

Title: Hamiltonian Gauge Theory With Corners II: memory as superselection in null YM theory

Speakers: Aldo Riello

Collection: Quantum Gravity Around the Corner

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Abstract: On Tuesday, M. Schiavina laid out the theoretical framework for the symplectic reduction of gauge theories in the presence of corners. In this talk I will apply this theoretical framework to Yang-Mills theory on a null boundary and show how a pair of soft charges controls the residual (corner) gauge symmetry after the first-stage symplectic reduction, and therefore the superselection structure of the theory after the second-stage symplectic reduction. I will also discuss the subtleties of the gauge $A_u = 0$, the interpretation of electromagnetic memory as superselection, and how the nonlinear structure of the non-Abelian theory complicates this picture.



Drawings: Guido Scarabottolo

Hamiltonian gauge theory with corners / 2
**Memory as superselection
in null Maxwell theory**

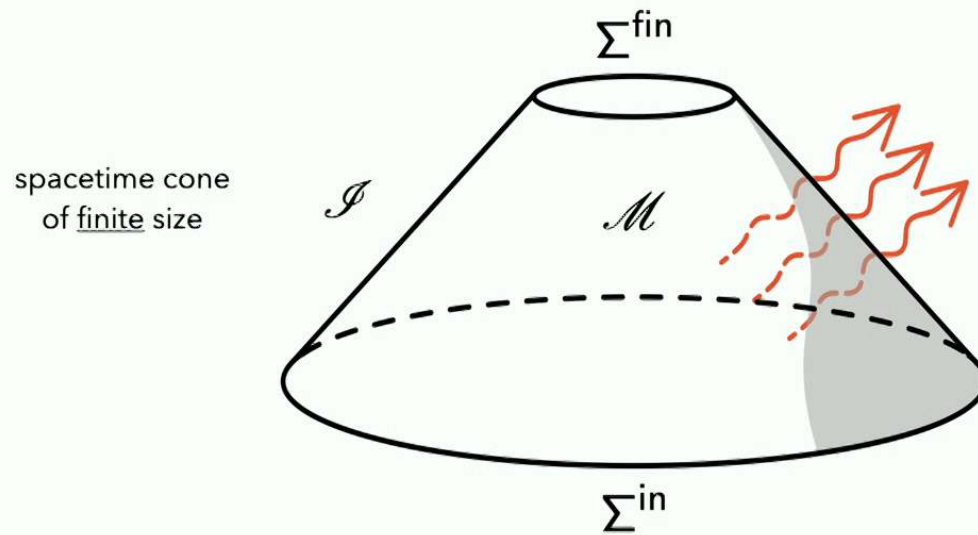
Aldo Riello

Based on:
AR & M. Schiavina 2207.00568v2
and more to come, stay tuned!



Quantum Gravity Around the Corner
Perimeter Institute, October 3-7, 2022

Spacetime manifold with corners

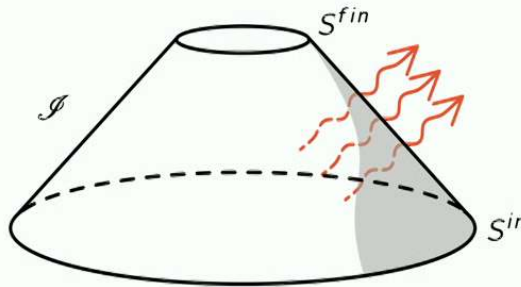


$$\partial \mathcal{M} = \Sigma^{\text{fin}} \cup \mathcal{I} \cup \Sigma^{\text{in}}$$

$$\partial \Sigma^{\text{fin/in}} \cong S_2^{\text{fin/in}}$$

$$\partial \mathcal{I} \cong S_2^{\text{fin}} \cup S_2^{\text{in}}$$

Punchline



even w.r.t. corner g.transf.

