Title: NLTS Hamiltonians from good quantum codes

Speakers: Anurag Anshu
Date: September 28, 2022 - 11:00 AM

URL: https://pirsa.org/22090094

Abstract: The NLTS (No Low-Energy Trivial State) conjecture of Freedman and Hastings [2014] posits that there exist families of Hamiltonians with all low energy states of non-trivial complexity (with complexity measured by the quantum circuit depth preparing the state). Our recent work https://arxiv.org/abs/2206.13228 (with Nikolas Breuckmann and Chinmay Nirkhe) proves this conjecture by showing that the recently discovered families of constant-rate and linear-distance QLDPC codes correspond to NLTS local Hamiltonians. This talk will provide background on the conjecture, its relevance to quantum many-body physics and quantum complexity theory, and touch upon the proof techniques.

Zoom link: https://pitp.zoom.us/j/94224635225?pwd=SUovNXA3MWlkRUJlcTIxV0pLQzQxdz09

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NLTS Hamiltonians from good quantum codes

Anurag Anshu

School of Engineering & Applied Sciences, Harvard University

(Joint with Nikolas Breuckmann & Chinmay Nirkhe)

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UC Berkeley → IBM

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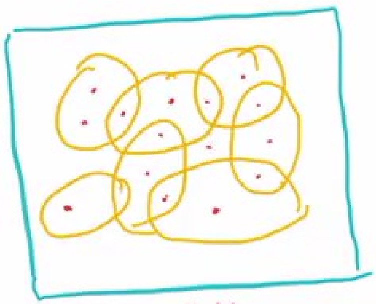
Local Hamiltonians



n qubits

$$H_n = \sum_{i=1}^{m} h_i \quad ; \quad h_i \succeq 0, \quad \|h_i\| \le 1.$$

$\lambda_{max}(H) \simeq O(m)$

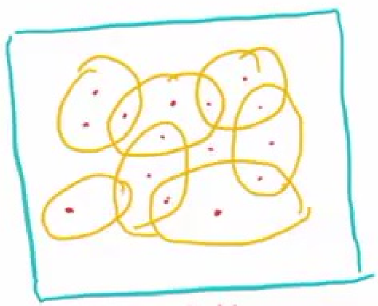
$\lambda_{min}(H) + \epsilon m$

ϵ- energy states $\{$

$\lambda_{min}(H) \rightarrow$ ground state

Motivating question: are there "simple" low-energy states?

Local Hamiltonians



n qubits

local terms

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$\lambda_{min}(H) + \epsilon m$

$\lambda_{min}(H) \rightarrow$ ground state

not just eigenstates $\rightarrow$ ϵ- energy states

Motivating question: are there "simple" low-energy states?

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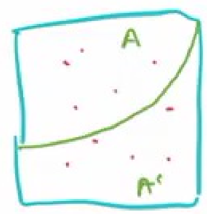
"Simple" low energy states?

- Depends on notion of simplicity.

Example: area law behaviour / tensor network of poly(n) bond dim.

Such ϵ- energy state exists: $\frac{e^{-\beta H}}{Tr(e^{-\beta H})}$ at $\beta \sim \frac{1}{\epsilon}$



[Wolf et.al. 2008] Area law: $I[A:A^c] \leq O(\beta |\partial A|)$

[Molnar et.al. 2015] Tensor network: $O(n^{\beta})$ PEPO

Not simple enough: local expectation value hard to compute beyond 1D.

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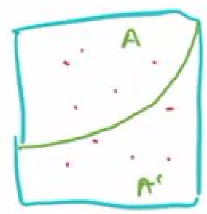

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Even "Simple" low-energy states?

- States prepared by constant depth quantum circuits
- States with small stabilizer rank.

