Title: Carrollian spaces at infinity: an embedding space picture

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Abstract: In this talk, I will discuss certain homogeneous spaces of the Poincaré group that correspond to the well-known asymptotic regions of asymptotically flat spacetimes: time-, space-, and light-like infinity. I will then show that all of these spaces admit a uniform description as surfaces embedded in a higher-dimensional space. I will conclude with some comments concerning the computation of correlators of conformal carrollian theories from this embedding space picture.

Carrollian Spaces at Infinity: An Embedding Space Picture

Jakob Salzer
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Quantum Gravity around the Corner, PI, Oct 3 - 7

Based on 2112.03319 w/ J. Figueroa O'Farrill, E. Have, S. Prohazka

and WIP w/ S. Prohazka

Introduction

- Holographic principle is our current key to understanding QG
- Best example : AdS/CFT
- How to generalize to asymptotically dS/ flat spaces?
- very basic feature of AdS/CFT : two spaces (of different dimension) w/ same symmetries $\Longrightarrow$ two homogeneous spaces for AdS group
- What is the corresponding situation for flat space? How many homogeneous spaces (of different dimension) are there?

Outline

Homogeneous Spaces of $ISO(3,1)$

Embedding Space Picture

Application: (Simplest) Correlators
of Carrollian CFTs

"Zeroth-order AdS/CFT"

AdS_4 as homog. space

$$AdS_4 \approx \frac{SO(3,2)}{\underbrace{SO(3,1)}} \iff \underbrace{(so(3,2), so(3,1))}_{\text{"Klein pair"}}$$

stabilizer subgroup: leaves points invariant

sanity check: $\dim(so(3,2)) - \dim(so(3,1)) = 10 - 6 = 4$ ✓

•) $\exists$ (low-dimensional) $so(3,1)$-invariant tensor fields on AdS_4?

Yes! $\exists$ (co-) metric w/ const. negative curvature.

•) Symmetries of this inv. structure?

$$\mathcal{L}_\xi g_{ab} = 0 \iff \xi \in so(3,2)$$

"Zeroth-order AdS/CFT"

	AdS_4	"3d-space"
Klein pair	$(\mathfrak{so}(3,2), \mathfrak{so}(3,1))$	$(\mathfrak{so}(3,2), \mathfrak{h})$ $\dim \mathfrak{h} = 7$ $(10-7=3)$
inv. structure	(co-)metric w/ negative curvature	
symmetries of inv. structure	$\mathcal{L}_\xi g_{ab} = 0 \quad \xi \in \mathfrak{so}(3,1)$	

"Zeroth-order AdS/CFT" [Patera, Sharp, Winternitz, Zassenhaus '76]

	AdS_4	conf. comp. $Mink_3$
Klein pair	$(\mathfrak{so}(3,2), \mathfrak{so}(3,1))$	$(\mathfrak{so}(3,2), \mathbb{R} \ltimes ISO(2,1))$ almost unique choice
inv. structure	(co-)metric w/ negative curvature	$\nexists$, but conformally inv. metric
symmetries of inv. structure	$\mathcal{L}_\xi g_{ab} = 0 \quad \xi \in \mathfrak{so}(3,2)$	$\mathcal{L}_\xi g_{ab} = 2\omega g_{ab} \Rightarrow \xi \in \mathfrak{so}(3,2)$

Homogeneous Spaces of ISO(3,1)

	4d Minkowski	$\mathscr{I}$
Klein pair	$(\mathfrak{iso}(3,1), \mathfrak{so}(3,1))$	$(\mathfrak{iso}(3,1), \underbrace{(\mathfrak{iso}(2) \oplus \mathbb{R}^3) \oplus \mathbb{R}}_{\text{unique* choice}})$
inv. structure	flat (co-) metric	$\nexists$, but conf. Carroll structure (n^a, q_{ab})
symmetries of inv. structure	$\mathcal{L}_\xi g_{ab} = 0$ $\xi \in \mathfrak{iso}(3,1)$	$\left.\begin{array}{l}\mathcal{L}_\xi n^a = \omega^{-1} n^a, \\ \mathcal{L}_\xi q_{ab} = 2\omega q_{ab}\end{array}\right\} \xi \in BMS$ * for effective action

Homogeneous Spaces of ISO(3,1)

