Title: Review Talk: A primer on the covariant phase space formalism cont.

Speakers: Adrien Fiorucci

Collection: Quantum Gravity Around the Corner

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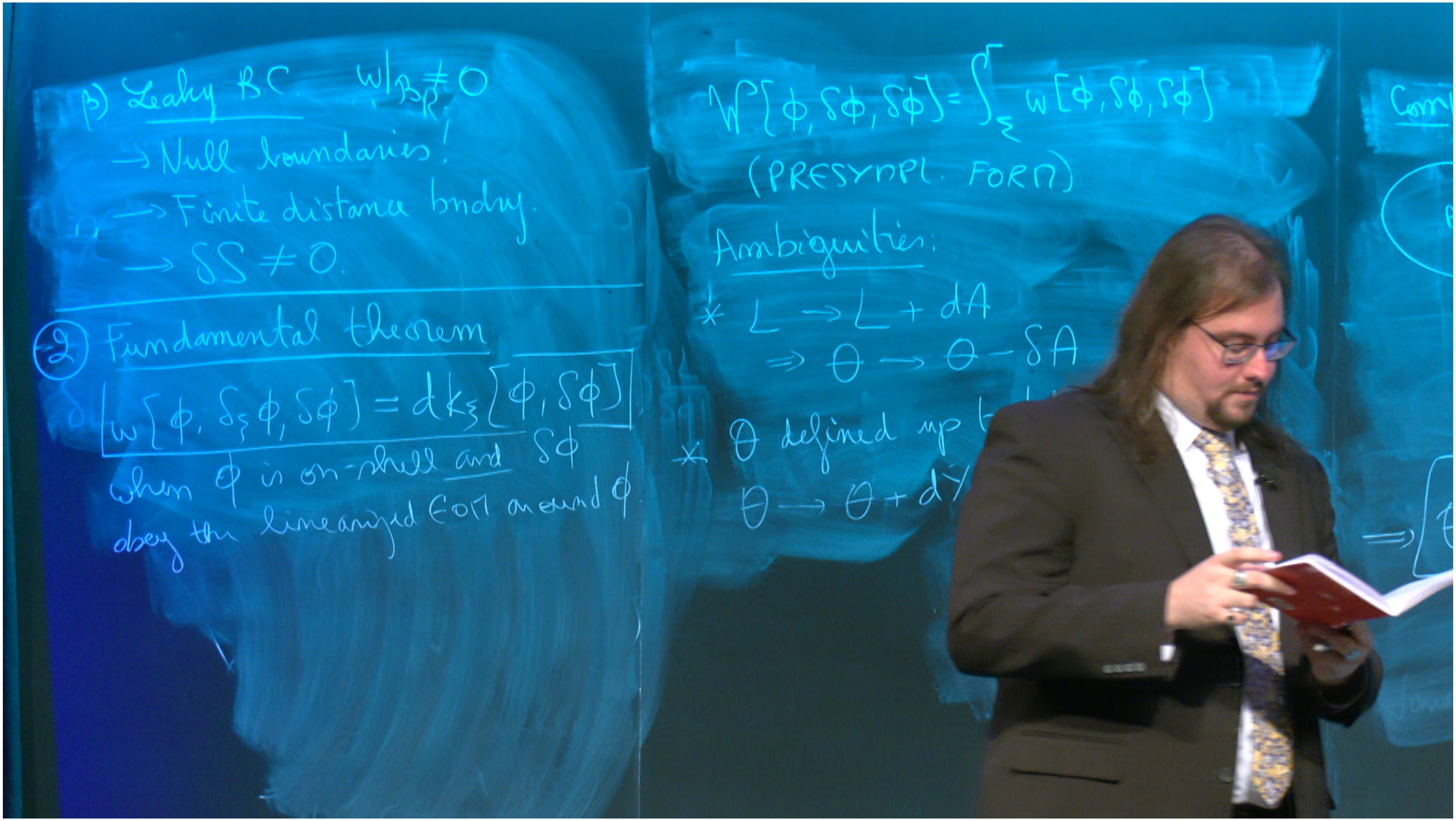

β) Leaky BC   ω|B_R ≠ 0
→ Null boundaries!
→ Finite distance bndry.
→ δS ≠ 0.
② Fundamental theorem
ω[φ, δ_ξ φ, δφ] = d k_ξ[φ, δφ]
when φ is on-shell and δφ
obey the linearized EOM around φ
W[φ, δφ, δφ] = ∫_Σ ω[φ, δφ, δφ]
(PRESYMPL. FORM)
Ambiguities:
* L → L + dA
⇒ Θ → Θ − δA
* Θ defined up
Θ → Θ + dY

β) Leaky BC $\quad \omega|_{B_R} \neq 0$

→ Null boundaries!

