Title: Review Talk: An introduction to coadjoint orbits cont.

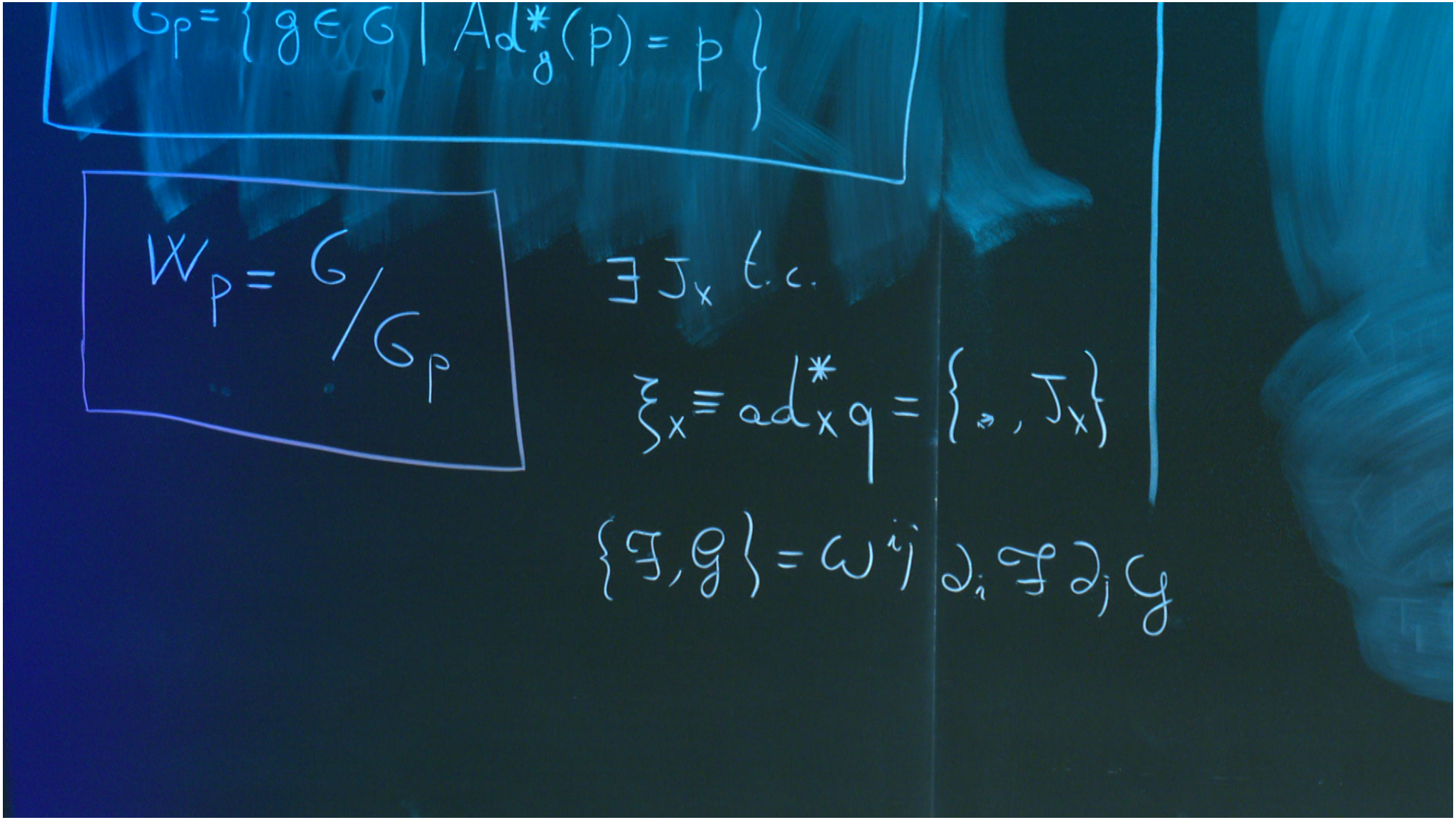
Speakers: Andrea Campoleoni

Collection: Quantum Gravity Around the Corner

Date: October 03, 2022 - 2:35 PM

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HAMILTONIAN FIELD & MOMENTUM MAP
MOMENTUM MAP FOR COADJ. O.
$J_x(p) = \langle p, x \rangle$


$G_p = \{ g \in G \mid Ad^*_g(p) = p \}$
$W_p = G/G_p$
$\exists J_x$ t.c.
$\xi_x \equiv ad^*_x q = \{ \cdot, J_x \}$
$\{ f, g \} = \omega^{ij} \partial_i f \partial_j g$

$$\{\mathcal{F},\mathcal{G}\}(p) = \langle p, [T_p\mathcal{F}, T_p\mathcal{G}]\rangle$$

($T_p\mathcal{F}$: differential of $\mathcal{F}$)

$$\mathcal{F} \to \xi_{\mathcal{F}} = -\{\mathcal{F}, \cdot\} \quad \& \quad [\xi_{\mathcal{F}}, \xi_{\mathcal{G}}] = \xi_{\{\mathcal{F},\mathcal{G}\}}$$

FROBENIUS $\longrightarrow$ $\mathfrak{g}^*$ is foliated in SYMPLECTIC LEAVE

$$\dot{\gamma}(t) = (\xi_H)_{\gamma(t)} \in T\mathfrak{g}^* \simeq \mathfrak{g}^*$$

$$\dot{\gamma}(t) = \mathrm{ad}^*_{T_{\gamma(t)}H}(\gamma(t))$$

$$\dot{\gamma}(t) = (\xi_H)_{\gamma(t)} \in T\mathfrak{g}^* \simeq \mathfrak{g}^*$$

$$\boxed{\dot{\gamma}(t) = \mathrm{ad}^*_{T_{\gamma(t)}H}(\gamma(t))}$$

$$(\xi_H)_p(\mathcal{F}) = -\{H, \mathcal{F}\} = -\langle p, [T_pH, T_p\mathcal{F}] \rangle$$

$$\langle (\xi_H)_p, T_p\mathcal{F} \rangle \qquad = -\langle p, \mathrm{ad}_{T_pH}(T_p\mathcal{F}) \rangle = \langle \mathrm{ad}^*_{T_pH}, T_p\mathcal{F} \rangle$$

