Title: Celestial higher spin charges and gravitational multipole moments

Speakers: Ali Seraj

Collection: Quantum Gravity Around the Corner

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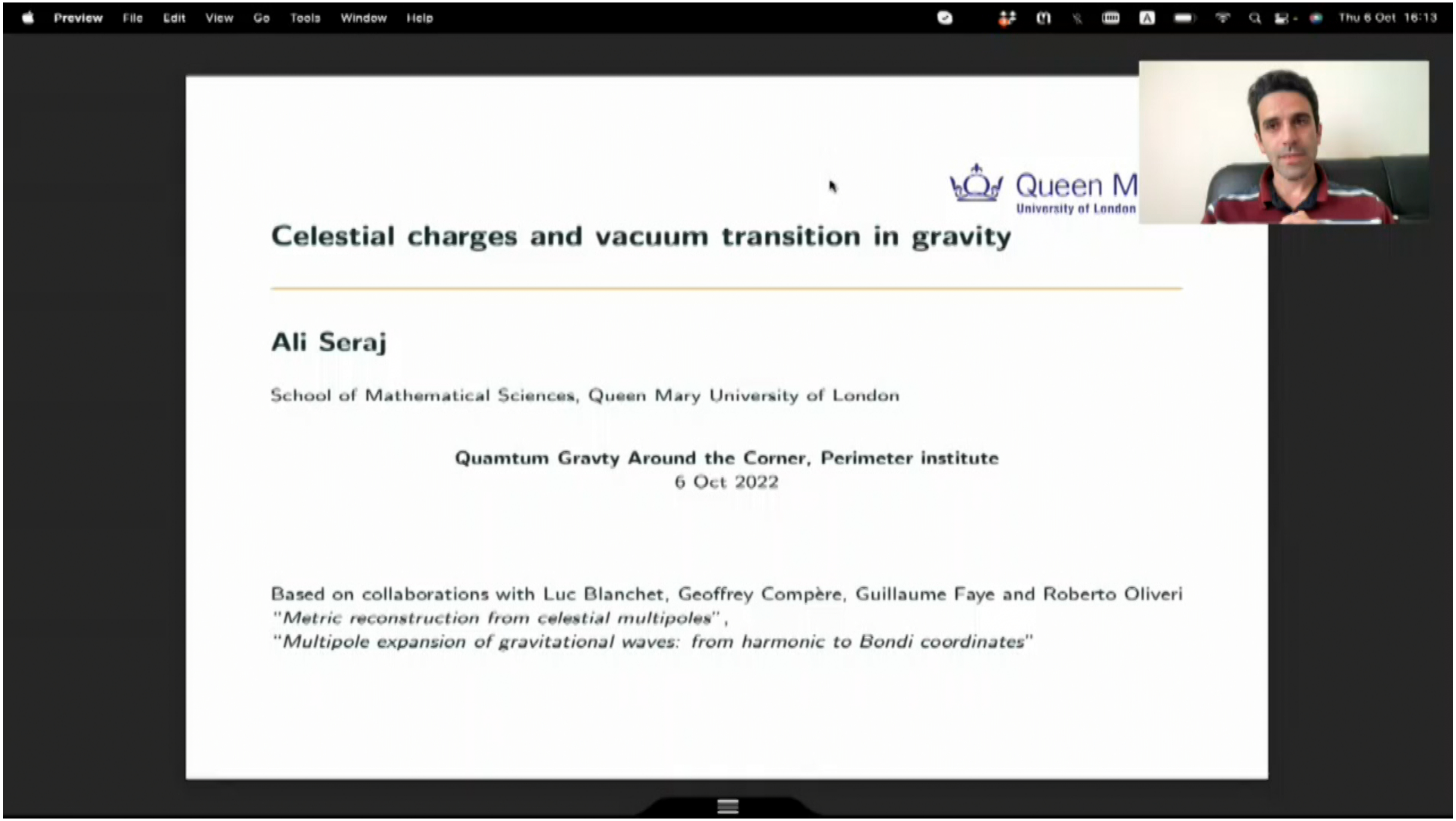
Abstract: In this talk, I will describe celestial higher spin charges as corner integrals, and their relationship with gravitational multipole moments. I will then explain that these charges uniquely label gravitational vacua and the corresponding flux-balance equations describe the transition caused by gravitational radiation among different vacua. This tak is based on arXiv:2206.12597.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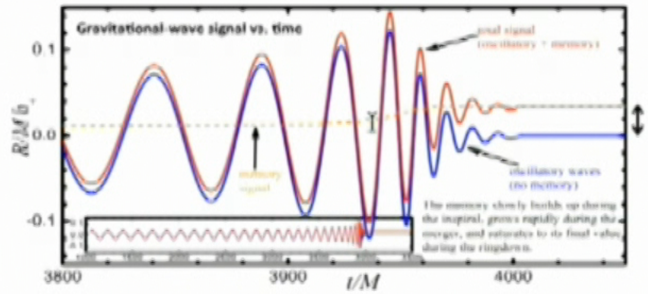
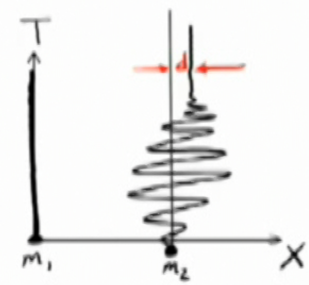
Queen M
University of London
Celestial charges and vacuum transition in gravity
Ali Seraj
School of Mathematical Sciences, Queen Mary University of London
Quamtum Gravty Around the Corner, Perimeter institute
6 Oct 2022
Based on collaborations with Luc Blanchet, Geoffrey Compère, Guillaume Faye and Roberto Oliveri
"Metric reconstruction from celestial multipoles",
"Multipole expansion of gravitational waves: from harmonic to Bondi coordinates"

