Title: BMS fluxes from every corner

Speakers: Laura Donnay

Collection: Quantum Gravity Around the Corner

Date: October 06, 2022 - 9:00 AM

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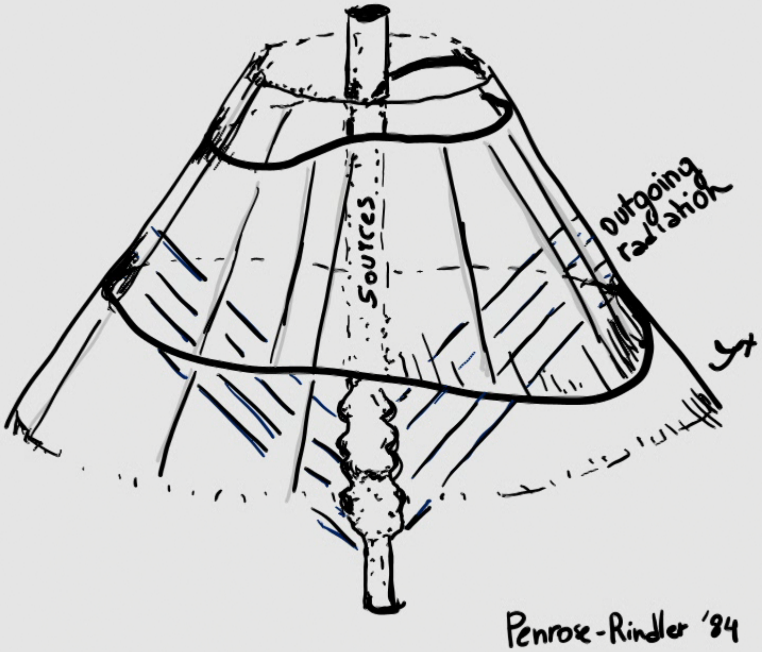
Abstract: In this talk, I will present an updated account on the prescription for BMS fluxes in asymptotically flat spacetimes, including their split into hard and soft pieces and the associated symplectic structure. Implications for flat space holography will be discussed.



BMS fluxes
from every **corner**

Laura DONNAY

Quantum Gravity around the Corner
Perimeter Institute
Oct 6 2022

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes $\longrightarrow$ vanishing cosmological constant

$$\Lambda = 0$$

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes → vanishing cosmological constant

$$\Lambda = 0$$

These spacetimes are relevant

from collider physics ...

... to astrophysics
(< cosmological scales)

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes

→ **Black holes**

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes

Black holes

Our understanding of quantum properties of black holes goes *hand-in-hand* with the **spectacular advances** of the **holographic** or **AdS/CFT correspondence**.

$$\boxed{S_{BH} = \frac{Ac^3}{4G\hbar}}$$

→ 'Primordial holographic relationship'

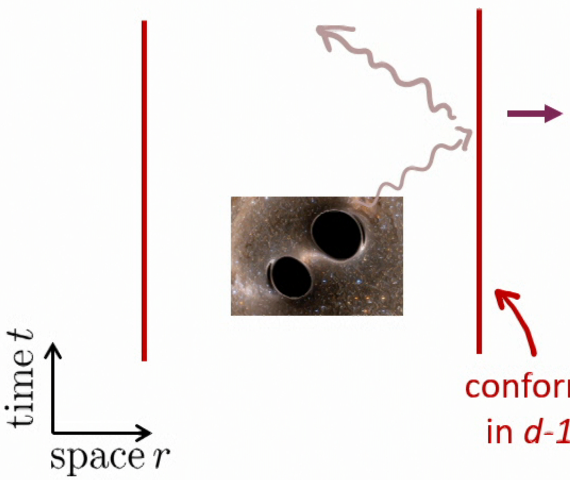
[Bekenstein][Hawking]

Problem: realistic black holes (e.g. Kerr) do not possess an AdS decoupling region.

→ need to develop a **holographic correspondence** for **flat spacetimes**

Holographic principle

Anti-de Sitter
in *d* dimensions



conformal boundary
in *d-1* dimensions

The holographic principle

Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

Holographic principle

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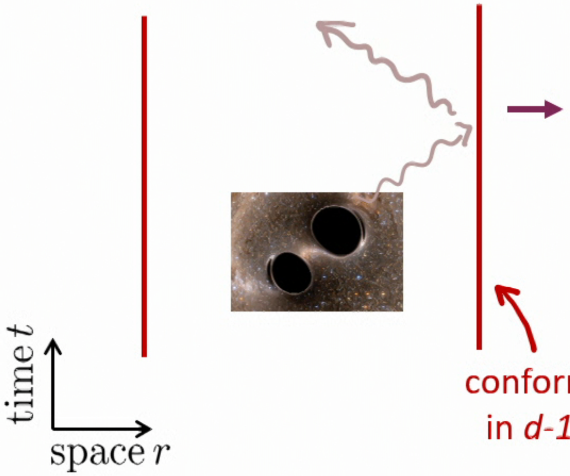

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The holographic principle → **How general is it?**

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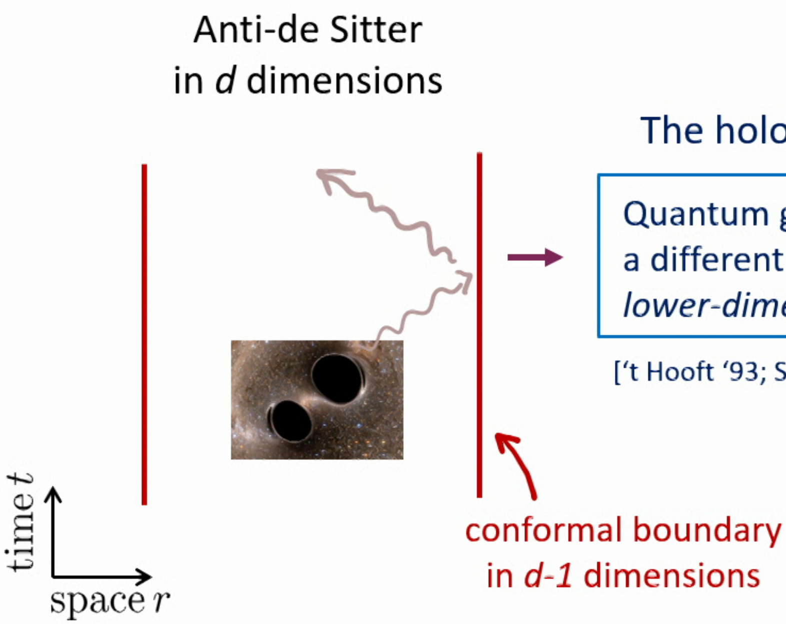
Go beyond the canonical cases!

Anti-de Sitter	vs	**Flat**
$\Lambda < 0$		$\Lambda = 0$
CFT		

Holographic principle



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Anti-de Sitter	vs	Flat
$\Lambda < 0$		$\boxed{\Lambda = 0}$
CFT		??
		this talk

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Early attempts:

[Susskind ‘99][Polchinski ‘99][Giddings ’99]
[de Boer, Solodukhin ‘03][Arcioni, Dappiaggi ’03 ’04]
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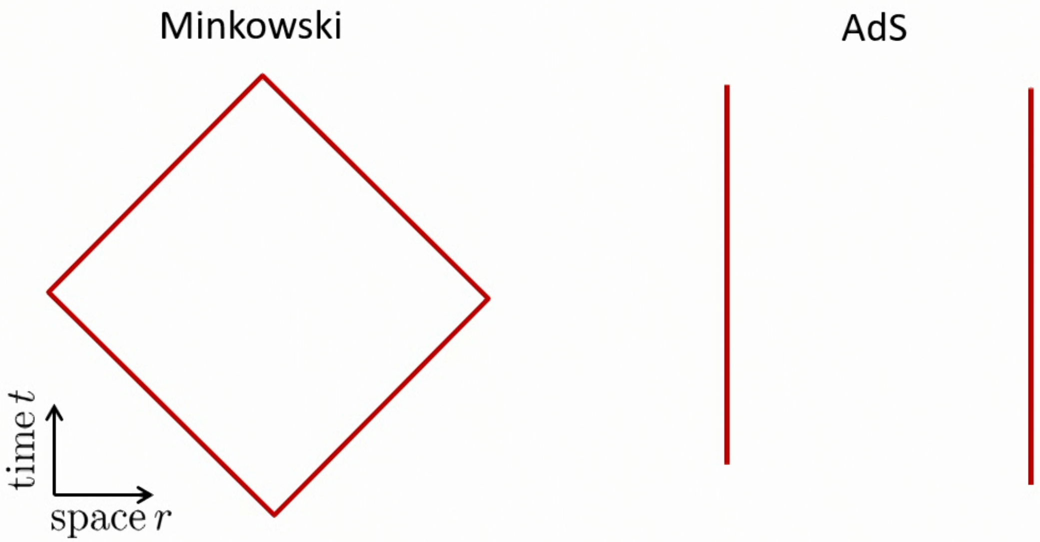

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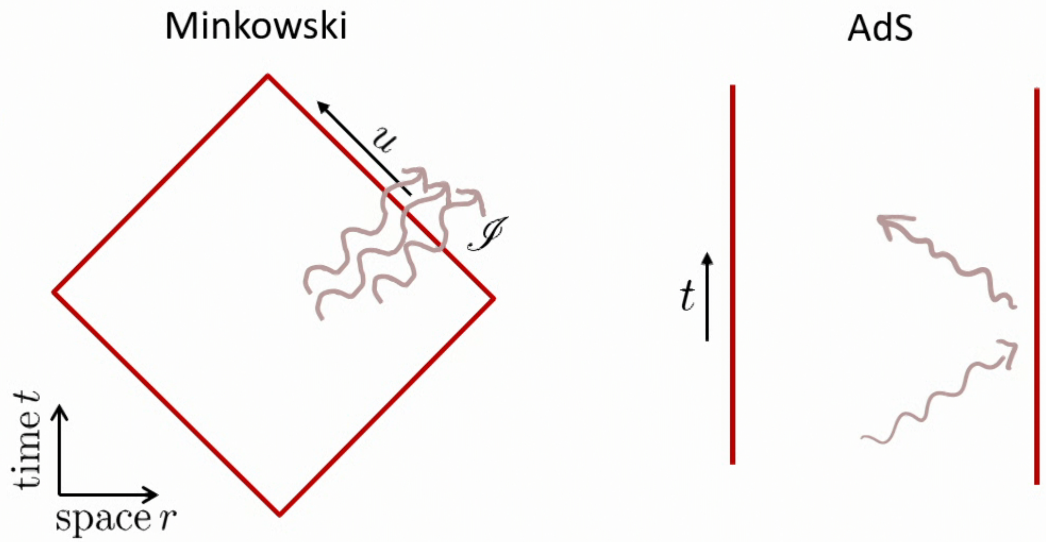

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1. The boundary is a **null** hypersurface

$$u = t - r$$

2. There are **fluxes** leaking out through the boundary

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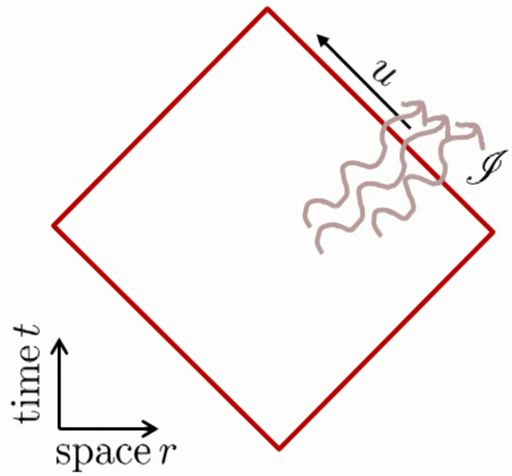
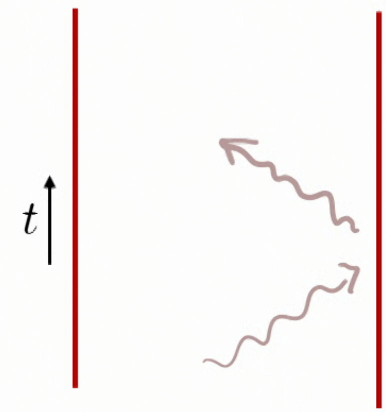


