Title: Cutting Corners with Celestial CFT

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Collection: Quantum Gravity Around the Corner

Date: October 06, 2022 - 2:15 PM

URL: https://pirsa.org/22100022

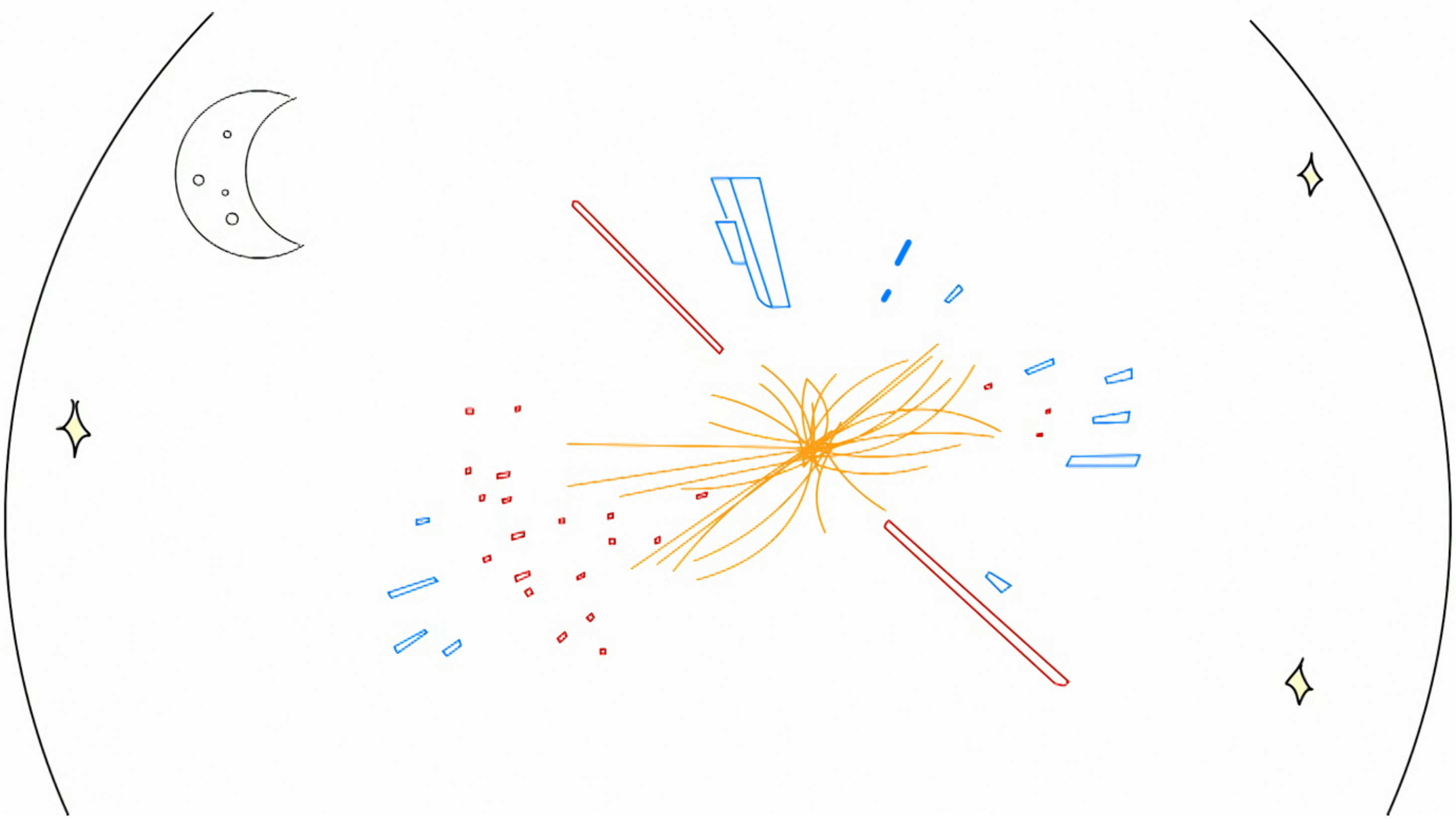
Abstract: Celestial Holography proposes a duality between gravitational scattering in asymptotically flat spacetimes and a conformal field theory living on the celestial sphere. The main motivation for this program comes from the connection between soft theorems and asymptotic symmetries in asymptotically flat spacetimes, and our ability to recast soft operators as currents in a codimension 2 CFT.

We present a streamlined route from asymptotic symmetries in 4D asymptotically flat spacetimes to currents in a 2D celestial CFT in a manner that makes their relation to QFT soft theorems manifest. We use this to re-examine the bulk picture of radial evolution in the 2D theory and reconcile the construction of charges in CCFT with the more familiar construction from AdS/CFT, despite the differing codimension.

In the process, we point out how the Carrollian and Celestial perspectives amount to slicing the bulk and boundary in different ways -- our celestial slices cut through the usual corner! -- and emphasize how these perspectives inform each other.

Based on 2201.06805, 2202.11127 and 2205.10901

Celestial Holography proposes a duality between gravitational scattering in asymptotically flat spacetimes and a CFT living on the celestial sphere.



Synopsis

The advantage of this program is that it reorganizes scattering in terms of symmetries.

In particular, an ∞-dimensional enhancement coming from the asymptotic symmetry group.

The symmetries are further enhanced when we consider all conformally soft modes.

These additional symmetries should put interesting constraints on the S-matrix.

