Title: Predictions for Quantum Gravitational Signatures from Inflation

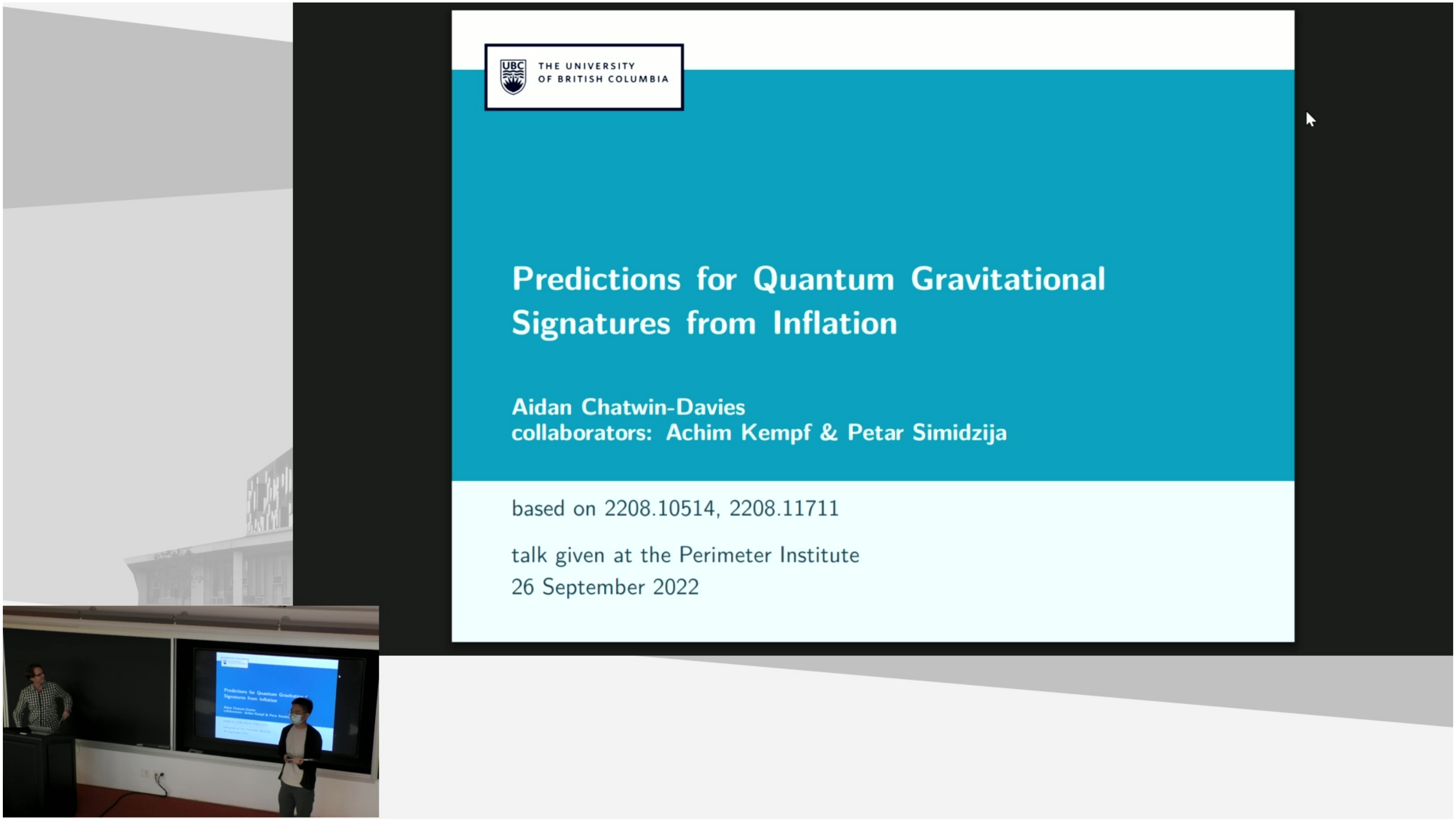
Speakers: Aidan Chatwin-Davies

Series: Cosmology & Gravitation

Date: September 26, 2022 - 12:00 PM

URL: https://pirsa.org/22090099

Abstract: The huge separation between the Planck scale and typical laboratory scales makes it extremely difficult to detect quantum gravitational effects; however, the situation is in principle much more favourable in cosmology. In particular, the Planck and Hubble scales were only separated by about 5 to 6 orders of magnitude during inflation. This motivates looking for present-day signatures of Planck-scale physics from the early universe. The question, then, is what quantum gravitational effects should we look for, and what are their observational signatures? Here I will discuss predictions for how a generic, quantum gravity-motivated, natural ultraviolet cutoff manifests in primordial power spectra. The cutoff is model-independent, both in the sense that it does not rely on a particular UV completion of quantum gravity, nor does it assume a particular model of inflation. The predicted signature consists of small oscillations that are superimposed on the conventional primordial power spectra, where the template waveform is parameterized by the location of the cutoff between the Planck and Hubble scales. This will allow experiments to place new rigorous bounds on the scale at which quantum gravity effects become important.


THE UNIVERSITY
OF BRITISH COLUMBIA
Predictions for Quantum Gravitational Signatures from Inflation
Aidan Chatwin-Davies
collaborators: Achim Kempf & Petar Simidzija
based on 2208.10514, 2208.11711
talk given at the Perimeter Institute
26 September 2022

Quantum gravity is hard to detect experimentally

- extreme separation of scales
- for instance, $\ell_{\mathrm{Planck}}/\ell_{\mathrm{LHC}} \sim 10^{-15}$
- Planck-scale effects suppressed like $(\ell_{\mathrm{Planck}}/\ell_{\mathrm{LHC}})^{\#}$ ...ouch!