Poisson (w/ simpl. leaves)

the fully gauge-reduced radiative phase space at S_{cri} is a collection of **superselection sectors** labelled by initial and final (gauge orbits of the) electric fluxes through the corners S^{in} and S^{fin}

- moreover... in the Abelian theory (EM)
(and in the absence of magnetic charges) we can take
one of the superselection labels to be the **EM memory**
- and... after *constraint* reduction (which is a partial reduction)
one obtains "extended phases space" where
soft charges generate the residual corner gauge action

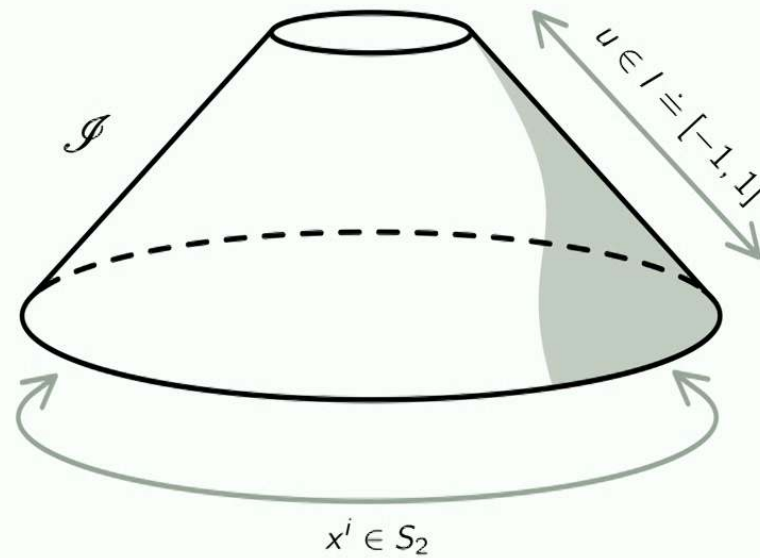
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Introduction

off-shell YM phase space at Scri
and its gauge symmetries

Guido Scarabottolo

Null boundary with corners



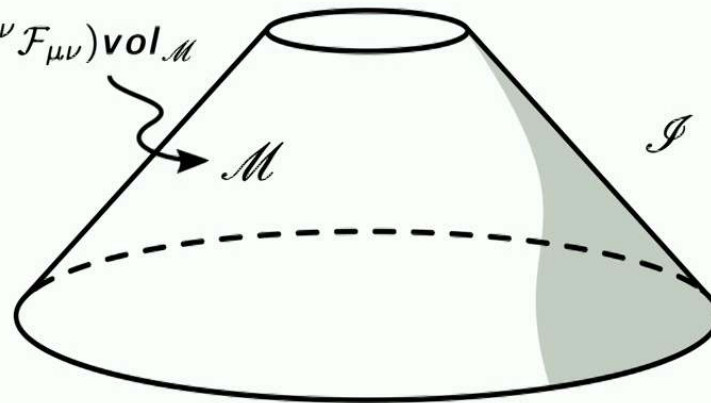
$$\mathcal{I} \cong I \times S_2 \ni (u, x)$$

$$\begin{aligned} \ell &= \partial_u \quad \text{null-vector} \\ ds^2 &= \gamma_{ij}(x) dx^i dx^j \end{aligned}$$

$$\text{vol}_{\mathcal{I}} \doteq \text{vol}_S \wedge du \doteq \sqrt{\gamma} d^2x \wedge du$$

Null Yang-Mills : (off-shell) phase space

$$\mathcal{S}[\mathcal{A}] = \frac{1}{4} \int_{\mathcal{M}} \text{tr}(\mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}) \text{vol}_{\mathcal{M}}$$



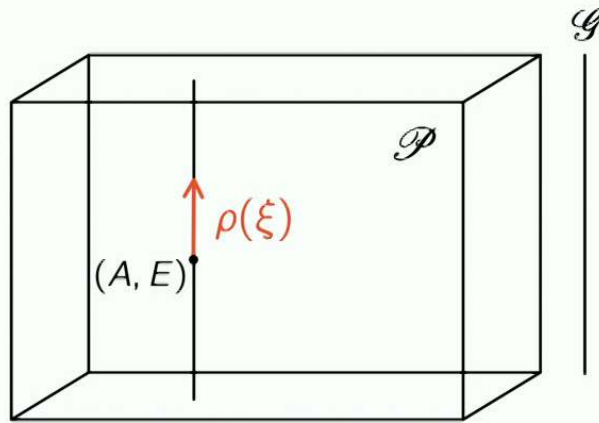
$$\tilde{\omega} = \text{tr}(d \star \mathcal{F} \wedge d\mathcal{A})$$