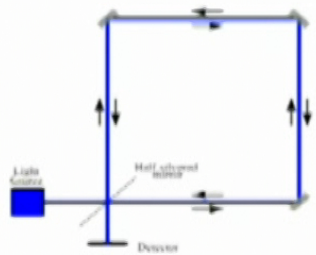
Motivation

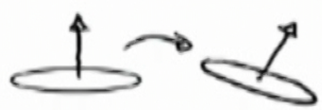
Gravitational wave memory effect [Zel'dovich, Polnarev, Braginsky, Christodoulou, Blanchet, Damour, Thorne, ...





Several other known memory effects, e.g. spin memory [Pasterski et al. '16] and gyroscopic memory [Seraj, Oblak '21].





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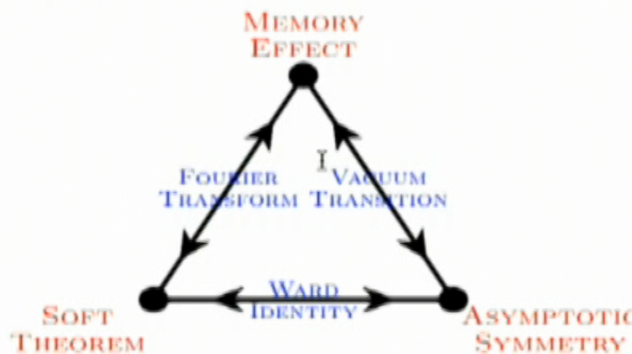
Introduction

Infrared Triangle *[Strominger'15]*



ASG $\longrightarrow$ Space of Vacua $\xrightarrow{\text{radiative process}}$ Vacuum transition $\longrightarrow$ Memory