→ Finite distance bndry.

→ $\delta S \neq 0$.

(2) Fundamental theorem

$$\boxed{\omega[\phi, \delta_\xi \phi, \delta\phi] = d k_\xi[\phi, \delta\phi]}$$

when ϕ is on-shell and $\delta\phi$ obey the linearized EOM around ϕ.

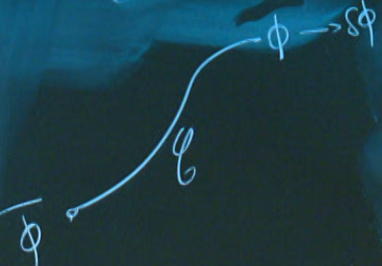
$$W[\phi, \delta_\xi\phi, \delta\phi] = \int_\Sigma \omega[\phi, \delta_\xi\phi, \delta\phi]$$

$$= \oint_{S=\partial\Sigma} k_\xi[\phi; \delta\phi]$$

$$\triangleq \delta\!\!\!/ H_\xi[\phi, \delta\phi]$$

$$\in \Omega^{(0,1)}(\mathcal{F})$$

variation of the charge between ϕ and $\phi + \delta\phi$



$$Q_\xi[\phi] = \int_\gamma \oint_S k_\xi + N_\xi[\bar\phi]$$

Con

$$W[\phi, \delta_\xi\phi, \delta\phi] = \int_\Sigma \omega[\phi; \delta_\xi\phi, \delta\phi]$$
$$= \oint_{S=\partial\Sigma} k_\xi[\phi; \delta\phi]$$
$$\triangleq \delta\!\!\!/ H_\xi[\phi, \delta\phi]$$
$$\in \Omega^{(0,1)}(\mathcal{F})$$

variation of the charge between ϕ^i and $\phi^i + \delta\phi^i$

$$\bar\phi \xrightarrow{\gamma} \phi \to \delta\phi$$

$$Q_\xi[\phi] = \int_\gamma \oint_S k_\xi + N_\xi[\bar\phi]$$

$$k_\xi[\phi; \delta\phi] = \delta q_\xi[\phi] - q_{\delta\xi}[\phi] - \iota_\xi \Theta[\phi; \delta\phi]$$

(IYER-WALD CHARGES)

$$dq_\xi = J_\xi - S_\xi$$

$$q_\xi = -I_\xi \Theta[\phi, \delta_\xi\phi]$$

integrating d

$$\Theta^\mu_{EH} = \frac{\sqrt{-g}}{16\pi G}\left(\nabla_\nu (\delta g)^{\mu\nu} - \nabla^\mu (\delta g)^\nu{}_\nu\right)$$

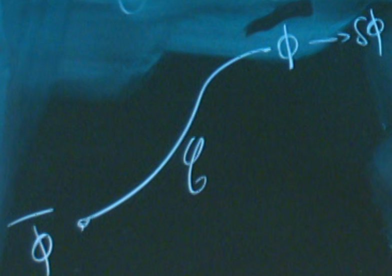
$$I_\xi \propto \xi^\alpha \frac{\partial}{\partial \partial_\mu \xi^\alpha} \frac{\partial}{\partial dx^\nu}(\ldots)$$

$$I_\xi d + d I_\xi = id$$

$$q_\xi$$

$$W[\phi,\delta_\xi\phi,\delta\phi] = \int_\Sigma \omega[\phi,\delta_\xi\phi,\delta\phi]$$
$$= \oint_{S=\partial\Sigma} k_\xi[\phi;\delta\phi]$$
$$\triangleq \oint H_\xi[\phi,\delta\phi]$$
$$\in \Omega^{(0,1)}(\mathcal{F})$$

variation of the charge between ϕ^i and $\phi^i+\delta\phi^i$



$$Q_\xi[\phi] = \int_{\mathcal{C}}\oint_S k_\xi + N_\xi[\bar\phi]$$

$$k_\xi[\phi;\delta\phi] = \delta q_\xi[\phi] - q_{\delta\xi}[\phi] - \iota_\xi\Theta[\phi;\delta\phi]$$

(IYER-WALD CHARGES)

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integrating d

$$\Theta^\mu_{EH} = \frac{\sqrt{-g}}{16\pi G}\left(\nabla_\nu(\delta g)^{\mu\nu} - \nabla^\mu(\delta g)^\nu{}_\nu\right)$$
$$I_\xi \propto \xi^\alpha \frac{\partial}{\partial\partial_\mu\xi^\alpha}\frac{\partial}{\partial dx^\mu}(\dots)$$
$$I_\xi d + dI_\xi = \mathrm{id}$$
$$q_\xi = (d^{n-2}x)_{\mu\nu}\frac{\sqrt{-g}}{16\pi G}\left(\nabla^\mu\xi^\nu - \nabla^\nu\xi^\mu\right)$$