\downarrow pullback to $\mathcal{S}cri$
 (+ presymplectic reduction)

$$\mathcal{P} = \mathcal{A} \times \mathcal{E} \doteq \Omega^1(\mathcal{I}, \mathfrak{g}) \times \Omega^0(\mathcal{I}, \mathfrak{g})$$

$$\omega = \text{tr}(dF_{ui} \wedge dA^i + dE \wedge dA_u) \text{vol}_{\mathcal{I}}$$

Gauge symmetries are locally Hamiltonian



gauge algebra & gauge group

$$\mathfrak{G} \simeq C^\infty(\mathcal{I}, \mathfrak{g})$$

$$\mathcal{G} \doteq \exp \mathfrak{G} \simeq C_0^\infty(\mathcal{I}, G)$$

action

$$\rho : \mathfrak{G} \times \mathcal{P} \rightarrow T\mathcal{P}$$

$$\rho(\xi)(A, E) = (\mathcal{D}\xi, [E, \xi])$$

$$\mathfrak{i}_{\rho(\xi)} \omega = \langle dH, \xi \rangle \quad \langle H, \xi \rangle = \text{tr}(F_u^i \mathcal{D}_i \xi + E \mathcal{D}_u \xi) \text{vol}_{\mathcal{I}}$$

the Hamiltonian flow equation holds at the level of densities
[starting point of Michele's talk]

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Superselection & memory

review and application of
symplectic reduction with corners

[cf. Michele's talk]

Guido Scarabottolo

Momentum, constraint, and flux forms

$$i_{\rho(\xi)}\omega = \langle dH, \xi \rangle \quad \langle H, \xi \rangle = \text{tr}(F_u^i \mathcal{D}_i \xi + E \mathcal{D}_u \xi) \text{vol}_{\mathcal{J}}$$

decomposition of the **momentum form**
(dual-valued local form)

$$H = H_o + dh$$

$$\langle H_o, \xi \rangle = -\text{tr}((\mathcal{D}_u E + \mathcal{D}^i F_u^i) \xi) \text{vol}_{\mathcal{J}}$$

Gauss constraint form

($C^\infty(\Sigma)$ -linear in ξ)

$$\langle dh, \xi \rangle = \partial_u \text{tr}(E \xi) \text{vol}_{\mathcal{J}} + \nabla_i \text{tr}(F_u^i \xi) \text{vol}_{\mathcal{J}}$$

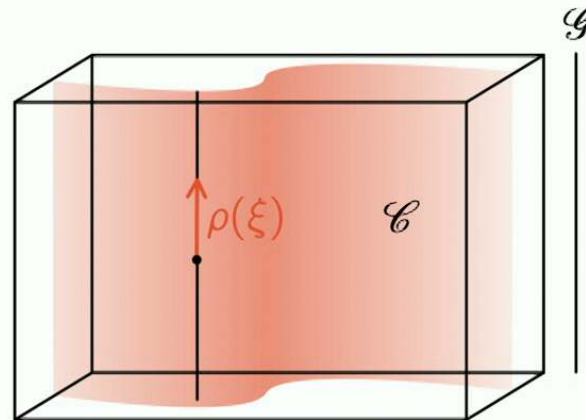
flux form

Constraint surface

Gauss constraint form

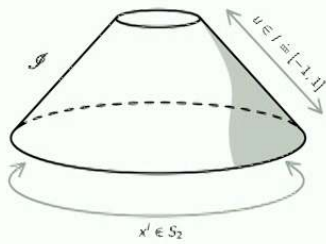
$$\langle H_o, \xi \rangle = -\text{tr}((\mathcal{D}_u E + \mathcal{D}^i F_u^i) \xi) \text{vol}_{\mathcal{I}}$$

