Title: Wald-Zoupas vs. improved Noether charge: anomalies as soft terms at scri

Speakers: Simone Speziale

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Abstract: In the last few years, various authors have extended the covariant phase space to include arbitrary anomalies, and a notion of improved Noether charge. After reviewing this construction and discussing examples of anomalies, I will point out that the covariance requirements of the seminal Wald-Zoupas paper permit the presence of a special class of anomalies. To illustrate the meaning of such anomalies, one can look at the case of future null infinity where they take the form of the soft terms in the flux-balance laws.

Wald-Zoupas prescription and i-Noether: anomalies as soft terms at Scri

Simone Speziale

PI, 7-10-22

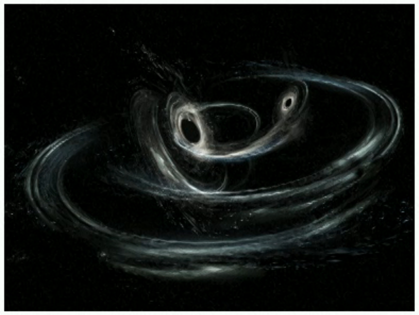
based on work done with Gloria Odak and Antoine Rignon-Bret

as well as many discussions with Abhay Ashtekar, Glenn Barnich, Adrien Fiorucci, Laurent Freidel, Roberto Oliveri, Daniele Pranzetti, Romain Ruzziconi, Anthony Speranza and Wolfgang Wieland

Centre de Physique Théorique

Motivational question

- Wald-Zoupas ('99) showed how the covariant phase space (CPS) can be used to derive (previously known) expressions for BMS charges and fluxes at future null infinity



$$\dot{M} = \frac{1}{4} D_A D_B N^{AB} - \frac{1}{8} N_{AB} N^{AB}$$

- In the years since, the CPS has been developed to include a general treatment of \
(i) field-dependent diffeomorphisms and (ii) anomalies
(Barnich-Compere, Hopfmuller-Freidel, Speranza, FOPS2, Chandrasekaran-Flanagan-Shehzad…)

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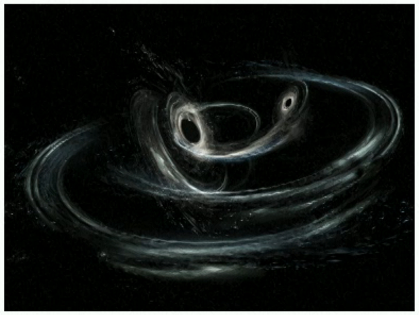

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- WZ never talk much about either aspects, yet the BMS group includes both; **so how come they get the right result?** **and, what new perspective bring the recent developments to the WZ prescription?**

Outline:

- Motivations and the inclusions of anomalies in the covariant phase space
- Charge prescriptions: WZ and anomalies
- The example of finite null hypersurfaces
- The example of null infinity: anomalies as soft terms

Field-dependent diffeomorphisms

There can be various reasons to consider field-dependent diffeomorphisms

1. Gauge-fixing
2. Manipulating the constraint algebra
3. `Slicing' in field space
4. …

Typical example of 1: BMS bulk extension

- the universal structure used at future null infinity *fixes the symmetry vector fields there as well as their first order extension away from it*; from the second order onwards, it is **arbitrary**

  e.g. Bondi-Sachs coordinates:

$$\xi = \underbrace{\tau\partial_u + Y^A\partial_A}_{\text{tangential part}} - \Omega\underbrace{(\frac{1}{2}D\cdot Y\partial_\Omega + \partial^A\tau\partial_A)}_{\text{fixed by the universal structure}} - \Omega^2\underbrace{(\frac{1}{2}\bar{\Delta}\tau\partial_r - \frac{1}{2}C^{AB}\partial_B\tau\partial_A)}_{\text{arbitrary (namely, fixed by the bulk coordinate choice)}} + \dots$$

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Remark: Two terms / One term

This 1 vs 2 **captures the difference** between the finite and the infinite-distance case; and it has a very clear origin that I will explain at the end

$$\xi = \underline{\tau\partial_u + Y^A\partial_A} + \underline{r\dot{\tau}\partial_r} + \underline{r^2(...)}$$

