Title: Horizons, fluid and what non-linearities might await

Speakers: Luis Lehner

Collection: Quantum Gravity Around the Corner

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Abstract: We analyze, first from a geometric point of view, the behavior
of dynamical horizons. We then connect with Carrolian fluids and discuss potential phenomena stemming from non-linearities
in the resulting equations
[This is joint work with Jaime Redondo Yuste]

Horizons, fluids and what non-linearities might away

Luis Lehner
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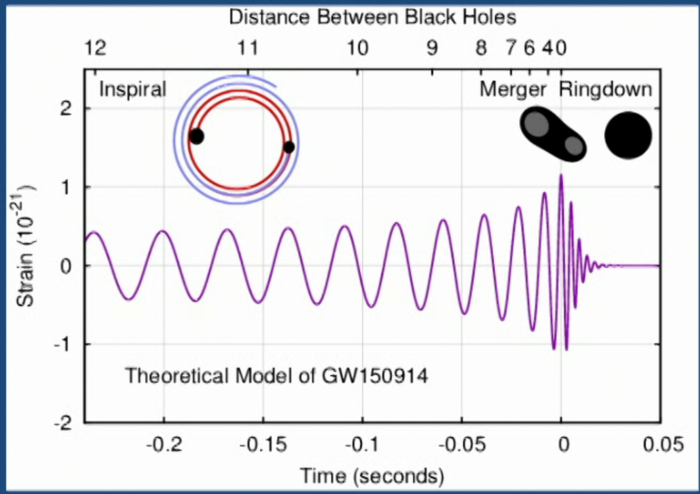
Outline

- Motivation
- Background observations
- Carrollian hydro <-> horizons
- some open qns

Motivations

- Gravitational waves and their simplicity



- Hydrodynamics and inverse cascade

2+1 hydrodynamics, enstrophy is special

[wikipedia]

In the case that the flow is incompressible, or equivalently that $\nabla \cdot \mathbf{u} = 0$, the enstrophy can be described as the integral of the square of the vorticity ω,[2]

$$\mathcal{E}(\boldsymbol{\omega}) \equiv \int_{\Omega} |\boldsymbol{\omega}|^2 \, dx$$

or, in terms of the flow velocity,

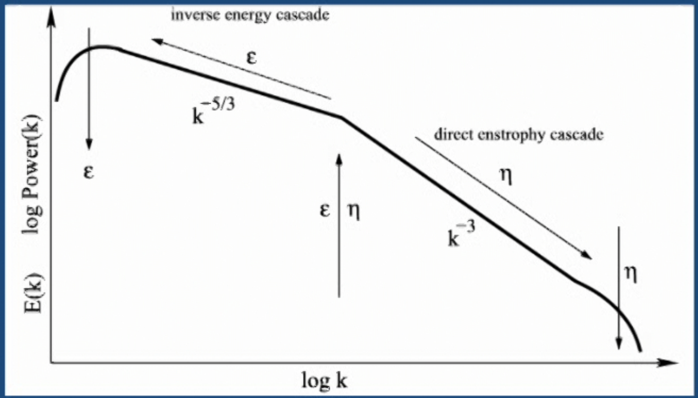
$$\mathcal{E}(\mathbf{u}) \equiv \int_{S} |\nabla \times \mathbf{u}|^2 \, dS \,.$$

In the context of the incompressible Navier-Stokes equations, enstrophy appears in the following useful result[1]

$$\frac{d}{dt}\left(\frac{1}{2}\int_{\Omega} |\mathbf{u}|^2\right) = -\nu\mathcal{E}(\mathbf{u}).$$

The quantity in parentheses on the left is the energy in the flow, so the result says that energy declines proportional to the kinematic viscosity ν times the enstrophy.

→ energy cascades primarily to the IR



[Ringler]

Fluid-gravity correspondence (in aAdS)

- But, AdS/CFT motivates (& make ‘rigorous’) a fluid/gravity correspondence, previously identified through the ‘membrane paradigm programme’ [Damour, Price-Thorne-McDonald]
  - Relies on a ‘non-standard’ [in GR] perturbative approach