constraint surface = vanishing locus of the constraint form



Constraint surface

$$\mathcal{C} \hookrightarrow \mathcal{P}$$



On-shell flux space

$$\langle dh, \xi \rangle = \partial_u \text{tr}(E\xi) \text{vol}_{\mathcal{F}} + \nabla_i \text{tr}(F_u^i \xi) \text{vol}_{\mathcal{F}}$$

the flux form integrates to the flux map:

$$\langle h, \xi \rangle \doteq \int_{\mathcal{F}} \langle dh, \xi \rangle = \int_S \text{tr}(E^{fin} \xi^{fin} - E^{in} \xi^{in}) \text{vol}_S$$

which we can think of as a map

$$h : \mathcal{P} \rightarrow \mathfrak{G}^*$$

and thus allows us to introduce the **on-shell flux space**

$$\mathfrak{F} \doteq \text{Im}(\iota_{\mathcal{F}}^* h) \subset \mathfrak{G}^*$$

Lemma:

$$\mathfrak{F} \simeq \begin{cases} \mathfrak{G}_{\partial}^* \simeq \mathfrak{G}_S^* \times \mathfrak{G}_S^* & G \text{ semisimple} \\ \text{Ann}_{\partial}(\mathfrak{g}) \subset \mathfrak{G}_{\partial}^* & G \text{ Abelian} \end{cases}$$

Main theorem

AR, Schiavina 2022

Given an on-shell flux f

$$f \equiv (f_{in}, f_{fin}) \in \mathfrak{F} \doteq \text{Im}(\iota_{\mathcal{C}}^* h) \subseteq \mathfrak{G}_S^* \times \mathfrak{G}_S^*$$

consider its gauge orbit,
and the set of **on-shell** configurations (A,E)
whose **flux** $h(A,E)$ belongs to the **given gauge orbit**:

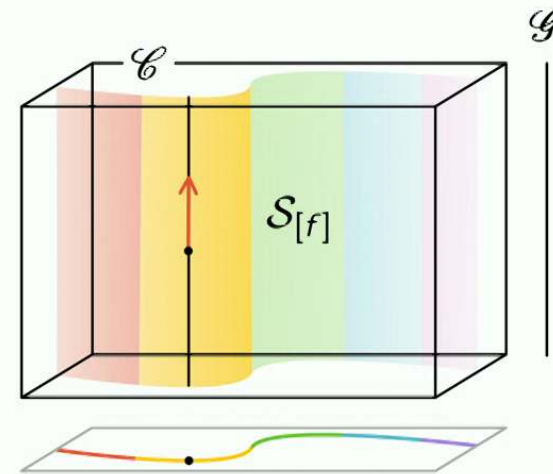
$$\mathcal{S}_{[f]} \doteq h^{-1}(\mathcal{O}_f) \cap \mathcal{C} \xrightarrow{\iota_{[f]}} \mathcal{P}$$

Then, the **flux superselection sector**

$$\underline{\mathcal{S}}_{[f]} \doteq \mathcal{S}_{[f]} / \mathcal{G} \quad \pi^* \underline{\omega}_{[f]} = \iota_{[f]}^* \omega$$

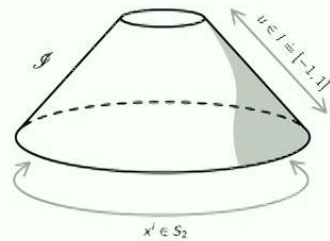
is a symplectic space.

$$\begin{array}{ccc}
 (\mathcal{P}, \omega) & \xrightarrow{\quad \quad \quad} & (\underline{\mathcal{S}}_{[f]}, \underline{\omega}_{[f]}) \\
 \swarrow \iota_{[f]} & & \searrow \pi \\
 & \mathcal{S}_{[f]} \doteq h^{-1}(\mathcal{O}_f) \cap \mathcal{C} &
 \end{array}$$



$$\underline{\mathcal{P}} \doteq \mathcal{C} / \mathcal{G} = \bigsqcup_{\mathcal{O}_f} \underline{\mathcal{S}}_{[f]}$$

Moreover,
the fully-reduced phase space is a Poisson space
whose symplectic leaves are the superselection sectors.