The radial quantization perspective plays a crucial role in each case.

Outline

1. Extrapolate Dictionary
2. Celestial Amplitudes
3. Radial = Rinder
4. Soft Thm = Ward Id
5. Fluxes as Charges
6. Soft and Hard Sectors
7. Towers of Symmetries
8. Celestial vs Carrollian

$$e^{ipX} = e^{-ip^0 u - ip^0 r(1-\cos\theta)} \rightarrow e^{-ip^0 u} \times \frac{1}{p^0 r} \frac{\delta(\theta)}{\sin\theta}$$

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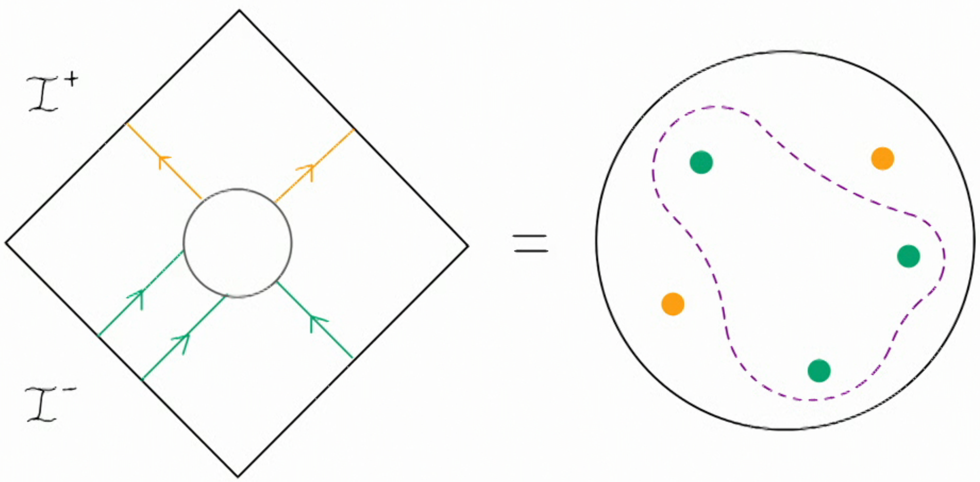
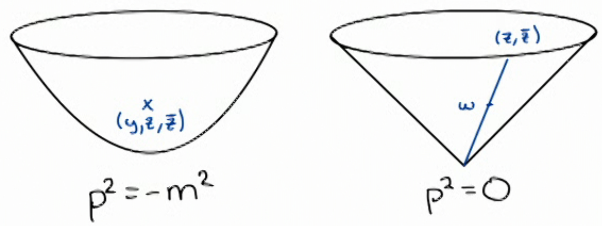
Goal: Apply the holographic principle to $\Lambda=0$ quantum gravity.

Plan: Celestial Holography proposes that the natural dual system lives on the celestial sphere.

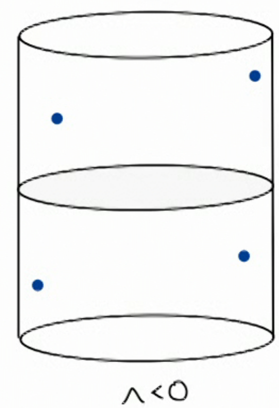


We can merge aspects of the two standard precedents for our hologram...

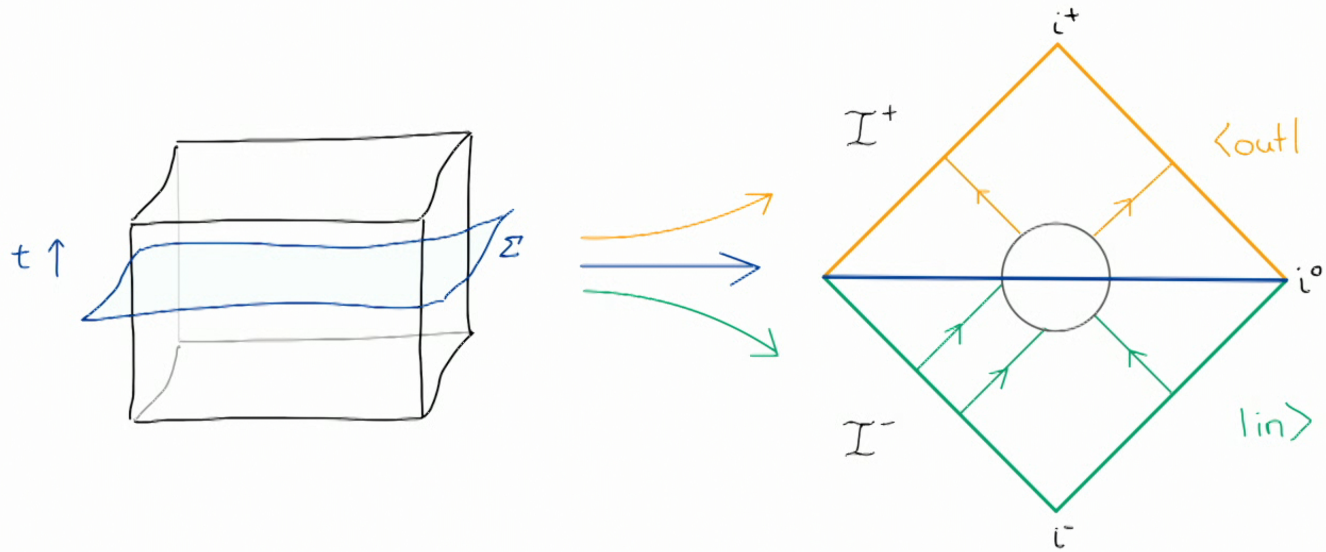
$\langle \text{out} | \mathcal{S} | \text{in} \rangle$



AdS/CFT



... by pushing our Cauchy slice to scri to prepare the in and out states with operators on the boundary.

