Title: The partial Bondi gauge

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Collection: Quantum Gravity Around the Corner

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Abstract: I will review the recent construction of an extended solution space for gravity, based on a so-called partial Bondi gauge fixing. This aims at investigating the possible relaxations of the boundary conditions, in order to include for example a cosmological constant, a polyhomogeneous expansion, and an arbitrary time-dependent boundary metric. I will also explain how to properly map these results to the Newman-Penrose formalism. Finally, I will discuss the application to three-dimensional gravity, where a new asymptotic symmetry can be revealed after working out all the subtleties of the covariant phase space formalism.

Introduction

Motivations

- Symmetries are a precious tool to understand classical and quantum gravity
- Many symmetry groups have been studied: BMS, EBMS, GBMS, BMSW, ΛBMS, UCS, ...
  [BMS, Barnich, Campiglia, Ciambelli, Compère, Fiorucci, Flanagan, Freidel, Laddha, Leigh, Nichols, Oliveri, Pranzetti, Ruzziconi, Speziale, Troessaert, ...]
- What is the largest symmetry group in gravity?
- How much "structure" show we relax and allow for in the solution space?
- What is the subleading / overleading structure?

Our goal [MG, Zwikel]

- Construct a framework including $\Lambda \neq 0$, time-dependent boundary sources, and log terms
- Unify and build up on [Barnich, Troessaert] [Compère, Fiorucci, Ruzziconi] [Mao] [Valiente Kroon] [Chrusciel, MacCallum, Singleton] [Godazgar, Long] [Pool, Skenderis, Taylor]
- Revisit the construction of the Newman–Unti gauge [Barnich, Lambert]

Setup

Partial Bondi gauge

- Pick Bondi coordinates (u, r, x^A) and impose the three conditions $g_{rr} = 0 = g_{rA}$ to get

$$\mathrm{d}s^2 = e^{2\beta}\frac{V}{r}\mathrm{d}u^2 - 2e^{2\beta}\mathrm{d}u\,\mathrm{d}r + g_{AB}\left(\mathrm{d}x^A - U^A\mathrm{d}u\right)\left(\mathrm{d}x^B - U^B\mathrm{d}u\right)$$

- There is still a freedom in redefining the radial coordinate
- For the transverse metric we assume [Chrusciel, MacCallum, Singleton] [Winicour]

$$g_{AB} = r^2 q_{AB} + rC_{AB} + D_{AB} + \frac{1}{r}\Big(E_{AB} + (\ln r)E^{\ell}_{AB}\Big) + \mathcal{O}(r^{-2})$$

- We'll let the Einstein equations decide from here on, and complete the gauge fixing later

Bondi–Sachs gauge

- Algebraic condition $\det g_{AB} = r^2 \det q^0_{AB}$ [BMS]
- Differential condition $\partial_r(r^{-4}\det g_{AB}) = 0$ allows for Weyl rescalings [Barnich, Troessaert]
- These conditions imply in particular $C = q^{AB}C_{AB} = 0$
- r is the luminosity distance

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Newman–Unti gauge

- Algebraic condition $\beta = 0$ [Newman, Unti]
- Differential condition $\partial_r\beta = 0$ allows for free boundary metric [MG, Zwikel]

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Newman–Unti gauge

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- Differential condition $\partial_r\beta = 0$ allows for free boundary metric [MG, Zwikel]
- The trace C is free
- r is the affine parameter for ∂_r

Solution space

Solving the Einstein equations

- We solve $\mathbf{E}_{\mu\nu} = G_{\mu\nu} + 6\Lambda g_{\mu\nu} = 0$ following the Bondi hierarchy
- $\mathbf{E}_{rr} \;\Rightarrow\; \beta = \beta_0 + \frac{1}{32r^2}\left(C_{AB}C^{AB} - 4D\right) + \mathcal{O}(r^{-3})$
- $\mathbf{E}_{rA} \;\Rightarrow\; U^A = U_0^A + \frac{U_1^A}{r} + \frac{U_2^A}{r^2} + \frac{1}{r^3}\left(N^A + (\ln r)e^{2\beta_0} D_B \mathfrak{L}^{AB}\right) + \mathcal{O}(r^{-4})$