BMSW group of arbitrary null surfaces

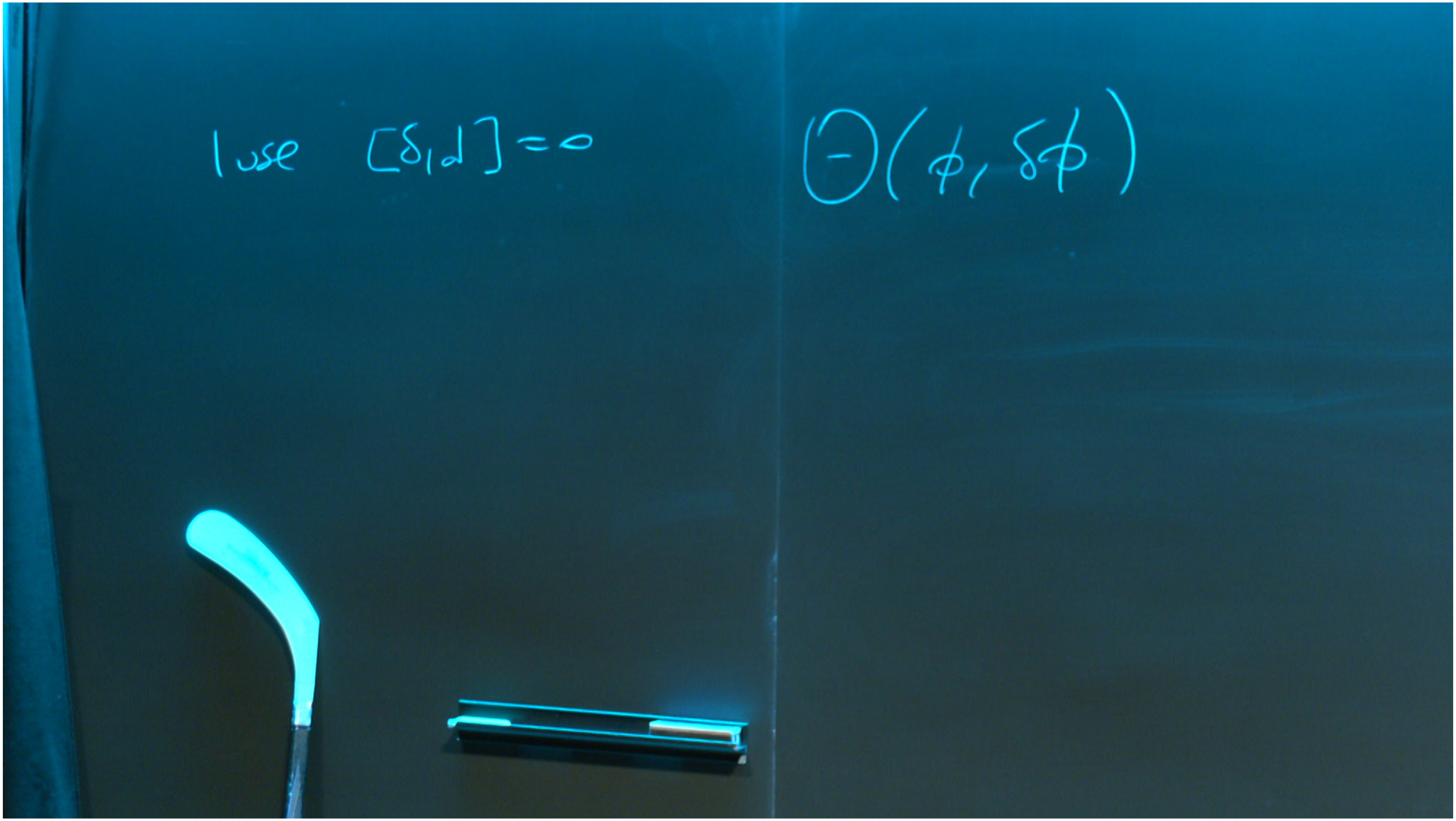
Anomalies

When Sachs '62 computed the transformation of the asymptotic shear under a BMS transformation, it was pretty clear that it was not a diffeomorphism: $\delta_\xi \sigma \neq \pounds_\xi \sigma$

In the modern CPS construction, this discrepancy is understood by the presence of background fields breaking covariance;

- in the BMS example, the conformal factor Ω;
- if spacetime has a boundary at finite distance, the field Φ localising the boundary

Notation I will use for the CPS: $(x^\mu,\ d,\ i,\ \pounds_\xi;\quad g,\ \delta,\ I,\ \delta_\xi)$


I use $[\delta, d] = 0$
$\Theta(\phi, \delta\phi)$

$$\Theta(\phi, \delta\phi) \longrightarrow \delta\phi \text{ 1 form} \Rightarrow \alpha = \alpha_\mu dx^\mu = \alpha(x; dx^\mu)$$

$\to \delta\phi$ 1 form $\Rightarrow$ $\alpha = \alpha_\mu dx^\mu = \alpha(x; dx^\mu)$

$\to \delta\phi$ vectorfield $\Rightarrow$ $\omega(\delta_1 ; \delta_2) = \delta_1 \Theta(\delta_2) - \delta_2 \Theta(\delta_1) - \Theta([\delta_1, \delta_2])$

$] = 0$

$$\Theta(\phi, \delta\phi) \begin{cases} \to \delta\phi \text{ 1 form } \Rightarrow \alpha = \alpha_\mu\, dx \\ \to \delta\phi \text{ vectorfield } \Rightarrow \omega(\delta_1, \delta \end{cases}$$

$d \to \delta \;;\; \Theta \qquad \omega = \delta\Theta$

$= 0$

$\Theta(\phi, \delta\phi)$ $\to$ $\delta\phi$ vector field $\Rightarrow$ $\omega(\delta_1, \delta_2)$

$d \leftarrow \delta \,,\quad \Theta \qquad \omega = \delta\Theta$

$$\hat{X}_\xi = \int dx\, \mathcal{L}_\xi \phi \frac{\delta}{\delta\phi(x)} \qquad I_{\hat{X}_\xi} = I_\xi$$

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- dynamical fields: $g, \qquad \delta_\xi g = \pounds_\xi g$
- background fields: $\Phi, \qquad \delta_\xi \Phi = 0 \neq \pounds_\xi \Phi$

Then for an arbitrary functional $\delta_\xi F(g,\Phi) = \partial_g F \delta_\xi g + \partial_\Phi \delta_\xi \Phi = \partial_g F \pounds_\xi g - \pounds_\xi F - \partial_\Phi F \pounds_\xi \Phi$

$$\Delta_\xi F = (\delta_\xi - \pounds_\xi) F = \partial_\Phi F \pounds_\xi \Phi$$

If it is a form in field-space $\delta_\xi [F(g,\Phi)\delta g] = \partial_g F \pounds_\xi g \delta g + F \delta \pounds_\xi g = \pounds_\xi [F\delta g] - \partial_\Phi F \pounds_\xi \Phi \delta g + F \pounds_{\delta \xi} g$

⇨ Anomaly operator (Speranza '18) $\boxed{\Delta_\xi := \delta_\xi - \pounds_\xi - I_{\delta\xi}}$

$$\Theta(\phi, \delta\phi) \quad \begin{cases} \delta\phi \text{ 1 form} \Rightarrow \alpha = \alpha_\mu dx^\mu = \alpha(x; dx \\ \delta\phi \text{ vectorfield} \Rightarrow \omega(\delta_1; \delta_2) = \delta_1 \Theta \end{cases}$$

$$\delta_\xi = I_\xi \delta + \delta I_\xi \; ; \quad \delta_\xi F = \partial_g F \underbrace{\delta_\xi g}_{\mathcal{L}_\xi g} + \partial_\Phi F \underbrace{\delta_\xi \Phi}_{=0} =$$

$$\hat{X}_\xi = \int dx\, \mathcal{L}_\xi \phi \frac{\delta}{\delta\phi(x)}$$

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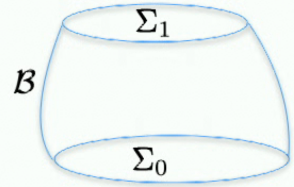
$$I_{\hat{\xi}} = I_\xi \qquad (\delta_\xi - \mathcal{L}_\xi) F = - \partial_\Phi F \, \mathcal{L}_\xi \Phi$$

Anomalies from a boundary

Consider a spacetime with a boundary $\mathcal{B}$ characterized by a Cartesian equation $\Phi(x^\mu)=0$

and normal 1-form $n_\mu = -f\partial_\mu\Phi$



Residual diffeomorphisms: the possible symmetries of the CPS associated with this boundary must be tangential to it in order to preserve it:

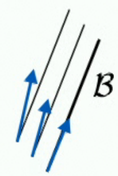
$$\xi\cdot n \stackrel{\mathcal{B}}{=} 0 \quad \Rightarrow \quad \xi^\mu = \bar{\xi}^\mu + \Phi\hat{\xi}^\mu, \quad \bar{\xi}^\Phi = 0$$