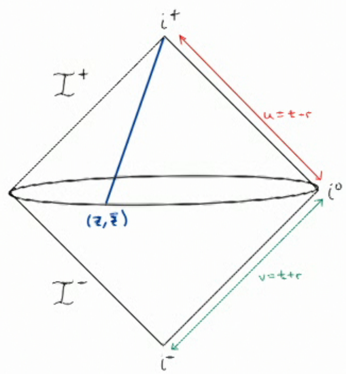


For example taking $r \to \infty$, u- fixed the plane wave localizes...

$$e^{ip\cdot X} = e^{-ip^0 u - ip^0 r(1-\cos\theta)} \longrightarrow e^{-ip^0 u} \times \frac{i}{p^0 r}\,\frac{\delta(\theta)}{\sin\theta}$$

... and we can prepare an m=0 momentum eigenstate with the boundary limit of our bulk operator smeared on a generator of $\mathcal{J}^+$.

$$h_{\mu\nu} = \sum_{\alpha=\pm} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p^0} \left[\varepsilon^{\alpha *}_{\mu\nu} a_\alpha e^{ip\cdot x} + \varepsilon^{\alpha}_{\mu\nu} a^\dagger_\alpha e^{-ip\cdot x} \right]$$



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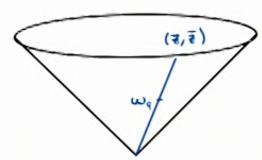
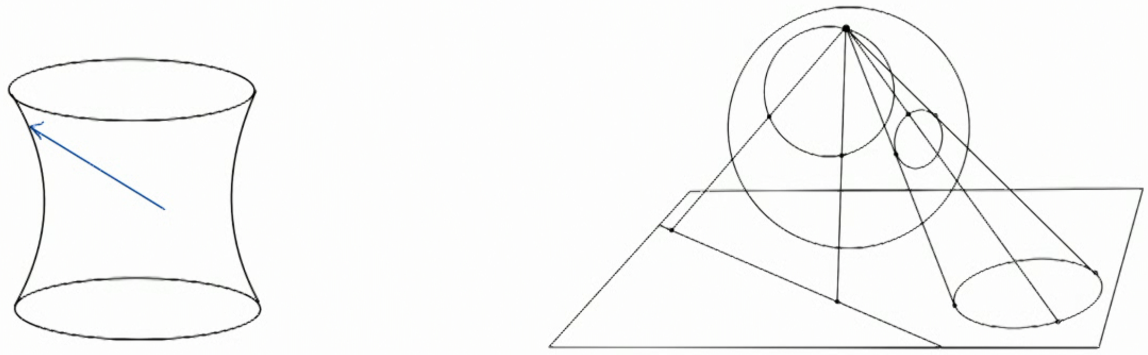


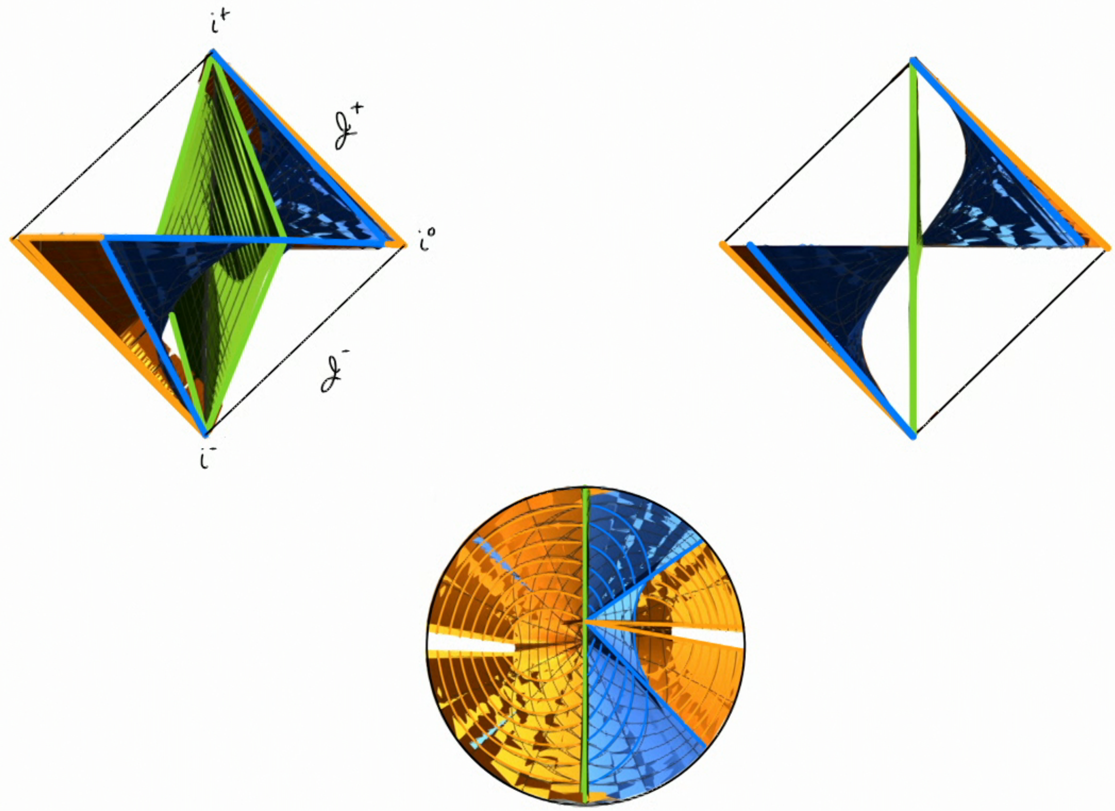
$p^2=0$

We can use this to see how the features of the S-matrix lift to the spacetime picture and vice versa.

