Title: Carroll symmetry in gravity and string theory

Speakers: Niels Obers

Collection: Quantum Gravity Around the Corner

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Abstract: I will discuss the small speed of light expansion of general relativity, utilizing the modern perspective on non-Lorentzian geometry. The leading order in the expansion leads to an action that corresponds to the electric Carroll limit of general relativity, of which I will highlight some interesting properties. The next-to-leading order will also be obtained, which exhibits a particular subsector that correspond to the magnetic Carroll limit, which features a solution that describes the Carroll limit of a Schwarzschild black hole. The incorporation of a cosmological constant in the Carroll (or ultra-local) expansion will also be commented on. Finally, I will describe how Carroll symmetry and geometry arises on the world-sheet of certain limits of string theory sigma models.

Carroll Symmetry in Gravity and String Theory

Quantum Gravity around the corner, PI, Waterloo, Oct. 4, 2022

Niels Obers (Nordita & Niels Bohr Institute)

DET FRIE FORSKNINGSRÅD
DANISH COUNCIL FOR INDEPENDENT RESEARCH

THE VELUX FOUNDATIONS
VILLUM FONDEN VELUX FONDEN

Vetenskapsrådet

based on work:

2110.02319 (Front. of Physics) (de Boer,Hartong, NO,Sybesma,Vandoren)
2112.12684 (SciPost) (Hansen,NO,Oling,Søgaard)

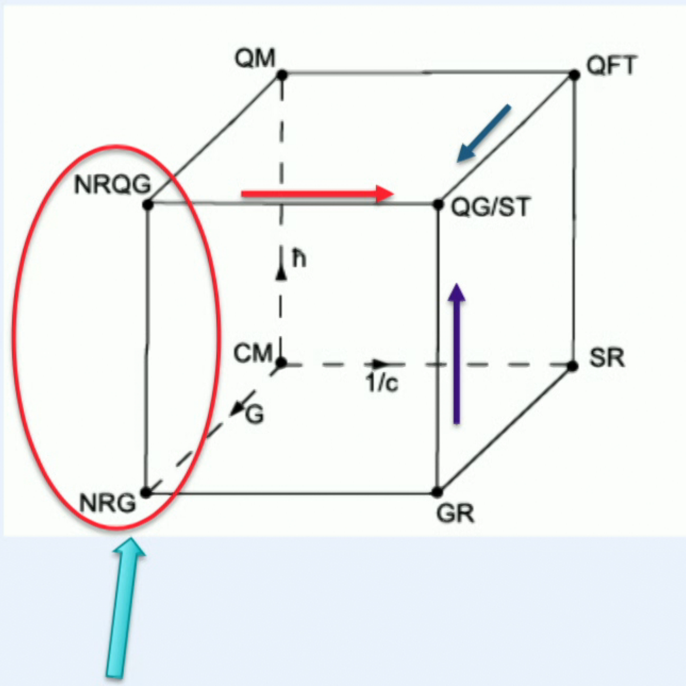
2107.006542 (JHEP) (Bidussi,Harmark,Hartong,NO,Oling)

to appear **2210.xxxxx/22??.yyyyy** ((Bidussi,Harmark,Hartong,NO,Oling)

& earlier works

Cube of physical theories

$(\hbar, G_N, 1/c)$



a third route towards (relativistic) quantum gravity

how does this fit with string theory/holography ?

already (classical) non-relativistic gravity (NRG) is more than just Newtonian gravity

-→ uses Newton-Cartan geometry (and torsional generalization)

What about small speed of light limit ?

Carroll limit:

zero speed of light contraction of Poincare

running (boosting) without moving: Levy-Leblond(1965)

Red Queens race of L. Carroll's Through the looking glass

→ expand relativistic theories around c=0:

what do we get ? what can we use it for ?

