Title: Carrollian Perspective on Celestial Holography

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Collection: Quantum Gravity Around the Corner

Date: October 04, 2022 - 11:00 AM

URL: https://pirsa.org/22100038

Abstract: The flat space holography program aims at describing quantum gravity in asymptotically flat spacetime in terms of a dual lower-dimensional field theory. Two different roads to construct flat space holography have emerged. The first consists of a 4d bulk / 3d boundary duality, called Carrollian holography, where 4d gravity is suggested to be dual to a 3d Carrollian CFT living on the null boundary of the spacetime. The second is a 4d bulk / 2d boundary duality, called celestial holography, where 4d gravity is dual to a 2d CFT living on the celestial sphere. I will argue that these two seemingly contradictory proposals are actually related. The Carrollian operators will be mapped to the celestial operators using an appropriate integral transform. The Ward identities of the sourced Carrollian CFT, encoding the gravitational flux-balance laws, will be shown to reproduce those of the 2d celestial CFT, encoding the bulk soft theorems.

Carrollian Perspective on Celestial Holography

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4th of October 2022

References

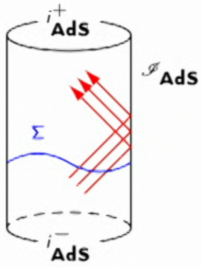
- Based on:

    1. Carrollian Perspective on Celestial Holography
    Laura Donnay, Adrien Fiorucci, Yannick Herfray, Romain Ruzziconi
    arXiv:2202.04702 Phys.Rev.Lett. (2022)

    2. Flat Space Holography: from Null Infinity to the Celestial Sphere
    Laura Donnay, Adrien Fiorucci, Yannick Herfray, Romain Ruzziconi
    arXiv:22xx.xxxxx

Motivations

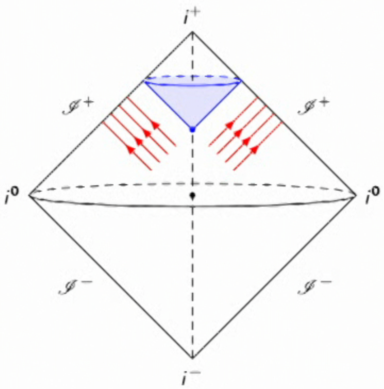
- Holographic principle:
  Gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region.
- Explicit realization of this principle: AdS/CFT correspondence.
  $\Longrightarrow$ "Quantum gravity in a box" (implemented by Dirichlet boundary conditions)
- How general is the holographic principle? Does it extend to asymptotically flat spacetimes?



Flat space holography program

(see *e.g.* [Susskind '99] [Polchinski '99] [Giddings '00] [de Boer-Solodukhin '03] [Arcioni-Dappiaggi '03] [Mann-Marolf '06] for early attempts).

- Asymptotic symmetries form the Bondi-van der Burg-Metzner-Sachs (BMS) group [Bondi-van der Burg-Metzner '62] [Sachs '62]
  $\Longrightarrow$ Broadly studied in the literature
  (see *e.g.* [Newman-Unti '62] [Penrose '65] [Geroch '77] [Ashtekar-Streubel '81] [Barnich-Troessaert '10])
- Important obstructions to flat space holography:
  1. Null nature of $\mathscr{I}^+$ and $\mathscr{I}^-$
  2. Radiation leaking through the conformal boundary
     $\Longrightarrow$ Open gravitational system
     $\Longrightarrow$ The BMS charges are not conserved



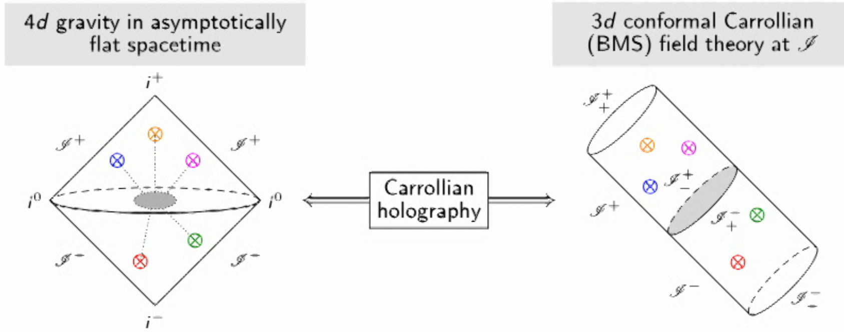
Holographic nature of null infinity

- How to construct the holographic dual of asymptotically flat spacetime? (bottom-up approach)
- *Two distinct but complementary visions of* $\mathscr{I}^+$:

Picture 1: Carrollian holography	**Picture 2: Celestial holography**
$\mathscr{I}^+$ is seen as a boundary along which there is an evolution with respect to u	$\mathscr{I}^+$ is seen as a portion of Cauchy hypersurface pushed to infinity
Describe the dynamics of the system	Describe the state of the system
Flux-balance laws	Scattering problem between $\mathscr{I}^-$ and $\mathscr{I}^+$
Suggests a $4d$ bulk / $3d$ boundary *Carrollian holography*	Suggests a $4d$ bulk / $2d$ boundary *celestial holography*
Dual: $3d$ BMS field theory	Dual: $2d$ Celestial CFT

Carrollian Holography

- *Carrollian holography*:



- BMS algebra $\simeq$ conformal Carrollian algebra [Duval-Gibbons-Horvathy '14].

  $\Longrightarrow$ Dual theory: Carrollian CFT in 3*d*.

- "Carroll" refers to the $c \to 0$ limit of the Poincaré group [Lévy-Leblond '65].

  $\Longrightarrow$ Carrollian geometry naturally induced on null hypersurfaces [Geroch '77] [Henneaux '79] [Ashtekar-Schmidt '79]:

$$(q_{ab}, n^c) \quad \text{with} \quad q_{ab} n^b = 0 \,.$$

- Carrollian holography follows a similar pattern than AdS/CFT correspondence: 4*d* bulk / 3*d* boundary duality.

  $\Longrightarrow$ Naturally arises from a flat limit procedure ($\Lambda \to 0$).

  $\Longrightarrow$ The flat limit in the bulk induces a Carrollian limit at the boundary.

Pros *vs* Cons of Carrollian holography

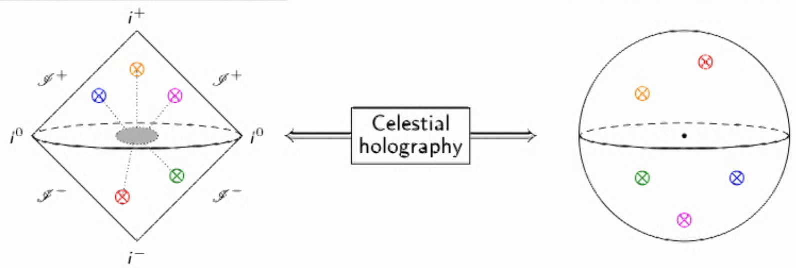
- Success of this approach:
  1. 3*d* gravity (entropy matching, entanglement entropy, effective action, correlation functions, Carroll anomaly . . .).
     [Barnich-Gomberoff-Gonzalez '12] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '13] [Bagchi-Fareghbal '12] [Detournay-Grumiller-Scholler-Simon '14] [Bagchi-Basu-Grumiller-Riegler '15] [Hartong '16] [Bagchi-Grumiller-Merbis '16] [Campoleoni-Ciambelli-Delfante-Marteau-Petropoulos-Ruzziconi '22]
  2. Fluid/gravity correspondence [See Marios' talk]:
     [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18]

Gravity in asymptotically flat spacetime $\Longleftrightarrow$ Carrollian fluid at the boundary

- Drawbacks:
  1. Few is known about quantum Carrollian CFTs.
  2. How to treat the non-conservation of the charges generated by outgoing radiation?

Celestial holography

- *Celestial holography:*



- $\mathcal{S}$-matrix elements in the bulk $\Longleftrightarrow$ Correlation functions in a $2d$ CFT

  [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Strominger '18] [Donnay-Puhm-Strominger '19] [Fotopoulos-Taylor '19]

- Massless scattering $\Longrightarrow$ Mellin transform: [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]

$$\langle \mathcal{O}_{\Delta_1, J_1}(z_1, \bar{z}_1) \dots \mathcal{O}_{\Delta_N, J_N}(z_N, \bar{z}_N) \rangle = \left(\prod_{i=1}^{N} \int_0^{+\infty} d\omega_i \, \omega_i^{\Delta_i - 1} \right) \mathcal{A}(\{\omega_i\}, \{z_i\}, \{\bar{z}_i\})$$

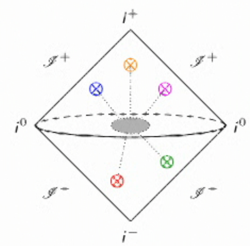
  where the CCFT operators $\mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i)$ are characterized by conformal dimension Δ_i and spin J_i.