An intuitive way to see this is to look at the boost images of both. In the bulk we sweep out hyperplanes through $X^{\mu}=0$ with space-like normal.



On the celestial sphere circles map to circles under Mobius transformations.


i^+
$\mathscr{I}^+$
i^0
$\mathscr{I}^-$
i^-

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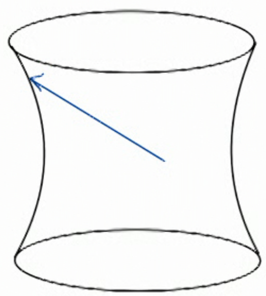
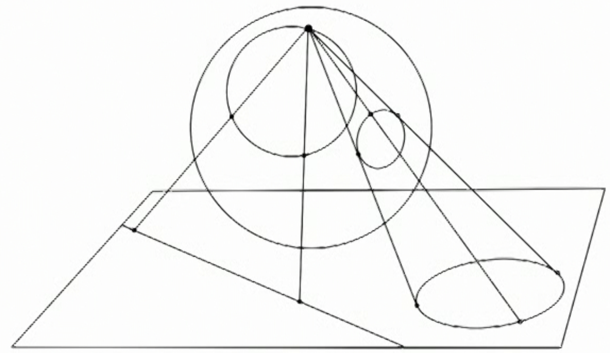




On the celestial sphere circles map to circles under Mobius transformations.

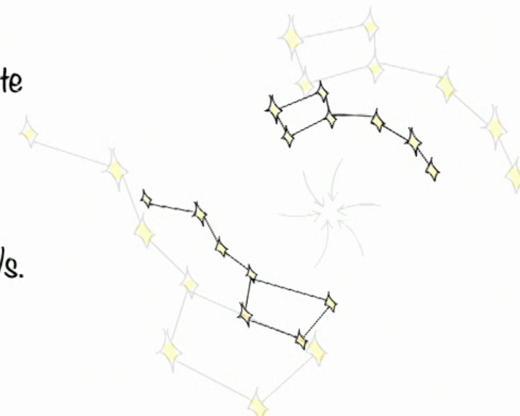
Soft Thm = Ward Id

Residual diffeomorphisms that preserve the falloffs and act non-trivially on the asymptotic data are part of the Asymptotic Symmetry Group.

$$ASG = \frac{\text{Allowed Symmetries}}{\text{Trivial Symmetries}}$$

The ASG will be much larger than the group of isometries of any given spacetime within this class.

	Poincaré	$\subset$	BMS
# generators:	10		∞



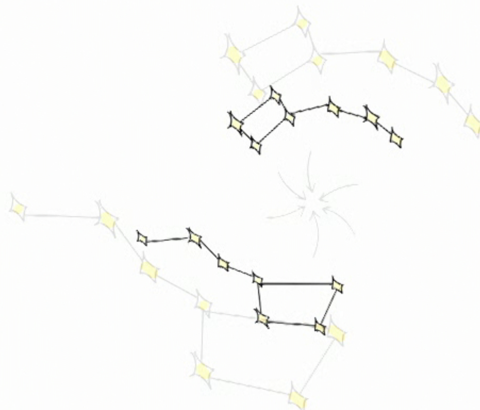
- Supertranslations induce angle-dependent shifts in the time coordinate

$$\xi|_{\mathcal{I}^+} = f(z,\bar{z})\,\partial_u$$

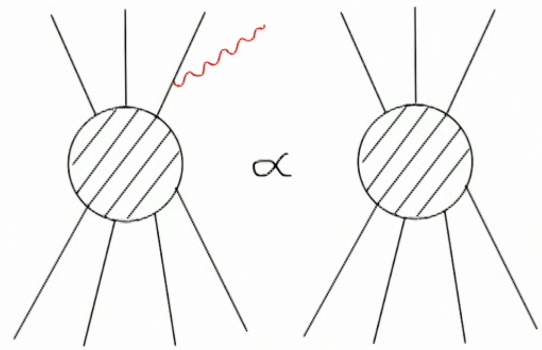
- Superrotations extend global conformal transformations to local CKVs.

$$\xi|_{\mathcal{I}^+} = Y^z(z)\,\partial_z + \frac{u}{2} D_z Y^z(z)\,\partial_u + c.c.$$

Once we've identified the residual gauge parameter there are quite a few steps to show



ASG Ward Id. = Soft Thm.



which is the starting point of the celestial holography program since these Ward identities further look like ones in a 2D CFT.