AdS



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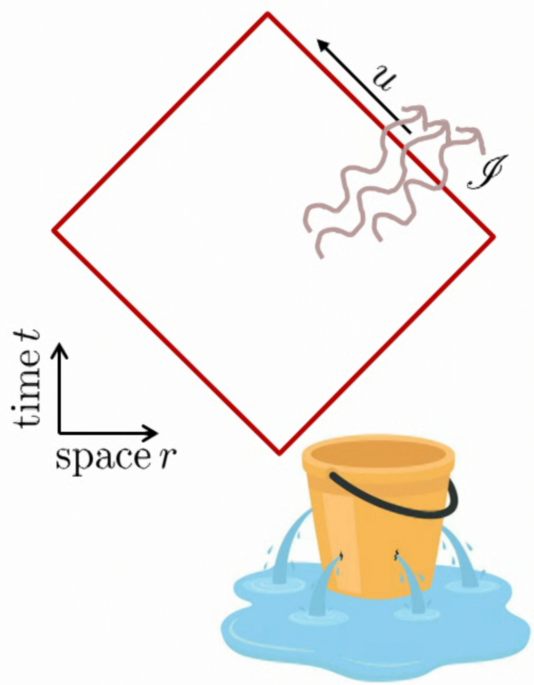

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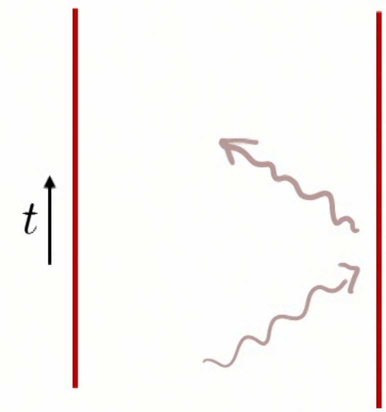

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two natural boundaries/proposals

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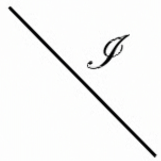

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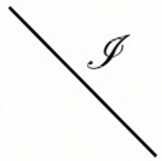

4d bulk/3d holography: ‘Carroll holography’

Dual: 3d ‘BMS field theory’

[Arcioni, Dappiaggi ’03 ’04] [Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06] [Bagchi, Basu, Kakkar, Melhra ’16] [Bagchi, Melhra, Nandi ’20] [LD, Fiorucci, Herfray, Ruzziconi ’22][Bagchi, Banerjee, Basu, Dutta ‘22][...]

celestial sphere

Euclidean 2-sphere

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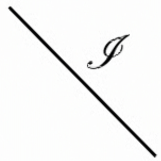

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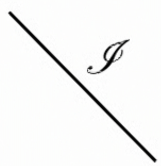

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treatment of fluxes ☹

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role of translations obscured ☹

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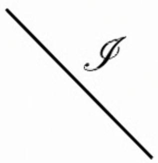

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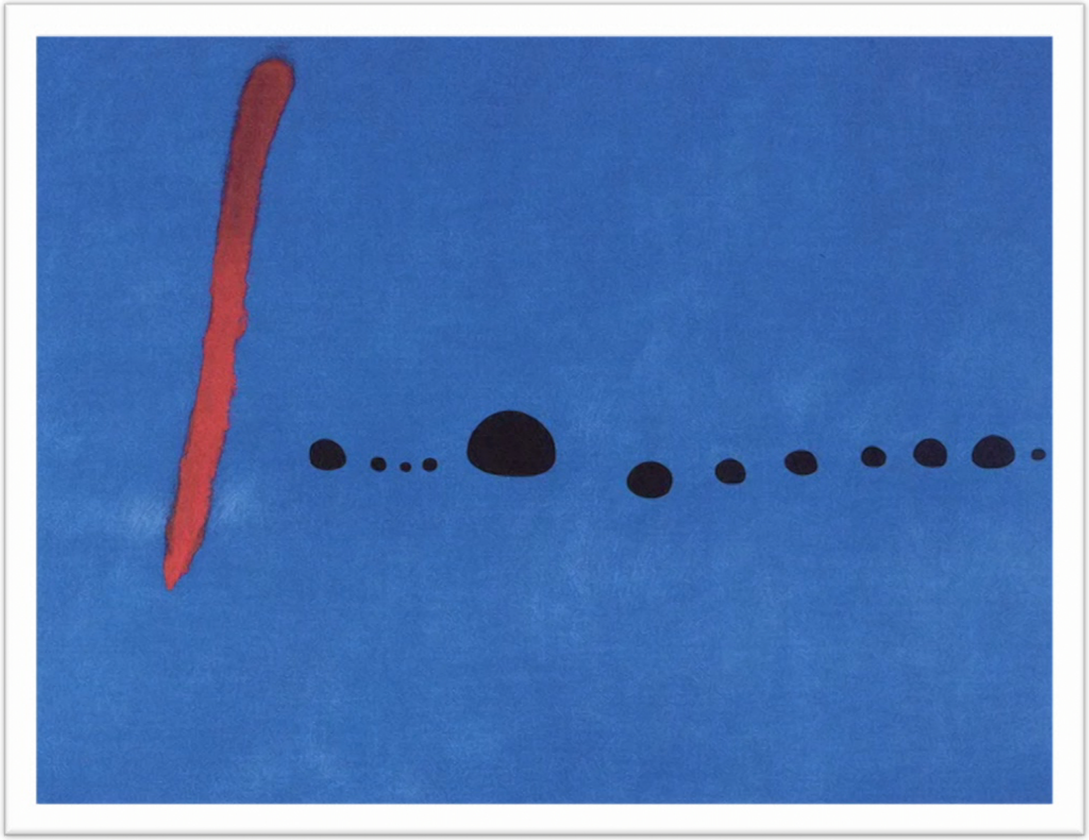


Joan Miró, *Bleu II* (1961)

Outline

1. BMS & the S-matrix
2. Celestial currents
3. BMS fluxes & phase space

based on

2108.11969 w/ **Romain RUZZICONI**

2205.11477 w/ **Kevin NGUYEN** & **Romain RUZZICONI**

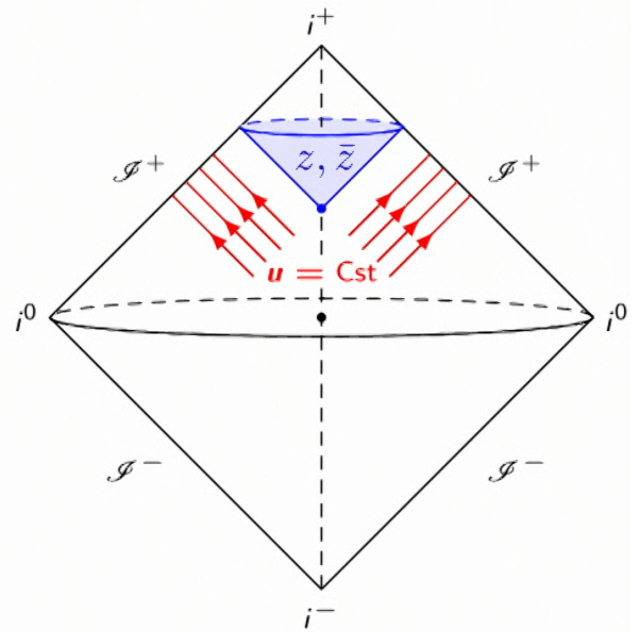
Gravitational solution space

[Bondi, van der Burg, Metzner '62] [Sachs '62]
[Barnich, Troessaert '10]

Asymptotically flat spacetimes in Bondi gauge:

$$r \to \infty \qquad\qquad (u, r, x^A)\,, \quad x^A = (z, \bar{z})$$

$$\boxed{\begin{aligned} ds^2 &= -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\ &\quad + \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^z C_{zz}\mathrm{d}u\mathrm{d}z \\ &\quad + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots \end{aligned}}$$



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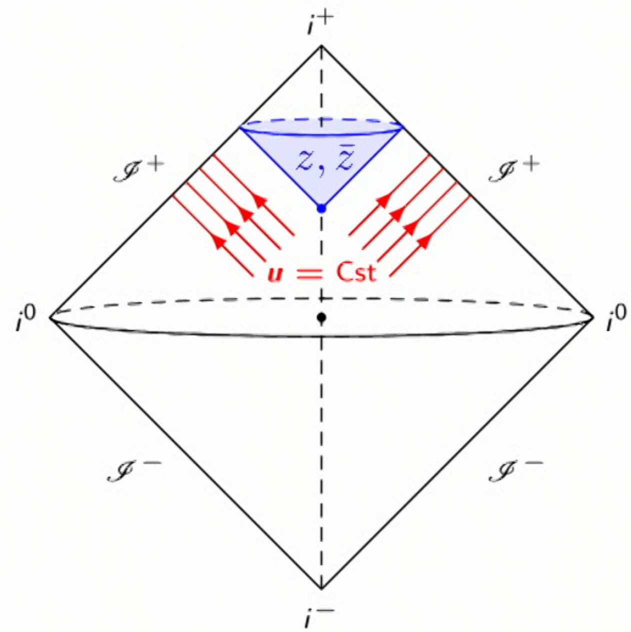

The Bondi **mass** and **angular momentum** aspects satisfy

$$\partial_u M = -\frac{1}{8}N_{AB}N^{AB} + \frac{1}{4}\partial_A\partial_B N^{AB}\,,$$

$$\begin{aligned} \partial_u N_A &= \partial_A M + \frac{1}{16}\partial_A(N_{BC}C^{BC}) - \frac{1}{4}N^{BC}\partial_A C_{BC} \\ &\quad - \frac{1}{4}\partial_B(C^{BC}N_{AC} - N^{BC}C_{AC}) - \frac{1}{4}\partial_B\partial^B\partial^C C_{AC} + \frac{1}{4}\partial_B\partial_A\partial_C C^{BC} \end{aligned}$$

$N_{AB} \equiv \partial_u C_{AB}$

Bondi news: encodes **gravitational waves!**

On the various extensions of BMS

$$\xi = \left(\mathcal{T}(z,\bar{z}) + \tfrac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \cdots$$

- Global BMS: $\boxed{\mathfrak{bms}_4^{\text{glob}} = \mathfrak{so}(3,1) \oplus\!\!\!\!+\; \mathfrak{supertranslations}}$ [Bondi, van der Burg, Metzner '62] [Sachs '62]

  supertranslations needed to include radiation

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  allows for non-globally well-defined transformations on the celestial sphere

- Generalized BMS: $\boxed{\mathfrak{bms}_4^{\text{gen}} = \mathfrak{diff}(S^2) \inplus \mathfrak{supertranslations}}$ [Campiglia, Laddha '14]

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- Weyl BMS: $\boxed{\mathfrak{bms}_4^{\text{Weyl}} = [\mathfrak{diff}(S^2) \inplus \text{Weyl}] \inplus \mathfrak{supertranslations}}$

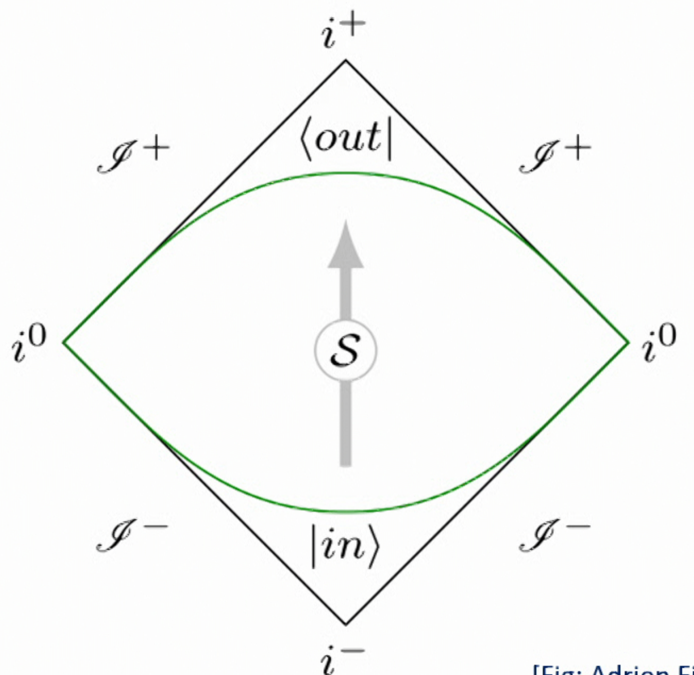
  [Barnich, Troessaert '10][Freidel, Oliveri, Pranzetti, Speziale '21]

  includes (on top of the rest) Weyl rescalings

BMS and the scattering problem

Seminal observation: BMS symmetries constrain the gravitational scattering problem! [Strominger ’14]