Scales are much closer in the **early universe**

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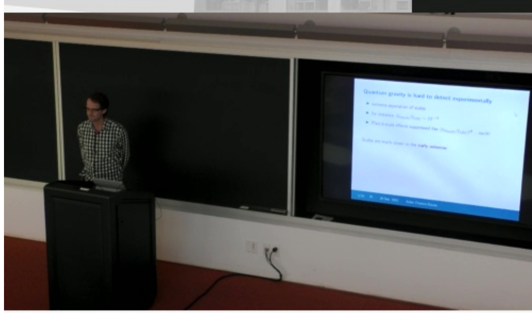


Some estimates

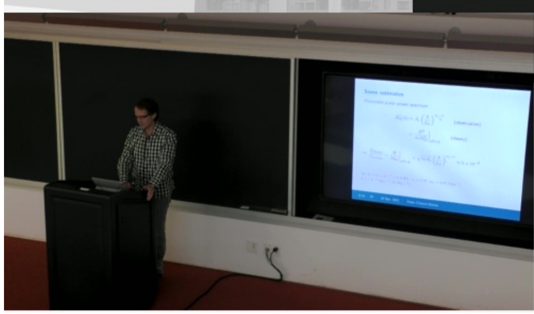
Primordial scalar power spectrum

$$\Delta^2_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_P}\right)^{n_s - 1} \qquad \text{(observation)}$$

$$= \left.\frac{H^2}{\pi \epsilon M_{\rm Pl}^2}\right|_{aH=k} \qquad \text{(theory)}$$

$$\Rightarrow \frac{\ell_{\rm Planck}}{\ell_{\rm Hubble}} \sim \left.\frac{H}{M_{\rm Pl}}\right|_{aH=k} = \sqrt{\pi \epsilon A_s} \left(\frac{k}{k_P}\right)^{n_s - 1} \approx 5 \times 10^{-6}$$

for $A_s \approx 2 \times 10^{-9}$, $\epsilon \approx 0.003$, $n_s \approx 0.97$, $k_P = 0.05\ \mathrm{Mpc}^{-1}$, $k \in (10^{-4}\ \mathrm{Mpc}^{-1}, 10\ \mathrm{Mpc}^{-1})$

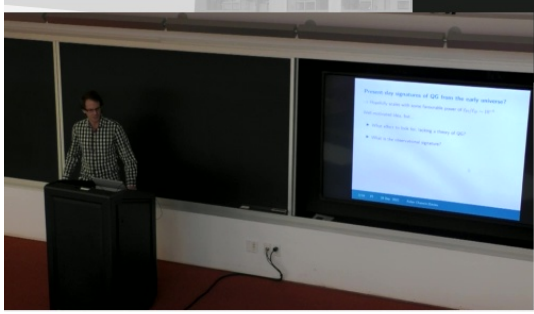


Present-day signatures of QG from the early universe?

→ Hopefully scales with some favourable power of $\ell_{\mathrm{Pl}}/\ell_H \sim 10^{-5}$

Well-motivated idea, but...

- What effect to look for, lacking a theory of QG?
- What is the observational signature?



Summary of the basic idea

- GR+QFT works *really well* for cosmological perturbations

∴ minimally modify QFT on curved spacetime apparatus

- what are the most dominant corrections as the Planck scale is approached from below?

→ focus on a generic prediction of quantum gravity

→ breakdown of distance at short scales, i.e. *natural UV cutoff*

- model covariantly

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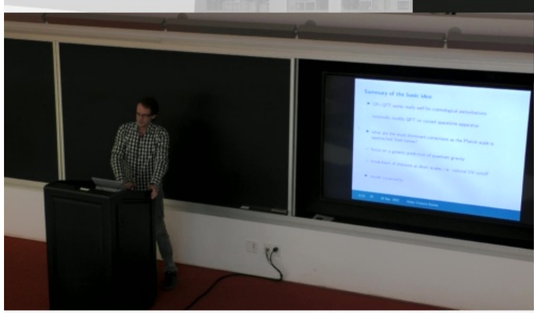
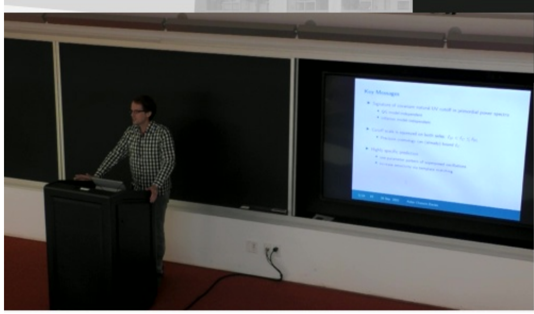


Key Messages

- Signature of covariant natural UV cutoff in primordial power spectra
  - QG model-independent
  - inflation model-independent
- Cutoff scale is squeezed on both sides: $\ell_H < \ell_C \leq \ell_{\mathrm{Pl}}$
  - Precision cosmology can (already) bound ℓ_C
- Highly specific prediction
  - one-parameter pattern of superposed oscillations
  - increase sensitivity via template matching



Outline

1. Introduction
2. Covariant Natural Ultraviolet Cutoffs
3. Calculation
4. Results
5. Conclusions and Outlook

Breakdown of distance

Generic expectation from most theories of quantum gravity:

Notion of distance breaks down at fine enough scales

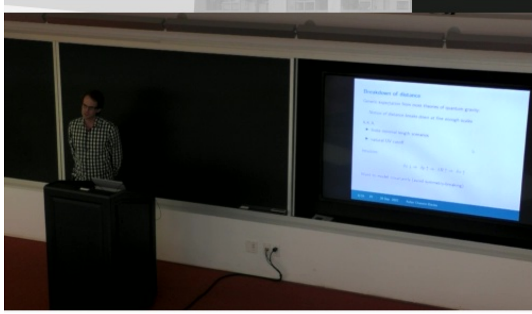
A.K.A.

