Title: Orbit Analysis of Corner Symmetries

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Collection: Quantum Gravity Around the Corner

Date: October 05, 2022 - 9:00 AM

URL: https://pirsa.org/22100043

Abstract: Corner symmetries are those diffeomorphisms that become physical in codimension two, in that they support non-zero Noether charges. Recently we have shown how to extend phase space so that all such charges are integrable and give a representation of the corner symmetry algebra on this extended phase space. More recently we have studied the coadjoint orbits of what we now call the universal corner symmetry. One finds that certain complementary subalgebras, the extended corner symmetry and the asymptotic corner symmetry, can be associated with finite-distance and asymptotic corners, respectively. There is a simple geometric interpretation here in terms of an Atiyah Lie algebroid over a corner, whose structure group is the universal corner symmetry. The local geometry of a classical spacetime is encoded in related geometric structures.

Orbit Analysis of Corner Symmetries

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based on work with Luca Ciambelli
arXiv:2207.06441

October 5, 2022
QG around the Corner
Perimeter Institute

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Corners and Quantum Gravity

- in gauge theories, charges have support in codimension-2 [Regge-Teitelboim '74]
  - in gravitational (or any diff-invariant) theories, we say that Diff(M) is a local symmetry
    - this in turn means that physical Noether charges are (almost always) zero
  - at best, only a few diffeomorphisms will support charges
    - this is true independent of any dynamical details of a given theory
- the world is quantum!
  - classical limits are just that, some effective emergent description
    - quantizing classical geometries has limited applicability
  - quantum theory, whatever it is, is likely to be organized by symmetry
- this leads us to the *corner program*

[Donnelly, Freidel, Geiller, Oliveri, Pranzetti, Speranza, Speziale, ... '16-'21]
[Luca C., RGL '21-'22]

Outline

1. **review how we think of *corner symmetries***
    - focus on geometric concepts
        - **e.g., embeddings of corners in spacetime**
    - leads to an extension of phase space to include modes supported on subspaces ← a modification of Donnelly-Freidel
        - **charges become integrable, symmetry algebra represented properly on extended phase space** [Luca C., RGL, P.-C. Pai '21] [Freidel '21]
2. **set up orbit analysis for corner symmetry**
    - we now call the full symmetry the *universal corner symmetry* (UCS)
        - **this is just group theory: corners and their properties are an interpretation**
    - two important subalgebras emerge:
        - **extended corner symmetry (ECS) — corresponds to finite-distance corners**
        - **asymptotic corner symmetry (ACS) — corresponds to asymptotic corners**

Outline.2

- the analysis leads to an interpretation in terms of a Lie algebroid over a corner, with the corner symmetry playing the role of structure group
- we identify simple moment maps between coadjoint orbits and a phase space, now interpreted in terms of a certain associated bundle over the corner
  - can be thought of as the building blocks of a classical spacetime
    - a bulk metric emerges from sections, connections on these corner bundles
- an incomplete discussion of quantum issues, …

UCS

- the universal corner symmetry is the maximal finitely-generated subalgebra of diff(M) associated with a corner

$$Diff(S) \ltimes GL(2,\mathbb{R}) \ltimes \mathbb{R}^2 \qquad \textbf{UCS}$$

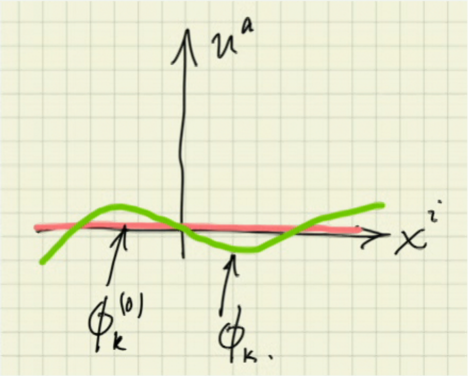
- an embedding is described by a map $\phi_k : S_k \to M$
  - described in local coordinates by $\phi_k : \sigma^\alpha \mapsto y^\mu(\sigma)$
  - local adapted coordinates on M: $y^\mu = (u^a, x^i)$
  - the trivial embedding has

$$\phi_k^{(0)} : \sigma^\alpha \mapsto (u^a(\sigma) = 0,\ x^i(\sigma) = \delta^i_\alpha \sigma^\alpha)$$