  with $\mathfrak{L}_{AB} := D_{\langle AB\rangle} - \frac{1}{4} C C_{\langle AB\rangle}$
- $\mathbf{E}_{ru} \;\Rightarrow\; V = \Lambda e^{2\beta_0} r^3 + r^2\left(\Lambda e^{2\beta_0} C - \partial_u \ln\sqrt{q} - D_A U_0^A\right) + rV_1 + 2M + \mathcal{O}(r^{-1})$
- $\mathbf{E}_{\langle AB\rangle}\big|_{\mathcal{O}(r)} \;\Rightarrow\; \Lambda e^{2\beta_0} C_{\langle AB\rangle} = (\partial_u - \partial_u \ln\sqrt{q}) q_{AB} + D_{\langle A} U^0_{B\rangle}$
- $\mathbf{E}_{\langle AB\rangle}\big|_{\mathcal{O}(r^0)} \;\Rightarrow\; \Lambda \mathfrak{L}_{AB} = 0$ — no logs in (A)dS
- $\mathbf{E}_{\langle AB\rangle}\big|_{\mathcal{O}(r^{-1})} \;\Rightarrow\; \partial_u \mathfrak{L}_{AB} = (\dots)$ — [Winicour]
- $\mathbf{E}_{\langle AB\rangle}\big|_{\mathcal{O}(r^{-2})} \;\Rightarrow\; \partial_u E_{AB} = (\dots)$ — [Grant, Nichols] [Freidel, Pranzetti, Raclariu]
- $\mathbf{E}_{\langle AB\rangle}\big|_{\mathcal{O}(r^{-n})} \;\Rightarrow\; \Lambda g^{n\geq 2}_{AB} = \partial_u g^{n-1}_{AB} + (\dots)$ — finite data in (A)dS
- $\mathbf{E}_{uA}\big|_{\mathcal{O}(r^{-2})} \;\Rightarrow\; \partial_u N_A = (\dots)$
- $\mathbf{E}_{uu}\big|_{\mathcal{O}(r^{-2})} \;\Rightarrow\; \partial_u M = (\dots)$

Solution space

Summary

- The solution space contains
  - cosmological constant Λ
  - boundary sources entering $\mathrm{d}s^2|_{\mathcal{I}^+} = 2\Lambda e^{4\beta_0}\mathrm{d}u^2 + q_{AB}(\mathrm{d}x^A - U_0^A)(\mathrm{d}x^B - U_0^B)$
  - time dependency $\partial_u q_{AB}$
  - logarithmic terms
  - trace mode C (differentiating the BS and NU gauges)
- To write the EOMs it is more efficient to switch to Newman–Penrose

Newman–Penrose formalism

Weyl scalars

- Usually the NP formalism is set in NU gauge (so that $\epsilon = \pi = \kappa = 0$), but this is not necessary
- Projecting the Weyl tensor gives

$$\Psi_0 = \frac{1}{r^4}\mathfrak{E}_{AB}m^A m^B + \frac{\ln r}{r^5}E^{\ell}_{\langle AB\rangle}m^A m^B + \frac{1}{r^5}\mathcal{E}_{AB}m^A m^B + \mathcal{O}(r^{-6})$$

$$\Psi_1 = \frac{\ln r}{r^4}D^B\mathfrak{E}_{AB}m^A + \frac{1}{r^4}\mathcal{P}_A m^A + \mathcal{O}(r^{-5})$$

$$\Psi_2 = \frac{1}{r^3}(\mathcal{M} + i\widetilde{\mathcal{M}}) + \mathcal{O}(r^{-4})$$

$$\Psi_3 = \frac{1}{r^2}\mathcal{J}^A\bar{m}_A + \mathcal{O}(r^{-3})$$

$$\Psi_4 = \frac{1}{r}\mathcal{N}_{AB}\bar{m}^A\bar{m}^B + \mathcal{O}(r^{-2})$$

- Peeling broken: log-asymptotic flatness [Christodoulou, Friedrich, Kehrberger, Klainerman, Winicour]

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- The "covariant functionals" are e.g. (turning off the boundary metric for simplicity)

$$\begin{aligned}
\mathcal{E}_{AB} &:= E_{\langle AB\rangle} - \frac{5}{6}E^{\ell}_{\langle AB\rangle} - \frac{1}{6}C\mathfrak{L}_{AB} + \frac{1}{16}C_{\langle AB\rangle}(C_{CD}C^{CD} - 4D)\\
\mathcal{M} &:= M + \frac{1}{16}(\partial_u + \partial_u \ln\sqrt{q})(4D - C_{AB}C^{AB}) + \frac{\Lambda}{4}(E_l - 4E)
\end{aligned}$$