- Conformal symmetries in CCFT induced by Lorentz transformations in the bulk: $\mathrm{Conf}(S^2) \simeq$ Lorentz.

Pros *vs* Cons of celestial holography

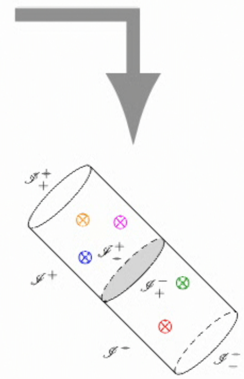
- Advantages and successes of this approach:
  1. Use the powerful techniques of CFT.
  2. Ward identities in the CCFT encode the soft theorems in the bulk.
  3. New $w_{1+\infty}$ symmetries uncovered in the CCFT OPEs [See Ana's talk].
     $\Longrightarrow$ Infinite tower of soft theorems in the bulk.
     $\Longrightarrow$ Provides an organization of the solution space in gravity.
     [Strominger '21] [Guevara-Himwich-Pate-Strominger '21] [Adamo-Mason-Sharma '21] [Freidel-Pranzetti-Raclariu '21] [Compère-Oliveri-Seraj '22] [Bu-Heuveline-Skinner '22]
- Drawbacks:
  1. Seems completely different from AdS_4/CFT_3.
  2. Information on the dynamics not manifest.
     (*Where is time in celestial holography? How to encode the flux-balance laws?*).

Objectives



- **From the bulk...** Gravity in 4d asymptotically flat spacetime:

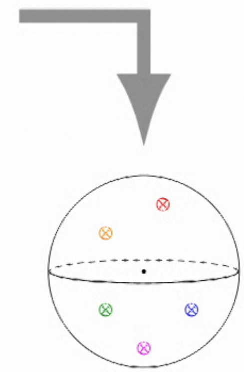
  $\Longrightarrow$ Review the BMS flux-balance laws.
  (non-conservation of the BMS charges at null infinity)



- **...to null infinity...** 3d Carrollian CFT at null infinity:

  $\Longrightarrow$ How to describe the BMS flux-blance laws from a holographic perspective?

  $\Longrightarrow$ Sourced Ward identities of the Carrollian CFT holographically encode the BMS flux-balance laws.



- **...to the celestial sphere.** 2d Celestial CFT:

  $\Longrightarrow$ How to relate Carrollian and celestial holographies?

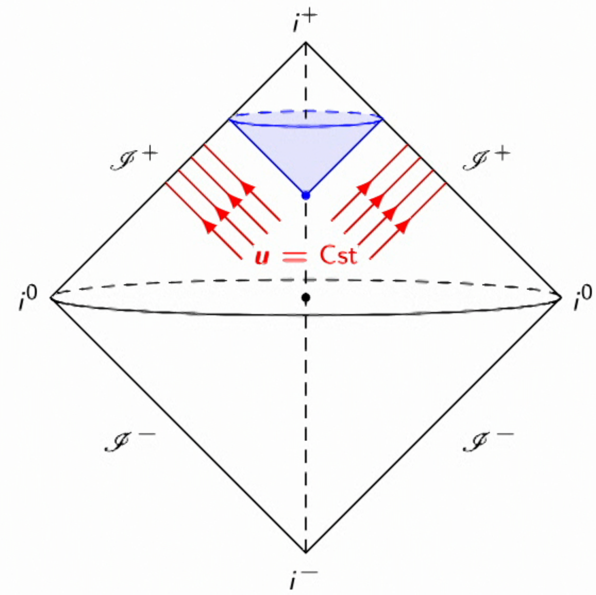
  $\Longrightarrow$ Map the Carrollian to the celestial operators.

  $\Longrightarrow$ Equivalence between Carrollian and celestial Ward identities.