- finite minimal length scenarios
- natural UV cutoff

Intuition:

$$\delta x \downarrow \Rightarrow \ \delta p \uparrow \Rightarrow \ \delta R \uparrow \Rightarrow \ \delta x \uparrow$$

Want to model *covariantly* (avoid symmetry-breaking)



How to make minimal length covariant?

$\rightarrow$ covariant generalization of maximum frequency, i.e. *bandlimit*

Ex: 1D function

$$f(x) = \int_{-\Omega}^{\Omega} \mathrm{d}k \; e^{ikx} F(k)$$

Notice: $-\partial_x^2(e^{ikx}) = k^2 e^{ikx}, \qquad k^2 \in [0, \Omega^2]$

- Lorentzian generalization: restrict spectrum of d'Alembertian, $\Box$

$$\phi(x) = \int_{\lambda \in [-\Omega^2, \Omega^2]} \mathrm{d}\mu(\lambda) \; u_\lambda(x) \; \Phi(\lambda)$$

where $\Box u_\lambda(x) = \lambda u_\lambda(x)$

[Kempf, Martin 0708.0062; ACD, Kempf, Martin 1210.0750]

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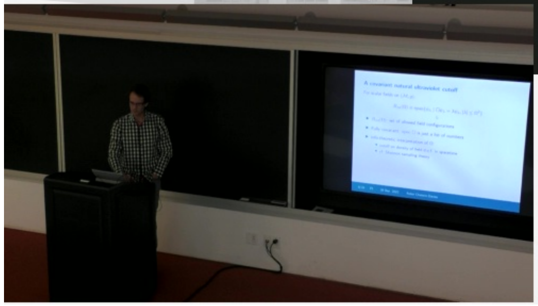
A covariant natural ultraviolet cutoff

For scalar fields on $(\mathcal{M}, g)$:

$$B_{\mathcal{M}}(\Omega) \equiv \mathrm{span}\{\psi_\lambda \mid \Box\psi_\lambda = \lambda\psi_\lambda, |\lambda| \leq \Omega^2\}$$

- $B_{\mathcal{M}}(\Omega)$: set of allowed field configurations
- Fully covariant: $\mathrm{spec}\,\Box$ is just a list of numbers
- info-theoretic interpretation of Ω:
  - cutoff on density of field d.o.f. in spacetime
  - cf. Shannon sampling theory



Implement via the QFT path integral

Ex: Feynman propagator

The usual expression:

$$iG_F(x,x') = \frac{\int \mathcal{D}\phi\, \phi(x)\phi(x')e^{iS[\phi]}}{\int \mathcal{D}\phi\, e^{iS[\phi]}}$$

Discard trans-Planckian contributions:

$$iG_F^{\Omega}(x,x') = \frac{\int_{B_{\mathcal{M}}(\Omega)} \mathcal{D}\phi\, \phi(x)\phi(x')e^{iS[\phi]}}{\int_{B_{\mathcal{M}}(\Omega)} \mathcal{D}\phi\, e^{iS[\phi]}}$$

Some comments

- operational interpretation:
  - discarding most off-shell (i.e. quantum) contributions to P.I.

Ex: massless scalar field

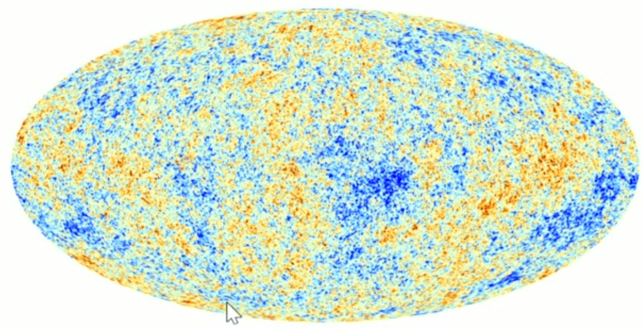
$$\Box\phi = 0 \qquad \text{on-shell}$$

$$\Box\phi = \lambda\phi \qquad \text{off-shell}$$

- softened cutoff (more on this later)

Goal

Look for signatures of Planck-scale cutoff in



Today:

- compute correction to *primordial power spectrum* (PPS)
- focus on *scalar perturbations*

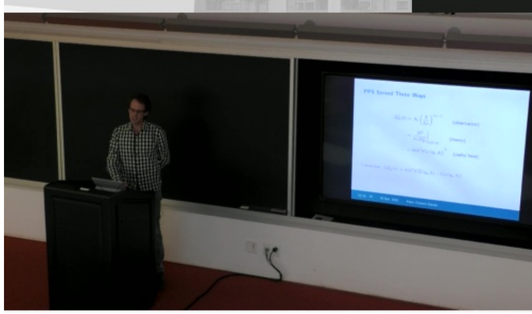
PPS Served Three Ways

$$\Delta^2_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_P}\right)^{n_s - 1} \qquad \text{(observation)}$$

$$= \left.\frac{H^2}{\pi \epsilon M_{\mathrm{Pl}}^2}\right|_{aH=k} \qquad \text{(theory)}$$

$$= 4\pi k^3 |G_F(\eta_k, k)| \qquad \text{(useful here)}$$

Correction: $\delta\Delta^2_{\mathcal{R}}(k) \equiv 4\pi k^3 |G^{\Omega}_F(\eta_k, k) - G_F(\eta_k, k)|$