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- Shear-free gravitational radiation is allowed since $\mathcal{N}_{AB}\Big|_{C_{AB}=0} \neq 0$

Newman–Penrose formalism

Evolution equations from the Bianchi identities

- For $\Lambda = 0$

$$\eth_u \mathcal{M} = \frac{1}{2} D_A \mathcal{J}^A + \frac{1}{4} C_{AB} \mathcal{N}^{AB}$$

$$\eth_u \mathcal{P}_A = \partial_A \mathcal{M} + \widetilde{\partial}_A \widetilde{\mathcal{M}} + C_{\langle AB \rangle} \mathcal{J}^B + \frac{1}{2} D^B (\partial_u \ln \sqrt{q}\, \mathfrak{L}_{AB})$$

$$\eth_u \mathcal{E}_{AB} = \frac{1}{2} D_{\langle A} \mathcal{P}_{B \rangle} + \frac{3}{2} C_{\langle AB \rangle} \mathcal{M}_{\mathbb{C}} + \frac{3}{2} \mathcal{E}^{\ell}_{\langle AB \rangle} \partial_u \ln \sqrt{q} - \frac{1}{4} \Big(C \partial_u \ln \sqrt{q} + 2 \partial_u C + 3R \Big) \mathfrak{L}_{AB}$$

- log terms should therefore modify the sub-leading and sub-sub-leading soft theorems
- For $\Lambda \neq 0$ (no analogue of Bondi mass loss)

$$\eth_u \mathcal{M} = \frac{1}{2} D_A (\mathcal{J}^A - \Lambda \mathcal{P}^A) + \frac{3\Lambda}{2} C \mathcal{M} + \frac{1}{4} C_{AB} (\mathcal{N}^{AB} + \Lambda^2 \mathcal{E}^{AB})$$

Symplectic structure

Symplectic renormalization

- r-divergency of the charges can be cured by symplectic renormalization
  [Compère, Marolf, Papadimitriou, Skenderis, Bianchi, Freedman, ...]
- Always possible with a corner term since

$$\delta L \approx \mathrm{d}\theta = \partial_u \theta^u + \partial_r \theta^r + D_A \theta^A \qquad \Rightarrow \qquad \theta^r_{\text{div}} \approx \delta L - D_A \theta^A - \partial_u \theta^u$$

$\Lambda = 0$ potential

- Flat limit is well-defined, no corner term needed contrary to [Compère, Fiorucci, Ruzziconi]
- For example in NU gauge, keeping only C and q_{AB} free we get

$$\theta^r_{\text{finite}} = \sqrt{q}\left[2N_{AB}\delta C^{\langle AB\rangle} - \delta q^{AB}\left(C_{\langle AB\rangle}R - 2D_A D^C C_{\langle BC\rangle} + D_A \partial_B C\right) + R\delta C\right]$$

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- The news is $N_{AB} = \partial_u C_{\langle AB\rangle} = \partial_u^{-1}\mathcal{N}_{AB}$ (the shear of ingoing n)

$\Lambda \neq 0$ potential

- We need to replace everywhere $C_{\langle AB\rangle} = \dfrac{1}{\Lambda}(\partial_u - \partial_u \ln\sqrt{q})q_{AB}$
- Recalling that $\mathcal{S}_{AB} := \mathcal{N}_{AB} + \Lambda^2 \mathcal{E}_{AB}$, we find e.g. the potential for Λ-BMSW

$$\theta^r_{\text{finite}} = -\frac{3}{\Lambda}\sqrt{q}\, q^{AB}\delta\mathcal{S}_{AB} + \delta\sqrt{q}\left(2\mathcal{M} + \frac{3}{\Lambda}D^2\partial_u \ln\sqrt{q}\right)$$

Asymptotic symmetries

Asymptotic Killing vectors

- As usual, preserving $g_{rr} = 0 = g_{rA}$ gives

$$\xi^u = f(u, x^A) \qquad \xi^A = Y^A(u, x^A) - \int_r^\infty \mathrm{d}r' \, e^{2\beta} g^{AB} \partial_B f$$