Solution space of 4d asymptotically flat spacetimes

- Asymptotically flat metric in Bondi coordinates to study $\mathscr{I}^+$: (u, r, x^A) where $x^A = (z, \bar{z})$ [Bondi-van der Burg-Metzner '62] [Sachs '62]:

$$ds^2 = \left(\frac{2M}{r} + \mathcal{O}(r^{-2})\right) du^2 - 2\left(1 + \mathcal{O}(r^{-2})\right) dudr + \left(r^2 \mathring{q}_{AB} + r\, C_{AB} + \mathcal{O}(r^0)\right) dx^A dx^B + \left(\frac{1}{2}\partial_B C^B_A + \frac{2}{3r}(N_A + \frac{1}{4} C^B_A \partial_C C^C_B) + \mathcal{O}(r^{-2})\right) dudx^A .$$



- Flat boundary metric: $\mathring{q}_{AB} dx^A dx^B = 2dzd\bar{z}$ ($ds^2_{\text{Mink}} = -2dudr + 2r^2 dzd\bar{z}$).
- *Time evolution/constraint equations* on the mass and angular momentum aspects

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4}\partial_A \partial_B N^{AB},$$

$$\partial_u N_A = \partial_A M + \frac{1}{16}\partial_A(N_{BC} C^{BC}) - \frac{1}{4} N^{BC}\partial_A C_{BC} - \frac{1}{4}\partial_B(C^{BC} N_{AC} - N^{BC} C_{AC}) - \frac{1}{4}\partial_B \partial^B \partial^C C_{AC} + \frac{1}{4}\partial_B \partial_A \partial_C C^{BC},$$

with $N_{AB} = \partial_u C_{AB}$ the Bondi news tensor.

Asymptotic symmetries

- Restriction at $\mathscr{I}^+$ of the asymptotic Killing vectors:

$$\xi|_{\mathscr{I}^+} = \bar{\xi}(\mathcal{Y}, \mathcal{Y}, \bar{\mathcal{Y}}) = \left[\mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

  where

  1. $\mathcal{T} = \mathcal{T}(z, \bar{z})$ is the supertranslation parameter;
  2. $\mathcal{Y} = \mathcal{Y}(z)$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}}(\bar{z})$ are the superrotation parameters satisfying the conformal Killing equation.

- Commutation relations:

$$[\bar{\xi}(\mathcal{T}_1, \mathcal{Y}_1, \bar{\mathcal{Y}}_1), \bar{\xi}(\mathcal{T}_2, \mathcal{Y}_2, \bar{\mathcal{Y}}_2)] = \bar{\xi}(\mathcal{T}_{12}, \mathcal{Y}_{12}, \bar{\mathcal{Y}}_{12}),$$

  with

$$\mathcal{T}_{12} = \mathcal{Y}_1\partial\mathcal{T}_2 - \frac{1}{2}\partial\mathcal{Y}_1\mathcal{T}_2 - (1 \leftrightarrow 2) + \text{c.c.}, \quad \mathcal{Y}_{12} = \mathcal{Y}_1\partial\mathcal{Y}_2 - (1 \leftrightarrow 2), \quad \bar{\mathcal{Y}}_{12} = \bar{\mathcal{Y}}_1\bar{\partial}\bar{\mathcal{Y}}_2 - (1 \leftrightarrow 2)$$

  where c.c. stands for complex conjugate terms. $\Longrightarrow$ $\boxed{\mathfrak{bms}_4 \simeq \text{Conformal Carroll algebra.}}$ [Duval-Gibbons-Horvathy '14]

  (Extended BMS: $\mathfrak{bms}_4^{\text{ext}} = (\text{Witt} \oplus \text{Witt}) \inplus \mathfrak{supertranslations}^*$ [Barnich-Troessaert '10])

BMS surface charges

- At a cut $\mathcal{S}_u \equiv \{u = \text{constant}\}$ of $\mathscr{I}^+$, one can construct "surface charges" associated with BMS symmetries using covariant phase space methods [Wald-Zoupas '99] [Barnich-Troessaert '10]. [See Adrien's lecture]
- BMS charges are non-integrable and non-conserved due to the outgoing radiation at $\mathscr{I}^+$.
  $\Longrightarrow$ Typical properties for a dissipative system.
- *Selection of a meaningful integrable part:*
  [Compère-Fiorucci-Ruzziconi '18] [Campiglia-Peraza '20] [Compère-Fiorucci-Ruzziconi '20] [Donnay-Ruzziconi '21] [Freidel-Pranzetti '21]:

$$\bar{H}_\xi[g] = \frac{1}{8\pi G}\int_{\mathcal{S}_u} d^2 z\, [2\mathcal{T}\mathscr{M} + \mathcal{Y}\bar{\mathscr{N}} + \bar{\mathcal{Y}}\mathscr{N}]$$

