Title: Hamiltonian Gauge Theory With Corners II: memory as superselection in null YM theory

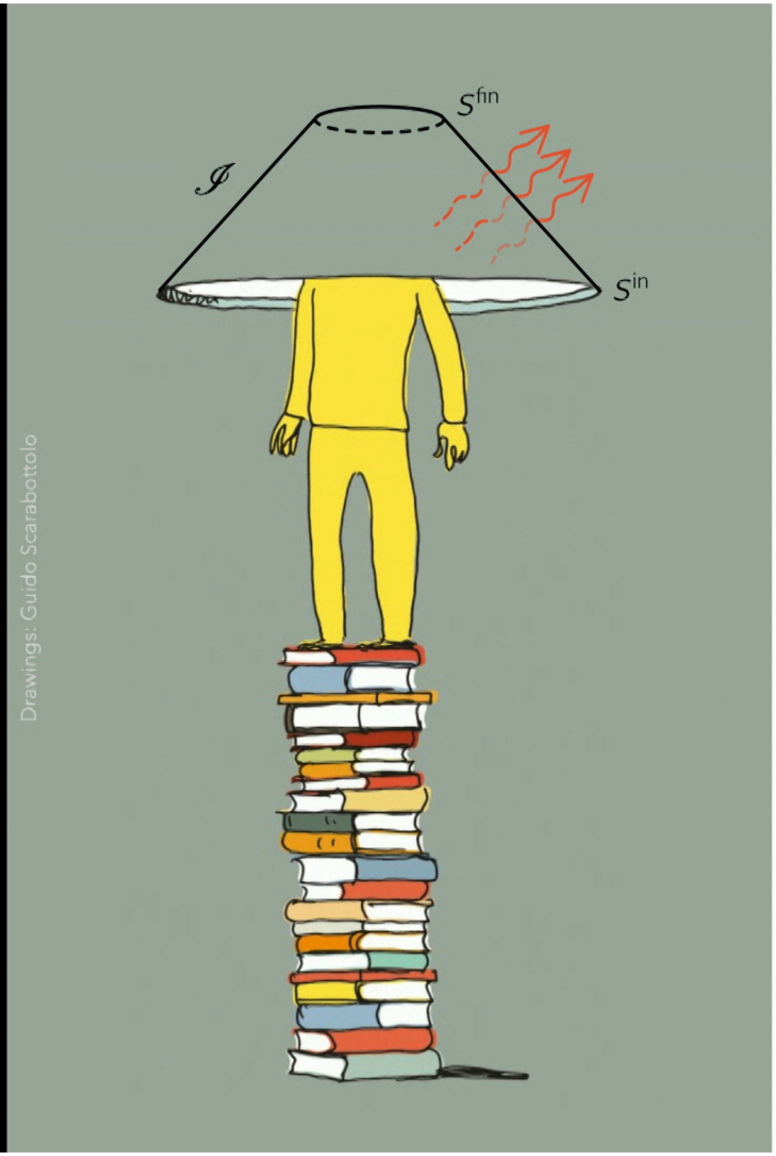
Speakers: Aldo Riello

Collection: Quantum Gravity Around the Corner

Date: October 07, 2022 - 11:00 AM

URL: https://pirsa.org/22100027

Abstract: On Tuesday, M. Schiavina laid out the theoretical framework for the symplectic reduction of gauge theories in the presence of corners. In this talk I will apply this theoretical framework to Yang-Mills theory on a null boundary and show how a pair of soft charges controls the residual (corner) gauge symmetry after the first-stage symplectic reduction, and therefore the superselection structure of the theory after the second-stage symplectic reduction. I will also discuss the subtleties of the gauge A_u = 0, the interpretation of electromagnetic memory as superselection, and how the nonlinear structure of the non-Abelian theory complicates this picture.