1. Take a Cauchy slice

2. Push it down to $\mathcal{I}^-$ to define $|in\rangle$

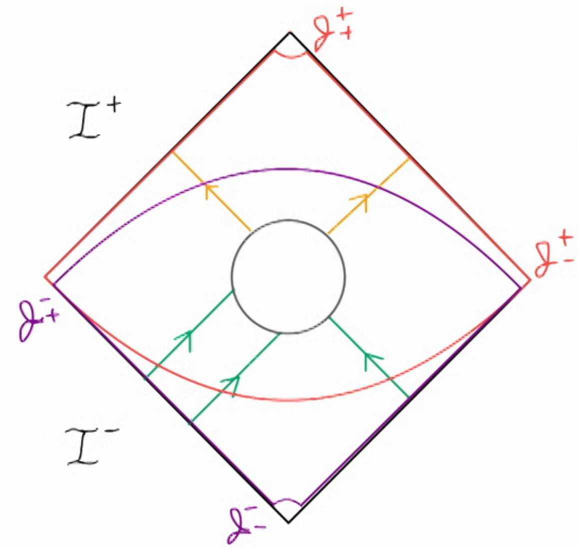
3. Push it up to $\mathcal{I}^+$ to define $\langle out|$

4. Evaluate the canonical charges on both slices

5. Use antipodal matching to equate $\mathcal{I}^+_-$ & $\mathcal{I}^-_+$ data

6. Use the constraint eq. & ibp to write Q as a flux

7. Insert this operator in S-matrix elements and use the soft thm to show



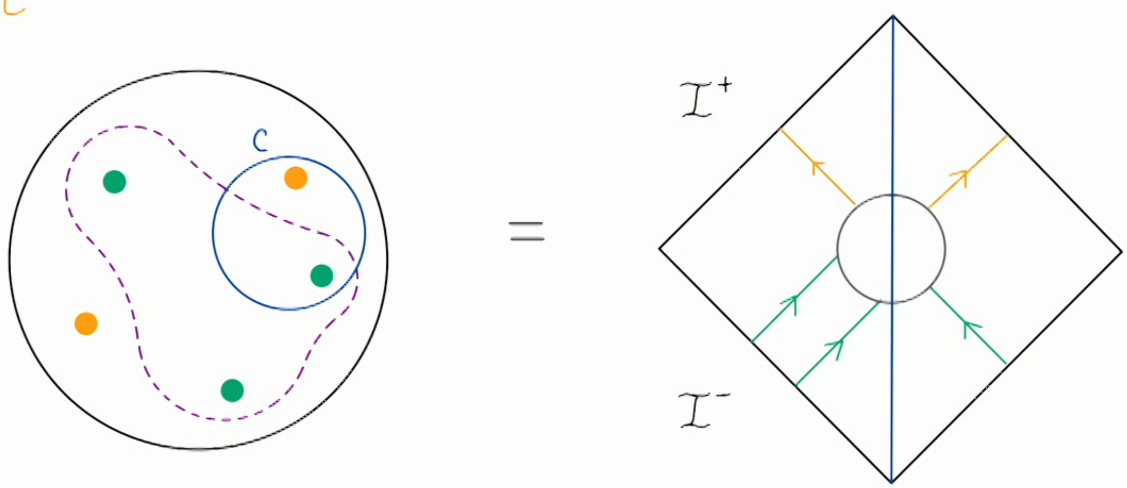
$$\langle out| (Q_S^+ + Q_H^+)\mathcal{S} - \mathcal{S}(Q_S^- + Q_H^-)|in\rangle = 0 .$$

The charges in gauge theory are co-dimension 2. If I lift a path C on the celestial sphere to a co-dimension 1 hypersurface Σ_C of the bulk that runs along the generators of $\mathscr{I}$ I have

$$\mathcal{J}_C(\lambda) = \int_{\partial\Sigma_C} \star\, \kappa(\lambda)$$

charge generating transformation on Celestial State defined on contour C

bulk 'charge' evaluated on hypersurface Σ_C



Let us use the U(1) case as an example. The gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \lambda$$

is generated by the canonical charge with 2-form

$$\kappa = \lambda F \qquad F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$$

For an ordinary Cauchy slice $\partial\Sigma = S^2 @ i^0$

$$Q^+(\lambda) = \lim_{r\to\infty} \int_{S^2} d^2 z \sqrt{\gamma}\, \lambda\, (r^2 F_{ru}) \quad \rightarrow 7 \text{ steps}$$

By contrast if we pull back this form to the bndy Σ_C we have

$$Q^C(\lambda) = -i \int du \oint_C dx^A \lambda\, \epsilon_{AC} \gamma^{CB} F_{Bu} + \mathcal{I}^- \text{ contrib.}$$

can evaluate with the soft thm. directly

On the equations of motion

$$\partial_\nu K^{\nu\mu} \overset{w}{=} j^\mu \Rightarrow \int_{\partial R} \star K = \int_R \star j$$

where j^μ is the Noether current

$$j_\mu = F_{\nu\mu}\partial^\nu \lambda + \lambda J^M_\mu$$

If we apply this to slices pushed to the conformal boundary

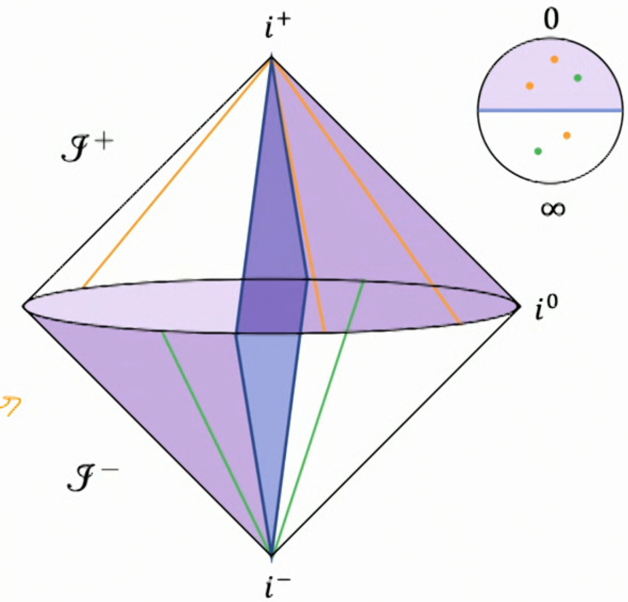
$$\partial M = \Sigma_{in} \cup \Sigma_{out} = \Sigma_N \cup \Sigma_S$$