Complexity theoretic picture:

Given H

QMA Merlin

← ... An ϵ-energy quantum state

Kitaev [1999]

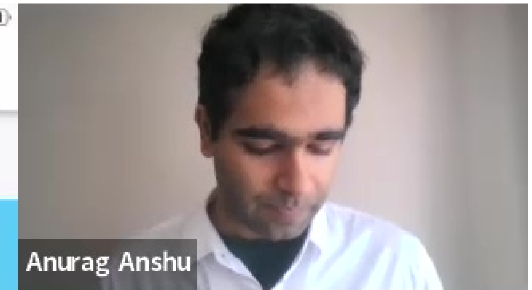
Arthur

Is $\lambda_{min}(H) < a$

or $\lambda_{min}(H) > a + \epsilon n$

NP Merlin

← classical description of a ϵ-energy state



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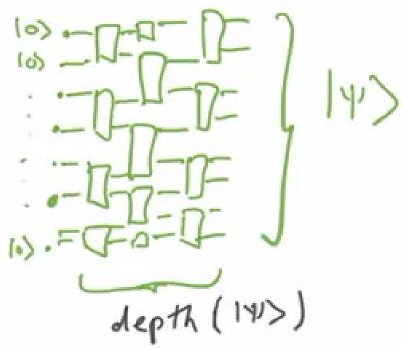
No Low-energy Trivial States conjecture

[Freedman – Hastings 2014]

$\exists\, \epsilon$ & $\{H_n\}_{n=1}^{\infty}$ such that all ϵ-energy states have quantum circuit depth $\Omega(\log\log n)$.



Moral: High depth corresponds to "long range entanglement" or "high complexity".

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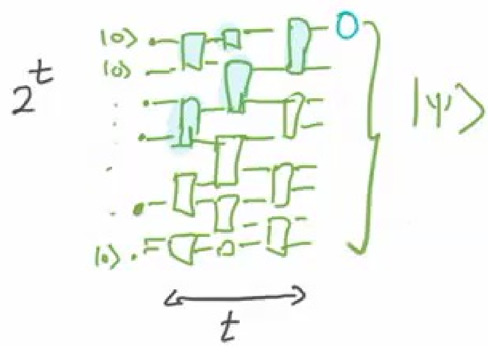
Why quantum circuit depth?



$\langle\psi| O |\psi\rangle$ can be computed in 2^{2^t} time.

$\Rightarrow \langle\psi| H |\psi\rangle$ can be computed in $O(n\, 2^{2^t})$ time.

$\sim$ poly(n) if $t = O(\log \log n)$

- If NLTS is false $\Rightarrow$ Quantum PCP conjecture is false (Assuming QMA $\neq$ NP).

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Main result

- Consider the $[n,k,d]$ quantum code of Leverrier & Zemor [2022] (also Panteleev & Kalachev [2021]).
  - $k = \Theta(n)$ & $d = \Theta(n)$.
  - Quantum Parity Checks are constant weight.
- The Hamiltonian H derived from this code has NLTS property:
  - Let $\epsilon = \min\left(\frac{k}{n}, \frac{d^2}{n^2}\right)$. Any ϵ-energy state has circuit depth $\Omega(\log n)$.

$$H = H_X + H_Z = \sum_{\substack{a \in \\ X\text{-checks}}} \frac{(I - X^a)}{2} + \sum_{\substack{b \in \\ Z\text{-checks}}} \frac{(I - Z^b)}{2}$$

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$a = 1100$; $X^a = X_1 \otimes X_2 \otimes I \otimes I$