$\langle out|S|in\rangle$



[Fig: Adrien Fiorucci]

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→ 2 key ingredients

1 Noether charges for BMS symmetries

[Barnich, Troessaert '10]

$$Q_{\mathcal{T}} = \frac{1}{4\pi G} \int d^2 z \sqrt{\gamma}\, \mathcal{T} M$$

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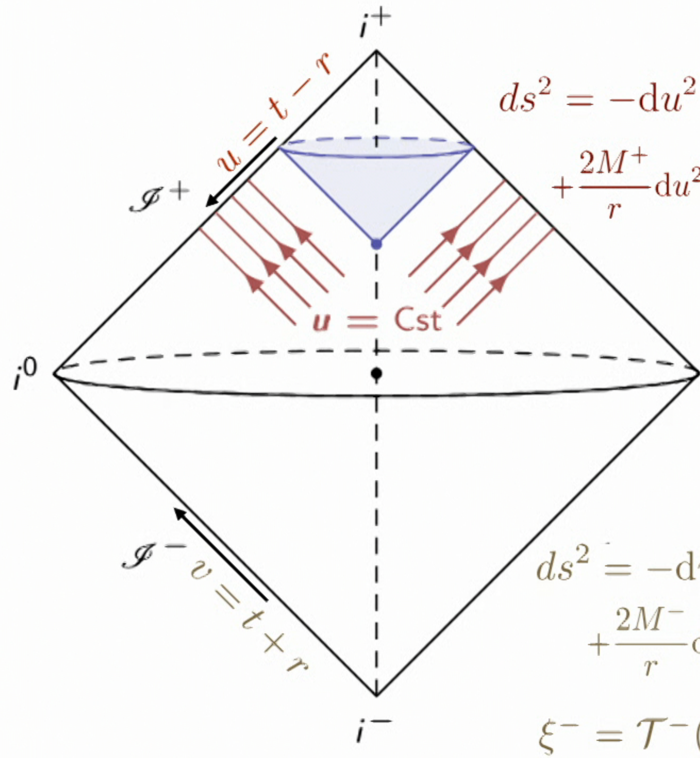
2 Relating the *past* and the *future*



$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}\,dzd\bar{z} + \frac{2M^+}{r}du^2 + rC^+_{zz}dz^2 + D^zC^+_{zz}dudz + \cdots$$

$$\xi^+ = \mathcal{T}^+(z,\bar{z})\partial_u$$

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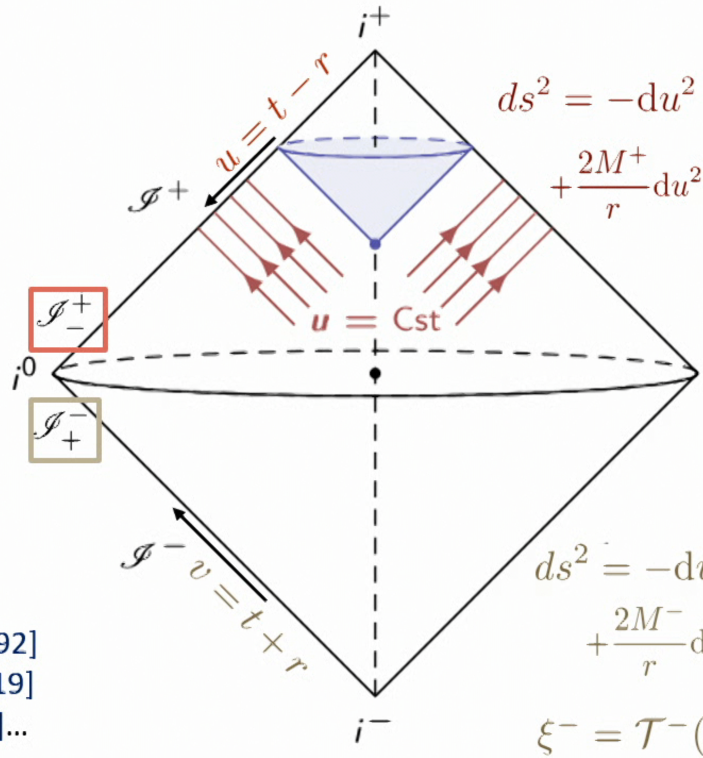
2. Relating the *past* and the *future*

Antipodal matching conditions

$$M^-(v,z,\bar z)|_{\mathscr{I}^-_+} = M^+(u,z,\bar z)|_{\mathscr{I}^+_-}$$

$$\mathcal{T}^-(z,\bar z)|_{\mathscr{I}^-_+} = \mathcal{T}^+(z,\bar z)|_{\mathscr{I}^+_-}$$

[Strominger '14]; see also [Herberthson, Ludvigsen '92]
[Troessaert '18][Henneaux, Troessaert '18][Prabhu '19]
[Kroon, Mohamed '21][Capone, Nguyen, Parisini '22]...



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$$\longrightarrow \boxed{Q^+_{\mathcal{T}} = Q^-_{\mathcal{T}}}$$

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$$\langle out|Q^+_{\mathcal{T}}\mathcal{S} - \mathcal{S}Q^-_{\mathcal{T}}|in\rangle = 0$$

infinite amount of conservation laws

3 languages for the same IR physics

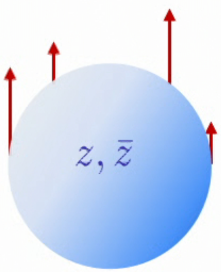
[Strominger '18]

Asymptotic symmetries

General Relativity

supertranslations

[Bondi-Metzner-Sachs '62]



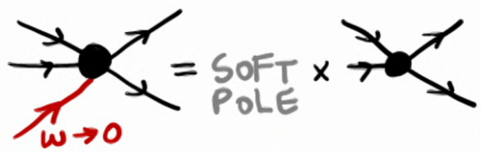
$\Delta C_{AB} \neq 0$

Soft theorems

Quantum Field Theory

leading soft graviton theorem

[Weinberg '65]

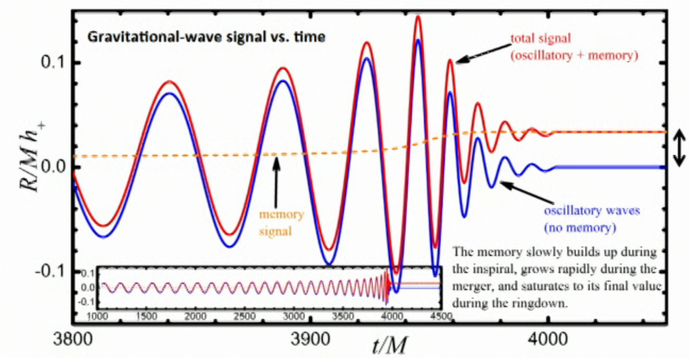


Memory effects

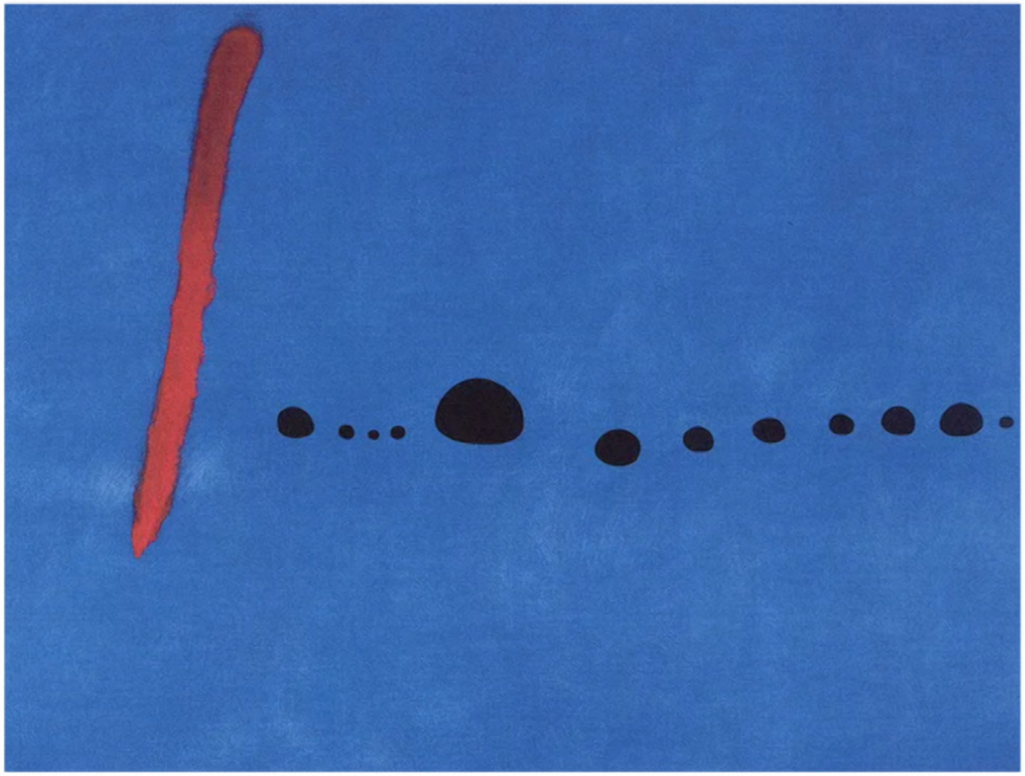
GW observation

displacement memory

[Zel'dovich, Polnarev, Braginskii, Thorne, Christodoulou] ... 70s – 90s



[Favata, '10]



Joan Miró, *Bleu II* (1961)

Outline

1. BMS & the S-matrix
2. Celestial currents
3. BMS fluxes & phase space

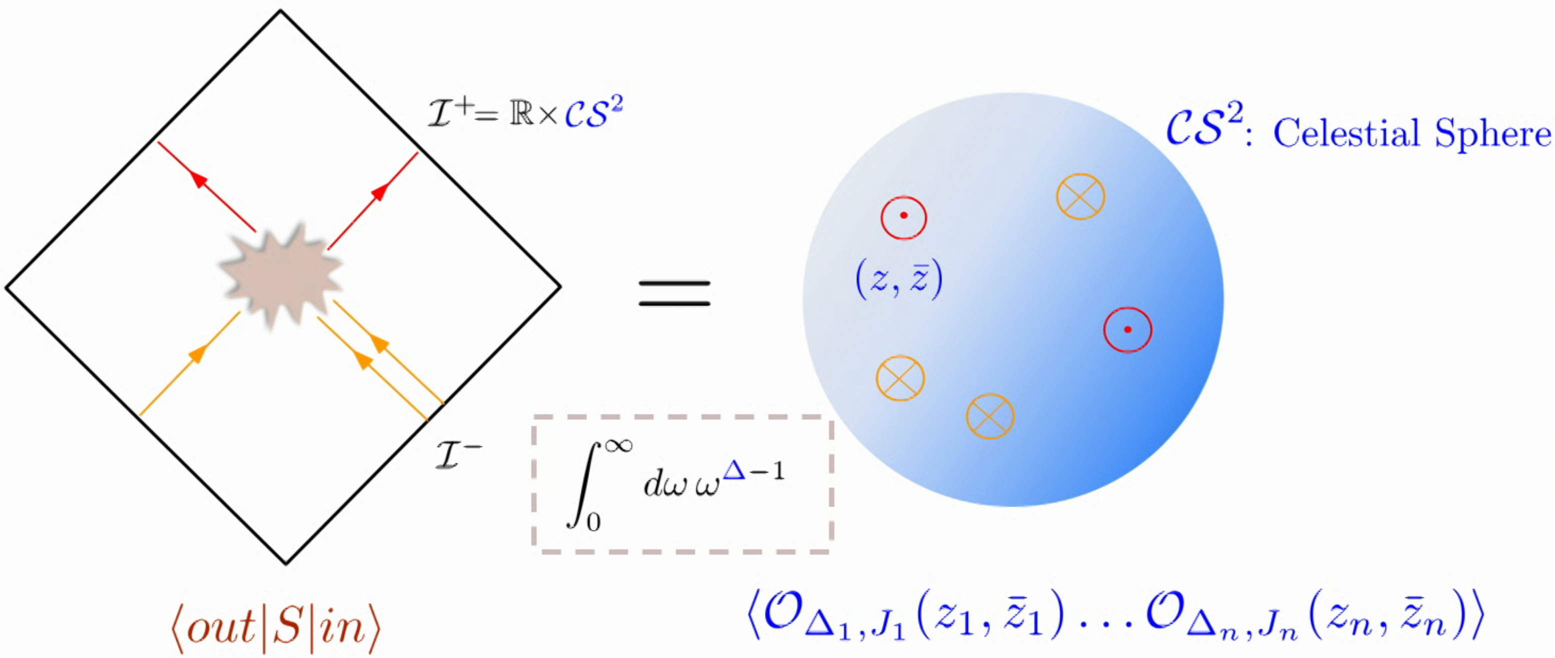
based on

2108.11969 w/ **Romain RUZZICONI**

2205.11477 w/ **Kevin NGUYEN** & **Romain RUZZICONI**

Celestial Holography

The 4d spacetime S-matrix is encoded in a 2d 'Celestial Conformal Field Theory'