Path integrals are unwieldy

$$iG_F^{\Omega}(x,x') = \frac{\int_{B_{\mathcal{M}}(\Omega)} \mathcal{D}\phi\, \phi(x)\phi(x')e^{iS[\phi]}}{\int_{B_{\mathcal{M}}(\Omega)} \mathcal{D}\phi\, e^{iS[\phi]}}$$

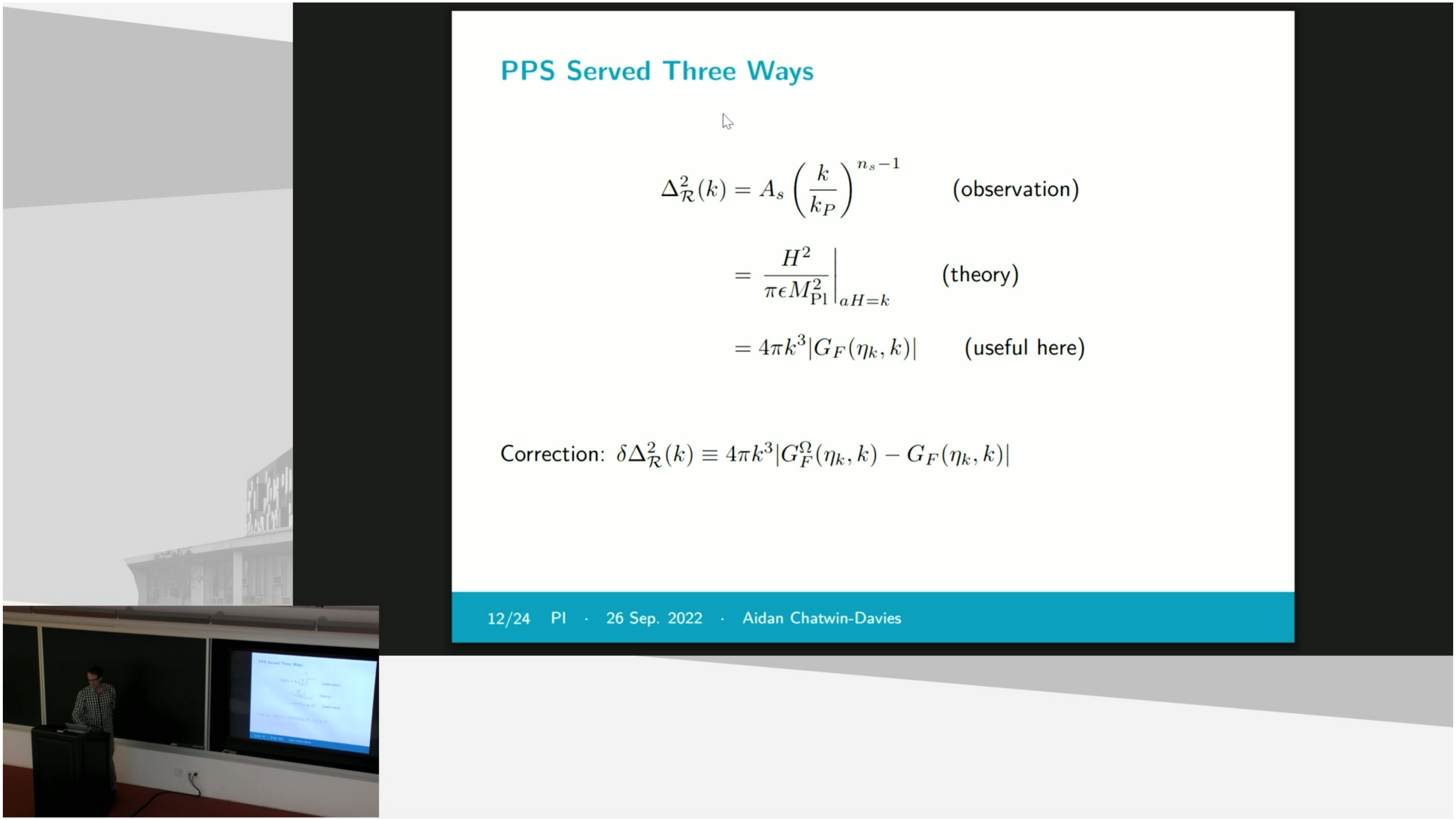
Equivalent definition via projectors:

$$G_F^{\Omega} = P_\Omega G_F P_\Omega$$

where, acting on a test function $u(x)$,

$$P_\Omega u(x) \equiv \sum_{\lambda \in \mathrm{spec}\,\Box} \theta(\Omega^2 - |\lambda|)\, \langle \psi_\lambda, u\rangle \psi_\lambda(x)$$

Remark: soften the *sharp* cutoff by smoothing the Heaviside step function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Calculation overview

Inputs:

- FLRW scale factor, $a(\eta)$

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right]$$

- assumption: single-field inflation

Steps:

1. Write down G_F for comoving curvature perturbation
2. Write down projectors P_Ω
3. Compute correction $\delta\Delta^2_{\mathcal{R}}(k)$