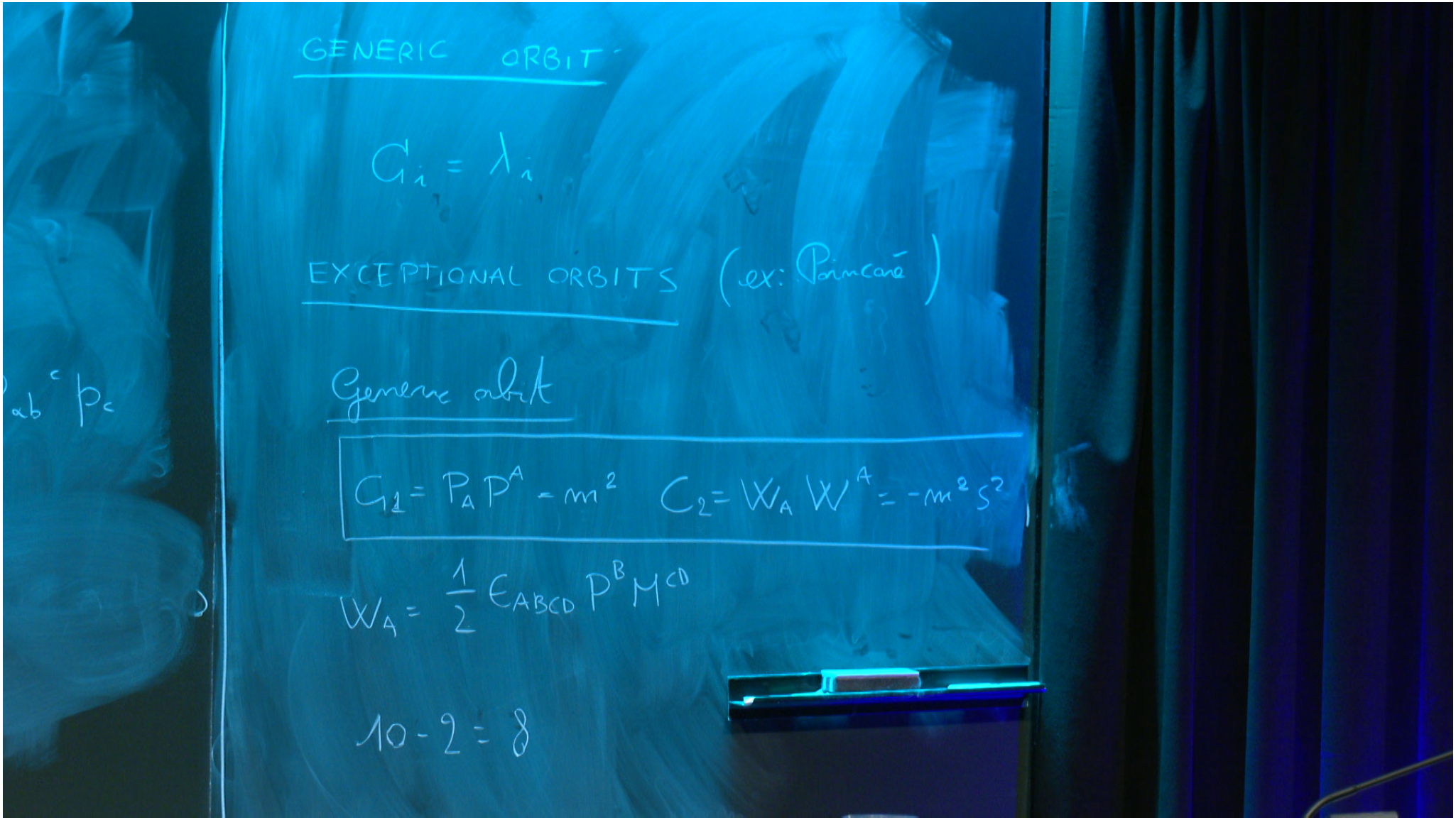
$$[t_a, t_b] = f_{ab}{}^c t_c$$

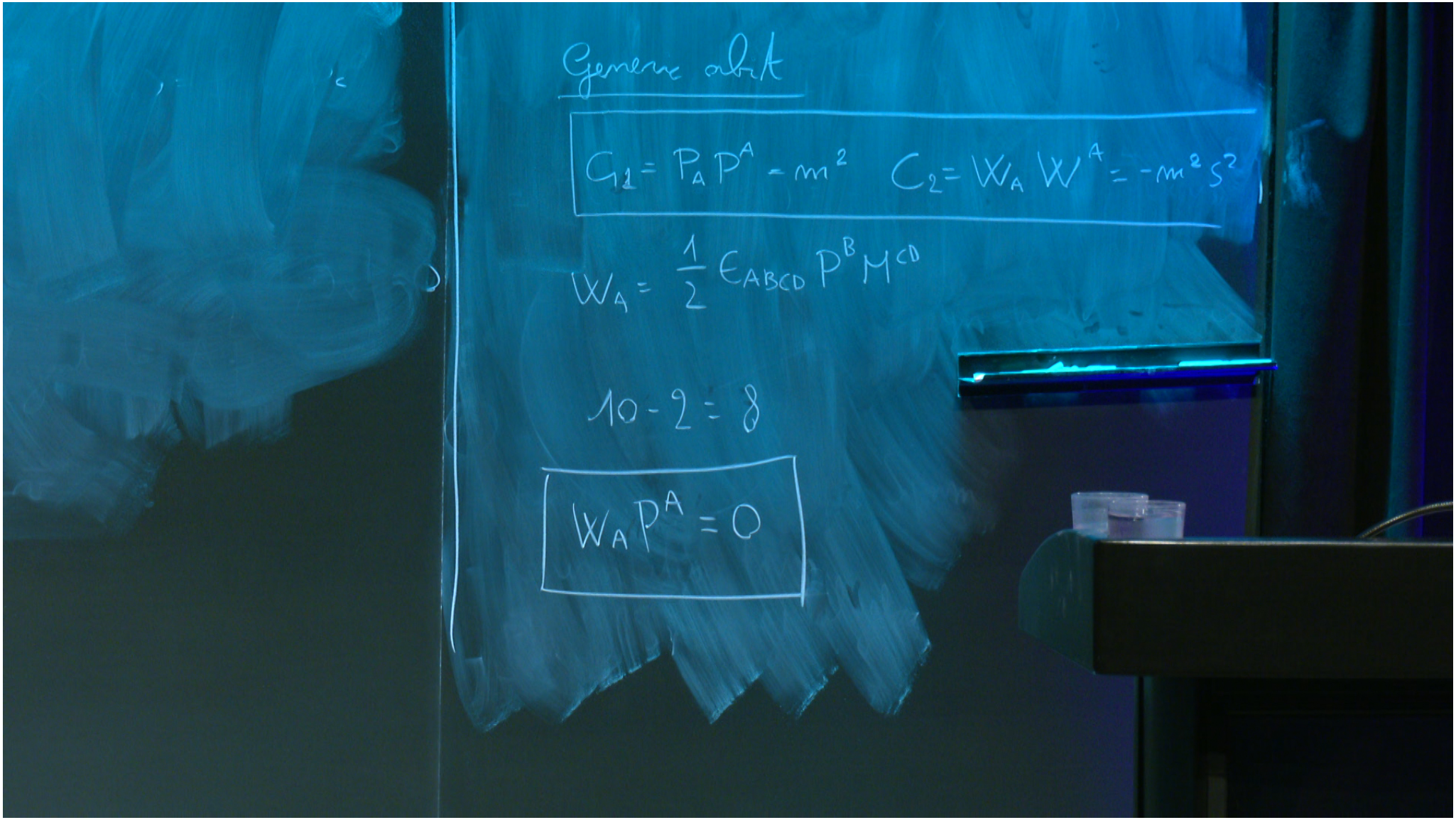
$$\langle (t^a)^*, t_b \rangle = \delta^a_b \qquad T_{p_a} \in (g^*)^* \simeq g$$