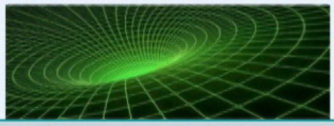
- Carroll symmetry as an organizing principle for a perturbative expansion around c=0

terminology: this is not ultra-relativistic limit ($v/c \to 1$)

rather ultra-local limit

Space-Time symmetries and Geometry

local
symmetries of space and time ←→ geometry of space and time



spacetime symmetries:

time translations, space translations, spatial rotations + …..

crucial difference → type of boosts

relativistic (Lorentz) $t \to \gamma(t + \vec{v}\vec{x}/c^2) \quad , \quad \vec{x} \to \gamma(\vec{x} + \vec{v}t)$

non-relativisistic (Galilean) $t \to t \quad , \quad \vec{x} \to \vec{x} + \vec{v}t$

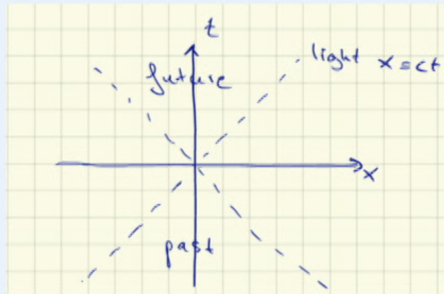
$c \to \infty$

ultra-relativistic (Carroll) $t \to t + \vec{\hat{v}}\vec{x} \quad , \quad \vec{x} \to \vec{x}$

$c \to 0$

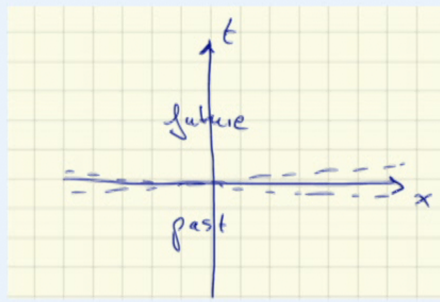
Lightcones & Geometry

Lorentz



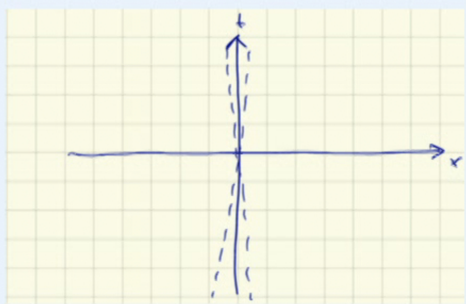
(pseudo-) Riemmanian geometry (GR)

Galilean



torsional Newton-Cartan geometry

Carroll



Carroll geometry

Non-Lorentzian geometries

recent progress in understanding non-relativistic
(large and small speed of light) corners of:
gravity, quantum field theory and string theory:

→ builds on improved understanding of non-Lorentzian geometries
= spacetimes with local symmetries other than Lorentz

NL geometries appear in:

- bdry geometries in non-AdS holography (e.g. Lifshitz flat space)
- covariant formulations of PN approximation in GR
- covariant formulations of non-Lorentzian fluids and CMT systems (FQHE, fractons, ..)
- Horava–Lifshitz gravity, non-relativistic versions of CS, JT
- double field theory
- **ultra-local GR (Carroll) - cosmology, black hole horizons**
- **non-relativistic corners of String Theory**

Recent examples of Carroll

- null hypersurfaces in Lorentzian geom exhibit Carroll

  → black hole membrane paradigm

  [Donnay,Marteau,2019],[Penna,2018]

- BMS isomorphic to conformal Carroll Duvall,Gibbons,Horvathy,2014

  → (3D/4D) flat space holography & celestial holography

  [Bagchi,Detournay,Fareghbal,Simon,2012],[Hartong,2015],
  [Ciambella,Marteau,Petkou,Petropoulos,Siampos,2018],
  [Donnay,Fiorucci,Herfray,Ruzziconi,2021]….

