Title: Quasinormal-mode filters and their impacts on future black-hole spectroscopy

Speakers: Sizheng Ma

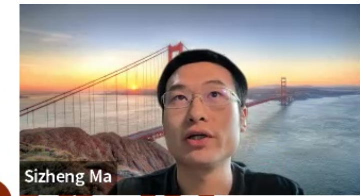
Series: Strong Gravity

Date: September 22, 2022 - 1:00 PM

URL: https://pirsa.org/22090090

Abstract: In this talk, I present a novel framework to do black-hole spectroscopy. This approach is based on a new technique so-called "quasinormal-mode filter", which can be classified into two subsets: rational filter and full filter. On the theoretical level, I explain how to use the rational filter to understand the ringdown of numerical-relativity waveforms. On the observational level, I introduce a way to incorporate the filter into Bayesian analysis. The new Bayesian framework not only allows us to analyze the ringdown of a real gravitational-wave event without Markov chain Monte Carlo, but also yields a natural estimate of the ringdown start time. By applying our method to GW150914, we find strong and self-consistent evidence for the first overtone from multiple perspectives. On the other hand, the relationship between the full filter and the metric reconstruction is discussed. Its connection to a numerical-relativity technique "Cauchy-characteristic Matching" is provided. In the final part of the talk, I also briefly present our recent progress on fully relativistic 3D Cauchy-characteristic Matching.

Zoom link: https://pitp.zoom.us/j/95593408817?pwd=U2RXZDV2WUtwNDlaRDVBVTJjTEdBQT09



The QNM filter: a novel framework in black-hole spectroscopy

Sizheng Ma (Caltech)

In collaboration w/: Ling Sun (OzGrav, ANU), Yanbei Chen (Caltech)

09/22/2022 @Perimeter Institute

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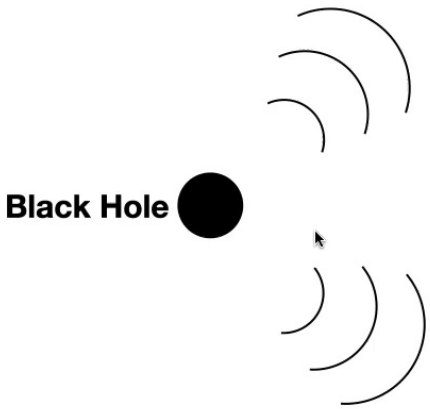
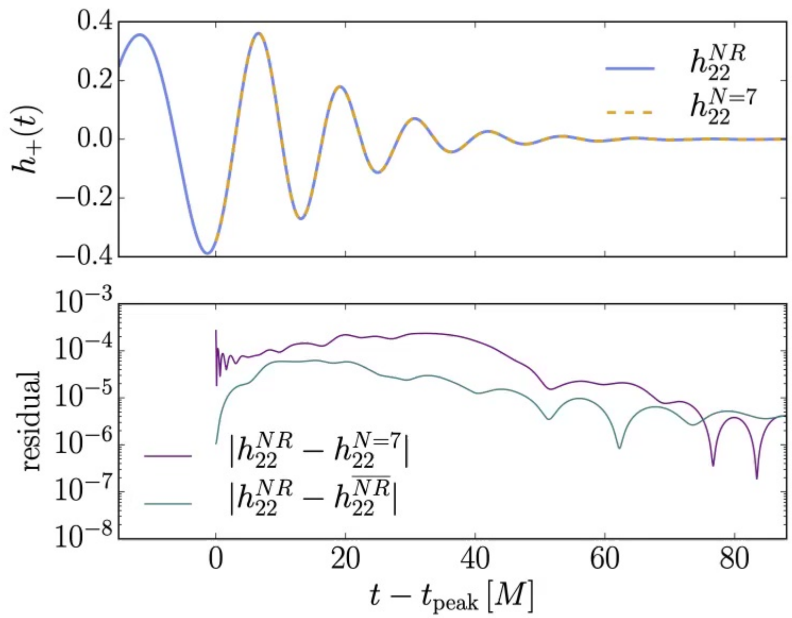
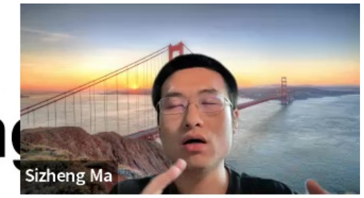
BH spectroscopy and ringdown modeling





- Overfitting?



$$\omega_{lmn}$$

$$h_{lm} = \sum_n A_n e^{-i\omega_{lmn}t}$$

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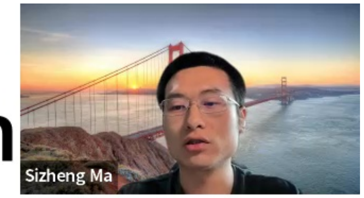
Giesler et al., arXiv: 1903.08284