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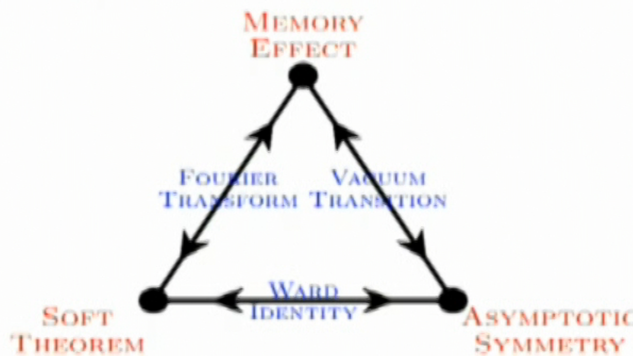
Introduction

Infrared Triangle *[Strominger'15]*



ASG $\longrightarrow$ Space of Vacua $\xrightarrow{\text{radiative process}}$ Vacuum transition $\longrightarrow$ Memory

Non-degenerate description of the "vacuum"? Or how to label the vacua?

Largest set of asymptotic charges?

Celestial charges

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GR in Bondi gauge

In Bondi coordinates (u, r, θ^a), imposing $g_{rr} = g_{ra} = 0$ implies

$$ds^2 = -e^{2\beta}(F du^2 + 2du\, dr) + g_{ab}\,(d\theta^a - \frac{U^a}{r^2}du)\,(d\theta^b - \frac{U^b}{r^2}du)$$

BC: $\lim_{r\to\infty}(r^{-2}g_{ab}) = \gamma_{ab}(\theta^a)$ metric on celestial sphere $D_c\gamma_{ab} = 0$

Determinant condition $\partial_r \det(r^{-2}g_{ab}) = 0$, implies

$$g_{ab} = r^2\sqrt{1 + \frac{\mathcal{C}_{cd}\mathcal{C}^{cd}}{2r^2}}\,\gamma_{ab} + r\,\mathcal{C}_{ab}\,,$$

[Grant, Nichols '21]

where $\mathcal{C}_{ab}$ is symmetric trace-free $\gamma^{ab}\mathcal{C}_{ab} = 0$

Asymptotic expansion

$$\mathcal{C}_{ab} = C_{ab} + \sum_{n=2}^{\infty} r^{-n}\underset{(n)}{E_{ab}}\,,$$

$$F = 1 - \frac{2m}{r} + \mathcal{O}(r^{-2}),$$

$$g_{ab}U^b = \tfrac{1}{2}D^bC_{ab} - \tfrac{1}{r}\left[-\tfrac{2}{3}N_a + \tfrac{1}{16}\partial_a(C^2)\right] + \mathcal{O}(r^{-2}),$$

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Einstein equations

Bondi free data

gravitational waveform $C_{ab}(u, \theta^a)$,

Corner d.o.f $m(\theta^a)$, $N_a(\theta^a)$, $\underset{(n)}{E_{ab}}(\theta^a)$ defined on some initial time

News tensor $N_{ab} = \partial_u C_{ab}$ measure of radiation. $N_{ab} = 0$ non-radiative spacetime.

Balance equations give the time evolution of corner quantities

$$\partial_u m = \underbrace{\frac{1}{4} D_a D_b N^{ab}}_{\text{soft flux}} - \underbrace{\frac{1}{8} N_{ab} N^{ab}}_{\text{hard flux}},$$

4

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m is conserved in non-radiative spacetimes

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Local flux-balance equations

Local flux-balance equations [Grant, Nichols '22]

$$n=0 \quad : \qquad \frac{1}{4} D_b D_c N^{bc} = -\mathcal{F}(u) + \partial_u m,$$

$$n=1 \quad : \qquad -\frac{u}{2} D_c D_{\langle a} D_{b\rangle} N^{bc} = -\mathcal{F}_a(u) + \partial_u \mathcal{N}_a,$$

$$n=2 \quad : \qquad \frac{u^2}{12} \mathsf{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(2)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(2)}{\mathcal{E}_{ab}},$$

$$n\geq 3 \quad : \qquad \frac{(-u)^n}{6\,n!} \mathcal{D}_{n-3} \cdots \mathcal{D}_0 \, \mathsf{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(n)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(n)}{\mathcal{E}_{ab}}\,.$$

where $\mathcal{N}_a, \underset{(n)}{\mathcal{E}_{ab}}$ are improved versions of $N_a, \underset{(n)}{E_{ab}}$