we have a natural split into hard and soft contributions on $\mathcal{J}^\pm$

Gauss's law on this surface

relates soft radiation & external

charge kinematics



The radial = Rindler picture let's us jump directly to the statement that soft operators in the bulk generate symmetries on the celestial sphere.

We also see that our co-dimension 2 hologram is consistent with AdS/CFT expectations for how bulk and boundary symmetries are related.

Can we further see why this basis is a better way to present the flat space hologram?

$$Q_S + Q_H = 0$$

Let us use the U(1) case as an example. The gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \lambda$$

is generated by the canonical charge with 2-form

$$k = \lambda F \qquad F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$$

For an ordinary Cauchy slice $\partial\Sigma = S^2 @ i^0$

$$Q^+(\lambda) = \lim_{r\to\infty} \int_{S^2} d^2 z \sqrt{\gamma}\, \lambda\, (r^2 F_{ru}) \quad \to 7 \text{ steps}$$

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can evaluate with the soft thm. directly

The choice $\lambda = \frac{1}{z-\omega}$ picks out a celestial current

$$j = j^+ + j^- \qquad j^+ = -4\pi \int du\, F_{uz}$$

generating a U(1) Kac-Moody symmetry

$$\langle j(z)\, \mathcal{O}_1(z_1,\bar{z}_1)\ldots \mathcal{O}_n(z_n,\bar{z}_n)\rangle = \sum_i \frac{Q_i}{z-z_i} \langle \mathcal{O}_1(z_1,\bar{z}_1)\ldots \mathcal{O}_n(z_n,\bar{z}_n)\rangle$$

This uses a shadow redundancy. In 4D we can also isolate a single helicity term using the magnetic dual

$$Q_{(A)SD} = \frac{1}{2}(Q \mp i\tilde{Q})$$

$$Q_S + Q_H = 0$$

Decompose into reps of:

$SL(2, \mathbb{C})$

Virasoro

BMS

...

built from spin-s radiative

built from matter + $\omega \neq 0$

Consider the $SL(2,\mathbb{C})$ multiplet structure. A primary state:

$$L_1|h,\bar h\rangle = \bar L_1|h,\bar h\rangle = 0 \qquad h=\tfrac{1}{2}(\Delta+J) \quad \bar h = \tfrac{1}{2}(\Delta-J)$$

will have a primary descendant at level-k when:

$$L_1(L_{-1})^k|h,\bar h\rangle = -k\overbrace{(2h+k-1)}^{0}(L_{-1})^{k-1}|h,\bar h\rangle = 0$$

similarly for $\bar L_{-1}$. When both conditions are met we get nested primaries:

$$\begin{tikzcd}
 & \left|\frac{1-k}{2},\frac{1-\bar k}{2}\right\rangle \arrow[dl, "(\bar L_{-1})^{\bar k}"'] \arrow[dr, "(L_{-1})^{k}"] & \\
\left|\frac{1-k}{2},\frac{1+\bar k}{2}\right\rangle \arrow[dr] & & \left|\frac{1+k}{2},\frac{1-\bar k}{2}\right\rangle \arrow[dl] \\
 & \left|\frac{1+k}{2},\frac{1+\bar k}{2}\right\rangle &
\end{tikzcd}$$

$k,\bar k \in \mathbb{Z}_{>}$

The statement that a primary descendant decouples gives a conserved current!

Lets stick to the linearized case and consider the subleading soft graviton.

Spin Memory

$$
\begin{tikzcd}
\Delta=-1 & & \varepsilon \arrow[dl] \arrow[dr] & \\
& T \arrow[dr] & & \lim\limits_{\Delta\to 0}\Delta\mathcal{O}_{\Delta,+2} := S \arrow[dl] \\
\Delta=3 & & Q_S\big|_{Y\propto\delta^{(2)}(z-w)} &
\end{tikzcd}
$$

Superrotations

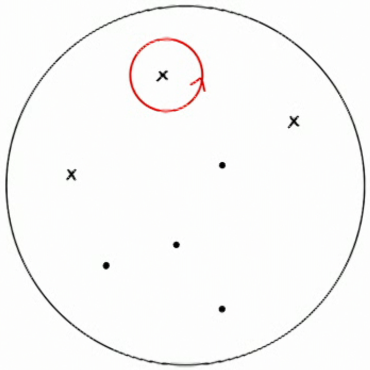
$$
\begin{tikzcd}
& \bar{Y}^{\bar{z}} \arrow[dl] \arrow[dr] & \\
\Theta_{\bar{z}\bar{z}} := N^{vac}_{\bar{z}\bar{z}} \arrow[dr] & & \partial\bar{Y} \arrow[dl] \\
& \partial\bar{\partial}^3\bar{Y} &
\end{tikzcd}
$$

The main interest in the boost basis comes from how these states couple to the subleading soft graviton.

