Title: Hamiltonian Gauge Theory With Corners I: General Theory

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Collection: Quantum Gravity Around the Corner

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Abstract: I will present an analysis of the Hamiltonian formulation of gauge theories on manifolds with corners in the particular, yet common, case in which they admit an equivariant momentum map.

In the presence of corners, the momentum map splits into a part encoding "Cauchy data" or constraints, and a part encoding the "flux" across the corner. This decomposition plays an important role in the construction of the reduced phase space, which then becomes an application of symplectic reduction in stages for local group actions.

The output of this analysis are natural "corner" Poisson structures, leading to the concept of (classical) flux superselection sectors as their symplectic leaves.

This is based on a collaboration with A. Riello. My talk will cover the general framework of corner superselection, while Riello's talk will deal with its application to null boundaries and soft charges.

Hamiltonian gauge theory with corners I: General Theory

Quantum Gravity Around the Corner '22

Michele Schiavina, Università di Pavia

UNIVERSITÀ DI PAVIA

Overview of Part I

Joint work with A. Riello 2207.00568 - Part II on Friday

Problem:
Describe reduced phase space for Hamiltonian gauge theories on manifolds with corners

- Hamiltonian reduction paradigm must be refined.
- Two stages: "constraint reduction" and "flux superselection".
- Adjusted paradigm: reduced phase space is **Poisson** manifold **foliated** by symplectic leaves (superselection sectors) **labeled** by the coadjoint orbits of "fluxes" - a.k.a. corner charges.

Non-null Yang–Mills theory as running example.
Null YM will be presented by Aldo in Part II.
General Relativity is in progress.

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Quick recap of Hamiltonian reduction

Let $(\mathcal{P}, \omega, \mathcal{G}, H)$ be a Hamiltonian $\mathcal{G}$-space with equivariant momentum map $H \colon \mathcal{P} \to \mathfrak{G}^*$, i.e. for all $\xi \in \mathfrak{G} = \mathrm{Lie}(\mathcal{G})$:

$$\iota_{\rho(\xi)}\omega = \langle \mathbb{d} H, \xi \rangle \qquad \text{Hamiltonian flow condition}$$

$$\mathbb{L}_{\rho(\xi)} H = \mathrm{ad}^*_\xi H \qquad \text{Equivariance}$$

Note: $\mathbb{d}$ will always denote de Rham on $\mathcal{P}$.

Theorem (Marsden, Weinstein; Meyer; Arms)

For every coadjoint orbit $\mathcal{O}_f \subset \mathfrak{G}^*$ we have a symplectic manifold:[1]

$$\underline{\mathcal{C}}_{[f]} \doteq H^{-1}(\mathcal{O}_f)/\mathcal{G}, \qquad e.g. \quad \underline{\mathcal{C}}_0 \doteq H^{-1}(0)/\mathcal{G},$$

Moreover $\mathcal{P}/\mathcal{G}$ is Poisson, and $\underline{\mathcal{C}}_{[f]}$ for all $f \in \mathfrak{G}^*$ its symplectic leaves.

[1] We need a free action for this to hold.

PARTITION FUNCTION OF ABELIAN CHERN-SIMONS THEORIES ON HANDLEBODIES

BASED ON
MP and Cedric Yu
JHEP 07 (2021) 194, arXiv:2104.12799, MP

- CHERN-SIMONS THEORIES ON OPEN MANIFOLDS: METRIC DEPENDENCE FROM BOUNDARY CONDITIONS
- PARTITION FUNCTIONS OF C-S ON OPEN MANIFOLDS AS WAVE FUNCTIONS
- AN EXPLICIT BASIS OF WAVE FUNCTION FOR ABELIAN C-S
- AN IMPLICIT BASIS OF PARTITION FUNCTIONS FOR ABELIAN AND NONABELIAN C-S
- RELATING THE BASES FOR HANDLEBODIES: 1) THE INITIAL CONDITION
- RELATING THE BASES: 2) RADIAL QUANTIZATION OF C-S AS PROJECTION OVER GAUGE INVARIANT WAVE FUNCTIONS

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[1] We need a free action for this to hold.