- tensionless limits of strings

  [Bagchi,2013], [Harmark et al]

- limits of GR/ulra-local behavior leads to solvable corners

  [Henneaux,1979],[Hartong,2015][Bergshoeff,Gomis,Rollier,ter Veldhuis,2017],
  [Henneaux,Salgado-Rebolledo,2021],[Perez,2021],[Hansen et al, 2021], ….

see also: Bekaert,Morand,2015/Figueroa-O'Farrill,Prohazka,2018/FigueroaOF et al 2022

- inflationary cosmology

  [de Boer,Hartong,Obers,Sybesma,Vandoren,2021]

Plan

- part I: Carroll symmetry in gravity

- Carroll geometry
- review: pre-ultra-local parametrization of GR
- leading-order action = electric theory
- next-to-leading action (includes magnetic theory)
- solutions of constraints and evolution equations

- part II: Carroll symmetry in string theory

- non-relativistic strings
- two new classes of sigma models from NRST
  → one with Carrollian world-sheet structure

Outlook

Carroll geometry

defined by

- timelike vector field v^μ
- spatial metric $h_{\mu\nu} = \delta_{ab} e_\mu{}^a e_\nu{}^b,$

transforming under Carroll boosts:

$$\delta_\lambda v^\mu = 0, \qquad \delta_\lambda h^{\mu\nu} = 2\lambda^{(\mu} v^{\nu)}, \qquad \lambda^\mu = h^{\mu\nu}\lambda_\nu$$

from gauging the Carroll algebra

orthogonality and completeness

$$\tau_\mu v^\mu = -1, \quad \tau_\mu h^{\mu\nu} = 0, \quad h_{\mu\nu} v^\nu = 0, \quad \delta^\mu_\nu = -v^\mu \tau_\nu + h^{\mu\rho} h_{\rho\nu}.$$

Carroll compatible connection:

$$\tilde{\nabla}_\mu v^\nu = 0, \qquad \tilde{\nabla}_\rho h_{\mu\nu} = 0.$$

$$\tilde{\Gamma}^\rho_{\mu\nu} = -v^\rho \partial_{(\mu}\tau_{\nu)} - v^\rho \tau_{(\mu} \mathcal{L}_v \tau_{\nu)} + \frac{1}{2} h^{\rho\lambda} \left[\partial_\mu h_{\nu\lambda} + \partial_\nu h_{\lambda\mu} - \partial_\lambda h_{\mu\nu}\right] - h^{\rho\lambda}\tau_\nu K_{\mu\lambda},$$

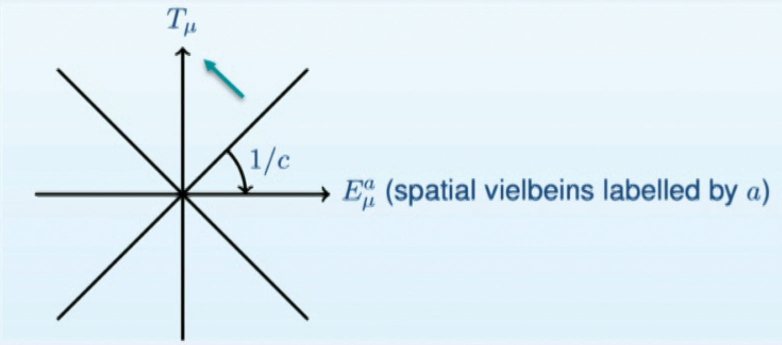
extrinsic curvature $K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_v h_{\mu\nu}$

- can be defined intrinsically or follows from small c limit in GR:

speed of light dependence in GR

metric: $g_{\mu\nu} = -c^2 T_\mu T_\nu + \Pi_{\mu\nu}$

T = time-like vielbein
Pi = spatial metric



small speed of light → light-cone closes up

assume the tensors T and Pi can be expanded as Taylor series: expand in c^2

(llarge speed of light → light-cone opens up)

PUL (pre-ultralocal) parametrization of GR

PUL (pre-ultralocal) parametrization of GR: time/space split

$$g_{\mu\nu} = -c^2 T_\mu T_\nu + \Pi_{\mu\nu}, \qquad g^{\mu\nu} = -\frac{1}{c^2} V^\mu V^\nu + \Pi^{\mu\nu}.$$

rewrite EH using PUL connection (adapted to Carroll when expanding)