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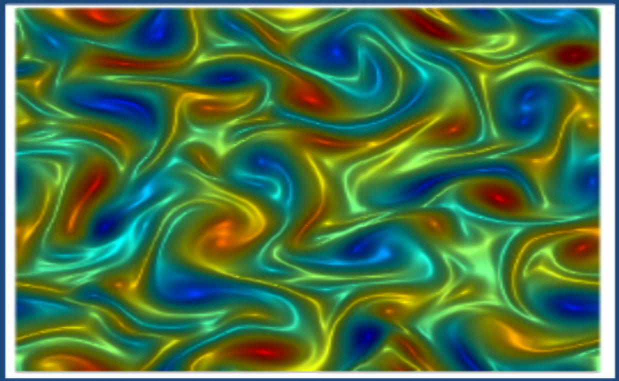
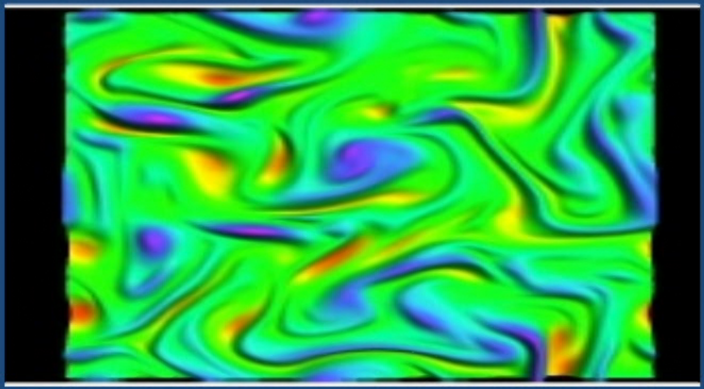


[Adams,Chesler,Liu. PRL 2014]



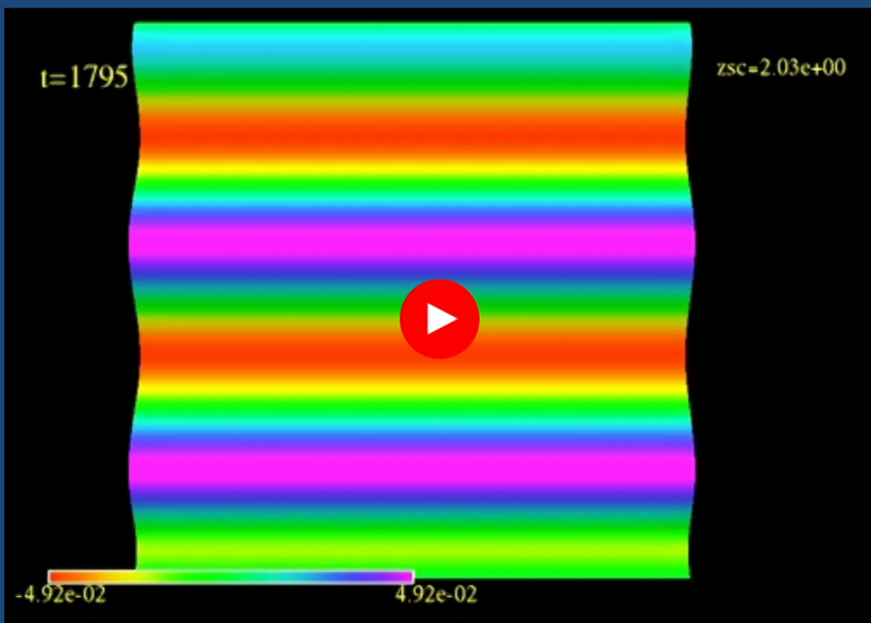
[Green,Carrasco,LL, PRX 2013]

$$\Omega_{\mu\nu} = \nabla_{[\mu}\rho^{1/d}u_{\nu]}\,,$$

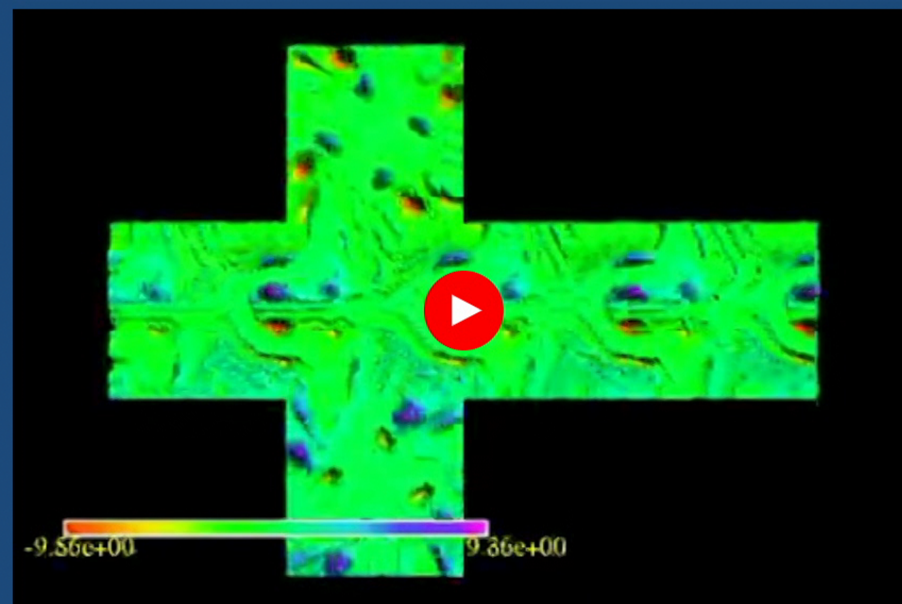
$$Z \equiv \int_{S^2} \rho^{-2/3}\Omega^2 u^0 d\Sigma = \int_{S^2} (\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}a_\mu a^\mu)\gamma d\Sigma$$

Viscous relativistic hydrodynamics dual to:

[Schwarzschild AdS, Poincare Patch:



Kerr AdS, global coordinates



[Carrasco,Green,LL,Myers]

- But this was in asymptotic AdS… does it carry over to AF spacetimes? [why would? Why wouldn't?]

