

Title: Carroll symmetry in gravity and string theory

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Collection: Quantum Gravity Around the Corner

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Abstract: I will discuss the small speed of light expansion of general relativity, utilizing the modern perspective on non-Lorentzian geometry. The leading order in the expansion leads to an action that corresponds to the electric Carroll limit of general relativity, of which I will highlight some interesting properties. The next-to-leading order will also be obtained, which exhibits a particular subsector that correspond to the magnetic Carroll limit, which features a solution that describes the Carroll limit of a Schwarzschild black hole. The incorporation of a cosmological constant in the Carroll (or ultra-local) expansion will also be commented on. Finally, I will describe how Carroll symmetry and geometry arises on the world-sheet of certain limits of string theory sigma models.

Carroll Symmetry in Gravity and String Theory

Quantum Gravity around the corner, PI, Waterloo, Oct. 4, 2022

Niels Obers (Nordita & Niels Bohr Institute)



THE VELUX FOUNDATIONS
VILLUM FONDEN & VELUX FONDEN



Vetenskapsrådet

based on work:

2110.02319 (Front. of Physics) (de Boer, Hartong, NO, Sybesma, Vandoren)

2112.12684 (SciPost) (Hansen, NO, Oling, Søgaard)

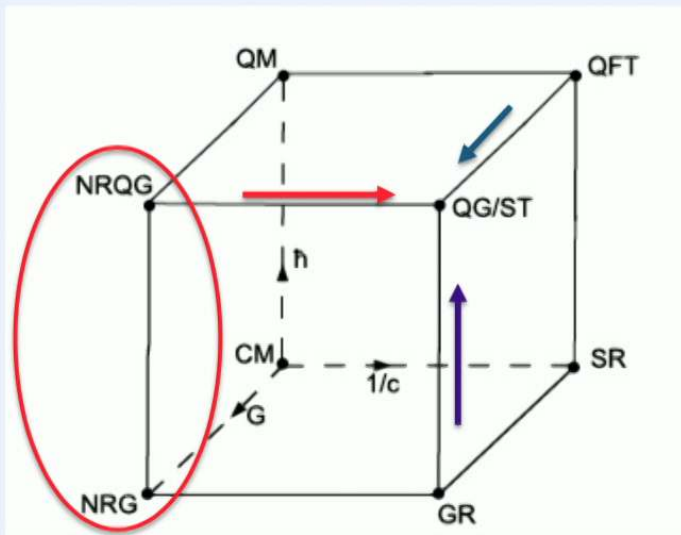
2107.006542 (JHEP) (Bidussi, Harmark, Hartong, NO, Oling)

to appear **2210.xxxxxx/22??**.yyyyy ((Bidussi, Harmark, Hartong, NO, Oling)

& earlier works

Cube of physical theories

$(\hbar, G_N, 1/c)$



a third route towards
(relativistic) quantum gravity

how does this fit with
string theory/holography ?

already (classical) non-relativistic gravity (NRG) is more than just
Newtonian gravity

-> uses Newton-Cartan geometry (and torsional generalization)

What about small speed of light limit ?

Carroll limit:

zero speed of light contraction of Poincare

running (boosting) without moving:

Levy-Leblond(1965)

Red Queens race of L. Carroll's Through the looking glass

→ expand relativistic theories around $c=0$:

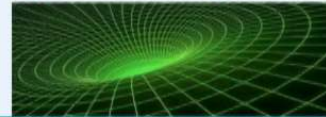
what do we get ? what can we use it for ?

- Carroll symmetry as an organizing principle for a perturbative expansion around $c=0$

terminology: this is not ultra-relativistic limit ($v/c \rightarrow 1$)
rather ultra-local limit

Space-Time symmetries and Geometry

local symmetries of space and time \leftrightarrow geometry of space and time



spacetime symmetries:

time translations, space translations, spatial rotations +

crucial difference \rightarrow type of boosts

relativistic (Lorentz)

$$t \rightarrow \gamma(t + \vec{v}\vec{x}/c^2) \quad , \quad \vec{x} \rightarrow \gamma(\vec{x} + \vec{v}t)$$

non-relativistic (Galilean)