$$\begin{aligned}\tilde{C}^\rho_{\mu\nu} &= -V^\rho \partial_{(\mu} T_{\nu)} - V^\rho T_{(\mu} \mathcal{L}_V T_{\nu)} \\ &+ \frac{1}{2} \Pi^{\rho\lambda} \left[\partial_\mu \Pi_{\nu\lambda} + \partial_\nu \Pi_{\lambda\mu} - \partial_\lambda \Pi_{\mu\nu}\right] - \Pi^{\rho\lambda} T_\nu \mathcal{K}_{\mu\lambda},\end{aligned}$$

$$\mathcal{K}_{\mu\nu} = -\tfrac{1}{2} \mathcal{L}_V \Pi_{\mu\nu}.$$

EH takes form

$$R : \approx \frac{1}{c^2} \left(\mathcal{K}^{\mu\nu} \mathcal{K}_{\mu\nu} - \mathcal{K}^2\right) + \Pi^{\mu\nu} \overset{(\tilde{C})}{R}_{\mu\nu} + \frac{c}{4} \Pi^{\mu\nu} \Pi^{\rho\sigma} (dT)_{\mu\rho} (dT)_{\nu\sigma} \,,$$

Carroll geometry from small c

expand in small c in the PUL form

$$V^\mu = v^\mu + c^2 M^\mu + \mathcal{O}(c^4), \qquad T_\mu = \tau_\mu + \mathcal{O}(c^2)$$

$$\Pi^{\mu\nu} = h^{\mu\nu} + c^2 \Phi^{\mu\nu} + \mathcal{O}(c^4), \qquad \Pi_{\mu\nu} = h_{\mu\nu} + \mathcal{O}(c^2),$$

LO fields=Carroll geometry

NLO fields = extra gauge fields

small c expansion of local Lorentz → Carroll boost transformations

- expansion of the PUL connection → Carroll compatible connection

$$\tilde{\Gamma}^\rho_{\mu\nu} = \tilde{C}^\rho_{\mu\nu}\Big|_{c=0}$$

- Remark: similar to the (previously considered) non-relativistic expansion

LO and NLO action

Hansen et al (2021)

- expand EH for small c: LO action = leading order part

$$\overset{(2)}{\mathcal{L}}_{\rm LO} = \frac{e}{16\pi G_N}\left[K^{\mu\nu}K_{\mu\nu} - K^2\right],$$

Henneaux,1979

$$\delta\overset{(2)}{\mathcal{L}}_{\rm LO} = \frac{e}{8\pi G_N}\left[\overset{(2)}{G}{}^v_\mu \delta v^\mu + \frac{1}{2}\overset{(2)}{G}{}^h_{\mu\nu}\delta h^{\mu\nu}\right],$$

$$\overset{(2)}{G}{}^v_\mu = -\frac{1}{2}\tau_\mu(K^{\rho\sigma}K_{\rho\sigma} - K^2) + h^{\gamma\lambda}\tilde{\nabla}_\lambda(K_{\mu\gamma} - Kh_{\mu\gamma}),$$
$$\overset{(2)}{G}{}^h_{\mu\nu} = -\frac{1}{2}h_{\mu\nu}(K^{\rho\sigma}K_{\rho\sigma} - K^2) + K(K_{\mu\nu} - Kh_{\mu\nu}) - v^\rho\tilde{\nabla}_\rho(K_{\mu\nu} - Kh_{\mu\nu}).$$

- NLO action (includes EOMs of LO)

$$\overset{(4)}{\mathcal{L}}_{\rm NLO} = \frac{e}{8\pi G_N}\left[\frac{1}{2}h^{\mu\nu}\tilde{R}_{\mu\nu} + \overset{(2)}{G}{}^v_\mu M^\mu + \frac{1}{2}\overset{(2)}{G}{}^h_{\mu\nu}\Phi^{\mu\nu}\right].$$

see also: Bergshoeff et al,2017

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$$\mathcal{K}_{\mu\nu} = -\tfrac{1}{2}\mathcal{L}_V \Pi_{\mu\nu}.$$

EH takes form

$$R :\approx \frac{1}{c^2}\left(\mathcal{K}^{\mu\nu}\mathcal{K}_{\mu\nu} - \mathcal{K}^2\right) + \Pi^{\mu\nu} \overset{(\tilde{C})}{R}_{\mu\nu} + \frac{c}{4}\Pi^{\mu\nu}\Pi^{\rho\sigma}(dT)_{\mu\rho}(dT)_{\nu\sigma},$$