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- Back to AF spacetimes, beyond linear perturbation? Take h1 (with l,m,n); a further mode on the 'time-dependent' background, obeys:

$$[\, \mathrm{Box}_{kerr} + O(h_1) \,]\, \Phi = 0.$$

- With the solution having the form: $e^{t(\alpha - \omega_l)}$ with

$$\alpha = \pm\sqrt{|H h_0(t)/Q m'|^2 - (\omega'_R - \omega_R/2)^2}\,,$$

- So exponentially growing solution if:

$$h_0(t)/(m'\omega'_I) - |Q/H| \sqrt{(\omega'_R - \omega_R/2)^2 / \omega'^2_I + 1} > 0\,.$$

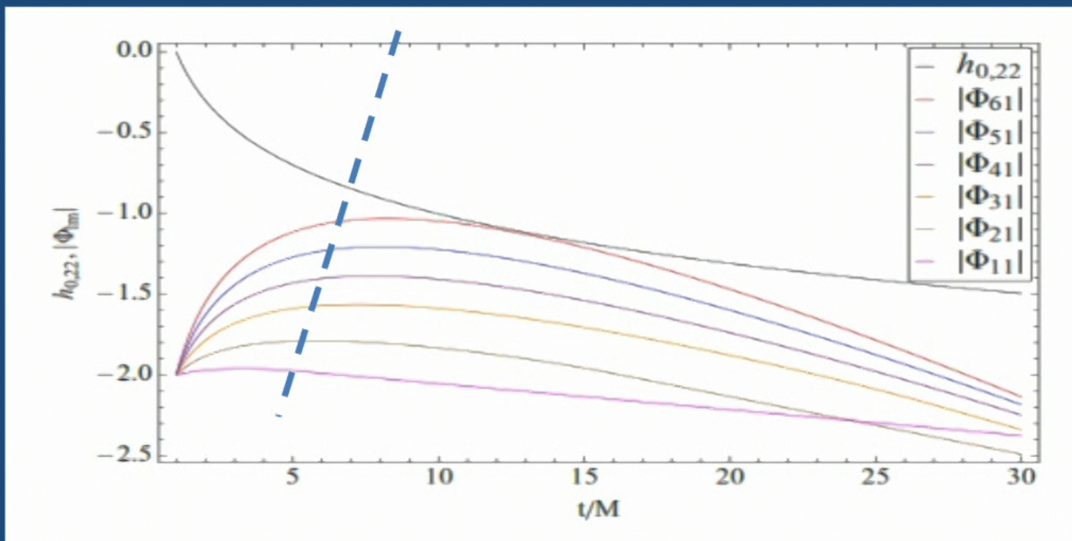
- if Φ has l, m/2 a parametric instability can turn on; *i.e. inverse cascade.*
- Further, one can find 'critical values' for growth onset.
- And can define a max value as:

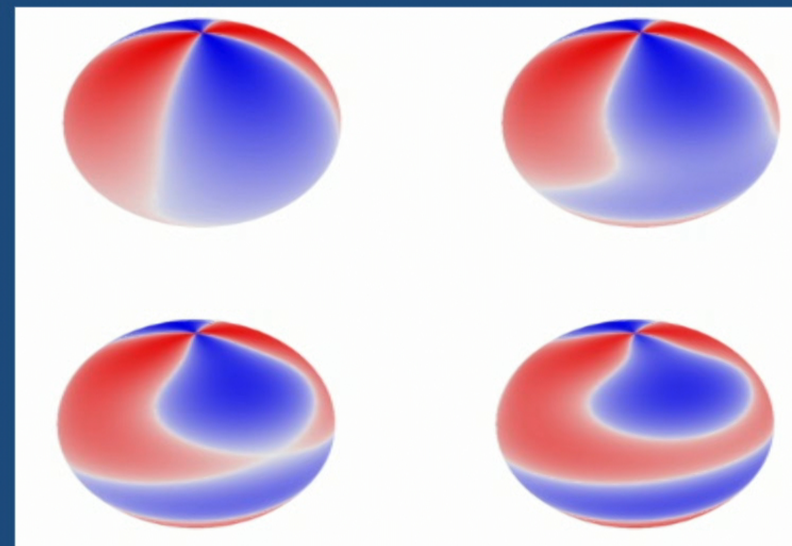
$$Re_g = h_o/(m\,\omega_\nu)$$

- identify λ<-> 1/m ; v <-> h_o ; η/ρ <-> ω_ν

$$\rightarrow Re_g = Re$$

'Turbulence' , AF spacetimes and 'beyond linear'





a = 0.998, perturbation ~ 0.02%, initial mode l=2,m=2

Could 'potentially' have observational consequences. Signal is different from what might be expected at the linear level

[Yang,Zimmemarn,LL '14]

Back to membrane paradigm → horizons

- [with Jaime Redondo-Yuste, in progress]

- (Horizon) null surface is a special surface. Brings robustness to MP.
- Carrollian symmetry → Carrollian hydro
  [Donnay,Ciambelli,C.Marteau,Petkou,Petropoulos, Siampos,Freidel,Jai-akson,Jafari,Speranza,Adami,Grumiller,Zwickel…]

- Can we make contact with previous discussion somehow?