Hamiltonian gauge theory with corners / 2

Memory as superselection in null Maxwell theory

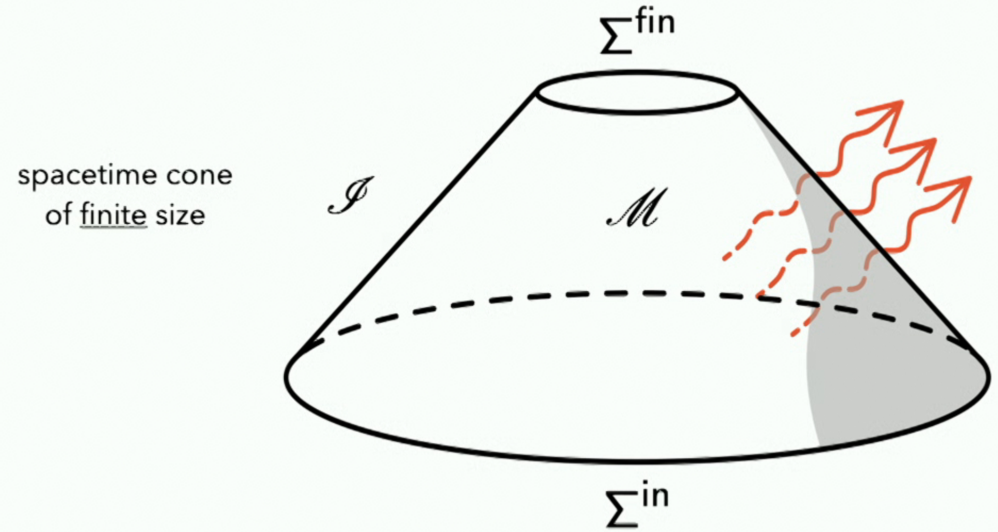
Aldo Riello

Based on:
AR & M. Schiavina 2207.00568v2
and more to come, stay tuned!

PERIMETER INSTITUTE

Quantum Gravity Around the Corner
Perimeter Institute, October 3-7, 2022

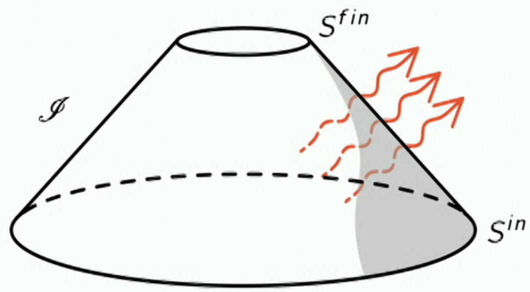
Spacetime manifold with corners



$$\partial \mathscr{M} = \Sigma^{\mathrm{fin}} \cup \mathscr{I} \cup \Sigma^{\mathrm{in}}$$

$$\partial \Sigma^{\mathrm{fin/in}} \cong S_2^{\mathrm{fin/in}} \qquad \partial \mathscr{I} \cong S_2^{\mathrm{fin}} \cup S_2^{\mathrm{in}}$$

Punchline



even w.r.t. corner g.transf.   Poisson (w/ sympl. leaves)

the fully gauge-reduced radiative phase space at Scri is a collection of **superselection** sectors labelled by initial and final (gauge orbits of the) electric fluxes through the corners S^{in} and S^{fin}

moreover… in the Abelian theory (EM) (and in the absence of magnetic charges) we can take one of the superselection labels to be the EM memory

and… after *constraint* reduction (which is a partial reduction) one obtains "extended phases space" where soft charges generate the residual corner gauge action

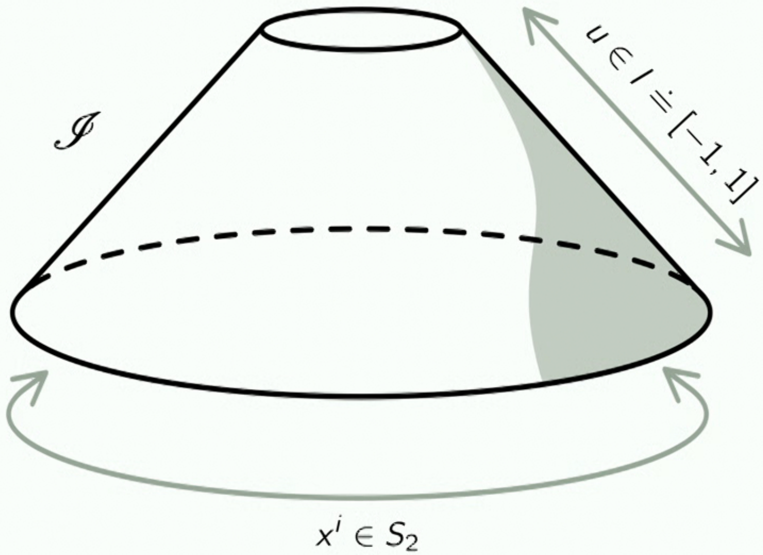
1

Introduction

off-shell YM phase space at Scri
and its gauge symmetries

Guido Scarabottolo

Null boundary with corners



$$\mathscr{I} \cong I \times S_2 \ni (u, x)$$

$\ell = \partial_u$ null-vector

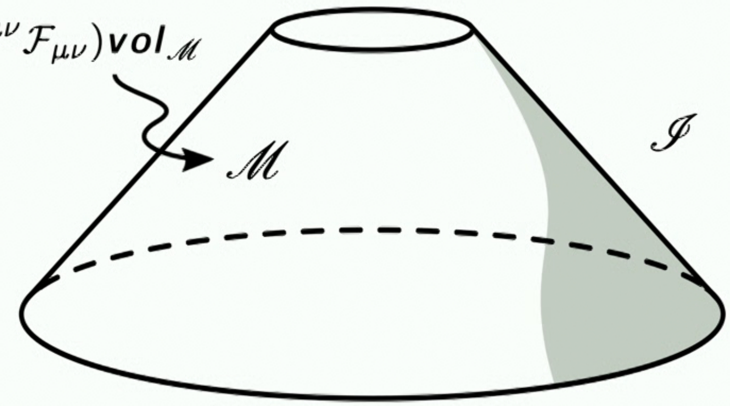
$ds^2 = \gamma_{ij}(x) dx^i dx^j$

$$\boldsymbol{vol}_{\mathscr{I}} \doteq \boldsymbol{vol}_S \wedge du \doteq \sqrt{\gamma} d^2x \wedge du$$

