(\frac{z_c}{z_H}\right)^{2d}\right)\mathcal{F}_1(d-2) + \left(a_2 + a_6\left(\frac{z_c}{z_H}\right)^d\right)\mathcal{F}_1(2d-2) + a_3\mathcal{F}_1(3d-2)$$
$$+ \left(a_4 + a_8\left(\frac{z_c}{z_H}\right)^d\right)\mathcal{F}_2(2)\left(\frac{z_c}{z_H}\right)^d + a_7\mathcal{F}_2(d+2)\left(\frac{z_c}{z_H}\right)^{2d}\Bigg\} \tag{14}$$

Where $\mathcal{F}_1(n) = z_c^n\left(\frac{2}{\ell^n} - \frac{1}{(2\ell+x)^n}\right)$ and $\mathcal{F}_2(n) = z_c^{-n}\left(2\ell^n - (2\ell+x)^n\right)$.

- Corrections are expansions with parameter $\left(\frac{z_c}{z_H}\right)$.
- Also, corrections depend on functions of entangling regions length.

$(z_H \gg \ell, x) \wedge (z_c \gg \ell, x)$

$$I(A,B) = \frac{2R^{d-1}}{4G_N^{(d+1)}}\left(\frac{L}{z_c}\right)^{d-2}\left\{-\frac{x}{z_c} + \frac{(d-1)^2}{24}\left(1-\left(\frac{z_c}{z_H}\right)^d\right)\left(\frac{x^3+3\ell(x+\ell)^2}{{z_c}^3}\right)\right\} \tag{15}$$

- It still shows a first-order phase transition.
- Critical separation x_{crit} depends on the cutoff.
- HMI is a decreasing function of the cutoff z_c.
- When $x \to 0$, in contrast to the zero cutoff case, the HMI remains finite.

$(x \ll z_H \ll \ell) \wedge (x \ll z_c \ll \ell)$

$$I(A,B) = \frac{2R^{d-1}}{4G_N^{(d+1)}}\left(\frac{L}{z_c}\right)^{d-2}\left\{\frac{1}{d-2} - \left(1+\left(\frac{z_c}{z_H}\right)^{d-1}\right)\left(\frac{x}{2z_c}\right) + \frac{(d-1)^2}{6}\left(1-\left(\frac{z_c}{z_H}\right)^{d}\right)\left(\frac{x}{2z_c}\right)^3 + \left(\frac{z_c}{z_H}\right)^{d-2}\left(S_{high} - \sqrt{\frac{d-1}{2d}}\left(2\epsilon_\ell - \epsilon_{2\ell+x}\right)\right) - \frac{1}{4}\left(\frac{z_c}{z_H}\right)^{d} + \frac{1}{2d}\left(\frac{z_c}{z_H}\right)^{2d-2} - \frac{3}{8(d+2)}\left(\frac{z_c}{z_H}\right)^{2d} + \frac{1}{8d}\left(\frac{z_c}{z_H}\right)^{3d-2}\right\} \quad (16)$$

- We see corrections in the form of $\left(\frac{z_c}{z_H}\right)$ and $\left(\frac{x}{2z_c}\right)$.
- There exist special isolated divergent terms for each disjoint regions as expressed in the ϵ.

Motivation

What is BV(-BFV)?

The BV formalism is a mathematical tool to describe and quantize gauge theories. It is a generalization of the Faddeev–Popov ghosts and BRST construction for more general types of symmetries.

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The BV formalism is a mathematical tool to describe and quantize gauge theories. It is a generalization of the Faddeev–Popov ghosts and BRST construction for more general types of symmetries.

Why BV-BFV?

- The BV-BFV formalism comes with a quantisation scheme for theories with gauge symmetries defined on manifold with boundary and corners.
- Furthermore, the classical BV-BFV formalism provides a cohomological description of the reduced phase space and gives rise to a Poisson structure on corners.

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- The BV-BFV formalism comes with a quantisation scheme for theories with gauge symmetries defined on manifold with boundary and corners.
- Furthermore, the classical BV-BFV formalism provides a cohomological description of the reduced phase space and gives rise to a Poisson structure on corners.