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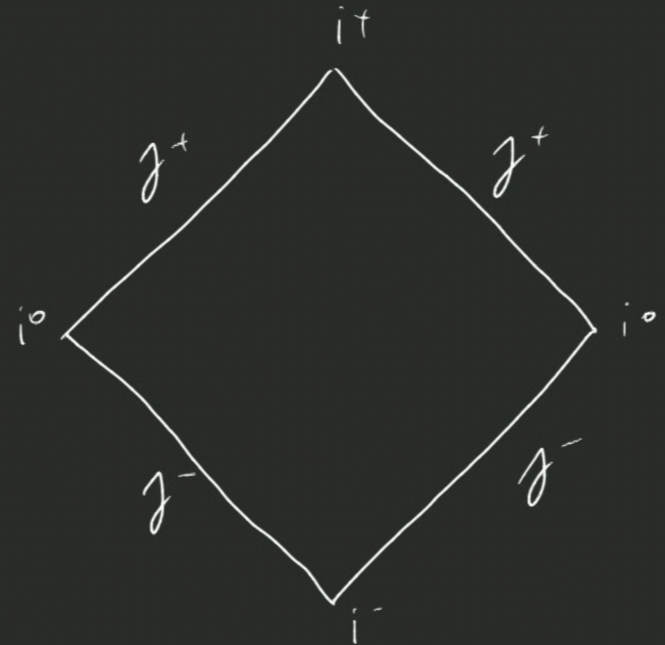
Homogeneous Spaces of $ISO(3,1)$



Where are the other asymptotic regions of Minkowski space?

Look at 4-dim. homogeneous spaces $\Longrightarrow$ subalgebras of $iso(3,1)$ of 6 dim.

Homogeneous Spaces of ISO(3,1)

?	
$(iso(3,1), iso(2,1))$ pseudo-Carrollian (q_{ab}, n^a) q_{ab}... lorentzian w/ pos. curvature $\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$ $\xi \in so(3,1) \ltimes f(dS_3)$	

Homogeneous Spaces of ISO(3,1)

Spi
$(\mathfrak{iso}(3,1), \mathfrak{iso}(2,1))$
pseudo-Carrollian (q_{ab}, n^a)
q_{ab} ... lorentzian w/ pos. curvature
$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$
$\xi \in \mathfrak{so}(3,1) \ltimes f(dS_3)$

Spi (Ashtekar, Hansen '78)

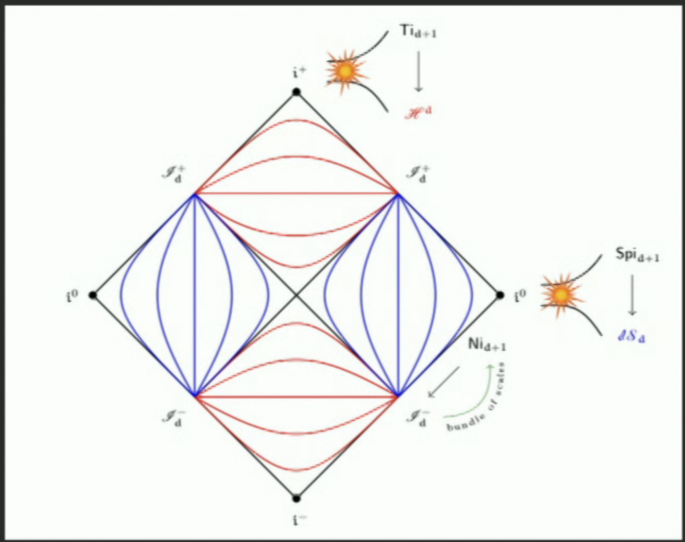
"blow-up" of spatial infinity

[see also Gibbons '19]

Homogeneous Spaces of $ISO(3,1)$

Spi	Ti = AdS-Carroll	Ni
$(iso(3,1), iso(2,1))$	$(iso(3,1), iso(3))$	$(iso(3,1), iso(2) \ltimes \mathbb{R}^3)$
pseudo-Carrollian	Carrollian	doubly-Carrollian
(q_{ab}, n^a)	(q_{ab}, n^a)	(q_{ab}, n^a, ℓ^b)
q_{ab}… Lorentzian	q_{ab}… Euclidean w/	$q_{ab} n^a = q_{ab} \ell^b = 0$
w/ pos. curvature	neg. curvature	q_{ab}… const. pos. curvature
$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$	$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$	$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = \mathcal{L}_\xi \ell^a = 0$
$\xi \in so(3,1) \ltimes f(dS_3)$	$\xi \in so(3,1) \ltimes f(\mathcal{H}_3)$	$\xi \in BMS$