  - a vector field may be written

$$\underline{\xi} = \xi^i(u,x)\underline{\partial}_i + \xi^a(u,x)\underline{\partial}_a$$

    and the coefficients expanded for points close to the corner



UCS as Maximal Embedding Algebra

- vector fields of the form

$$\xi^b(u,x) = \xi^b_{(0)}(x) + \xi_{(1)}{}^b{}_{a_1}(x)u^{a_1} + \cancel{\tfrac{1}{2}\xi_{(2)}{}^b{}_{a_1a_2}(x)u^{a_1}u^{a_2}} + \ldots$$

$$\xi^i(u,x) = \xi^i_{(0)}(x) \cancel{+\xi^i_{(1)a_1}(x)u^{a_1}} + \cancel{\tfrac{1}{2}\xi^i_{(2)a_1a_2}(x)u^{a_1}u^{a_2}} + \ldots$$

- close under the Lie bracket to $\underbrace{Diff(S)}_{\text{diffs of corner}} \ltimes \underbrace{GL(2,\mathbb{R}) \ltimes \mathbb{R}^2}_{\text{linear transformations of 2 normal directions}}$

$$\boxed{\begin{aligned}
\underbrace{[\underline{\xi}_1, \underline{\xi}_2]}_{\mathcal{A}_k} \;=\; & \underbrace{[\hat{\underline{\xi}}_{(0)1}, \hat{\underline{\xi}}_{(0)2}]^j \underline{\partial}_j}_{\mathfrak{diff}(S)} \\
& + \Big[\underbrace{\hat{\underline{\xi}}_{(0)1}(\xi^b_{(0)2}) - \hat{\underline{\xi}}_{(0)2}(\xi^b_{(0)1})}_{\mathfrak{diff}(S) \text{ acts on } \mathbb{R}^k} + \underbrace{\xi_{(1)2}{}^b{}_a \xi^a_{(0)1} - \xi_{(1)1}{}^b{}_a \xi^a_{(0)2}}_{\mathfrak{gl}(k,\mathbb{R}) \text{ acts on } \mathbb{R}^k} \Big] \underline{\partial}_b \\
& + u^c \Big[- \underbrace{[\xi_{(1)1}, \xi_{(1)2}]^b{}_c}_{\mathfrak{gl}(k,\mathbb{R})} + \underbrace{\hat{\underline{\xi}}_{(0)1}(\xi_{(1)2}{}^b{}_c) - \hat{\underline{\xi}}_{(0)2}(\xi_{(1)1}{}^b{}_c)}_{\mathfrak{diff}(S) \text{ acts on } \mathfrak{gl}(k,\mathbb{R})} \Big] \underline{\partial}_b .
\end{aligned}}$$

Corner Symmetry

- in a diff-invariant theory, Diff(M) is a gauge symmetry
- the interpretation of the universal corner symmetry is that it contains the diffeomorphisms that could become physical in the presence of a corner
    - support charges, realized as Hamiltonians on a suitable phase space
    - "could" because in specific examples only a subalgebra is supported
    - we will see that this is because the non-zero charges in a given theory are associated with specific coadjoint orbits
- indeed, in the orbit analysis we will see that certain subalgebras come to prominence

UCS vs. ECS or ACS

- **in E-H gravity, not all of UCS is supported on corners**
  - *finite distance corner*: generally supports (orbit of) ECS

$$Diff(S) \ltimes \underline{SL(2,\mathbb{R}) \ltimes \mathbb{R}^2} = Diff(S) \ltimes H_s$$

[Freidel - et al '17-21]
[Speranza '17]
[Ciambelli, RGL 2104]

  - e.g., at a boundary of some subregion
    - **no scaling requirements, fields uncharged under Weyl subgroup — decouples**

  - *asymptotic corner*: generally supports ACS

$$Diff(S) \ltimes \underline{GL(1,\mathbb{R}) \ltimes \mathbb{R}^2} = Diff(S) \ltimes H_w$$

  - bulk diffeomorphisms restricted to respect some asymptotic structure
    - **(e.g., second order pole in metric) — fields charged under Weyl subgroup**
    - **relevant to asym-AdS, asym-flat, …**

- **depending on context/beliefs/desires, one may also reduce *Diff(S)***

The Orbit Method

- extending the phase space to include the embedding of a corner in a classical spacetime leads to integrable charges
  - a purely geometric effect
  - extends to all gauge theories
- this seems tied to particular classical spacetimes, covariant phase space technology, etc.
- it is of interest to study the corner symmetries directly, without reference to a classical system
- i.e., we release the UCS from its defining representation, and study its features abstractly
  - corners will then re-emerge in terms of representations

related work:
[Barnich-Ruzziconi '21]
[Freidel et al '21]