In this talk

- Brief introduction to classical BV-BFV
- Reduced phase space of gravity
- Poisson structure on the boundary

BV-BFV formalism I

Definition

A BV theory is a quadruple
$$\mathfrak{F} = (\mathcal{F}, S, \varpi, Q)$$
where

- $\mathcal{F}$ is a graded manifold (called space of fields);
- ϖ is a degree-(-1) exact symplectic form on $\mathcal{F}$;
- $S : \mathcal{F} \to \mathbb{R}$ is a degree k functional (the action);
- Q is an odd vector field on $\mathcal{F}$ such that $[Q, Q] = 0$

such that
$$\iota_Q \varpi = \delta S.$$

Example: Palatini–Cartan gravity

- $$\mathcal{F} = T^*[-1] \left(\Omega^1_{nd}(M,\mathcal{V}) \oplus \mathcal{A}(M) \oplus \Omega^0[1](M,\wedge^2\mathcal{V}) \oplus \Gamma[1]TM \right)$$

- $$\varpi = \int_M \delta e^\dagger \delta e + \delta\omega^\dagger \delta\omega + \delta c^\dagger \delta c + \iota_{\delta\xi}\delta\xi^\dagger$$

- $$S = \int_M \frac{1}{2}e^2 F_\omega - \frac{1}{4!}\Lambda e^4 + e^\dagger \left(L_\xi^\omega e - [c,e] \right) + \omega^\dagger \left(\iota_\xi F_\omega - d_\omega c \right)$$
$$+ \frac{1}{2} c^\dagger \left(\iota_\xi \iota_\xi F_\omega - [c,c] \right) + \frac{1}{2}\iota_{[\xi,\xi]}\xi^\dagger$$

- Cohomological vector field Q, from $\iota_Q \varpi = \delta$.

Example: Palatini–Cartan gravity

- Cotangent bundle (antifields) → $T^*[-1]$; Tetrad (e) → $\Omega^1_{nd}(M,\mathcal{V})$; Connection ($\omega$) → $\mathcal{A}(M)$; Ghosts encoding symmetries (c, ξ) → $\Omega^0[1](M,\wedge^2\mathcal{V}) \oplus \Gamma[1]TM$

$$\mathcal{F} = T^*[-1] \left(\Omega^1_{nd}(M,\mathcal{V}) \oplus \mathcal{A}(M) \oplus \Omega^0[1](M,\wedge^2\mathcal{V}) \oplus \Gamma[1]TM \right)$$

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- Cohomological vector field Q, from $\iota_Q\varpi = \delta$.

I. Plan and Motivations

- Study of the classical phase space of 3D asymptotically AdS gravity:

  Select the allowed metric fluctuations at infinity [Brown-Henneaux '86]

- No requirement to fix any particular gauge but it is often convenient

  For example: Fefferman–Graham, Bondi gauge

- In this talk: covariant Bondi gauge, allow for a smooth flat-space limit

  Originally from fluid/gravity correspondence

  Study holographically Lorentz and Carroll-boost anomalies

II. Covariant Bondi gauge in AdS

- Key idea: relax the AdS Bondi gauge → dependence on the boundary dyad

$$\mathrm{d}s^2_{\mathrm{AdS}} = \frac{2}{k^2}\,\mathrm{u}\,(\mathrm{d}r + r\,\mathrm{A}) + r^2 g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{8\pi\mathcal{G}}{k^4}\,\mathrm{u}\,(\varepsilon\,\mathrm{u} + \chi * \mathrm{u})$$

- Boundary metric and Cartan frame:

$$g_{\mu\nu} = \frac{1}{k^2}\left(-u_\mu u_\nu + *u_\mu * u_\nu\right)$$

Weyl connection: [Loganayagam '08]

$$\mathrm{A} = \frac{1}{k^2}\left(\Theta^* * \mathrm{u} - \Theta\,\mathrm{u}\right), \qquad \Theta = \nabla_\mu u^\mu, \qquad \Theta^* = \nabla_\mu * u^\mu$$