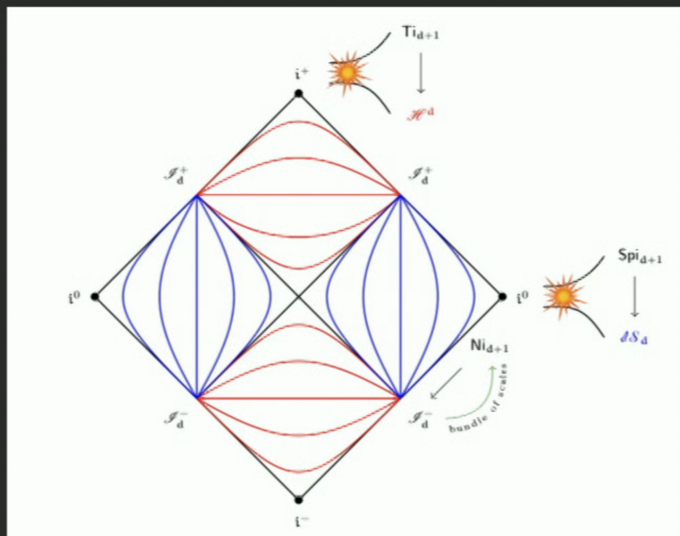
Homogeneous Spaces of ISO(3,1)



$$\begin{tikzcd}[column sep=small]
\mathbb{M} & \mathrm{Spi} \arrow[d] & \mathrm{Ti} \arrow[d] & & \mathrm{Ni} \arrow[dl] \arrow[dr] & & d+1 \\
& dS & \mathscr{H} & \mathscr{I} \arrow[dr] & & \mathscr{L} \arrow[dl] & d \\
& & & & \mathrm{CS} & & d-1
\end{tikzcd}$$

Spi	Ti = AdS-Carroll	Ni
$(iso(3,1), iso(2,1))$	$(iso(3,1), iso(3))$	$(iso(3,1), iso(2) \ltimes \mathbb{R}^3)$
pseudo-Carrollian (q_{ab}, n^a)	Carrollian (q_{ab}, n^a)	doubly-Carrollian (q_{ab}, n^a, ℓ^b)
q_{ab}… lorentzian w/ pos. curvature	q_{ab}… Euclidean w/ neg. curvature	$q_{ab} n^a = q_{ab} \ell^b = 0$ q_{ab}… const. pos. curvature
$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$ $\xi \in so(3,1) \ltimes f(dS_3)$	$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$ $\xi \in so(3,1) \ltimes f(\mathscr{H}_3)$	$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = \mathcal{L}_\xi \ell^a = 0$ $\xi \in BMS$

Homogeneous Spaces of ISO(3,1)



$$\begin{tikzcd}
\mathbb{M} & \mathrm{Spi} \arrow[d] & \mathrm{Ti} \arrow[d] & & \mathrm{Ni} \arrow[dl] \arrow[dr] & & d+1 \\
& d\mathcal{S} & \mathscr{H} & \mathscr{I} \arrow[dr] & & \mathscr{L} \arrow[dl] & d \\
& & & & \mathrm{CS} & & d-1
\end{tikzcd}$$

Spi	Ti = AdS-Carroll	Ni
$(iso(3,1), iso(2,1))$	$(iso(3,1), iso(3))$	$(iso(3,1), iso(2) \ltimes \mathbb{R}^3)$
pseudo-Carrollian (q_{ab}, n^a)	Carrollian (q_{ab}, n^a)	doubly-Carrollian (q_{ab}, n^a, ℓ^b)
q_{ab}... lorentzian w/ pos. curvature	q_{ab}... Euclidean w/ neg. curvature	$q_{ab} n^a = q_{ab} \ell^b = 0$ q_{ab}... const. pos. curvature
$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$ $\xi \in so(3,1) \ltimes f(dS_3)$	$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = 0$ $\xi \in so(3,1) \ltimes f(\mathscr{H}_3)$	$\mathcal{L}_\xi q_{ab} = \mathcal{L}_\xi n^a = \mathcal{L}_\xi \ell^a = 0$ $\xi \in BMS$

Outline

Homogeneous Spaces of ISO(3,1)

Embedding Space Picture

Application: (Simplest) Correlators of Carrollian CFTs

Embedding Space

Can we find a simple geometric description of Spi, T_i, N_i, $\mathcal{J}$?