$$t \rightarrow t \quad , \quad \vec{x} \rightarrow \vec{x} + \vec{v}t$$

$$c \rightarrow \infty$$

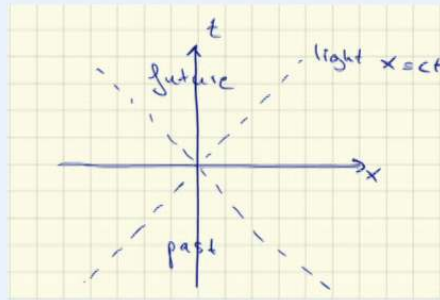
ultra-relativistic (Carroll)

$$t \rightarrow t + \vec{v}\vec{x} \quad , \quad \vec{x} \rightarrow \vec{x}$$

$$c \rightarrow 0$$

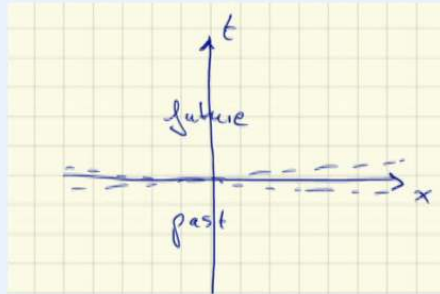
Lightcones & Geometry

Lorentz



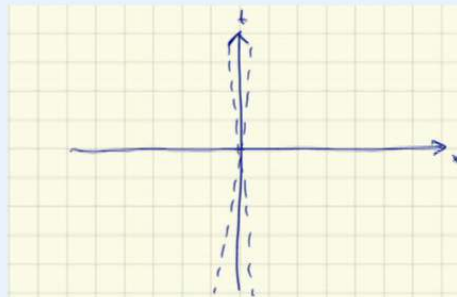
(pseudo-) Riemmanian
geometry (GR)

Galilean



torsional Newton-Cartan
geometry

Carroll



Carroll
geometry

Non-Lorentzian geometries

recent progress in understanding non-relativistic
(large and small speed of light) corners of:
gravity, quantum field theory and string theory:

→ builds on improved understanding of non-Lorentzian geometries
= spacetimes with local symmetries other than Lorentz

NL geometries appear in:

- bdry geometries in non-AdS holography (e.g. Lifshitz flat space)
- covariant formulations of PN approximation in GR
- covariant formulations of non-Lorentzian fluids and CMT systems (FQHE, fractons, ..)
- Horava–Lifshitz gravity, non-relativistic versions of CS, JT
- double field theory
- **ultra-local GR (Carroll) - cosmology, black hole horizons**
- **non-relativistic corners of String Theory**

Recent examples of Carroll

- null hypersurfaces in Lorentzian geom exhibit Carroll
→ black hole membrane paradigm
[Donnay,Marteau,2019],[Penna,2018]
- BMS isomorphic to conformal Carroll Duvall,Gibbons,Horvathy,2014
→ (3D/4D) flat space holography & celestial holography
[Bagchi,Detournay,Fareghbal,Simon,2012],[Hartong,2015],
[Ciambella,Marteau,Petkou,Petropoulos,Siampos,2018],
[Donnay,Fiorucci,Herfray,Ruzziconi,2021]. . . .
- tensionless limits of strings
[Bagchi,2013], [Harmark et al]
- limits of GR/ultra-local behavior leads to solvable corners
[Henneaux,1979],[Hartong,2015][Bergshoeff,Gomis,Rollier,ter Veldhuis,2017],
[Henneaux,Salgado-Rebolledo,2021],[Perez,2021],[Hansen et al, 2021],
see also: Bekaert,Morand,2015/Figueroa-O’Farrill,Prohazka,2018/FigueroaOF et al 2022
- inflationary cosmology
[de Boer,Hartong,Obers,Sybesma,Vandoren,2021]

Plan

- part I: Carroll symmetry in gravity

- Carroll geometry
- review: pre-ultra-local parametrization of GR
- leading-order action = electric theory
- next-to-leading action (includes magnetic theory)
- solutions of constraints and evolution equations

- part II: Carroll symmetry in string theory

- non-relativistic strings
- two new classes of sigma models from NRST
 - one with Carrollian world-sheet structure

Outlook

Carroll geometry

defined by

- timelike vector field

$$v^\mu$$

- spatial metric

$$h_{\mu\nu} = \delta_{ab} e_\mu^a e_\nu^b,$$

transforming under Carroll boosts:

$$\delta_\lambda v^\mu = 0,$$

$$\delta_\lambda h^{\mu\nu} = 2\lambda^{(\mu} v^{\nu)}, \quad \lambda^\mu = h^{\mu\nu} \lambda_\nu$$

from gauging the
Carroll algebra

orthogonality and completeness

$$\tau_\mu v^\mu = -1, \quad \tau_\mu h^{\mu\nu} = 0, \quad h_{\mu\nu} v^\nu = 0, \quad \delta_\nu^\mu = -v^\mu \tau_\nu + h^{\mu\rho} h_{\rho\nu}.$$