Memory as superselection

Abelian G

$$\mathcal{O}_f = \{f_{in}\} \times \{f_{fin}\}$$

alternative labeling:

$$(f_{fin}, f_{fin} - f_{in}) = (f_{fin}, \Delta\Phi)$$

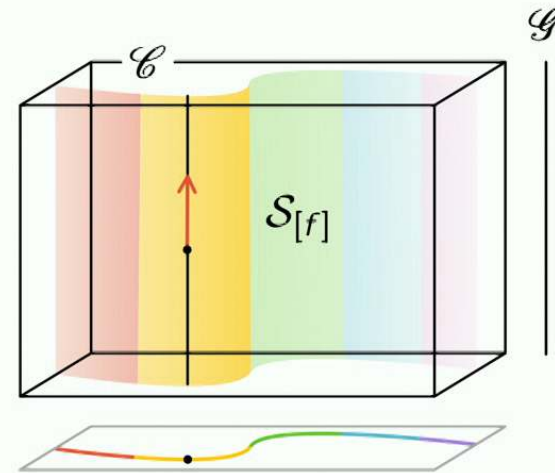
Bieri-Garfinkle memory*
 $\vec{v}^{fin} - \vec{v}^{in} = \vec{\nabla}\Phi$

Non-Abelian G

$$\mathcal{O}_f = \mathcal{O}_{in} \times \mathcal{O}_{fin}$$

orbits are **non-pointlike!**
 flux differences
 do not have a neat geometrical meaning

$$\mathcal{S}_{[f]} \doteq h^{-1}(\mathcal{O}_f) \cap \mathcal{C} \xrightarrow{U[f]} \mathcal{P}$$



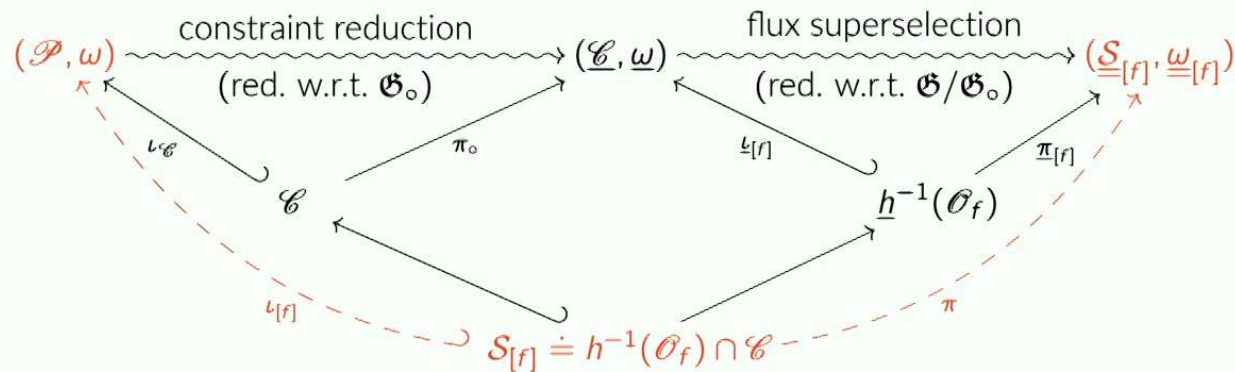
$$\underline{\mathcal{P}} \doteq \mathcal{C}/\mathcal{G} = \bigsqcup_{\mathcal{O}_f} \mathcal{S}_{[f]}$$

* Provided: $F_{ij}^{in} = F_{ij}^{fin} = 0$

Reduction by stages

An important subalgebra of gauge transformations is the **flux annihilator ideal** :

$$\begin{aligned}\mathfrak{G}_o &= \text{Ann Im}(\iota_{\mathcal{C}}^* h) \equiv \text{Ann}(\mathfrak{F}) \\ &\sim \{\xi : \xi|_{\partial\mathcal{I}} = 0\}\end{aligned}$$