$\mathbb{R}^{4,2}$ with $ds^2 = g_{IJ}\, dX^I dX^J = \eta_{\mu\nu}\, dx^\mu\, dx^\nu + 2\, dx^+\, dx^-$

$$X^I = \begin{pmatrix} x^\mu \\ x^+ \\ x^- \end{pmatrix} \qquad Q_\varepsilon := x_\mu x^\mu + 2x^+ x^- = \varepsilon$$

$$\left.\begin{array}{l} Q_{\rho^2} = dS_5 \text{ w/ signature } (3,2) \\ Q_0 = \text{"generalized Light-cone"} \\ Q_{-\rho^2} = AdS_5 \end{array}\right\} \text{preserved by } SO(4,2)$$

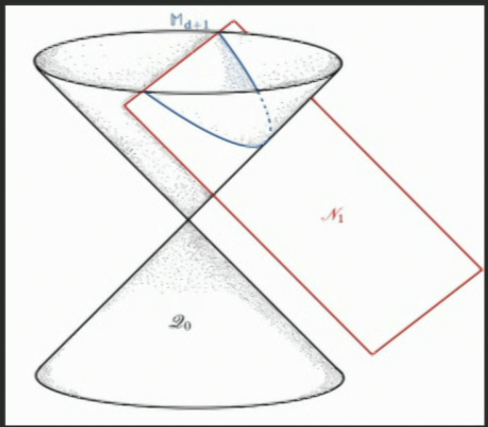
Embedding Space

break $SO(4,2) \longrightarrow ISO(3,1)$: introduce null surface

$\mathcal{N}_\sigma : X^- = \sigma$ and consider $\mathcal{N}_\sigma \cap Q_\varepsilon$

$\sigma \neq 0 : \mathcal{N}_\sigma \cap Q_\varepsilon \Rightarrow \mathrm{Mink}_4$

$$X' = \begin{pmatrix} x^\mu \\ \varepsilon/(2\sigma) \\ \sigma \end{pmatrix}$$



Embedding Space

$Q_{\rho^2} \cap \mathcal{N}_0 :\quad \mathrm{Spi} \qquad X'_{\mathrm{Spi}} = \begin{pmatrix} x^\mu \\ x^+ \\ 0 \end{pmatrix} \quad x_\mu x^\mu = \rho^2$

$Q_{-\rho^2} \cap \mathcal{N}_0 :\quad \mathrm{Ti} \qquad X'_{\mathrm{Ti}} = \begin{pmatrix} x^\mu \\ x^+ \\ 0 \end{pmatrix} \quad x_\mu x^\mu = -\rho^2$

$Q_0 \cap \mathcal{N}_0 :\quad \mathrm{Ni} \qquad X'_{\mathrm{Ni}} = \begin{pmatrix} x^\mu \\ x^+ \\ 0 \end{pmatrix} \quad x_\mu x^\mu = 0$

$(Q_0 \cap \mathcal{N}_0)/\mathbb{R}^+ :\quad \mathscr{I} \qquad [X'_{\mathscr{I}}] : X'_{\mathrm{Ni}} \sim \lambda X'_{\mathrm{Ni}}$

Reconstructing Minkowski

Pick a point $v^\mu \in \mathrm{Mink}_4 \Longrightarrow$ embedded as $(\mathcal{N}_1 \cap Q_0)$

$$\Updownarrow$$

$$X' = \begin{pmatrix} v^\mu \\ -\frac{1}{2} v\cdot v \\ 1 \end{pmatrix} \longrightarrow \begin{cases} \begin{pmatrix} x^\mu \\ -\frac{1}{2} + x\cdot v \\ 0 \end{pmatrix}, & x\cdot x = -1 \text{ in } \mathrm{Ti} \\[2ex] \begin{pmatrix} x^\mu \\ x\cdot v \\ 0 \end{pmatrix}, & x\cdot x = 0 \text{ in } \mathrm{Ni} \\[2ex] \begin{pmatrix} x^\mu \\ +\frac{1}{2} + x\cdot v \\ 0 \end{pmatrix}, & x\cdot x = 1 \text{ in } \mathrm{Spi} \end{cases}$$

Outline

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Embedding Space Picture

Application: (Simplest) Correlators of Carrollian CFTs

Outline

Homogeneous Spa

Embedding Spa

Application: (Simplest) Correlators of Carrollian CFTs (WIP)*

*Hic sunt leones.