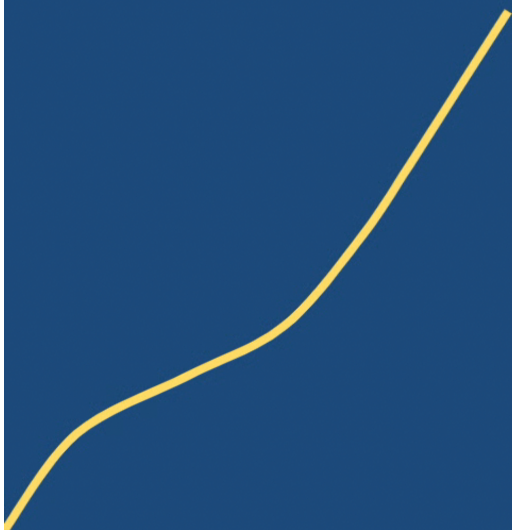
$$ds^2 = -V dv^2 + 2 dv d\rho + 2\Upsilon_A dv dx^A + \gamma_{AB} dx^A dx^B, \quad (1)$$

where v is the advanced null time, ρ parametrizes the distance to the surface, and x^A are the angular coordinates on the sphere, $A = 2, 3$. We expand the metric in powers of the distance to the surface

$$\begin{aligned} V = 2\kappa\rho + O(\rho^2), \quad \Upsilon_A = U_A\rho + O(\rho^2), \\ \gamma_{AB} = \Omega_{AB} - 2\lambda_{AB}\rho + O(\rho^2). \end{aligned} \quad (2)$$

$$\dot{\theta}^{(l)} - \kappa\theta^{(l)} + \frac{1}{2}\theta^{(l)2} + N^{(l)}_{AB} N^{AB\,(l)} = 0,$$

$$\dot{\mathcal{H}}_A + \theta^{(l)}\mathcal{H}_A - \nabla_A\kappa - \frac{1}{2}\nabla_A\theta^{(l)} + \nabla^B N^{(l)}_{AB} = 0,$$



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$$\Sigma_{AB} N^{(l)AB} = 0,$$

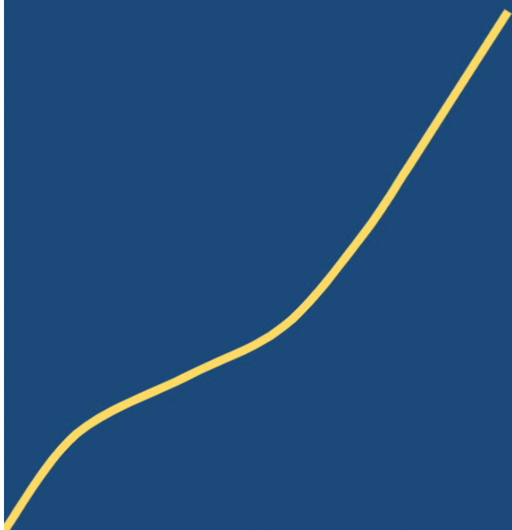
$$\dot{\mathcal{J}}^A + \theta^{(l)}\mathcal{J}^A + (\hat{\nabla}_B + \varphi_B)\Sigma^{AB} = 0.$$

$$\dot{e} + \theta^{(l)} e + \alpha(\hat{\nabla}_A + 2\varphi_A)\mathcal{J}^A +$$

$$\left(N^{(l)}_{AB} + \frac{1}{2}\theta\Omega_{AB}\right)(\Pi^{AB} + p\Omega^{AB}) = 0,$$

$$\dot{\pi}_B + \theta^{(l)}\pi_B + \alpha\left(\hat{\nabla}_A + \varphi_A\right)(\Pi^A_B + p\delta^A_B) +$$

$$+ \alpha\varphi_B e - \varpi^{AB}\mathcal{J}_A = 0.$$



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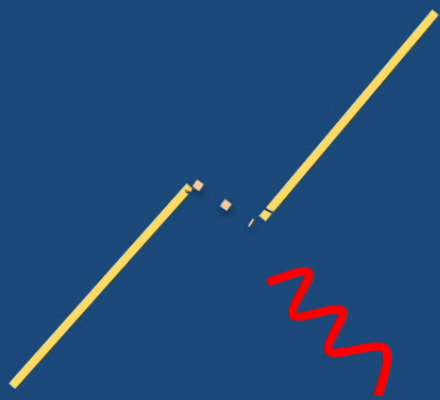
‘dictionary’