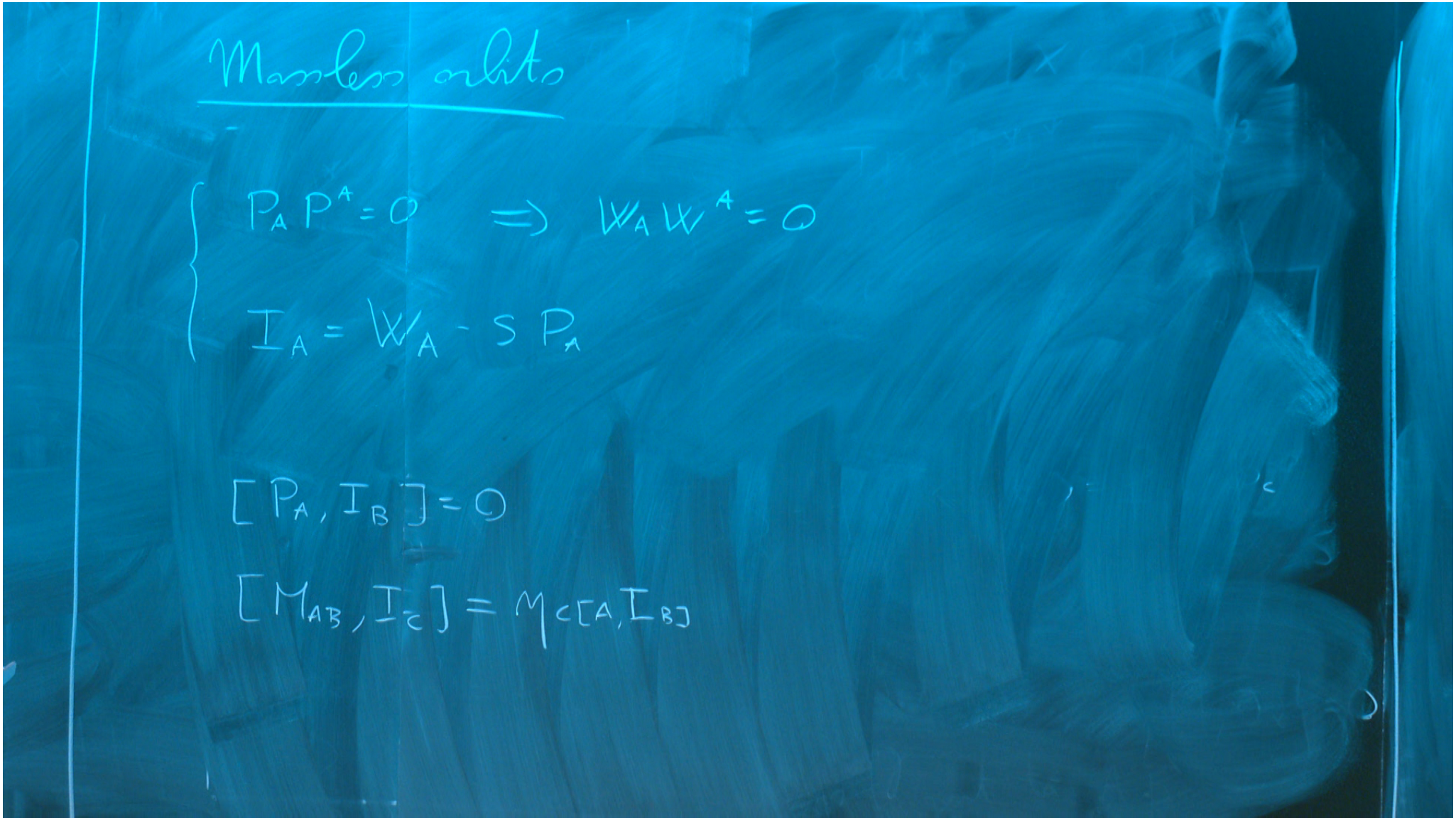
$$T_q p_a \left(\frac{\partial}{\partial p_b} \right) = \frac{\partial p_a}{\partial p_b} = \delta^b_a \implies T_{p_a} \simeq t_a$$