5

Null Yang-Mills : (off-shell) phase space

$$\mathcal{S}[\mathcal{A}] = \frac{1}{4}\int_{\mathscr{M}} \mathrm{tr}(\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu})\,\boldsymbol{vol}_{\mathscr{M}}$$



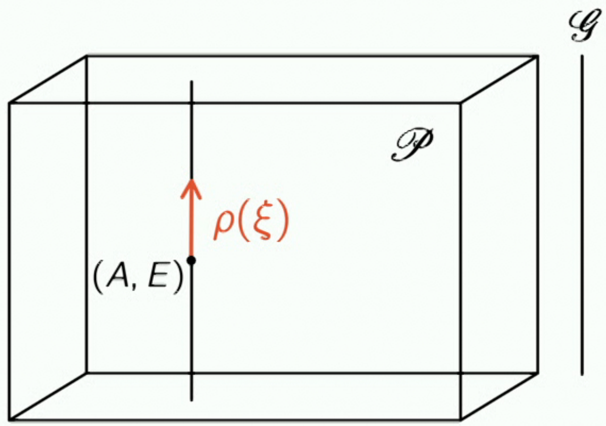
$$\tilde{\boldsymbol{\omega}} = \mathrm{tr}(\mathbb{d}\star\mathcal{F}\wedge\mathbb{d}\mathcal{A})$$

↓ pullback to *Scri*
(+ presymplectic reduction)

$$\mathscr{P} = \mathscr{A}\times\mathscr{E} \doteq \Omega^1(\mathscr{I},\mathfrak{g})\times\Omega^0(\mathscr{I},\mathfrak{g})$$

$$\boldsymbol{\omega} = \mathrm{tr}(\mathbb{d}F_{ui}\wedge\mathbb{d}A^i + \mathbb{d}E\wedge\mathbb{d}A_u)\,\boldsymbol{vol}_{\mathscr{I}}$$

Gauge symmetries are locally Hamiltonian



gauge algebra & gauge group

$$\mathfrak{G} \simeq C^\infty(\mathscr{I}, \mathfrak{g})$$
$$\mathscr{G} \doteq \exp \mathfrak{G} \simeq C_0^\infty(\mathscr{I}, G)$$

action

$$\rho : \mathfrak{G} \times \mathscr{P} \to T\mathscr{P}$$
$$\rho(\xi)(A, E) = (\mathscr{D}\xi, [E, \xi])$$

$$\mathbb{i}_{\rho(\xi)}\boldsymbol{\omega} = \langle \mathbb{d}\boldsymbol{H}, \xi\rangle \qquad \langle \boldsymbol{H}, \xi\rangle = \mathrm{tr}(F_u^i \mathscr{D}_i \xi + E\mathscr{D}_u\xi)\boldsymbol{vol}_{\mathscr{I}}$$

the Hamiltonian flow equation holds at the level of densities
[starting point of Michele's talk]

2

Superselection
& memory

review and application of
symplectic reduction with corners
[cf. Michele's talk]

Guido Scarabottolo

Momentum, constraint, and flux forms

$$\mathbb{i}_{\rho(\xi)}\boldsymbol{\omega} = \langle \mathbb{d}\boldsymbol{H}, \xi \rangle \qquad \langle \boldsymbol{H}, \xi \rangle = \mathrm{tr}(F_u^i \mathscr{D}_i \xi + E \mathscr{D}_u \xi)\boldsymbol{vol}_{\mathscr{I}}$$

decomposition of the **momentum form**
(dual-valued local form)

$$\boldsymbol{H} = \boldsymbol{H}_\circ + d\boldsymbol{h}$$

$$\langle \boldsymbol{H}_\circ, \xi \rangle = -\mathrm{tr}((\mathscr{D}_u E + \mathscr{D}^i F_u^i)\xi)\boldsymbol{vol}_{\mathscr{I}}$$

Gauss constraint form

($C^\infty(\Sigma)$-linear in ξ)

$$\langle d\boldsymbol{h}, \xi \rangle = \partial_u \mathrm{tr}(E\xi)\boldsymbol{vol}_{\mathscr{I}} + \nabla_i \mathrm{tr}(F_u^i \xi)\boldsymbol{vol}_{\mathscr{I}}$$