Nonlinear fluxes $\underset{(n)}{\mathcal{F}_{ab}}(u)$ vanish when $N_{ab}=0$

Bondi data $C_{ab}, m, \mathcal{N}_a, \underset{(n)}{\mathcal{E}_{ab}}$ are conserved in non-radiative spacetimes

$\mathcal{D}_n \equiv -\frac{n+2}{2(n+1)(n+4)}(D^2 + n^2 + 5n + 2).$

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Tower of charges

It is natural to define charges by two towers of charges

$$\mathcal{Q}^{+}_{0,\phi}(u) \equiv \oint_S m\phi, \qquad \mathcal{Q}^{-}_{0,\phi}(u) \equiv \oint_S \tilde{m}\phi$$

$$\mathcal{Q}^{+}_{1,\phi}(u) \equiv \oint_S \mathcal{N}^a D_a\phi, \qquad \mathcal{Q}^{-}_{1,\phi}(u) \equiv \oint_S \tilde{\mathcal{N}}^a D_a\phi$$

$$\mathcal{Q}^{+}_{n,\phi}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}}^{ab}\, D_a D_b\phi, \qquad \mathcal{Q}^{-}_{n,\phi}(u) \equiv \oint_S \underset{(n)}{\tilde{\mathcal{E}}}^{ab}\, D_a D_b\phi.$$

for arbitrary function $\phi(\theta^a)$ on the sphere,

$$\tilde{m} \equiv \frac{1}{4} D_c D_d \tilde{C}^{cd}\,, \qquad \tilde{C}_{ab} \equiv \epsilon_a{}^c C_{cb} \qquad \tilde{\mathcal{N}}^a = \epsilon^a{}_b \mathcal{N}^b$$

First line: Supermomenta[Bondi et al., Sachs], dual supermomenta [Godazgar[3], Pope; Porrati et al.]

Second line: Super-angular momenta, super center of mass [Campiglia, Laddha, Compère et al.]

Third line: "Higher spin" charges [Grant, Nichols'21; Freidel, et al. '21; Compère, Oliveri, AS'22]

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Celestial charges [Compère, Oliveri, Seraj '22]

$$\mathcal{Q}^\pm_{n,L} \equiv \mathcal{Q}^\pm_n[\phi = n_{\langle L\rangle}]$$

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Metric reconstruction

Celestial charges can be used to reconstruct a nonradiative spacetime

Subtlety: Supertranslation frame is ambiguity

$$C_{ab} = -2D_{\langle a}D_{b\rangle}C^{+} + 2\tilde{D}_{(a}D_{b)}C^{-}\,, \qquad \delta_T C^{+} = T(\theta^a)$$

C^{-} is fixed by dual supermomenta, while C^{+} is unconstrained

One can choose a supertranslations frame in which $C^{+} = 0$

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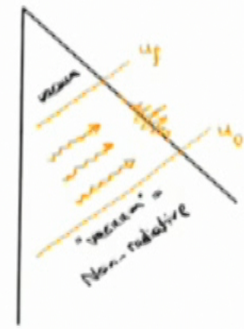
Radiative vacuum transitions

Sandwich spacetimes: Radiation N_{ab} decays exponentially outside a time interval



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Radiative vacuum transitions

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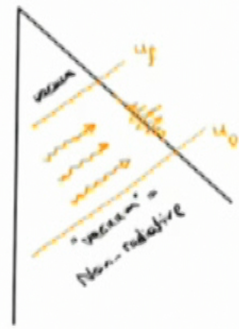

Nonradiative spacetime= **radiative vacuum** (no graviton)

The system is in vacuum state before and after radiation

The vacua are completely characterized by celestial charges $Q^{I}_{h,L}$.