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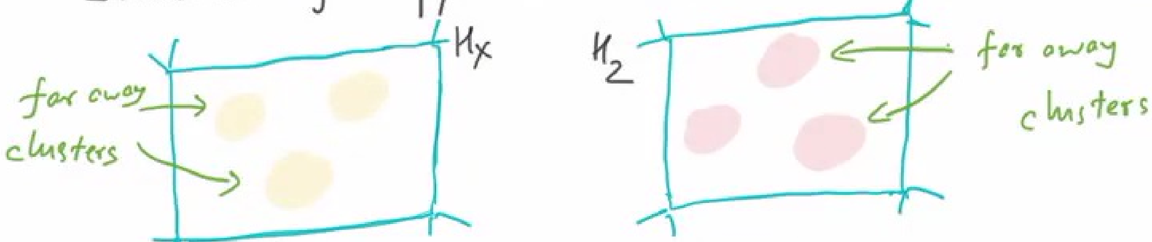
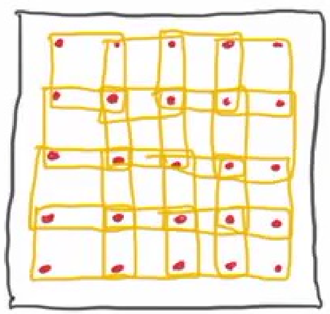
Key Steps

- Structure of approximate codewords of H_X & H_Z

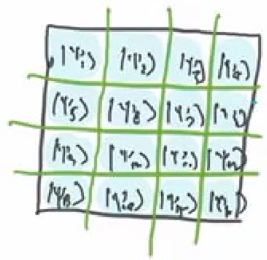


- Main message: expansion $\Rightarrow$ clustering
- Q Circuit lower bound based on "spread of" output distribution.
- Uncertainty principle to enforce spread
  - Inspired by [Eldar-Harrow 2016].
  - Key difference: we need high rate of quantum code. [see also A., Nirkhe, 2020].

Expansion seems necessary: a non-NLTS example



- H_n is on a square lattice
- $\exists$ ϵ-energy state with circuit depth $\sim 2^{1/\epsilon^2}$.



NLTS property requires boundary to scale as volume, which happens in expander hyper-graphs

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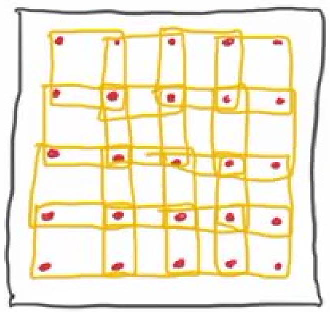
Expansion seems necessary: a non-NLTS example



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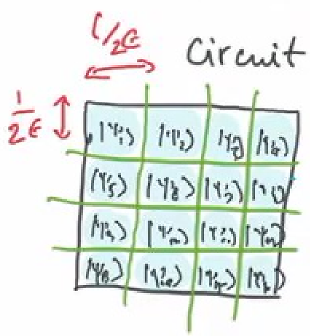

$|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \ldots$

NLTS property requires boundary to scale as volume which happens in expander hyper-graphs

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Expansion ⇒ Clustering ; an Illustrative Example

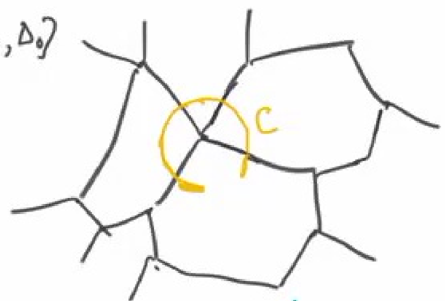
[A., Breuckmann 2022]

- Tanner Code T (Sipser-Spielman)

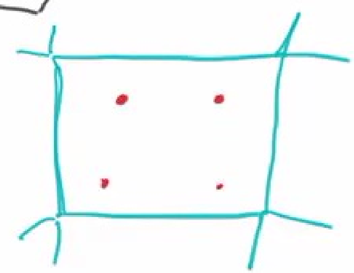
degree d $G=(V,E)$, $C_0=[d,k_0,\Delta_0]$

$\Downarrow$

$\lambda = \max(|\lambda_2|,|\lambda_n|)$.



- Distance $\sim \Theta(|E|)$ if $\Delta_0 \geq 2\lambda$



- what about approx codewords?

