

Title: Conformal Carroll Scalar Actions

Speakers: Gerben Oling

Collection: Quantum Gravity Around the Corner

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Abstract: I will construct explicit actions that are invariant under BMS/conformal Carroll symmetries. Over the last few years, we have developed a systematic procedure for constructing non-Lorentzian theories from limits and expansions of Lorentzian theories. To obtain explicit examples of candidate dual field theories for flat space holography, I apply this procedure to the conformally coupled scalar action, which leads to two classes of actions that are invariant under conformal Carroll symmetries. Time permitting, I will briefly mention ongoing work on the computation and classification of Weyl anomalies in such theories.

Conformal Carroll Scalar Actions

Gerben Oling

Nordita (Stockholm University and KTH)

Based mainly on 2207.03468 with Stefano Baiguera, Watse Sybesma and Benjamin Søgaard

Quantum Gravity around the Corner @ Perimeter, October 6th 2022

Carroll limits and flat holography

Are used to 'relativistic' **Lorentz** boosts

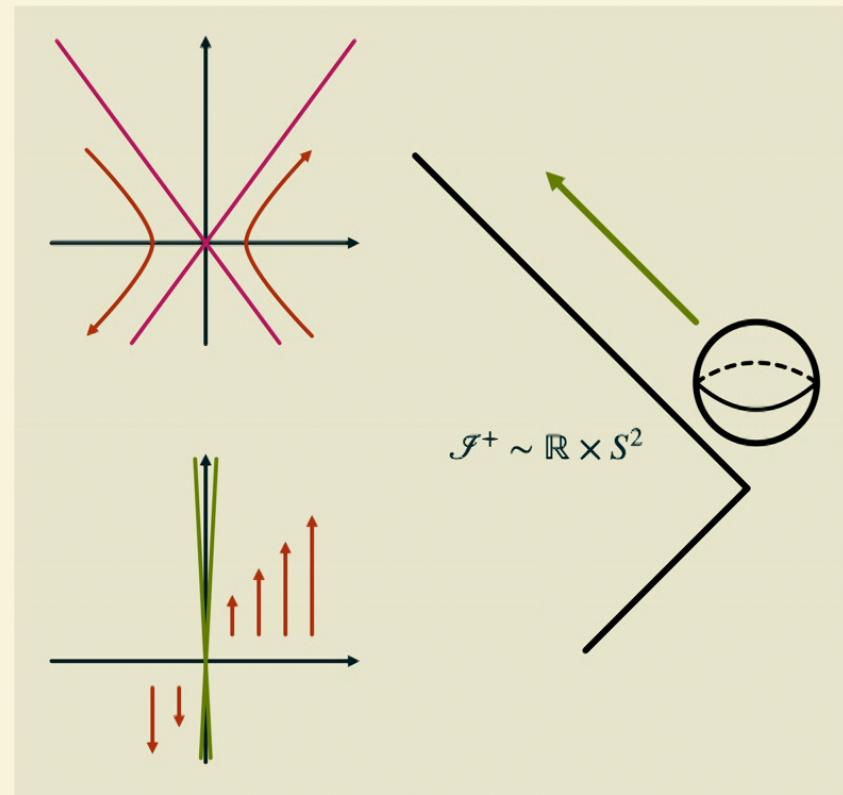
$$t \rightarrow t + \beta x, \quad x \rightarrow x + \beta t$$

Taking $c \rightarrow 0$ limit gives **Carroll** boosts [Levy-Leblond] [Sen Gupta]

$$t \rightarrow t + \lambda x \quad x \rightarrow x$$

Not obviously physical, but:

- **ultra-local behavior** leads to solvable systems such as integrable BKL-type dynamics in GR [see Niels' talk]
- **BMS** = conformal Carroll algebra at \mathcal{J}^+ [Duval, Gibbons, Horvathy, Zhang]
- Flat space holography, relation to celestial approach [Donnay, Fiorucci, Herfray, Ruzziconi] [Bagchi, Banerjee, Basu, Dutta]



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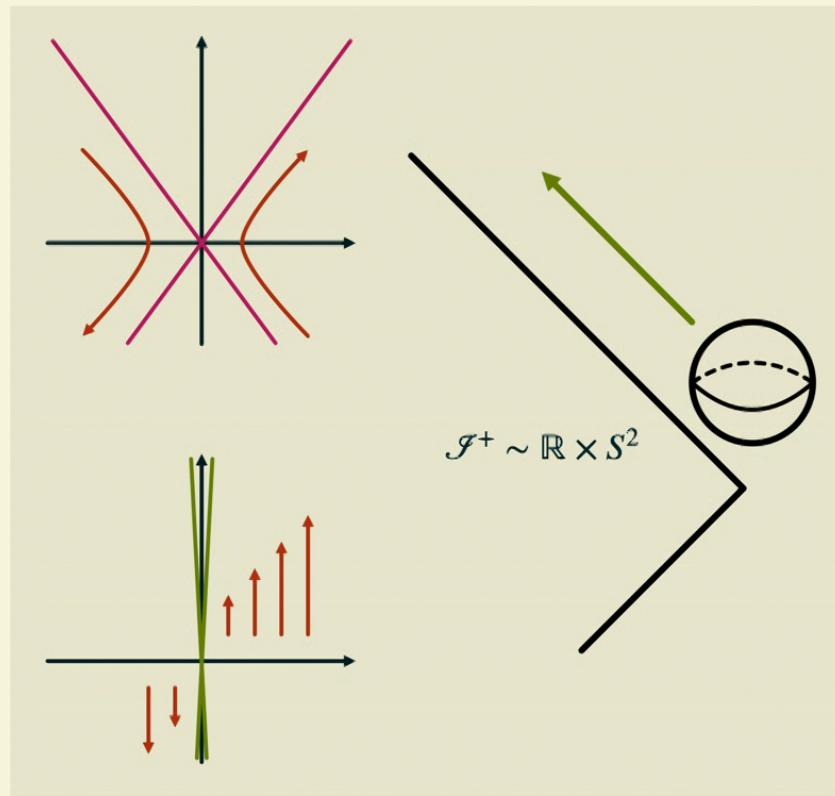
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Expand Lorentz-invariant actions to get Carroll-invariant actions