Because Φ is a fixed background structure in the CPS, we have an anomaly: $\Delta_\xi := \delta_\xi - \pounds_\xi - I_{\delta\xi}$

$$\Delta_\xi n_\mu = (\Delta_\mu \ln f - \hat{\xi}^\Phi)n_\mu$$

- E.g. For a gradient, f=1, the anomaly comes entirely from $\hat{\xi}^\Phi$

diffeos tangent at $\mathcal{B}$ but not at the other leaves of the Φ foliations



- There is one choice for which the anomaly vanishes: pick $f=f(g)$ such that $n^2=\pm 1$

So what is the anomaly really capturing? the *foliation-dependence,* which breaks covariance;
and it is possible to remove anomalies working with the foliation-independent choice of unit-norm
⇨ we can anticipate that anomalies will be prominent on null boundaries

Anomalies on a null boundary

On an *arbitrary* null boundary, there is no preferred choice of f $\qquad l_\mu = -f\partial_\mu \Phi$

any quantity explicitly dependent on f will be anomalous, namely non-covariant;
e.g., the inaffinity k of the normal

(Chandrasekaran-Speranza '19)

- There is no choice of normal for which the anomaly vanishes $\qquad \Delta_\xi l_\mu = w_\xi l_\mu$

l-dep. quantities will inherit the anomaly $\qquad \Delta_\xi \theta_{(l)} = w_\xi \theta_{(l)} \quad \Delta_\xi \epsilon_{\mathcal{N}} = -w_\xi \epsilon_{\mathcal{N}}$

Nonetheless, the pull-back of the EH symplectic potential is not anomalous:
(Parattu et al '15, Myers et al '16, Hopfmuller-Freidel '16, Oliveri-Speziale '18, ...)

$$\Theta = \int_{\mathcal{N}} \left[\left(\sigma^{\mu\nu} - \frac{1}{2}\left(\theta + 2k\right)\gamma^{\mu\nu}\right)\delta\gamma_{\mu\nu} + 2(\eta_\mu - \theta n_\mu)\delta l^\mu + \frac{1}{2}\partial_n l^2 n^\mu \delta l_\mu \right]\epsilon_{\mathcal{N}} - \delta\ell^{\mathrm{D}} + \int_{\partial\mathcal{N}} \vartheta^{\mathrm{EH}}$$

where $\ell^{\mathrm{D}} = -2(\theta_{(l)} + k_{(l)})\epsilon_{\mathcal{N}}$ $\qquad$ One can check that : $\Delta_\xi \Theta = 0$

But any individual term in general **not** covariant:
(as observed for instance in Myers et al 16, Chandrasekaran-Speranza '19)

- the boundary Lagrangian by itself is not covariant
- the Dirichlet flux here identified is not covariant
  (unless we do restrictions on the variations, we will come back to this)

CPS with field-dependent diffeos and anomalies

$$-I_\xi \omega = d(\delta q_\xi - i_\xi \theta) \;\Rightarrow\; -I_\xi \omega = d(\delta q_\xi - i_\xi \theta - q_{\delta\xi}) \;\Rightarrow\; -I_\xi \omega = d(\delta q_\xi - i_\xi \theta - q_{\delta\xi} - A_\xi)$$

Iyer-Wald '94, WZ '99

Barnich-Brandt '01

Barnich-Compere '05

Freidel-Oliveri-Pranzetti-SS '21

Chandrasekaran-Flanagan-Shehzad-Speranza '21

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Only assumption: the anomaly is a boundary term, in other words, there always exist a choice of covariant bulk Lagrangian $\quad \Delta_\xi L = da_\xi \quad \Leftrightarrow \quad L = L^c + d\ell, \quad \Delta_\xi \ell = a_\xi$

(interesting to go beyond this assumption, but not for this talk)

a: Lagrangian anomaly

It follows that $\Delta_\xi\theta = \delta a_\xi - a_{\delta\xi} + dA_\xi$

A: symplectic anomaly

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A: symplectic anomaly

The two anomalies enter respectively the Noether charge and canonical generator calculations,

$$j_\xi := I_\xi\theta - i_\xi L - a_\xi \mathrel{\hat=} dq_\xi$$

$$\not\delta h_\xi := -I_\xi\omega = \delta I_\xi\theta - \delta_\xi\theta = \delta I_\xi\theta - \pounds_\xi\theta - \Delta_\xi\theta - I_{\delta\xi}\theta$$
$$\mathrel{\hat=} d\left(\delta q_\xi - i_\xi\theta - q_{\delta\xi} - A_\xi\right)$$

Let us comment on the two roles separately

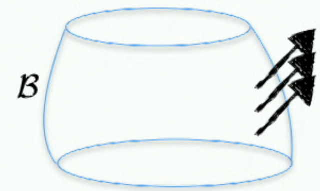
Flux of the Noether charge

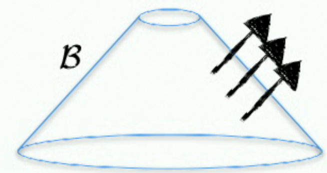
The Lagrangian anomaly enters the variation of the Noether charge

$$j_\xi := I_\xi \theta - \cancel{i_\xi L} - a_\xi \hat{=} dq_\xi \qquad \Delta_\xi L = da_\xi \qquad \Leftrightarrow \qquad L = L^c + d\ell, \quad \Delta_\xi \ell = a_\xi$$

Consider a lateral boundary B (can be time-like or null)
Restricting to tangent diffeos that preserve the boundary,
the second term vanishes in the pull-back
Then, two contributions to the flux:

- the symplectic flux
- the Lagrangian anomaly





The anomaly introduces a contribution to the flux which is ``polluted'' by non-geometric quantities
Clarifying the meaning of the anomaly contribution to the flux is a goal of this talk
We will see below an example of this: the soft terms, where the non-geometric ``pollution'' is their dependence on ST

Non-integrability and charge prescriptions

The symplectic anomaly contributes to the obstruction to integrability

$$\not\delta h_\xi := -I_\xi \omega \,\hat{=}\, d\left(\delta q_\xi - i_\xi \theta - q_{\delta\xi} - A_\xi\right) \longrightarrow h_\xi$$

Examples of different prescriptions in the literature:

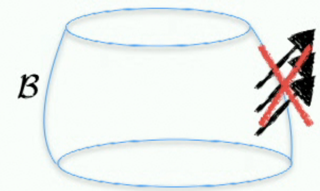
1. Improved Noether charge
   (Iyer-Wald '95, Harlow '19, Freidel-Geiller-Pranzetti '20, Margalef-Villasenor '20, Freidel-Oliveri-Pranzetti-SS '21, Chandrasekaran-Flanagan-Shehzad-Speranza '21, …)
2. Wald-Zoupas
   (Wald-Zoupas '99, Chandrasekaran-Flanagan-Prabhu '18, Ashtekar-Khera-Kolanowski-Lewandowski '21, …)
3. Path-independence in field space
   (Barnich-Brandt, Troessaert, Henneaux, Compere…)
4. Slicing
   (Barnich-Troessaert, Grumiller, Sheikh-Jabbari, Zwikel, Geiller, Adami…)
5. Other requirements… e.g. taking Ψ_1 instead of DS as angular momentum
   (Strominger et al '15, Compere-Fiorucci-Ruzziconi '20, Freidel-Pranzetti-Raclariu '21,…)
6. Extending the phase space, or adding a symmetric part to the symplectic form
   (Ciambelli-Leigh '21, Freidel '21, Wieland '22)