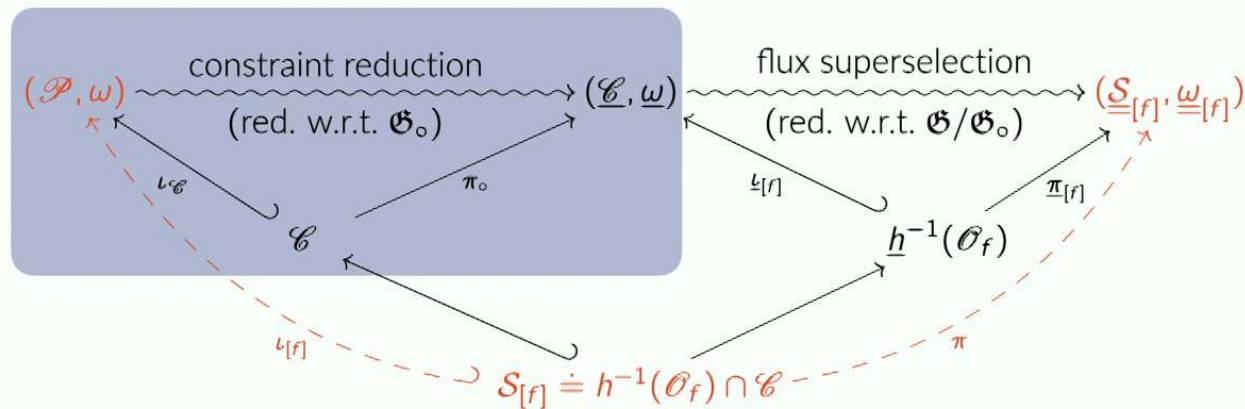
a.k.a. **constraint g.transf.** \approx subgroup of g.transf. generated by the Gauss constraint only

$$i_{\rho(\xi_o)} \omega = \langle dH_o, \xi_o \rangle$$

Reduction by stages

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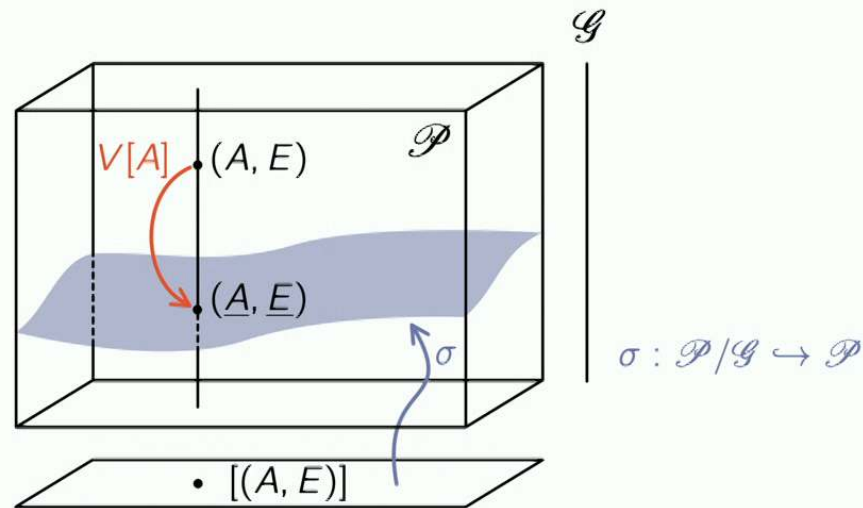
$$\mathfrak{i}_{\rho(\xi_o)} \omega = \langle dH_o, \xi_o \rangle$$

Dressing

$$V : \mathcal{P} \rightarrow \mathcal{G}$$

$$(A, E) = (\underline{A}, \underline{E})^{V[A]}$$

(dressing)



Choose as "gauge condition"

$$\underline{A}_u = 0$$

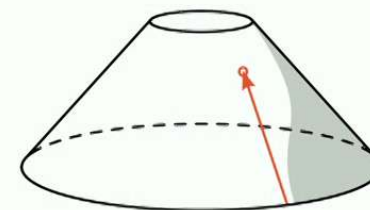
Choose as "dressing factor":

$$V[A](u, x) \doteq \text{Pexp} \int_{-1}^u du' A_{u'}(u', x)$$

$$\Lambda[A](x) \doteq V[A](+1, x)$$

a bi-local object,
invariant w.r.t. \mathfrak{G}_0

[cf. zero modes in Sucheta Majumdar's talk]