Ω is an **unknown parameter**

$\rightarrow$ to be fixed by comparing with data

$\rightarrow$ expect $H < \Omega < M_{\mathrm{Pl}}$

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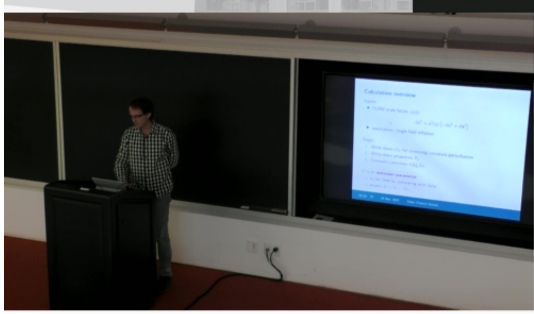


How to fix the FLRW geometry?

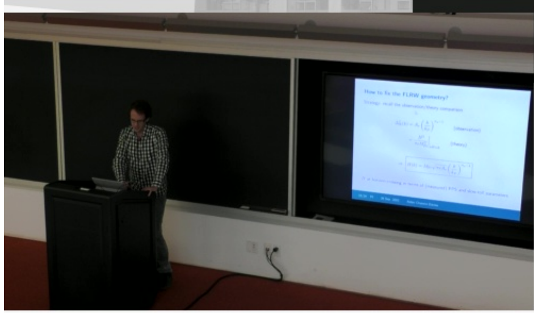
Strategy: recall the observation/theory comparison

$$\Delta^2_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_P}\right)^{n_s - 1} \qquad \text{(observation)}$$

$$= \left.\frac{H^2}{\pi \epsilon M_{\mathrm{Pl}}^2}\right|_{aH=k} \qquad \text{(theory)}$$

$$\Rightarrow \boxed{H(k) = M_{\mathrm{Pl}} \sqrt{\pi \epsilon A_s} \left(\frac{k}{k_P}\right)^{n_s - 1}}$$

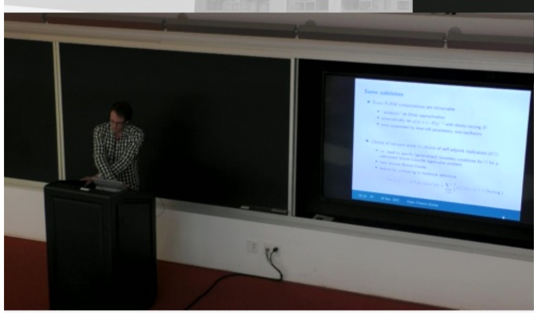
H at horizon-crossing in terms of (measured) PPS and slow-roll parameters



Some subtleties

- Exact FLRW computations are intractable
  - "adiabatic" de Sitter approximation
  - schematically, let $a(\eta) = (-H\eta)^{-1}$ with slowly-varying H
  - error suppressed by slow-roll parameters, non-oscillatory
- Choice of vacuum state $\leftrightarrow$ choice of self-adjoint realization of $\Box$
  - i.e. need to specify (generalized) boundary conditions for $\Box$ for a well-posed Sturm-Liouville eigenvalue problem
  - here assume Bunch-Davies
  - deduce by comparing to textbook definition

$$G_F(x,x') = \langle 0|\mathcal{T}\hat{\phi}(x)\hat{\phi}(x')|0\rangle \stackrel{!}{=} \sum_{\lambda\neq 0}\frac{1}{\lambda}\psi^*_\lambda(x)\psi_\lambda(x') + \text{(homog.)}$$



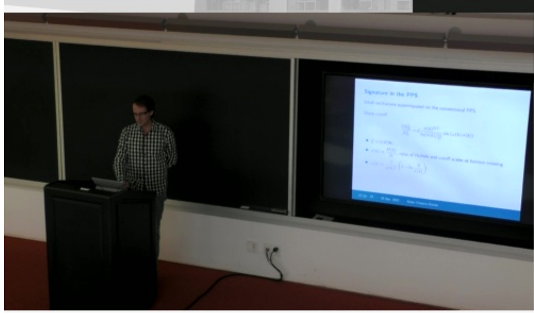
Signature in the PPS

Small oscillations superimposed on the conventional PPS

Sharp cutoff:

$$\frac{\delta\Delta^2_{\mathcal{R}}}{\Delta^2_{\mathcal{R}}} = \mathcal{C}\frac{\sigma(k)^{3/2}}{\ln(\sigma(k)/2)}\sin\left(\omega(k)\,\sigma(k)\right)$$

- $\mathcal{C} = 0.8796...$
- $\sigma(k) \equiv \dfrac{H(k)}{\Omega}$, ratio of Hubble and cutoff scales at horizon crossing
- $\omega(k) \equiv \dfrac{1}{\sigma(k)^2}\left(1 - \ln\dfrac{2}{\sigma(k)}\right)$

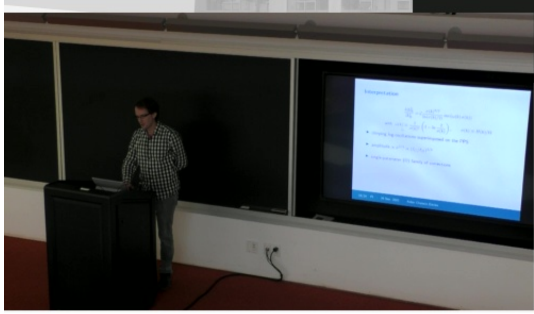


Interpretation

$$\frac{\delta\Delta_{\mathcal{R}}^2}{\Delta_{\mathcal{R}}^2} = \mathcal{C}\,\frac{\sigma(k)^{3/2}}{\ln(\sigma(k)/2)}\,\sin\left(\omega(k)\,\sigma(k)\right)$$

$$\text{with}\quad \omega(k) \equiv \frac{1}{\sigma(k)^2}\left(1 - \ln\frac{2}{\sigma(k)}\right), \qquad \sigma(k) \equiv H(k)/\Omega$$