Basis for celestial holography

Holographic basis:

$$\boxed{\int_0^\infty d\omega\, \omega^{\Delta-1}}$$

Mellin transform

ω: energy

de Boer, Solodukhin
Cheung, de la Fuente, Sundrum
Pasterski, Shao, Strominger

Plane waves (null momentum $p^\mu = \omega q^\mu(z,\bar{z})$) get mapped to

$$\Psi^{\pm}_{\Delta}(X;z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i p\cdot X}$$

$\Delta = h + \bar{h}$: conformal dimension
$(z,\bar{z})$: a point on $\mathcal{CS}^2$

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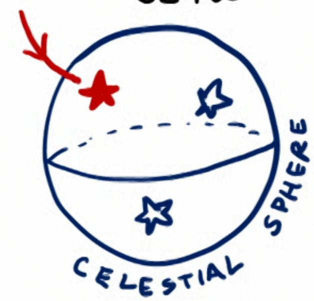
'conformal primary wavefunctions' which transform under $SL(2,\mathbb{C})$

$$\Psi_{h,\bar{h}}(z,\bar{z}) \to (cz+d)^{2h}(\bar{c}\bar{z}+\bar{d})^{2\bar{h}}\Psi_{h,\bar{h}}(z,\bar{z})$$

$z \to \frac{az+b}{cz+d}$



as primaries of weights

$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

2d spin J

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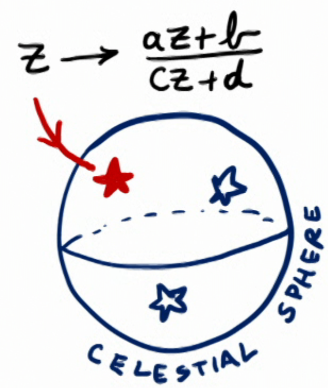

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Basis for celestial holography

- **Celestial operators** are defined as [LD, Pasterski, Puhm '20]

$$\boxed{\mathcal{O}_\Delta(z,\bar{z}) = (\Phi(X), \Psi_\Delta(X; z, \bar{z}))}$$

bulk operator

$(\cdot,\cdot)$: Klein-Gordon inner product pushed at $\mathscr{I}$

X : a point in the bulk

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bulk operator

conformal primary wavefunction

$(\cdot,\cdot)$: Klein-Gordon inner product pushed at $\mathscr{I}$

Recall:

$$\Psi^\pm_\Delta(X;z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i p\cdot X}$$

X : a point in the bulk

$p^\mu = \omega q^\mu(z,\bar{z})$

NB: for simplicity, I consider here only scalar operators (and some labels are sometimes omitted)

Basis for celestial holography - let's repeat

- Momentum basis

$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[a(p)e^{ip\cdot X} + a(p)^\dagger e^{-ip\cdot X}\right]$$

$$p^\mu = \omega q^\mu(\vec{w})$$

Basis for celestial holography - let's repeat

- Momentum basis

$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[a(p)e^{ip\cdot X} + a(p)^\dagger e^{-ip\cdot X}\right] \qquad p^\mu = \omega q^\mu(\vec{w})$$

$$= \int \frac{d^2\vec{w}}{2(2\pi)^3}\omega d\omega \left[a(\omega,\vec{w})e^{i\omega q\cdot X} + a(\omega,\vec{w})^\dagger e^{-i\omega q\cdot X}\right]$$

- Celestial basis

$$\Phi(X) = \int \frac{d^2\vec{w}}{2(2\pi)^3} \overbrace{\int_{c-i\infty}^{c+i\infty} \frac{d\Delta}{i2\pi}}^{\text{inverse Mellin}} \left[a_{2-\Delta}(\vec{w})\Psi^+_\Delta(X;\vec{w}) + a_{2-\Delta}(\vec{w})^\dagger \Psi^-_\Delta(X;\vec{w})\right]$$

Basis for celestial holography - let's repeat

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Mellin of plane waves — Mellin of ladder operators

Ladder operators

$$a(p) = (\Phi(X), e^{ip\cdot X})$$

Celestial operators

$$a_\Delta(\vec{w}) = (\Phi(X), \Psi^+_\Delta(X;\vec{w})) \equiv \mathcal{O}_{\Delta,J=0}(\vec{w})$$

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Celestial currents

$$\int_0^\infty d\omega\, \omega^{\Delta-1}$$

$$\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

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weights of the celestial operators

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weights of the celestial operators

Celestial currents are obtained by taking 'conformally soft' limits $\Delta \to \mathbb{Z}$

[LD, Puhm, Strominger '18]

- dual notion to energetically soft limit $\omega \to 0$ -

QED $(J=1)$**:**

$\Delta \to 1$

- U(1) Kac-Moody current

$$J(z) = \mathcal{O}_{\Delta=1,J=1} \; : (1,0)$$

$$\langle J(z)\mathcal{O}_1 \cdots \mathcal{O}_n\rangle = \sum_{k=1}^n \frac{1}{(z-z_k)} \langle \mathcal{O}_1 \cdots \mathcal{O}_n\rangle$$

Celestial version of Weinberg's soft photon theorem!

Celestial currents

$$\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^{h} \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

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Gravity $(J = 2)$**:**

$\Delta \to 1$

- Supertranslation current

$$P(z,\bar{z}) = \partial_{\bar{z}} \mathcal{O}_{\Delta=1, J=2}$$

$$(\tfrac{3}{2}, -\tfrac{1}{2} + 1) = (\tfrac{3}{2}, \tfrac{1}{2})$$

Celestial currents

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$$P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)} \mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$$

Celestial version of Weinberg's (leading) soft graviton theorem!

[Strominger '14][He, Lysov, Mitra, Strominger '15]
[LD, Puhm, Strominger '18][Stieberger, Taylor '19]

Celestial currents

$$\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^{h} \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

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Gravity $(J = 2)$**:**

- 2d stress tensor $T(z)$: $(2,0)$!!

$$\boxed{T(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{h}{(z-w)^2}\mathcal{O}_{h,\bar{h}}(w,\bar{w}) + \frac{\partial \mathcal{O}_{h,\bar{h}}(w,\bar{w})}{z-w}}$$

[Kapec, Mitra, Raclariu, Strominger][Cheung, de la Fuente, Sundrum]
[LD, Puhm, Strominger][Stieberger, Taylor][Fotopoulos, Stieberger,Taylor]

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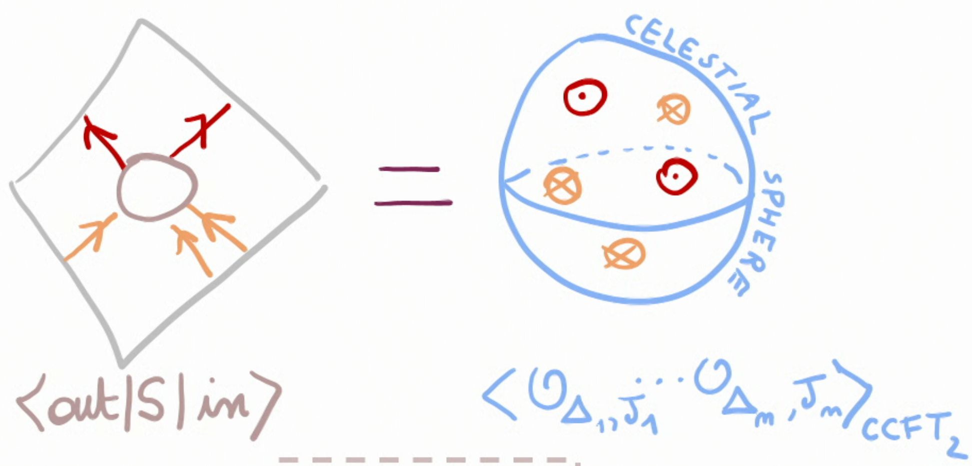
[Kapec, Mitra, Raclariu, Strominger][Cheung, de la Fuente, Sundrum]
[LD, Puhm, Strominger][Stieberger, Taylor][Fotopoulos, Stieberger,Taylor]

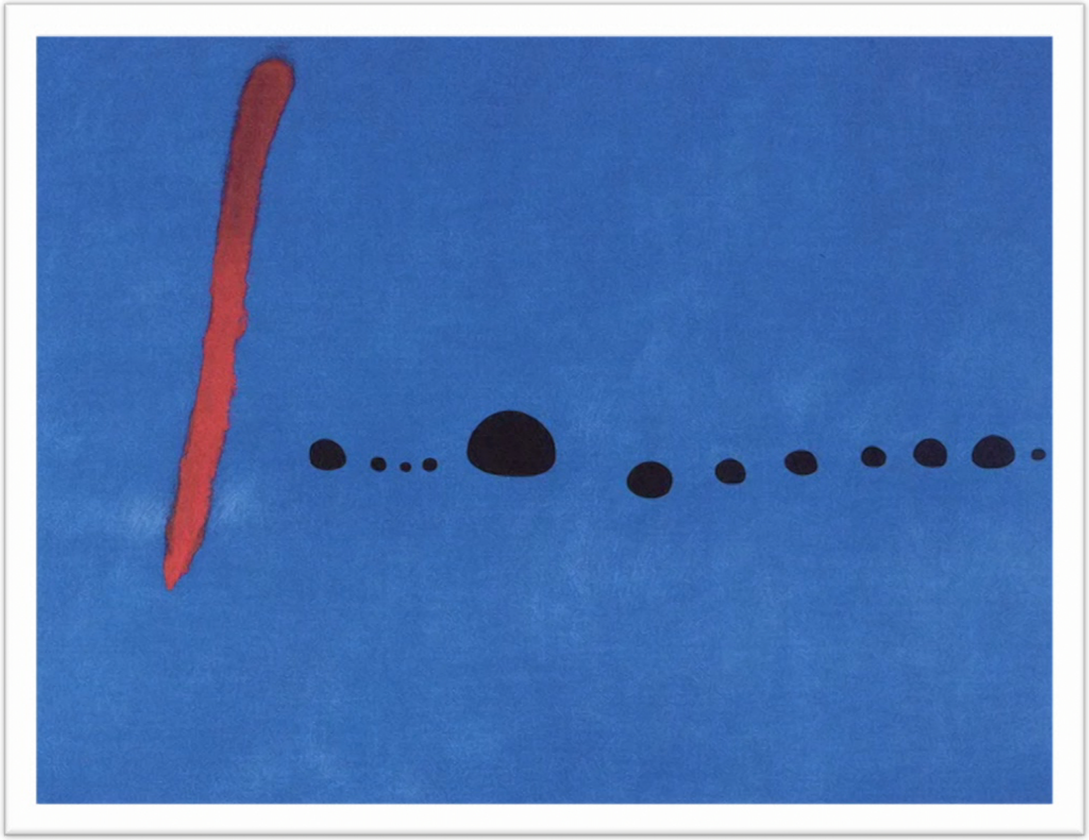
This promotes celestial operators to full **Virasoro** primaries on the celestial sphere!