⇒ use this to **construct explicit flat space dual field theories** for example from limits of top-down AdS/CFT settings?



Main goal: **find explicit actions for conformal Carroll theories**

- Obtain Carroll geometry from $c \rightarrow 0$ expansion of Lorentzian
- Discuss **Carroll boost symmetries** and their consequences
- Use expansion to **construct conformal Carroll action** from

$$S = -\frac{1}{2} \int d^d x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{d-2}{4(d-1)} R \phi^2 \right)$$

Lorentzian review

Local Lorentz transformations act on vielbeine E^A_μ and inverse Θ_A^μ as

$$\delta_\Lambda E^A_\mu = \Lambda^A_B E^B_\mu \quad , \quad \delta_\Lambda \Theta_A^\mu = -\Lambda^B_A \Theta_B^\mu$$

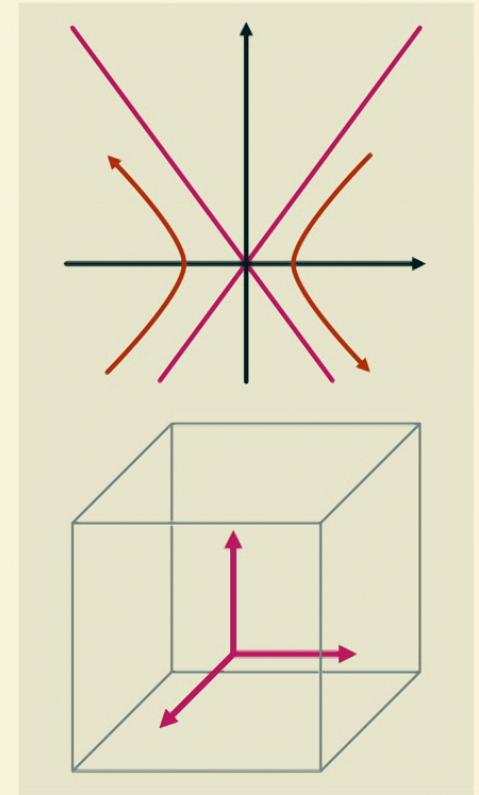
Define energy-momentum tensor using vielbein variation

$$T^\mu_A = \frac{1}{E} \frac{\delta S}{\delta E^A_\mu}$$

Then Ward identity for local Lorentz symmetries is

$$0 = \delta_\Lambda S = \int d^d x E \left(T^\mu_A \delta_\Lambda E^A_\mu \right) \quad \Rightarrow \quad 0 = T^\mu_A \Lambda^A_B E^B_\mu = T^{AB} \Lambda_{AB}$$

so energy-momentum tensor is symmetric



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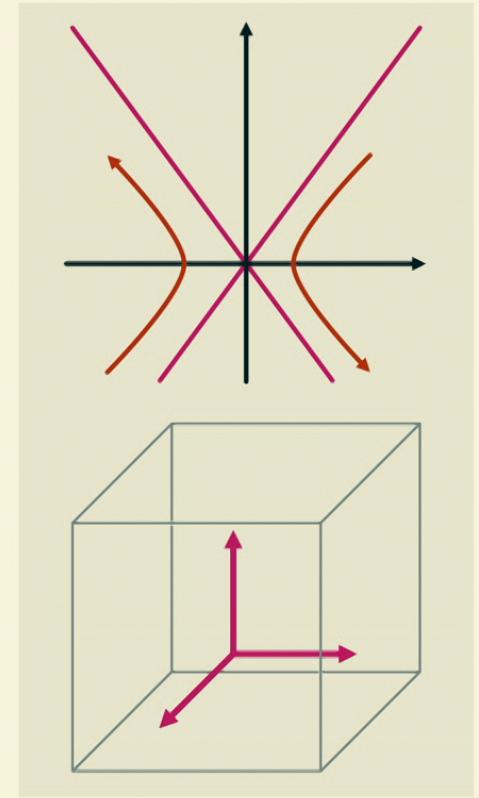
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Likewise, Weyl symmetries $\delta_\Omega E^A_\mu = \Omega E^A_\mu$ imply it is traceless, $T^\mu_\mu = 0$

Both hold for conformal scalar $S = -\frac{1}{2} \int d^d x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{d-2}{4(d-1)} R \phi^2 \right)$