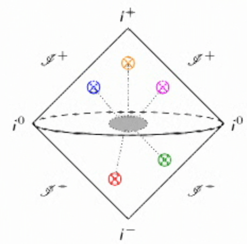
$$\mathscr{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}}), \qquad \mathscr{N} = N_{\bar{z}} - u\bar{\partial}\mathscr{M} + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz})\Big]$$
$$+ \frac{u}{4}\bar{\partial}\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar{z}} - \Big(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\Big)C^{\bar{z}}_z\Big].$$

- Remark: $\mathscr{M} = -\text{Re}\Psi^0_2$, $\mathscr{N} = -\Psi^0_1 + u\eth\Psi^0_2$ [Newman-Penrose '62] [Newman-Unti '62].
- Properties: (i) conserved when $N_{AB} = 0$, (ii) generate canonically the transformations on the radiative phase space, (iii) form a representation of BMS algebra at $\mathscr{I}^\pm$ [See Laura's talk].
- BMS flux-balance laws:

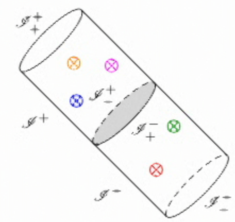
$$\boxed{\frac{d}{du}\bar{H}_\xi[g] = \mathcal{F}_\xi[g] \neq 0\,, \quad \mathcal{F}_\xi[g]|_{N_{AB}=0} = 0.}$$

How to describe the BMS flux-blance laws from a Carrollian holography perspective?

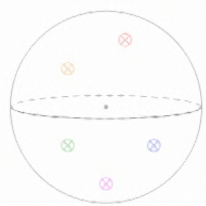
From the bulk...



...to null infinity...



...to the celestial sphere.



Sourced Ward identities

- Goal: Establish a framework that holographically encodes the leaks through the conformal boundary.

$\Longrightarrow$ Key ingredient: coupling with external sources [Troessaert '15] [Wieland '20] [Barnich-Fiorucci-Ruzziconi, *to appear*].

- Consider a QFT on a manifold $\mathscr{M}$ with coordinates x^a.
- Fields: $\Phi^i(x)$, symmetries: $\delta_K \Phi^i = K^i[\Phi]$, conserved Noether currents: $\partial_a j^a_K(x) = 0$.
- Couple the theory with external sources $\sigma(x)$:

$\Longrightarrow$ Classically, generically breaks the Noetherian symmetries;
$\Longrightarrow$ Noether currents are no longer conserved:

$$\partial_a j^a_K(x) = F_K(x), \qquad F_K(x)|_{\sigma=0} = 0\,.$$

- At the quantum level, sourced Ward identities (key result) [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\boxed{\partial_a \langle j^a_K(x) X\rangle + \frac{\hbar}{i}\sum_{i=1}^{N} \delta^{(n)}(x-x_i)\,\delta_{K^i}\,\langle X\rangle = \langle F_K(x) X\rangle}$$

with

1. $X \equiv \Phi^{i_1}(x_1)\dots\Phi^{i_N}(x_N)$: insertions of operators;
2. $\delta_{K^i}\,\langle X\rangle \equiv \langle \Phi^{i_1}(x_1)\dots K^i[\Phi(x_i)]\dots\Phi^{i_N}(x_N)\rangle$.

$\Longrightarrow$ With no field insertion: $\partial_a\langle j^a_K(x)\rangle = \langle F_K(x)\rangle$ (reproduces the classical equation);
$\Longrightarrow$ In absence of sources: $\partial_a\langle j^a_K(x)X\rangle + \frac{\hbar}{i}\sum_{i=1}^N \delta^{(n)}(x-x_i)\,\delta_{K^i}\,\langle X\rangle = 0$ (standard result).

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Sourced Carrollian CFT

- Consider a 3d Carrollian CFT. $x^a = (u, z, \bar{z})$. Carrollian structure: $ds^2 = 0\, du^2 + 2dzd\bar{z}$ and $n^a\partial_a = \partial_u$.
- Noether currents:

$$j^a_{\bar{\xi}} = \mathcal{C}^a{}_b \bar{\xi}^b, \qquad \mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{M} & \mathcal{N}_\mathcal{B} \\ \mathcal{B}^A & \mathcal{A}^A{}_B \end{bmatrix}.$$

$\Longrightarrow$ $\mathcal{C}^a{}_b$: Carrollian stress-tensor; $\mathcal{M}$, $\mathcal{N}_\mathcal{B}$, $\mathcal{B}^A$, $\mathcal{A}^A{}_B$: Carrollian momenta.
[Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [de Boer, Hartong, Obers, Sybesma, Vandoren '18] [Ciambelli-Marteau '18] [Donnay-Marteau '19] [Chandrasekaran-Flanagan-Shehzad-Speranza '21] [Freidel-Pranzetti '21]