Summary: Celestial Holography



$$\boxed{\int_0^\infty d\omega\, \omega^{\Delta - 1}}$$

The soft sector of scattering is captured by celestial currents $\Delta \to \mathbb{Z}$



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Question

Which objects from the gravitational phase space

$$
\begin{aligned}
ds^2 = & -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\
& + \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^z C_{zz}\mathrm{d}u\mathrm{d}z \\
& + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots
\end{aligned}
$$

transform as conformal primaries under the action of extended BMS symmetries?

$$
\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \rightarrow \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})
$$

BMS extended symmetries

BMS (supertanslations + superrotations) symmetries act as

$$\delta_{(f,Y)} C_{AB} = [f\partial_u + \mathcal{L}_Y - \frac{1}{2} D_C Y^C] C_{AB} - 2 D_A D_B f + \mathring{q}_{AB} D_C D^C f,$$

$$\delta_{(f,Y)} N_{AB} = [f\partial_u + \mathcal{L}_Y] N_{AB} - (D_A D_B D_C Y^C - \frac{1}{2} \mathring{q}_{AB} D_C D^C D_D Y^D),$$

$$\delta_{(f,Y)} M = [f\partial_u + \mathcal{L}_Y + \frac{3}{2} D_C Y^C] M + \frac{1}{8} D_C D_B D_A Y^A C^{BC} + \frac{1}{4} N^{AB} D_A D_B f + \frac{1}{2} D_A f D_B N^{AB},$$

$$\begin{aligned}\delta_{(f,Y)} N_A = {} & [f\partial_u + \mathcal{L}_Y + D_C Y^C] N_A + 3 M D_A f - \frac{3}{16} D_A f N_{BC} C^{BC} \\ & - \frac{1}{32} D_A D_B Y^B C_{CD} C^{CD} + \frac{1}{4} (2 D^B f + D^B D_C D^C f) C_{AB} \\ & - \frac{3}{4} D_B f (D^B D^C C_{AC} - D_A D_C C^{BC}) + \frac{3}{8} D_A (D_C D_B f C^{BC}) \\ & + \frac{1}{2} (D_A D_B f - \frac{1}{2} D_C D^C f \mathring{q}_{AB}) D_C C^{BC} + \frac{1}{2} D_B f N^{BC} C_{AC}.\end{aligned}$$

$$\xi^u = \mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}}) \equiv f$$

$$\xi^z = \mathcal{Y} + \mathcal{O}(r^{-1}), \qquad \xi^{\bar{z}} = \bar{\mathcal{Y}} + \mathcal{O}(r^{-1}),$$

$$\xi^r = -\frac{r}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}}) + \mathcal{O}(r^0),$$

BMS symmetry generators

(nb: from now on, I will work with the flat 2d metric, for simplicity)

Question

Which objects from the gravitational phase space

$$
\begin{aligned}
ds^2 &= -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\
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\end{aligned}
$$

transform as conformal primaries under the action of extended BMS symmetries?

$$
\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})
$$

How are they related to the celestial CFT currents?
What is their algebra?

$$P(z,\bar{z}) : (\tfrac{3}{2}, \tfrac{1}{2})$$
$$T(z) : (2,0)$$

BMS charges and fluxes

- At each cut $\{u = \text{constant}\}$ of $\mathscr{I}^+$, the prescription for BMS charges is

[Barnich, Troessaert '11] [He, Lysov, Mitra, Strominger '14] [Kapec, Lysov, Pasterski, Strominger '14]
[Compère, Fiorucci, Ruzziconi '19 '20] [Campiglia, Peraza '20] [LD, Ruzziconi '21]
[Fiorucci '21] [Freidel, Pranzetti '21][Freidel, Pranzetti, Raclariu '21] [LD, Nguyen, Ruzziconi '22]

$$Q_\xi = \frac{1}{8\pi G}\int_S d^2 z\,[2\mathcal{T}\mathcal{M} + \mathcal{Y}\bar{\mathcal{N}} + \bar{\mathcal{Y}}\mathcal{N}]\,,$$

$$\mathcal{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}})$$

$$\mathcal{N} = N_{\bar{z}} - u\bar{\partial}\mathcal{M} + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz}) + \frac{u}{4}\bar{\partial}\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar{z}} - \Big(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\Big)C^{\bar{z}}_z\Big]$$

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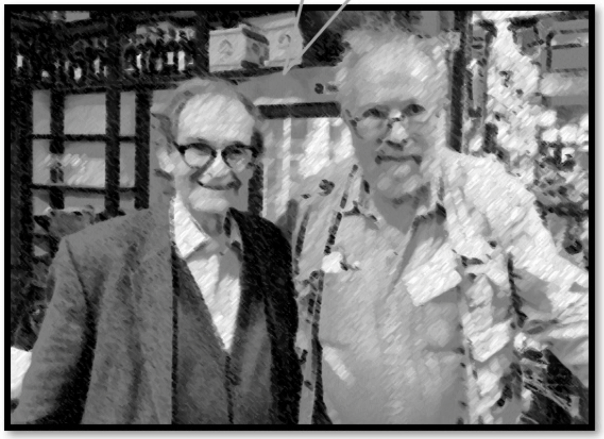
$$\mathcal{M} = -\frac{1}{2}(\Psi_2^0 + \bar{\Psi}_2^0)$$
$$\mathcal{N} = -\Psi_1^0 + u\bar{\partial}\Psi_2^0$$

$$\boxed{\begin{aligned} Q_\xi &= \frac{1}{8\pi G}\int_S d^2z\,[2\mathcal{T}\mathcal{M} + \mathcal{Y}\bar{\mathcal{N}} + \bar{\mathcal{Y}}\mathcal{N}]\,, \\ \mathcal{M} &= M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}}) \\ \mathcal{N} &= N_{\bar{z}} - u\bar{\partial}\mathcal{M} + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz}) \\ &\quad + \frac{u}{4}\bar{\partial}\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar{z}} - \Big(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\Big)C^{\bar{z}}_z\Big] \end{aligned}}$$



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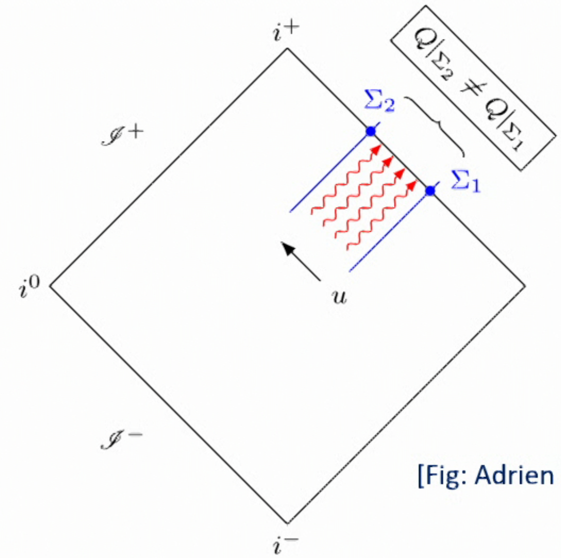
$$\mathcal{N} = N_{\bar z} - u\bar\partial\mathcal{M} + \frac{1}{4}C_{\bar z\bar z}\bar\partial C^{\bar z\bar z} + \frac{3}{16}\bar\partial(C_{zz}C^{zz}) + \frac{u}{4}\bar\partial\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar z} - \Big(\bar\partial^2 - \frac{1}{2}N_{\bar z\bar z}\Big)C^{\bar z}_z\Big]$$



[Fig: Adrien Fiorucci]

- Outgoing radiation ⟶ BMS charges are not conserved but satisfy flux balance laws

- The BMS fluxes are

$$F_\xi = \int_{-\infty}^{+\infty} du\,\partial_u Q_\xi = \int_S d^2z\,[\mathcal{T}\mathcal{P} + \mathcal{Y}\bar{\mathcal{J}} + \bar{\mathcal{Y}}\mathcal{J}]$$

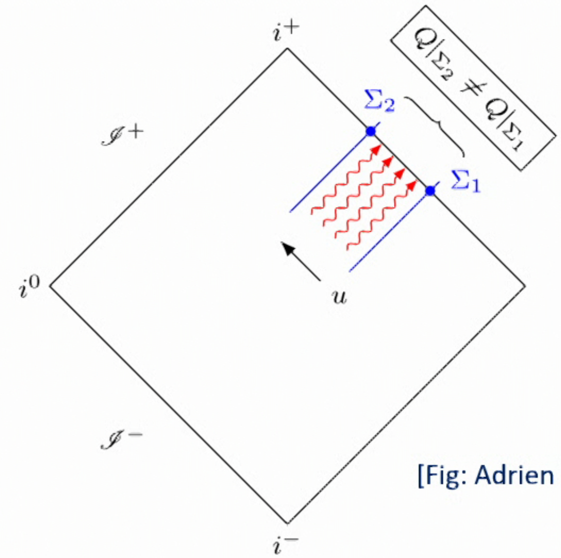
BMS charges and fluxes

- At each cut $\{u = \text{constant}\}$ of $\mathscr{I}^+$, the prescription for BMS charges is

$$Q_\xi = \frac{1}{8\pi G}\int_S d^2z\,[2\mathcal{T}\mathcal{M} + \mathcal{Y}\bar{\mathcal{N}} + \bar{\mathcal{Y}}\mathcal{N}],$$

$$\mathcal{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar z\bar z}N^{\bar z\bar z})$$

$$\begin{aligned}\mathcal{N} = {} & N_{\bar z} - u\bar\partial\mathcal{M} + \frac{1}{4}C_{\bar z\bar z}\bar\partial C^{\bar z\bar z} + \frac{3}{16}\bar\partial(C_{zz}C^{zz}) \\ & + \frac{u}{4}\bar\partial\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar z} - \Big(\bar\partial^2 - \frac{1}{2}N_{\bar z\bar z}\Big)C^{\bar z}_z\Big]\end{aligned}$$



[Fig: Adrien Fiorucci]

- Outgoing radiation $\longrightarrow$ BMS charges are not conserved but satisfy flux balance laws

- The BMS fluxes are

$$F_\xi = \int_{-\infty}^{+\infty} du\,\partial_u Q_\xi = \int_S d^2z\,[\mathcal{T}\mathcal{P} + \mathcal{Y}\bar{\mathcal{J}} + \bar{\mathcal{Y}}\mathcal{J}]$$

$$\mathcal{P} = \frac{1}{4\pi G}\int_{-\infty}^{+\infty} du\,\partial_u\mathcal{M}$$: 'supermomentum flux'

$$\mathcal{J} = \frac{1}{8\pi G}\int_{-\infty}^{+\infty} du\,\partial_u\mathcal{N}$$: 'super angular momentum flux'

BMS fluxes as primaries

- So, we have constructed BMS fluxes

$$\mathcal{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}})$$

$$\mathcal{N} = N_{\bar{z}} - u\bar{\partial}\mathcal{M} + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz}) + \frac{u}{4}\bar{\partial}\left[\left(\partial^2 - \frac{1}{2}N_{zz}\right)C^z_{\bar{z}} - \left(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\right)C^{\bar{z}}_z\right]$$

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- They transform as primary fields under the action of superrotations! [Barnich, Ruzziconi '21] [LD, Ruzziconi '21]

$$\delta_{(\mathcal{Y},\bar{\mathcal{Y}})}\phi_{h,\bar{h}} = (\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + h\partial\mathcal{Y} + \bar{h}\bar{\partial}\bar{\mathcal{Y}})\phi_{h,\bar{h}}$$

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Using the phase space infinitesimal transformations

$$\delta_{(f,Y)}M = [f\partial_u + \mathcal{L}_Y + \tfrac{3}{2}D_CY^C]M + \tfrac{1}{8}D_CD_BD_AY^AC^{BC} + \tfrac{1}{4}N^{AB}D_AD_Bf + \tfrac{1}{2}D_Af\,D_BN^{AB}$$

$$\delta_{(f,Y)}C_{AB} = [f\partial_u + \mathcal{L}_Y - \tfrac{1}{2}D_CY^C]C_{AB} - 2D_AD_Bf + \mathring{q}_{AB}D_CD^Cf$$

one can check indeed that

$$\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})}\mathcal{P} = \Big[\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \frac{3}{2}\partial\mathcal{Y} + \frac{3}{2}\bar{\partial}\bar{\mathcal{Y}}\Big]\mathcal{P} \quad : \Big(\frac{3}{2},\frac{3}{2}\Big) \text{ primary}$$

$$\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})}\mathcal{J} = \Big[\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + 2\partial\mathcal{Y} + 1\bar{\partial}\bar{\mathcal{Y}}\Big]\mathcal{J} + \frac{1}{2}\mathcal{T}\bar{\partial}\mathcal{P} + \frac{3}{2}\bar{\partial}\mathcal{T}\mathcal{P} \quad : (2,1) \text{ primary}$$

Summary …and a remaining question

- Which objects from the gravitational phase space transform as conformal primaries under the action of extended BMS symmetries?