$$T_{zz} \propto \int d^2z' \frac{1}{(z-z')^4} \lim_{\omega \to 0} (1+\omega\partial_\omega)[a_-(\omega\hat{x}') - a_+^\dagger(\omega\hat{x}')]$$

Our quasi-primaries get promoted to Virasoro primaries when we turn on gravity.

$$\langle T_{zz} \mathcal{O}_1^+ \dots \mathcal{O}_n^- \rangle = \sum_k \left[\frac{h_k}{(z-z_k)^2} + \frac{\partial_{z_k}}{z-z_k} \right] \langle \mathcal{O}_1^+ \dots \mathcal{O}_n^- \rangle$$



The candidate CCFT stress tensor corresponds to a memory observable symplectically paired with the superrotation Goldstone mode

$$\bar{T}^{CFT}_{\bar{z}\bar{z}} = \frac{-3i}{8\pi G}\int d^2z\sqrt{\gamma}\,\frac{1}{(\bar{z}-\bar{w})^4}\underline{\int du\,u\,\partial_u C^{\bar{z}\bar{z}}} \qquad C^{vac}_{\bar{z}\bar{z}} = (u+C)\,\underline{\Theta_{\bar{z}\bar{z}}} - 2D_{\bar{z}}^2 C$$

Demanding the soft and hard operators transforms covariantly under the full ASG as a conformal field with these weights provides higher order corrections.

leading soft graviton

$$\bar{T} = \oint \frac{dz}{2\pi i}\, J_{soft}(z,\bar{z}) \qquad J_{soft} = -\bar{\mathcal{D}}^3 N^{(1)} - \bar{\mathcal{D}}^3 \mathcal{C} N^{(0)} - 3\bar{\mathcal{D}}^2\mathcal{C}\,\bar{\mathcal{D}} N^{(0)}$$

subleading soft graviton

where

$$\mathcal{D}\phi_{h,\bar{h}} = [D_z - h\partial\Phi]\,\phi_{h,\bar{h}} \qquad \Theta_{zz} = \frac{1}{2}D_z\Phi D_z\Phi - D_z^2\Phi$$

[Donnay, Ruzziconi '21]

Now we commonly considered scattering around the trivial superrotation vacuum

$$\Theta_{zz} = 0 \implies \mathcal{D} \longrightarrow D_z$$

The remaining terms almost look like the proposed loop correction

$$\Delta \bar{T} \propto \underbrace{\frac{1}{\varepsilon}}_{\text{IR div.}} \underbrace{\int d^2w \frac{1}{z-w}}_{\text{contour}} \Big(\underbrace{\bar{\mathcal{D}} \bar{N}^{(0)} N^{(0)} + 3 \bar{N}^{(0)} \bar{\mathcal{D}} N^{(0)}}_{\text{memory vs. Goldstone}} \Big)$$

[He, Kapec, Raclariu, Strominger '17]

Trust the symmetries!

That derivation predated our understanding of the supertranslation vertex operators.

$$\mathcal{O}_k = W_k \tilde{\mathcal{O}}_k \qquad W_k = e^{i\eta_k \omega_k C(z_k, \bar{z}_k)}$$

This leading soft sector exhibits a Kac-Moody like symmetry

$$P = \frac{1}{4G}\bar{\mathcal{D}} N^{(0)}, \quad \tilde{P} = i\mathcal{D}C : \quad P_z P_w \sim 0 \quad P_z \tilde{P}_w \sim \frac{1}{(z-w)^2} \quad \tilde{P}_z \tilde{P}_w \sim -\frac{1}{\varepsilon}\frac{G}{\pi}\frac{\bar{z}-\bar{w}}{z-w}$$

[Himwich, Narayanan, Pate, Paul, Strominger '20]

The off diagonal level structure implies an ambiguity when there are no other soft insertions

$$\mathcal{D}^2 C \Leftrightarrow -\frac{1}{2\pi\varepsilon} N^{(0)}$$

Similar to our $T^{CFT} \propto \Theta$ sectors!

Towers of Symmetries

Consider the $SL(2,\mathbb{C})$ multiplet structure. A primary state:

$$L_1|h,\bar{h}\rangle = \bar{L}_1|h,\bar{h}\rangle = 0 \qquad h=\tfrac{1}{2}(\Delta+J) \quad \bar{h}=\tfrac{1}{2}(\Delta-J)$$

will have a primary descendant at level-k when:

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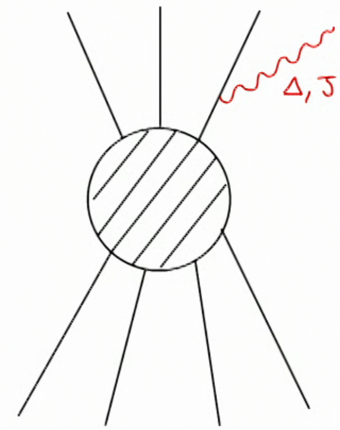
\end{tikzcd}$$

$k,\bar{k} \in \mathbb{Z}_{>}$

The statement that primary descendants decouple gives us a tower of symmetries!