Local Gauge Theory with Corners

Local Lagrangian field theory on $\Sigma \times \mathbb{R}$. Assume $\partial\Sigma \neq \emptyset$. $\dim(\Sigma) \doteq n$
Hamiltonian formulation[2] yields $(\mathcal{P}, \boldsymbol{\omega}, \boldsymbol{H}, \mathcal{G})$ locally Hamiltonian $\mathcal{G}$-space:

1. $\mathcal{P} = \Gamma(\Sigma, F)$ sections of a vector bundle (for simplicity),
2. $\boldsymbol{\omega} \in \Omega^{2,\mathrm{top}}_{\mathrm{loc}}(\mathcal{P} \times \Sigma)$ a local symplectic density on $\mathcal{P}$,
3. $\boldsymbol{H} \in \Omega^{0,\mathrm{top}}_{\mathrm{loc}}(\mathcal{P} \times \Sigma)$ a local functional (density) on $\mathcal{P}$,
4. $\mathcal{G}$ a local lie group with a local action on $\mathcal{P}$.

Flow and equivariance now hold pointwise: for $\xi \in \mathfrak{G} = \mathrm{Lie}(\mathcal{G})$

$$\iota_{\rho(\xi)}\boldsymbol{\omega} = \langle \mathbb{d}\boldsymbol{H}, \xi\rangle \qquad \text{local Hamiltonian form}$$

$$\mathbb{L}_{\rho(\xi)}\boldsymbol{H} = \mathrm{ad}^*_\xi \boldsymbol{H} + d\boldsymbol{k}(\xi) \qquad \text{Equivariance up to corners}$$

Note 1: *Local* pairing $\langle \mathbb{d}\boldsymbol{H}, \xi\rangle$: may depend on derivatives $\partial\xi$.
⇝ Generally not $C^\infty(\Sigma)$-linear!
Note 2: Integrate $\omega \doteq \int_\Sigma \boldsymbol{\omega}$ and $H \doteq \int_\Sigma \boldsymbol{H}$.
⇝ Equivariant Momentum map, up to corners (CE cocycle).

[2] Canonical approach, Kijowski–Tulczyjew, covariant... up to obstructions

Running Example I: (Spacelike) Yang–Mills Theory

Consider G abelian or semisimple, with inner product $\mathrm{tr}\colon \mathfrak{g}\times\mathfrak{g}\to\mathbb{R}$, and look at G-connections $A\in\mathcal{A}\doteq \mathrm{Conn}(P\to\Sigma)$ with Σ spacelike.
We have the *geometric/canonical phase space*:

$$\mathcal{E}\doteq\Omega^{n-1}(\Sigma,\mathfrak{g}),\qquad \mathcal{P}\doteq T^\vee\mathcal{A}=\mathcal{A}\times\mathcal{E}\ni(A,E),\qquad \boldsymbol{\omega}=\mathrm{tr}(\mathbb{d}A\mathbb{d}E).$$

The gauge action of $\mathcal{G}\doteq C_0^\infty(\Sigma,G)$ reads

$$(A,E,\xi)\longmapsto\rho(\xi)(A,E)=(d_A\xi,\mathrm{ad}(\xi)\cdot E),\qquad \xi\in\mathfrak{G}=C^\infty(\Sigma,\mathfrak{g}).$$

It is locally Hamiltonian with (equivariant) momentum map

$$\iota_{\rho(\xi)}\boldsymbol{\omega}=\langle\mathbb{d}\boldsymbol{H},\xi\rangle,\qquad \langle\boldsymbol{H},\xi\rangle=\mathrm{tr}(Ed_A\xi).$$

Note: G semisimple: restrict to $A\in\mathcal{A}$ irreducible.
$\rightsquigarrow$ Denote $\mathfrak{G}_A=\{\xi\in\mathfrak{G}\mid d_A\xi=0\}$. Then $\mathfrak{G}_A=0$.
G abelian: all $A\in\mathcal{A}$ are reducible by constants $\xi\in\mathfrak{g}\hookrightarrow\mathfrak{G}$.