A new approach — a frequency-domain

$$h(t) = e^{-i\omega_{lmn}(t-t_0)}\,\Theta(t-t_0)$$

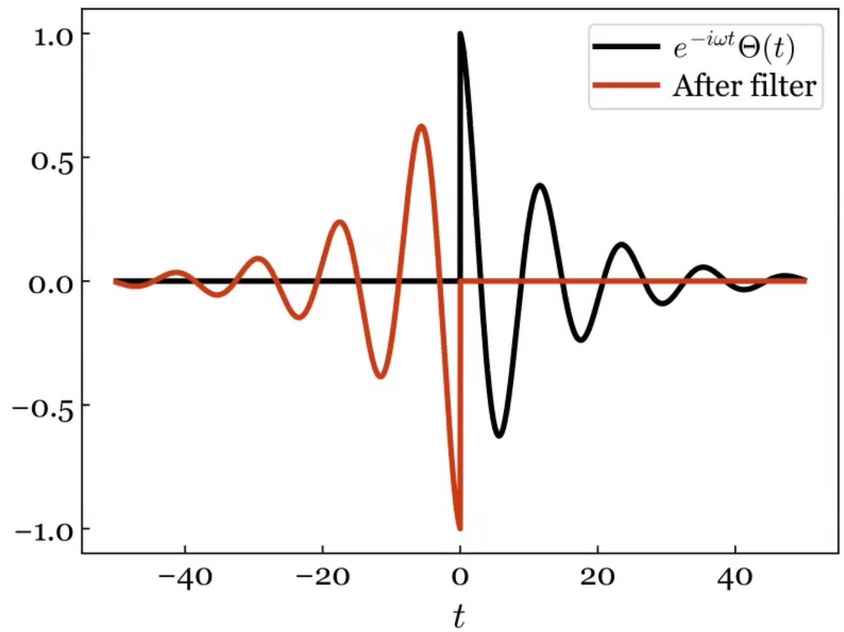
Time domain operator: $\left(\frac{d}{dt} + i\omega_{lmn}\right) h(t) = 0$