$\{x : |Tx| \leq \epsilon |E|\}$

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Anurag Anshu

Expansion ⇒ Clustering ; an Illustrative Example

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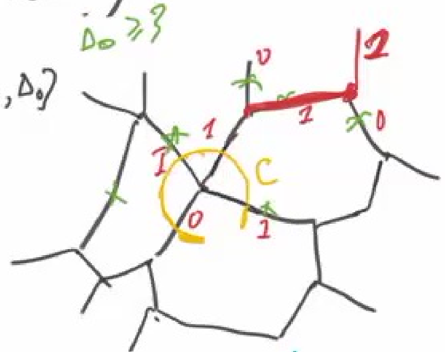
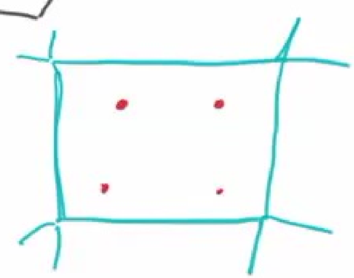


- Distance ~ $\Theta(|E|)$ if $\Delta_0 \geq 2\lambda$



- what about approx codewords?

$\{x : |Tx| \leq \epsilon |E|\}$

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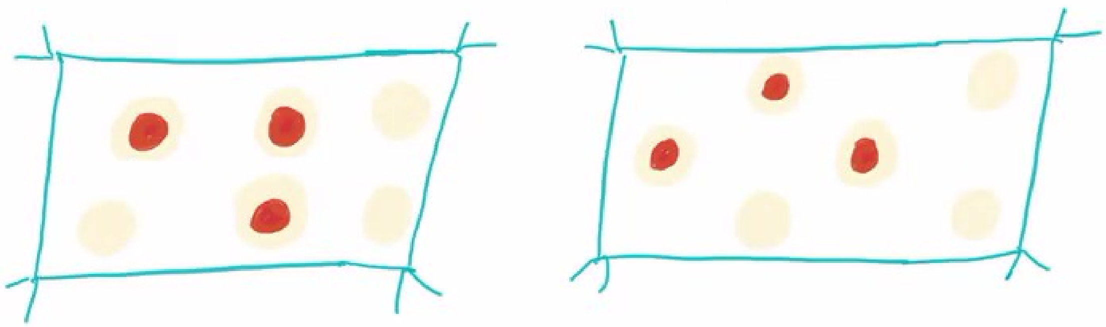
Structure of good quantum codes.

- Leverrier - Zemor [2022] & Panteleev - Kalachev [2021]
- Distance = $\Theta(n)$; proof relies on expansion of the X and Z tanner graphs.
- Same proof gives the clustering of approx codewords.



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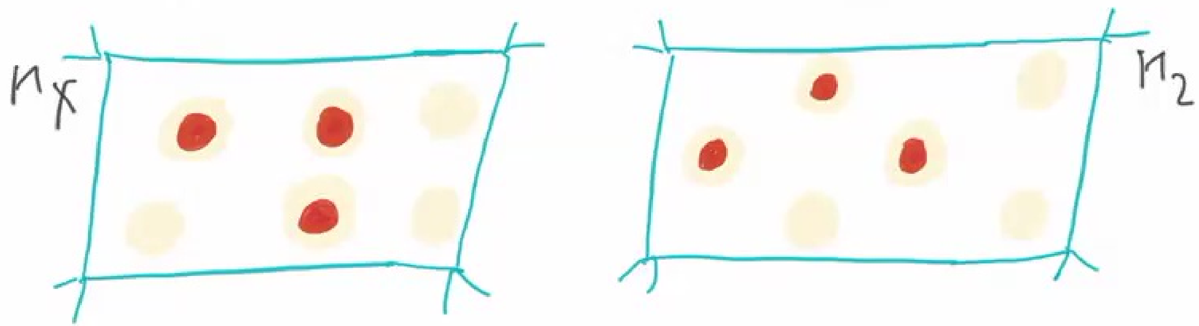