Carroll compatible connection:

$$\tilde{\nabla}_\mu v^\nu = 0, \quad \tilde{\nabla}_\rho h_{\mu\nu} = 0.$$

$$\tilde{\Gamma}_{\mu\nu}^\rho = -v^\rho \partial_{(\mu} \tau_{\nu)} - v^\rho \tau_{(\mu} \mathcal{L}_v \tau_{\nu)} + \frac{1}{2} h^{\rho\lambda} [\partial_\mu h_{\nu\lambda} + \partial_\nu h_{\lambda\mu} - \partial_\lambda h_{\mu\nu}] - h^{\rho\lambda} \tau_\nu K_{\mu\lambda},$$

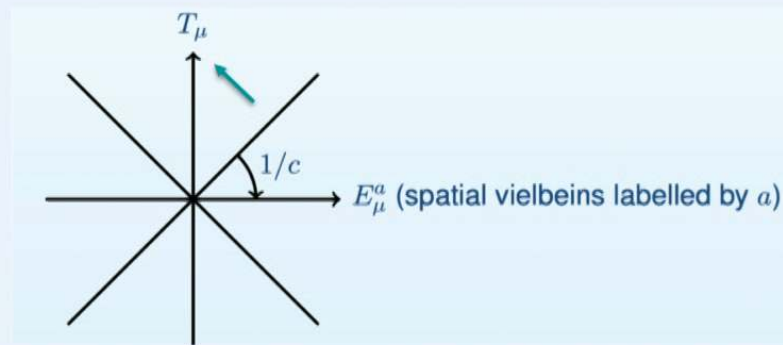
extrinsic curvature $K_{\mu\nu} = -\frac{1}{2} \mathcal{L}_v h_{\mu\nu}$

- can be defined intrinsically or follows from small c limit in GR:

speed of light dependence in GR

metric: $g_{\mu\nu} = -c^2 T_\mu T_\nu + \Pi_{\mu\nu}$

T = time-like vielbein
 Π = spatial metric



small speed of light \rightarrow light-cone closes up

assume the tensors T and Π can be expanded as Taylor series: expand in c^2

(**large speed of light** \rightarrow light-cone opens up)

PUL (pre-ultralocal) parametrization of GR

PUL (pre-ultralocal) parametrization of GR: time/space split

$$g_{\mu\nu} = -c^2 T_\mu T_\nu + \Pi_{\mu\nu}, \quad g^{\mu\nu} = -\frac{1}{c^2} V^\mu V^\nu + \Pi^{\mu\nu}.$$

rewrite EH using PUL connection (adapted to Carroll when expanding)

$$\begin{aligned} \tilde{C}_{\mu\nu}^\rho &= -V^\rho \partial_{(\mu} T_{\nu)} - V^\rho T_{(\mu} \mathcal{L}_V T_{\nu)} \\ &\quad + \frac{1}{2} \Pi^{\rho\lambda} [\partial_\mu \Pi_{\nu\lambda} + \partial_\nu \Pi_{\lambda\mu} - \partial_\lambda \Pi_{\mu\nu}] - \Pi^{\rho\lambda} T_\nu \mathcal{K}_{\mu\lambda}, \end{aligned}$$

$$\mathcal{K}_{\mu\nu} = -\frac{1}{2} \mathcal{L}_V \Pi_{\mu\nu}.$$

EH takes form

$$R : \approx \frac{1}{c^2} (\mathcal{K}^{\mu\nu} \mathcal{K}_{\mu\nu} - \mathcal{K}^2) + \Pi^{\mu\nu} \overset{\circ}{R}_{\mu\nu} + \frac{c}{4} \Pi^{\mu\nu} \Pi^{\rho\sigma} (dT)_{\mu\rho} (dT)_{\nu\sigma},$$