$$\{p_a, p_b\} = \langle p, [t_a, t_b] \rangle = f_{ab}{}^c \langle p, t_c \rangle = f_{ab}{}^c p_c$$

$$\boxed{\{p_a, p_b\} = f_{ab}{}^c p_c}$$


GENERIC ORBIT
$G_i = \lambda_i$
EXCEPTIONAL ORBITS (ex: Poincaré)
Generic orbit
$C_1 = P_A P^A = m^2$ $C_2 = W_A W^A = -m^2 s^2$
$W_A = \frac{1}{2} \epsilon_{ABCD} P^B M^{CD}$
$10 - 2 = 8$


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$W_A = \frac{1}{2} \epsilon_{ABCD} P^B M^{CD}$
10 - 2 = 8
$W_A P^A = 0$


Massless orbits
$P_A P^A = 0 \Rightarrow W_A W^A = 0$
$I_A = W_A - S P_A$
$[P_A, I_B] = 0$
$[M_{AB}, I_C] = M_{C[A} I_{B]}$

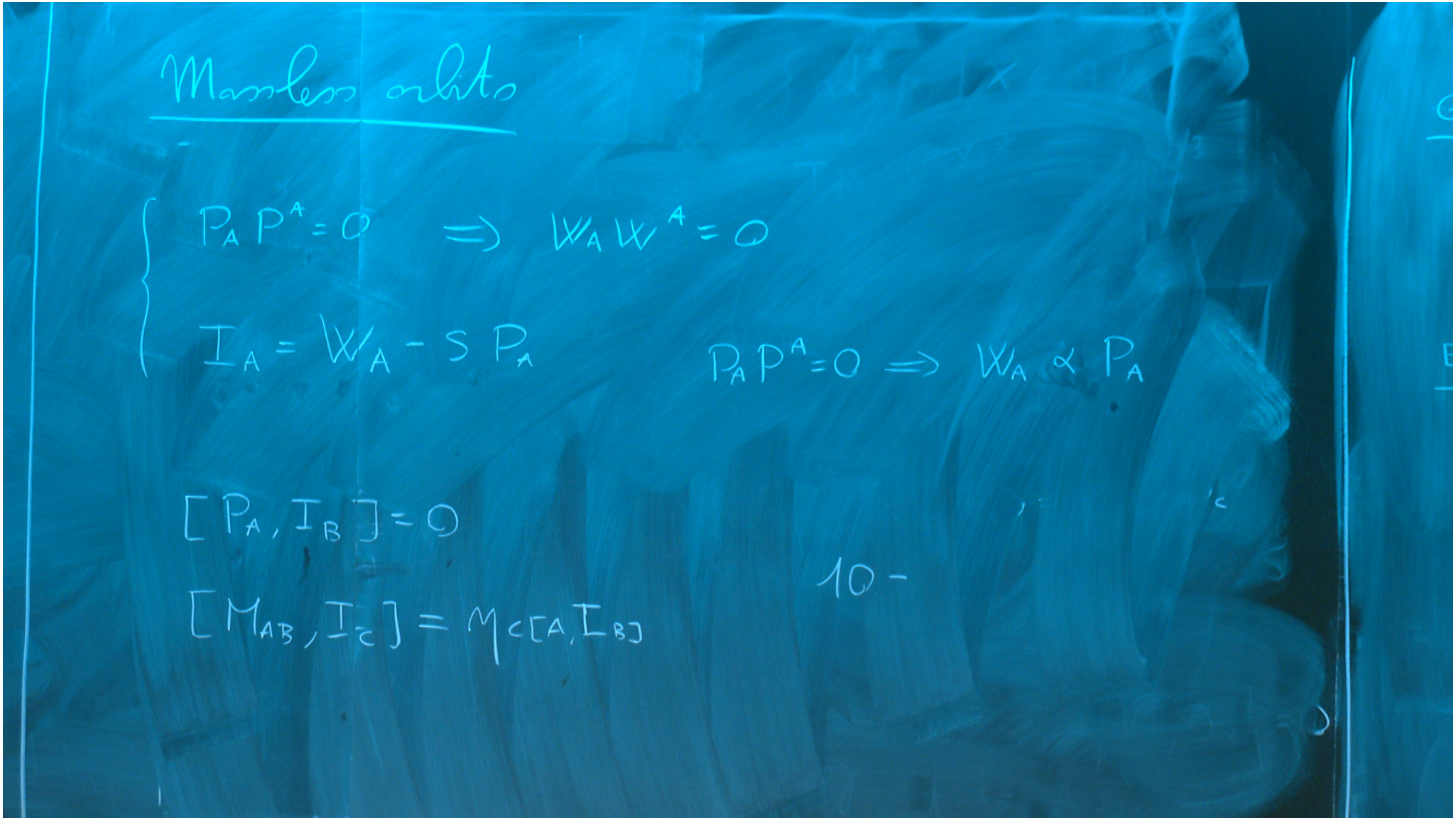
Massless orbits

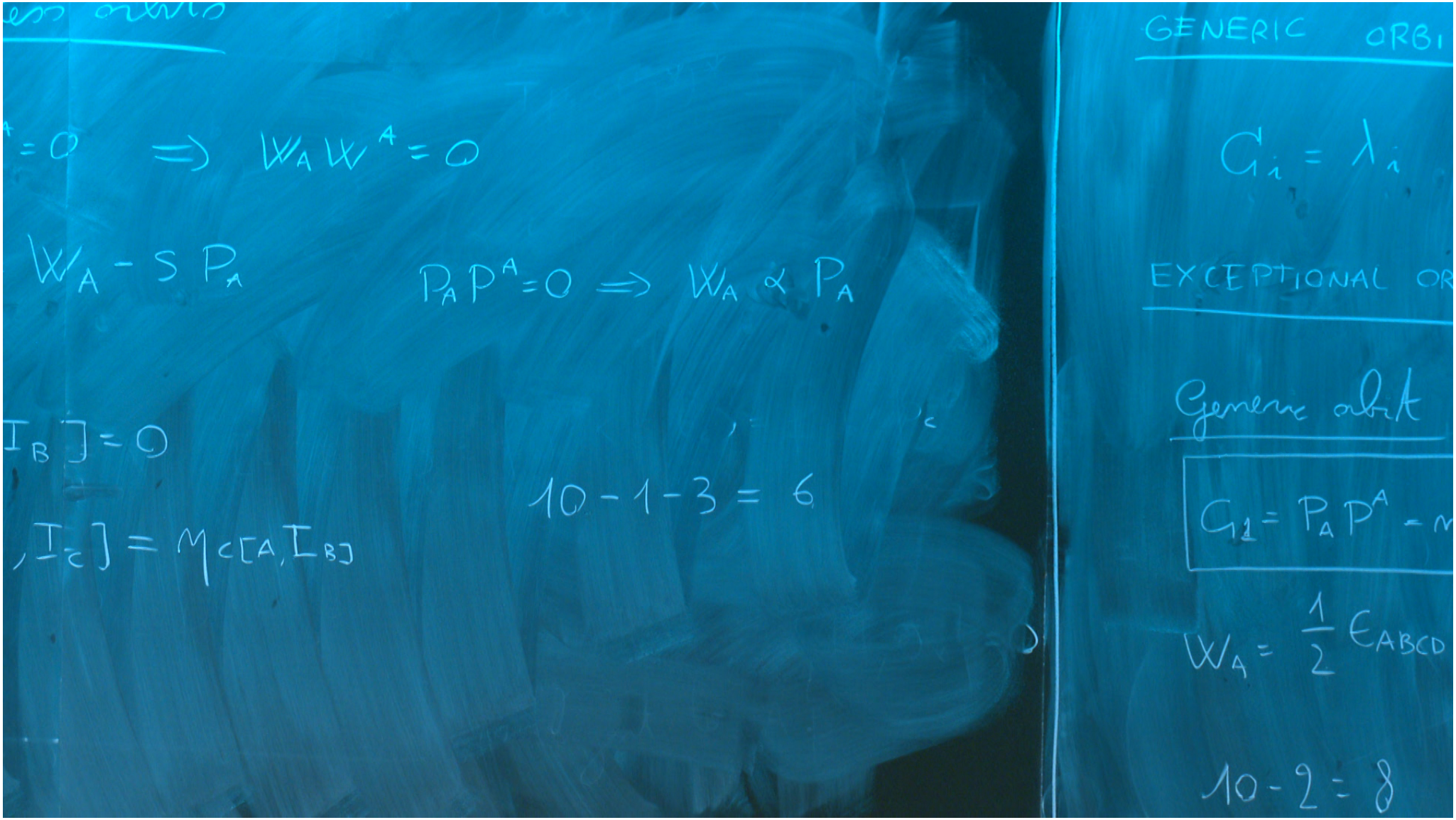
$$\begin{cases} P_A P^A = 0 \implies W_A W^A = 0 \\ I_A = W_A - S P_A \end{cases}$$