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see also: Bergshoeff et al,2017

LO theory = timelike (electric) theory

EOMs

$$\overset{(2)}{G}{}^{v}_{\mu} = -\frac{1}{2}\tau_\mu\left(K^{\rho\sigma}K_{\rho\sigma} - K^2\right) + h^{\nu\rho}\tilde{\nabla}_\rho\left(K_{\mu\nu} - K h_{\mu\nu}\right),$$
$$\overset{(2)}{G}{}^{h}_{\mu\nu} = -\frac{1}{2}h_{\mu\nu}\left(K^{\rho\sigma}K_{\rho\sigma} - K^2\right) + K\left(K_{\mu\nu} - K h_{\mu\nu}\right) - v^\rho\tilde{\nabla}_\rho\left(K_{\mu\nu} - K h_{\mu\nu}\right).$$

projections on time and space

$$K^{\mu\nu}K_{\mu\nu} - K^2 = 0,$$
$$h^{\rho\sigma}\tilde{\nabla}_\rho(K_{\sigma\mu} - K h_{\sigma\mu}) = 0,$$
$$\mathcal{L}_v K_{\mu\nu} = -2K_\mu{}^\rho K_{\rho\nu} + K K_{\mu\nu},$$

constraint equations

evolution equations

solutions of electric theory

evolution of (arbitrary) initial data

depends only on v^μ -derivatives (ultra local)

can integrate in suitable adapated coordinates

$$h_{ij}(t) = h_{(0)ik} \exp[-2t\, h^{kl}_{(0)} K_{(0)lj}].$$

Hansen,NO,Oling,Soegaard
see also: Dautcourt; Niedermaier

→ much simpler then evolution in full GR !

construct initial data using 3+1 methods (Bowen-York type solutions)

$$h_{0,ij} = \psi^4 \delta_{ij},$$
$$K_{0,ij} = \psi^{-2}\bar{L}X_{ij} + \frac{1}{3}K_0\psi^4\delta_{ij},$$

$$\psi = \left[\frac{3}{2K_0^2}\bar{L}X_{ij}\bar{L}X^{ij}\right]^{1/12},$$
$$\bar{L}X^{ij} = \frac{3}{2r^3}\left[x^iP^j + x^jP^i - \left(\delta^{ij} - \frac{x^ix^j}{r^2}\right)P_kx^k\right] + \frac{3}{r^5}\left[\epsilon^{ik}{}_lJ_kx^lx^j + \epsilon^{jk}{}_lJ_kx^lx^i\right].$$

have physical boundary charges (linear momentum and angular momentum)
can be computed from charge integrand involving:

$$\Theta^\mu = \frac{e}{8\pi G}\left[(Kh^\mu_\nu - K^\mu{}_\nu)\delta v^\nu - \frac{1}{2}(Kh_{\sigma\rho} - K_{\sigma\rho})v^\mu\delta h^{\sigma\rho}\right].$$ presymplectic potential

$$Q^{[\mu\nu]} = \frac{e}{4\pi G}(v^{[\mu}K^{\nu]}{}_\sigma\xi^\sigma - v^{[\mu}\xi^{\nu]}K).$$ Noether-Wald charge

no mass charge at LO !

Hansen,NO,Oling,Soegaard
see also: Perez

solutions of electric theory

evolution of (arbitrary) initial data

depends only on v^μ -derivatives (ultra local)

can integrate in suitable adapated coordinates

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→ much simpler then evolution in full GR !

