Title: Asymptotic algebra of quantum electrodynamics from the BV-BFV perspective

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Abstract: This talk focuses on classical features of asymptotic QED, i.e. the limit of QED at null and time-like infinity. The BV-BFV formalism allows one to view this as a boundary theory of bulk QED and carries a natural notion of what it means to be a symmetry of the model. I will make the connection between this perspective and the earlier findings of Herdegen (JMP 1996) and Strominger et.al. (JHEP 2014) concerning large gauge symmetries in QED. This is based on a joint paper with Michele Schiavina (CMP 2021).

# Asymptotic algebra of quantum electrodynamics from the BV-BFV perspective

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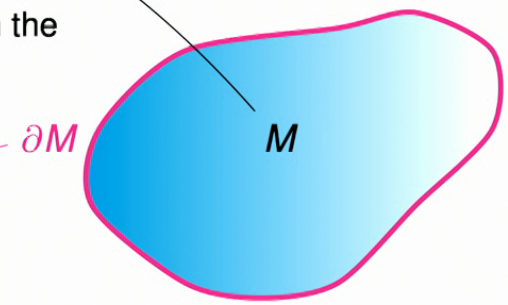
QGAC, 06.10.2022

[1] joint work with Michele Schiavina

# BV-BFV data

- $(\mathcal{F}, \Omega, S, Q)$
  - $(-1)$-symplectic graded manifold $(\mathcal{F}, \Omega)$.
  - Degree 0 action functional $S$
  - An odd vector field $Q$ on $\mathcal{F}$ of degree 1 with the cohomological property $[Q, Q] = 0$.
- $(\mathcal{F}^\partial, \Omega^\partial, S^\partial, Q^\partial)$



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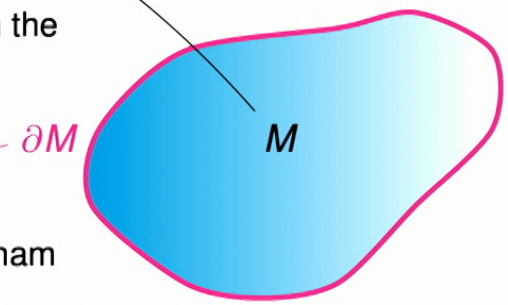
  - $(-1)$-symplectic graded manifold $(\mathcal{F}, \Omega)$.
  - Degree 0 action functional $S$
  - An odd vector field $Q$ on $\mathcal{F}$ of degree 1 with the cohomological property $[Q, Q] = 0$.
- $(\mathcal{F}^\partial, \Omega^\partial, S^\partial, Q^\partial)$
  - Exact (0)-symplectic graded manifold $(\mathcal{F}^\partial, \Omega^\partial = \delta\alpha^\partial)$, where $\delta$ denotes the de Rham differential on the space of local forms,
  - Degree 1 local action functional $S^\partial$ on $\mathcal{F}^\partial$,

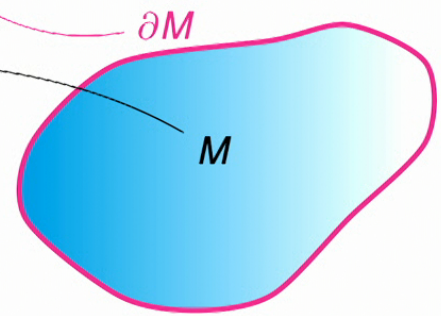


# BV-BFV data

$(\mathcal{F}, \Omega, S, Q)$ and $(\mathcal{F}^\partial, \Omega^\partial, S^\partial, Q^\partial)$ connected by

$$\pi : \mathcal{F} \to \mathcal{F}^\partial$$

such that:



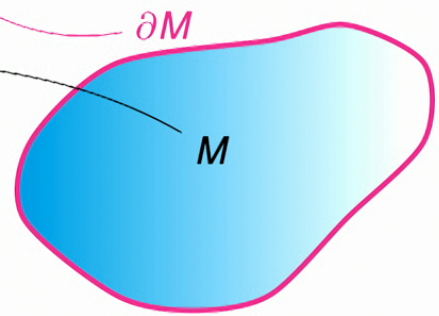
# BV-BFV data

$(\mathcal{F}, \Omega, S, Q)$ and $(\mathcal{F}^\partial, \Omega^\partial, S^\partial, Q^\partial)$ connected by

$$\pi : \mathcal{F} \to \mathcal{F}^\partial$$

such that:

- $\iota_Q \Omega = \delta S + \pi^* \alpha^\partial$
- $\frac{1}{2} \iota_Q \iota_Q \Omega = \pi^* S^\partial$
- $\iota_{Q^\partial} \Omega^\partial = \delta S^\partial$
- $\frac{1}{2} \iota_{Q^\partial} \iota_{Q^\partial} \Omega^\partial = 0$




We can generalize this and assign data to corners, then: $\iota_{Q^\partial} \Omega^\partial = \delta S^\partial + \pi^* \alpha^{\partial\partial}$, $\frac{1}{2} \iota_{Q^\partial} \iota_Q \Omega^\partial = \pi^* S^{\partial\partial}$, etc.

# Generalization to asymptotic boundary

- Take the limit where $\partial M$ is the boundary "at infinity" while imposing falloff conditions on fields.
- Aim: systematic mathematical description of asymptotic observables and symmetries.

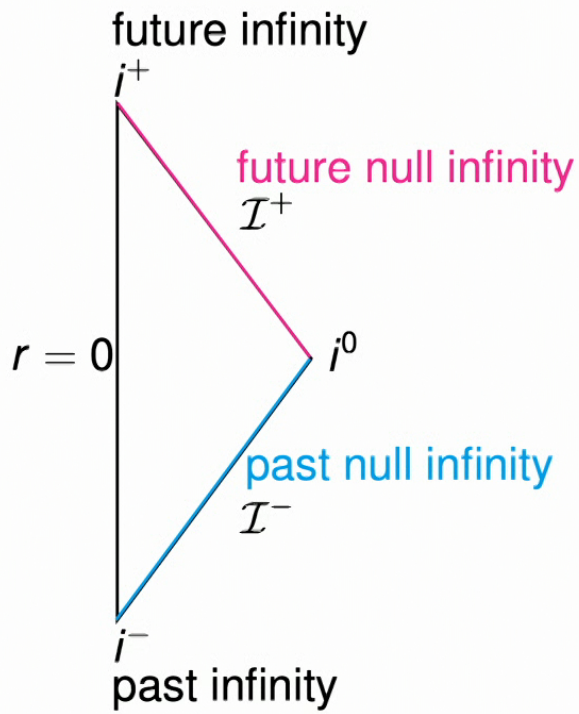
# Some (known and not so known) literature


- A. Herdegen, *Asymptotic algebra for charged particles and radiation*, JMP **37** (1996), 100.
- A. Herdegen, *Semidirect product of CCR and CAR algebras and asymptotic states in quantum electrodynamics*, JMP **39** (1998), 1788–42.
- T. He, P. Mitra, A. P. Porfyriadis, and A. Strominger, *New symmetries of massless QED*, JHEP **10** (2014),112.
- D. Kapec, M. Pate, and A. Strominger, *New symmetries of QED*. Advances in Theoretical and Mathematical Physics **21** (2017), 1769–1785.
- A. Herdegen, *Asymptotic structure of electrodynamics revisited*, LMP 107 (2017) 1439–1470.
- KR and M. Schiavina, *Asymptotic symmetries in the BV-BFV formalism*, CMP **385** (2021), 1083–1132.

# A little controversy...

- In his paper *Asymptotic structure of electrodynamics revisited* (2016), Herdegen claims that there is no new gauge symmetry of QED, since the "large gauge transformations" (LGTs) of Strominger are not symmetries of the asymptotic structure.
- On the other hand, if one applies the Noether procedure to these transformations, one gets conservation laws...
- So are LGTs symmetries or not?
- A neat answer is provided within the BV-BFV framework.

# Asymptotic structure of (scalar) QED




**Variables**:

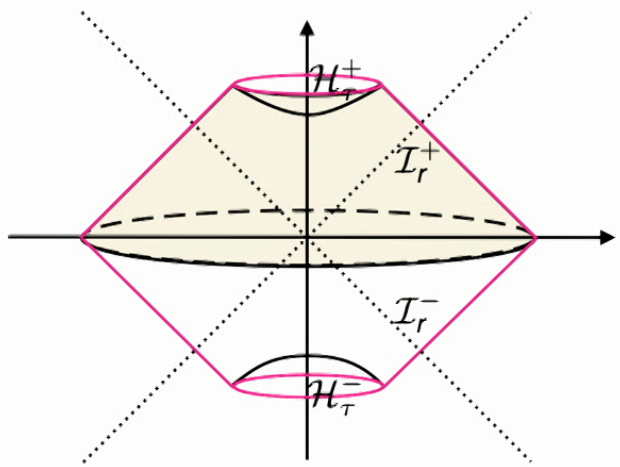
- We focus on Minkowski spacetime $M$ (see the diagram on the left).
- Photons $A \in \Omega^1(M)$ fall off like $1/R$ along null vectors and have asymptotes living on $\mathcal{I}^\pm$.
- Matter fields $\varphi \in \mathcal{C}^\infty(M, \mathbb{C})$ have asymptotes living on $i^\pm$.