The Orbit Method

- **the orbit method can be thought of as a tool for studying representations of (certain) groups**
  - for particle physicists, it is essentially the little group analysis that leads to a codification of particle quantum states
    - in this case, the Poincaré group, $SO(1, d-1) \ltimes \mathbb{R}^d$
  - the groups that we will study here are simple generalizations

- **what are usually taken as symmetries (e.g., BMSW) are *contained* in these algebras**
  - in fact there is a concise interpretation for this fact in the orbit analysis

- it turns out to be useful to first study the groups

$$Diff(S) \ltimes GL(2,\mathbb{R}) \ltimes \mathbb{R}^2$$

$$\downarrow$$

$$H = GL(2,\mathbb{R}) \ltimes \mathbb{R}^2$$

$$H_S = SL(2,\mathbb{R}) \ltimes \mathbb{R}^2 \qquad \uparrow \qquad H_w = GL(1,\mathbb{R}) \ltimes \mathbb{R}^2$$

$$\nwarrow \mathfrak{h}_s \qquad \mathfrak{h} \qquad \mathfrak{h}_w \nearrow$$

Lie algebras

- this can be thought of as working at a point in S
- later we will add Diff(S) back in
  - a simple interpretation in terms of an Atiyah Lie algebroid with structure group H
  - so for these groups, Lie algebras are replaced by Lie algebroids

Coadjoint Orbits

- the orbit method just utilizes the fact that a Lie algebra acts on itself

$$ad_{\underline{\mu}} : \underline{\nu} \to ad_{\underline{\mu}}\underline{\nu} \equiv [\underline{\mu}, \underline{\nu}]_{\mathfrak{g}}, \qquad \forall \underline{\mu}, \underline{\nu} \in \mathfrak{g}$$

- the dual of the Lie algebra $\mathfrak{g}^* : \mathfrak{g} \to \mathbb{R}$

$$m(\underline{\mu}) \in \mathbb{R}, \qquad \forall m \in \mathfrak{g}^*; \quad \underline{\mu} \in \mathfrak{g}$$

- the Lie algebra also acts on the dual via the coadjoint

$$ad^*_{\underline{\mu}} : m \to ad^*_{\underline{\mu}} m, \qquad ad^*_{\underline{\mu}} m(\underline{\nu}) = -m(ad_{\underline{\mu}}\underline{\nu}), \qquad \forall m \in \mathfrak{g}^*; \quad \underline{\mu}, \underline{\nu} \in \mathfrak{g}$$

- should regard $ad^*_{\underline{\mu}} m \equiv \delta_{\underline{\mu}} m \in T_m \mathfrak{g}^*$ as a tangent vector
  - basic fact: exponentiating this action leads to a coadjoint orbit $O_m \subset \mathfrak{g}^*$
  - at a given point m, $ad^*_{\underline{\mu}} m$ will trace out a subspace $\mathcal{O}_m \subset T_m \mathfrak{g}^*$

- while $\mathfrak{g}^*$ is generally Poisson, $\mathcal{O}_m$ is symplectic, carrying the KKS symplectic form

Kirillov-Kostant-Souriau

ucs Coadjoint Orbits

- for ucs, the Lie algebra can be written in terms of a basis

$$[\underline{t}^a{}_b, \underline{t}^c{}_d] = \delta^c{}_b \underline{t}^a{}_d - \delta^a{}_d \underline{t}^c{}_b, \qquad [\underline{t}^a{}_b, \underline{t}_c] = -\pi^a{}_c{}^d{}_b \underline{t}_d, \qquad [\underline{t}_b, \underline{t}_c] = \underline{0}.$$

- and the dual basis satisfies

$$t^a{}_b(\underline{t}^c{}_d) = \pi^a{}_d{}^c{}_b, \qquad t^a{}_b(\underline{t}_c) = 0, \qquad t^a(\underline{t}^b{}_c) = 0, \qquad t^a(\underline{t}_b) = \delta^a{}_b.$$

- typical elements then written as

$$\mathfrak{h} \ni \underline{\mu} = \theta^a{}_b \underline{t}^b{}_a + b^a \underline{t}_a \qquad \mathfrak{h}^* \ni m = J^a{}_b t^b{}_a + P_a t^a$$