- Energy-momentum tensor: [Brown-York '93]

$$\mathrm{T} = \mathrm{T}(\varepsilon, \chi) \; : \qquad \nabla_\mu T^{\mu\nu} = 0\,, \qquad T^\mu{}_\mu = \frac{R}{16\pi\mathcal{G}k}$$

II. Covariant Bondi gauge in AdS: residual symmetries

- Asymptotic Killing vectors: [Ciambelli-Marteau-Petropoulos-Ruzziconi '20]

$$v = \left(\xi^\mu - \frac{1}{k^2 r}\,\eta * u^\mu\right)\partial_\mu + \left(r\,\sigma + \frac{1}{k^2}\left(*u^\nu\,\partial_\nu\eta + \Theta^*\eta\right) + \frac{4\pi\mathcal{G}}{k^2 r}\,\chi\,\eta\right)\partial_r$$

$\hookrightarrow$ bdy diffeomorphisms $\xi^\mu(x)$, Weyl rescalings $\sigma(x)$ and Lorentz boosts $\eta(x)$

$$\delta_{(\xi,\sigma,\eta)}\mathrm{u} = \mathcal{L}_\xi\mathrm{u} + \sigma\,\mathrm{u} + \eta\,*\mathrm{u}\,, \quad \delta_{(\xi,\sigma,\eta)}*\mathrm{u} = \mathcal{L}_\xi\,*\mathrm{u} + \sigma\,*\mathrm{u} + \eta\,\mathrm{u}$$

where

$$\delta_{(\xi,\sigma,\eta)}g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} + 2\,\sigma\,g_{\mu\nu}$$

and

$$\begin{pmatrix}\mathrm{u}'\\ *\mathrm{u}'\end{pmatrix} = \begin{pmatrix}\cosh\eta & \sinh\eta\\ \sinh\eta & \cosh\eta\end{pmatrix}\begin{pmatrix}\mathrm{u}\\ *\mathrm{u}\end{pmatrix}$$

- Question: What are the asymptotic symmetries?

II. Covariant Bondi gauge in AdS: symplectic structure

- Einstein–Hilbert presymplectic potential: [Iyer-Wald '94]

$$\Theta_{\text{EH}}[G;\delta G] = \frac{\sqrt{-G}}{32\pi\mathcal{G}}\left[\nabla^N \delta G_{PN}\, G^{PM} - \nabla^M \delta G_{PN}\, G^{PN}\right] \epsilon_{MQS}\, \mathrm{d}x^Q \wedge \mathrm{d}x^S$$

Radial divergences: need for renormalization

$$\Theta^{(r)}_{\text{EH}}[G;\delta G] = r^2\, \Theta_{(2)} + r\, \Theta_{(1)} + \Theta_{(0)} + \mathcal{O}\left(r^{-1}\right)$$

Ambiguous definition:

$$\Theta_{\text{EH}}[G;\delta G] \to \Theta_{\text{EH}}[G;\delta G] + \delta Z[G] - \mathrm{d}Y[G;\delta G]$$

- Choices of prescription:
  i. same results as obtained in FFG [de Haro-Solodukhin-Skenderis (2000)]
  ii. presymplectic potential that remains finite in the flat-space limit

II. Covariant Bondi gauge in AdS: surface charges

- Conformal gauge: conformally flat bdy metric ($x^{\pm} = \phi \pm k\, u$)

$$\mathrm{d}s^2 = \mathrm{e}^{2\varphi}\, \mathrm{d}x^+ \mathrm{d}x^-$$

Parametrization of the Cartan frame: ($\varphi = \varphi(x^+, x^-)$, $\zeta = \zeta(x^+, x^-)$)

$$\mathrm{u} = -\frac{k}{2}\, \mathrm{e}^{\varphi} \left(\mathrm{e}^{\zeta}\, \mathrm{d}x^+ - \mathrm{e}^{-\zeta}\, \mathrm{d}x^- \right), \qquad *\mathrm{u} = \frac{k}{2}\, \mathrm{e}^{\varphi} \left(\mathrm{e}^{\zeta}\, \mathrm{d}x^+ + \mathrm{e}^{-\zeta}\, \mathrm{d}x^- \right)$$