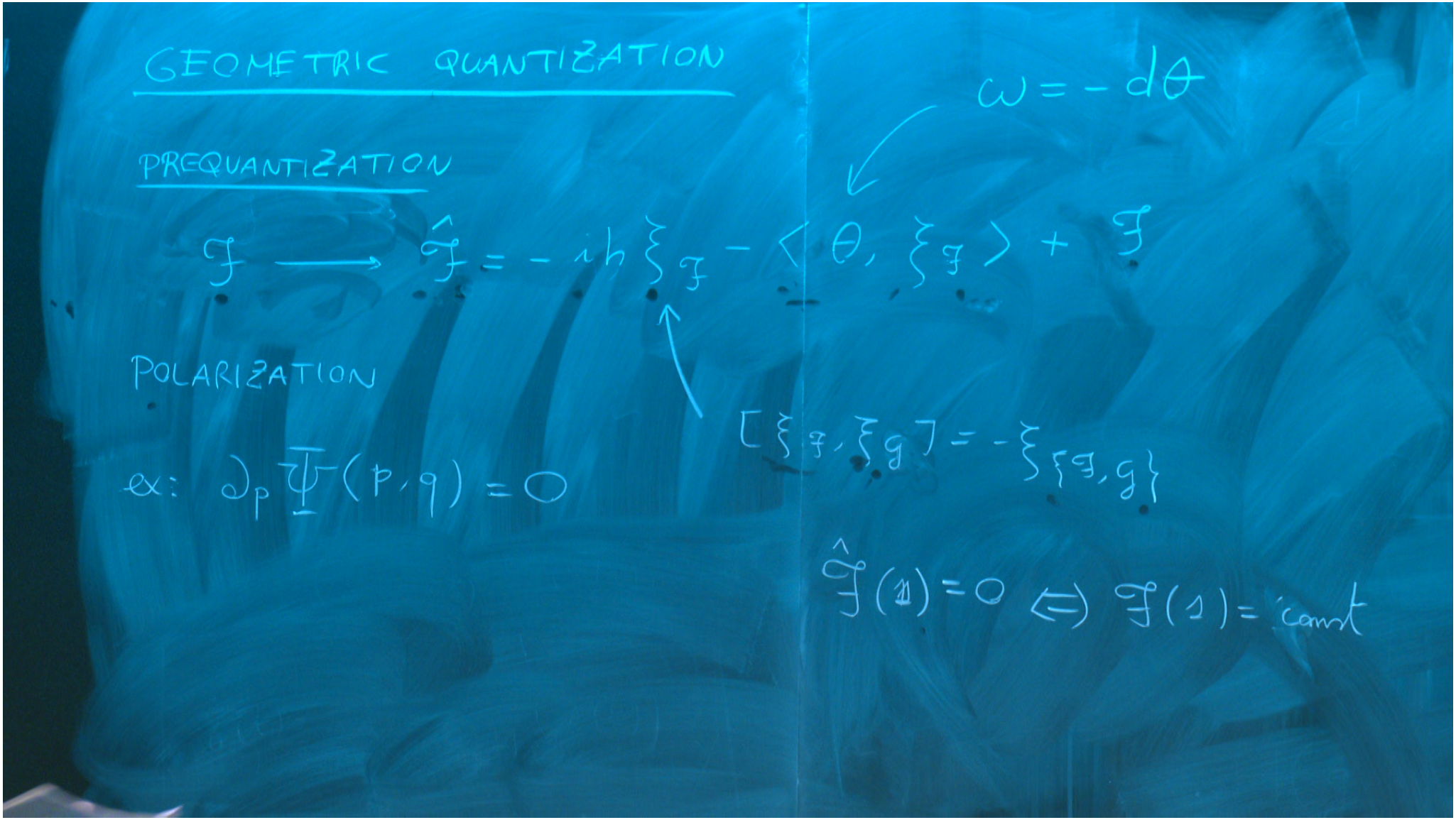
$P_A P^A = 0 \implies W_A \propto P_A$

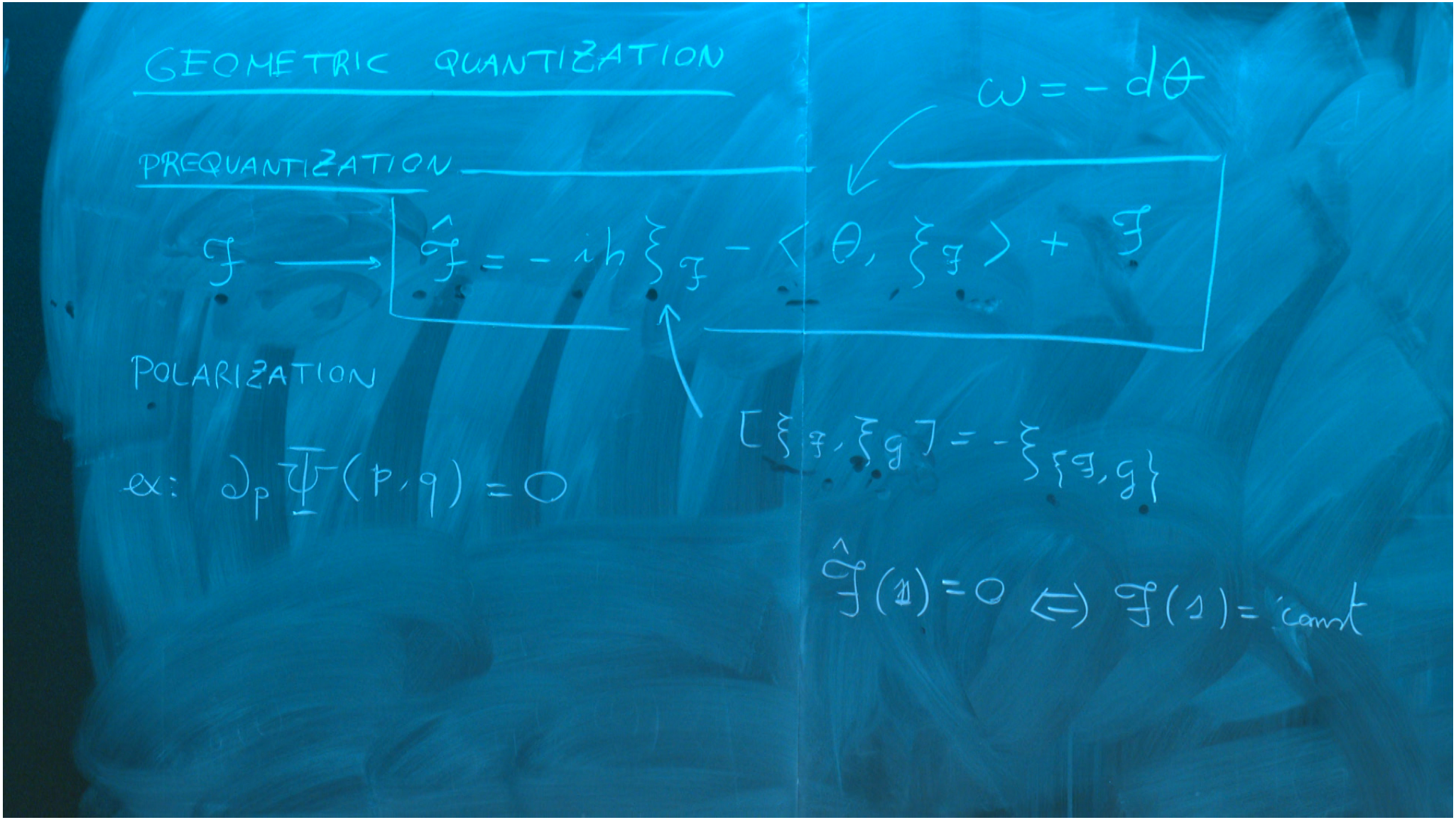
$[P_A, I_B] = 0$

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$I_A = W_A - S P_A$
$P_A P^A = 0 \Rightarrow W_A \propto P_A$
$[P_A, I_B] = 0$
$[M_{AB}, I_C] = \eta_{C[A} I_{B]}$
10 -


$W^A = 0 \Rightarrow W_A W^A = 0$
$W_A \propto P_A$
$P_A P^A = 0 \Rightarrow W_A \propto P_A$
$I_B] = 0$
$I_C] = \eta_{C[A} I_{B]}$
$10 - 1 - 3 = 6$
GENERIC ORBI