- and so the pairing reads

$$m(\underline{\mu}) = \theta^b{}_a J^a{}_b + b^a P_a.$$

- we then find that a tangent vector at the point m has the form

$$ad^*_{\underline{\mu}} m = \Big([J, \theta]^a{}_b - b^a P_b\Big) t^b{}_a + P_b \theta^b{}_c t^c.$$

UCS Coadjoint Orbits

- equivalently, we can think of this in terms of the components of the tangent vector,

$$\delta_{\underline{\mu}} J^a{}_b = [J,\theta]^a{}_b - b^a P_b, \qquad \delta_{\underline{\mu}} P_a = P_b \theta^b{}_a.$$

- in the case of UCS, this is invertible:

$$b^a = -\delta_{\underline{\mu}} J^a{}_b J^b{}_c \kappa^{cd} P_d - J^a{}_b \delta_{\underline{\mu}} J^b{}_c \kappa^{cd} P_d + \mathcal{J} \delta_{\underline{\mu}} J^a{}_b \kappa^{bc} P_c,$$

$$\theta^a{}_b = -\delta_{\underline{\mu}} J^a{}_c \kappa^{cd} P_d P_b + \left(P_e \delta_{\underline{\mu}} J^e{}_c \kappa^{cd} P_d\right) \delta^a{}_b + \delta_{\underline{\mu}} P_c \kappa^{cd} P_d J^a{}_b - P_e J^e{}_d \kappa^{cd} \delta_{\underline{\mu}} P_c \delta^a{}_b.$$

$$\kappa^{ab} = \frac{\varepsilon^{ab}}{C_3}, \qquad C_3 := P_a J^a{}_b \varepsilon^{bc} P_c.$$

- this invertibility means that UCS has no Casimirs, and that generic orbits are 6-dimensional and (locally) symplectic
  - the KKS symplectic form is given by

$$\Omega^{(\mathfrak{h})}_m(\underline{\mu},\underline{\nu}) = -J^a{}_b[\theta,\theta']^b{}_a - P_a(\theta^a{}_c b'^c - \theta'^a{}_c b^c),$$

UCS Coadjoint Orbits

- in that derivation, we made use of the UCS invariant tensor

$$\kappa^{ab} = \frac{\varepsilon^{ab}}{C_3}$$

  - ε^{ab} is an invariant tensor of ECS, while C_3 is a cubic Casimir of ECS

$$C_3 := P_a J^a{}_b \varepsilon^{bc} P_c .$$

- it's useful to rewrite the symplectic form in terms of the variations,

$$\Omega_m^{(\mathfrak{h})}(\underline{\mu}, \underline{\nu}) = \delta_{\underline{\mu}} P_a (\delta_{\underline{\nu}} \bar{J}^a{}_b \bar{J}^b{}_c + \bar{J}^a{}_b \delta_{\underline{\nu}} \bar{J}^b{}_c) \kappa^{cd} P_d - \tfrac{1}{2} (P_a \delta_{\underline{\mu}} \bar{J}^a{}_b) \kappa^{bc} (P_d \delta_{\underline{\nu}} \bar{J}^d{}_c) + \tfrac{1}{2} \delta_{\underline{\mu}} \log C_3 \delta_{\underline{\nu}} \mathcal{J} - (\underline{\mu} \leftrightarrow \underline{\nu})$$

- where we separated out the trace pieces,

$$J^a{}_b = \bar{J}^a{}_b + \tfrac{1}{2} \mathcal{J} \delta^a{}_b, \qquad \theta^a{}_b = \bar{\theta}^a{}_b + \tfrac{1}{2} w \delta^a{}_b, \qquad w = \theta^a{}_b \delta^b{}_a,$$

$$\delta_{\underline{\mu}} \log C_3 = w := tr\ \theta$$

- this separates the symplectic form into 4+2
  - we would like an explanation for this fact

ces Coadjoint Orbits

- **consider repeating the analysis for ecs**
  - the dual space is 5-dimensional
    - orbits can't be 5d, because they will be symplectic
  - a generic orbit is 4d, along which C_3 is constant
  - can think of the dual space as foliated by symplectic leaves, each labeled by the cubic Casimir
- **symplectic form on the leaves can be obtained locally**
  - the relations between parameters and tangent vectors can no longer be inverted directly, since the tangent vector components are not independent

$$\Omega_m^{(\mathfrak{h}_s)}(\underline{\mu},\underline{\nu}) = \frac{2P_0\delta_{\underline{\mu}}\bar{J}_3\delta_{\underline{\nu}}\bar{J}^0{}_1 + P_1\delta_{\underline{\mu}}\bar{J}^1{}_0\delta_{\underline{\nu}}\bar{J}^0{}_1 + \delta_{\underline{\mu}}P_1\delta_{\underline{\nu}}J_C^2}{2(\bar{J}^0{}_1P_0 - \bar{J}_3P_1)} - (\underline{\mu} \leftrightarrow \underline{\nu}).$$