consistent subsector of NLO theory
= spacelike (magnetic) theory

NLO EOMs complicated
simplify by setting NLO fields to zero (need to be on-shell in LO theory)
action derivation:
rewrite PUL for of EH using Lagrange multiplier

$$S = \frac{c^4}{16\pi G}\int_M \left[-\frac{c^2}{4} G^{\mu\nu\rho\sigma}\chi_{\mu\nu}\chi_{\rho\sigma} + G^{\mu\nu\rho\sigma}\chi_{\mu\nu}\mathcal{K}_{\rho\sigma} + \Pi^{\mu\nu}\overset{(C)}{R}_{\mu\nu} + \frac{c^2}{4}\Pi^{\mu\nu}\Pi^{\rho\sigma}(dT)_{\mu\rho}(dT)_{\nu\sigma}\right] E\, d^d x.$$

$$G^{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Pi^{\mu\rho}\Pi^{\nu\sigma} + \Pi^{\mu\sigma}\Pi^{\nu\rho} - 2\Pi^{\mu\nu}\Pi^{\rho\sigma}\right),$$

$$G_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{d-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right).$$

DeWitt metric

small c limit gives:

$$\overset{(4)}{S}_{\text{mag}} = \frac{1}{16\pi G}\int_M \left[\phi^{\mu\nu}K_{\mu\nu} + h^{\mu\nu}\tilde{R}_{\mu\nu}\right] e\, d^d x,$$

Langrangian version of Henneaux ,Selgado-Rebolledo

Lagrange multiplier implementing zero extrinsic curvature

EOMs

$$0 = \tilde{\nabla}_\rho \phi^\rho{}_\mu - \upsilon^\rho \tau_{\rho\nu}\phi^\nu{}_\mu + \tau_\mu h^{\rho\sigma}\tilde{R}_{\rho\sigma} + \frac{1}{2}h^{\nu\sigma}h^\lambda_\mu \tau_{\nu\lambda}\upsilon^\kappa \tau_{\kappa\sigma} - \tilde{\nabla}_\nu \left(h^{\nu\sigma}\tau_{\mu\sigma}\right),$$

$$0 = -\frac{1}{2}\upsilon^\rho \tilde{\nabla}_\rho \phi_{\mu\nu} + \tilde{R}_{\mu\nu} - \frac{1}{2}h_{\mu\nu}h^{\sigma\rho}\tilde{R}_{\sigma\rho} + \frac{1}{2}h_{\mu\nu}\tilde{\nabla}_\lambda (h^{\lambda\gamma}\upsilon^\kappa \tau_{\kappa\gamma}).$$

go to boost frame that allows for spatial foliation:

$$0 = h^{\mu\nu}\hat{R}_{\mu\nu},$$

$$0 = \hat{\nabla}_\nu \phi^\nu{}_\mu,$$

constraint eq.

$$\frac{1}{2}\mathcal{L}_\upsilon \phi_{\mu\nu} = \hat{R}_{\mu\nu} - \hat{\nabla}_{(\mu}a_{\nu)} - a_\mu a_\nu + h_{\mu\nu}h^{\rho\sigma}\left(\hat{\nabla}_{(\rho}a_{\sigma)} + a_\rho a_\sigma\right),$$

evolution eq.

solutions with cosmological constant

$$S_\Lambda = \frac{c^4}{16\pi G}\int d^{d+1}x\, E(-2\Lambda).,$$

$$\Lambda = \frac{1}{c^2}\overset{(-2)}{\Lambda} + \overset{(0)}{\Lambda} + \cdots\ ,$$

LO NLO

- LO theory initial data $h_{(0)ij}, \quad K_{(0)ij} = -Hh_{(0)ij},$

$$h_{\mu\nu}(t) = e^{2Ht}h_{(0)\mu\nu}, \qquad K_{\mu\nu}(t) = -Hh_{\mu\nu}(t).$$

simple physical example: $h_{(0)ij}$ standard flat metric

corresponds to Carroll limit of dS in planar coordinates

- NLO theory: both postive and negative Lambda possible

e.g. Carroll limit of (anti)de Sitter in static coordinates

Outlook (part I)

- solutions: full NLO theory (and beyond), numerics/analytics..
- apply 3+1 techniques to NR expansion
- Cosmology and Carroll gravity
- Carroll strings
- Tensionless strings
- Flat space Holography
- Carroll fluids in curved spacetime [de Boer et al, to appear]
  de Boer et al (2017)/Ciambelli,Marteau,Petkou,Petropoulos,Siampos (2018, 2018)
  Donnay,Marteau (2019) Ciambelli,Marteau,Petropoulos,Ruzziconi (2020),…
  Petkou,Petropoulos,Rivera,Betancour,Siampos(2022) Freidel,Jai-akson(2022)
- applications to supersonic behavior ?