Answer: The following BMS fluxes

$$\boxed{\mathcal{P} = \frac{1}{4\pi G}\int_{-\infty}^{+\infty} du\, \partial_u \mathcal{M}} \qquad (h, \bar{h}) = \left(\frac{3}{2}, \frac{3}{2}\right)$$

$$\boxed{\mathcal{J} = \frac{1}{8\pi G}\int_{-\infty}^{+\infty} du\, \partial_u \mathcal{N}} \qquad (2, 1)$$

Summary ...and a remaining question

- Which objects from the gravitational phase space transform as conformal primaries under the action of extended BMS symmetries?

  Answer: The following BMS fluxes

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$$\boxed{\mathcal{J} = \frac{1}{8\pi G}\int_{-\infty}^{+\infty} du\, \partial_u \mathcal{N}} \qquad (2,1)$$

- How are they related to the celestial CFT currents? What is their algebra?

  The answer to these questions requires

  1. a refined analysis of the radiative phase space with superrotations

Extended radiative phase space

(1) Refined analysis of the radiative phase space with superrotations

- the "shifted news" is defined so as stay zero for any vacuum configuration
  [Compère, Long '16][Compère, Fiorucci, Ruzziconi '18]

$$\widetilde{N}_{zz}(u,x) \equiv N_{zz}(u,x) - N^{vac}_{zz}(x)$$

tracefree part of the Geroch tensor

Extended radiative phase space

① Refined analysis of the radiative phase space with superrotations

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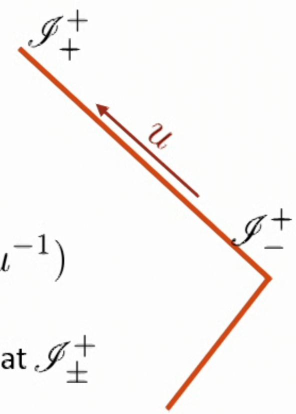
tracefree part of the Geroch tensor

$$\delta N_{zz} = (f\partial_u + \mathcal{L}_{\mathcal{Y}})N_{zz} - \partial^3\mathcal{Y}$$

$$\delta \widetilde{N}_{zz} = (f\partial_u + \mathcal{L}_{\mathcal{Y}})\widetilde{N}_{zz}$$

- fall-offs as $u \to \pm\infty$: $\quad N_{zz} = N^{vac}_{zz} + o(u^{-2}) \qquad C_{zz} = (u + C_\pm)N^{vac}_{zz} - 2\partial^2 C_\pm + o(u^{-1})$

value of the supertranslation field at $\mathscr{I}^+_\pm$



Extended radiative phase space

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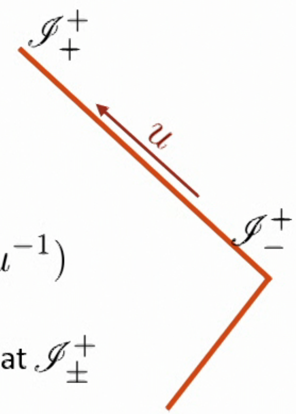


- fall-offs as $u \to \pm\infty$: $N_{zz} = N^{vac}_{zz} + o(u^{-2})$ $\quad C_{zz} = (u + C_\pm)N^{vac}_{zz} - 2\partial^2 C_\pm + o(u^{-1})$

  NB: no "gravitational tails"

  value of the supertranslation field at $\mathscr{I}^+_\pm$

- we also define (please bear with me) [Compère, Fiorucci, Ruzziconi '18][Campiglia, Laddha '21][LD, Nguyen, Ruzziconi '22]

$$\widetilde{C}_{zz} \equiv C_{zz} - uN^{vac}_{zz} - C^{(0)}_{zz}$$
$$C^{(0)}_{zz} \equiv -\mathscr{D}^2(C_+ + C_-)$$

$\mathscr{D}$: "superrotation covariant derivative" $\quad \mathscr{D}\phi_{h,\bar{h}} = (\partial - h\partial\varphi)\phi_{h,\bar{h}}$

[Campiglia, Peraza '20][LD, Ruzziconi '21][Barnich, Ruzziconi '21]
[Freidel, Pranzetti, Raclariu '21]

Extended radiative phase space

1. Refined analysis of the radiative phase space with superrotations

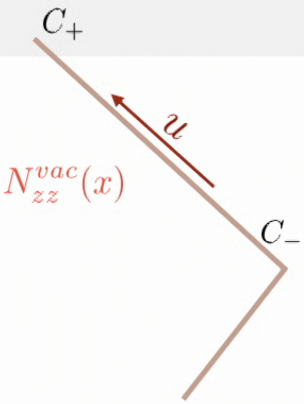
- Recall that the *leading* and *subleading* soft gravitons operators are

$$\tilde{N}_{zz}(u,x) \equiv N_{zz}(u,x) - N^{vac}_{zz}(x)$$

$$\mathcal{N}^{(0)}_{zz} = \int du\, \tilde{N}_{zz} \qquad \mathcal{N}^{(1)}_{zz} = \int du\, u\tilde{N}_{zz}$$

Extended radiative phase space

$$C \equiv \frac{1}{2}(C_+ + C_-)$$



$$C^{(0)}_{zz} \equiv -2\mathscr{D}^2 C$$

$$\tilde{N}_{zz}(u,x) \equiv N_{zz}(u,x) - N^{vac}_{zz}(x)$$

(1) Refined analysis of the radiative phase space with superrotations

- Recall that the *leading* and *subleading* soft gravitons operators are

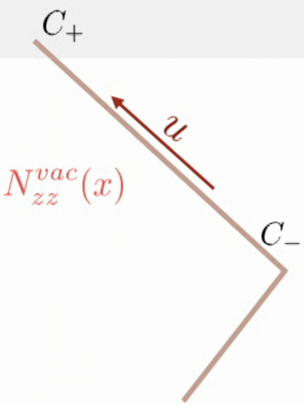
$$\mathcal{N}^{(0)}_{zz} = \int du\, \tilde{N}_{zz} \qquad \mathcal{N}^{(1)}_{zz} = \int du\, u\tilde{N}_{zz}$$

- Symplectic structure

$$\Omega = \Omega_{hard} + \Omega_{soft} \qquad \Omega_{hard} = \frac{1}{32\pi G}\int dud^2z \left[\delta\tilde{N}_{zz} \wedge \delta\tilde{C}_{\bar{z}\bar{z}} + c.c.\right]$$

Extended radiative phase space

$$C \equiv \frac{1}{2}(C_+ + C_-)$$



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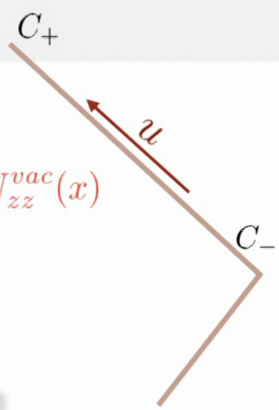
$$\Omega_{soft} = \frac{1}{32\pi G}\int d^2z\left[\delta\mathcal{N}^{(0)}_{zz}\wedge\delta C^{(0)}_{\bar{z}\bar{z}} + \delta\Pi_{zz}\wedge\delta N^{vac}_{\bar{z}\bar{z}} + c.c.\right]$$

$$\Pi_{zz} \equiv 2\mathcal{N}^{(1)}_{zz} + C\mathcal{N}^{(0)}_{zz}$$

Extended radiative phase space

$$C \equiv \frac{1}{2}(C_+ + C_-)$$



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1. Refined analysis of the radiative phase space with superrotations

- Recall that the *leading* and *subleading* soft gravitons operators are

$$\mathcal{N}^{(0)}_{zz} = \int du\, \tilde{N}_{zz} \qquad \mathcal{N}^{(1)}_{zz} = \int du\, u\tilde{N}_{zz}$$

- Symplectic structure

$$\Omega = \Omega_{hard} + \Omega_{soft}$$

$$\boxed{\begin{aligned} \Omega_{hard} &= \frac{1}{32\pi G}\int dud^2z \left[\delta\tilde{N}_{zz}\wedge\delta\tilde{C}_{\bar{z}\bar{z}} + c.c.\right] \\ \Omega_{soft} &= \frac{1}{32\pi G}\int d^2z \left[\delta\mathcal{N}^{(0)}_{zz}\wedge\delta C^{(0)}_{\bar{z}\bar{z}} + \delta\Pi_{zz}\wedge\delta N^{vac}_{\bar{z}\bar{z}} + c.c.\right]\end{aligned}}$$

$$\Pi_{zz} \equiv 2\mathcal{N}^{(1)}_{zz} + C\mathcal{N}^{(0)}_{zz}$$

- It differs from Ashtekar-Streubel's symplectic structure by a *corner* term

$$\Omega = \Omega_{AS} + \int_{\mathscr{I}^+} d\delta Y \qquad \delta Y = \frac{1}{32\pi G}\left[\delta(C\tilde{C}_{zz})\wedge\delta N^{vac}_{\bar{z}\bar{z}} + c.c.\right] dz\wedge d\bar{z}$$

$$\Omega_{AS} = \frac{1}{32\pi G}\int dud^2z\,[\delta N_{zz}\wedge\delta C_{\bar{z}\bar{z}} + c.c.]$$

Soft fluxes and celestial currents

(2) Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries

[LD, Ruzziconi '21]

$$\mathcal{P}^{hard} = \frac{1}{16\pi G}\int du\, \tilde{N}_{zz}\tilde{N}_{\bar{z}\bar{z}}$$

$$\mathcal{P}^{soft} = \frac{1}{8\pi G}\mathscr{D}^2 \mathcal{N}^{(0)}_{\bar{z}\bar{z}} \qquad \mathcal{N}^{(0)}_{\bar{z}\bar{z}} = \int du\, \tilde{N}_{\bar{z}\bar{z}}$$

leading soft graviton operator

Soft fluxes and celestial currents

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$$\mathcal{P}^{soft} = \frac{1}{8\pi G}\mathscr{D}^2 \mathcal{N}^{(0)}_{\bar{z}\bar{z}} \qquad \mathcal{N}^{(0)}_{\bar{z}\bar{z}} = \int du\, \tilde{N}_{\bar{z}\bar{z}} \quad : (h,\bar{h}) = \left(\frac{3}{2}, -\frac{1}{2}\right)$$

$\left(\frac{3}{2},\frac{3}{2}\right)$

leading soft graviton operator

✓ The associated supertranslation fluxes generate the transformations on the radiative phase space

$$F^{soft}_{\mathcal{T}} = \int_{-\infty}^{+\infty} du\, \partial_u Q^{soft}_{\mathcal{T}} = \int_{\mathcal{S}} d^2z\, \mathcal{T}\mathcal{P}^{soft}$$

$$F^{hard}_{\mathcal{T}} = \int_{-\infty}^{+\infty} du\, \partial_u Q^{hard}_{\mathcal{T}} = \int_{\mathcal{S}} d^2z\, \mathcal{T}\mathcal{P}^{hard}$$