- Charges associated with the Weyl–Lorentz symmetries: ($\delta_v \varphi = \varpi$, $\delta_v \zeta = h$)

$$Q_{(\varpi,h)} = \frac{1}{4\pi \mathcal{G} k} \int_0^{2\pi} \mathrm{d}\phi \left(h \left(\partial_- - \partial_+ \right) \zeta \right)$$

$\hookrightarrow$ integrable and non-conserved: Lorentz is anomalous, Weyl is pure gauge

Outline

I. Plan and Motivations

II. Covariant Bondi gauge in AdS and holographic frames

III. Flat limit and boundary Carroll frames

IV. Summary

III. Flat limit: surface charges and anomalies

- Key idea: timelike AdS bdy $\underset{k\to 0}{\longrightarrow}$ null manifold, Carrollian geometry
- Conformal gauge: parametrization of the Carrollian dyad ($\beta = \lim_{k\to 0} \frac{\zeta}{k}$)

$$\mu = \lim_{k\to 0} \frac{\mathrm{u}}{k^2} = -\mathrm{e}^{\varphi}\left(\mathrm{d}u + \beta\,\mathrm{d}\phi\right), \quad \mu^* = \lim_{k\to 0} \frac{*\mathrm{u}}{k} = \mathrm{e}^{\varphi}\,\mathrm{d}\phi$$

Charges associated with the Weyl–boost symmetries: ($\delta_v \varphi = \varpi$, $\delta_v \beta = \tilde{h}$)

$$Q_{(\varpi,\tilde{h})} = \frac{1}{4\pi\mathcal{G}} \int_0^{2\pi} \mathrm{d}\phi \left(\partial_u \tilde{h}\,\beta\right)$$

$\hookrightarrow$ integrable, non-conserved: Carroll boost is anomalous, Weyl is pure gauge

- Anomalies: ($\mathcal{A} = \lim_{k\to 0} \mathrm{A}$, $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, $\lambda = \lim_{k\to 0} \frac{\eta}{k}$)

$$\delta_{(\xi,\sigma,\lambda)} S_{\mathrm{C}} = \int \left(\lambda\,\frac{\mathcal{F}}{8\pi\mathcal{G}}\right) \mathrm{vol}_{\partial\mathcal{M}}$$

$\hookrightarrow$ new holographic prediction, calling for further investigation

IV. Summary

Main goal:

- Explore the charges of 3D (AdS or flat) gravity in covariant Bondi gauge

  $\hookrightarrow$ bdy diffeomorphisms, Weyl rescalings and local frame boosts

Results:

- Divergences in the symplectic structure
- Renormalization via ambiguities
- Surface charges and anomalies
- New holographic Carrollian prediction

Future possibilities:

- Relate to asymptotic corner group [Donnelly-Freidel '16, Freidel-Geiller-Pranzetti '20, Ciambelli-Leigh-Pai '21]
- Connect to the celestial holography proposal [Strominger '17, Pasterski-Pate-Raclariu '21, Donnay-Fiorucci-Herfray-Ruzziconi '22]
- Extension to higher dimensions [Petkou-Petropoulos-Betancour-Siampos '22]

IV. Summary

"I'm Late", Alice in Wonderland, White Rabbit, by Sir John Tenniel

Thank you for listening!

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dS_2 as excitation of AdS_2

Florian Ecker

October 3, 2022

based on 2204.00045

with Daniel Grumiller and Robert McNees

Motivation

2D dilaton gravity as a playground for

- Classical and quantum gravity
- Black holes
- Holography

$\rightarrow$ Explore specific model with interesting features

Outline

1. A 2D gravity model with state-dependent curvature
2. Solution space
3. Thermodynamics

2D dilaton gravity with state-dependent curvature

- Dilaton gravity models: The UV-family ($\kappa^2 = 8\pi G$)

$$\Gamma_{\text{cl}} = \frac{1}{2\kappa^2}\int_M \mathrm{d}^2x\sqrt{-g}\,\big(X\,R - U(X)\,(\nabla X)^2 - 2V(X)\big) + \Gamma_\partial$$