- chirping log-oscillations superimposed on the PPS
- amplitude $\propto \sigma^{3/2} \propto (\ell_C/\ell_H)^{3/2}$
- single-parameter (Ω) family of corrections



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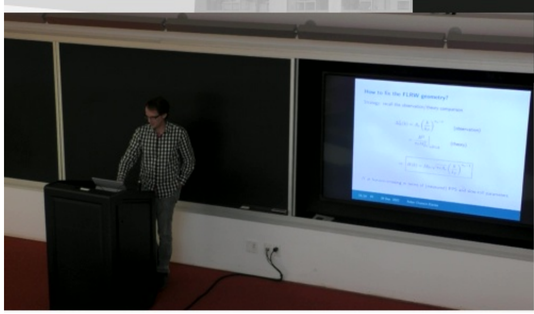

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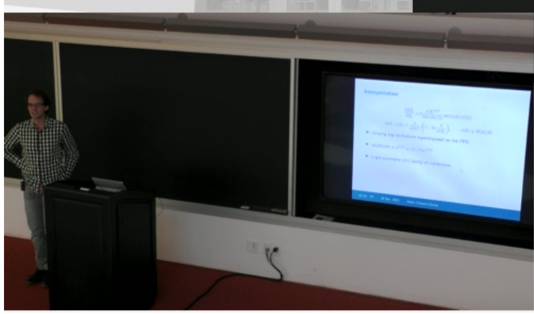
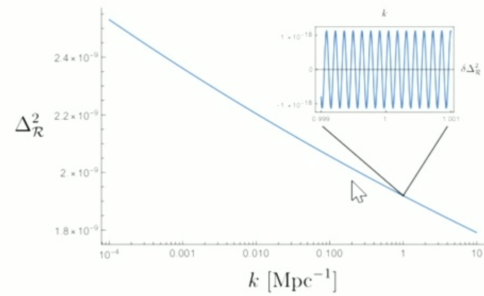
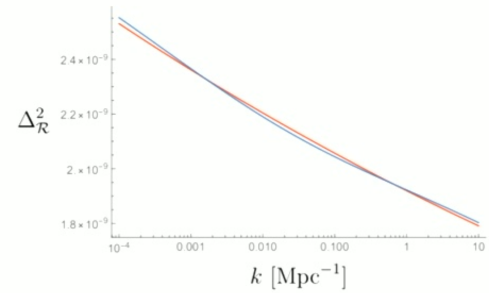


Visualization

$\Omega = M_{\mathrm{Pl}}$



$\Omega = (0.5 \times 10^{-4}) M_{\mathrm{Pl}}$



Conclusion: left is probably imperceptible, right is probably too drastic

$(A_s = 2 \times 10^{-9},\ \epsilon = 0.003,\ n_s = 0.97,\ k_P = 0.05\ \mathrm{Mpc}^{-1})$

More general cutoffs

- considered simplest case of single-parameter sharp cutoff, but can also soften the cutoff:

$$P_\Omega u(x) \equiv \sum_{\lambda \in \mathrm{spec}\Box} \theta(\Omega^2 - |\lambda|)\langle \psi_\lambda, u\rangle \psi_\lambda(x) \;\to\; \sum_{\lambda \in \mathrm{spec}\Box} f(\lambda)\langle \psi_\lambda, u\rangle \psi_\lambda(x)$$

- in principle functional d.o.f.
- frequency unchanged, amplitude damps, phase shifts
- heuristically: 3-parameter family to explore