Carroll geometry from small c

expand in small c in the PUL form

$$\begin{aligned} V^\mu &= v^\mu + c^2 M^\mu + \mathcal{O}(c^4), & T_\mu &= \tau_\mu + \mathcal{O}(c^2) \\ \Pi^{\mu\nu} &= h^{\mu\nu} + c^2 \Phi^{\mu\nu} + \mathcal{O}(c^4), & \Pi_{\mu\nu} &= h_{\mu\nu} + \mathcal{O}(c^2), \end{aligned}$$

LO fields = Carroll geometry

NLO fields = extra gauge fields

small c expansion of local Lorentz \rightarrow Carroll boost transformations

- expansion of the PUL connection \rightarrow Carroll compatible connection

$$\tilde{\Gamma}_{\mu\nu}^\rho = \tilde{C}_{\mu\nu}^\rho \Big|_{c=0}$$

- Remark: similar to the (previously considered) non-relativistic expansion

LO and NLO action

Hansen et al (2021)

- expand EH for small c: LO action = **leading order part**

$$\mathcal{L}_{\text{LO}}^{(2)} = \frac{e}{16\pi G_N} \left[K^{\mu\nu} K_{\mu\nu} - K^2 \right],$$

Henneaux,1979

$$\delta\mathcal{L}_{\text{LO}}^{(2)} = \frac{e}{8\pi G_N} \left[G_{\mu}^{(2)v} \delta v^{\mu} + \frac{1}{2} G_{\mu\nu}^{(2)h} \delta h^{\mu\nu} \right],$$

$$G_{\mu}^{(2)v} = -\frac{1}{2} \tau_{\mu} (K^{\rho\sigma} K_{\rho\sigma} - K^2) + h^{\gamma\lambda} \tilde{\nabla}_{\lambda} (K_{\mu\gamma} - K h_{\mu\gamma}),$$

$$G_{\mu\nu}^{(2)h} = -\frac{1}{2} h_{\mu\nu} (K^{\rho\sigma} K_{\rho\sigma} - K^2) + K (K_{\mu\nu} - K h_{\mu\nu}) - v^{\rho} \tilde{\nabla}_{\rho} (K_{\mu\nu} - K h_{\mu\nu}).$$

- NLO action (includes EOMs of LO)

$$\mathcal{L}_{\text{NLO}}^{(4)} = \frac{e}{8\pi G_N} \left[\frac{1}{2} h^{\mu\nu} \tilde{R}_{\mu\nu} + G_{\mu}^{(2)v} M^{\mu} + \frac{1}{2} G_{\mu\nu}^{(2)h} \Phi^{\mu\nu} \right].$$

see also: Bergshoeff et al,2017

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see also: Bergshoeff et al,2017

LO theory = timelike (electric) theory

EOMs

$$\overset{(2)}{G}_{\mu}^{\nu} = -\frac{1}{2}\tau_{\mu} \left(K^{\rho\sigma} K_{\rho\sigma} - K^2 \right) + h^{\nu\rho} \tilde{\nabla}_{\rho} (K_{\mu\nu} - K h_{\mu\nu}),$$

$$\overset{(2)}{G}_{\mu\nu}^h = -\frac{1}{2}h_{\mu\nu} \left(K^{\rho\sigma} K_{\rho\sigma} - K^2 \right) + K (K_{\mu\nu} - K h_{\mu\nu}) - v^{\rho} \tilde{\nabla}_{\rho} (K_{\mu\nu} - K h_{\mu\nu}).$$

projections on time and space

$$K^{\mu\nu} K_{\mu\nu} - K^2 = 0,$$

$$h^{\rho\sigma} \tilde{\nabla}_{\rho} (K_{\sigma\mu} - K h_{\sigma\mu}) = 0,$$

$$\mathcal{L}_v K_{\mu\nu} = -2K_{\mu}^{\rho} K_{\rho\nu} + K K_{\mu\nu},$$

constraint equations

evolution equations

solutions of electric theory

evolution of (arbitrary) initial data

depends only on v^μ -derivatives (ultra local)

can integrate in suitable
adapted coordinates

$$h_{ij}(t) = h_{(0)ik} \exp[-2t h_{(0)}^{kl} K_{(0)lj}].$$

Hansen,NO,Oling,Soegaard
see also: Dautcourt; Niedermaier

→ much simpler then evolution in full GR !