Reduction with corners is reduction in stages

Problem: $\boldsymbol{H}^{-1}(0)$ *is not the correct constraint locus!*
Additional "corner conditions" imposed by $\boldsymbol{H} = 0$ *"kill the flux."*

Proposition (Constraint / Flux splitting [Riello, MS])

There is a natural bulk/boundary splitting:

$$\boldsymbol{H} = \boldsymbol{H}_\circ + d\boldsymbol{h}$$

with $\mathcal{C} \doteq \boldsymbol{H}_\circ^{-1}(0)$ *the constraint set of the theory.*
We call $\boldsymbol{H}_\circ$ *the constraint form and* $\boldsymbol{h}$ *the flux form.*

Note: $\boldsymbol{H}_\circ$ is NOT a equivariant momentum map for $\mathcal{G}$ anymore!
$\mathcal{C}$ is still coisotropic: the reduction $\underline{\mathcal{C}} \doteq \mathcal{C}/\mathcal{C}^\omega$ is symplectic (if smooth).
We call $\underline{\mathcal{C}}$ the **constraint-reduced** phase space.

Question: Is there a subgroup $\mathcal{G}_\circ$ for which $\mathcal{C}$ is zero level set of the associated momentum map $J_\circ \colon \mathcal{P} \to \mathfrak{G}_\circ^*$, so that $\underline{\mathcal{C}} = \mathcal{C}/\mathcal{G}_\circ$?
Answer: Yes!

First Stage: Constraint Reduction

Theorem (Constraint reduction [Riello MS])

Let $h_{\mathcal{C}} \doteq \iota_{\mathcal{C}}^* \int_\Sigma d\boldsymbol{h}$*. Under certain regularity assumptions:*

1. *The subspace* $\mathfrak{G}_\circ \doteq \mathrm{AnnIm}(h_{\mathcal{C}}) \subset \mathfrak{G}$ *is the maximal Lie ideal whose associated momentum map* $J_\circ$ *is such that* $J_\circ^{-1}(0) = \mathcal{C}$*.*

   Normal subgroup $\mathcal{G}_\circ \subset \mathcal{G}$*:* **constraint gauge group**.
   Quotient group $\underline{\mathcal{G}} \doteq \mathcal{G}/\mathcal{G}_\circ$*:* **flux gauge group**

2. *Hamiltonian action* $\underline{\mathcal{G}} \circlearrowright \underline{\mathcal{C}} = \mathcal{C}/\mathcal{G}_\circ$*, with momentum map* $\underline{h}\colon \underline{\mathcal{C}} \to \underline{\mathfrak{G}}^*$*, such that* $h_{\mathcal{C}} = \pi_\circ^* \underline{h}$*.*

   We call $\underline{h}$ *the* **flux map** *and* $\mathfrak{F} \doteq \mathrm{Im}(\underline{h})$ *the* **flux space**.

3. *Equivariance controlled by the Chevalley-Eilenberg cocycle* $k \doteq \int d\boldsymbol{k}$*. [Recall:* $\boldsymbol{H}$ *was equivariant up to corner]*

Comments:
Reduction $\underline{\mathcal{C}}$ *is not enough* in the presence of corners.
Functions $C^\infty(\underline{\mathcal{C}})$ are NOT observables: residual gauge symmetry $\underline{\mathcal{G}} \circlearrowright \underline{\mathcal{C}}$.
Need second stage reduction!

Yang–Mills II: Constraint/flux split

The Hamiltonian momentum map splits as:

$$\boldsymbol{H} = \boldsymbol{H}_\circ + d\boldsymbol{h}, \qquad \langle \boldsymbol{H}_\circ, \xi \rangle = \mathrm{tr}(d_A E \xi), \quad \langle d\boldsymbol{h}, \xi \rangle = -d\mathrm{tr}(E\xi),$$

and $\mathcal{C} = \boldsymbol{H}_\circ^{-1}(0) = \{(A, E) \in \mathcal{P} \mid d_A E = 0\}$: *Gauss' Constraint*.