$k_\xi[\phi;\delta\phi] = \delta q_\xi[\phi] - q_{\delta\xi}[\phi] - \iota_\xi \Theta[\phi;\delta\phi]$

(IYER-WALD CHARGES)

$dq_\xi = J_\xi - S_\xi$

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integrating d

$\Theta^\mu_{EH} = \frac{\sqrt{-g}}{16\pi G}\left(\nabla_\nu(\delta g)^{\mu\nu} - \nabla^\mu(\delta g)^\nu{}_\nu\right)$

$I_\xi \propto \xi^\alpha \frac{\partial}{\partial\partial_\mu\xi^\alpha}\frac{\partial}{\partial dx^\nu}(\ldots)$

$I_\xi d + d I_\xi = id$

$q_\xi = (d^{n-2}x)_{\mu\nu}\frac{\sqrt{-g}}{16\pi G}(\nabla^\mu\xi^\nu - \nabla^\nu\xi^\mu)$

$* \; d\omega = 0$

$\Longrightarrow \int_{\Sigma_2} i_{\delta_\xi\phi}\omega = \int_{\Sigma_1} i_{\delta_\xi\phi}\omega + \int_{B} i_{\delta_\xi\phi}\omega$

$\delta H_\xi|_{S_2} = \delta H_\xi|_{S_1} + \int_B i_{\delta_\xi\phi}\omega$

$*$ Non-integrability:

(1) $\delta\xi = 0$

$\delta\oint_S k_\xi = -\oint_S \delta\,\iota_\xi\Theta = \oint_S i_\xi\omega \neq 0$

(2) Field-dependance

$$\not\delta H_\xi = \delta H_\xi + \Xi_\xi[\phi, \delta\phi]$$

$$[\delta_{\xi_1}, \delta_{\xi_2}]\phi^i = -\delta_{[\xi_1,\xi_2]_*}\phi^i$$

Algebra of A.S

<u>Assuming integrability</u>

$$\delta_{\xi_2} H_{\xi_1} \triangleq \{H_{\xi_1}, H_{\xi_2}\}$$

$$\overset{\text{Thm}}{=} H_{[\xi_1,\xi_2]_*} + \underbrace{K_{\xi_1,\xi_2}}_{\text{central extension}}$$

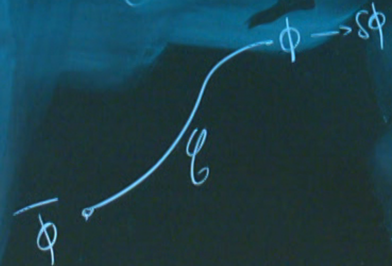
$$W[\phi, \delta_\xi\phi, \delta\phi] = \int_\Sigma \omega[\phi, \delta_\xi\phi, \delta\phi]$$

$$= \oint_{S=\partial\Sigma} k_\xi[\phi;\delta\phi]$$

$$\triangleq \not\delta H_\xi[\phi;\delta\phi]$$

$$\in \Omega^{(0,1)}(\mathcal{F})$$

variation of the charge between ϕ^i and $\phi^i+\delta\phi^i$



$$Q_\xi[\phi] = \int_{\mathcal{C}} \oint_S k_\xi + N_\xi[\bar\phi]$$

$k_\xi[$

$(\mathrm{IY}$

dq_ξ

q_ξ

Θ

I

$\delta\!\!\!/ H_\xi = \delta H_\xi + \Xi_\xi[\phi, \delta\phi]$

$[\delta_{\xi_1}, \delta_{\xi_2}]\phi^i = -\delta_{[\xi_1,\xi_2]_*}\phi^i$

Algebra of A.S

Assuming integrability

$\delta_{\xi_2} H_{\xi_1} \triangleq \{H_{\xi_1}, H_{\xi_2}\}$

$\overset{\text{Thm}}{=} H_{[\xi_1,\xi_2]_*} + \underbrace{K_{\xi_1,\xi_2}[\bar{\Phi}]}_{\text{central extension}}$

In presence of non-int charges

→ Split $\delta\!\!\!/ H_\xi = \delta H_\xi + \Xi_\xi$

→ $\{H_{\xi_1}, H_{\xi_2}\}_* \triangleq \delta_{\xi_2} H_{\xi_1} + \Xi_{\xi_2}[\phi, \delta_{\xi_1}\phi]$

$\{H_{\xi_1}, H_{\xi_2}\}_* = H_{[\xi_1,\xi_2]_*}$
$+ \int_{\mathcal{C}} \left(\delta_{\xi_1}\Xi_{\xi_2} - \delta_{\xi_2}\Xi_{\xi_1} - \Xi_{[\xi_1,\xi_2]_*}\right)$
$+ K_{\xi_1,\xi_2}[\bar{\Phi}]$