Prominent members:

- JT-model $\quad U = 0 \quad V \propto X$

  [Teitelboim '83, Jackiw '84]

- CGHS-model $\quad U = -\frac{1}{X} \quad V \propto X$

  [Callan, Giddings, Harvey, Strominger '92]

2D dilaton gravity with state-dependent curvature

- Dilaton gravity models: The UV-family ($\kappa^2 = 8\pi G$)

$$\Gamma_{\text{cl}} = \frac{1}{2\,\kappa^2}\int_M \mathrm{d}^2x\sqrt{-g}\,\big(X\,R - U(X)\,(\nabla X)^2 - 2V(X)\big) + \Gamma_\partial$$

- Interesting 1-parameter subfamily:

$$U = -\frac{2}{X} \qquad V = -X^3 - bX^2 \qquad b \in \mathbb{R}^- \, .$$

  - Conformally related to Almheiri–Polchinski-model [Almheiri, Polchinski '14]

$$g^{AP}_{\mu\nu} = X^2\, g_{\mu\nu}$$

  $\rightarrow$ EOM are the same

2D dilaton gravity with state-dependent curvature

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$$g^{AP}_{\mu\nu} = X^2\, g_{\mu\nu}$$

$\rightarrow$ EOM are the same
$\rightarrow$ However: Different thermodynamics
Different behaviour when coupled to matter

Solution space

Static patch solutions: $\mathrm{d}s^2 = -\xi(r)\,\mathrm{d}t^2 + \frac{1}{\xi(r)}\,\mathrm{d}r^2$

$$\xi(r) = (1 + b\,r)^2 - \mu r^2\,, \qquad X = \frac{1}{r}$$

Remarks:

- Labeled by one constant of motion: μ
- State-dependent constant curvature

$$R = 2(\mu - b^2)$$

- Points at large X are in deep interior of spacetime

  $\Rightarrow$ Solutions share this region but have different asymptotics!

  $\Rightarrow$ Some solutions have Killing horizons, others do not

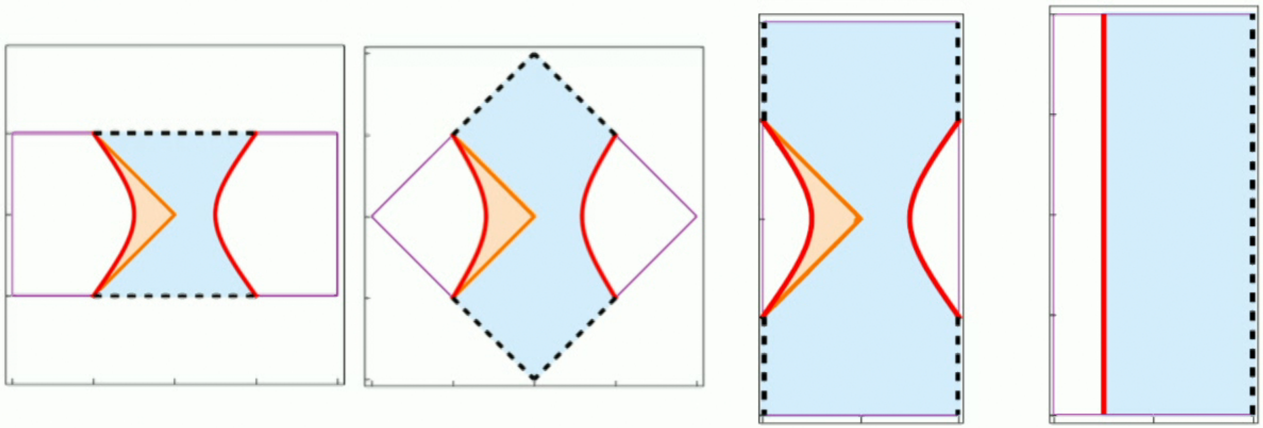
Solution space

$$R = 2(\mu - b^2)$$

$\mu > b^2$: static patch of dS
$\mu = b^2$: Rindler patch in flat space
$0 \leq \mu < b^2$: AdS-Rindler patch