$$\begin{aligned} & e = \theta^{(l)}, \quad p = -\kappa, \\ & \Pi_{AB} = 2\eta N^{(l)}_{AB} + \zeta\theta^{(l)}\Omega_{AB}, \quad \eta = \frac{1}{2}, \quad \zeta = -\frac{1}{2}, \\ & \pi^A = -\frac{1}{\kappa}\left(D_A\kappa - \mathcal{H}^B\dot{\Omega}_{AB} - \frac{1}{\kappa}\mathcal{H}_A\dot{\kappa}\right), \\ & \Sigma^{AB} = \mathcal{J}^A = 0. \end{aligned} \tag{18}$$



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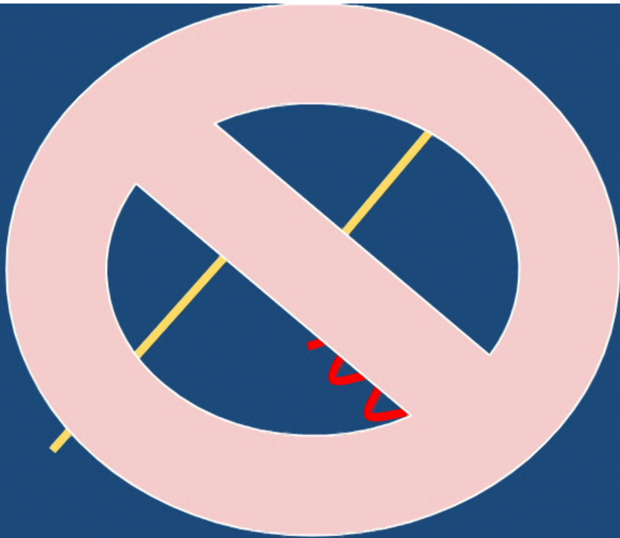
- But what are N, k? both *gauge* and physics in them. Consider perturbing Schwarzschild (vacuum case)

$$\kappa = \frac{1}{4m} + \epsilon\tilde{\kappa}, \qquad U_A = \epsilon\tilde{V}_A,$$
$$\Omega_{AB} = 4m^2 q_{AB} + \epsilon C_{AB}, \quad \lambda_{AB} = -2m q_{AB} + \epsilon S_{AB}, \tag{25}$$

$$\theta^{(n)} = -1/m - \epsilon^2 C^{AB}(C_{AB} + S_{AB}),$$
$$N^{(n)}_{AB} = \epsilon(C_{AB} + S_{AB}) - \frac{1}{2}\epsilon^2 C^{CD}(C_{CD} + S_{CD})q_{AB}.$$



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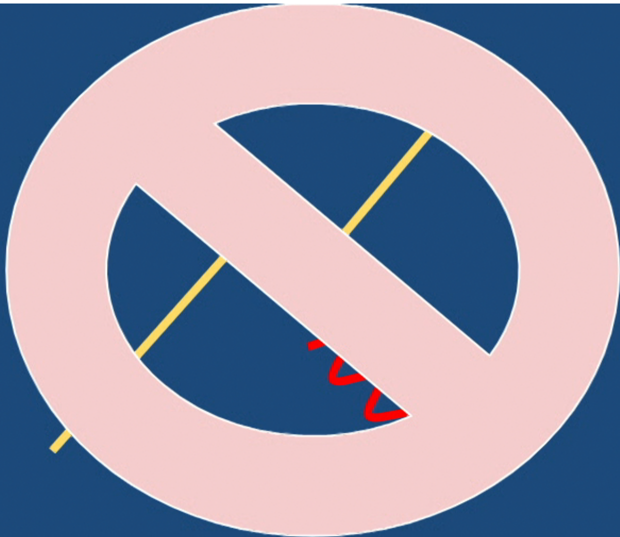

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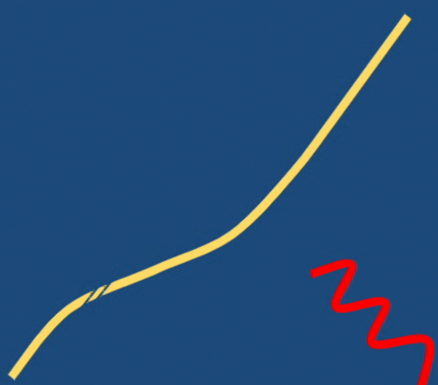

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$$\quad - \frac{1}{2}\epsilon^2 C^{CD}(C_{CD} + S_{CD})q_{AB}.$$

Involve the rest of EEs, which link horizon to bulk behavior. To 2nd order…

- {k,N} functions of (H,C,S,θ) , and evoln eqn for $C_{,t}$=F(H,C,S,θ)