Improved Noether charge prescription

The idea is to work with a symplectic potential that **vanishes** when **conservative boundary conditions** are imposed: that way the system is made Hamiltonian and the charges are integrable

To achieve that, we take the pull-back on B of whatever ϑ we are starting with, and decompose it as follows:

$$\boxed{\underset{\leftarrow}{\theta} = \theta' - \delta\ell + d\vartheta}$$



where $\theta' = p\delta q$ for some choice of polarization of the phase space

That way, the new symplectic flux will vanish if:

- $\theta'|_{\delta q=0} \stackrel{\mathcal{B}}{=} 0$ restricting the variations throughout the phase space:
  ⇨ useful for ***conservative*** boundary conditions
- $\theta'|_{p=0} \stackrel{\mathcal{B}}{=} 0$ no flux for *arbitrary* variations
  around special `stationary' configurations in the phase space
  ⇨useful for ***leaky*** boundary conditions

(both options can and have been used in the literature)

Improved Noether charge prescription

By changing the symplectic potential to $\theta' = \theta + \delta\ell - d\vartheta$

we obtain a new Noether charge $j'_\xi := I_\xi\theta' - i_\xi L' - a'_\xi \mathrel{\hat{=}} dq'_\xi$

This turns out to be related to the initial one by :

$$\boxed{q'_\xi = q_\xi + i_\xi\ell - I_\xi\vartheta}$$

improved Noether charge

Why improved?

- If ϑ was the bare potential of the EH action and ℓ is GHY, then the result turns the Komar formulas into the Brown-York formulas, and this fixes the factors of two mismatch in the masses at spatial and null infinity

Cohomological ambiguities

Hoping that I am doing well with time, it is useful to pause for a second here in order to clarify the relation between two seemingly opposite approaches that can be found in the literature:

Freidel-Oliveri-Pranzetti-SS ’21
Chandrasekaran-Flanagan-Shehzad-Speranza ’21

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The difference has to do with the way the cohomological ambiguities are handled:

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The difference has to do with the way the cohomological ambiguities are handled:

To have a **unique** charge:

- FOPS2: Use a prescription to fix the cohomology ambiguities once and for all
  (e.g. Anderson's homotopy operator, or just hand-pick the `bare' potentials $d\theta \mapsto \theta$)
  Then,
  $$(L,\ell) \mapsto !(\theta,\theta',\vartheta)$$
- CFSS: Fix θ' via a choice of boundary conditions, **plus** fix the corner ambiguity by prescribing ℓ
  Then,
  $$!(\theta',\ell), \quad (\theta,\vartheta)$$ arbitrary but ambiguity affects only *q* and not *q'*

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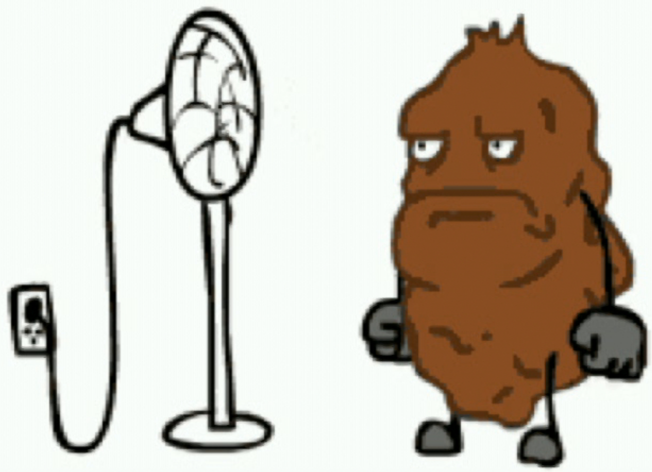
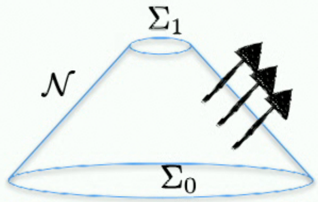


My viewpoint: it does … $+ i_\xi \ell - I_\xi \vartheta$

A notational advantages of the homotopy procedure however is that the formula is completely symmetric

Barnich-Troessaert bracket for the charges

Suppose we fix a prescription for the charges:



$$-I_\xi \omega = \not{\delta} h_\xi = \delta q_\xi - \mathcal{F}_\xi$$

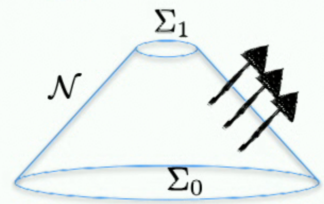
integrable piece   flux term

now we can ask: Is the algebra of charges correctly represented?

Recall that in standard situations, $\quad -I_\chi I_\xi \omega = \delta_\chi h_\xi = \{h_\chi, h_\xi\} \equiv h_{[\chi,\xi]}$

But in the presence of flux, this equation fails

Barnich-Troessaert bracket for the charges



Suppose we fix a prescription for the charges:

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- Barnich-Troessaert '11:

  new prescription for the bracket,

$$\{q_\chi, q_\xi\} := \delta_\chi q_\xi + I_\xi \mathcal{F}_\chi \neq -I_\chi I_\xi \omega$$

Cocycle and anomalies

$$\{q_\chi, q_\xi\} := \delta_\chi q_\xi + I_\xi \mathcal{F}_\chi \equiv q_{[\chi,\xi]} + K_{(\chi,\xi)}$$

Freidel-Oliveri-Pranzetti-S '21: we can compute K for an arbitrary theory, verify it satisfies Jacobi

$$K_{\xi,\chi} := i_\xi i_\chi L + i_\xi a_\chi - i_\chi a_\xi$$

Cocycle and anomalies

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Freidel-Oliveri-Pranzetti-S '21: we can compute *K* for an arbitrary theory, verify it satisfies Jacobi

$$K_{\xi,\chi} := i_\xi i_\chi L + i_\xi a_\chi - i_\chi a_\xi$$

previously identified in Speranza '17 (see also Chandra-Speranza '20, Chandra-Flanagan-Shehzad-Speranza '22:)