prelude: Carroll (scalar) field theories

see talk Gerben Oling

$$\mathcal{L} = \frac{1}{2c^2}(\partial_t\phi)^2 - \frac{1}{2}(\partial_i\phi)^2 - V(\phi)\,. \quad \Leftrightarrow \quad \mathcal{L} = -\frac{c^2}{2}\chi^2 + \chi\partial_t\phi - \frac{1}{2}(\partial_i\phi)^2 - V(\phi)\,.$$

↓ Carroll limit

↓ chi = Lagrange multiplier

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \tilde{V}(\phi)$$

(timelike theory)

$$\mathcal{L} = \chi\dot{\phi} - \frac{1}{2}(\partial_i\phi)^2 - V(\phi)\,.$$

spacelike theory)

$$\delta\phi = \vec{b}\cdot\vec{x}\,\dot{\phi}\,, \qquad \delta\chi = \vec{b}\cdot\vec{x}\,\dot{\chi} + \vec{b}\cdot\vec{\partial}\phi\,.$$

boost Ward identies imply:

$$\langle O(t,\vec{x})O(0,0)\rangle = F(t)\delta(\vec{x})$$

$$\langle O(t,\vec{x})O(0,0)\rangle = f(|\vec{x}|),$$

→ 2D field theories (string theory worldsheets) allow for more general Carroll theories

can be found by considering worldsheet limits of non-relativistic string theory

Main message:
two new classes of sigma models from NRST

non-relativistic strings on curved spactime:
torsional string Newton-Cartan geometry

two distinct
scaling limits

Galilean string

physical constraint

`particle-string'

Galilean worldsheet structure
torsional Galilean geometry (target space)

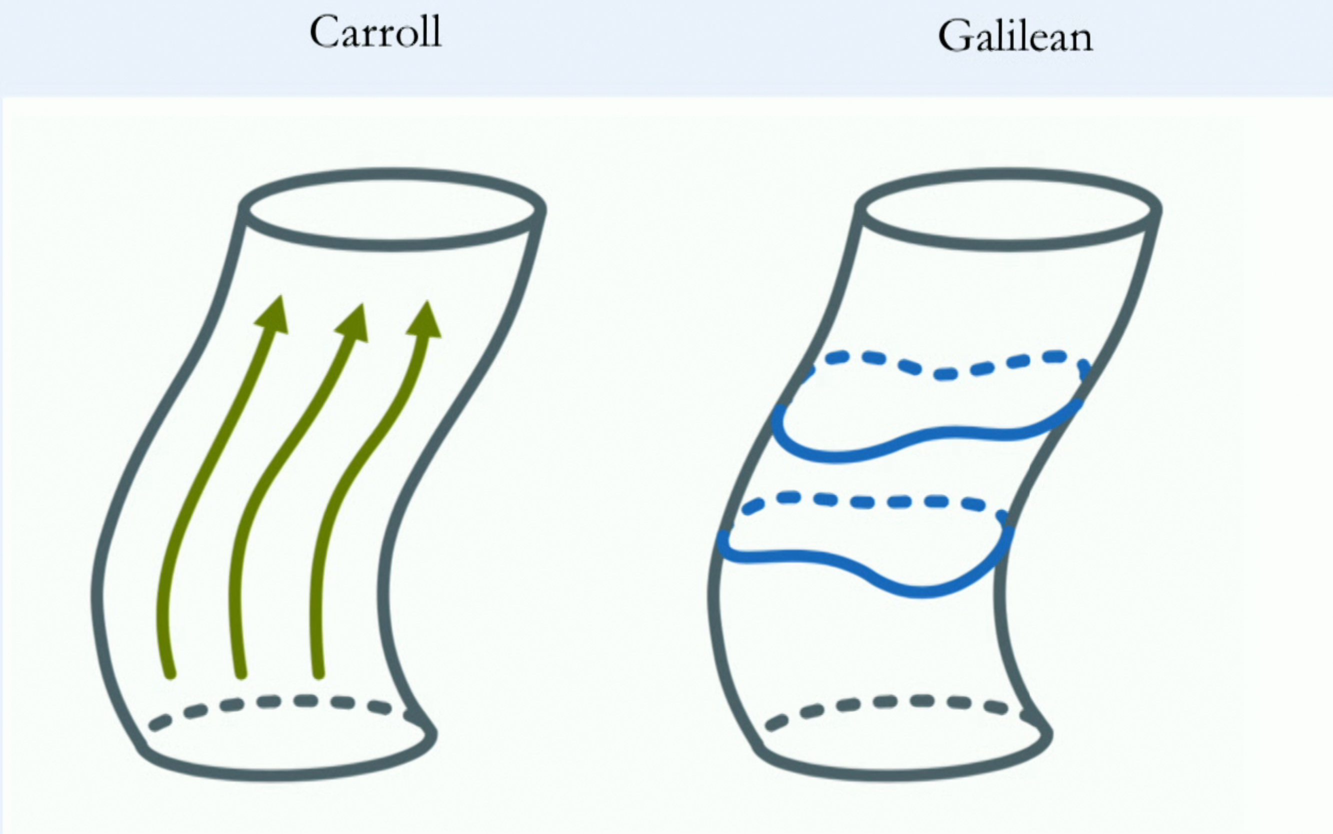
Virasoro x R (subalgebra of GCA)

