Title: Towers of soft operators and celestial holography

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Collection: Quantum Gravity Around the Corner

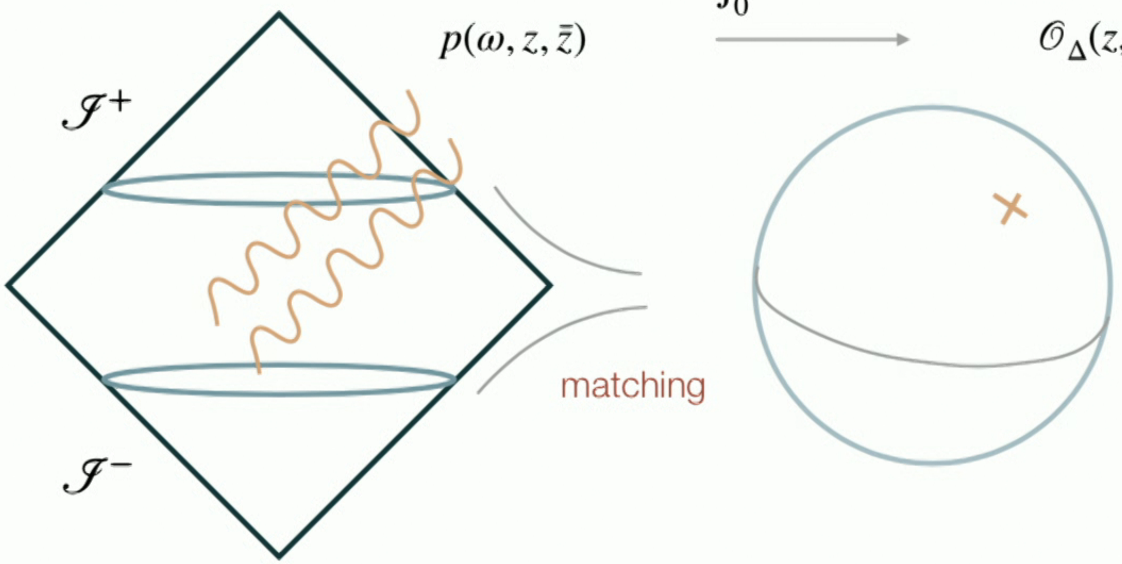
Date: October 05, 2022 - 10:00 AM

URL: https://pirsa.org/22100044

Abstract: The tree-level soft theorems were recently shown to arise from the conservation of infinite towers of charges extracted from the asymptotic Einstein equations. There is evidence this tower promotes the extended BMS algebra to an infinite higher-spin symmetry algebra. In this talk I will introduce towers of canonically conjugate memory and Goldstone operators, highlighting their role in parameterizing the gravitational phase space. I will discuss the conditions under which these towers provide a complete set of scattering states and demonstrate that they are the building blocks of both soft and hard charges. I will finally show that the tower of tree level soft symmetries can be used to extend the Dirac (Faddeev-Kulish) dressings to include the infinite towers of Goldstones and comment on their implications for the gravitational S-matrix.

# Motivation

$$p(\omega, z, \bar{z}) \xrightarrow{\int_0^\infty d\omega\, \omega^{\Delta-1}} \mathcal{O}_\Delta(z, \bar{z})$$