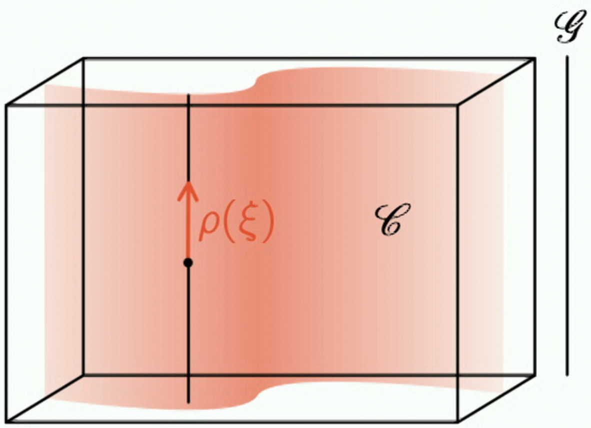
flux form

Constraint surface

Gauss constraint form

$$\langle \boldsymbol{H}_\circ, \xi \rangle = -\mathrm{tr}((\mathscr{D}_u E + \mathscr{D}^i F^i_u)\xi)\boldsymbol{vol}_{\mathscr{I}}$$

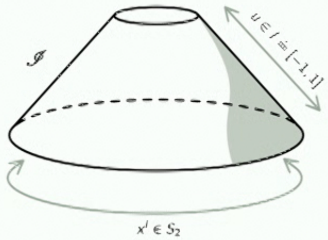
constraint surface = vanishing locus of the constraint form



Constraint surface

$$\mathscr{C} \hookrightarrow \mathscr{P}$$

10



On-shell flux space

$$\langle d\boldsymbol{h}, \xi\rangle = \partial_u \mathrm{tr}(E\xi)\boldsymbol{vol}_{\mathscr{I}} + \nabla_i \mathrm{tr}(F^i_u \xi)\boldsymbol{vol}_{\mathscr{I}}$$

the *flux form* form integrates to the **flux map**:

$$\langle h, \xi\rangle \doteq \int_{\mathscr{I}} \langle d\boldsymbol{h}, \xi\rangle = \int_S \mathrm{tr}(E^{fin}\xi^{fin} - E^{in}\xi^{in})\boldsymbol{vol}_S$$

which we can think of as a map

$$h : \mathscr{P} \to \mathfrak{G}^*$$

and thus allows us to introduce the **on-shell flux space**

$$\mathfrak{F} \doteq \mathrm{Im}(\iota^*_{\mathscr{C}} h) \subset \mathfrak{G}^*$$

Lemma:

$$\mathfrak{F} \simeq \begin{cases} \mathfrak{G}^*_\partial \simeq \mathfrak{G}^*_S \times \mathfrak{G}^*_S & G \text{ semisimple} \\ \mathrm{Ann}_\partial(\mathfrak{g}) \subset \mathfrak{G}^*_\partial & G \text{ Abelian} \end{cases}$$

Main theorem

AR, Schiavina 2022

Given an on-shell flux f

$$f \equiv (f_{in}, f_{fin}) \in \mathfrak{F} \doteq \mathrm{Im}(\iota_{\mathscr{C}}^* h) \subseteq \mathfrak{G}_S^* \times \mathfrak{G}_S^*$$

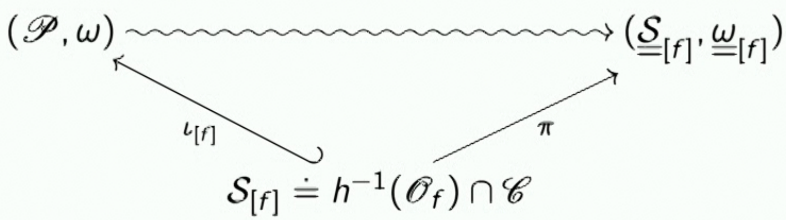
consider its gauge orbit,
and the set of **on-shell** configurations (A,E)
whose **flux** h(A,E) belongs to the **given gauge orbit**:

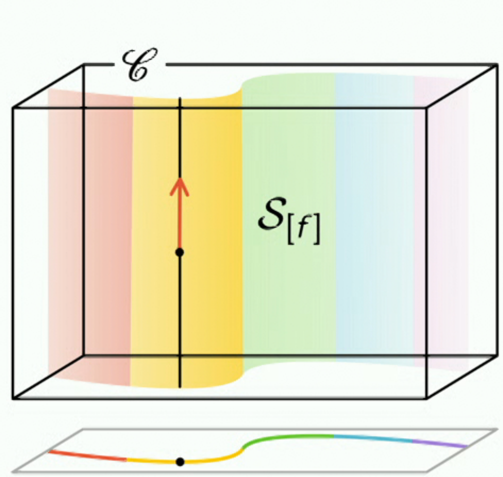
$$\mathcal{S}_{[f]} \doteq h^{-1}(\mathscr{O}_f) \cap \mathscr{C} \xhookrightarrow{\iota_{[f]}} \mathscr{P}$$

Then, the **flux superselection sector**

$$\underline{\underline{\mathcal{S}}}_{[f]} \doteq \mathcal{S}_{[f]}/\mathscr{G} \qquad \pi^* \underline{\underline{\omega}}_{[f]} = \iota_{[f]}^* \omega$$

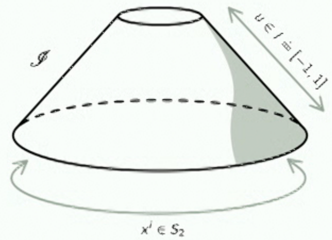
is a symplectic space.