construct initial data using 3+1 methods (Bowen-York type solutions)

$$h_{0,ij} = \psi^4 \delta_{ij},$$

$$K_{0,ij} = \psi^{-2} \bar{L}X_{ij} + \frac{1}{3} K_0 \psi^4 \delta_{ij},$$

$$\psi = \left[\frac{3}{2K_0^2} \bar{L}X_{ij} \bar{L}X^{ij} \right]^{1/12},$$

$$\bar{L}X^{ij} = \frac{3}{2r^3} \left[x^i P^j + x^j P^i - \left(\delta^{ij} - \frac{x^i x^j}{r^2} \right) P_k x^k \right]$$

$$+ \frac{3}{r^5} \left[\epsilon^{ik} J_k x^l x^j + \epsilon^{jk} J_k x^l x^i \right].$$

have physical boundary charges (linear momentum and angular momentum)
can be computed from charge integrand involving:

$$\Theta^\mu = \frac{e}{8\pi G} \left[(K h_\nu^\mu - K^\mu_\nu) \delta v^\nu - \frac{1}{2} (K h_{\sigma\rho} - K_{\sigma\rho}) v^\mu \delta h^{\sigma\rho} \right]. \quad \text{presymplectic potential}$$

$$Q^{[\mu\nu]} = \frac{e}{4\pi G} (v^{[\mu} K^{\nu]}_\sigma \xi^\sigma - v^{[\mu} \xi^{\nu]} K). \quad \text{Noether-Wald charge}$$

no mass charge at LO !

Hansen,NO,Oling,Soegaard
see also: Perez

solutions of electric theory

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Hansen,NO,Oling,Soegaard
see also: Dautcourt; Niedermaier

→ much simpler than evolution in full GR !

consistent subsector of NLO theory
 = spacelike (magnetic) theory

NLO EOMs complicated

simplify by setting NLO fields to zero (need to be on-shell in LO theory)

action derivation:

rewrite PUL for of EH using Lagrange multiplier

$$S = \frac{c^4}{16\pi G} \int_M \left[-\frac{c^2}{4} G^{\mu\nu\rho\sigma} \chi_{\mu\nu} \chi_{\rho\sigma} + G^{\mu\nu\rho\sigma} \chi_{\mu\nu} \mathcal{K}_{\rho\sigma} + \Pi^{\mu\nu} \overset{\circ}{R}_{\mu\nu} + \frac{c^2}{4} \Pi^{\mu\nu} \Pi^{\rho\sigma} (dT)_{\mu\rho} (dT)_{\nu\sigma} \right] E d^d x.$$

$$G^{\mu\nu\rho\sigma} = \frac{1}{2} (\Pi^{\mu\rho} \Pi^{\nu\sigma} + \Pi^{\mu\sigma} \Pi^{\nu\rho} - 2\Pi^{\mu\nu} \Pi^{\rho\sigma}),$$

$$G_{\mu\nu\rho\sigma} = \frac{1}{2} \left(\Pi_{\mu\rho} \Pi_{\nu\sigma} + \Pi_{\mu\sigma} \Pi_{\nu\rho} - \frac{2}{d-1} \Pi_{\mu\nu} \Pi_{\rho\sigma} \right).$$

DeWitt metric

small c limit gives:

$$S_{\text{mag}}^{(4)} = \frac{1}{16\pi G} \int_M \left[\phi^{\mu\nu} K_{\mu\nu} + h^{\mu\nu} \tilde{R}_{\mu\nu} \right] e d^d x,$$

Langrangian version of
Henneaux ,Selgado-Rebolledo

Lagrange multiplier implementing zero extrinsic curvature

EOMs

$$0 = \tilde{\nabla}_\rho \phi^\rho{}_\mu - v^\rho \tau_{\rho\nu} \phi^\nu{}_\mu + \tau_\mu h^{\rho\sigma} \tilde{R}_{\rho\sigma} + \frac{1}{2} h^{\nu\sigma} h^\lambda{}_\mu \tau_{\nu\lambda} v^\kappa \tau_{\kappa\sigma} - \tilde{\nabla}_\nu (h^{\nu\sigma} \tau_{\mu\sigma}),$$

$$0 = -\frac{1}{2} v^\rho \tilde{\nabla}_\rho \phi_{\mu\nu} + \tilde{R}_{\mu\nu} - \frac{1}{2} h_{\mu\nu} h^{\sigma\rho} \tilde{R}_{\sigma\rho} + \frac{1}{2} h_{\mu\nu} \tilde{\nabla}_\lambda (h^{\lambda\gamma} v^\kappa \tau_{\kappa\gamma}).$$

go to boost frame that allows for spatial foliation:

$$0 = h^{\mu\nu} \hat{R}_{\mu\nu},$$

$$0 = \hat{\nabla}_\nu \phi^\nu{}_\mu,$$

$$\frac{1}{2} \mathcal{L}_v \phi_{\mu\nu} = \hat{R}_{\mu\nu} - \hat{\nabla}_{(\mu} a_{\nu)} - a_\mu a_\nu + h_{\mu\nu} h^{\rho\sigma} \left(\hat{\nabla}_{(\rho} a_{\sigma)} + a_\rho a_\sigma \right),$$

constraint eq.