Wilson line

Constraint reduction

$$\omega(A, E) = \omega(\underline{A}^{V[A]}, \underline{E}^{V[A]})$$

↓
[on-shell]

= ...

$$\approx \int_{\mathcal{I}} \text{tr}(\partial_u d\underline{A}^i \wedge d\underline{A}_i) \text{vol}_{\mathcal{I}} + \int_S \text{tr}(E^{fin} d\Lambda[A] \wedge [A]^{-1}) \text{vol}_S$$

Ashtekar–Streubel
radiative phase space

$$\hat{\mathcal{A}}_{AS}$$

"hard sector"

[cf. Laura Donnay's talk]

one sphere-worth extension
(bi-local!)

$$T^*\mathcal{G}_S$$

"soft sector"

[cf. Kasia Rejzner's talk]

"corner BV-BFV symplectic struct."

Theorem:

$$\underline{\mathcal{C}} \doteq \mathcal{C}/\mathcal{G}_0 \simeq_{loc} \hat{\mathcal{A}}_{AS} \times T^*\mathcal{G}_S$$

$$\ni (\underline{A}_i, E^{fin}, \Lambda)$$

Global result in Abelian case: $\mathcal{C} \simeq \hat{\mathcal{A}}_{AS} \times T^*\mathcal{G}_S$

Residual gauge action, soft charges, and superselection

$$\underline{\mathcal{C}} \doteq \mathcal{C}/\mathcal{G}_0 \simeq_{loc} \hat{\mathcal{A}}_{AS} \times T^*\mathcal{G}_S$$

Residual gauge transformations

$$\underline{\mathfrak{G}} \doteq \mathfrak{G}/\mathfrak{G}_0 \sim \mathfrak{G}_S \times \mathfrak{G}_S \ni (\xi_{in}, \xi_{fin})$$

with residual action

$$\rho(\xi_{in}, \xi_{fin}) \begin{pmatrix} \underline{A}_j \\ E^{fin} \\ \wedge \end{pmatrix} = \begin{pmatrix} \underline{\mathcal{D}}_j \xi_{in} \\ [E^{fin}, \xi_{fin}] \\ \xi_{fin} \wedge - \wedge \xi_{in} \end{pmatrix}$$

and Hamiltonian generator the **soft charge**

$$\langle \underline{h}, (\xi_{in}, \xi_{fin}) \rangle = \int_S \text{tr} (E_{fin} (\xi_{fin} - \text{Ad}_\wedge \xi_{in}) + (\underline{\mathcal{D}}^i \partial_u \underline{A}_i)^0 \xi_{in}) \text{vol}_S$$

Residual gauge action, soft charges, and superselection

$$\underline{\mathcal{C}} \doteq \mathcal{C}/\mathcal{G}_0 \simeq_{loc} \hat{\mathcal{A}}_{AS} \times T^*\mathcal{G}_S$$

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Non-local in u
(\underline{A} is not a connection)

(bi-local)
left & right actions on
 $T^*\mathcal{G}_S \simeq \mathcal{G}_S \times \mathcal{G}_S^*$

and Hamiltonian generator the **soft charge**

$$\langle \underline{h}, (\xi_{in}, \xi_{fin}) \rangle = \int_S \text{tr} (E_{fin} (\xi_{fin} - \text{Ad}_{\wedge} \xi_{in}) + (\underline{\mathcal{D}}^i \partial_u \underline{A}_i)^0 \xi_{in}) \text{vol}_S$$

Abelian: $\Delta\Phi = (\nabla^i \underline{A}_i)^{diff}$
(Bieri-Garfinkle memory*)

* Provided: $F_{ij}^{in} = F_{ij}^{fin} = 0$

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Residual gauge action, soft charges, and superselection

$$\underline{\mathcal{C}} \doteq \mathcal{C}/\mathcal{G}_0 \simeq_{loc} \hat{\mathcal{A}}_{AS} \times T^*\mathcal{G}_S$$

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in general
 $= \text{Ad}_\wedge^{-1} E^{fin} - E^{in}$