- Consider the sourced Ward identities with (quasi) conformal Carrollian primary fields insertions:

$$\delta_{\bar{\xi}} \Phi_{(k,\bar{k})} = \left[\left(\mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}}) \right) \partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\, \partial\mathcal{Y} + \bar{k}\, \bar{\partial}\bar{\mathcal{Y}} \right] \Phi_{(k,\bar{k})} \,.$$

- Sourced Ward identities for 3d Carrollian CFT:

$$\partial_u \langle \mathcal{M}\, X \rangle + \frac{\hbar}{i} \sum_i \delta^{(3)}(x - x_i) \partial_{u_i} \langle X \rangle = \langle F_u\, X \rangle \ ,$$

$$\partial_u \langle \mathcal{N}_z\, X \rangle - \frac{1}{2}\partial \langle \mathcal{M}\, X \rangle + \bar{\partial} \langle \mathcal{A}^{\bar{z}}{}_z\, X \rangle + \frac{\hbar}{i} \sum_i \left[\delta^{(3)}(x - x_i)\partial_i \langle X \rangle - \partial \left(\delta^{(3)}(x - x_i)\, k_i \langle X \rangle \right) \right] = \langle F_z\, X \rangle \ ,$$

$$\langle \mathcal{B}^A\, X \rangle = 0 \,, \qquad \langle (\mathcal{A}^z{}_z + \frac{1}{2}\mathcal{M}) X \rangle + \frac{\hbar}{i} \sum_i \delta^{(3)}(x - x_i)\, k_i \, \langle X \rangle = 0 \,.$$

- Claim: The sourced Ward identities holographically encode the BMS flux-balance laws.

BMS surface charges

- At a cut $\mathcal{S}_u \equiv \{u = \text{constant}\}$ of $\mathscr{I}^+$, one can construct "surface charges" associated with BMS symmetries using covariant phase space methods [Wald-Zoupas '99] [Barnich-Troessaert '10]. [See Adrien's lecture]
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- Properties: (i) conserved when $N_{AB} = 0$, (ii) generate canonically the transformations on the radiative phase space, (iii) form a representation of BMS algebra at $\mathscr{I}^\pm$ [See Laura's talk].
- BMS flux-balance laws:

$$\boxed{\frac{d}{du}\bar{H}_\xi[g] = \mathcal{F}_\xi[g] \neq 0\,, \quad \mathcal{F}_\xi[g]|_{N_{AB}=0} = 0.}$$

Holographic correspondence

- Correspondence between boundary Carrollian momenta and bulk gravitational data at $\mathscr{I}^+$ [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\langle \mathcal{M} \rangle = \frac{1}{4\pi G}\left[M + \frac{1}{8}(C_{AB}N^{AB}) \right], \quad \langle \mathcal{A}^A{}_B \rangle + \frac{1}{2}\delta^A{}_B \langle \mathcal{M} \rangle = 0\,,$$

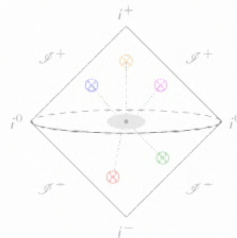
$$\langle \mathcal{N}_A \rangle = \frac{1}{8\pi G}\left(N_A + \frac{1}{4} C_A^B \partial_C C_B^C + \frac{3}{32}\partial_A (C_B^C C_C^B) + \frac{u}{4}\partial^B(\partial_B \partial_C - \frac{1}{2}N_{BC})C_A^C - \frac{u}{4}\partial^B(\partial_A \partial_C - \frac{1}{2}N_{AC})C_B^C \right)\,.$$

- Fixed by requiring compatibility between boundary Noether currents and bulk gravitational charges.
- Similar to the AdS/CFT dictionary where the holographic stress-energy tensor of the CFT is identified with some subleading order in the expansion of the bulk metric. [Balasubramanian-Kraus '99] [de Haro-Solodukhin-Skenderis '01]
- External sources identified with the news: $\sigma_{AB} = N_{AB}$.
- Dissipation through some fluxes $F_u[\sigma]$, $F_A[\sigma]$ such that $F_u[\sigma = 0] = 0 = F_A[\sigma = 0]$.