} horizon at $r_h = \frac{1}{\sqrt{\mu} - b}$

$\mu < 0$: AdS without horizon



Thermodynamics

- Switch to Euclidean signature $\tau = it$ and impose additional boundary condition

$$\tau \sim \tau + \beta \qquad\qquad \beta = T^{-1} \,.$$

- Saddle point approximation: $\Gamma_{\text{cl}}^{\text{Eucl}} = \beta F$
- Impose regularity at the horizon $\Rightarrow \mu = 4\pi^2 T^2 > 0$ for a given T
- Horizonless solutions ($\mu < 0$) are consistent with any T
  $\Rightarrow$ canonical ensemble: 1 state w. horizon + horizonless continuum

States with horizon are ground states at any temperature

$$F_{\text{hor}} = -\frac{1}{2\,\kappa^2}\left(2\pi T - b\right)^2 \leq -\frac{b^2}{2\kappa^2} \qquad F_{\text{no hor}} = \frac{3\left(b^2 - \mu\right)}{2\kappa^2} > 0$$

Thermodynamics

$T > \lvert b\rvert/2\pi$	$T = \lvert b\rvert/2\pi$	$0 < T < \lvert b\rvert/2\pi$
dS_2	flat space	AdS_2

$\Rightarrow$ The high T phase is dominated by dS_2, the low T phase by AdS_2

Various checks of stability:

- Tunneling to the continuum
- Inclusion of classical matter (bubble nucleation)
- Semiclassical backreaction of N massless scalar fields ϕ

  Define path integral measure:

$$1 = \int \mathcal{D}\,\delta\phi\, e^{-\|\delta\phi\|^{2}} \qquad \|\delta\phi\|^{2} := \int_M \mathrm{d}^2x\sqrt{g}\; X^2\,\delta\phi^2$$

$\Rightarrow$ Picture does not change qualitatively.

Further directions

- Solve for looser boundary conditions
  $\rightarrow$ Investigate asymptotic symmetries
- Look for implementations of holography
  $\rightarrow$ Is dS_2 dual to high-temperature SYK? [Susskind '21]
- Is the model related to some higher dimensional theory?
  (like e.g. JT to near-extremal RN black hole)

Thank You!

Brown-York charges on null boundaries

with alternative boundary conditions

Gloria Odak

October 3 2022

CPT Marseille

* dependence of energy on boundary conditions was anticipated by Iyer&Wald '95

* recent attention around the improved Noether charge prescription and the role of boundary lagrangians has given new interest to this question.

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$$L = L^{bulk} + \ell \longrightarrow \theta = \theta^{bulk} + \delta\ell - d\vartheta$$

"pdq" for a choice of polarization of the phase space

corner symplectic potential induced by ℓ

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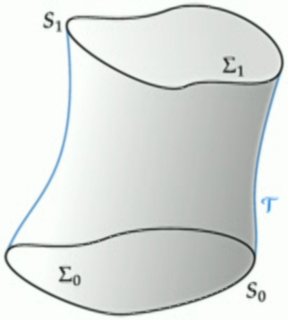
* recent attention around the improved Noether charge prescription and the role of boundary lagrangians has given new interest to this question.

$$L = L^{bulk} + \ell \qquad\qquad \theta = \theta^{bulk} + \delta\ell - d\vartheta$$

$$\delta H_\xi = -\int_\Sigma I_\xi \delta\theta$$

$$H_\xi = \int_S q_\xi^{bulk} + i_\xi \ell - I_\xi \vartheta$$

non-null boundaries



Pullback of the EH symplectic potential to the non-null boundary

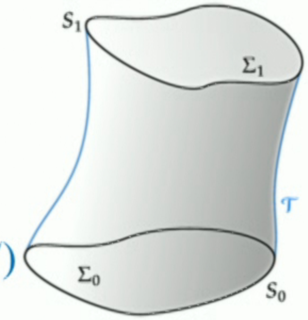
$$\theta^{\rm EH} = s\left(K_{\mu\nu}\delta q^{\mu\nu} - 2\delta K\right)\epsilon_{\Sigma} + d\vartheta^{\rm EH}$$

non-null boundaries

Pullback of the EH symplectic potential to the non-null boundary

$$\theta^{\rm EH} = s\left(K_{\mu\nu}\delta q^{\mu\nu} - 2\delta K\right)\epsilon_\Sigma + d\vartheta^{\rm EH} = s q_{\mu\nu}\delta\tilde{\Pi}^{\mu\nu} d^3x + d\vartheta^{\rm EH}$$