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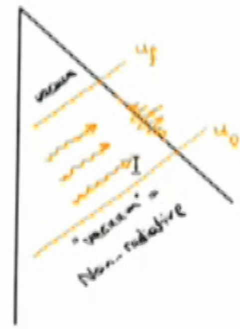
Radiative vacuum transitions

Sandwich spacetimes: Radiation N_{ab} decays exponentially outside a time interval



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The vacua are completely characterized by celestial charges $\mathcal{Q}_{n,L}$.
Vacuum transition specified by $\Delta\mathcal{Q}_{n,L}$ controlled by **flux balance equations**.

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Flux balance equations

Vacuum transition and balance equations

Flux balance equation

$$n = 0 \quad : \quad \partial_u m = \frac{1}{4} D_b D_c N^{bc} + \mathcal{F}(u),$$

$$n = 1 \quad : \quad \partial_u \mathcal{N}_a = -\frac{u}{2} D_c D_{\langle a} D_{b\rangle} N^{bc} + \mathcal{F}_a(u),$$

$$n \geq 2 \quad : \quad \partial_u \underset{(n)}{\mathcal{E}_{ab}} = \frac{(-u)^n}{6\, n!} \mathcal{D}_{n-3} \cdots \mathcal{D}_0\, \mathsf{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] + \underset{(n)}{\mathcal{F}_{ab}}(u).$$

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Vacuum transition and balance equations

Flux balance equation integrated over time

$$n=0 \quad : \quad \Delta m = \frac{1}{4} D_b D_c \mathcal{M}_0(N^{bc}) + \int du \mathcal{F}(u),$$

$$n=1 \quad : \quad \Delta \mathcal{N}_a = -\frac{1}{2} D_c D_{\langle a} D_{b\rangle} \mathcal{M}_1(N^{bc}) + \int du \mathcal{F}_a(u),$$

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Flux-balance equations for celestial charges $n \geq 3$

$$\Delta \mathcal{Q}^{+}_{n,L} = \frac{n-1}{2^n n!(n+1)!} \prod_{k=0}^{n-1} [-\ell(\ell+1) + k(k+1)] \oint_S \hat{n}_L D_a D_b \mathcal{M}_n(N^{ab}) + \text{hard flux},$$

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Vacuum transition and balance equations

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Vacuum transition given by

$\mathcal{M}_n(N_{ab}) = \int du u^n N_{ab}$ Melling tranform of the news, or sub-n soft graviton operator

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Nonlinear fluxes of hard gravitons

Classification of flux-balance equations

Memory-full Soft term nonzero.

$$\text{Memory} = \Delta Q - \text{hard flux}$$

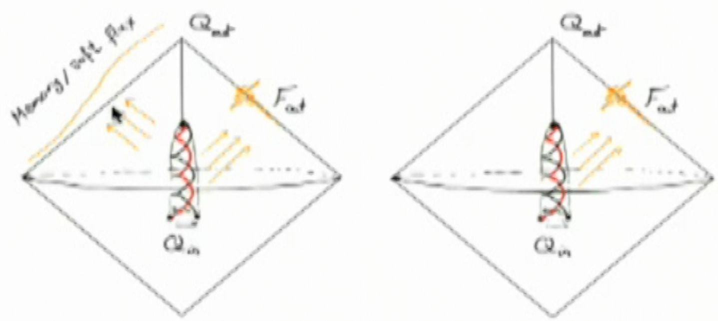
Relates memory to charges and fluxes

Soft sector adapts to the hard process to satisfy conservation

Memory-less Soft (memory) term vanishes.