# Geometrical setup

Consider the theory of a finite region $\mathcal{W}_R \subset \mathbb{M}$, bounded by a piecewise-null and piecewise spacelike boundary

$$\partial\mathcal{W}_R := \mathcal{I}_r^+ \cup \mathcal{I}_r^- \cup \mathcal{H}_\tau^+ \cup \mathcal{H}_\tau^-$$

with $r$ (radial coordinate) and $\tau$ (hyperbolic time $\tau = \sqrt{t^2 - r^2}$) get scaled by a parameter $R$ that we send to $R \to \infty$.

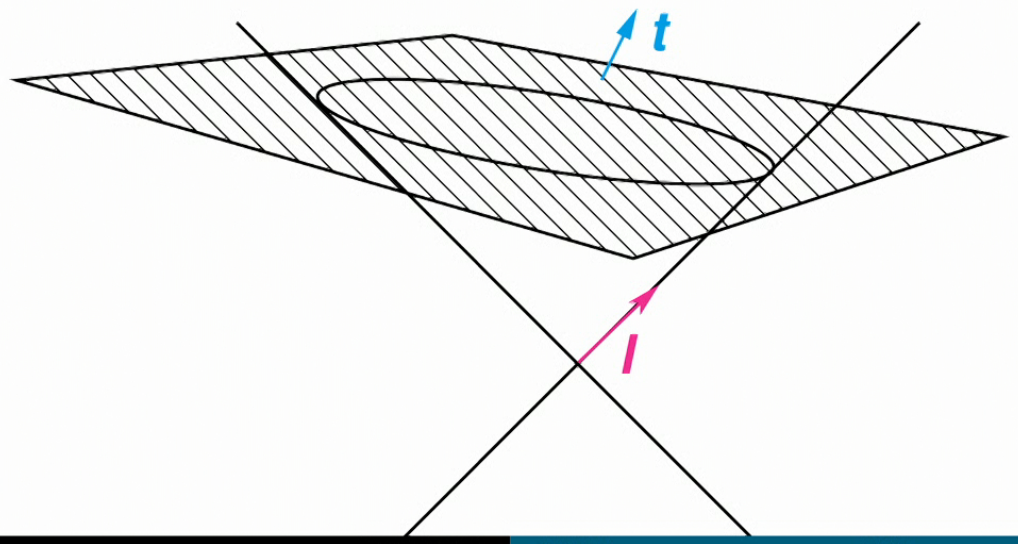


# Asymptotic fields I

- Following Herdegen (JMP 95), we introduce variables $R$, $s$, $\boldsymbol{l}$:

$$x = R\boldsymbol{l} + s\frac{\boldsymbol{t}}{\boldsymbol{t}\cdot\boldsymbol{l}},$$

for a fixed time-like vector $\boldsymbol{t}$.