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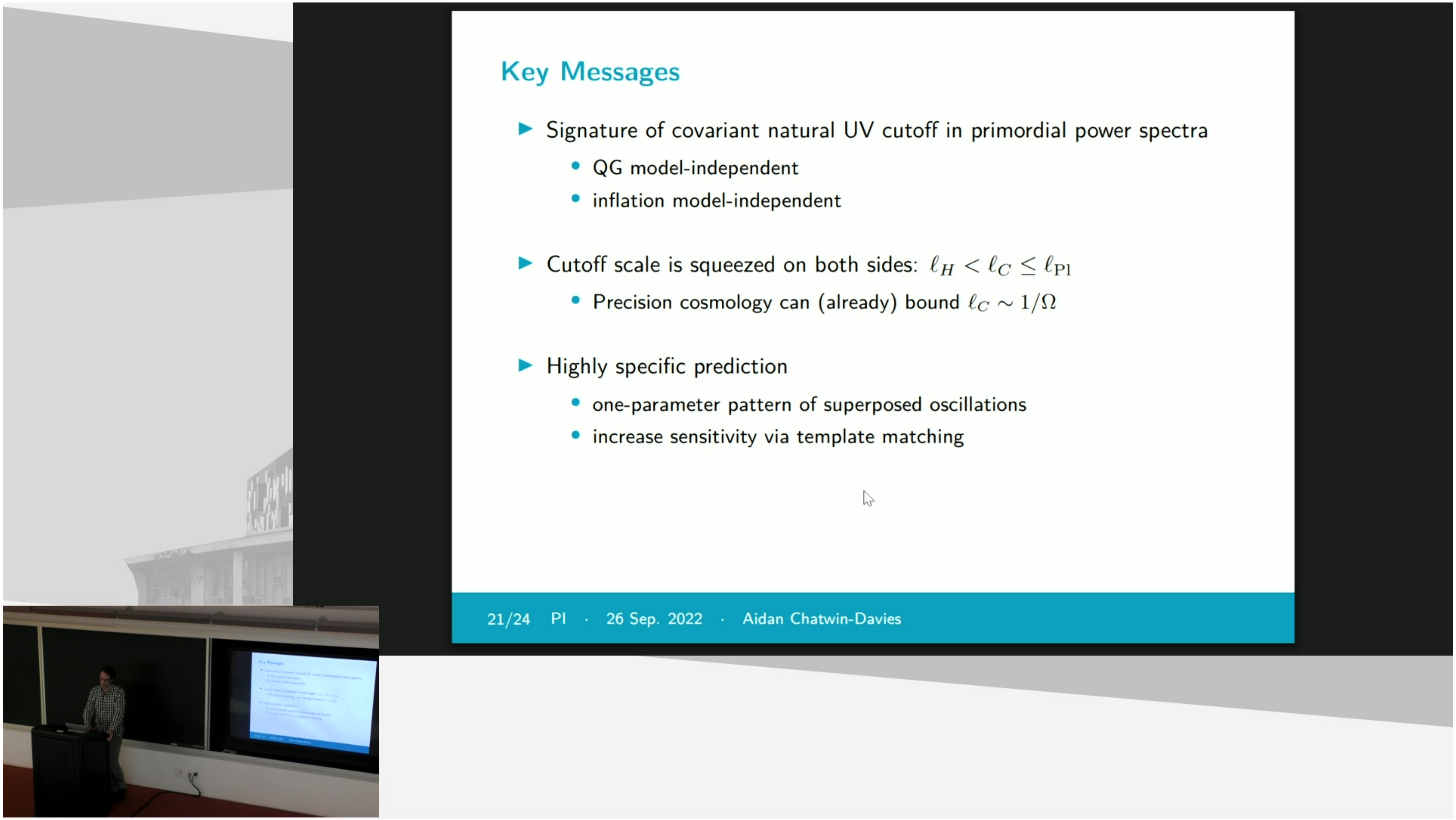
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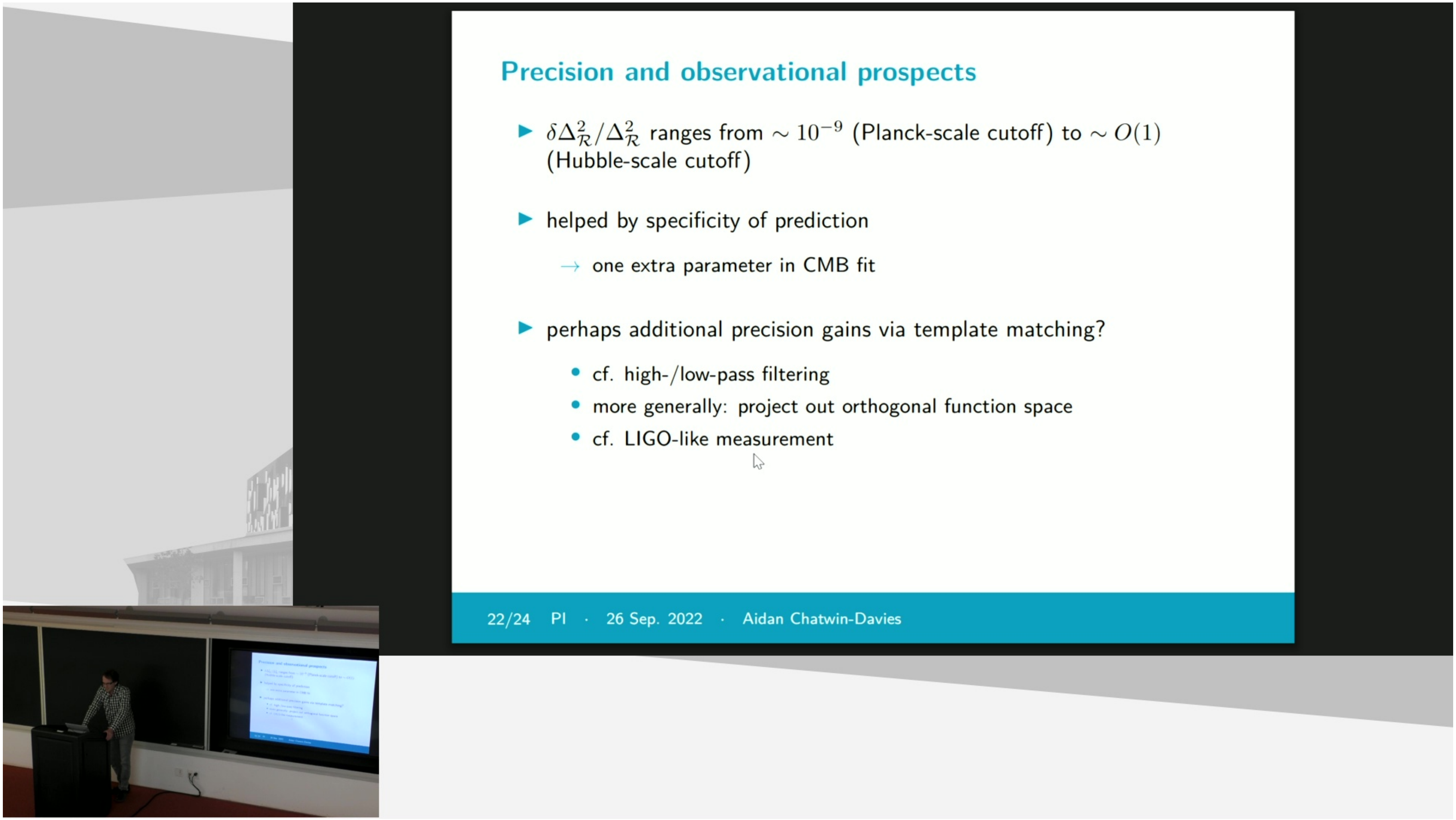
Precision and observational prospects
▶ $\delta\Delta_{\mathcal{R}}^2/\Delta_{\mathcal{R}}^2$ ranges from $\sim 10^{-9}$ (Planck-scale cutoff) to $\sim O(1)$ (Hubble-scale cutoff)
▶ helped by specificity of prediction
→ one extra parameter in CMB fit
▶ perhaps additional precision gains via template matching?
• cf. high-/low-pass filtering
• more generally: project out orthogonal function space