$\longrightarrow$

$$\{F^{soft}_{\mathcal{T}}, C^{(0)}_{zz}\} = \delta_{\mathcal{T}} C^{(0)}_{zz}$$
$$\{F^{soft}_{\mathcal{T}}, \Pi_{zz}\} = \delta_{\mathcal{T}} \Pi_{zz}$$
$$\{F^{soft}_{\mathcal{T}}, \widetilde{N}_{zz}\} = 0$$

soft sector

$$\{F^{hard}_{\mathcal{T}}, C^{(0)}_{zz}\} = 0$$
$$\{F^{hard}_{\mathcal{T}}, \Pi_{zz}\} = 0$$
$$\{F^{hard}_{\mathcal{T}}, \widetilde{N}_{zz}\} = \delta_{\mathcal{T}} \widetilde{N}_{zz}$$

hard sector

Soft fluxes and celestial currents

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$$\underbrace{\mathcal{P}^{soft}}_{\left(\frac{3}{2},\frac{3}{2}\right)} = \frac{1}{8\pi G}\mathscr{D}^2\mathcal{N}^{(0)}_{\bar{z}\bar{z}} \qquad \mathcal{N}^{(0)}_{\bar{z}\bar{z}} = \int du\, \tilde{N}_{\bar{z}\bar{z}} \quad : (h,\bar{h}) = \left(\frac{3}{2},-\frac{1}{2}\right)$$

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$$F^{hard}_{\mathcal{T}} = \int_{-\infty}^{+\infty} du\,\partial_u Q^{hard}_{\mathcal{T}} = \int_{\mathcal{S}} d^2z\,\mathcal{T}\mathcal{P}^{hard}$$

Nb:

$$i_{\delta_{\mathcal{T}}}\Omega^{soft} = \delta F^{soft}_{\mathcal{T}}$$

$$i_{\delta_{\mathcal{T}}}\Omega^{hard} = \delta F^{hard}_{\mathcal{T}}$$

Soft fluxes and celestial currents

(2) Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries [LD, Ruzziconi '21]

$$\mathcal{P}^{hard} = \frac{1}{16\pi G}\int du\, \tilde{N}_{zz}\tilde{N}_{\bar{z}\bar{z}} \qquad \delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})}\mathcal{P}^{hard/soft} = \left[\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \frac{3}{2}\partial\mathcal{Y} + \frac{3}{2}\bar{\partial}\bar{\mathcal{Y}}\right]\mathcal{P}^{hard/soft}$$

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$\left(\frac{3}{2},\frac{3}{2}\right)$

leading soft graviton operator

✓ The associated supertranslation fluxes generate the transformations on the radiative phase space

✓ The soft and hard sectors factorize [Campiglia-Laddha '21][LD, Ruzziconi '21]

$$\{F^{soft}_{\mathcal{T}}, F^{hard}_{\mathcal{T}}\} = 0$$

Soft fluxes and celestial currents

(2) Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries

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$\left(\frac{3}{2}, \frac{3}{2}\right)$

leading soft graviton operator

- Link with celestial holography? Remember: 'supertranslation current'

$$\Delta \to 1 \quad P(z,\bar{z}) : \left(\tfrac{3}{2}, \tfrac{1}{2}\right) \qquad P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$$

$$\boxed{\mathcal{P}^{soft}(z,\bar{z}) = \bar{\mathcal{D}}\, P(z,\bar{z}) + \mathcal{D}\,\bar{P}(z,\bar{z})}$$

see also celestial diamonds of [Pasterski, Puhm, Trevisani '21]

Soft fluxes and celestial currents

2 Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries [LD, Ruzziconi '21]

$(2,1)$

$$\bar{\mathcal{J}}^{hard} = \frac{1}{16\pi G}\int du\left[\frac{3}{2}\tilde{C}_{zz}\partial\tilde{N}_{\bar{z}\bar{z}} + \frac{1}{2}\tilde{N}_{\bar{z}\bar{z}}\partial\tilde{C}_{zz} + \frac{u}{2}\partial(\tilde{N}_{zz}\tilde{N}_{\bar{z}\bar{z}})\right]$$

$$\bar{\mathcal{J}}^{soft} = \frac{1}{16\pi G}\left[-\mathscr{D}^3\mathcal{N}^{(1)}_{\bar{z}\bar{z}} + \frac{3}{2}C^{(0)}_{zz}\mathscr{D}\mathcal{N}^{(0)}_{\bar{z}\bar{z}} + \frac{1}{2}\mathcal{N}^{(0)}_{\bar{z}\bar{z}}\mathscr{D}C^{(0)}_{zz}\right]$$

$\mathcal{N}^{(0)}_{\bar{z}\bar{z}} = \int du\, \tilde{N}_{\bar{z}\bar{z}}$ $\leftarrow$ leading soft graviton operator $\left(\frac{3}{2}, -\frac{1}{2}\right)$

$\mathcal{N}^{(1)}_{\bar{z}\bar{z}} = \int du\, u\tilde{N}_{\bar{z}\bar{z}}$ $\leftarrow$ *sub*leading soft graviton operator $(1,-1)$

Soft fluxes and celestial currents

2 Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries [LD, Ruzziconi '21]

$$(2,1)\quad \bar{\mathcal{J}}^{hard} = \frac{1}{16\pi G}\int du \left[\frac{3}{2}\tilde{C}_{zz}\partial\tilde{N}_{\bar{z}\bar{z}} + \frac{1}{2}\tilde{N}_{\bar{z}\bar{z}}\partial\tilde{C}_{zz} + \frac{u}{2}\partial(\tilde{N}_{zz}\tilde{N}_{\bar{z}\bar{z}})\right]$$

$$\bar{\mathcal{J}}^{soft} = \frac{1}{16\pi G}\left[-\mathscr{D}^3\mathcal{N}^{(1)}_{\bar{z}\bar{z}} + \frac{3}{2}C^{(0)}_{zz}\mathscr{D}\mathcal{N}^{(0)}_{\bar{z}\bar{z}} + \frac{1}{2}\mathcal{N}^{(0)}_{\bar{z}\bar{z}}\mathscr{D}C^{(0)}_{zz}\right]$$

$$\mathcal{N}^{(0)}_{\bar{z}\bar{z}} = \int du\, \tilde{N}_{\bar{z}\bar{z}}$$ ← leading soft graviton operator $\left(\frac{3}{2}, -\frac{1}{2}\right)$

$$\mathcal{N}^{(1)}_{\bar{z}\bar{z}} = \int du\, u\tilde{N}_{\bar{z}\bar{z}}$$ ← *sub*leading soft graviton operator $(1, -1)$

✓ Again, one can check that the associated superrotation fluxes generate the transformations on the radiative phase space

Soft fluxes and celestial currents

(2) Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries
[LD, Ruzziconi '21]

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Soft fluxes and celestial currents

(2) Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries [LD, Ruzziconi '21]

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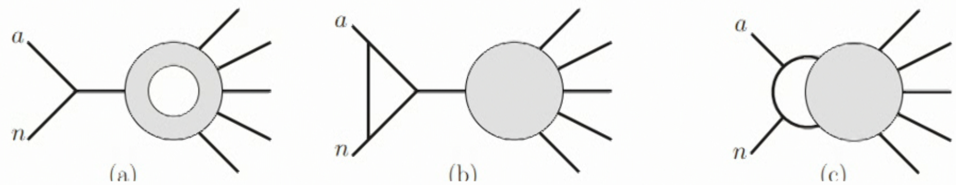

On Loop Corrections to Subleading Soft Behavior of Gluons and Gravitons

Zvi Bern, Scott Davies and Josh Nohle

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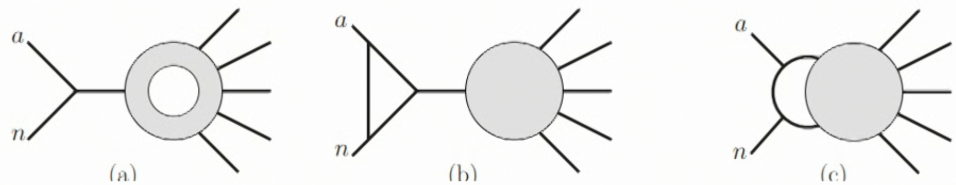

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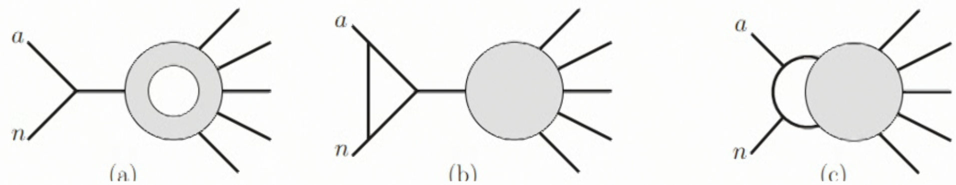

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Since it is one-loop exact, this shows that superrotations are genuine symmetries of the gravitational
S-matrix beyond semiclassical level.

Notice: see however [Cachazo, Yuan ‘14]

Celestial CFT OPE from BMS flux algebra

- Finally, one can deduce the following OPE for the celestial CFT [LD, Ruzziconi '21]

$$P(z,\bar{z})P(w,\bar{w}) \sim 0$$

$$P(z,\bar{z})\bar{P}(w,\bar{w}) \sim 0$$

$$T(z)P(w,\bar{w}) \sim \frac{1}{(z-w)}\partial_w P(w,\bar{w}) + \frac{3/2}{(z-w)^2}P(w,\bar{w})$$

$$\bar{T}(\bar{z})P(w,\bar{w}) \sim \frac{1}{(\bar{z}-\bar{w})}\partial_{\bar{w}} P(w,\bar{w}) + \frac{1/2}{(\bar{z}-\bar{w})^2}P(w,\bar{w})$$

$$P(z,\bar{z})T(w) \sim \frac{1/2}{(z-w)}\partial_w P(w,\bar{z}) + \frac{3/2}{(z-w)^2}P(w,\bar{z}),$$

$$\boxed{T(z)T(w) \sim \frac{1}{(z-w)}\partial_w T(w) + \frac{2}{(z-w)^2}T(w)} \longrightarrow \boxed{c=0}$$

$$\bar{T}(\bar{z})T(w) \sim 0$$

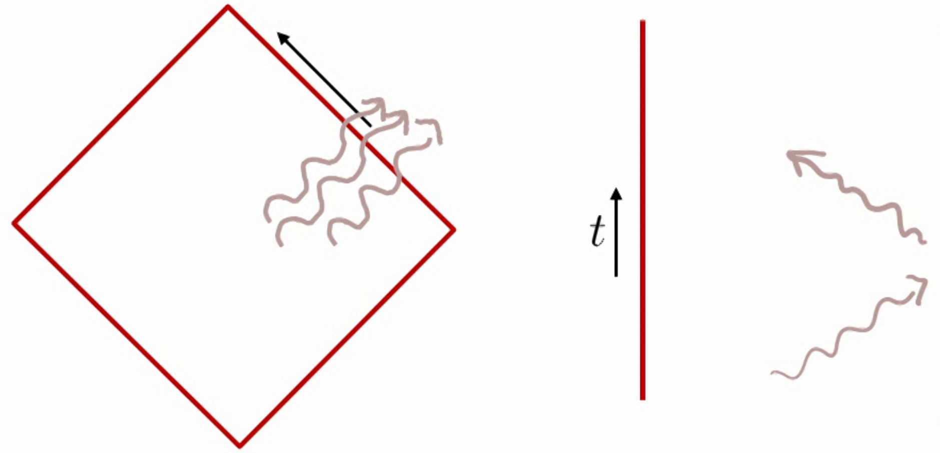
see also [Fotopoulos, Stieberger Taylor, Zhu '19]

Conclusion and outlook

Flat space holography

Very different from holography in Anti-de Sitter (AdS acts like a box)!

Flat holography forces us to deal with leaks of radiation through the boundary.