4D asymptotically flat spacetime

2D celestial sphere

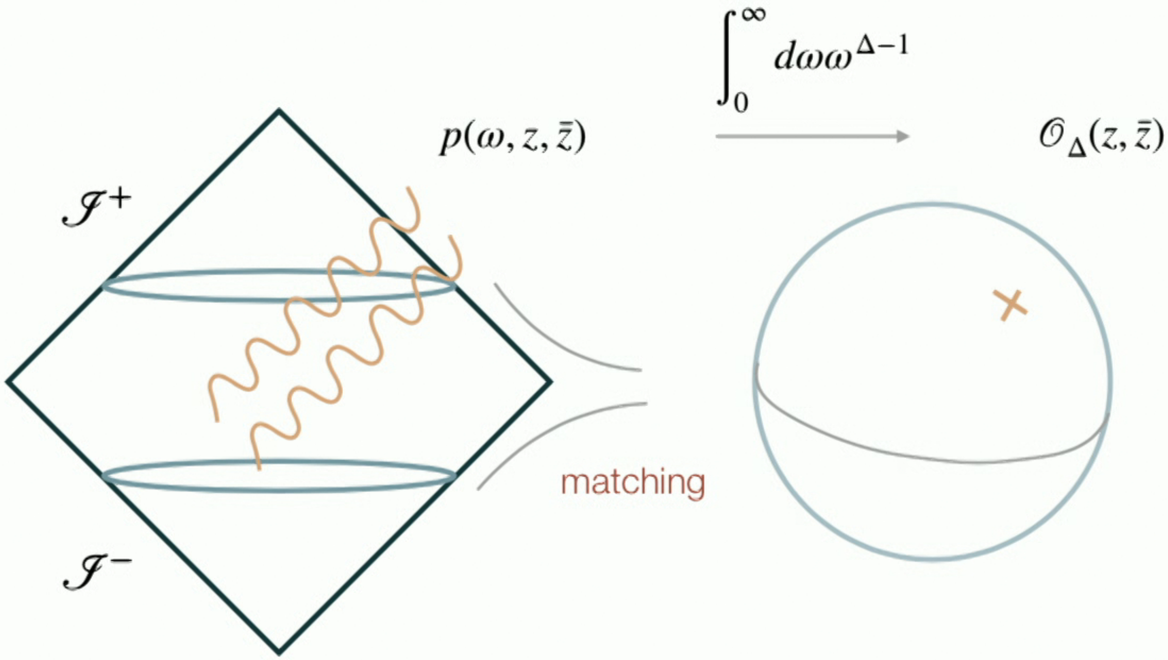
Plane wave → conformal primary basis

$$e^{\pm i\omega \hat{p}\cdot X} \to \varphi_\Delta(\hat{p}; X_\pm)$$

$$\pm\omega \to \Delta = 1 + i\lambda, \quad \lambda \in \mathbb{R}$$

[Pasterski, Shao, Strominger '16; Pasterski, Shao '17, Romain's talk]

# Motivation




4D asymptotically flat spacetime

2D celestial sphere

Translations $\hat{P}$ shift $\Delta$:

$$\hat{P}\mathcal{O}_\Delta = \mathcal{O}_{\Delta+1}$$

Q: Integer $\Delta$ basis?

# Outline

- Tower of soft theorems from charge conservation [Laurent's talk]
- Memories, Goldstones and the discrete basis
- Applications:
  - Soft and hard charges
  - Dressed states and the soft S matrix

# From soft theorems to conservation laws

System of differential equations extracted from asymptotic EE: $\mathcal{Q}_s^k = D\partial_u^{-1}(\mathcal{Q}_{s-1}^k) + \frac{(s+1)}{2}\partial_u^{-1}(C\mathcal{Q}_{s-2}^{k-1})$

integral $\int^u du'$

- $Q_0$ — complex mass aspect
- $Q_1$ — angular momentum aspect
- $Q_{s\geq 2}$ — transverse metric/$\mathbf{\Psi}_0$ expansion

Primaries with respect to extended BMS

Charges are highly non-linear; admit an expansion $\mathcal{Q}_s = \mathcal{Q}_s^1 + \mathcal{Q}_s^2 + \cdots$

quadratic

Charge action at $\mathcal{I}^+_-$ is ill defined $\quad \lim_{u\to-\infty} \{\mathcal{Q}_s(u,z), C(u',z')\} = \infty \ldots$

[Freidel, Pranzetti '21; Freidel, Pranzetti, A.R. '21]

# From soft theorems to conservation laws

Charge action at $\mathscr{I}^{+}_{-}$ becomes well defined for $q_s(z) = \lim_{u \to -\infty} \sum_{n=0}^{s} \frac{(-u)^{s-n}}{(s-n)!} D^{s-n} \mathcal{Q}_n(u,z)$

Linear charges related to sub-leading soft gravitons:

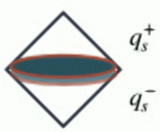
$$q_s^1(z) = \frac{\kappa}{16\pi} \frac{(-i)^s}{s!} \lim_{\omega \to 0^+} (\partial_\omega)^{s-1}(1+\omega\partial_\omega) D^{s+2} \left( a_+^{\text{out}\dagger}(\omega\hat{x}) + (-1)^s a_-^{\text{out}}(\omega\hat{x}) \right)$$

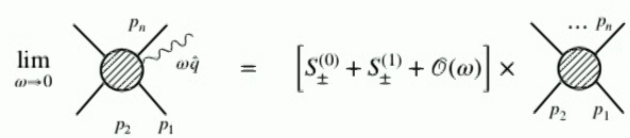
Action of quadratic charges related to the soft factor:

$1 + u'\partial_{u'}$

$$\{q_s^2(z), C(u',z')\} = \frac{\kappa^2}{8} \sum_{n=0}^{s} (-1)^{s+n} \frac{(n+1)(\Delta+2)_{s-n}}{(s-n)!} \partial_{u'}^{1-s} D_{z'}^n C(u',z') D_z^{s-n} \delta(z,z')$$

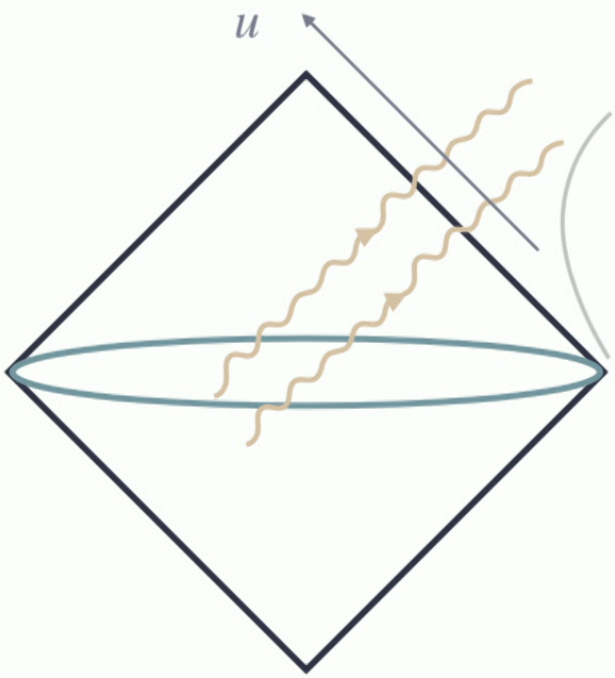
Conservation laws truncated to quadratic order $\langle \text{out} | [q_s^1, \mathcal{S}] | \text{in} \rangle = - \langle \text{out} | [q_s^2, \mathcal{S}] | \text{in} \rangle \implies$ tower of soft theorems!




[Freidel, Pranzetti, A.R. '21]

# From soft theorems to conservation laws




Charges well defined provided

$$N \sim |u|^{-1-s-\epsilon}, \quad u \to \pm\infty$$

**IR condition**

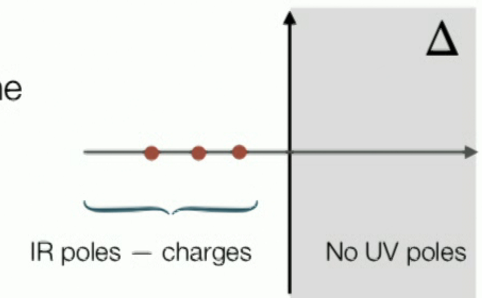
Assume exponential suppression as $\omega \to \infty$

**UV condition**

$$\widetilde{N}_{-}(\omega) = \int du e^{i\omega u} \partial_u C^*(u)$$

No poles in RH complex $\Delta$ plane

$$\widehat{N}(\Delta) = \int_0^\infty d\omega \omega^{\Delta-1} \widetilde{N}_{-}(\omega)$$




[Arkani-Hamed, Pate, A.R., Strominger '20]

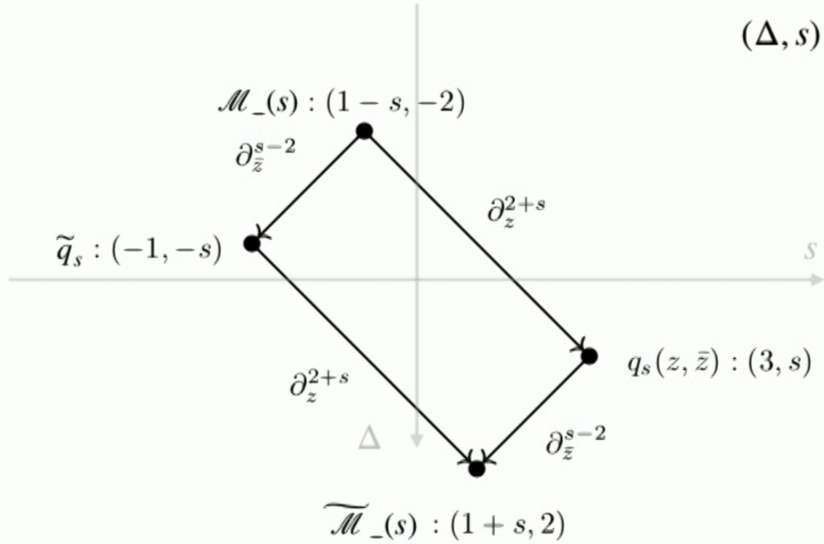
# Memories and Goldstones

Tower of soft/memory operators:

$$\mathscr{M}_\pm(n) \equiv \mathrm{Res}_{\Delta=-n}\, \widehat{N}_\pm(\Delta), \quad n \in \mathbb{Z}_+ .$$