$$\downarrow$$

$$\omega - \omega_{lmn}$$

$$\downarrow$$

$$\frac{\omega - \omega_{lmn}}{\omega - \omega^*_{lmn}}$$

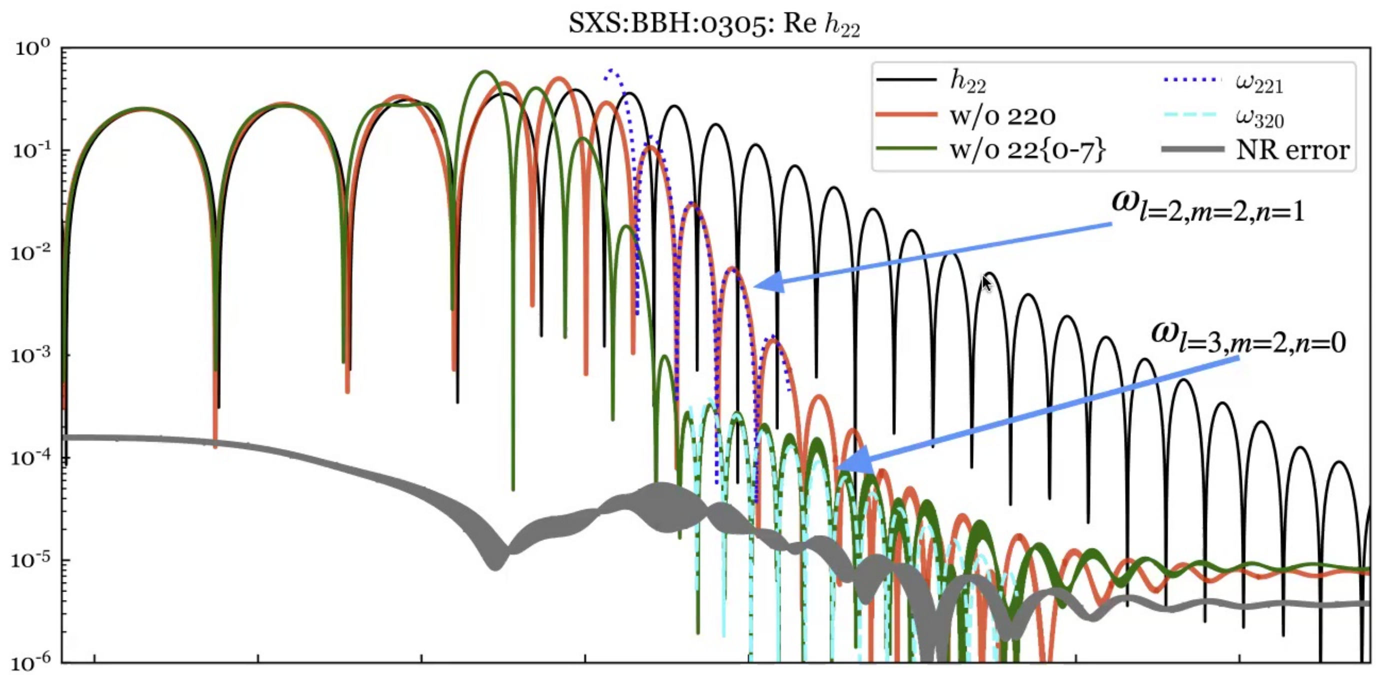


arXiv: 2207.10870

The spherical-spheroidal mixing m

$$\text{Filter} \sim \prod_{lmn} (\omega - \omega_{lmn})$$

GW150914-like
numerical-relativity simulation



arXiv: 2207.10870

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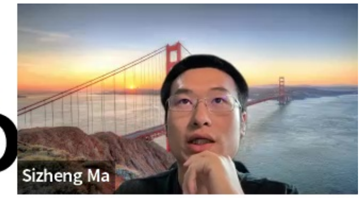
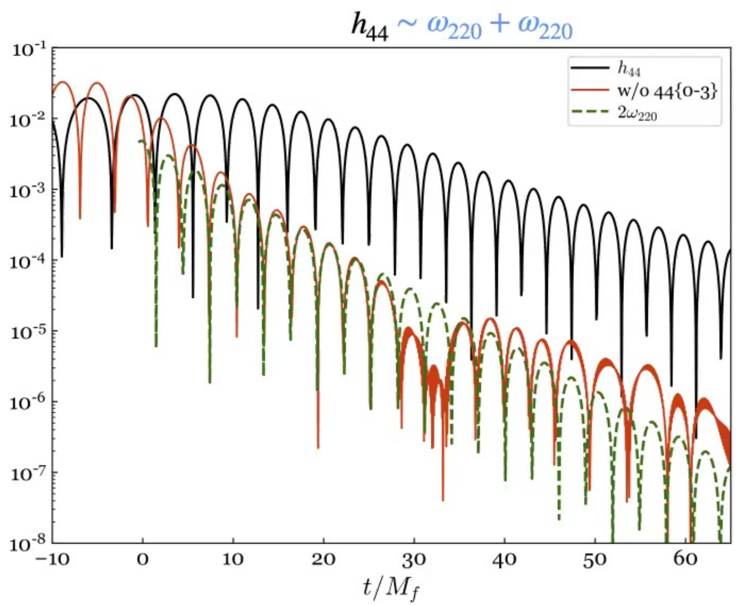
Second-order quasinormal mo

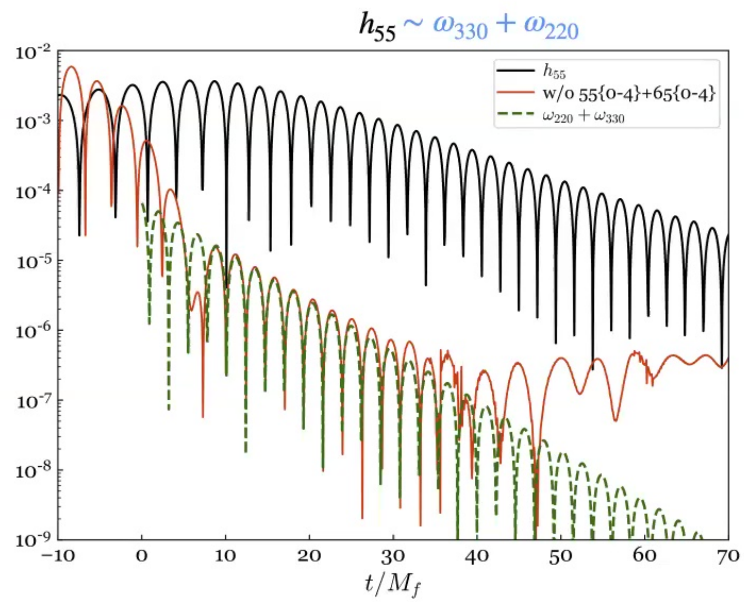


$$h_{lm} = \sum_n A_n e^{-i\omega_{lmn}t} \quad + Be^{-i(\omega_{l_1m_1n_1}+\omega_{l_2m_2n_2})t}$$