Carrollian CFTs

[Bagchi, Banerjee, Basu, Dutta; Donnay, Fiorucci, Herfray, Ruzziconi; Bagueira, Oling, Sybesma, Søgaard; Ciambelli, Leigh, Marteau, Petkou, Petropoulos..]

field theory on conformal Carroll background $\simeq \mathcal{J}$

conf. Car symmetries $\Longleftrightarrow$ ext BMS $\{ \mathcal{T}(z,\bar{z}),\ Y(z), \bar{Y}(\bar{z}) \}$

primary field: $\delta_g \Phi(u,z,\bar{z}) = \mathcal{T}(z,\bar{z})\partial_u \Phi + (Y(z)\partial + \frac{u}{2}\partial Y + k\,\partial Y) + c.c.$

carrollian weight

$\boxed{\text{conf Carroll correlators} \Longleftrightarrow \text{scattering amplitudes in AF space}}$

embedding space picture for 4d CFT : $\mathbb{R}^{4,2}$

field $\Phi(X)$ defined on $X^2=0$: $\Phi(\lambda X) = \lambda^{-\Delta}\,\Phi(X)$

correlators written in terms of inner products: $X_1 \cdot X_2$

e.g. 2pt-correlator:

$$\langle \Phi(X_1)\,\Phi(X_2)\rangle = \frac{\text{const}}{(X_1\cdot X_2)^{\Delta}} \longrightarrow \frac{\text{const}'}{(x_1-x_2)^{2\Delta}}$$

field in physical space: $\phi(x^\mu) = \Phi(X)\Big|_{X\in Q_0\cap N_1}$

Q_0

$$Q_0\cap N_1 : \; X = \begin{pmatrix} x^\mu \\ -\frac{1}{2}\,x\cdot x \\ 1 \end{pmatrix}$$

Remember: $\quad Q_0 \cap \mathcal{N}_0 : \quad \mathcal{N}_i \qquad X'_{\mathcal{N}_i} = \begin{pmatrix} x^\mu \\ x^+ \\ 0 \end{pmatrix} \quad x_\mu x^\mu = 0$

$$(Q_0 \cap \mathcal{N}_0)/\mathbb{R}^+ : \mathcal{J} \qquad [X'_{\mathcal{J}}] : X'_{\mathcal{N}_i} \sim \lambda X'_{\mathcal{N}_i}$$

$\boxed{\text{Use different cut of } Q_0 \implies \text{Carrollian correlator}}$

$$X = \begin{pmatrix} \omega q^\mu \\ x^+ \\ 0 \end{pmatrix} \overset{\omega \neq 0}{\sim} \begin{pmatrix} q^\mu \\ u \\ 0 \end{pmatrix} \qquad q^\mu = (1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z})$$

$$X_1 \cdot X_2 \sim |z_{12}|^2$$

e.g.: 2-pt correlator

$$\langle \phi(u_1, z_1, \bar{z}_1)\, \phi(u_2, z_2, \bar{z}_2) \rangle = \frac{\text{const}}{(X_1 \cdot X_2)^\Delta} = \frac{\text{const}'}{|z_{12}|^{2\Delta}}$$

convert more general 4d CFT correlators in embedding space

Where is u-dependence? Include $i\varepsilon$ prescription!

$$X' = \begin{pmatrix} q^\mu \\ u \\ 0 \end{pmatrix} + i\varepsilon \begin{pmatrix} \vdots \\ \vdots \\ 1 \end{pmatrix} \qquad X_1 \cdot X_2 = |z_{12}|^2 + i\varepsilon_{21}(u_2 - u_1)$$

$$\langle \phi(X_2)\, \phi(X_1) \rangle = \frac{\text{const}}{(X_1 \cdot X_2)^\Delta} = \frac{\text{const}}{(|z_{12}|^2 + i\varepsilon_{21}(u_2-u_1))^\Delta}, \quad \varepsilon_{21} > 0$$

$$\text{for } \Delta = 1: \langle \phi(X_2)\, \phi(X_1) \rangle = \delta(z_{12})\, \delta(\bar{z}_{12}) \int_0^\infty \frac{d\omega}{\omega} e^{i\omega(u_2-u_1)}$$