The memory operators appear as coefficients in the Taylor expansion of the Fourier transformed news:

$$\widetilde{N}_\pm(\omega) = \sum_{n=0}^{\infty} \omega^n \mathscr{M}_\pm(n)$$




[Puhm, Pasterski, Trevisani '21]

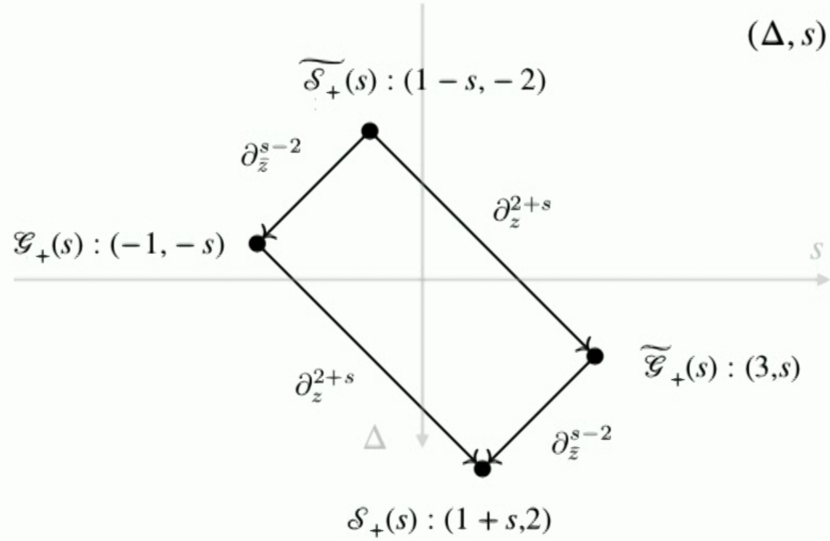
[Donnay, Puhm, Pasterski '22]

## Memories and Goldstones

Tower of Goldstone operators:

$$\mathcal{S}_{\pm}(n) \equiv \lim_{\Delta \to n} \widehat{N}_{\pm}(\Delta), \quad n \in \mathbb{Z}_{+}$$

$$\mathcal{S}_{+}(n) = D_z^{n+2} \mathcal{G}_{+}(n), \qquad \mathcal{S}_{-}(n) = D_{\bar{z}}^{n+2} \mathcal{G}_{-}(n) \,.$$




The Goldstone operators appear as coefficients in the Taylor expansion of the shear:

$$C_{\pm}(u) = \frac{i}{2\pi} \sum_{n=0}^{\infty} \frac{(-iu)^n}{n!} \mathcal{S}_{\pm}(n)$$

[Puhm, Pasterski, Trevisani '21]

[Donnay, Puhm, Pasterski '22]

# A discrete scattering basis

Gravitational symplectic potential $\Theta = \frac{2}{\kappa^2}\int_{-\infty}^{+\infty} du \int_S d^2z\sqrt{q}N(u)\delta C(u) \implies \{N(u,z), C(u',z')\} = -i\frac{\kappa^2}{2}\delta(u-u')\delta^{(2)}(z-z')$

Split shear and news into positive and negative energy components

$$C(u) = C_+(u) + C_-^*(u), \qquad N(u) = \partial_u C^*(u) = N_-(u) + N_+^*(u) \implies \Theta = \Theta_+ + \Theta_-$$

$$\left.\begin{aligned} C_\pm(u) &= \frac{i}{2\pi}\sum_{n=0}^{\infty}\frac{(-iu)^n}{n!}\mathcal{S}_\pm(n) \\ \mathcal{M}_\pm(n) &= \frac{i^n}{n!}\int_{-\infty}^{+\infty} du u^n N_\pm(u), \end{aligned}\right\} \longrightarrow \boxed{\Theta_+ = \frac{1}{i\kappa^2\pi}\sum_{n=0}^{\infty}\int_S d^2z\sqrt{q}\mathcal{M}_+(n)\delta\mathcal{S}_+^*(n)}$$