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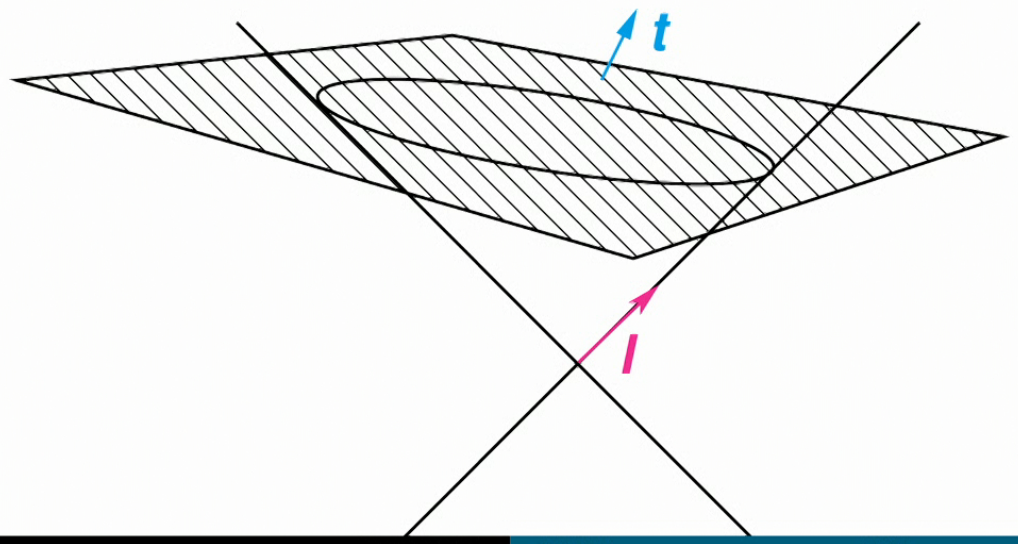
$$x = R\boldsymbol{l} + s\frac{\boldsymbol{t}}{\boldsymbol{t}\cdot\boldsymbol{l}},$$

  for a fixed time-like vector $\boldsymbol{t}$.
- Null asymptotics are obtained by taking $R \to \infty$.



# BV data for scalar QED I

- The canonical shifted symplectic structure $\Omega$ on $\mathcal{F}$ is

$$\Omega = \int_M \delta A \delta A^\ddagger + \delta c \delta c^\ddagger + \delta\varphi\delta\varphi^\ddagger + \delta\overline{\varphi}\delta\overline{\varphi}^\ddagger,$$

- The BV-extended action functional is given by:

$$S = \int_M \left(-\tfrac{1}{8\pi} F_A \wedge \star F_A + \tfrac{1}{2}\left(d_A\overline{\varphi} \wedge \star d_A\varphi + m^2\overline{\varphi}\varphi\right)\right) + \text{antifields}$$

where $d_A\varphi = d\varphi + iqA\varphi$ and $d_A\overline{\varphi} = d\overline{\varphi} - iqA\overline{\varphi}$.

- The BV operator $Q$ is given by

$$QA = d_A c \quad QA^\ddagger = -\frac{1}{4\pi} d_A \star F_A - iq\,\overline{\varphi} \star d_A\varphi + iq \star d_A\overline{\varphi}\varphi$$

$$Q\varphi = c\varphi \qquad Q\varphi^\ddagger = (-d_A \star d_A + m^2)\overline{\varphi} + \varphi^\ddagger c$$

$$Q\overline{\varphi} = -c\overline{\varphi} \qquad Q\overline{\varphi}^\ddagger = (-d_A \star d_A + m^2)\varphi - \overline{\varphi}^\ddagger c$$

$$Qc = 0 \qquad Qc^\ddagger = 0$$

# Equations of motion

- Denote the matter current by $J := -iq\,\overline{\varphi} \star d_A\varphi + iq \star d_A\overline{\varphi}\varphi$.
- The classical equations of motion correspond to the image of $Q$ in degree zero, so on the level of cohomology (after quotienting by the image of $Q$), in other words on-shell, we have that:

$$d_A \star F_A = J$$
$$(-d_A \star d_A + m^2)\varphi = 0$$
$$(-d_A \star d_A + m^2)\overline{\varphi} = 0$$

  Note that $d_A J = 0$ on-shell, since $[F_A, \star F_A] = 0$.
- We assume that $J \sim \tau^{-3}$.

# BV data for scalar QED II

- The boundary action is given by

$$S^\partial = \frac{1}{2}\iota_Q\iota_Q\Omega = \int_M d\left[c\left(-\frac{1}{4\pi}d_A\star F_A + J\right)\right], \qquad (*)$$

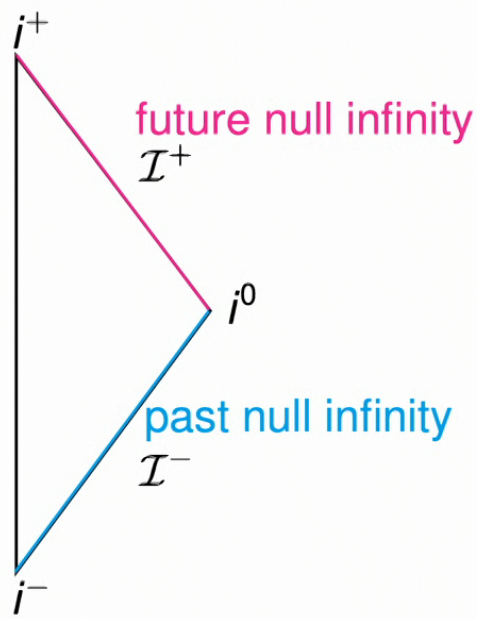
  which is a degree 1 functional on $\mathcal{F}^\partial$.
- $\mathcal{F}^\partial$ is equipped with the symplectic form $\Omega^\partial$ given by