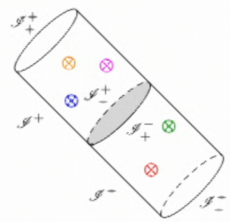
$\Longrightarrow$ With these identifications, the sourced Ward identities reproduce the BMS flux-balance laws.

How to relate Carrollian and celestial holographies?

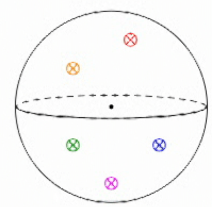
From the bulk...



...to null infinity...



...to the celestial sphere.

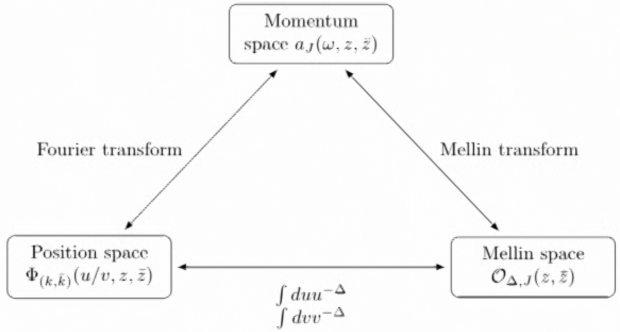


Relation between Carrollian and celestial operators

- Relation between (quasi) conformal Carrollian primary operators and CCFT operators [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\left.\begin{aligned}\mathcal{O}^{out}_{\Delta_i,J_i}(z_i,\bar{z}_i) &= i^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} du_i\, u_i^{-\Delta_i}\, \Phi^{out}_{(k_i,\bar{k}_i)}(u_i,z_i,\bar{z}_i)\,,\\ \mathcal{O}^{in}_{\Delta_j,J_j}(z_j,\bar{z}_j) &= i^{\Delta_j}\Gamma[\Delta_j]\int_{-\infty}^{+\infty} dv_j\, v_j^{-\Delta_j}\, \Phi^{in}_{(k_j,\bar{k}_j)}(v_j,z_j,\bar{z}_j)\,.\end{aligned}\right\}\quad \text{(Fourier + Mellin transforms)}$$

$\Longrightarrow$ Exchange between time and conformal dimension.
$\Longrightarrow$ Extrapolate dictionary [Pasterski-Puhm-Trevisani '21].
$\Longrightarrow$ Scattering bases [Donnay-Pasterski-Puhm '22].



- Matching between Carrollian weights $(k,\bar{k})$ and celestial spin J: $k=\frac{1}{2}(1+J)\,,\bar{k}=\frac{1}{2}(1-J)$.
- Correlation functions ($N=m+n$):

$$\Big\langle \prod_{i=1}^{m}\mathcal{O}^{out}_{\Delta_i,J_i}(z_i,\bar{z}_i)\prod_{j=1}^{n}\mathcal{O}^{in}_{\Delta_j,J_j}(z_j,\bar{z}_j)\Big\rangle$$

$$= \left(\prod_{i=1}^{m} i^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty}du_i\,u_i^{-\Delta_i}\right)\left(\prod_{j=1}^{n} i^{\Delta_j}\Gamma[\Delta_j]\int_{-\infty}^{+\infty}dv_j\,v_j^{-\Delta_j}\right)\langle\underbrace{\Phi^{out}_{(k_1,\bar{k}_1)}(x_1)\dots\Phi^{out}_{(k_m,\bar{k}_m)}(x_m)}_{\text{Insertions at }\mathscr{I}^+}\underbrace{\Phi^{in}_{(k_1,\bar{k}_1)}(x_1)\dots\Phi^{in}_{(k_n,\bar{k}_n)}(x_n)}_{\text{Insertions at }\mathscr{I}^-}\rangle\,.$$

Relation between Carrollian and celestial Ward identities

- Integrated version of the sourced Carrollian CFT:

$$\delta_{\bar{\xi}} \langle X \rangle = \frac{i}{\hbar} \Big\langle \left(\int_{\mathscr{I}^- \sqcup \mathscr{I}^+} \mathbf{F}_{\bar{\xi}} - \int_{\mathscr{I}^+_+} \mathbf{j}_{\bar{\xi}} + \int_{\mathscr{I}^-_-} \mathbf{j}_{\bar{\xi}} \right) X \Big\rangle$$

  where $X \equiv \Phi^{out}_{(k_1,\bar{k}_1)}(x_1) \dots \Phi^{out}_{(k_m,\bar{k}_m)}(x_m) \Phi^{in}_{(k_1,\bar{k}_1)}(x_1) \dots \Phi^{in}_{(k_n,\bar{k}_n)}(x_n)$.