Carroll geometry from expansion

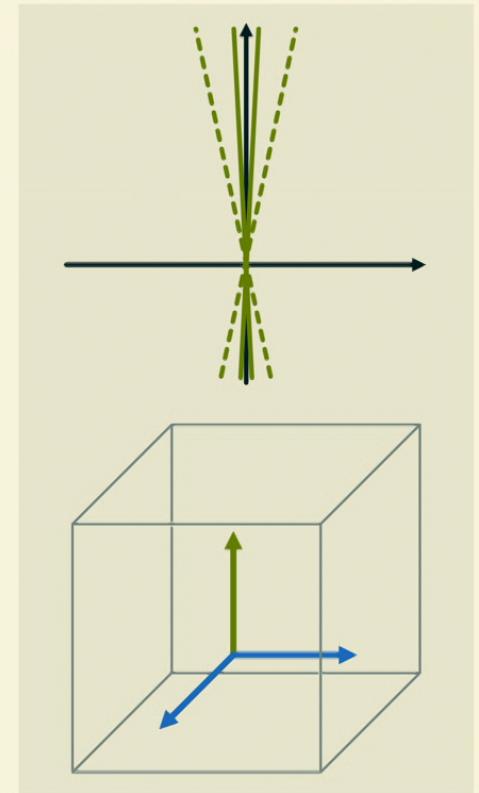
To do ultra-local $c \rightarrow 0$ expansion, first split off factors of c in vielbeine

$$E^A{}_\mu = \left(cT_\mu, E^a{}_\mu \right) , \quad \Theta_A{}^\mu = \left(-\frac{1}{c}V^\mu, \Theta_a{}^\mu \right)$$

Under Lorentz transformations $\Lambda^0{}_a$ these variables transform as

$$\delta_\Lambda T_\mu = \frac{1}{c}\Lambda^0{}_a E^a{}_\mu \quad \delta_\Lambda V^\mu = c\Lambda^a{}_0 T_\mu$$

$$\delta_\Lambda E^a{}_\mu = c\Lambda^a{}_0 T_\mu \quad \delta_\Lambda \Theta_a{}^\mu = \frac{1}{c}\Lambda^0{}_a V^\mu$$



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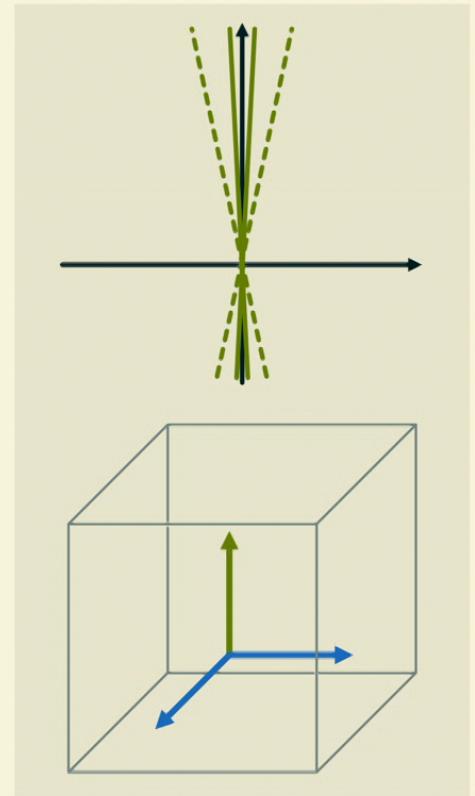
Then expand these variables around $c \rightarrow 0$ as

$$\begin{aligned} T_\mu &= \tau_\mu + \mathcal{O}(c^2) & V^\mu &= v^\mu + \mathcal{O}(c^2) \\ E^a{}_\mu &= e^a{}_\mu + \mathcal{O}(c^2) & \Theta_a{}^\mu &= \theta^a{}_\mu + \mathcal{O}(c^2) \end{aligned}$$

Limit of Lorentz boosts gives local Carroll boosts with $\Lambda^0{}_a = c\lambda_a + \dots$

$$\begin{aligned} \delta_\lambda \tau_\mu &= \lambda_a e^a{}_\mu & \delta_\lambda v^\mu &= 0 \\ \delta_\lambda e^a{}_\mu &= 0 & \delta_\lambda \theta^a{}_\mu &= \lambda_a v^\mu \end{aligned}$$

In the $c \rightarrow 0$ limit, local Carroll boosts **follow inevitably from local Lorentz boosts!**



Carroll boosts and energy flux

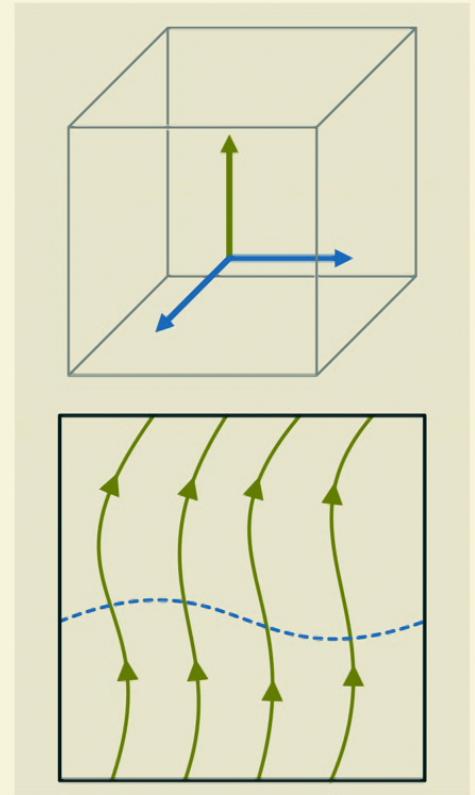
Fundamental Carroll data is time vector field v^μ and spatial metric $h_{\mu\nu} = \delta_{ab} e^a{}_\mu e^b{}_\nu$
⇒ fiber bundle with vertical v^μ and base metric $h_{\mu\nu}$