$$\Omega^\partial = \int_{\partial M} \frac{1}{4\pi}\delta A\delta[\star F_A]_{\partial M} + \delta c\delta A^\ddagger + \delta\overline{\varphi}\delta[\star d_A\varphi]_{\partial M} + \delta\varphi\delta[\star d_A\overline{\varphi}]_{\partial M},$$

- The BFV operator $Q^\partial$ is the Hamiltonian vector field of $S^\partial$, i.e.

$$\iota_{Q^\partial}\Omega^\partial = \delta S^\partial.$$

# Asymptotic fields II




**Asymptotic variables**:

- $\lim_{R\to\infty} RA(x+R\boldsymbol{l}) = V(\underbrace{x\cdot\boldsymbol{l}}_{s\in\mathbb{R}},\boldsymbol{l})$
- $\boldsymbol{l}\in S^2$ are null directions and $s$ changes along $\mathcal{I}=\mathcal{I}^-\cup\mathcal{I}^+$
- $\lim_{R\to\infty} RA(x-R\boldsymbol{l}) = V'(\underbrace{x\cdot\boldsymbol{l}}_{s\in\mathbb{R}},\boldsymbol{l})$
- For $J=0$, we have $V(+\infty,\boldsymbol{l}) = V'(-\infty,\boldsymbol{l}) = 0$. In general these asymptotes are given in terms of $J$.
- Matching condition: $V(-\infty,\boldsymbol{l}) = V'(+\infty,\boldsymbol{l})$

# Large gauge transformations

- Start with $A$ in the Lorenz gauge and consider $\hat{A} = A + d\Lambda$, with $\Lambda$ the gauge parameter.
- Assume the following asymptotics at $\mathcal{I}^+$ (analogous on $\mathcal{I}^-$):
  $\Lambda = \epsilon^+(I) + \mathcal{O}(1/R)$ and $d\Lambda = \frac{1}{R} V^{\epsilon^+}(I) + \dots$.
- We also need $\Lambda$ to satisfy Laplace equation on the hyperboloid at timelike infinity (so the asymptote can be reconstructed from the corner data, living on $S^2$)
- Asymptotes of $\Lambda$ have to satisfy a matching condition at $i^0$.
- Observation (due to Herdegen): for non-trivial asymptotic charge to exist, $\hat{A}$ cannot be in the Lorenz gauge.
- Such gauge transformations (often called large gauge transformations) do not preserve the symplectic structure on $\partial M$, but the failure to do so is governed by data assigned to the corner, i.e. $\partial\partial M$.

# The answer is around the corner

- The symplectic structure, at the asymptotic boundary, explicitly given by:

$$\Omega^{\partial} = \int_{\partial M} \frac{1}{4\pi} \delta A \delta[\star F_A]$$

- It is not invariant under LGT, instead, we have:

$$\mathcal{L}_{Q^{\partial}} \Omega^{\partial} = \int_{\partial M} d\left(\delta c \delta[\star F_A]\right) ,$$

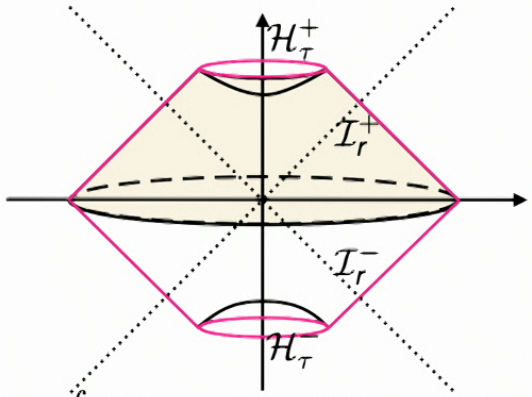
where $Q^{\partial}$ has to be evaluated at $\Lambda$, which is and LGT.