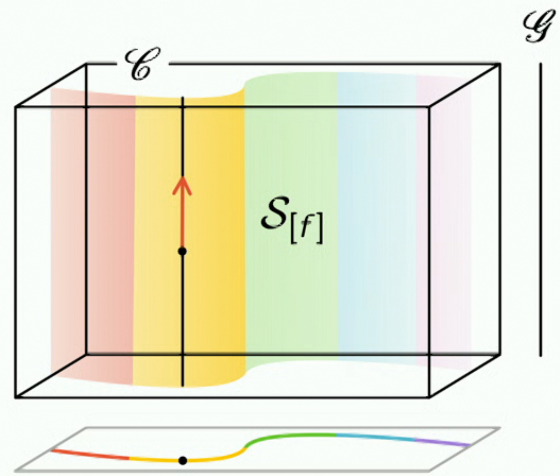
$$\underline{\underline{\mathscr{P}}} \doteq \mathscr{C}/\mathscr{G} = \bigsqcup_{\mathcal{O}_f} \underline{\underline{\mathcal{S}}}_{[f]}$$

Moreover,
the fully-reduced phase space is a Poisson space whose symplectic leaves are the superselection sectors.



Memory as superselection

$$\mathcal{S}_{[f]} \doteq h^{-1}(\mathscr{O}_f) \cap \mathscr{C} \xhookrightarrow{\iota_{[f]}} \mathscr{P}$$



Abelian G

$$\mathscr{O}_f = \{f_{in}\} \times \{f_{fin}\}$$

alternative labeling:

$$(f_{fin}, f_{fin} - f_{in}) = (f_{fin}, \boxed{\Delta\Phi})$$

Bieri-Garfinkle memory*
$\vec{v}^{fin} - \vec{v}^{in} = \vec{\nabla}\Phi$

Non-Abelian G

$$\mathscr{O}_f = \mathscr{O}_{in} \times \mathscr{O}_{fin}$$

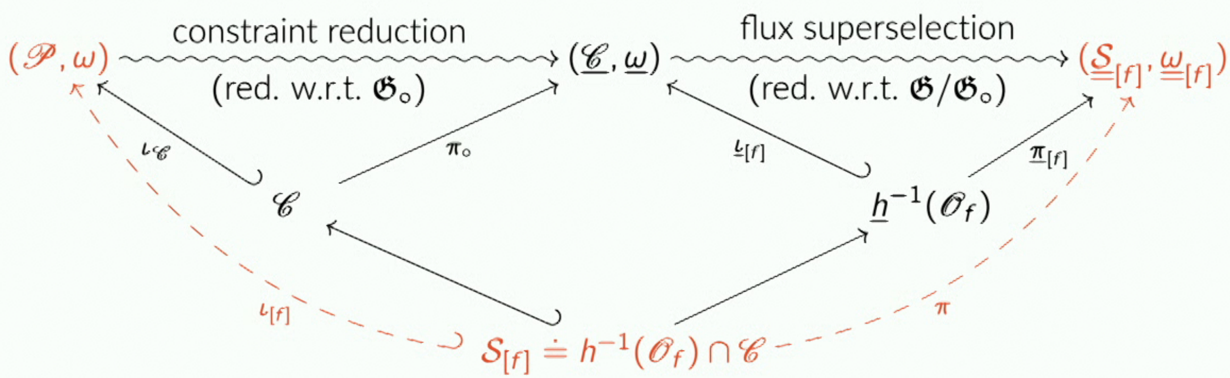
orbits are **non**-pointlike!
flux differences
do not have a neat ☞
geometrical meaning

$$\underline{\underline{\mathscr{P}}} \doteq \mathscr{C}/\mathscr{G} = \bigsqcup_{\mathscr{O}_f} \underline{\underline{\mathcal{S}}}_{[f]}$$

* Provided: $F^{in}_{ij} = F^{fin}_{ij} = 0$

An important subalgebra of gauge transformations is the **flux annihilator ideal** :

$$\mathfrak{G}_\circ = \mathrm{Ann}\,\mathrm{Im}(\iota_{\mathscr{C}}^* h) \equiv \mathrm{Ann}(\mathfrak{F})$$
$$\sim \{\xi \;:\; \xi|_{\partial\mathscr{I}} = 0\}$$