Remarks

- The cocycle can not be reabsorbed in the definition of the bracket if desired, so to have

$$\{h_\chi, h_\xi\}^L \equiv h_{[\chi,\xi]}$$

- Significantly, there exist an **off-shell** version of this formula, given by $\{q_\chi, q_\xi\}^L \equiv q_{[\chi,\xi]} + i_\xi C_\chi$

  ⇨ imposing the closure of the algebra implies (projections of) the Einstein's equations

  → **the larger the algebra, the more equations can be obtained**

  This reversed logic provides an independent motivation to look for enlargements of the asymptotic symmetry group (See Laurent's talk)

- More recent developments relating these brackets to Poisson brackets

(Ciambelli-Leigh '22, Freidel '22, Wieland '22; Chandrasekaran-Flanagan-Shehrad-Speranza '22)

Wald-Zoupas prescription

Let's look again at the formula for the Hamiltonian generators,

$$\not\delta h_\xi := -I_\xi \omega \,\hat{=}\, d\left(\delta q_\xi - i_\xi \theta - q_{\delta\xi} - A_\xi\right)$$

Situations where integrability occurs discussed in the WZ paper:

WZ's Case I: $i_\xi \theta + q_{\delta\xi} + A_\xi = \delta X$

e.g. spatial infinity

WZ's Case II: $i_\xi \theta + q_{\delta\xi} + A_\xi = i_\xi \bar{\theta} + \delta X$

e.g. null infinity

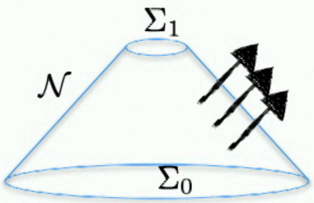
where $i_\xi \bar{\theta}$ is some **physically recognizable and unambiguous flux,** then we can shove it on the left-hand side

If X is not zero, we have a shift wrt Noether charge

As we already remarked, such a shift may actually welcomed, since e.g. starting from the EH Lagrangian the bare Noether charge would be given by the Komar formula which has well-known shortcomings, such as giving wrong factors of 2 at both spatial and future null infinity

The WZ prescription: flux

Case II. To identify the preferred potential $\bar{\theta}$, proceed as follows: start from the pull-back of the symplectic 2-form on the boundary, and require



0. $\underset{\leftarrow}{\omega} = \delta\bar{\theta} \quad \Rightarrow \quad \bar{\theta} = \underset{\leftarrow}{\theta} + \delta b$

1. it must be a local and covariant functional of the dynamical fields and background structure
2. is must vanish for arbitrary perturbations around stationary solutions
3. additional requirements that may be needed in case the first two requirements are not enough to select a single preferred one

Remarks:

- Concerning θ: You can pick it with the homotopy, you can pick it at random; it doesn't matter
  - If you pick it a la FOPS, then the prescription is: use the freedom to change the boundary Lagrangian to identify the physical flux
  - If you pick it random, then the prescription is: use the ambiguities to change it
- Concerning 3: requirements 1 and 2 are enough for the cases studied so far
- Starting from the EH action and its `bare' θ, one generically gets a non-zero *b*

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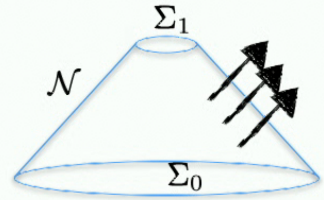


0. $\underset{\leftarrow}{\omega} = \delta\bar{\theta}$ ⇨ $\bar{\theta} = \underset{\leftarrow}{\theta} + \delta b$

1. it must be a local and covariant functional of the dynamical fields and background structure
2. **is must vanish for arbitrary perturbations around stationary solutions**
3. additional requirements that may be needed in case the first two requirements are not enough to select a single preferred one

Notice that there is no discussion of boundary conditions nor of boundary Lagrangian; so this is not an improved Noether charge a priori.

However, let us zoom in on condition 2:

⇨ It means that it has to be in the form $\bar{\theta} = p\delta q$ where $p(g_{\text{stationary}}) = 0$

This suggest that we can use this prescription to define an improved Noether charge with :

$$\theta' = \bar{\theta} \qquad (\ell, \vartheta) = (b + dc, 0 + \delta c)$$

But let us see first what is the prescription given by WZ for the charge

The WZ prescription: charges

Since we have identified the physical flux through $\bar{\theta} = \underset{\leftarrow}{\theta} + \delta b$,
that is the only quantity that should be subtracted in order to obtain an integrable charge:

$$\boxed{\not{\delta} \underset{\leftarrow}{\not{d}} q_\xi^{\mathrm{WZ}} := - I_\xi \omega + d i_\xi \bar{\theta}}$$

Comparing this to the general formula : $\not{\delta} h_\xi := -I_\xi \omega \,\hat{=}\, d\left(\delta q_\xi - \underline{i_\xi \theta} - \underline{q_{\delta\xi} - A_\xi}\right)$

- we see that we are shoving the flux on the LHS;
- we can already anticipate a potential discrepancy if it happens that $q_{\delta\xi} + A_\xi = \delta X \neq 0$

This potential discrepancy turns out to be exactly the anomaly of *b*

To prove this, we need to go into some details, starting with a bit of archeology:

do WZ allow for anomalies and $\delta\xi \neq 0$ *?*

WZ covariance and soft anomalies

case of null infinity, $\boldsymbol{\Theta}$ is required to be conformally invariant). Our proposal is the following: Let $\mathcal{H}_\xi$ satisfy[12]