# Application I: soft and hard charges

- The discrete basis allows us to decompose the asymptotic charges. For $s=0$, $m(z) = m_+ + m_-^*$

$$m_\pm(z) = \frac{1}{2} D^2 \mathcal{M}^*_\pm(0,z) - \frac{1}{8\pi}\sum_{n=0}^{\infty} \mathcal{S}_\pm(n+1,z)\mathcal{M}^*_\pm(n,z)$$

Leading soft graviton

- For arbitrary $s$ : $Q_s(\tau) = Q_s^1 + Q_s^2 + \cdots$

Linear charge: $Q_s^1(\tau) = Q_{s+}^{1*}(\tau) + Q_{s-}^1(\tau)$

$$Q_{s\pm}^1(\tau) = -\frac{(-i)^s}{2}\int_S d^2z\sqrt{q}\, D^{s+2}\tau(z)\mathcal{M}_\pm(s,z)$$

[to appear Freidel, Pranzetti, A.R.]

# Application I: soft and hard charges

- The discrete basis allows us to decompose the asymptotic charges. For $s=0$, $m(z) = m_+ + m_-^*$

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Hard charge

- For arbitrary $s$ : $Q_s(\tau) = Q_s^1 + Q_s^2 + \cdots$ Quadratic charge: $Q_s^2(\tau) = Q_{s+}^2(\tau) + Q_{s-}^{2*}(\tau)$

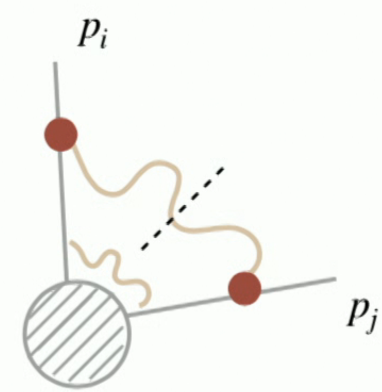
$$Q_{s+}^2(\tau) = \frac{i^s}{8\pi}\sum_{n=0}^{\infty}\sum_{\ell=0}^{s}\frac{(\ell+1)(3-n)_{s-\ell}}{(s-\ell)!}\int_S d^2z\sqrt{q}\,D^{s-\ell}\tau_s(z)\mathcal{S}_+^*(n,z)D^\ell\mathcal{M}_+(n+s-1,z)$$

[to appear Freidel, Pranzetti, A.R.]

# Application II: Dirac dressing

[Dirac '31; Weinberg '65; Chung; Kibble; Faddeev, Kulish…]

$$\langle p | \mathscr{D}_{\pm} \equiv \langle p | \exp \left\{ \sum_{s=0}^{\infty} \frac{(-1)^s}{\pi \kappa^2} \int d^2 w \int \underbrace{d^2 z}\, Q_{\mp}(s, w; p) \underbrace{G^{\pm}_{s+2}(w; z)}_{\mathscr{G}_{\pm}(s, w)} \left[ \mathscr{S}_{\pm}(s, z) - \mathscr{S}^{*}_{\mp}(s, z) \right] \right\}$$




- Diagonalizes the tower of soft charges $q_s$
- Generalizes the Faddeev-Kulish (s = 0) dressing
- Generalizes the soft S matrix by replacing "eikonal"/soft vertices by soft tower

$$\mathscr{S}_{\text{soft}} = \prod_{i,j} \mathscr{S}^{ij}_{\text{soft}} = \prod_{i,j} \exp \left\{ \frac{1}{2(2\pi)^3} \sum_{s=0}^{\infty} \int \frac{d^3 \overrightarrow{p}}{2\omega} \omega^{2s-2} S^{(s)}_{-,i}(p; p_i) S^{(s)}_{+,j}(p; p_j) \right\} \to \prod_{i,j} \exp \left\{ \sum_{s=0}^{\infty} \sum_{m=0}^{s} C^m_{ij} |z_{ij}|^{2(m+1)} \log |z_{ij}|^2 \partial^m_{z_i} \partial^m_{\bar{z}_j} \right\}$$

# Summary

Charges from EE & tower of soft theorems from charge conservation

Tower of memory/Goldstone modes forms a basis for `sufficiently localized" wavepackets

Soft and hard charges can be expanded in terms of the memory and Goldstone modes

Soft symmetries $\Longrightarrow$ Dirac/Faddeev-Kulish dressings include the entire tower of Goldstone modes

# Outlook

Implications for celestial amplitudes - constraints on amplitudes from integer $\Delta$ insertions?

Physical interpretation of IR-finite terms in dressings/soft S-matrix; higher dimensions?

Logarithmic soft theorems/modes?

Loop corrections?