$k_\xi[\phi; \delta\phi$

(IYER-W

$dq_\xi =$

$q_\xi = -$

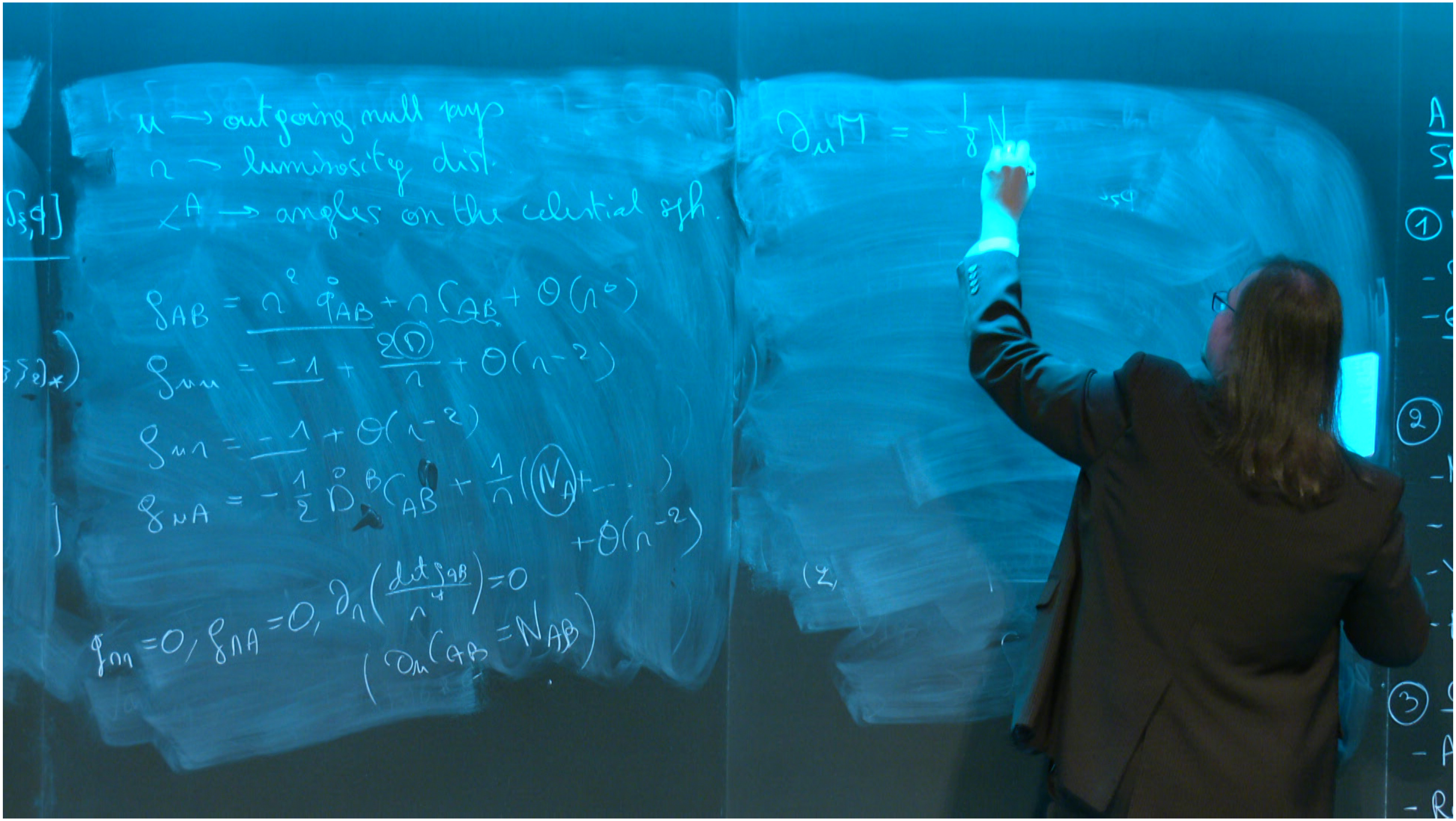
in

$\Theta^\mu_{EH} = \frac{\sqrt{}}{16}$

$I_\xi \propto$

$I_\xi d$

$q_\xi =$


u → outgoing null rays
r → luminosity dist.
x^A → angles on the celestial sph.
g_AB = r^2 q̊_AB + r C_AB + O(r^0)
g_uu = −1 + 2M/r + O(r^−2)
g_ur = −1 + O(r^−2)
g_uA = −1/2 D̊^B C_AB + 1/r (N_A + ...) + O(r^−2)
g_rr = 0, g_rA = 0, ∂_r (det g_AB / r^4) = 0
(∂_u C_AB = N_AB)
∂_u M = −1/8 N
(2,
①
②
③

$u \to$ outgoing null rays

$r \to$ luminosity dist.