- Preserving the expansion of g_{AB} leads to

$$\xi^r = r\xi^r_{+1} + \xi^r_0 + \frac{\xi^r_1}{r} + \frac{1}{r^2}\Big(\xi^r_2 + (\ln r)\xi^r_{\ell 2}\Big) + \mathcal{O}(r^{-3})$$

- To determine the free functions in ξ^r we need to reduce the partial Bondi gauge to BS or NU

$$\xi^r\big|_{\mathsf{BS}} = \frac{r}{2}\Big(2h(u, x^A) - f\partial_u \ln\sqrt{q} + U_0^A \partial_A f\Big) + \frac{1}{2}e^{2\beta_0}\Big(4(\partial^A\beta_0)(\partial_A f) + D_A\partial^A f\Big) + \mathcal{O}(r^{-1})$$

$$\xi^r\big|_{\mathsf{NU}} = \xi^r\big|_{\mathsf{BS}} + k(u, x^A)$$

- The algebra is $\big(\mathsf{diff}(\mathcal{I}^+) \oplus\!\!\!\!\!\!\in \mathbb{R}_h\big) \oplus\!\!\!\!\!\!\in \mathbb{R}_k$

Transformation laws

- Weyl $\delta_\xi \sqrt{q} = 2h + D_A Y^A$
- Translations

$$(\delta_\xi C_{AB})\big|_{\mathsf{NU}} = (\delta_\xi C_{AB})\big|_{\mathsf{BS}} + 2k\, q_{AB}$$

$$\delta_\xi C = \Big(f\partial_u + \pounds_Y - h + \frac{1}{2} f\partial_u \ln\sqrt{q} - \frac{1}{2}U_0^A\partial_A f\Big)C + 4k$$

- Explicitly (legally?) used in [Barnich, Lambert] [Blanchet, Compère, Faye, Oliveri, Seraj]
- We can go further in 3d [MG, Goeller, Zwikel]

3d case

Gauge choice

- Partial Bondi gauge

$$\mathrm{d}s^2 = e^{2\beta}\frac{V}{r}\,\mathrm{d}u^2 - 2e^{2\beta}\,\mathrm{d}u\,\mathrm{d}r + r^2 W(\mathrm{d}\phi - U\,\mathrm{d}u)^2$$

- Differential NU gauge

$$\partial_r\beta = 0 \Rightarrow \beta = \beta_0(u,\phi) \qquad\qquad \partial_r W \neq 0$$

Solution space

- Exact solution to $R_{\mu\nu} + 2\Lambda g_{\mu\nu} = 0$

$$W = e^{2\varphi_0}\left(1 - \frac{C}{r}\right)^2$$

$$U = U_0 + \frac{e^{2(\beta_0-\varphi_0)}}{(r-C)^2}\left(2(r-2C)\beta_0' - N\right)$$

$$V = \text{lengthy function of } (M, N, C, U_0, \beta_0, \varphi_0)$$

- The (uu) and $(u\phi)$ Einstein equations determine the evolution $\partial_u M$ and $\partial_u N$
- The function $C(u,\phi)$ is the exact analogue of the trace C in 4d

3d case

Asymptotic symmetries

- Generated by four functions (f, g, h, k) of (u, ϕ)

$$\xi^u = f \qquad \xi^\phi = g - \frac{e^{2(\beta_0 - \varphi_0)}}{r - C} f' \qquad \xi^r = rh + k + \mathcal{O}(r^{-1})$$

- (f, g, h, k) are supertranslations, superrotations, Weyl rescalings, and radial translations
- In particular we have

$$\delta_h (g_{\mu\nu})\big|_{\mathcal{I}+} = h (g_{\mu\nu})\big|_{\mathcal{I}+} \qquad \delta_\xi C = f\partial_u C + gC' + k$$

- The algebra is

$$\big(\mathsf{diff}(\mathcal{I}^+) \ltimes \mathbb{R}_k\big) \oplus \mathbb{R}_h$$

$$\big(\mathsf{diff}(\mathcal{I}^+) \ltimes \mathbb{R}_h\big) \ltimes \mathbb{R}_k \quad \text{in 4d}$$