Residual gauge action, soft charges, and superselection

$$\underline{\mathcal{C}} \doteq \mathcal{C}/\mathcal{G}_0 \simeq_{loc} \hat{\mathcal{A}}_{AS} \times T^*\mathcal{G}_S$$

Residual gauge transformations

$$\underline{\mathfrak{G}} \doteq \mathfrak{G}/\mathfrak{G}_0 \sim \mathfrak{G}_S \times \mathfrak{G}_S \ni (\xi_{in}, \xi_{fin})$$

with residual action

$$\rho(\xi_{in}, \xi_{fin}) \begin{pmatrix} \underline{A}_j \\ E^{fin} \\ \wedge \end{pmatrix} = \begin{pmatrix} \underline{\mathcal{D}}_j \xi_{in} \\ [E^{fin}, \xi_{fin}] \\ \xi_{fin} \wedge - \wedge \xi_{in} \end{pmatrix}$$

and Hamiltonian generator the **soft charge**

$$\langle \underline{h}, (\xi_{in}, \xi_{fin}) \rangle = \int_S \text{tr} (E_{fin} (\xi_{fin} - \text{Ad}_{\wedge} \xi_{in}) + (\underline{\mathcal{D}}^i \partial_u \underline{A}_i)^0 \xi_{in}) \text{vol}_S$$

Superslections
labeled by coadjoint orbits of
the initial and final* flux

in general

$$= \text{Ad}_{\wedge}^{-1} E^{fin} - E^{in}$$

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Conclusions

- The fully-reduced radiative phase space at Scri is a **Poisson mfd** [AR, Schiavina 2022].
- Its symplectic leaves are the **Flux Superselection Sectors** which are indexed by the gauge orbits of the initial and final electric fluxes.
- **Superselection** is a consequence of “tracing over” the dof in the complementary region w.r.t. a closed Cauchy surface [Bartlett, Rudolph, Spekkens 2007; Gomez, AR 2017-21]. It shows up in entanglement entropy structure [Donnelly 2011; Casini, Huerta, Rosabal 2014; AR in prep.]
- **Memory**: [Bieri, Garfinkle 2014; Pasterski 2017; Pate, Raclariu, Strominger 2017] In the Abelian case, with no magnetic charges, memory is a good SSS label. Not so in the non-Abelian case.
- **Constraint reduction** yields a *semi*-reduced ph.space which is given by the extension of Ashtekar–Streubel by a single T^*G_S (bi-local!).
- This space supports *residual corner gauge symmetries* generated by **soft charges**.
- **Hint?** soft theorems are a statement of “residual corner gauge covariance”, and soft charge “conservation” reread as statement that scattering unfolds within a given superselection sector determined by fluxes “through i^0 ”

THE LEADING TOWERS

REMINDER: $Q_K = \int_{\Sigma_2} d^2x \sqrt{a} (\kappa + b_i K^i) \quad \xi \in \text{BMS}_4$

ELECTRIC $\kappa = \xi^i \pi_i - \xi^{\hat{t}} \varepsilon \quad K^i = \frac{\varepsilon}{2} \xi^i - \frac{1}{8\pi G} (\xi^j * \Psi_j^i + \xi^{\hat{t}} * \chi^i)$

MAGNETIC $\kappa = \xi^i \psi_i - \xi^{\hat{t}} c \quad K^i = \frac{c}{2} \xi^i - \xi^j \Psi_j^i - \xi^{\hat{t}} \chi^i$

SELF-DUAL $\kappa = \xi^i \chi_i \quad K^i = -\xi^j \chi_j^i$

$c, \psi_i, \chi_i, \Psi_{ij}, \chi_i$ - Carrollian Cotton tensors

ε - Bondi mass aspect

π_i - angular momentum aspect

COMPARISON: BMS STANDARD APPROACH [SEE E.G. L. DONNAY' TALK]

$Q_\xi \sim \int_{S^2} dz \wedge d\bar{z} (2M\mathcal{T} + \mathcal{N}\bar{\mathcal{Y}} + \bar{\mathcal{N}}\mathcal{Y}) \rightarrow \text{electric tower}$