evolution eq.

solutions with cosmological constant

$$S_\Lambda = \frac{c^4}{16\pi G} \int d^{d+1}x E(-2\Lambda),$$

$$\Lambda = \frac{1}{c^2} \overset{(-2)}{\Lambda} + \overset{(0)}{\Lambda} + \dots,$$

LO NLO

- LO theory initial data

$$h_{(0)ij}, \quad K_{(0)ij} = -Hh_{(0)ij},$$

$$h_{\mu\nu}(t) = e^{2Ht} h_{(0)\mu\nu}, \quad K_{\mu\nu}(t) = -Hh_{\mu\nu}(t).$$

simple physical example: $h_{(0)ij}$ standard flat metric

corresponds to Carroll limit of dS in planar coordinates

- NLO theory: both positive and negative Lambda possible

e.g. Carroll limit of (anti)de Sitter in static coordinates

Outlook (part I)

- solutions: full NLO theory (and beyond), numerics/analytics..
- apply 3+1 techniques to NR expansion
- Cosmology and Carroll gravity
- Carroll strings
- Tensionless strings
- Flat space Holography
- Carroll fluids in curved spacetime [de Boer et al, to appear]
de Boer et al (2017)/Ciambelli,Marteau,Petkou,Petropoulos,Siampos (2018, 2018)
Donnay,Marteau (2019) Ciambelli,Marteau,Petropoulos,Ruzziconi (2020),...
Petkou,Petropoulos,Rivera,Betancour,Siampos(2022) Freidel,Jai-akson(2022)
- applications to supersonic behavior ?

prelude: Carroll (scalar) field theories

see talk Gerben Oling

$$\mathcal{L} = \frac{1}{2c^2} (\partial_t \phi)^2 - \frac{1}{2} (\partial_i \phi)^2 - V(\phi).$$



$$\mathcal{L} = -\frac{c^2}{2} \chi^2 + \chi \partial_t \phi - \frac{1}{2} (\partial_i \phi)^2 - V(\phi).$$



Carroll limit

$$\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - \tilde{V}(\phi)$$

(timelike theory)



chi = Lagrange multiplier

$$\mathcal{L} = \chi \dot{\phi} - \frac{1}{2} (\partial_i \phi)^2 - V(\phi).$$

(spacelike theory)

$$\delta \phi = \vec{b} \cdot \vec{x} \dot{\phi}, \quad \delta \chi = \vec{b} \cdot \vec{x} \dot{\chi} + \vec{b} \cdot \vec{\partial} \phi.$$