Momentum space soft theorems give poles in the celestial amplitude

$$\lim_{\Delta \to -n} (\Delta + n) \int_0^\infty d\omega\, \omega^{\Delta - 1} \sum_k \omega^k A^{(k)} = A^{(n)}$$



Factorizations at various orders in $\omega \sim 0$ imply factorizations of the residues as $\Delta \to$ special values

$$\langle \text{out} | a_-(\omega q)\, S | \text{in} \rangle = \sum_k \left(\frac{1}{\omega} \frac{(p_k \cdot \varepsilon^-)^2}{p_k \cdot q} - i \frac{p_k \cdot \varepsilon^- \varepsilon^{-\nu} q^\lambda J_{k\lambda\nu}}{p_k \cdot q} \right) \langle \text{out} | S | \text{in} \rangle + \mathcal{O}(\omega)$$

$\hookrightarrow \frac{1}{\Delta - 1}$ $\hookrightarrow \frac{1}{\Delta}$

This can yield a much richer symmetry structure. For instance the single helicity sector of gravity

$$H^{-2p+4}(z,\bar{z}) = \lim_{\varepsilon\to 0} \varepsilon\, \mathcal{O}_{-2p+4+\varepsilon,2}(z,\bar{z}) = \sum_{n=1-p}^{p-1} \frac{\bar{z}^{p-n-1}}{\Gamma(p-n)\Gamma(p+n)} w_n^p(z)$$

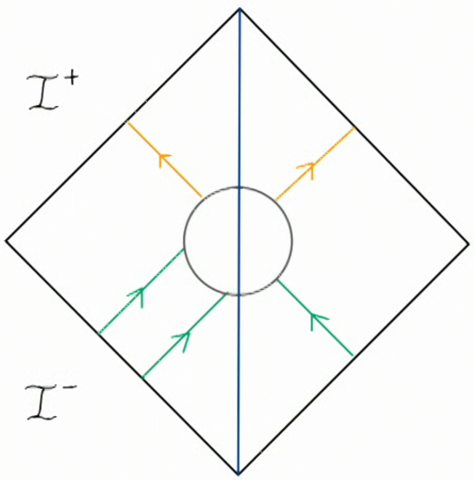
exhibits an $Lw_{1+\infty}$ symmetry

$$\hat{w}_n^p = \oint \frac{dz}{2\pi i} w_n^p(z) \qquad [\hat{w}_m^p, \hat{w}_n^q] = [m(q-1)-n(p-1)]\hat{w}_{m+n}^{p+q-2}$$

where these commutators are computed from the celestial OPEs using a complexification of the standard radial quantization prescription.

Celestial vs Carrollian

As we've emphasized here the Celestial program looks at Rindler evolution of the bulk and boundary.



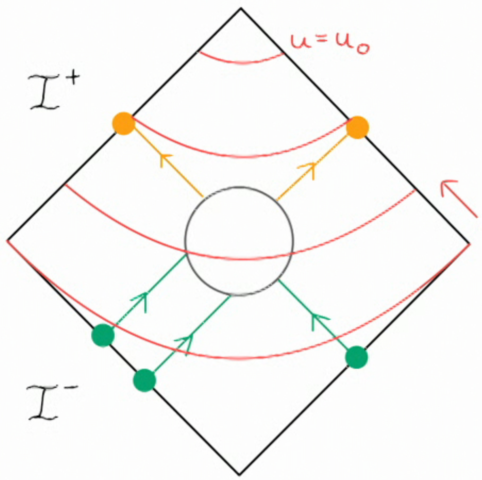
Rindler = radial

Benefits

- Soft thms are canonical charges
- In and out come together antipodally
- Decoupling of primary descendants implies infinite tower of symmetries

vs checking universality of $\omega \to 0$ expansion
+ augmenting phase space w/ overleading

By contrast, if we Fourier transform our in and out states we have correlators on a null 3-manifold.



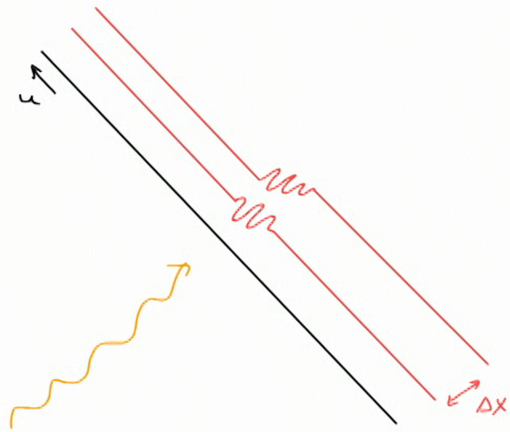
Issues

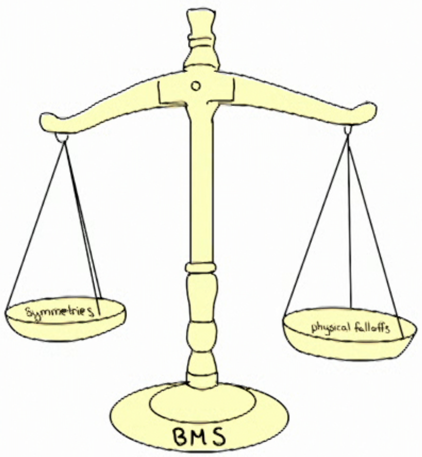
- Connecting in and out
- Null time coordinate
- Charge non-conservation

Raju et. al & Carrollian perspectives

The position basis lends itself to

- going from boundary to bulk
- ASG identification
- memory effect experiments





Is it physical to introduce over leading gauge transformations?

Synopsis

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In particular, an ∞-dimensional enhancement coming from the asymptotic symmetry group.

The symmetries are further enhanced when we consider all conformally soft modes.

These additional symmetries should put interesting constraints on the S-matrix.

The radial quantization perspective plays a crucial role in each case.