- The right hand side of the above formula can be interpreted as the canonical 1-form on $\mathcal{F}^{\partial\partial}_{\partial\partial M}$ (part of the corner data), denoted by $\Omega^{\partial\partial}$, so we have:

$$\mathcal{L}_{Q^{\partial}} \Omega^{\partial} = \pi^{*} \Omega^{\partial\partial} .$$

# Calculating the charge from the boundary action

$$S^{\partial} = \lim_{R\to\infty} S^{\partial}_{\mathcal{W}_R} = \underbrace{-\frac{1}{4\pi}\lim_{R\to\infty}\int_{\partial\mathcal{W}_R} c d_A \star F_A}_{S^{\partial}_{\text{soft}}} + \underbrace{\lim_{R\to\infty}\int_{\partial\mathcal{W}_R} cJ}_{S^{\partial}_{\text{hard}}},$$




(we used $\int_M d(cd_A \star F_A) = \int_M d(d_A c \wedge \star F_A)$ in (*) to rewrite the first term.)

# Asymptotic fields III

- Recall that $\Box A = 4\pi J$ and there exist unique retarded/advanced Green functions $\Delta^{\mathrm{R/A}}$. Define $A^{\mathrm{R/A}} = 4\pi\Delta^{\mathrm{R/A}}J$ as the retarded/advanced solutions and split:

$$A = A^{\mathrm{R}} + A^{\mathrm{in}} = A^{\mathrm{A}} + A^{\mathrm{out}}$$

- Here incoming/outgoing fields $A^{\mathrm{in/out}}$ are solutions to the homogeneous equation.
- The Pauli-Jordan function is defined by $\Delta = \Delta^{\mathrm{R}} - \Delta^{\mathrm{A}}$. The retarded/advanced solutions are $A^{\mathrm{R/A}} = 4\pi\int\Delta^{\mathrm{R/A}}(x-y)J(y)dy$, so

$$A^{\mathrm{rad}}(x) = 4\pi\int\Delta(x-y)J(y)dy = -\frac{1}{2\pi}\int\dot{V}_J(x\cdot \boldsymbol{l}, \boldsymbol{l})d^2\boldsymbol{l}\,,$$

where $V_J(s,\boldsymbol{l}) = \int dy\delta(s - y\cdot\boldsymbol{l})J(y)$.

# Asymptotic fields IV

- $\lim_{R\to\infty} RA^{\mathrm{R}}(x - R\boldsymbol{l}) = V_J(-\infty, \boldsymbol{l})$, $\lim_{R\to\infty} RA^{\mathrm{A}}(x + R\boldsymbol{l}) = V_J(+\infty, \boldsymbol{l})$.
- For $J \sim \tau^{-3} v\rho_\pm(v)$, we have

$$V_J(\pm\infty, \boldsymbol{l}) = \int_{\mathcal{H}_\pm} \frac{v\rho_\pm(v)}{v\cdot \boldsymbol{l}} d\mu(v)\,,$$

- Hence

$$V(s, \boldsymbol{l}) = V_J(s, \boldsymbol{l}) + V^{\mathrm{in}}(s, \boldsymbol{l}) = V_J(+\infty, \boldsymbol{l}) + V^{\mathrm{out}}(s, \boldsymbol{l})$$
$$V'(s, \boldsymbol{l}) = V_J(-\infty, \boldsymbol{l}) + V^{\mathrm{in}'}(s, \boldsymbol{l}) = V_J(s, \boldsymbol{l}) + V^{\mathrm{out}'}(s, \boldsymbol{l})$$

- So in the corners we have

$$V(+\infty, \boldsymbol{l}) = V_J(+\infty, \boldsymbol{l})\,, \qquad V'(-\infty, \boldsymbol{l}) = V_J(-\infty, \boldsymbol{l})\,,$$

# Soft charge

- The soft contribution to $S^{\partial}$ is:

$$S^{\partial}_{\text{soft}} = -\frac{1}{4\pi} \int\limits_{\mathcal{I}^+\cup\mathcal{I}^-} \lim_{R\to\infty} R^2 d_A c \wedge \star F_A \equiv S^{\partial}_{\text{soft},\mathcal{I}^+} + S^{\partial}_{\text{soft},\mathcal{I}^-} \,,$$