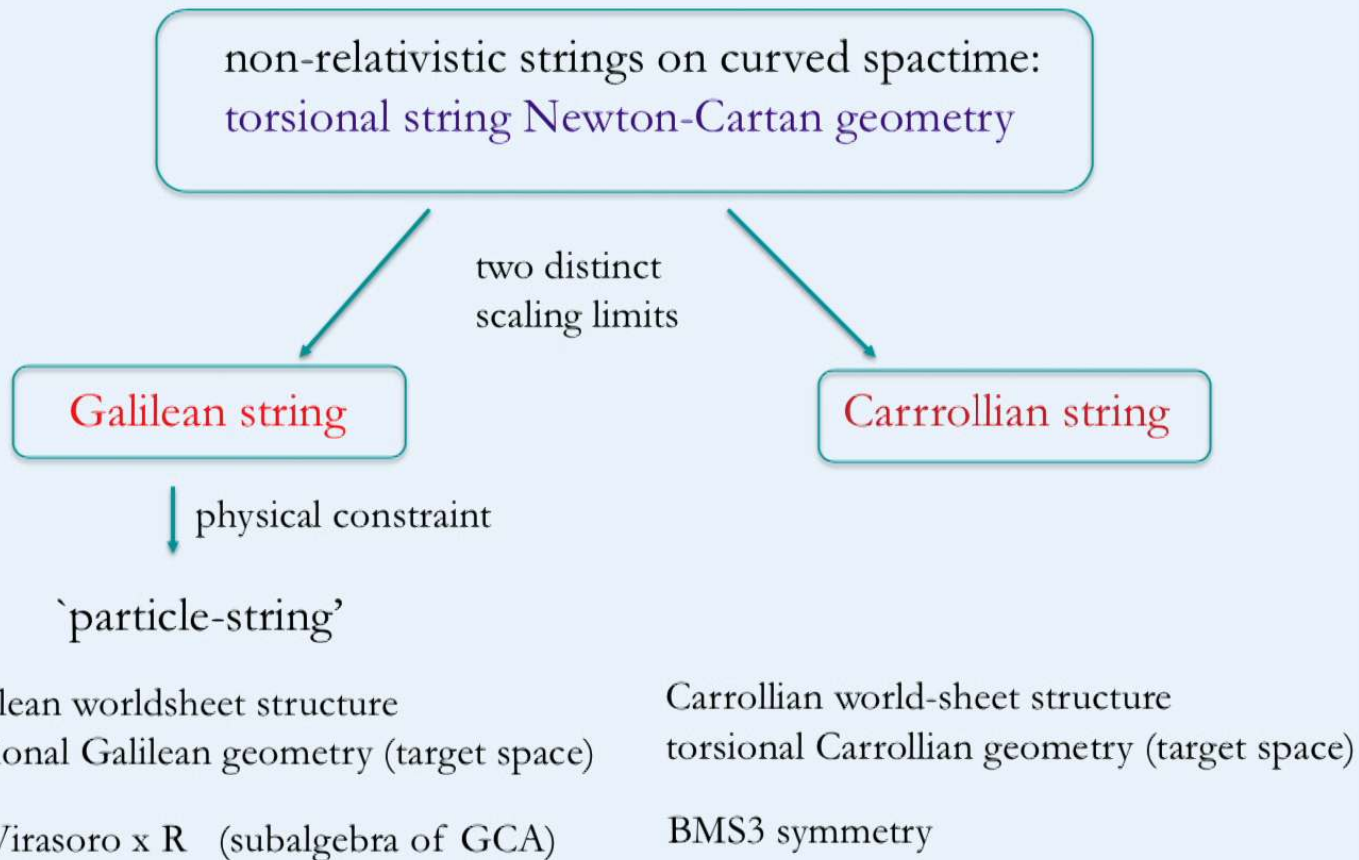
boost Ward identities imply:

$$\langle O(t, \vec{x}) O(0, 0) \rangle = F(t) \delta(\vec{x})$$

$$\langle O(t, \vec{x}) O(0, 0) \rangle = f(|\vec{x}|),$$

→ 2D field theories (string theory worldsheets) allow for **more general Carroll theories**
 can be found by considering worldsheet limits of non-relativistic string theory