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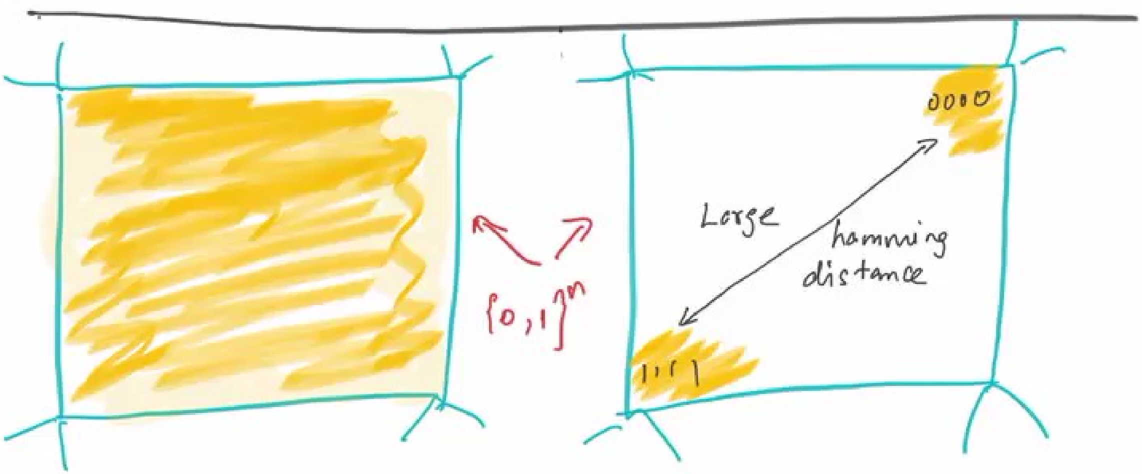
Circuit lower bound toolkit.

Example 1: $|+\rangle|+\rangle|+\rangle\ldots|+\rangle$; $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$

Example 2: $\frac{1}{\sqrt{2}}\left(|0\ldots0\rangle + |1\ldots1\rangle\right)$



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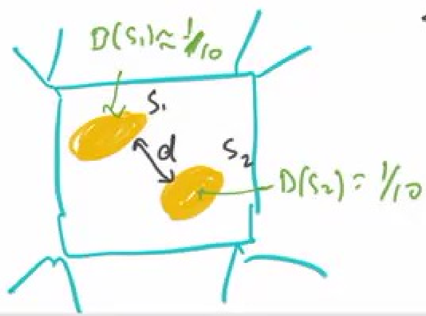
Circuit lower bound toolkit

Thm: Given $|\psi\rangle = U|0\rangle^{\otimes n}$, where $\mathrm{depth}(|\psi\rangle) = t$.

Let D be the distribution when $|\psi\rangle$ is measured in computational basis. If S_1, S_2 are two "heavy" sets with distance d, then

$$t = \Omega\left(\log\left(\frac{d^2}{n}\right)\right)$$



Proof: Polynomial method

[Kuwahara, Arad, Amico, Vedral 2015]

[Eldar, Harrow 2016]

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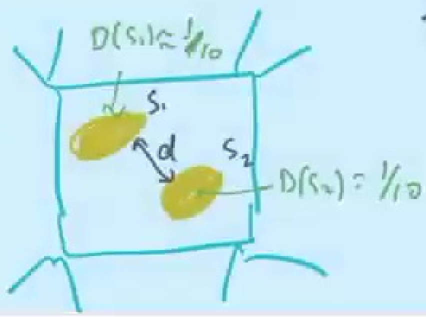

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[Kuwahara, Arad, Amico, Vedral 2015]
[Eldar, Harrow 2016]

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OPEN QUESTIONS

- NLTS property for circuit depth + stabilizer rank
- Fermionic NLTS property (Hastings & O'Donnell 2021)
- Are every good quantum LDPC codes NLTS?
- Quantum PCP??