$$\Delta Q = \text{hard flux}$$

Quantify radiation reaction



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Memory-less modes

Memory-less modes

$n = 0 \quad \ell = 0, 1$ $\qquad \mathcal{E} = \mathcal{Q}^{+}_{0,0}, \quad \mathcal{P}_i = \mathcal{Q}^{+}_{0,i}$

$n = 1 \quad \ell = 1$ $\qquad \mathcal{J}_i = \mathcal{Q}^{+}_{1,i}, \quad \mathcal{K}_i = \mathcal{Q}^{-}_{1,i}$

$n = 2 \quad \varnothing$

$n = 3 \quad \ell = 2$ $\qquad$ Newman-Penrose charges [Newman, Penrose '62][Godazgar et al.'18]

$n > 3 \quad \ell \leq n - 1$ $\qquad$ Generalized NP charges

Applications for radiation reaction [Peters, Mathews'63, . . . , Damour'20]

Memory-full modes

$n = 0 \quad \ell \geq 2$ $\qquad$ Displacement memory [Christodoulou]

$n = 1 \quad \ell \geq 2$ $\qquad$ spin and center of mass memory [Pasterski et al.; Nichols]

$n \geq 2 \quad \ell \geq n$ $\qquad$ subleading memory effects, e.g. "curve deviation" [Grant, Nichols]

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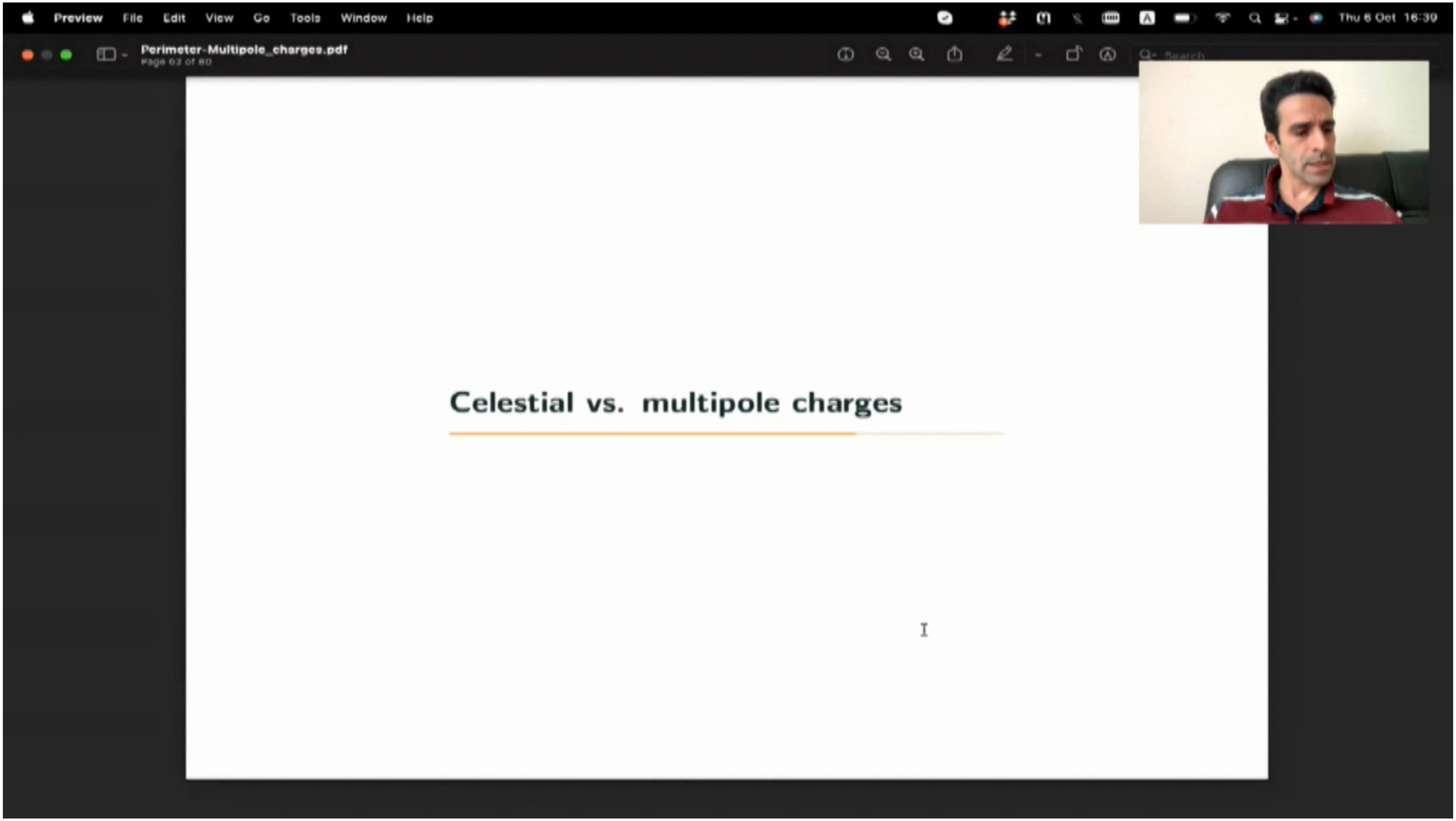
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Celestial vs. multipole charges