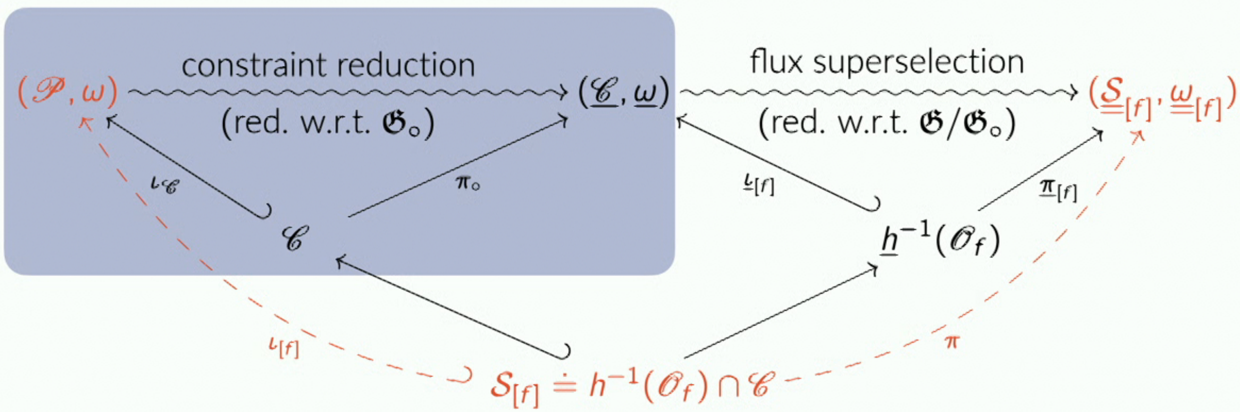


a.k.a. **constraint g.transf.** ≈ subgroup of g.transf. generated by the Gauss constraint only

$$\mathbb{i}_{\rho(\xi_\circ)}\boldsymbol{\omega} = \langle \mathbb{d}\boldsymbol{H}_\circ, \xi_\circ \rangle$$

An important subalgebra of gauge transformations is the **flux annihilator ideal** :

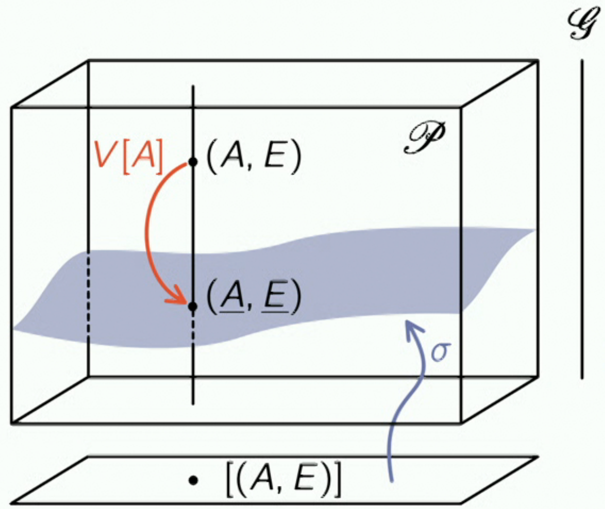
$$\mathfrak{G}_\circ = \mathrm{Ann}\,\mathrm{Im}(\iota_{\mathscr{C}}^* h) \equiv \mathrm{Ann}(\mathfrak{F})$$
$$\sim \{\xi \; : \; \xi|_{\partial\mathscr{I}} = 0\}$$



a.k.a. **constraint g.transf.** ≈ subgroup of g.transf. generated by the Gauss constraint only

$$\mathbb{i}_{\rho(\xi_\circ)}\boldsymbol{\omega} = \langle \mathbb{d}\boldsymbol{H}_\circ, \xi_\circ\rangle$$

Dressing



$V : \mathscr{P} \to \mathscr{G}$

$$(A, E) = (\underline{A}, \underline{E})^{V[A]}$$

(dressing)

$\sigma : \mathscr{P}/\mathscr{G} \hookrightarrow \mathscr{P}$

Choose as "gauge condition"

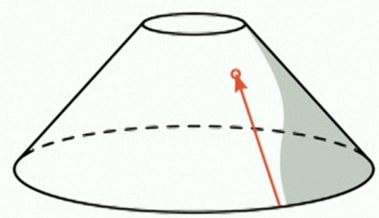
$$\underline{A}_u = 0$$

Choose as "dressing factor":

$$V[A](u, x) \doteq \mathrm{Pexp} \int_{-1}^{u} du' \, A_{u'}(u', x)$$

$$\Lambda[A](x) \doteq V[A](+1, x)$$

a bi-local object, invariant w.r.t. $\mathfrak{G}_\circ$



Wilson line

[cf. zero modes in Sucheta Majumdar's talk]

16

Constraint reduction

$$\omega(A,E) = \omega(\underline{A}^{V[A]}, \underline{E}^{V[A]})$$

[on-shell]

$$= \cdots$$

$$\approx \int_{\mathscr{I}} \mathrm{tr}(\partial_u \mathbb{d}\underline{A}^i \wedge \mathbb{d}\underline{A}_i)\boldsymbol{vol}_{\mathscr{I}} + \mathbb{d}\int_S \mathrm{tr}(E^{fin}\mathbb{d}\Lambda[A]\Lambda[A]^{-1})\boldsymbol{vol}_S$$

Ashtekar–Streubel radiative phase space

$\hat{\mathscr{A}}_{AS}$

"hard sector"

one sphere-worth extension (bi-local!)