Note: Imposing $\boldsymbol{H} = 0$ forces $E|_{\partial\Sigma} = 0$: *zero flux*.
Indeed $\langle h, \xi \rangle \doteq \int_{\partial\Sigma} \iota^*_{\partial\Sigma} \mathrm{tr}(E\xi)$ is the (smeared) "electric" flux.

Denote $\xi \in \mathfrak{G}_A \iff d_A \xi = 0$. The constraint gauge ideal $\mathfrak{G}_\circ$ reads:

$$\mathfrak{G}_\circ = \mathrm{Ann}(\mathfrak{F}) = \begin{cases} \{\xi \in \mathfrak{G} \mid \xi|_{\partial\Sigma} = 0\} & \text{G semisimple} \\ \{\xi \in \mathfrak{G} \mid \exists \chi \in \mathfrak{G}_A : \xi|_{\partial\Sigma} = \chi|_{\partial\Sigma}\} & \text{G Abelian} \end{cases}$$

and thus the flux gauge algebra $\underline{\mathfrak{G}}$ reads

$$\underline{\mathfrak{G}} = \mathfrak{G}/\mathfrak{G}_\circ = \begin{cases} C^\infty(\partial\Sigma, \mathfrak{g}) & G \text{ semisimple} \\ C^\infty(\partial\Sigma, \mathfrak{g})/\mathfrak{g} & G \text{ Abelian} \end{cases}$$

Yang–Mills III: Constraint reduction

[Singer; Narasimhan, Ramadas; Gomes, Hopfmüller, Riello...]

Radiative/Coulombic (Helmholz/Hodge) orthogonal decomposition: $E = E_{\text{rad}} + \star d\varphi$, with $\varphi \in C^\infty(\Sigma, \mathfrak{g}^*)$ is the Coulombic potential parametrised by $E_\partial \in \mathcal{E}_\partial = \Omega^{\text{top}}(\partial\Sigma, \mathfrak{g})$:

$$\begin{cases} \Delta_A \varphi = \star d_A E \approx 0 & \text{in } \Sigma, \\ \mathrm{n} \cdot d_A \varphi = E_\partial & \text{at } \partial\Sigma. \end{cases}$$

If $\mathcal{H}_A$ denotes radiative electric fields ($d_A E_{\text{rad}} = 0 = (E_{\text{rad}})_\partial$):

$$\mathcal{C} \simeq_{\text{loc}} \underbrace{\mathcal{H}_A \times \mathcal{A}}_{\mathcal{P}_{\text{rad}}} \times \mathcal{E}_\partial \quad \Longrightarrow \quad \underline{\mathcal{C}} \simeq_{\text{loc}} \frac{\mathcal{H}_A \times \mathcal{A}}{\mathcal{G}_\circ} \times \mathcal{E}_\partial$$

For G Abelian, $A = A_{\text{rad}} + d\varsigma$, with $\varsigma \in C^\infty(\Sigma, \mathfrak{g})$ solution of Neumann–Laplace, one obtains $\underline{\mathcal{C}} \simeq \underline{\mathcal{P}}_{\text{rad}} \times T^*\underline{\mathfrak{G}}$ (globally!)

$$\underline{\omega} \overset{\text{ab}}{=} \int_\Sigma \mathbb{d}E_{\text{rad}} \wedge \mathbb{d}A_{\text{rad}} + \int_{\partial\Sigma} \mathbb{d}E_\partial \wedge \mathbb{d}\varsigma_\partial,$$

with $\underline{\mathcal{P}}_{\text{rad}} \doteq \underline{\mathcal{P}}_{\text{rad}}/\underline{\mathcal{G}} \ni (A_{\text{rad}}, E_{\text{rad}})$.

Second Stage: Flux Superselection

First stage reduction output: $\underline{\mathcal{C}}$ symplectic.
Flux map $\underline{h}$ is a momentum map for $\underline{\mathcal{G}} \circlearrowright \underline{\mathcal{C}}$.