Can add inverse $\tau_\mu \sim$ Ehresmann connection and $h^{\mu\nu} \sim$ horizontal projector $h^{\mu\rho} h_{\rho\nu}$

These transform under Carroll boosts $\lambda_\mu = e^a{}_\mu \lambda_a$

$$\delta_\lambda \tau_\mu = \lambda_\mu, \quad \delta_\lambda h^{\mu\nu} = \lambda^\mu v^\nu + v^\mu \lambda^\nu \quad \delta_\lambda v^\mu = 0, \quad \delta_\lambda h_{\mu\nu} = 0$$

(Boosts ~ ambiguity in Ehresmann connection and horizontal section)



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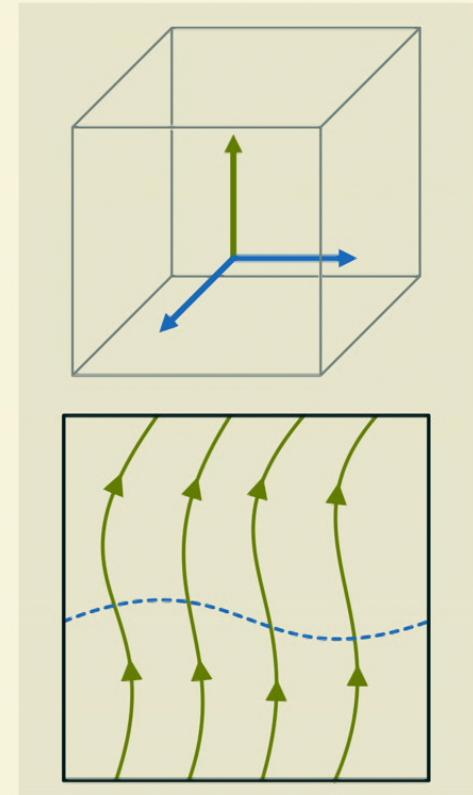
(Boosts \sim ambiguity in Ehresmann connection and horizontal section)

From $c \rightarrow 0$ limit get Carroll boost Ward identity

$$0 = T^\mu{}_\nu \left(E^A{}_\mu \Lambda^A{}_B \Theta_B{}^\nu \right) = T^\mu{}_\nu \left(-E^a{}_\mu \frac{1}{c} \Lambda^0{}_a V^\nu + T_\mu c \Lambda^b{}_0 \Theta_b{}^\nu \right) = -\lambda_\mu T^\mu{}_\nu v^\nu + \mathcal{O}(c^2)$$

Since $\lambda_\mu = e^a{}_\mu \lambda_a$ is spatial, this means $T^i{}_0 = 0$, vanishing energy flux

[De Boer, Hartong, Obers, Sybesma, Vandoren]



To boost or not to boost?

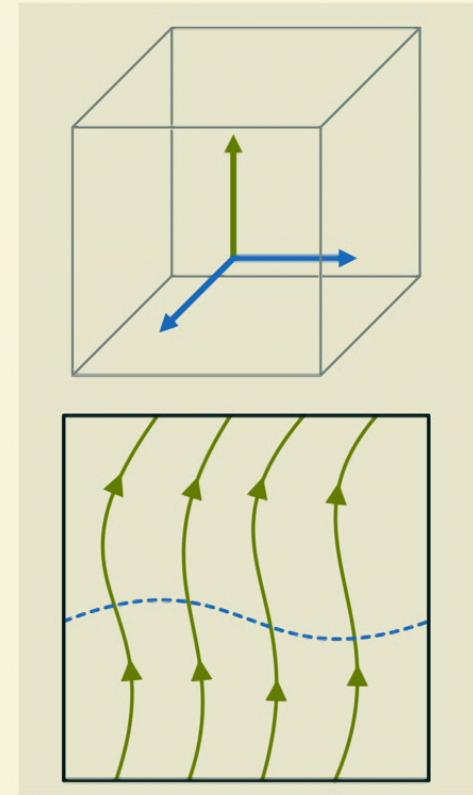
Local Carroll boost symmetry implies vanishing energy flux $T^i_0 = 0$

inevitable when taking limit of Lorentz-invariant theory

Boosts also constrain $\langle \phi(t, x) \phi(0, 0) \rangle = \begin{cases} f(t)\delta(x) \\ g(x) \end{cases}$, 'timelike' and 'spacelike' branches

[Henneaux, Salgado-Rebodello] [De Boer, Hartong, Obers, Sybesma, Vandoren]

Timelike branch reproduces CCFT correlators [Bagchi, Banerjee, Basu, Dutta]