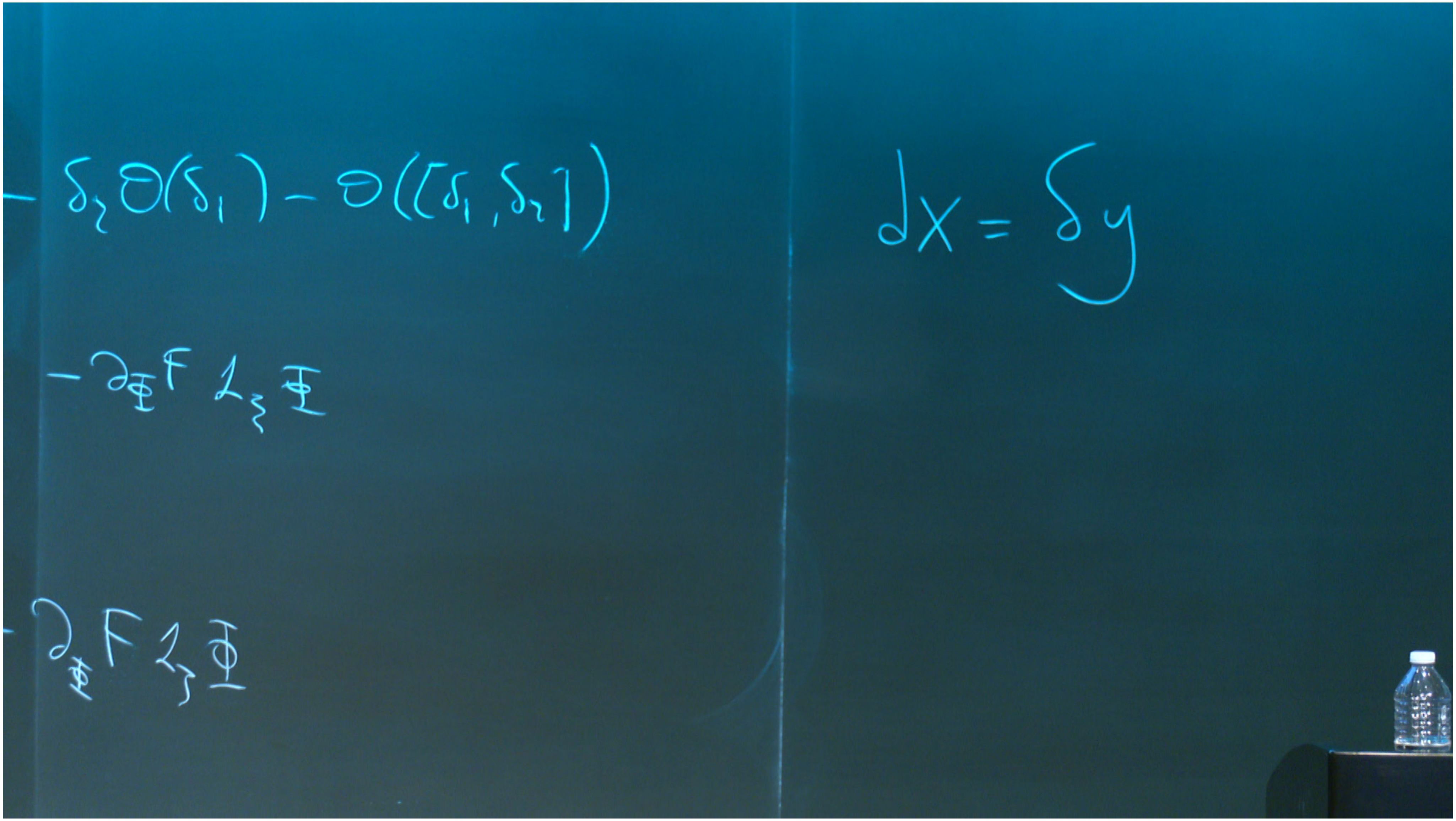
$$\delta \mathcal{H}_\xi = \int_{\partial\Sigma} [\delta \mathbf{Q} - \xi \cdot \theta] + \int_{\partial\Sigma} \xi \cdot \boldsymbol{\Theta} \qquad (26)$$

[9]More precisely, by "locally constructed" we mean the following: Suppose that $\chi : M \cup \mathcal{B} \to M \cup \mathcal{B}$ is a diffeomorphism which preserves the universal background structure. Suppose $(\phi, \delta\phi)$ and $(\phi', \delta\phi')$ are such that there exists an open (in $M \cup \mathcal{B}$) neighborhood, $\mathcal{O}$, of $p \in \mathcal{B}$ such that for all $x \in M \cap \mathcal{O}$ we have $\phi = \chi_* \phi'$ and $\delta\phi = \chi_* \delta\phi'$, where χ_* denotes the pullback map on tensor fields associated with the diffeomorphism χ. Then we require that at p we have $\boldsymbol{\Theta} = \chi_* \boldsymbol{\Theta}'$.

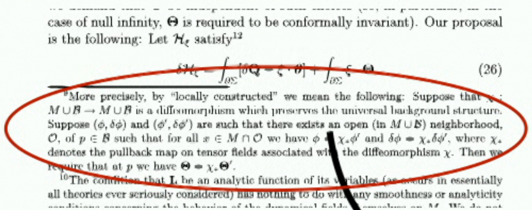
[10]The condition that $\mathbf{L}$ be an analytic function of its variables (as occurs in essentially all theories ever seriously considered) has nothing to do with any smoothness or analyticity

$$\Delta_\xi \bar{\theta} = 0$$

$$I_{\delta\xi} \bar{\theta} = 0$$


$-\delta_2\Theta(\delta_1) - \Theta([\delta_1, \delta_2])$
$dx = \delta y$
$-\partial_\Phi F \, \mathcal{L}_\xi \Phi$
$-\partial_\Phi F \, \mathcal{L}_\xi \Phi$

WZ covariance and soft anomalies



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$$\Delta_\xi\bar\theta = 0 \quad\Rightarrow\quad \Delta_\xi \underset{\leftarrow}{\theta} + \delta\Delta_\xi b - \Delta_{\delta\xi} b = \delta\bar a_\xi - \bar a_{\delta\xi} + dA_\xi = 0,$$

$$I_{\delta\xi}\bar\theta = 0 \quad\Rightarrow\quad \bar a_{\delta\xi} = -d\bar q_{\delta\xi}.$$

Putting together the two requirements, we find

$$\boxed{\delta\bar a_\xi = -d(\bar q_{\delta\xi} + A_\xi)}$$

1. WZ is defined also with a class of anomalies and field-dep. diffeos: *mild/soft anomalies*
2. When they are present, the prescription is to shift the Noether charge so to remove the anomalous flux:

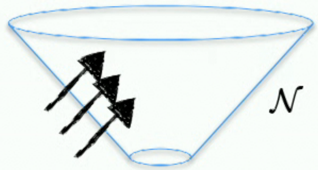
$$d\bar q_\xi = I_\xi\bar\theta - \bar a_\xi \qquad\longrightarrow\qquad dq^{\rm WZ}_\xi := d\bar q_\xi + \bar q_\xi = I_\xi\bar\theta$$

First example:
null hypersurface at finite distance

CFP covariant phase space on a null hypersurface

Chandrasekaran-Flanaga-Prabhu '18

There is an interesting story about the pull-back on null hypersurfaces, and various covariant phase spaces have been studied



$l_\mu = -f\partial_\mu\Phi$

For lack of time, I will just go straight to CFP and refer you to a paper we are posting very soon for a more general discussion

In CFP, we restrict $\delta l^\mu = \delta l_\mu = \delta k = 0$

then:

$$\underset{\leftarrow}{\theta} = \underbrace{\left(\sigma^{\mu\nu} - \frac{\theta_{(l)}}{2}\gamma^{\mu\nu}\right)\delta\gamma_{\mu\nu}\,\epsilon_{\mathcal{N}}}_{\text{candidate } \bar{\theta}} + 2\delta(\theta_{(l)}\epsilon_{\mathcal{N}})$$

We want to identify a **preferred** potential via $\bar{\theta} = \underset{\leftarrow}{\theta} + \delta b$, satisfying:

1. **Covariance** can be satisfied using the kinematical freedom to get rid of the inaffinity and of the spin-1 momentum : $\Delta_\xi\bar{\theta} = 0$

2. **Stationarity** satisfied by non-expanding horizons : $\sigma^{\mu\nu} = \theta_{(l)} = 0$

$\Rightarrow$ $b = -2\theta_{(l)}\epsilon_{\mathcal{N}} \;= -2d\epsilon_S$

The CFP anomalies and charges

Preferred WZ potential: $\bar{\theta} = \left(\sigma^{\mu\nu} - \frac{\theta_{(l)}}{2}\gamma^{\mu\nu}\right)\delta\gamma_{\mu\nu}\,\epsilon_{\mathcal{N}}$

Charge shift: $b = -2\theta_{(l)}\epsilon_{\mathcal{N}}$

Recall that $\Delta_\xi\theta_{(l)} = w_\xi\theta_{(l)}$ $\Delta_\xi\epsilon_{\mathcal{N}} = -w_\xi\epsilon_{\mathcal{N}}$ $\Rightarrow$ $\Delta_\xi b = 0.$

no anomaly shift

Furthermore, $\Delta_\xi\theta = 0$ hence $\bar{a}_\xi = 0$

We conclude that :

- no anomalous flux
- the WZ charge obtained by CFP is an improved Noether charge with $\ell = b, \;\; \vartheta = 0$

In particular, no anomaly contribution to the i-Noether fluxes $d\bar{q}_\xi = I_\xi\bar{\theta}$

Comments

On the importance of the covariance requirement:
without it we can for instance keep k, and then the charges will depend on it

$$\bar{\theta} = \Big(\sigma^{\mu\nu} - \frac{\theta_{(l)}}{2}\gamma^{\mu\nu}\Big)\delta\gamma_{\mu\nu}\,\epsilon_{\mathcal{N}} \qquad \longrightarrow \qquad \bar{\theta} = \Big(\sigma^{\mu\nu} - \frac{\theta_{(l)} + 2k_{(l)}}{2}\gamma^{\mu\nu}\Big)\delta\gamma_{\mu\nu}\,\epsilon_{\mathcal{N}}$$

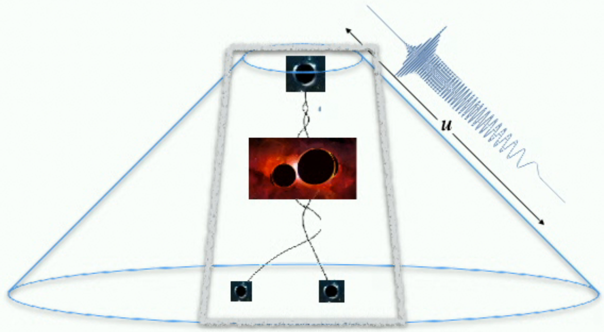
$$\Delta_\xi b \neq 0$$

(the problem here is that the inaffinity does not capture properties of the geometry of the hypersurface, but depends on non-geometric choices such as the scaling and the extension

On the importance of the stationarity requirement:

It allows us to get charges that automatically preserved on non-expanding horizons, such as the area, and all multipole moments (AKKL '21)

on the other hand, such charges are not great beyond the NEH case;
for instance, the area changes on a null cone in Minkowski, even though there is no physical flux;
one may then be interested in an alternative definition that gives conserved charges in this case
This can be obtained playing with the boundary conditions (with Odak and Rignon-ret, to appear)



Second example: future null infinity

Disclaimer: We *already know* that the WZ charges at null infinity are Noether charges for a specific boundary Lagrangian (BMSW '21)

Here I want to show how they can be *derived* as an improved Noether charge, and how this requires discussing anomalies

Scri symplectic potential

Pull-back of the EH symplectic potential at Scri, in Bondi coordinates:

$$\underset{\leftarrow}{\theta} = -\Big(2\delta M - \frac{1}{2}\delta(D_A D_B C^{AB}) + \frac{1}{2}N_{AB}\delta C^{AB} - \frac{1}{8}\delta(N_{AB}C^{AB})\Big)\epsilon_{\mathcal{I}}$$

To identify the **preferred** symplectic potential:

1. **covariance**
2. **stationarity**

At first sight, we could identify it as $N\delta C$: it vanishes for all perturbations are stationary spacetimes, thus 2 is satisfied. But 1 ? $\Delta_\xi(N\delta C\epsilon_{\mathcal{I}}) = 0?$

To answer that, we have to look at the anomalies at Scri.

Anomalies and Geroch's news

Residual diffeomorphisms:

$\xi = \tau\partial_u + Y^A\partial_A \qquad \tau = T + uW$

Starting from the general framework of BMSW,

the transformations on the phase space are:

$$\begin{aligned}
\delta_\xi\, \bar{q}_{AB} &= (\pounds_Y - 2\dot{\tau})\bar{q}_{AB}, \\
\delta_\xi\, C_{AB} &= (\tau\partial_u + \pounds_Y - \dot{\tau})C_{AB} - 2\bar{D}_{\langle A}\partial_{B\rangle}\tau, \\
\delta_\xi\, N_{AB} &= (\tau\partial_u + \pounds_Y)N_{AB} - 2\bar{D}_{\langle A}\partial_{B\rangle}\dot{\tau},
\end{aligned}$$

These quantities live on the bundle $\mathbb{R}\times S^2$, whose Lie derivative is $L_\xi = \tau\partial_u + \pounds_Y$

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\quad \Rightarrow \quad
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\Delta_{\bar{\xi}}\, \bar{q}_{AB} &= -2\dot{\tau}\bar{q}_{AB}, \\
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Remark: the anomalous transformations are identical for the BMS case.