- On-shell we find that:

$$S^{\partial}_{\text{soft},\mathcal{I}^+} \approx \frac{1}{4\pi} \int_{S^2} V^{\epsilon^+}(\boldsymbol{l}) V^{\text{out}}(-\infty, \boldsymbol{l}) d^2\boldsymbol{l} \equiv Q^{\text{soft},+}_{\epsilon^+}$$

$$S^{\partial}_{\text{soft},\mathcal{I}^-} \approx -\frac{1}{4\pi} \int_{S^2} V^{\epsilon^-}(\boldsymbol{l}) V'^{\text{in}}(+\infty, \boldsymbol{l}) d^2\boldsymbol{l} \equiv -Q^{\text{soft},-}_{\epsilon^-}$$

- Here we integrated a total derivative in the $s$ variable and used the fact that $V^{\text{out}}(+\infty, \boldsymbol{l}) = V'^{\text{in}}(-\infty, \boldsymbol{l}) = 0$.
- As a function of $\Lambda$ this is the flux map from Michele's talk (electric fluxes through the corners):

$$-4\pi S^{\partial,\text{soft}}_{\mathcal{I}^+\cup\mathcal{I}^-}[\Lambda] = -4\pi \left[Q^{+\text{soft}}_{\epsilon} - Q^{-\text{soft}}_{\epsilon}\right]$$

$$\approx -\int_{\mathcal{I}^+} du_+ \gamma_{z\bar{z}} dz d\bar{z} \Lambda \partial_z(\gamma^{z\bar{z}} F^{(0)}_{u_+\bar{z}}) + \int_{\mathcal{I}^-} du_- \gamma_{z\bar{z}} dz d\bar{z} \Lambda \partial_z(\gamma^{z\bar{z}} F^{(0)}_{u_-\bar{z}}) + c.c.$$

# Hard charge I

- The hard charge is computed in the similar way, by splititng

$$\lim_{R\to\infty} \int_{\partial\mathcal{W}_R} cJ$$

into contributions from $\mathcal{H}^+$ and $\mathcal{H}^-$, the hyperboloids at future/past timelike infinity.

$$S^{\partial,\text{hard}}_{\mathcal{H}^+\cup\mathcal{H}^-}[\Lambda] = \int_{\mathcal{H}^+} \Lambda_{\mathcal{H}^+}(v)\rho_+(v)dv - \int_{\mathcal{H}^-} \Lambda_{\mathcal{H}^-}(v)\rho_-(v)dv\,,$$

- Using the fact that $\Lambda_{\mathcal{H}^+}$ solves Laplace's equation on $\mathcal{H}^+$ with corner data given by $\varepsilon^+$, we can write

$$\Lambda_{\mathcal{H}^+}(v) = \int G(v,\boldsymbol{l})\varepsilon^+(\boldsymbol{l})d^2\boldsymbol{l}\,,$$

where $G$ is the Green function.

# Total charge

- The matching conditions (at spacelike infinity $i^0$) for the asymptotes of $\Lambda$ imply that $\epsilon^+(\boldsymbol{l}) = \epsilon^-(\boldsymbol{l}) \equiv \epsilon(\boldsymbol{l})$.
- The total hard charge is: $Q_\varepsilon^{\text{hard}} = Q_\varepsilon^{\text{hard}+} - Q_\varepsilon^{\text{hard}-}$.
- Recall

$$V(-\infty, \boldsymbol{l}) = V_J(+\infty, \boldsymbol{l}) + V^{\text{out}}(-\infty, \boldsymbol{l}),$$
$$V'(+\infty, \boldsymbol{l}) = V_J(-\infty, \boldsymbol{l}) + V^{\text{in}'}(+\infty, \boldsymbol{l}),$$

- Defining $Q_\epsilon^\pm \doteq Q_{\epsilon^\pm}^{\text{soft},\pm} + Q_{\epsilon^\pm}^{\text{hard},\pm}$, we conclude that

$$Q_\varepsilon^+ = \frac{1}{4\pi}\int d^2\boldsymbol{l}\, V^\varepsilon(\boldsymbol{l})\, V(-\infty, \boldsymbol{l})$$
$$Q_\varepsilon^- = \frac{1}{4\pi}\int d^2\boldsymbol{l}\, V^\varepsilon(\boldsymbol{l})\, V'(+\infty, \boldsymbol{l}),$$

- Crucially, we have

$$S^\partial \approx 0 \Leftrightarrow Q_\epsilon^+ \approx Q_\epsilon^- \Leftrightarrow V(-\infty, \boldsymbol{l}) = V'(+\infty, \boldsymbol{l}).$$

Thank you very much for your attention!