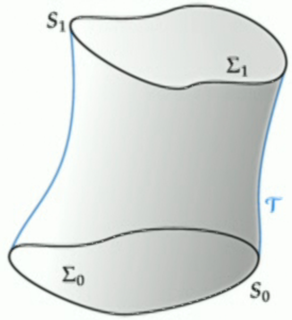
$$\tilde{\Pi}^{\mu\nu} = \sqrt{q}(K^{\mu\nu} + K q^{\mu\nu})$$



DIRICHLET: $\delta q^{\mu\nu} = 0$ $\ell^D = 2sK\epsilon_\Sigma$

NEUMANN: $\delta\tilde{\Pi}_{\mu\nu} = 0$ $\ell^N = 0$

non-null boundaries



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$$= -\tilde{P}^{\mu\nu}\delta\hat{q}_{\mu\nu} + \frac{4}{3}\delta K - \frac{2}{3}\delta(K\delta q)$$

$$\hat{q}_{\mu\nu} = q^{-\frac{1}{3}} q_{\mu\nu}$$

DIRICHLET:	$\delta q^{\mu\nu} = 0$	$\ell^D = 2sK\epsilon_\Sigma$
NEUMANN:	$\delta\tilde{\Pi}_{\mu\nu} = 0$	$\ell^N = 0$
YORK:	$\delta\hat{q}_{\mu\nu} = 0 = \delta K$	$\ell^Y = \frac{2}{3}sK\epsilon_\sigma$

non-null boundaries

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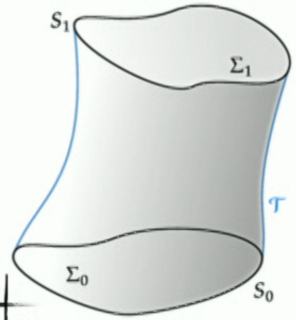

$$\ell^b = bK\epsilon_\Sigma$$

$$L = L^{EH} + d\ell^b$$

$$\theta := \theta^{EH} + \delta\ell^b - d\vartheta^{EH}$$

(always with the same corner symplectic potential)

$$\vartheta^{\rm EH} := -u_\mu \delta n^\mu \epsilon_S = u^\mu n^\nu \delta g_{\mu\nu}\epsilon_S$$

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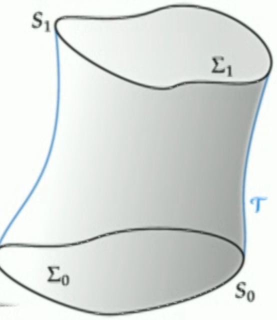

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DIRICHLET:	$\delta q^{\mu\nu} = 0$	$\ell^D = 2sK\epsilon_\Sigma$
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YORK:	$\delta\hat{q}_{\mu\nu} = 0 = \delta K$	$\ell^Y = \frac{2}{3}sK\epsilon_\sigma$

$$H^b_\xi = \int_S q^{\rm EH}_\xi + i_\xi\ell^b - I_\xi\vartheta^{EH} = \ldots = -2\int_S n^\mu\xi^\nu\left(\bar{K}_{\mu\nu} - \frac{b}{2}\bar{q}_{\mu\nu}\bar{K}\right)\epsilon_S$$

null boundary

- KNOWN CONSTRAINT-FREE DATA
- DON'T HAVE TO RESTRICT TO CONSERVATIVE B.C.
- LEAKY B.C. NOT AMBIGUOUS

Page 18 sur 25

null boundary

Pullback of the symplectic potential on a null boundary

$$\Theta = \int_{\mathcal{N}} \left[B^{\mu\nu}\delta\gamma_{\mu\nu} + 2\delta(\theta + k) + 2\omega_\mu \delta l^\mu + \partial_n l^2 n^\mu \delta l_\mu \right] \epsilon_{\mathcal{N}} + \int_{\partial\mathcal{N}} \vartheta^{\mathrm{EH}}$$