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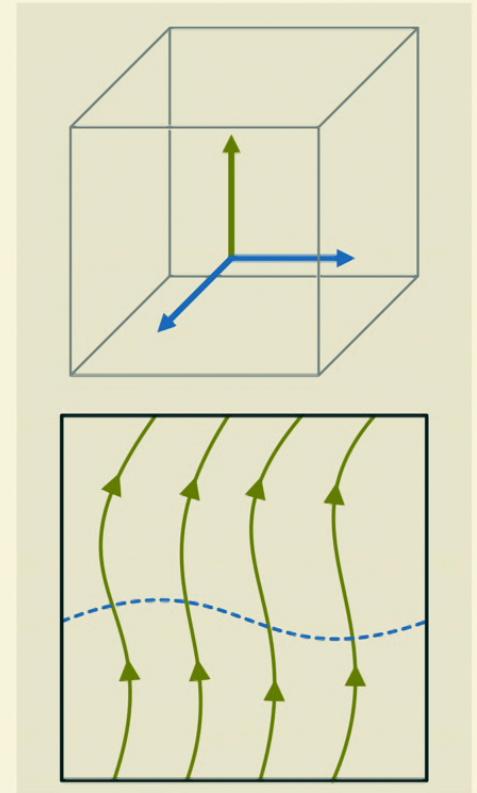
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However maybe **Carroll boosts not always desired** in flat space holography?

- known holographic fluids with $T^i_0 \neq 0$ [Ciambelli, Marteau, Petkou, Petropoulos, Siampos]
- focus instead on $(v^\mu, h_{\mu\nu})$ fiber structure?
[Ciambelli, Leigh, Marteau, Siampos] [Petkou, Petropoulos, Rivera Betancour, Siampos] [Freidel, Jai-akson]...
- go to Lorentz-breaking frame before taking flat/Carroll limit in AdS/CFT?
cf [Campoleoni, Ciambelli, Delfante, Marteau, Petropoulos, Ruzziconi]



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$$S = -\frac{1}{2} \int d^d x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{d-2}{4(d-1)} R \phi^2 \right)$$

Carroll connection and curvature

For now return to Carroll geometry with boosts, from $c \rightarrow 0$ of Lorentzian geometry

Introduce **extrinsic curvature** $K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_v h_{\mu\nu}$ and **acceleration** $a_\mu = 2v^\rho \partial_{[\mu} \tau_{\rho]}$

Both boost-invariant and spatial tensors (so $K^{\mu\nu} = h^{\mu\rho} h^{\nu\sigma} K_{\rho\sigma}$ etc)

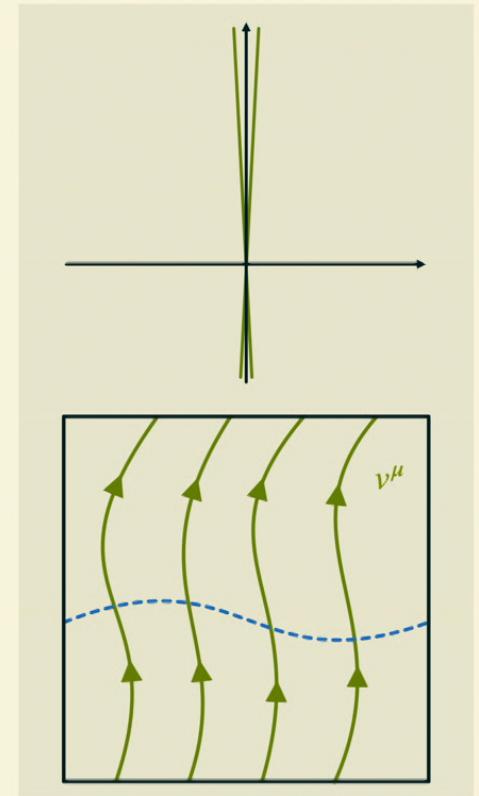
Want **connection** that satisfies $\tilde{\nabla}_\mu v^\nu = 0$ and $\tilde{\nabla}_\rho h_{\mu\nu} = 0$, choose

$$\tilde{\Gamma}_{\mu\nu}^\rho = -v^\rho \partial_{(\mu} \tau_{\nu)} - v^\rho \tau_{(\mu} \mathcal{L}_v \tau_{\nu)} + \frac{1}{2} h^{\rho\sigma} \left(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu} \right) - h^{\rho\sigma} \tau_\nu K_{\mu\sigma}$$

Has non-zero **torsion** $\tilde{T}^\rho_{\mu\nu} = 2h^{\rho\sigma} \tau_{[\mu} K_{\nu]\sigma}$ [Bekaert, Morand] [Hansen, Obers, GO, Søgaard]

Define its Riemann curvature in the usual way

$$\tilde{R}_{\mu\nu\sigma}^\rho = -\partial_\mu \tilde{\Gamma}_{\nu\sigma}^\rho + \partial_\nu \tilde{\Gamma}_{\mu\sigma}^\rho - \tilde{\Gamma}_{\mu\lambda}^\rho \tilde{\Gamma}_{\nu\sigma}^\lambda + \tilde{\Gamma}_{\nu\lambda}^\rho \tilde{\Gamma}_{\mu\sigma}^\lambda$$