$x^A \to$ angles on the celestial sph.

$$g_{AB} = r^2 \mathring{q}_{AB} + r C_{AB} + O(r^0)$$

$$g_{uu} = -1 + \frac{2M}{r} + O(r^{-2})$$

$$g_{ur} = -1 + O(r^{-2})$$

$$g_{uA} = -\frac{1}{2}\mathring{D}^B C_{AB} + \frac{1}{r}(N_A + \dots) + O(r^{-2})$$

$$g_{rr} = 0, \quad g_{rA} = 0, \quad \partial_r\left(\frac{\det g_{AB}}{r^4}\right) = 0$$

$$(\partial_u C_{AB} = N_{AB})$$

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \mathring{D}_A(\dots)^A$$

$$\partial_u N_A = \partial_A M + \frac{1}{4}\mathring{D}^B \mathring{D}_{[A}\mathring{D}^C C_{B]C} + \text{quad. } N_{AB}, C_{AB}$$

$$\xi^\mu \partial_\mu \big|_{\mathscr{I}^+} = \left(T(x^A) + \frac{u}{2}\mathring{D}_C R^C\right)\partial_u + \mathring{R}^A \partial_A$$

$$(T, \mathring{q}_{AB}) \longrightarrow (T, \mathring{q}_{AB}) \longrightarrow (T)$$

BMS$_4$ Generalized BMS$_4$ Weyl BMS$_4$

$$\Theta^{u}_{EH} = O(r^{-2})$$

$$\Theta^{r}_{EH} = \frac{\sqrt{\mathring{q}}}{16\pi G}\, \frac{1}{2} N^{AB} \delta C_{AB} + \delta(\ldots) + O(r^{-1})$$

$$\Longrightarrow \omega^{r}_{EH}\Big|_{\mathscr{I}^+} = \frac{\sqrt{\mathring{q}}}{16\pi G}\, \delta N^{AB} \wedge \delta C_{AB}$$

$$\delta_\xi C_{AB} = \left[f\partial_u + \mathcal{L}_R - \frac{1}{2} \mathring{D}_C \mathring{R}^C \right] C_{AB} - 2\left(D_A D_B f\right)^{TF}$$

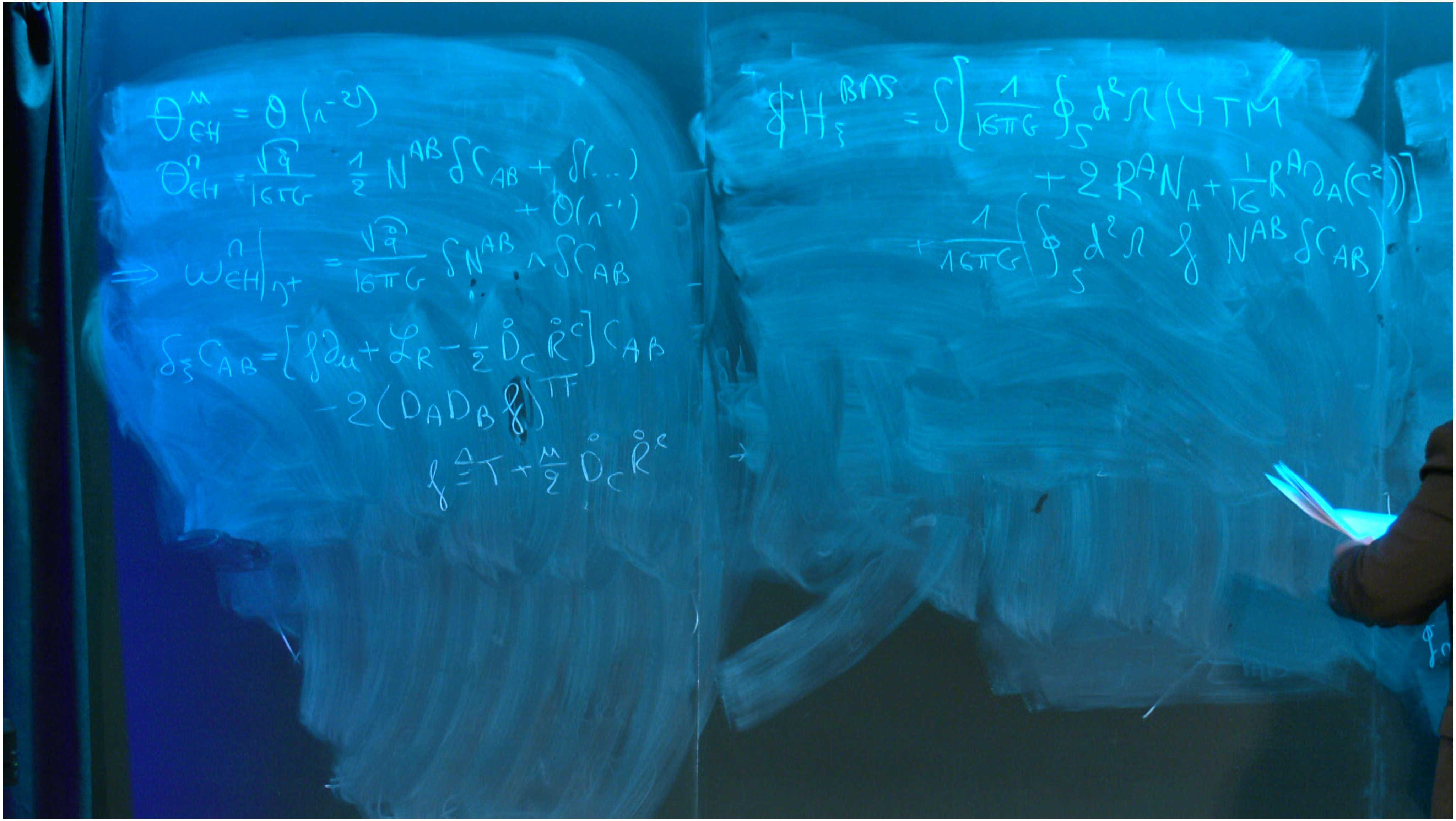
$$f \triangleq T + \frac{u}{2} \mathring{D}_C \mathring{R}^C$$