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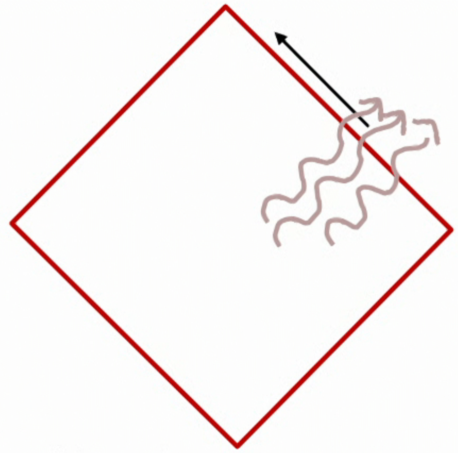

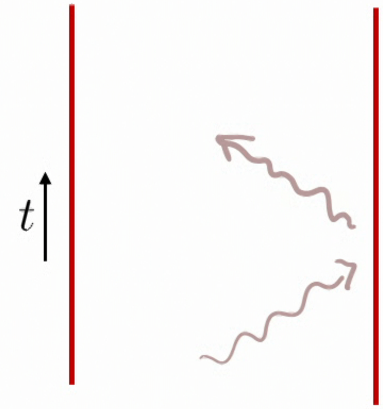

Infinitely many **symmetry constraints** beyond conformal invariance.

e.g. constraints coming from supertranslation symmetry have no analog in usual holography.

$$P(z)\mathcal{O}_{\Delta}(w,\bar{w}) \sim \frac{1}{z-w}\mathcal{O}_{\Delta+1}(w,\bar{w})$$

Key objects: BMS **fluxes** (give primaries, celestial OPEs, etc.)

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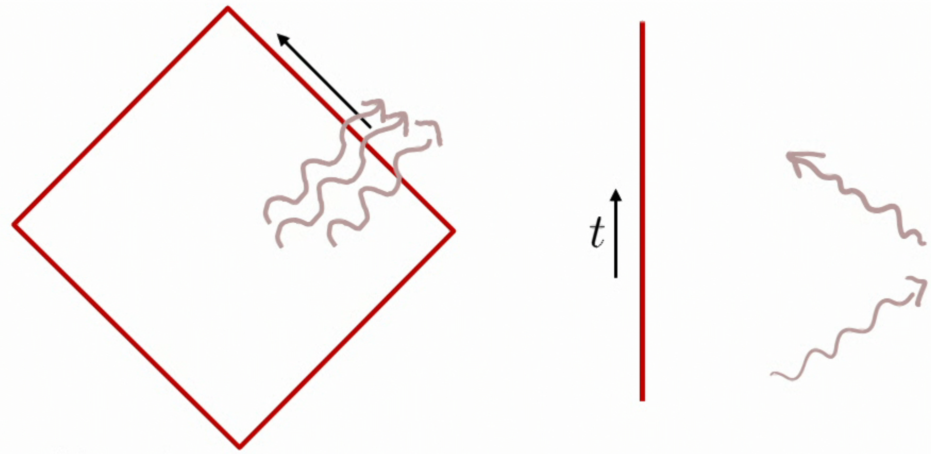

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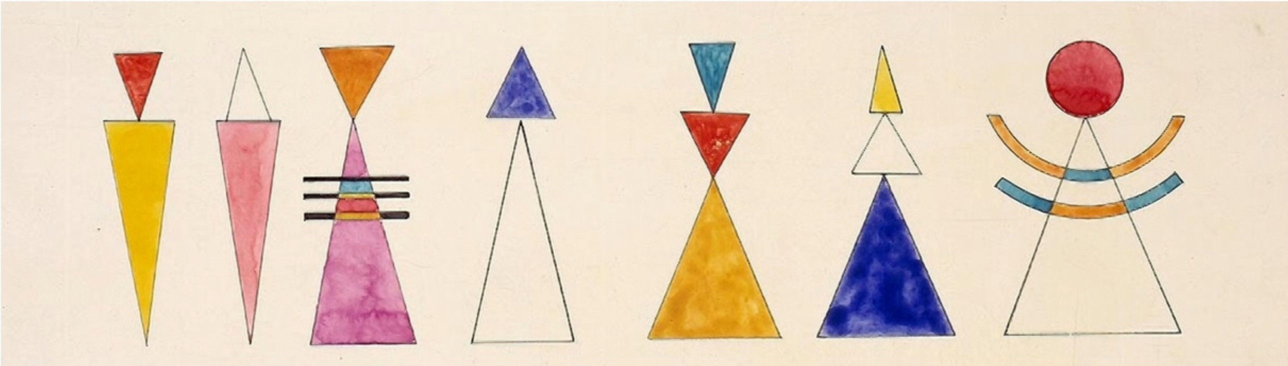
Outstanding challenges: **what is a Celestial CFT? what is a** conformal Carrollian **QFT?**
Beyond kinematics? Top-down constructions?

Let's not remain each in our corner…



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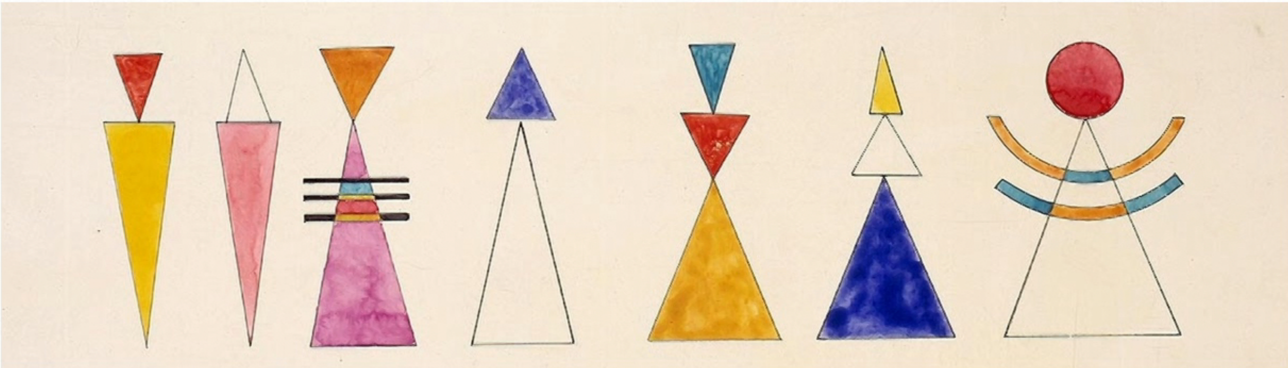

mathematical GR

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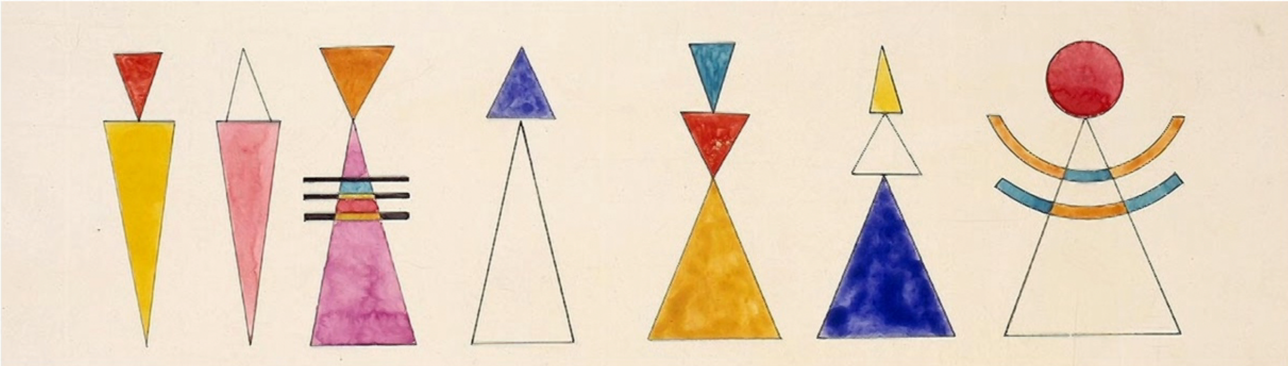

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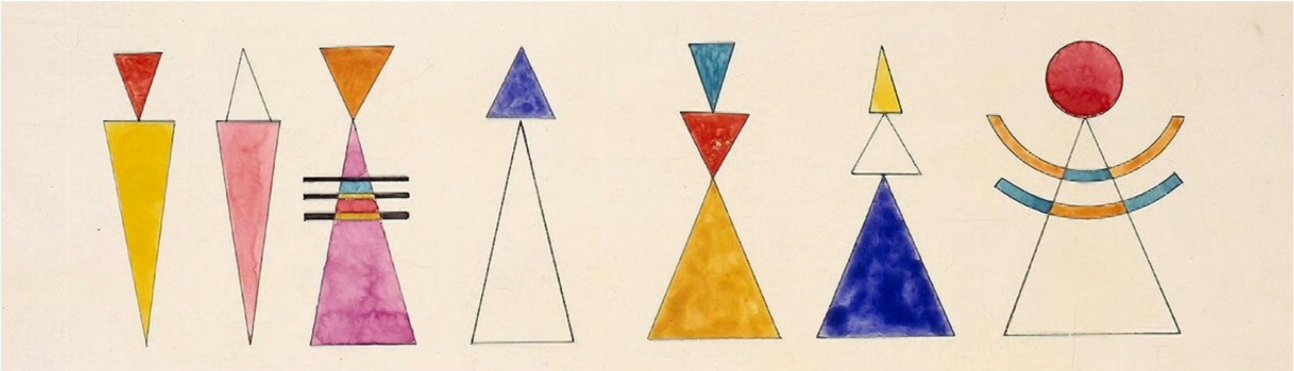

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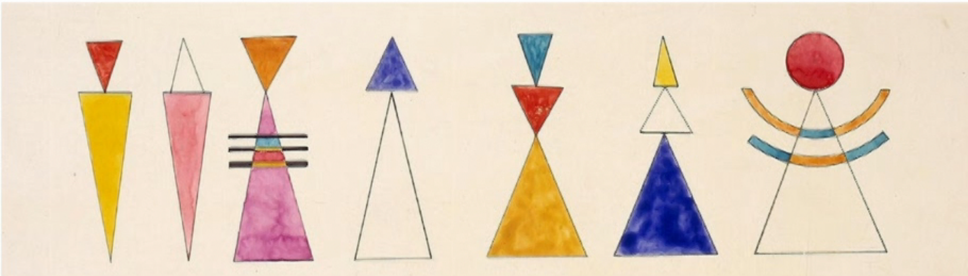

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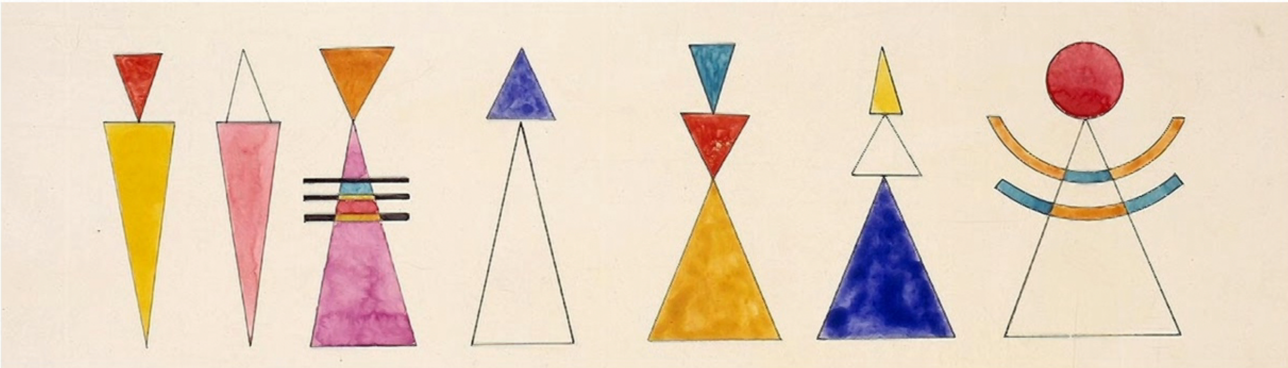

The number of pages in this book is exactly infinite.
No page is the first; none the last.

J.-L. Borges, *The book of sand (1975)*

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Thank you