Carroll geometry from Lorentzian

Carroll connection $\tilde{\Gamma}_{\mu\nu}^\rho$ can be obtained from Levi-Civita connection,

$$\Gamma_{\mu\nu}^\rho = \frac{1}{c^2} S_{(-2)}{}^\rho{}_{\mu\nu} + \tilde{C}_{\mu\nu}^\rho + S_{(0)}{}^\rho{}_{\mu\nu} + c^2 S_{(2)}{}^\rho{}_{\mu\nu},$$

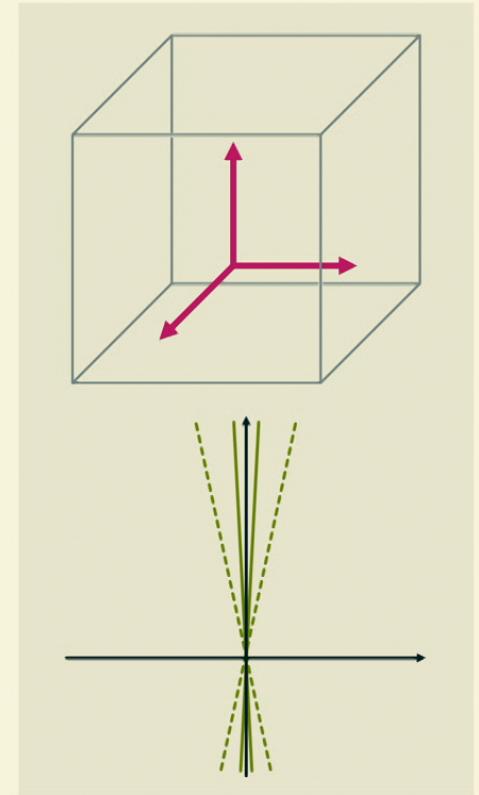
where the $S_{\mu\nu}^\rho$ are known tensors and $\tilde{C}_{\mu\nu}^\rho = \tilde{\Gamma}_{\mu\nu}^\rho + \dots$

Then Levi-Civita Ricci scalar is

$$R = \frac{1}{c^2} \left[\mathcal{K}^{\mu\nu} \mathcal{K}_{\mu\nu} + \mathcal{K}^2 - 2V^\mu \partial_\mu \mathcal{K} \right] + \Pi^{\mu\nu} \tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu A^\mu + c^2 \Pi^{\mu\rho} \Pi^{\nu\sigma} \partial_{[\mu} T_{\nu]} \partial_{[\rho} T_{\sigma]}$$

where $\mathcal{K}_{\mu\nu} = K_{\mu\nu} + \dots$, $\Pi^{\mu\nu} = h^{\mu\nu} + \dots$, $\Pi_{\mu\nu} = h_{\mu\nu} + \dots$ and $A_\mu = a_\mu + \dots$

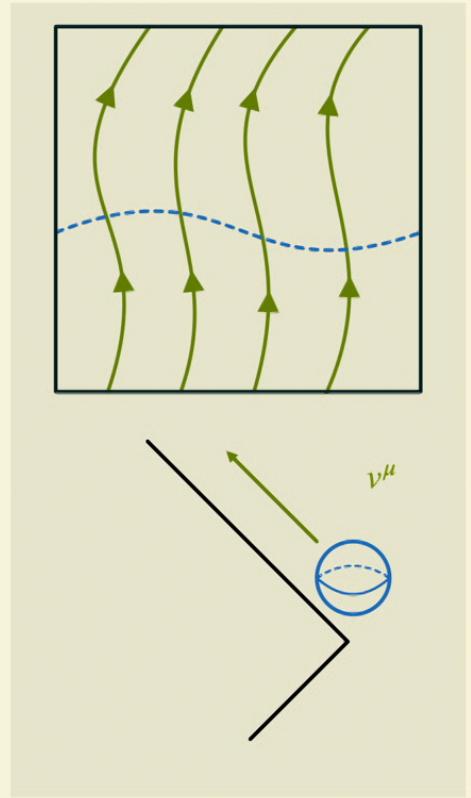
Finally, $\sqrt{-g} = cE = ce + \dots$ where $E = \det(T_\mu, \Pi_{\mu\nu})$ and $e = \det(\tau_\mu, h_{\mu\nu})$



Conformal scalar actions: timelike

Rewrite Lorentzian conformal scalar action,

$$\begin{aligned} S &= -\frac{1}{2} \int d^d x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{d-2}{4(d-1)} R \phi^2 \right) \\ &= -\frac{c}{2} \int d^d x E \left[\left(-\frac{1}{c^2} V^\mu V^\nu + \Pi^{\mu\nu} \right) \partial_\mu \phi \partial_\nu \phi \right. \\ &\quad \left. + \frac{d-2}{4(d-1)} \left(\frac{1}{c^2} [\mathcal{K}^{\mu\nu} \mathcal{K}_{\mu\nu} + \mathcal{K}^2 - 2V^\mu \partial_\mu \mathcal{K}] + \Pi^{\mu\nu} \tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu A^\mu + c^2 \Pi^{\mu\rho} \Pi^{\nu\sigma} \partial_{[\mu} T_{\nu]} \partial_{[\rho} T_{\sigma]} \right) \phi^2 \right] \end{aligned}$$