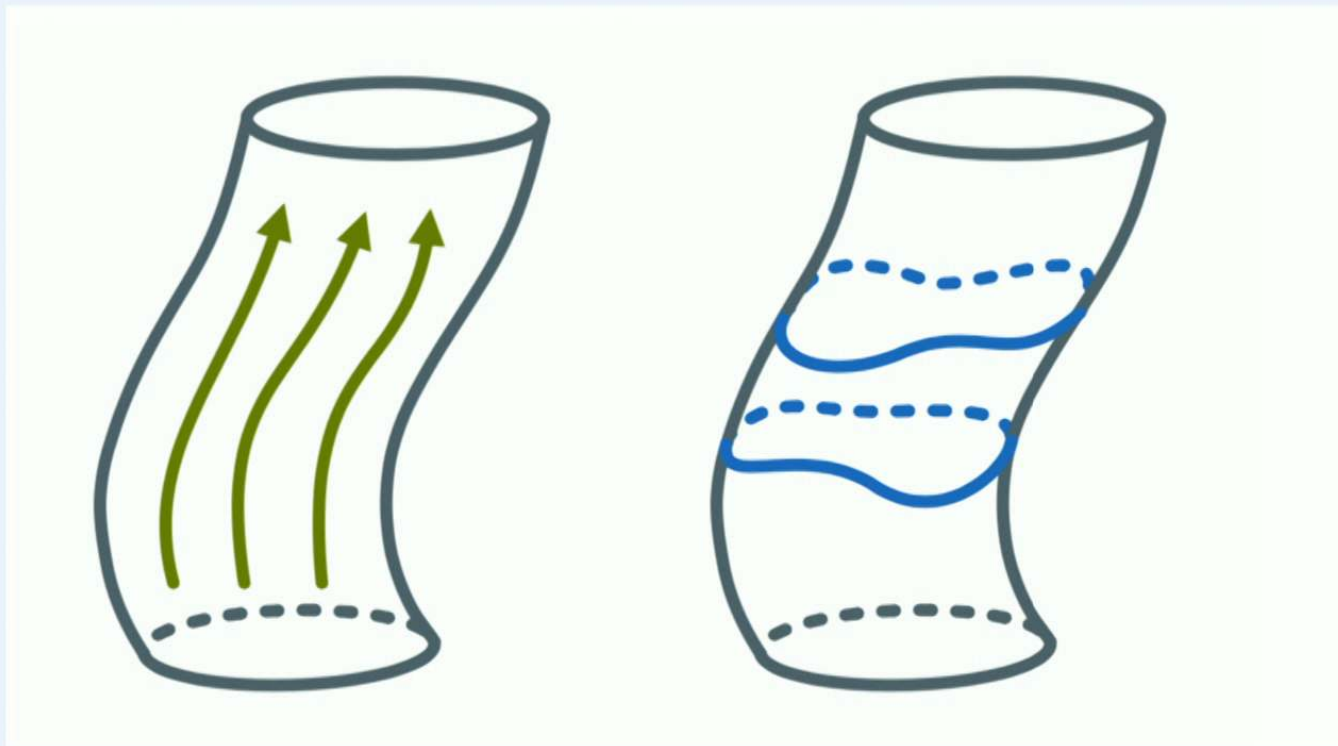
Main message: two new classes of sigma models from NRST



Visualisation of world-sheet

Carroll

Galilean



(drawing: courtesy of Gerben Oling)

Polyakov action for Carrollian strings

$$S = -\frac{T}{2} \int d^2\sigma \left[\left(-e e_0^\alpha e_0^\beta h_{MN} + \epsilon^{\alpha\beta} m_{MN} \right) \partial_\alpha X^M \partial_\beta X^N + \omega \epsilon^{\alpha\beta} e^1_\alpha \tau^1_\beta + \psi \epsilon^{\alpha\beta} (e^0_\alpha \tau^1_\beta + e^1_\alpha \tau^0_\beta) \right]$$

- exhibits **Carrollian world-sheet structure**
- Weyl and local (2D) **Carrollian boost** act on the zweibeine:

$$e^0_\alpha \rightarrow f e^0_\alpha + \hat{f} e^1_\alpha, \quad e^1_\alpha \rightarrow f e^1_\alpha, \quad \omega \rightarrow \frac{1}{f} \omega - \frac{\hat{f}}{f^2} \psi, \quad \psi \rightarrow \frac{1}{f} \psi$$

- no problem with (wrong-sign) kinetic terms
- Hamiltonian analysis for (subclass) of target spacetimes

$$x^M = (x^\mu, v),$$

$v = \text{compact isometry}$

$$\tau^0_M = \delta^t_M, \quad \tau^1_M = \delta^v_M, \quad m_{\mu\nu} = 0, \quad m_{v\mu} = -m_\mu$$

gauge-fixed action has residual symmetry: **BMS₃**
(contraction of Vir x Vir)

Outlook

- further study (quantization) of Carrollian sigma model
 - Hamiltonian analysis

Kluson (2021), Bidussi, Harnmark, Hartong, NO, Oling (to appear)

- obtain **beta functions**
- relation to limits of AdS/CFT

Harnmark, Wintergerst (2019), Baiguera, Harnmark, Wintergerst (2020)

Baiguera, Harnmark, Lei, Wintergerst (2020)

- connections to Carrollian (small speed of light) gravity

Henneaux (1979), Bergshoeff, Gomis, Rollier, ter Veldhuis (2017), Hartong (2015)

Henneaux, Salgado-Rebolledo (2021),

de Boer, Hartong, NO, Sybesma, Vandoren (2021), Perez (2021), Hansen, NO, Oling, Soegaard (2021)

- further examine Carroll geometry/gravity & Carrollian field theories and connect to holography
 - connections with corner symmetry...