  - can be thought of as coming from first term in ucs symplectic form

Corner Symmetries

ces Coadjoint Orbits

- **consider repeating the analysis for ecs**
  - the dual space is 5-dimensional
    - orbits can't be 5d, because they will be symplectic
  - a generic orbit is 4d, along which C_3 is constant
  - can think of the dual space as foliated by symplectic leaves, each labeled by the cubic Casimir
- **symplectic form on the leaves can be obtained locally**
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  - can be thought of as coming from first term in ucs symplectic form

acs Coadjoint Orbits

- **one can perform a similar analysis for acs**
  - in this case, the dual space is 3d and a generic orbit is 2d
  - the symplectic form on the orbits (on a coord patch with $P_0 \neq 0$) is

$$\Omega_m^{(\mathfrak{h}_w)}(\underline{\mu}, \underline{\nu}) = \frac{\delta_{\underline{\mu}} P_0 \delta_{\underline{\nu}} \mathcal{J} - \delta_{\underline{\mu}} \mathcal{J} \delta_{\underline{\nu}} P_0}{P_0},$$

  which can be thought of as coming from the second term in the ucs symplectic form

- **in fact, $H_w = GL(1,\mathbb{R}) \ltimes \mathbb{R}^2$ and $H_s = SL(2,\mathbb{R}) \ltimes \mathbb{R}^2$ are ideals in** $H = GL(2,\mathbb{R}) \ltimes \mathbb{R}^2$
  - their orbits are immersed in $\mathfrak{h}^*$
  - at a given point, they are complementary
  - the acs orbits have (under suitable circumstances) a subalgebra acting as BMSW

The Full UCS

- **Now we put back in *diff(S)*:**
  - given coordinates on S
$$\underline{\mathfrak{X}} = \xi^\beta(\sigma)\underline{\partial}_\beta + \theta^a{}_b(\sigma)\underline{t}^b{}_a + b^a(\sigma)\underline{t}_a$$
$$M = \alpha_\beta(\sigma)d\sigma^\beta + J^a{}_b(\sigma)t^b{}_a + P_a(\sigma)t^a.$$
  - and the invariant pairing $\langle\ ,\rangle : UCS^* \otimes UCS \to \mathbb{R}$ takes the form
$$M(\underline{\mathfrak{X}}) = i_{\underline{\xi}}\alpha + \theta^a{}_b J^b{}_a + b^a P_a, \qquad \langle M, \underline{\mathfrak{X}}\rangle = \int_S vol_S\, M(\underline{\mathfrak{X}}),$$

- **interpret this as the image under a moment(um) mapping of an integrated charge $H_{\underline{\mathfrak{X}}}$**

- **the *ad* action gives rise to the algebra of charges**
$$\langle ad^*_{\underline{\mathfrak{X}}} M, \underline{\mathfrak{Y}}\rangle = -\langle M, ad_{\underline{\mathfrak{X}}}\underline{\mathfrak{Y}}\rangle.$$

read as
$\delta_{\underline{\mathfrak{X}}} H_{\underline{\mathfrak{Y}}} = -H_{[\underline{\mathfrak{X}},\underline{\mathfrak{Y}}]}$

Remarks

- **there are a number of technical requirements for this to work out**
  - charges should be integrable and Hamiltonian
    - **need extended phase space ideas**

[Ciambelli, RGL 2104]
[Ciambelli, RGL, P.-C. Pai 2112]

  - in some cases there are central charges which may appear depending on the nature of the moment map and equivariant lifting
    - **(work in progress)**

[M. Klinger, P.C. Pai, RGL, Luca C.]