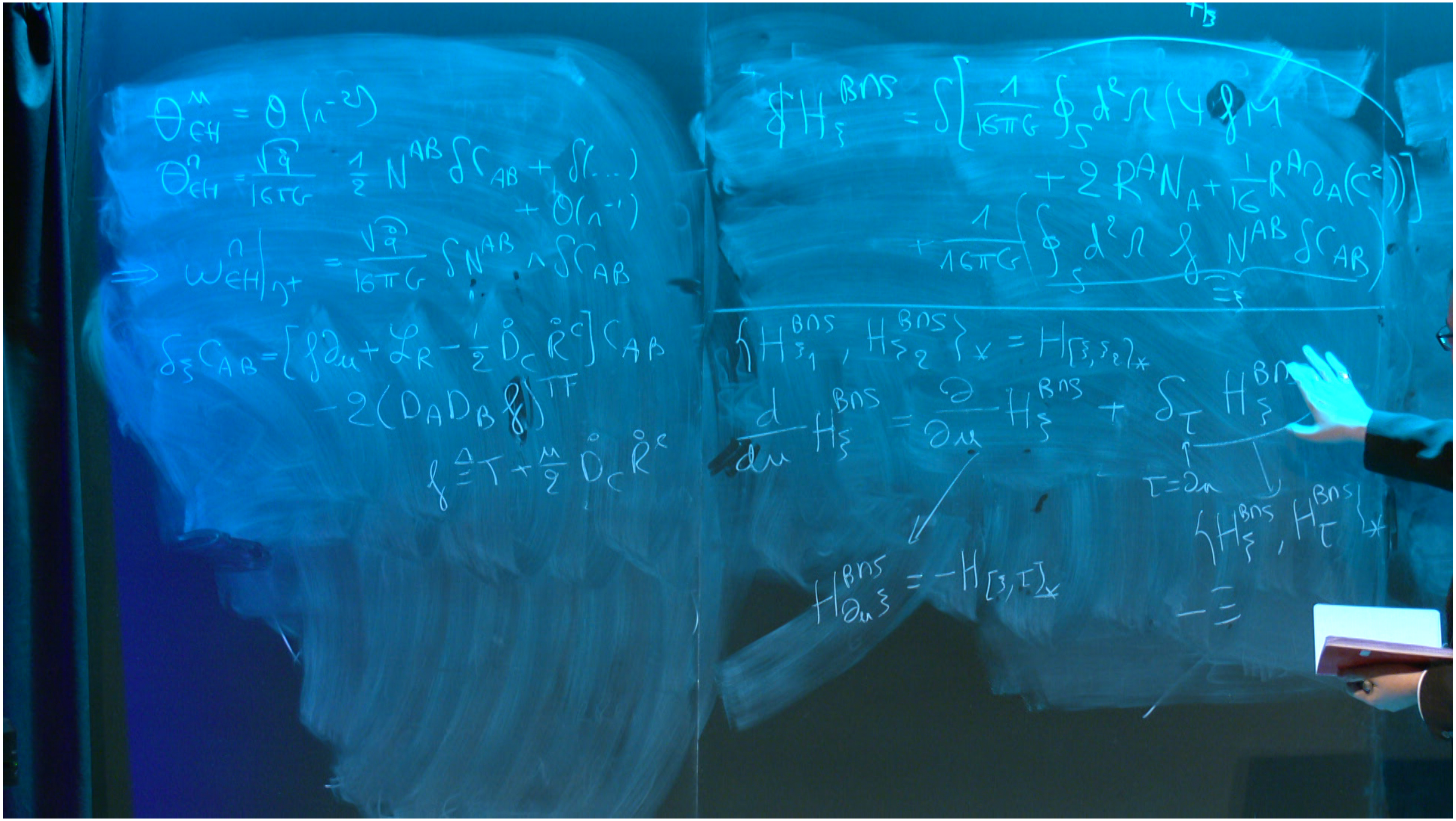
In presence of non-int. charges

$\longrightarrow$ Split $\delta\!\!\!/ H_\xi = \delta H_\xi + \Xi_\xi$

$$\longrightarrow \{H_{\xi_1}, H_{\xi_2}\}_* \triangleq \delta_{\xi_2} H_{\xi_1} + \Xi_{\xi_2}[\phi; \delta_{\xi_1}\phi]$$

$$\{H_{\xi_1}, H_{\xi_2}\}_* = H_{[\xi_1,\xi_2]_*} + \int_{\mathcal{S}} \left(\delta_{\xi_1} \Xi_{\xi_2} - \delta_{\xi_2} \Xi_{\xi_1} - \Xi_{[\xi_1,\xi_2]_*} \right) + K_{\xi_1,\xi_2}[\Phi]$$


$\Theta^{u}_{EH} = O(r^{-2})$
$\Theta^{r}_{EH} = \frac{\sqrt{q}}{16\pi G} \frac{1}{2} N^{AB} \delta C_{AB} + \delta(\ldots) + O(r^{-1})$
$\Rightarrow \omega^{r}_{EH}\big|_{\mathcal{I}^+} = \frac{\sqrt{\mathring{q}}}{16\pi G} \delta N^{AB} \wedge \delta C_{AB}$
$\delta_\xi C_{AB} = \left[f\partial_u + \mathcal{L}_R - \frac{1}{2}\mathring{D}_C \mathring{R}^C \right] C_{AB} - 2(D_A D_B f)^{TF}$
$f \triangleq T + \frac{u}{2} \mathring{D}_C \mathring{R}^C$
$\delta H^{BMS}_\xi = \delta\Big[\frac{1}{16\pi G} \oint_S d^2\Omega \,(4TM + 2R^A N_A + \frac{1}{16} R^A \partial_A (C^2))\Big]$