$G_i = \lambda_i$
EXCEPTIONAL OR
Generic orbit
$G_1 = P_A P^A$
$W_A = \frac{1}{2} \epsilon_{ABCD}$
$10 - 2 = 8$


GEOMETRIC QUANTIZATION
PREQUANTIZATION
$\omega = -d\theta$
$f \longrightarrow \hat{f} = -ih\xi_f - \langle \theta, \xi_f \rangle + f$
POLARIZATION
$[\xi_f, \xi_g] = -\xi_{\{f,g\}}$
ex: $\partial_p \Psi(p,q) = 0$
$\hat{f}(\psi) = 0 \Leftrightarrow f(\psi) = \text{const}$


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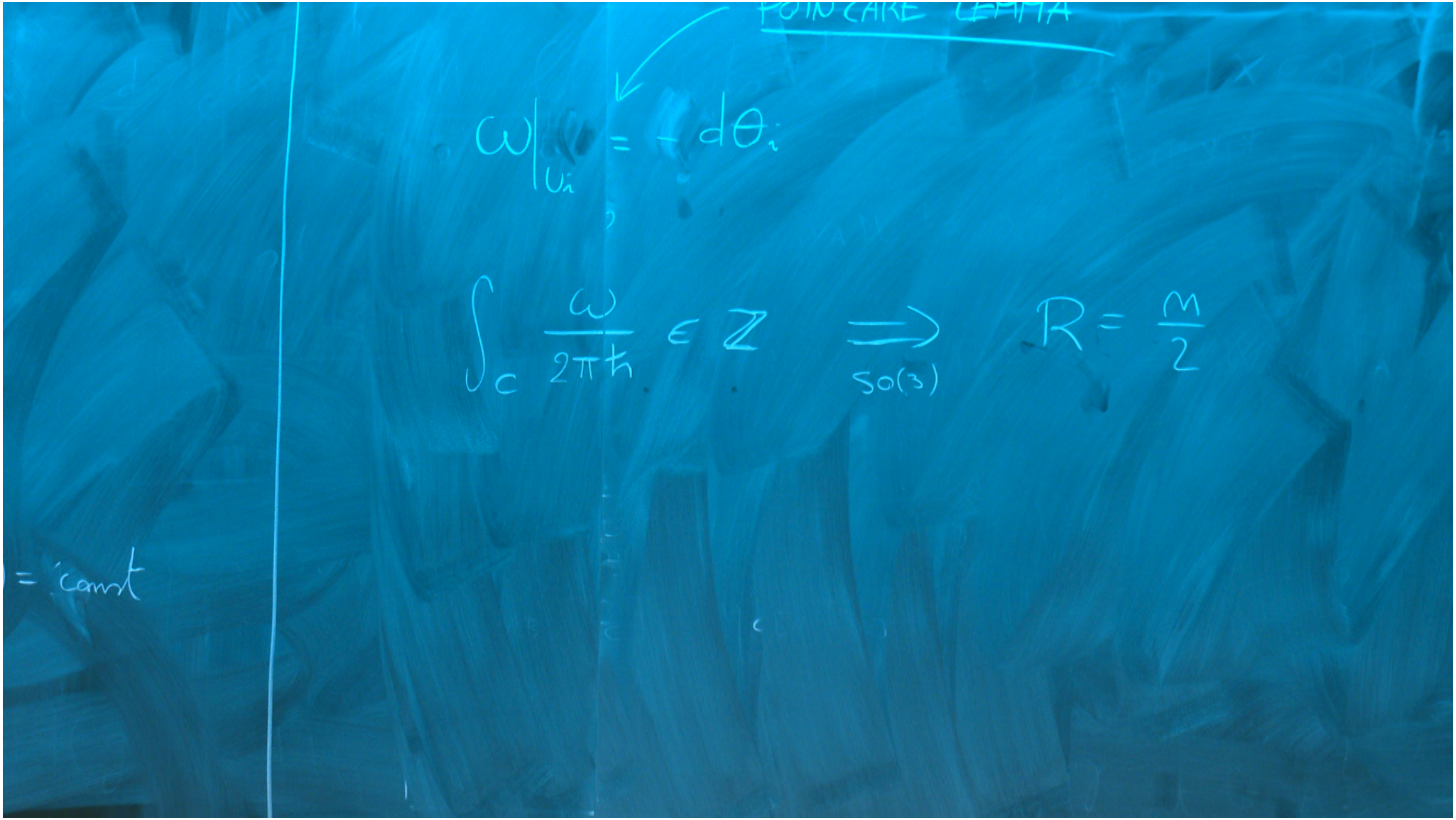
$\{f, g\}$