Post-Monkowskian expansion

Harmonic gauge $\qquad \partial_\mu h^{\alpha\mu} = 0\,, \qquad h^{\alpha\beta} \equiv \sqrt{-g}g^{\alpha\beta} - \eta^{\alpha\beta}$

Einstein equation outside sources

$$\Box h^{\alpha\beta} = \frac{16\pi G}{c^4}\tau^{\alpha\beta}\,, \qquad \tau^{\alpha\beta} = |g|T^{\alpha\beta} + \frac{c^4}{16\pi G}\Lambda^{\alpha\beta}(h^2, h^3, \cdots)$$

Post-Minkowskian expansion $h^{\alpha\beta} = \sum_{n=1}^{\infty} G^n h^{\alpha\beta}_{(n)}$.

At leading order (outside sources)

$$\Box h^{\alpha\beta}_{(1)} = 0, \qquad \partial_\mu h^{\alpha\mu}_{(1)} = 0$$

whose canocical solution is specified by mass M_L and spin S_L multipole moments

$$h_1^{00} = -4\sum_{\ell=0}^{+\infty}\frac{(-)^\ell}{\ell!}\bar{\partial}_L\Big(\frac{M_L(\tilde{u})}{\tilde{r}}\Big),$$

$$h_1^{0j} = 4\sum_{\ell=1}^{+\infty}\frac{(-)^\ell}{\ell!}\Big[\tilde{\partial}_{L-1}\Big(\frac{M^{(1)}_{jL-1}(\tilde{u})}{\tilde{r}}\Big) + \frac{\ell}{\ell+1}\tilde{\partial}_{pL-1}\Big(\frac{\varepsilon_{jpq}S_{qL-1}(\tilde{u})}{\tilde{r}}\Big)\Big],$$

$$h_1^{jk} = -4\sum_{\ell=2}^{+\infty}\frac{(-)^\ell}{\ell!}\Big[\tilde{\partial}_{L-2}\Big(\frac{M^{(2)}_{jkL-2}(\tilde{u})}{\tilde{r}}\Big) + \frac{2\ell}{\ell+1}\tilde{\partial}_{pL-2}\Big(\frac{\varepsilon_{pq(j}S^{(1)}_{k)qL-2}(\tilde{u})}{\tilde{r}}\Big)\Big],$$

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Nonradiative spacetimes (vacua)