$+ \frac{1}{16\pi G}\Big(\oint_S d^2\Omega\, f\, N^{AB} \delta C_{AB}\Big)$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$\delta_\xi C_{AB} = [f\partial_u + \mathcal{L}_R - \frac{1}{2}\mathring{D}_C \mathring{R}^C] C_{AB} - 2(D_A D_B f)^{TF}$
$f \hat{=} T + \frac{u}{2}\mathring{D}_C \mathring{R}^C$
$\delta H^{BMS}_\xi = \delta\Big[\frac{1}{16\pi G}\oint_S d^2\Omega\,(4 f M + 2 R^A N_A + \frac{1}{16} R^A \partial_A (C^2))\Big] + \frac{1}{16\pi G}\Big(\oint_S d^2\Omega\, f N^{AB}\delta C_{AB}\Big)$
$\{H^{BMS}_{\xi_1}, H^{BMS}_{\xi_2}\}_* = H_{[\xi_1,\xi_2]_*}$
$\frac{d}{du} H^{BMS}_\xi = \frac{\partial}{\partial u} H^{BMS}_\xi + \delta_\tau H^{BMS}_\xi$
$\tau = \partial_u$
$H^{BMS}_{\partial_u \xi} = -H_{[\xi,\tau]_*}$
$\{H^{BMS}_\xi, H^{BMS}_\tau\}_*$
$= \Xi$

$$\Theta^{u}_{EH} = \mathcal{O}(r^{-2})$$

$$\Theta^{n}_{EH} = \frac{\sqrt{\mathring{q}}}{16\pi G}\,\frac{1}{2} N^{AB}\,\delta C_{AB} + \delta(\dots) + \mathcal{O}(r^{-1})$$

$$\Longrightarrow\ \omega^{n}_{EH}\big|_{\mathscr{I}^+} = \frac{\sqrt{\mathring{q}}}{16\pi G}\,\delta N^{AB}\wedge\delta C_{AB}$$

$$\delta_\xi C_{AB} = \left[f\partial_u + \mathcal{L}_R - \frac{1}{2}\mathring{D}_C \mathring{R}^C\right] C_{AB} - 2\left(D_A D_B f\right)^{TF}$$