- Assumption of massless scattering: $\mathbf{j}_{\bar{\xi}}|_{\mathscr{I}^+_+} = 0 = \mathbf{j}_{\bar{\xi}}|_{\mathscr{I}^-_-}$.
- Incoming flux = outgoing flux: $\int_{\mathscr{I}^-} \mathbf{F}_{\bar{\xi}} = - \int_{\mathscr{I}^+} \mathbf{F}_{\bar{\xi}}$ (constraint on the sources).
- With these assumptions $\Longrightarrow$ invariance of the correlators under BMS symmetries: $\boxed{\delta_{\bar{\xi}} \langle X \rangle = 0 \,.}$

  [Donnay-Herfray-Fiorucci-Ruzziconi '22]

- Supertranslations:
  1. Define the supertranslation current [Strominger '13] :

$$P(z,\bar z) = \frac{1}{4G}\left(\int_{-\infty}^{+\infty} du + \int_{-\infty}^{+\infty} dv\right)\bar\partial \Pi_{zz}\,, \qquad \Pi_{zz} = \partial_{u/v}\Phi_{zz}\,, \qquad P(z,\bar z) = (\frac{3}{2},\frac{1}{2}).$$

  2. Perform the integral transforms on X:

$$\boxed{\delta_{\mathcal{T}}\langle X\rangle = 0 \quad \Longleftrightarrow \quad \Big\langle P(z,\bar z)\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\Big\rangle + \hbar\sum_{q=1}^{N}\frac{1}{z-z_q}\Big\langle \dots \mathcal{O}_{\Delta_q+1,J_q}(z_q,\bar z_q)\dots\Big\rangle = 0\,.}$$

(leading soft graviton theorem)

- Superrotations:
  1. Define the 2d stress-tensor [Kapec-Mitra-Raclariu-Strominger '17] :

$$T(z) = -\frac{i}{8\pi G}\int \frac{dw d\bar w}{z-w}\left(\int_{-\infty}^{+\infty} du\, u + \int_{-\infty}^{+\infty} dv\, v\right)\partial^3\Pi_{\bar w\bar w}\,, \qquad T(z) : (2,0)\,.$$

  2. Perform the integral transforms on X:

$$\boxed{\delta_{\mathcal{Y}}\langle X\rangle = 0 \quad \Longleftrightarrow \quad \Big\langle T(z)\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\Big\rangle + \hbar\sum_{q=1}^{N}\Big[\frac{\partial_q}{z-z_q} + \frac{h_q}{(z-z_q)^2}\Big]\Big\langle \prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar z_i)\Big\rangle = 0\,.}$$

(subleading soft graviton theorem) $h_q = \frac{1}{2}(\Delta_q + J_q)$

Perspectives

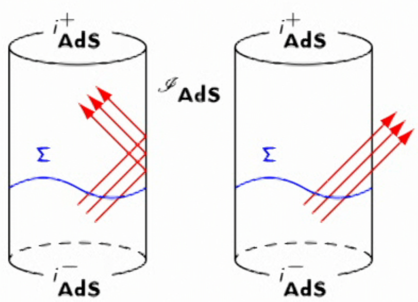
- Two complementary approaches to flat space holography:

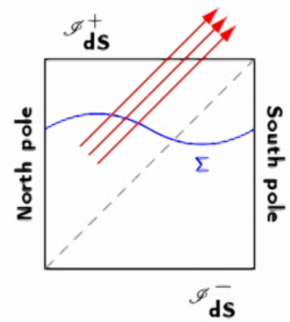
  Carrollian holography $\Longleftrightarrow$ Celestial holography

  $\Longrightarrow$ Deduce more insights in Carrollian holography from celestial holography and reciprocally.
  $\Longrightarrow$ Duality between holographic 3d Carrollian CFT and 2d CCFT?

- Relation with (A)dS/CFT correspondence?

  $\Longrightarrow$ In the bulk, the flat limit works provided one starts with leaky boundary conditions.
  $\Longrightarrow$ Λ-BMS symmetries and phase space. [Compère-Fiorucci-Ruzziconi '19] [Fiorucci-Ruzziconi '21]
  $\Longrightarrow$ Obtain the sourced Carrollian CFT in the ultra-relativistic limit of a sourced CFT.





Thank you!

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AUSTRIAN SCIENCE FUND