From harmonic to Bondi gauge *[Blanchet, Compère, Faye, Oliveri, Seraj '20]*

$$C_{ab} = 4Ge^i_{\langle a}e^j_{b\rangle}\sum_{\ell=2}^{+\infty}\frac{n_{L-2}}{\ell!}\left[M^{(\ell)}_{ijL-2} - \frac{2\ell}{\ell+1}\varepsilon_{ipq}n_p S^{(\ell)}_{jqL-2}\right] + \mathcal{O}(G)$$

Nonradiative spacetime

$$N_{ab} = 0 \qquad \Leftrightarrow \qquad \partial_u^{\ell+1}M_L = 0, \qquad \partial_u^{\ell+1}S_L = 0$$

$$M_L^{\text{nonrad}}(u) = \sum_{k=0}^{\ell} M_{L,k}u^k, \qquad S_L^{\text{nonrad}}(u) = \sum_{k=0}^{\ell} S_{L,k}u^k,$$

A non-radiative spacetime is characterized by multipole charges

$$M_{L,k}, \quad S_{L,k}, \qquad \ell \geq 0, \quad 0 \leq k \leq \ell$$

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Non-radiative spacetimes and charges

In the linearized theory, Bondi data turn out to be [Blanchet, Compère, Faye, Oliveri, Seraj '20]

$$m = \sum_{\ell=0}^{+\infty} \frac{(\ell+1)(\ell+2)}{2\ell!} n_L M_L^{(\ell)} + O(G)$$

$$\underset{(n)}{\mathcal{N}_a} = \epsilon_a^i \sum_{\ell \geq 1} \frac{(\ell+1)(\ell+2)}{2(\ell-1)!} n_{L-1} \left(1 - \frac{u}{n}\partial_u\right) \left[M_{iL-1}^{(\ell-1)} + \frac{2\ell}{\ell+1} \epsilon_{ipq} n_p S_{qL-1}^{(\ell-1)} \right] + O(G),$$

$$\underset{(n-2)}{E_{ab}} = 4 e^i_{\langle a} e^j_{b\rangle} \frac{n-1}{n+1} \sum_{\ell=n}^{\infty} \frac{1}{\ell!} a_{n\ell} n_{L-2} \left[M_{ijL-2}^{(\ell-n)} + \frac{2\ell}{\ell+1} \epsilon_{ipq} n_p S_{jqL-2}^{(\ell-n)} \right] + O(G)$$

Computing charges in non-radiative spacetimes gives [Compère, Oliveri, Seraj '22]

$$Q^+_{n,L} = a_{n,\ell}(\ell-n)! M_{L,\ell-n} + O(G),$$

$$Q^-_{n,L} = a_{n,\ell}(\ell-n)! \frac{2\ell}{\ell+1} S_{L,\ell-n} + O(G), \qquad \ell \geq n.$$

Generalized NP charges vanish at linear order

$$Q^{\pm}_{n,L} = 0 + O(G), \qquad 0 \leq \ell < n$$

Equivalence between celestial charges $Q^{\pm}_{n,L}$ and multipole charges $M_{L,k}, S_{L,k}$

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Perimeter-Multipole_charges.pdf
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Summary and Outlook

- Compute generalized Newman-Penrose charges at quadratic order [*Blanchet, Compère, Fay… Oliveri, Seraj, to appear*]

  Implications for binary systems?

- **Gravitational tail effects** imply that the sandwich model is not appropriate

$$C_{ab} = \sum_{n=1}^{\infty} \frac{c_{ab}}{u^n}\,, \qquad u \to +\infty$$

  Leads to **simple poles in the integer Mellin transform** of the news

- Gravitational EM duality [*Henneaux, Teitelboim '05*]

$$W_{\mu\nu\alpha\beta} \leftrightarrow *W_{\mu\nu\alpha\beta}\,, \qquad M_L \leftrightarrow \frac{2\ell}{\ell+1} S_L\,, \qquad C_{ab} \leftrightarrow \tilde{C}_{ab}$$

Thank you for your attention!

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