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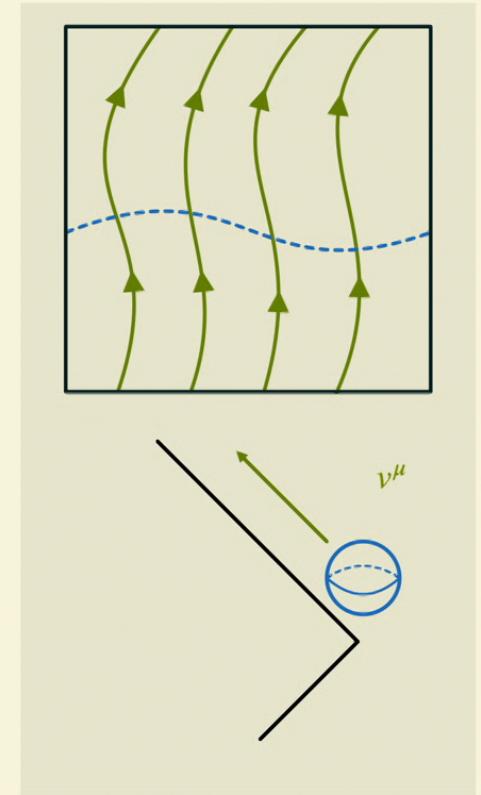
In Carroll limit $c \rightarrow 0$, leading-order terms give [Baiguera, GO, Sybesma, Søgaard]

$$S_t = -\frac{1}{2} \int d^d x e \left[-(v^\mu \partial_\mu \phi)^2 + \frac{(d-2)}{4(d-1)} (K^{\mu\nu} K_{\mu\nu} + K^2 - 2v^\mu \partial_\mu K) \phi^2 \right]$$

This is **timelike conformal Carroll scalar**, flat space propagator $\sim t \delta(x)$

Carroll boost-invariant and Weyl-invariant, so $T^i_0 = 0$ and $T^\mu_\mu = 0$

Also considered from no-boost approach in [Gupta, Suryanarayana] [Rivera-Betancour, Vilatte]



Conformal scalar actions: spacelike

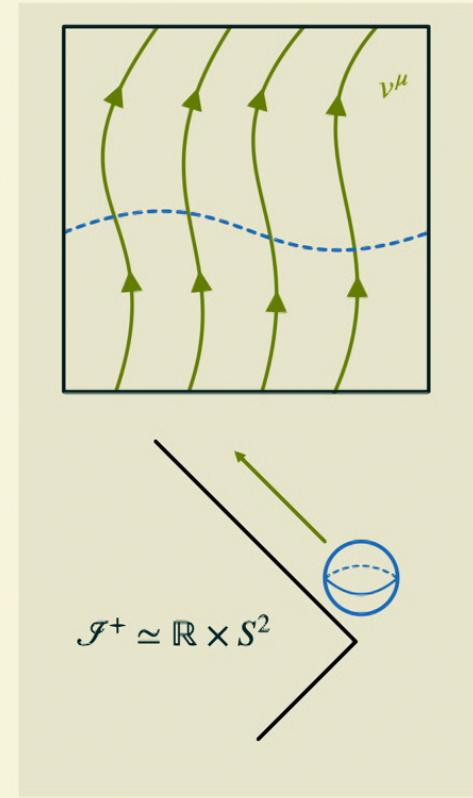
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Can take alternative Carroll limit $c \rightarrow 0$ using Lagrange multipliers,

$$S_s = -\frac{1}{2} \int d^d x e \left[h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{(d-2)}{4(d-1)} \left(h^{\mu\nu} \tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu a^\mu \right) + \chi \left(v^\mu \partial_\mu \phi + \frac{(d-2)}{4(d-1)} K \right) + \chi^{\mu\nu} \check{K}_{\mu\nu} \phi \right]$$

This is **spacelike conformal Carroll scalar**. [Baiguera, GO, Sybesma, Søgaard]



- also invariant under boosts and Weyl transformations
- extrinsic curvature must be pure trace $K_{\mu\nu} = \frac{h_{\mu\nu}}{d-1} K$
- time-dependence $v^\mu \partial_\mu \phi$ is fixed, so only **spacelike dynamics**

Conformal scalar actions: spacelike

Spacelike conformal Carroll scalar from next-to-leading-order terms,

$$S_s = -\frac{1}{2} \int d^d x e \left[h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{(d-2)}{4(d-1)} \left(h^{\mu\nu} \tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu a^\mu \right) \phi^2 + \chi \left(v^\mu \partial_\mu \phi + \frac{(d-2)}{4(d-1)} K \right) + \chi^{\mu\nu} \check{K}_{\mu\nu} \phi \right]$$

- invariant under local Carroll boosts and Weyl transformations
- energy-momentum tensor satisfies $T^i_0 = 0$ and $T^\mu_\mu = 0$