$T^*\mathscr{G}_S$

"soft sector"

[cf. Laura Donnay's talk]

[cf. Kasia Rejzner's talk]

"corner BV-BFV symplectic struct."

Theorem:

$$\underline{\mathscr{C}} \doteq \mathscr{C}/\mathscr{G}_\circ \simeq_{loc} \hat{\mathscr{A}}_{AS} \times T^*\mathscr{G}_S$$

$$\ni (\underline{A}_i, E^{fin}, \Lambda)$$

Global result in Abelian case: $\mathscr{C} \simeq \hat{\mathscr{A}}_{AS} \times T^*\mathfrak{G}_S$

Residual gauge action, soft charges, and superselection

$$\underline{\mathscr{C}} \doteq \mathscr{C}/\mathscr{G}_\circ \simeq_{loc} \hat{\mathscr{A}}_{AS} \times T^*\mathscr{G}_S$$

Residual gauge transformations

$$\underline{\mathfrak{G}} \doteq \mathfrak{G}/\mathfrak{G}_\circ \sim \mathfrak{G}_S \times \mathfrak{G}_S \ni (\xi_{in}, \xi_{fin})$$

with residual action

$$\rho(\xi_{in}, \xi_{fin}) \begin{pmatrix} \underline{A}_i \\ E^{fin} \\ \Lambda \end{pmatrix} = \begin{pmatrix} \underline{\mathscr{D}}_i \xi_{in} \\ [E^{fin}, \xi_{fin}] \\ \xi_{fin}\Lambda - \Lambda\xi_{in} \end{pmatrix}$$

and Hamiltonian generator generator the **soft charge**

$$\langle \underline{h}, (\xi_{in}, \xi_{fin}) \rangle = \int_S \mathrm{tr}\big(E_{fin}(\xi_{fin} - \mathrm{Ad}_\Lambda \xi_{in}) + (\underline{\mathscr{D}}^i \partial_u \underline{A}_i)^0 \xi_{in}\big) \boldsymbol{vol}_S$$

18

Residual gauge action, soft charges, and superselection

$$\underline{\mathscr{C}} \doteq \mathscr{C}/\mathscr{G}_\circ \simeq_{loc} \hat{\mathscr{A}}_{AS} \times T^*\mathscr{G}_S$$

Residual gauge transformations

$$\underline{\mathfrak{G}} \doteq \mathfrak{G}/\mathfrak{G}_\circ \sim \mathfrak{G}_S \times \mathfrak{G}_S \ni (\xi_{in}, \xi_{fin})$$

with residual action

$$\rho(\xi_{in}, \xi_{fin}) \begin{pmatrix} \underline{A}_i \\ E^{fin} \\ \Lambda \end{pmatrix} = \begin{pmatrix} \underline{\mathscr{D}}_i \xi_{in} \\ [E^{fin}, \xi_{fin}] \\ \xi_{fin}\Lambda - \Lambda\xi_{in} \end{pmatrix}$$

Non-local in u
($\underline{A}$ is not a connection)

(bi-local)
left & right actions on
$T^*\mathscr{G}_S \simeq \mathscr{G}_S \times \mathfrak{G}^*_S$

and Hamiltonian generator generator the **soft charge**

$$\langle \underline{h}, (\xi_{in}, \xi_{fin}) \rangle = \int_S \mathrm{tr}\big(E_{fin}(\xi_{fin} - \mathrm{Ad}_\Lambda \xi_{in}) + (\underline{\mathscr{D}}^i \partial_u \underline{A}_i)^0 \xi_{in}\big)\, \boldsymbol{vol}_S$$

Abelian: $\Delta\Phi = (\nabla^i \underline{A}_i)^{diff}$
(Bieri-Garfinkle memory*)

* Provided: $F^{in}_{ij} = F^{fin}_{ij} = 0$

Residual gauge action, soft charges, and superselection

$$\underline{\mathscr{C}} \doteq \mathscr{C}/\mathscr{G}_\circ \simeq_{loc} \hat{\mathscr{A}}_{AS} \times T^*\mathscr{G}_S$$