Carrrollian string

Carrollian world-sheet structure
torsional Carrollian geometry (target space)

BMS3 symmetry

Visualisation of world-sheet



(drawing: courtesy of Gerben Oling)

Polyakov action for Carrollian strings

$$S=-\frac{T}{2}\int d^2\sigma\Big[\Big(-e\,e_0{}^{\alpha}e_0{}^{\beta}h_{MN}+\epsilon^{\alpha\beta}m_{MN}\Big)\partial_\alpha X^M\partial_\beta X^N+\omega\epsilon^{\alpha\beta}e^1{}_\alpha\tau^1{}_\beta+\psi\epsilon^{\alpha\beta}(e^0{}_\alpha\tau^1{}_\beta+e^1{}_\alpha\tau^0{}_\beta)\Big]$$

- exhibits Carrollian world-sheet structure
- Weyl and local (2D) Carrollian boost act on the zweibeine:

$$e^0{}_\alpha\to fe^0{}_\alpha+\hat{f}e^1{}_\alpha\,,\quad e^1{}_\alpha\to fe^1{}_\alpha\,,\quad \omega\to\frac{1}{f}\omega-\frac{\hat{f}}{f^2}\psi\,,\quad \psi\to\frac{1}{f}\psi$$

- no problem with (wrong-sign) kinetic terms
- Hamiltonian analysis for (subclass) of target spacetimes $x^M=(x^\mu,v)$,

v = compact isometry

$$\tau^0{}_M=\delta^t_M\,,\quad \tau^1{}_M=\delta^v_M\,,\quad m_{\mu\nu}=0\,,\quad m_{v\mu}=-m_\mu$$

gauge-fixed action has residual symmetry: BMS_3
(contraction of Vir x Vir)

Outlook

- further study (quantization) of Carrollian sigma model
  - Hamiltonian analysis

  Kluson (2021), Bidussi,Harmark,Hartong,NO,Oling (to appear)
- obtain beta functions
- relation to limits of AdS/CFT

  Harmark,Wintergerst (2019),Baiguera,Harmark,Wintergerst(2020)
  Baiguera,Harmark,Lei,Wintergerst (2020)
- connections to Carrollian (small speed of light) gravity

  Henneaux (1979), Bergshoeff,Gomis,Rollier,ter Veldhuis(2017),Hartong(2015)
  Henneaux,Salgado-Rebolledo(2021),
  de Boer,Hartong,NO,Sybesma,Vandoren(2021),Perez(2021),Hansen,NO,Oling,Soegaard(2021)

→ further examine Carroll geometry/gravity & Carrollian field theories and connect to holography
  - connections with corner symmetry…