$$f \stackrel{\Delta}{=} T + \frac{u}{2}\mathring{D}_C \mathring{R}^C$$

$$\Xi_\xi$$

$$\delta H^{BMS}_\xi = \delta\left[\frac{1}{16\pi G}\oint_S d^2\Omega \left(4 f M + 2 R^A N_A + \frac{1}{16} R^A \partial_A (C^2)\right)\right] + \underbrace{\frac{1}{16\pi G}\oint_S d^2\Omega\, f\, N^{AB}\,\delta C_{AB}}_{=\Xi_\xi}$$

$$\left\{H^{BMS}_{\xi_1}, H^{BMS}_{\xi_2}\right\}_* = H_{[\xi_1,\xi_2]_*}$$

$$\frac{d}{du} H^{BMS}_\xi = \frac{\partial}{\partial u} H^{BMS}_\xi + \delta_\tau H^{BMS}_\xi$$

$$\tau = \partial_u$$

$$H^{BMS}_{\partial_u \xi} = -H_{[\xi,\tau]_*}$$

$$\left\{H^{BMS}_\xi, H^{BMS}_\tau\right\}_* - \Xi_\tau[g;\delta_\xi g]$$

$$\Theta^{u}_{EH} = \mathcal{O}(r^{-2})$$

$$\Theta^{r}_{EH} = \frac{\sqrt{\mathring{q}}}{16\pi G}\, \frac{1}{2} N^{AB}\,\delta C_{AB} + \delta(\ldots) + \mathcal{O}(r^{-1})$$

$$\Rightarrow \omega^{r}_{EH}\Big|_{\mathscr{I}^+} = \frac{\sqrt{\mathring{q}}}{16\pi G}\, \delta N^{AB} \wedge \delta C_{AB}$$

$$\delta_\xi C_{AB} = \left[f\partial_u + \mathcal{L}_R - \frac{1}{2}\mathring{D}_C \mathring{R}^C \right] C_{AB} - 2(D_A D_B f)^{TF}$$

$$f \triangleq T + \frac{u}{2}\mathring{D}_C \mathring{R}^C$$

$$\delta H^{BMS}_\xi = \delta\left[\frac{1}{16\pi G}\oint_S d^2\Omega \left(4 f M + 2 R^A N_A + \frac{1}{16} R^A \partial_A (C^2)\right)\right] + \frac{1}{16\pi G}\underbrace{\left(\oint_S d^2\Omega\, f\, N^{AB}\,\delta C_{AB}\right)}_{\Xi_\xi}$$

$$\{H^{BMS}_{\xi_1}, H^{BMS}_{\xi_2}\}_* = H_{[\xi_1,\xi_2]_*}$$

$$\frac{d}{du} H^{BMS}_\xi = \frac{\partial}{\partial u} H^{BMS}_\xi + \delta_\tau H^{BMS}_\xi$$

$$\tau = \partial_u$$

$$H^{BMS}_{\partial_u \xi} = \cancel{-H_{[\xi,\tau]_*}}$$

$$\cancel{\{H^{BMS}_\xi, H^{BMS}_\tau\}_*} - \Xi_\tau[g;\delta_\xi g]$$

$$\delta H_\xi^{BMS} = \int \Big[\overbrace{\frac{1}{16\pi G}\oint_S d^2\Omega \left(4\, \delta M + 2 R^A N_A + \frac{1}{16} R^A \partial_A (C^2)\right)}^{H_\xi} \Big] + \frac{1}{16\pi G}\underbrace{\oint_S d^2\Omega\, \delta N^{AB}\,\delta C_{AB}}_{\Xi_\xi}$$

$$\{H_{\xi_1}^{BMS}, H_{\xi_2}^{BMS}\}_* = H_{[\xi_1,\xi_2]_*}$$

$$\frac{d}{du} H_\xi^{BMS} = \frac{\partial}{\partial u} H_\xi^{BMS} + \delta_T H_\xi^{BMS}, \qquad T = \partial_u$$

$$H^{BMS}_{\partial_u \xi} = \cancel{-H_{[\xi,T]_*}}$$

$$\cancel{\{H_\xi^{BMS}, H_T^{BMS}\}_*} - \Xi_T[g; \delta_\xi g]$$

$$\frac{d}{du} H_\xi^{BMS} = -\Xi_T[g; \delta_\xi g] = -\oint_S d^2\Omega \frac{1}{16\pi G}\frac{1}{2} N^{AB}\, \delta_\xi C_{AB}$$

For $\xi = T$,

$$\frac{d}{du}\oint_S d^2\Omega \frac{M}{4\pi G} = -\frac{1}{32\pi G}\oint_S d^2\Omega \times N_{AB} N^{AB}$$

BONDI MASS-LOSS FORMULA.