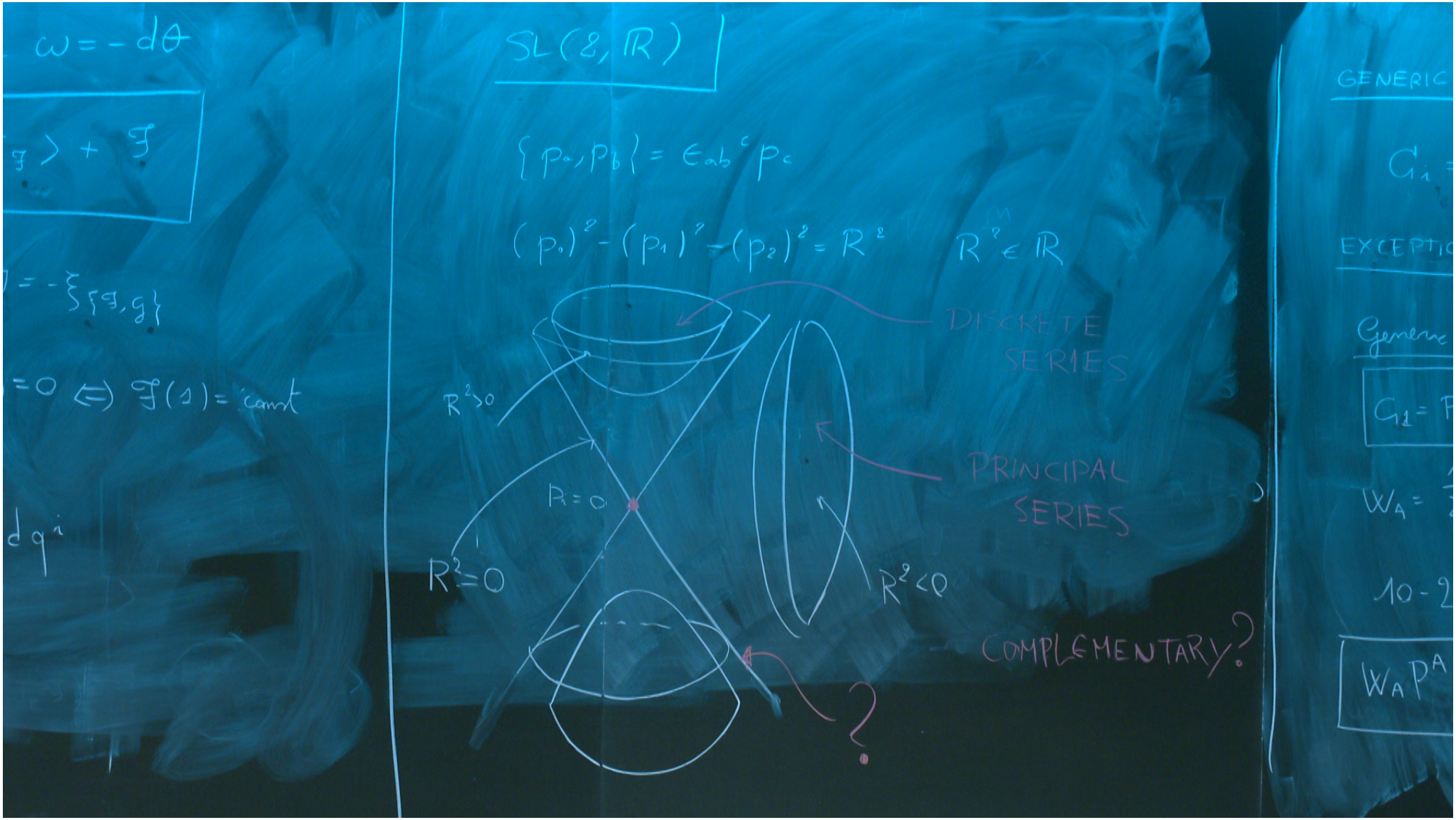
$\hat{\mathcal{F}}(\mathbb{1}) = 0 \Leftrightarrow \mathcal{F}(1) = \text{const}$

Example: Darboux coords.

$\omega = dq^i \wedge dp_i \longrightarrow \theta = p_i dq^i$

$$\begin{cases} \hat{q}^j = i\hbar \dfrac{\partial}{\partial p_j} + q^j \\ \hat{p}_j = -i\hbar \dfrac{\partial}{\partial q^j} \end{cases}$$


POINCARE LEMMA
$\omega|_{U_i} = -d\theta_i$
$\int_C \frac{\omega}{2\pi\hbar} \in \mathbb{Z} \implies R = \frac{n}{2}$
SO(3)
= const


$\omega = -d\theta$
$SL(2,\mathbb{R})$
$\{P_a, P_b\} = \epsilon_{ab}{}^c P_c$
$(P_0)^2 - (P_1)^2 - (P_2)^2 = R^2$
$R^2 \in \mathbb{R}$
DISCRETE SERIES
PRINCIPAL SERIES
COMPLEMENTARY?
$R^2 > 0$
$R = 0$
$R^2 = 0$
$R^2 < 0$
?
GENERIC
EXCEPTIO
Generic
$W_A P^A$