Residual gauge transformations

$$\underline{\mathfrak{G}} \doteq \mathfrak{G}/\mathfrak{G}_\circ \sim \mathfrak{G}_S \times \mathfrak{G}_S \ni (\xi_{in}, \xi_{fin})$$

with residual action

$$\rho(\xi_{in}, \xi_{fin}) \begin{pmatrix} \underline{A}_i \\ E^{fin} \\ \Lambda \end{pmatrix} = \begin{pmatrix} \underline{\mathscr{D}}_i \xi_{in} \\ [E^{fin}, \xi_{fin}] \\ \xi_{fin}\Lambda - \Lambda\xi_{in} \end{pmatrix}$$

and Hamiltonian generator generator the **soft charge**

$$\langle \underline{h}, (\xi_{in}, \xi_{fin}) \rangle = \int_S \mathrm{tr}\big(E_{fin}(\xi_{fin} - \mathrm{Ad}_\Lambda \xi_{in}) + (\underline{\mathscr{D}}^i \partial_u \underline{A}_i)^0 \xi_{in}\big) \boldsymbol{vol}_S$$

in general

$$= \mathrm{Ad}_\Lambda^{-1} E^{fin} - E^{in}$$

19

Residual gauge action, soft charges, and superselection

$$\underline{\mathscr{C}} \doteq \mathscr{C}/\mathscr{G}_\circ \simeq_{loc} \hat{\mathscr{A}}_{AS} \times T^*\mathscr{G}_S$$

Residual gauge transformations

$$\underline{\mathfrak{G}} \doteq \mathfrak{G}/\mathfrak{G}_\circ \sim \mathfrak{G}_S \times \mathfrak{G}_S \ni (\xi_{in}, \xi_{fin})$$

with residual action

$$\rho(\xi_{in}, \xi_{fin}) \begin{pmatrix} \underline{A}_i \\ E^{fin} \\ \Lambda \end{pmatrix} = \begin{pmatrix} \underline{\mathscr{D}}_i \xi_{in} \\ [E^{fin}, \xi_{fin}] \\ \xi_{fin}\Lambda - \Lambda\xi_{in} \end{pmatrix}$$

and Hamiltonian generator generator the **soft charge**

$$\langle \underline{h}, (\xi_{in}, \xi_{fin}) \rangle = \int_S \mathrm{tr}\big(E_{fin}(\xi_{fin} - \mathrm{Ad}_\Lambda \xi_{in}) + (\underline{\mathscr{D}}^i \partial_u \underline{A}_i)^0 \xi_{in} \big) \boldsymbol{vol}_S$$

in general

$$= \mathrm{Ad}_\Lambda^{-1} E^{fin} - E^{in}$$

Superslections
labeled by coadjoint orbits of
the initial and final* flux

4

Conclusions

- The fully-reduced radiative phase space at Scri is a Poisson mfd [AR, Schiavina 2022].
- Its symplectic leaves are the Flux Superselection Sectors
  which are indexed by the gauge orbits of the initial and final electric fluxes.
- Superselection is a consequence of
  "tracing over" the dof in the complementary region w.r.t. a closed Cauchy surface
  [Bartlett, Rudolph, Spekkens 2007; Gomez, AR 2017-21].
  It shows up in entanglement entropy structure
  [Donnelly 2011; Casini, Huerta, Rosabal 2014; AR in prep.]
- Memory: [Bieri, Garfinkle 2014; Pasterski 2017; Pate, Raclariu, Strominger 2017]
  In the Abelian case, with no magnetic charges, memory is a good SSS label.
  Not so in the non-Abelian case.
- *Constraint* reduction yields a *semi*-reduced ph.space
  which is given by the extension of Ashtekar–Streubel by a single $T^*\mathcal{G}_S$ (bi-local!).
- This space supports *residual corner gauge symmetries* generated by soft charges.
- Hint? soft theorems are a statement of "residual corner gauge covariance",
  and soft charge "conservation" reread as statement that
  scattering unfolds within a given superselection sector determined by
  fluxes "through i^0"

The leading towers

Reminder: $Q_K = \int_{\Sigma_2} \mathrm{d}^2x \sqrt{a}\left(\kappa + b_i K^i\right) \qquad \xi \in \mathrm{BMS}_4$

ELECTRIC $\kappa = \xi^i \pi_i - \xi^{\hat{t}} \varepsilon \quad K^i = \frac{\varepsilon}{2}\xi^i - \frac{1}{8\pi G}\left(\xi^j * \Psi^i{}_j + \xi^{\hat{t}} * \chi^i\right)$

MAGNETIC $\kappa = \xi^i \psi_i - \xi^{\hat{t}} c \quad K^i = \frac{c}{2}\xi^i - \xi^j \Psi^i{}_j - \xi^{\hat{t}} \chi^i$

SELF-DUAL $\kappa = \xi^i \chi_i \quad K^i = -\xi^j X^i{}_j$

$c, \psi_i, \chi_i, \Psi_{ij}, X_{ij}$ – Carrollian Cotton tensors

ε – Bondi mass aspect

π_i – angular momentum aspect

Comparison: BMS standard approach [see e.g. L. Donnay' talk]

$Q_\xi \sim \int_{S^2} \mathrm{d}z \wedge \mathrm{d}\bar{z}\left(2\mathcal{M}\mathcal{T} + \mathcal{N}\bar{\mathcal{Y}} + \bar{\mathcal{N}}\mathcal{Y}\right) \to$ electric tower