Corner terms

- We are interested in the algebra of integrable charges (topological theory), which here requires
  - symplectic renormalization with a corner term $\vartheta_{\mathsf{ren}} = re^{\varphi_0}\delta\beta_0$
  - use of a corner ambiguity $\vartheta_{\mathsf{c}} = -e^{\varphi_0} C \delta\beta_0$
  - use of an integrable slicing [Adami, Ruzziconi, Sheikh-Jabbari, Taghiloo, Yavartanoo, Zwikel]
- The corner terms needed for this procedure are encoded in the relative corner term

$$\vartheta_{\mathsf{EH/EC}} = \varepsilon_{ijk} e^i_\phi e^{j\alpha} \delta e^k_\alpha = \vartheta_{\mathsf{ren}} + \vartheta_{\mathsf{c}}$$

[De Paoli, Speziale] [Freidel, MG, Pranzetti]

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3d case

Properties of the charge

- The final charge is given in terms of the covariant mass and angular momentum by

$$\mathcal{Q} = \oint_S f\mathcal{M} + g\mathcal{N} + hC + ke^{\varphi_0}$$

- As expected in 3d these charges are r-independent
- Non-conserved because "leaky boundary conditions" (unless we take $U_0 = \beta_0 = \varphi_0 = 0$)
- The algebra at every u is

$$\{\mathcal{Q}[\xi_1], \mathcal{Q}[\xi_2]\} = \mathcal{Q}[\xi_1, \xi_2]_* + \oint_S f_1 g_2''' - f_2 g_1''' - \oint_S h_1 k_2 - h_2 k_1$$

$$\mathfrak{vir} \oplus \mathfrak{vir} \oplus \text{Heisenberg} \xrightarrow{\Lambda \to 0} \mathfrak{bms}_3 \oplus \text{Heisenberg}$$

- The Heisenberg component can be removed with a corner term $\vartheta = C\delta e^{\varphi_0}$

TMG

- 3d massive gravity by adding a Chern–Simons term for $\Gamma[g]$ [Deser, Jackiw, Templeton]

$$L_{\text{TMG}} = L_{\text{EH}}[g] + L_{\text{CS}}[\Gamma] \qquad\qquad L = e \wedge F + e \wedge e \wedge e + e \wedge \mathrm{d}_\omega e + L_{\text{CS}}[\omega]$$

- Weyl component in TMG cannot be removed by a corner

$$\mathcal{Q}_{\text{TMG}} = \oint_S \Lambda f\mathcal{N} + g\mathcal{M} + h\varphi_0'$$

- Dualities in 4d should also be interesting with respect to these extra boundary structures

Conclusion

We have seen that

- A complete solution space can be derived in partial Bondi gauge (with Λ, $\partial_u\sqrt{q}$, sources, logs)
- This extends to 4d the construction which can be carried out from A to Z in 3d
- Lots of extra structure to play with

Interesting prospects

- Charge algebra in partial Bondi gauge [MG, Zwikel, wip]
- Detailed logarithmic structure and log charges [MG, Zwikel, wip]
- Relationship with log soft theorems [Campiglia, Laddha, Sahoo, Sen, ...]
- Implications for celestial and Carrolian holography
  [Donnay, Fiorucci, Herfray, Pasterski, Raclariu, Ruzziconi, Strominger, ...]
- Coupling to matter [MG, Majumdar, wip]
- (A)dS radiation and memory effects
- Breathing memory and scalar tensor theories [Seraj]
- Further relaxations of the gauge (in particular g_{rA})
  [Campoleoni, Ciambelli, Delfante, Marteau, Petkou, Petropoulos, Ruzziconi, Siampos, ...]

Thanks!

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- We are interested in the algebra of integrable charges (topological theory), which here requires
  - symplectic renormalization with a corner term $\vartheta_{\text{ren}} = re^{\varphi_0}\delta\beta_0$
  - use of a corner ambiguity $\vartheta_{\text{c}} = -e^{\varphi_0} C \delta\beta_0$
  - use of an integrable slicing [Adami, Ruzziconi, Sheikh-Jabbari, Taghiloo, Yavartanoo, Zwikel]
- The corner terms needed for this procedure are encoded in the relative corner term

$$\vartheta_{\text{EH/EC}} = \varepsilon_{ijk} e^i_\phi e^{j\alpha} \delta e^k_\alpha = \vartheta_{\text{ren}} + \vartheta_{\text{c}}$$

[De Paoli, Speziale] [Freidel, MG, Pranzetti] (with a minus sign!)