• cf. LIGO-like measurement
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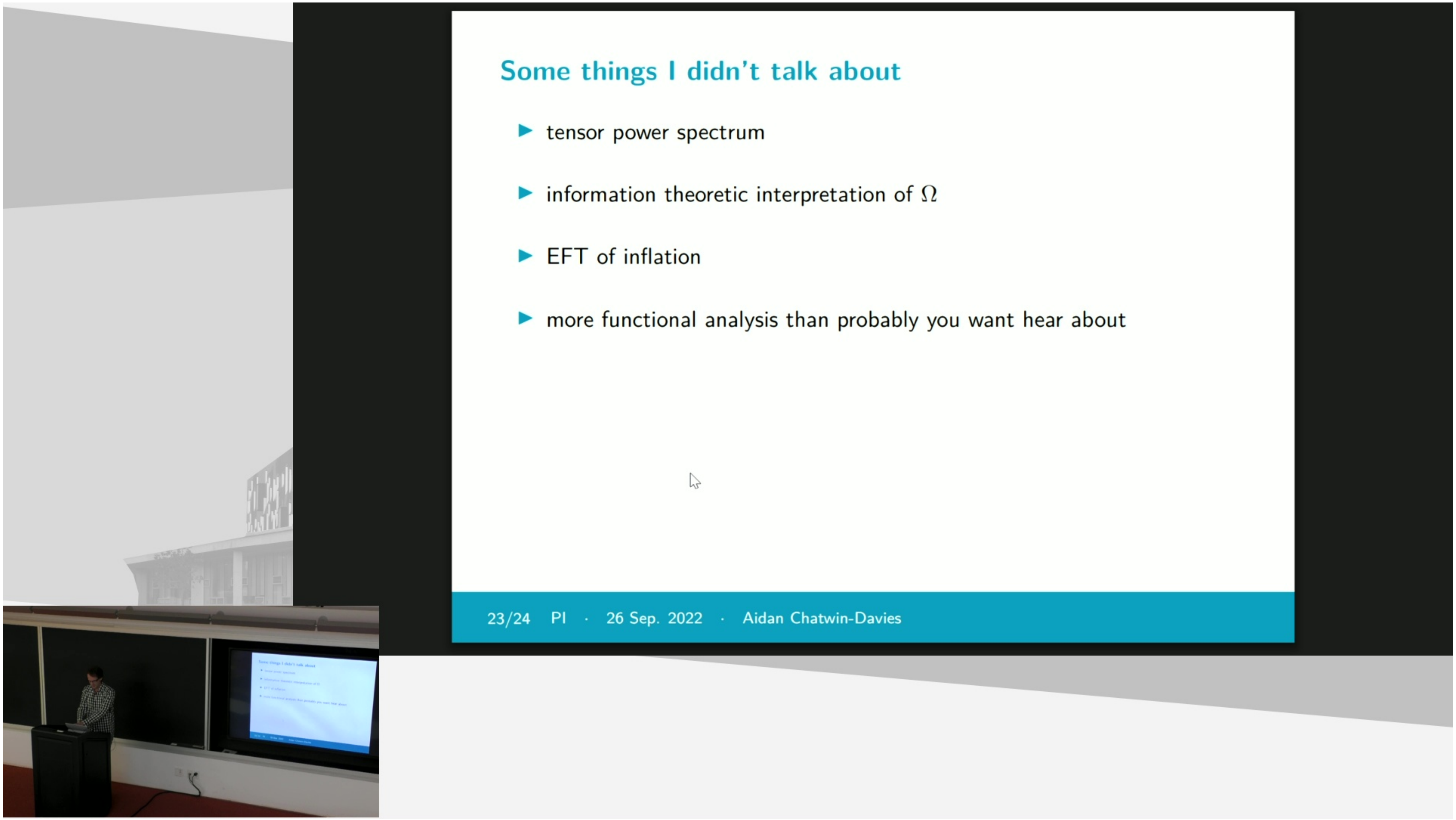
Some things I didn't talk about
tensor power spectrum
information theoretic interpretation of Ω
EFT of inflation
more functional analysis than probably you want hear about
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Some questions for you

- Thoughts, comments, impressions?
- People to talk with, resources to consult re: data analysis?
- Strategy? (E.g. propagate prediction forward to CMB vs. inferred PPS?)

Thanks!

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