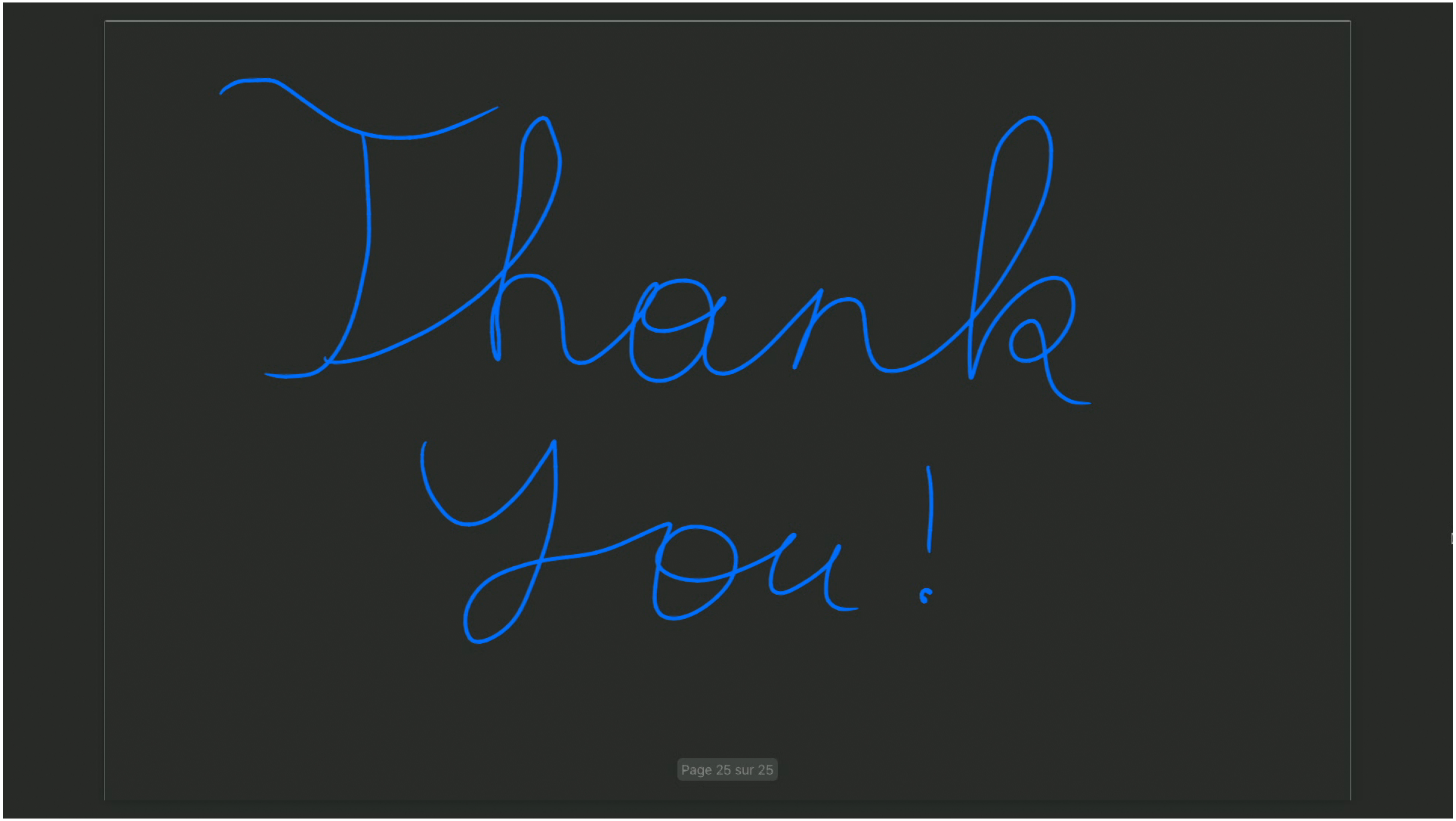
$$\boxed{\langle [\phi(X_2), \phi(X_1)] \rangle = \operatorname{sgn}(u_2-u_1)\, \delta(z_{12})\, \delta(\bar{z}_{12})}$$

reproduced from bulk correlator [Liu, Long]

Apply this prescription more generally to reproduce carCFT correlators

Conclusion

-) homogeneous spaces of $ISO(3,1)$ classify possible (holographic) dual spaces for AF spacetimes:
  $3d$: $\mathscr{I}$; $4d$: Spi, Ti, (Ni)
  $2d$: CS_2 but translations act non-effectively
-) Can be embedded in $\mathbb{R}^{4,2}$ à la Penrose–Rindler
-) Use this to write down correlators for field theories on these spaces; e.g. conf. Carroll on $\mathscr{I}$
-) Write down (low-point) correlators for theory on $\mathscr{I}$
  $\Longleftrightarrow$ compare to (low-point) amplitudes (WIP)
-) Use this to import CFT techniques to Carrollian CFT by appropriate limits in embedding space


Thank you!
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