→ replacing:

$$\begin{aligned}\dot{H}_A = & -\theta H_A - \nabla^2 H_A - \nabla^B \nabla_A H_B - \\ & - H_B \nabla_A H^B - H^B \nabla_B H_A - H_A \nabla^B H_B + \\ & + \frac{1}{2}\left[\nabla^B \nabla_C H^C + \nabla_C H^C \nabla^B\right](S_{AB} + 2C_{AB}) + \ldots\end{aligned} \tag{B4}$$

Which now relates more to NS

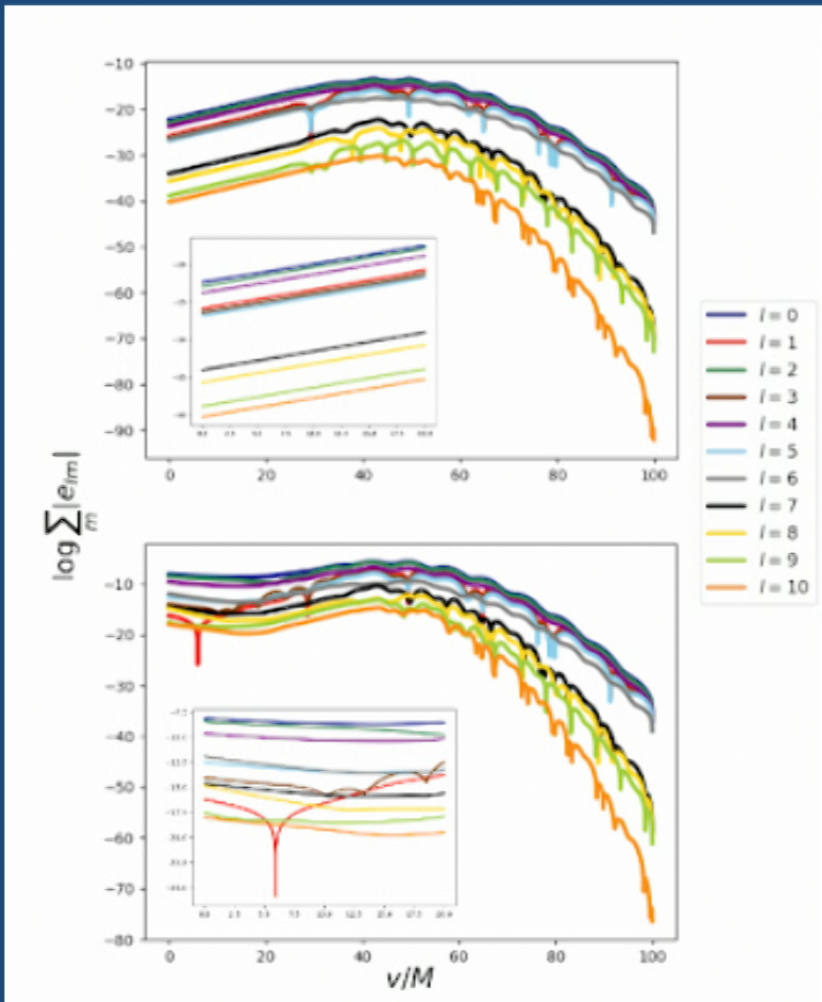
$$U_{,t} \simeq U \partial U + \eta/\rho\, \partial^2 U$$

and…being very 'liberal' with dimensions,

$$\dot{H}_A = -\theta H_A - \nabla^2 H_A - \nabla^B \nabla_A H_B - \\ - H_B \nabla_A H^B - H^B \nabla_B H_A - H_A \nabla^B H_B + \\ + \frac{1}{2}\left[\nabla^B \nabla_C H^C + \nabla_C H^C \nabla^B\right](S_{AB} + 2C_{AB}) + \ldots \quad \text{(B4)}$$

- Dissipation rate → $\theta + \nabla^2$
- Re → $H\lambda/(\theta\lambda^2+1)$ [recall, in NS, Re -> $\lambda u/(\eta/\rho)$]

Does it make sense?