null boundary

Pullback of the symplectic potential on a null boundary

$$\Theta = \int_{\mathcal{N}} \left[B^{\mu\nu}\delta\gamma_{\mu\nu} + 2\delta(\theta + k) + 2\omega_\mu \delta l^\mu + \partial_n l^2 n^\mu \delta l_\mu \right] \epsilon_{\mathcal{N}} + \int_{\partial\mathcal{N}} \vartheta^{\mathrm{EH}}$$

$$= \int_{\mathcal{N}} \left[\left(\sigma^{\mu\nu} - \frac{1}{2}(\theta + 2k)\gamma^{\mu\nu} \right) \delta\gamma_{\mu\nu} + 2\left(\eta_\mu - \theta n_\mu \right) \delta l^\mu + \frac{1}{2}\partial_n l^2 n^\mu \delta l_\mu \right] \epsilon_{\mathcal{N}} - \delta\ell^{\mathrm{D}} + \int_{\partial\mathcal{N}} \vartheta^{\mathrm{EH}}$$

$$= \int_{\mathcal{N}} \left[\hat{\sigma}^{\mu\nu}\delta\hat{\gamma}_{\mu\nu} + \delta(\theta + 2k) + 2\left(\eta_\mu - 2k n_\mu \right) \delta l^\mu + \frac{1}{2}\partial_n l^2 n^\mu \delta l_\mu \right] \epsilon_{\mathcal{N}} - \delta\ell^{\mathrm{Y}} + \int_{\partial\mathcal{N}} \vartheta^{\mathrm{EH}}$$

$\gamma^{\mu\rho}\gamma^{\nu\sigma}\delta\gamma_{\rho\sigma} = 0$	$\ell = -2k\epsilon_{\mathcal{N}}$	$\vartheta = \vartheta^{EH} + 2\delta\epsilon_S$
$\delta\tilde{P}^{\mu\nu} = 0$	$\ell = 0$	$\vartheta = \vartheta^{EH} + \delta\epsilon_S$
$\delta\hat{\gamma}_{\mu\nu} = 0 = \delta(\theta + 2k)$	$\ell = 0$	$\vartheta = \vartheta^{EH} + \delta\epsilon_S$
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null boundary

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$$\ell^b = -bk\epsilon_{\mathcal{N}}$$

$$L = L^{EH} + d\ell^b$$

$$\vartheta^c = \vartheta^{EH} - c\delta\epsilon_S$$

$$\vartheta^{\mathrm{EH}} := n_\mu \delta l^\mu \epsilon_S$$

$$\theta := \theta^{EH} + \delta\ell^b - d\vartheta^c$$

$$H_\xi = \int_S q_\xi^{\mathrm{EH}} + i_\xi \ell^b - I_\xi \vartheta^c = \ldots = -2\int_S \xi_\mu n_\nu \left(\nabla^\nu l^\mu - \frac{1}{2}\left(bk_{(n)} + c\theta_{(n)}\right) g^{\mu\nu} \right) \epsilon_S$$

null boundary

$\gamma^{\mu\rho}\gamma^{\nu\sigma}\delta\gamma_{\rho\sigma} = 0$	$\ell = -2k\epsilon_{\mathcal{N}}$	$\vartheta = \vartheta^{EH} + 2\delta\epsilon_S$
$\delta\tilde{P}^{\mu\nu} = 0$	$\ell = 0$	$\vartheta = \vartheta^{EH} + \delta\epsilon_S$
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* unlike the timelike case, for b=2=c, there is an anomaly discrepancy between the improved Noether charge and BY: $H_\xi = H_\xi^{BY} - 2n^\mu \Delta_\xi l_\mu$
* none of the b.c. studied here remove this discrepancy and matching is found only for non-anomalous diffeos