Let's focus on the BMS case from now on

$\alpha(x; dx^\mu)$

$$\mathcal{L}_\xi g_{\mu\nu} = \delta_\xi g_{\mu\nu} = \delta_\xi\left[\Omega^2 \tilde{g}_{\mu\nu}\right]$$

$$= \delta_1 \Theta(\delta_2) - \delta_2 \Theta(\delta_1) - \Theta([\delta_1, \delta_2])$$

$$dx = \delta y$$

$$\delta_\xi \Phi = \mathcal{L}_\xi F - \partial_\Phi F \, \mathcal{L}_\xi \Phi$$

$$= 0$$

$$C_{AB} = f(g_{\mu\nu}, \Omega)$$

$$- \mathcal{L}_\xi) F = - \partial_\Phi F \, \mathcal{L}_\xi \Phi$$

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- The **metric anomaly** is well known:
  in BMS the Bondi frames are fixed, so $\delta\bar q_{AB} = \delta_\xi \bar q_{AB} = 0$
  but the (boosts of the) BMS group act on it, $\varphi_\xi(\bar q_{AB}) = e^{-2\dot\tau}\bar q_{AB}$
- The **News anomaly** is also well known: as shown by Geroch '77, one can define the Geroch news as (in this conference often simply called *covariant* News)

$$\hat N_{AB} = \mathcal{N}_{AB} := \boxed{N_{AB} - \rho_{\langle AB\rangle}} \qquad \Delta_\xi(N-\rho) = 0$$

Anomalies and Geroch's news

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to be fair, I think that the better notation would be simply $\hat{N}\mapsto N, N\mapsto\partial_u C$ See Marios' talk

Preferred symplectic potential at Scri

Pull-back of the EH symplectic potential at Scri, in Bondi coordinates:

$$\underset{\leftarrow}{\theta} = -\Big(2\delta M - \frac{1}{2}\delta(D_A D_B C^{AB}) + \frac{1}{2}N_{AB}\delta C^{AB} - \frac{1}{8}\delta(N_{AB}C^{AB})\Big)\epsilon_{\mathcal{I}}$$

Identify the **preferred** symplectic potential:

1. **covariance**
2. **stationarity**

At first sight, we could identify it as NδC : it vanishes for all perturbations are stationary spacetimes, thus 2 is satisfied. But 1 ? $\Delta_\xi(N\delta C\epsilon_{\mathcal{I}}) = 0?$

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At first sight, we could identify it as $N\delta C$: it vanishes for all perturbations are stationary spacetimes, thus 2 is satisfied. But 1 ? $\Delta_\xi(N\delta C\epsilon_{\mathcal{I}}) = 0$?

The answer is no! this candidate preferred potential is anomalous:

$$\Delta_\xi(N\delta C\epsilon_{\mathcal{I}}) = -(2DD\dot{\tau})\delta C\epsilon_{\mathcal{I}}$$

To solve this, add and subtract Geroch's tensor: then, $\Delta_\xi\big((N-\rho)\delta C\epsilon_{\mathcal{I}}\big) = 0$

As a result, $b = \Big(2M + D_A\bar{U}^A - \frac{1}{8}N_{AB}C^{AB} + \frac{1}{2}\rho_{AB}C^{AB}\Big)\epsilon_{\mathcal{I}}$

$16\pi G = 1$

Preferred symplectic potential at Scri

$$b = \left(2M + D_A \bar{U}^A - \frac{1}{8} N_{AB} C^{AB} + \frac{1}{2} \rho_{AB} C^{AB}\right) \epsilon_{\mathcal{I}}$$

Recall that the (trace-less part of the) Geroch tensor vanishes in Bondi frames: $\rho_{\langle AB \rangle} = 0$

So one may think the shift is irrelevant; **but it is not, this shift is crucial to get the right charges!**
(just like $f(x_0) = 0, \;\; f'(x_0) \neq 0$)

Dray-Streubel's

Explicitly, $\bar{q}_\xi := q_\xi + i_\xi b = \tau(4M - \frac{1}{4} DDC) + 2Y^A \bar{P}_A$

so this i-Noether charge satisfies $\quad d\bar{q}_\xi = I_\xi \bar{\theta} - \bar{a}_\xi$

But as explained earlier, satisfying the WZ requirements guarantees that

$$\bar{a}_\xi = \Delta_\xi b = ds_\xi$$

so we can shove the anomaly in the definition of the charge and get the WZ flux.

Preferred symplectic potential at Scri

$$q^{\mathrm{WZ}} = q_\xi + i_\xi b + s_\xi \qquad \Delta_\xi b = ds_\xi \qquad s_\xi = \frac{1}{4}\tau DDC$$

We conclude that :

- there are soft anomalies at Scri
- the `naive' improved Noether charge based on $\ell = b, \quad \vartheta = 0$ is not the WZ charge, and would have a flux that includes anomalies
- The difference is a soft term; working with the naive Noether charge we would get different numerical factors in the memory effects
- In other words, the WZ prescription uniquely selects the corner Lagrangian

$$\ell = b + dc \qquad c = -\frac{1}{16}C^2$$

Further remarks:

Same conclusions were already reached in Chandrasekaran-Flanagan-Shehzad-Speranza '21
our contribution is a more explicit analysis in terms of anomalies

Caveat: this corner term is slightly different from the analysis done in BMSW '21,
because there we used the tetrad formulation, and it is known that the two differ by corner terms
(De Paoli-S '18, Oliveri-S '19)

A historical remark

Of course, WZ computed the charges without ever talking about anomalies

What we have shown here is:

- how the WZ result can be framed in the current language of CPS with anomalies
- how in return this allows to understand the role that anomalies have in the construction of charges

However, we can also make a useful historical remark

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Let's go back to the general form of a mild/soft anomaly:

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In the BMS case, $A_\xi = 0$ hence $s_\xi = -\bar{q}_{\delta\xi}$

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So even if WZ could get away without ever mentioning explicitly anomalies, they should have at least talked about field-dependent diffeos… **and in fact they do**, claiming (albeit w/o detailed proof) that the field-dependence of the Geroch-Winicour extension integrates to zero

- WZ '99 They give an ok indirect argument using Geroch-Winicour extension
- BT '11 Explicit consistent calculation, no cheating!
- FN '15 They use the Winicour-Tamburino extension and give a wrong argument
- GPS '21 They use the Winicour-Tamburino extension and give no argument

Scri as a NEH in the unphysical spacetime

Because Scri is **very different** from a null surface in spacetime:
it is a null surface in the **unphysical** spacetime
the conformal compactification involved changes the nature of the background structure

- Φ : location of the boundary
- Ω : location of the boundary *and* conformal factor

Specifically, the universal structure is

- Φ : $[l = \omega l]$
- Ω : $[(q, l) = (\omega^{-2} q, \omega l)]$

These qualitative considerations can be made very precise, and one can show that starting from an abstract spacetime and write general formulas for the fluxes and the charges;
when specified to these different universal structures it reproduces the two different settings.
(Ashtekar and S, *Scri as a NEH*, to appear)
In particular, the extra structure made available by the conformal factor ***eliminates the dilatation*** *from the symmetry group, and thus the area from the charges (in standard BMS)*

Along these lines, I learned from Luca that the Carrollian framework also allows to treat general $[(q, l) = (\omega^n q, \omega l)]$ and thus encompass both Scri and finite surfaces as two cases of a unique framework, which I think it is very nice and shows convergence

References

For more details:

Gloria Odak, Antoine Rignon-Bret and S,
Revisiting the WZ prescription: soft terms as anomalies, to appear

Gloria Odak, Antoine Rignon-Bret and S,
Exploring alternative boundary conditions on null hypersurfaces, to appear