- **all of this has a concise interpretation in terms of an Atiyah Lie algebroid over the corner *S***
  - here, the structure group is $H = GL(2,\mathbb{R}) \ltimes \mathbb{R}^2$

Lie Algebroid Primer

- **given a principal G-bundle $\pi : P \to M$ with a principal connection**
    - TP is a bundle over P, whereas $A = TP/G$ is a vector bundle over M
    - principal connection descends to a connection on the Lie algebroid A
- **neatly combines gauge transformations with diffeomorphisms**

$$0 \longrightarrow L \longrightarrow A \longrightarrow TM \longrightarrow 0$$

    - the connection defines a global split $A = H \oplus V$ with H corresponding to diffs, and V an image of the Lie algebra bundle L
- **one can use such structure to describe gauge theories (gravity) on a spacetime M (particularly useful in the context of subregions…)**
- **here the role of M is played by a corner S, and G is $GL(2, \mathbb{R}) \ltimes \mathbb{R}^2$**
    - the Lie algebra element $\underline{\mathfrak{x}}$ of full UCS is interpreted as a section of A
    - dual of Lie algebra promoted to dual bundle A^*

Affine Structures

- this is quite powerful, because other geometric structures can be identified with associated bundles
  - this is how representation theory then enters
  - one can imagine a functional integral quantization as a gauge theory over corners
    - (I'm hopeful that this is work in progress…)
- an important example of a representation is a rank-2 affine bundle $\mathcal{B}$

$$(\sigma_i^\alpha, \psi_i^a) = (\sigma_i^\alpha(\sigma_j), \underbrace{R_{ij}{}^a{}_b}_{GL(2,\mathbb{R})}\psi_j^b + \underbrace{b_{ij}{}^a}_{\mathbb{R}^2})$$

- a connection on this bundle is a pair $(a_{i,\alpha}^{(0)a}, a_{i,\alpha}^{(1)a}{}_b)$
- a section $M \in A^*$ can be thought of as in $Der(\mathcal{B})^* \sim T^*U \oplus End(\mathcal{B})^*$

Moment Maps

- moment maps relate a phase space to an orbit $\mu_s : \chi \to A_s^*$
- here they exist by restricting to ECS and ACS
  - e.g., for ECS, the invariant tensor ε gives a map $\varepsilon : End(\mathcal{B}_s)^* \to S^2\mathcal{B}_s$.

$$({J^a}_b, P_a) \longrightarrow (h^{ab}_{(0)}, h^a_{(1)})$$

  - in fact, all of the pieces of a metric on a classical spacetime (decomposed with respect to a corner) are present

$$N^b{}_a = \sqrt{-\det h^{(0)}}\, h^{bc}_{(0)} \varepsilon_{ca}, \qquad b_j = -N^b{}_a a_j^{(1)a}{}_b, \qquad p_d = \tfrac{1}{2} N^a{}_c h^{cb}_{(0)} (h^{(1)}_{db,a} - h^{(1)}_{da,b}).$$

$$h^{(1)}_a = \tfrac{1}{2} \varepsilon^{bc} (h^{(1)}_{ab,c} - h^{(1)}_{ac,b}).$$

$$H_{\underline{\mathfrak{x}}} = \mu_s\Big(\int_S vol_S \Big[\theta^a{}_b N^b{}_a + \xi^j b_j + b^a p_a\Big]\Big)$$

$$\mu_s(N^a{}_b) = J^a{}_b \qquad \mu_s(p_a) = P_a \qquad \mu_s(b_i) = \alpha_i.$$

Incomplete Remarks on Quantum

- the coadjoint orbit analysis for UCS is quite tractable, and can be interpreted in terms of a Lie algebroid over a corner
- associated representations contain the ingredients to build classical geometries (in which corners are codimension-2)
- can fit this into an interpretation of any diff-invariant classical theory
- but can also imagine it is the starting point for a quantum theory…
  - the ultimate *bulk reconstruction*
  - a semi-classical spacetime as a "condensate of corners"
- alternatively, perhaps a simpler idea is to try to figure out what corresponds to a unitary representation
  - 'canonical quantization'
  - this e.g. would presumably map onto asymptotic quantization of linearized gravity

Even More Incomplete Remarks

- when we fix structures, patterns emerge
  - the fluid picture is an example
  - Casimirs of $SL(2,\mathbb{R})$ are rarely Casimirs of $SL(2,\mathbb{R}) \times \mathbb{R}^2$
    - we've seen that the corner charges are well-defined invariants
    - other examples include the volume of S (because 1 is a local invariant), as well as integrated curvature invariants
    - these map onto what Laurent called enstrophies, etc.
- one of the appealing features of the corner program is the close built-in relationship with entanglement, non-factorizability, and perhaps even the information puzzle…