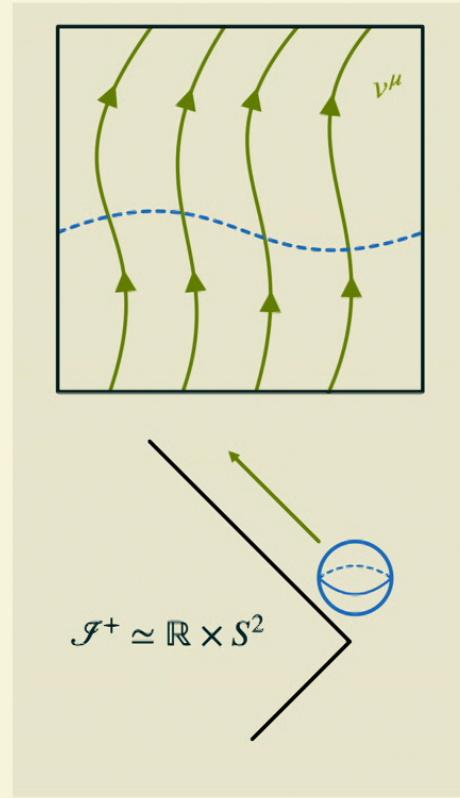
Flat space propagator $\sim \log(x)^2$ of spacelike Euclidean free boson

Remarkably, can dimensionally reduce the action explicitly using constraints,

$$S_s = -\frac{1}{2} \int d^{d-1} x \sqrt{h} \left(h^{ij} \partial_i \hat{\phi} \partial_j \hat{\phi} + \frac{(d-3)}{4(d-2)} \hat{R} \hat{\phi}^2 + \frac{1}{4(d-1)(d-2)} A^{-2} \hat{R}_{A^{-2} h_{ij}} \right)$$

where $\hat{\phi} = A^{1/2} \phi$ and the background field $A = \int_v \tau$ encodes former 'Carroll time'

but otherwise this is $(d-1)$ -dimensional Euclidean conformal scalar!



Conformal scalar actions: spacelike

Dimensional reduction to Euclidean theory

$$\begin{aligned} S_s &= -\frac{1}{2} \int d^d x e \left[h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{(d-2)}{4(d-1)} \left(h^{\mu\nu} \tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu a^\mu \right) \phi^2 + \chi \left(v^\mu \partial_\mu \phi + \frac{(d-2)}{4(d-1)} K \right) + \chi^{\mu\nu} \check{K}_{\mu\nu} \phi \right] \\ &= -\frac{1}{2} \int d^{d-1} x \sqrt{h} \left(h^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} + \frac{(d-3)}{4(d-2)} \hat{R} \hat{\phi}^2 + \frac{1}{4(d-1)(d-2)} A^{-2} \hat{R}_{A^{-2} h_{ij}} \right) \end{aligned}$$

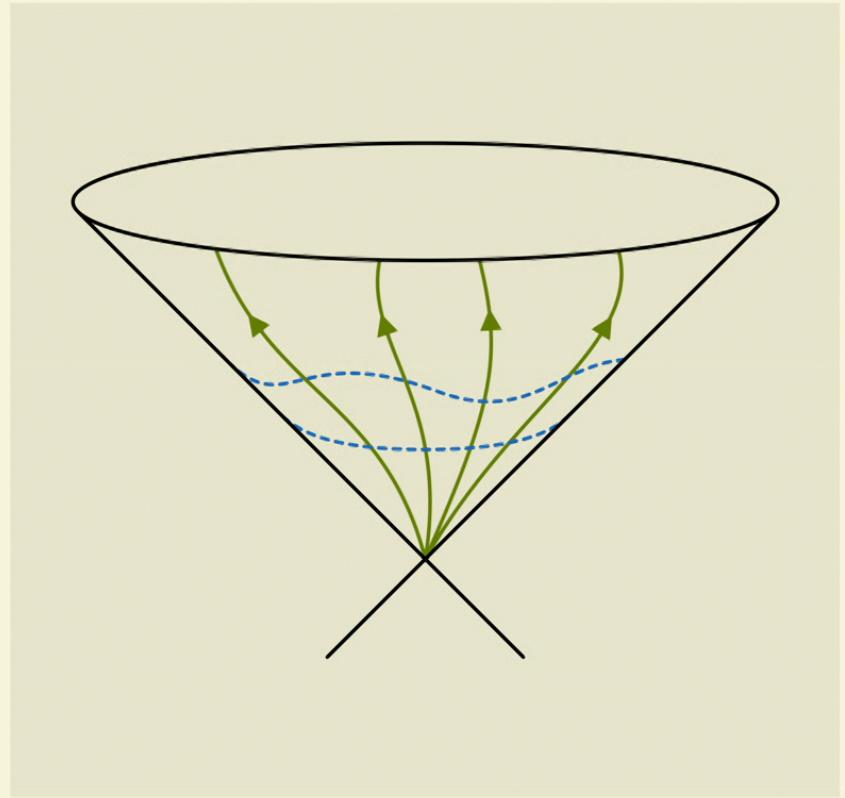
Reminiscent of **embedding space** formalism!

Get $(d-1)$ -dim conformal $SO(d,1)$ representations
from $(d+1)$ -dim Lorentz representations in $\mathbb{R}^{1,d}$

Restriction to light cone

- ⇒ Carrollian **spacelike** theory
- ⇒ Euclidean theory

Similar procedure for other spacelike Carroll theories?



Summary and outlook

Constructed timelike and spacelike conformal Carroll scalar actions

Allow explicit computations using only basic QFT techniques

Ongoing and future work:

- study sources and breaking of boosts ~ supertranslations
- complete general anomaly classification
- fermions?
- direct computation of scalar anomalies?

Build up conformal Carroll \leftrightarrow CCFT dictionary

Top-down flat holography from $c \rightarrow 0$ limit of AdS/CFT?