Consider the coadjoint orbit $\mathcal{O}_f \in \mathfrak{G}^*$ of a flux $f \in \mathfrak{F} = \mathrm{Im}(\underline{h}) \subset \mathfrak{G}^*$.
All on-shell configurations whose flux is in $\mathcal{O}_f$ are acted upon by $\underline{\mathcal{G}}$:[3]

$$\underline{\mathcal{S}}_{[f]} = \underline{h}^{-1}(\mathcal{O}_f) \quad \rightsquigarrow \quad \underline{\underline{\mathcal{S}}}_{[f]} = \underline{\mathcal{S}}_{[f]}/\underline{\mathcal{G}} \qquad \text{Superselection sector (SSS)}$$

Theorem (Flux Superselection [Riello, MS])

The fully-reduced phase space $\underline{\underline{\mathcal{C}}} = \underline{\mathcal{C}}/\underline{\mathcal{G}} = \mathcal{C}/\mathcal{G}$ is a Poisson manifold whose symplectic leaves are the superselection sectors:

$$\underline{\underline{\mathcal{C}}} = \bigsqcup_{f \in \underline{\mathfrak{G}}^*} \underline{\underline{\mathcal{S}}}_{[f]}$$

[3] Ignoring multiple connected components. Note: this action is always free.

Running Example IV: Flux Superselection

Radiative/Coulombic split leads to constraint reduction:

$$\underline{\mathcal{C}} \simeq \underline{\mathcal{P}}_{\text{rad}} \times \mathcal{E}_\partial, \qquad \underline{\mathcal{P}}_{\text{rad}} = \mathcal{P}_{\text{rad}}/\mathcal{G}_\circ.$$

$\underline{\mathcal{G}}$ acts (freely) on $\underline{\mathcal{P}}_{\text{rad}}$. Then $\underline{\mathcal{C}}$ is a fibre bundle

$$\underline{\mathcal{C}} \simeq_{\text{loc}} \mathcal{E}_\partial \times \underline{\mathcal{G}} \times \underline{\underline{\mathcal{P}}}_{\text{rad}} \simeq T^*\underline{\mathcal{G}} \times \underline{\underline{\mathcal{P}}}_{\text{rad}}, \qquad \underline{\underline{\mathcal{P}}}_{\text{rad}} \doteq \underline{\mathcal{P}}_{\text{rad}}/\underline{\mathcal{G}}$$

Note: *Extended phase space* (Donnelly–Freidel), without any extension!

Flux map $\underline{h} : E_\partial \mapsto \int_{\partial\Sigma} \mathrm{tr}(E_\partial \cdot)$ identifies $\mathcal{E}_\partial \simeq \underline{\mathfrak{G}}^* = C^\infty(\partial\Sigma, \mathfrak{g})^*$.

Looking at $\underline{\mathcal{S}}_{[f]} = \underline{h}^{-1}(\mathcal{O}_f)$, since $\mathcal{O}_\xi \hookrightarrow \mathcal{E}_\partial \simeq \mathfrak{G}^*$

$$\underline{\mathcal{S}}_{[f]} \simeq_{\text{loc}} \mathcal{O}_f \times \underline{\mathcal{G}} \times \underline{\underline{\mathcal{P}}}_{\text{rad}} \hookrightarrow \mathcal{E}_\partial \times \underline{\mathcal{G}} \times \underline{\underline{\mathcal{P}}}_{\text{rad}} \simeq_{\text{loc}} \underline{\mathcal{C}},$$

Clearly, then, we have a foliation: for every *flux* $f \in \mathfrak{F} = \mathrm{Im}(\underline{h}) \simeq \underline{\mathfrak{G}}^*$

$$\underline{\underline{\mathcal{S}}}_{[f]} \simeq_{\text{loc}} \mathcal{O}_f \times \underline{\underline{\mathcal{P}}}_{\text{rad}} \hookrightarrow \mathcal{E}_\partial \times \underline{\underline{\mathcal{P}}}_{\text{rad}} \simeq_{\text{loc}} \underline{\underline{\mathcal{C}}}.$$

Off-Shell Corner Data

Working on-shell is often times cumbersome. Induce off-shell corner data:

$$\pi_\partial \colon \mathbb{A} = \mathcal{P} \times \mathfrak{G} \to \mathbb{A}_\partial \doteq \mathcal{P}_\partial \times \mathfrak{G}_\partial, \qquad \varpi_\partial = \langle \check{\mathbb{d}} h, \check{\mathbb{d}} \xi \rangle$$

We can recover on-shell data $\mathcal{C}_\partial = \pi_\partial(\underline{\mathcal{C}}) \subset \mathcal{P}_\partial$, and relate $\mathfrak{G}_\partial$ with $\underline{\mathfrak{G}}$.