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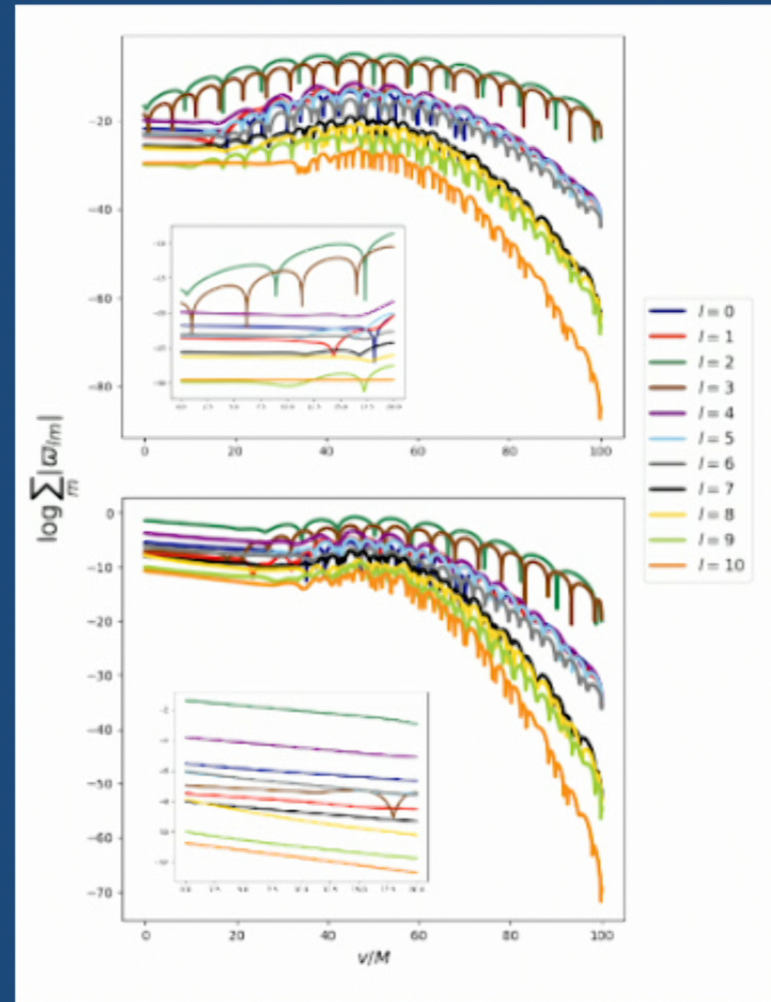

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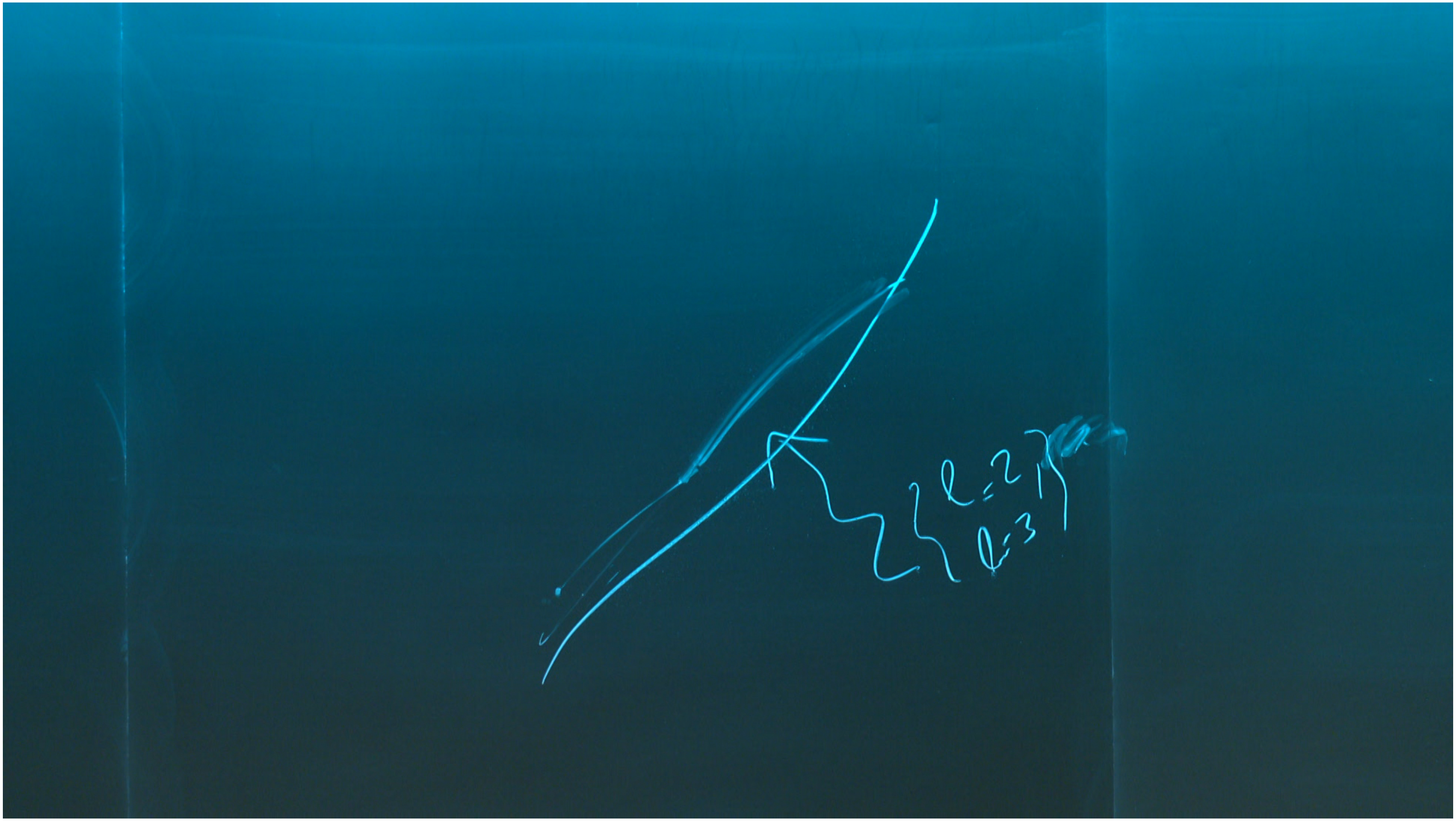

l=2
l=3

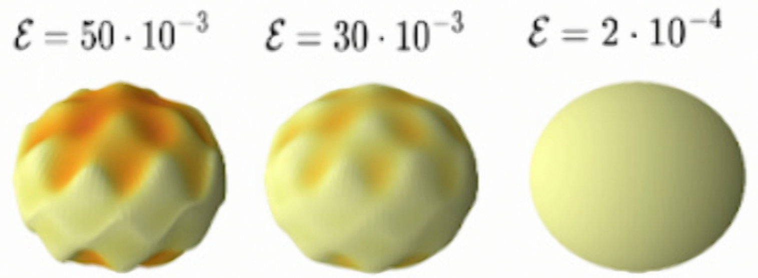
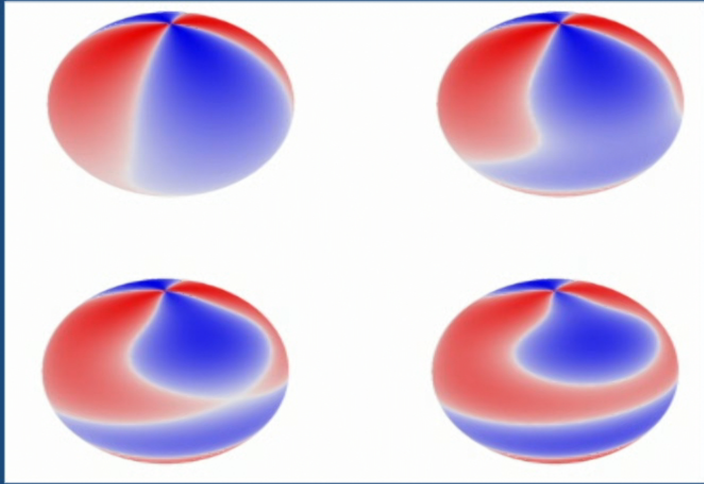
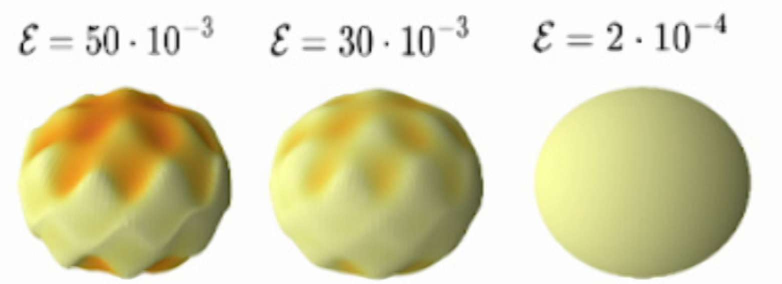


FIG. 8. Isometric embedding of the horizon metric Ω_{AB} for different values of $\mathcal{E}$, as indicated in the image. The colormap acts as a topographic indicator.





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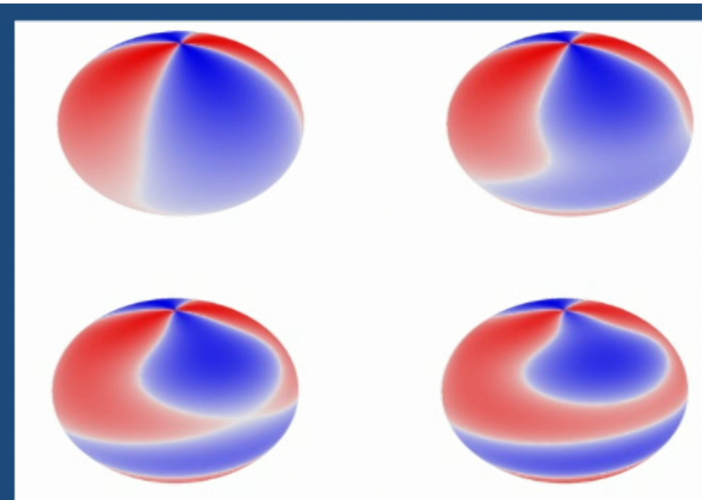

- Negative pressure or Negative energy? Negative entropy? (e + p = T s)

- Interesting regime for 'non-simple' waveforms?

- Message for other null surfaces and their structure? & beyond GR?