Theorem (Compatibility of Corner Data [Riello, MS])

$\mathbb{A}_\partial$ is a symplectic Lie algebroid; $\mathcal{C}_\partial \hookrightarrow (\mathcal{P}_\partial, \Pi_\partial)$ Poisson (sub)manifolds. There exists a commuting diagram of Poisson manifolds

$$\begin{array}{ccc} \underline{\mathcal{C}} & \xrightarrow{\ \pi\ } & \underline{\underline{\mathcal{C}}} \\ {\scriptstyle \pi_{\partial,\underline{\mathcal{C}}}}\big\downarrow & & \big\downarrow{\scriptstyle \underline{q}} \\ \mathcal{C}_\partial & \xrightarrow{\ q_\partial\ } & \mathcal{B} \end{array}$$

Noether Charge algebra $\rightsquigarrow$ Poisson structure $(\mathcal{C}_\partial, \Pi_{\mathcal{C}_\partial}) \hookrightarrow (\mathcal{P}_\partial, \Pi_\partial)$.
The Casimirs of Π_∂ label superselections.

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$$\begin{cases} \Pi_\partial^{YM} = \int_{\partial\Sigma} \mathrm{tr}\left(E_\partial \left[\frac{\delta}{\delta E_\partial}, \frac{\delta}{\delta E_\partial} \right] \right) & \textit{Yang–Mills} \\ \Pi_\partial^{CS} = \int_{\partial\Sigma} \mathrm{tr}\left(a \left[\frac{\delta}{\delta a}, \frac{\delta}{\delta a} \right] \right) + \mathrm{tr}\left(\frac{\delta}{\delta a} d \frac{\delta}{\delta a} \right) & \textit{Chern–Simons, w. central extension} \end{cases}$$

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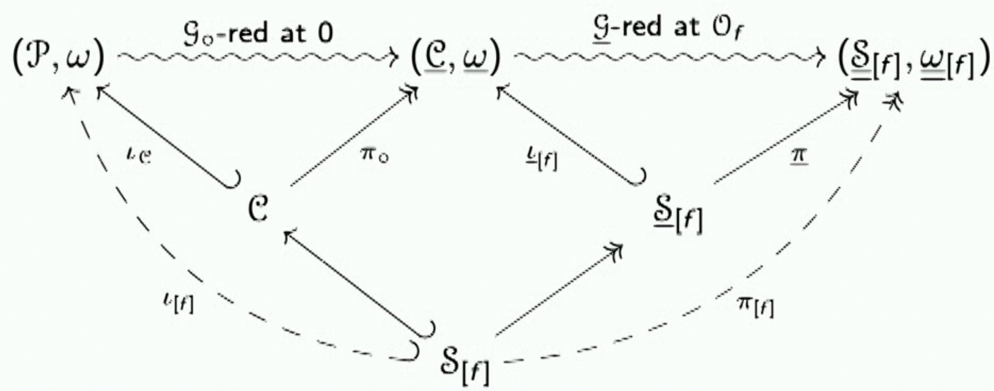
Conclusions



1. $\underline{\mathcal{C}}$ partial reduction - extended phase space with $T^*\underline{\mathcal{G}}$.
2. Full reduction: Poisson manifold with flux superselection leaves.
3. Corner Noether charge algebra given by corner Poisson structure.
4. Extension to Gravity in progress. Difficulties related to absence of clear momentum map picture. We have ideas.
5. Relation to soft charges/memory will be evident in Aldo's talk. See also Kasia's talk, where flux map is given in BV-BFV language.

Thanks!