linear effects

second-order effects





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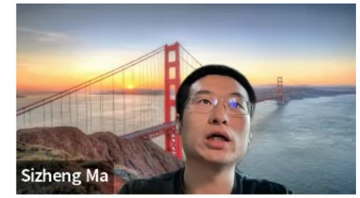
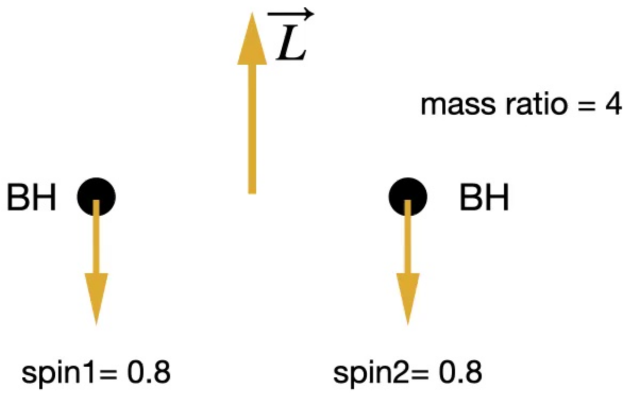
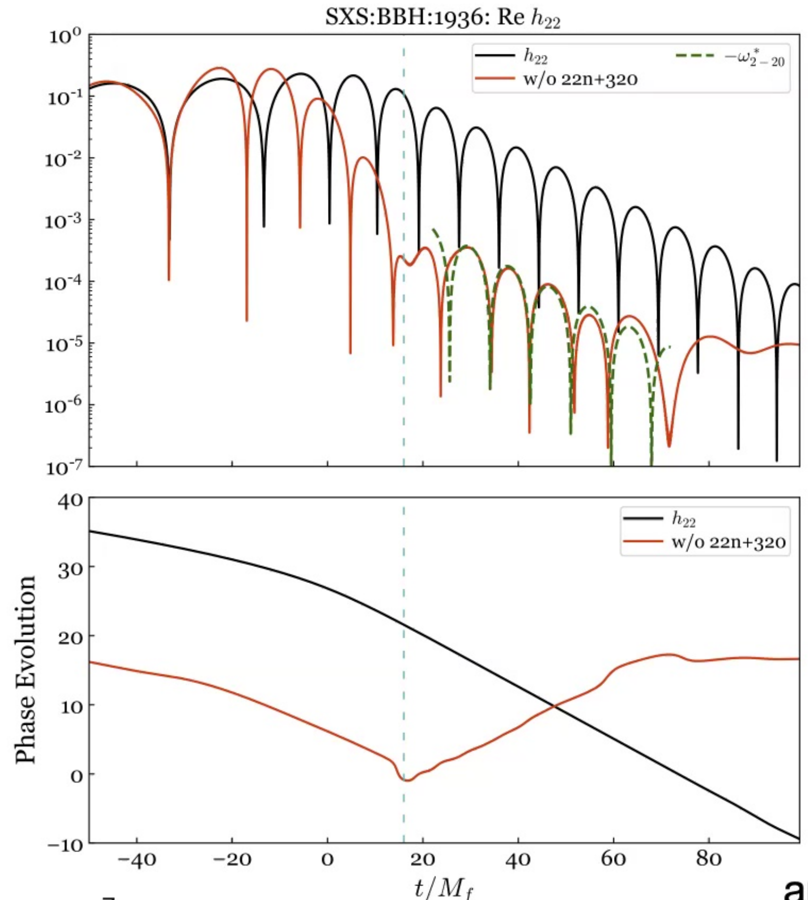
arXiv: 2207.10870, 2208.07380, 2208.07374, 2208.07379

Retrograde modes







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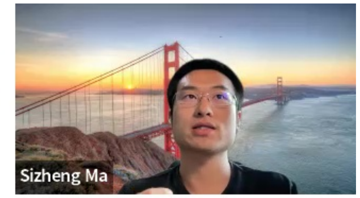
arXiv: 2207.10870

Takeaway messages



- The goal is to reveal subdominant QNMs by removing the leading one
- The filter is imposed on the entire waveform (avoid spectral leakage)

    1. Its impact on inspiral portion: a trivial negative time and phase shift (see 2207.10870 for details)
    2. Its impact on the corresponding QNM: flipping it (see 2207.10870 for details)
    3. Its impact on other QNMs: reducing their amplitudes (Ma et al. in prep)

- The corresponding QNM can be removed regardless of its amplitude and phase, as well as inclination and polarization angles!

arXiv: 2207.10870

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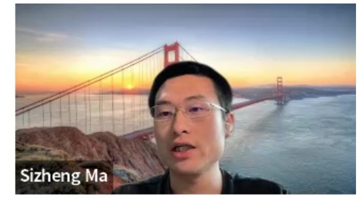
Outline



- Properties of the rational filter in terms of numerical-relativity waveforms (arXiv: *2207.10870*)

- Incorporating the rational filter into Bayesian inference (Ma, et al., in prep.)

- The full filter (briefly, arXiv: *2207.10870*, 2203.03174)

- Cauchy-Characteristic Matching (Ma, et al., in prep.)

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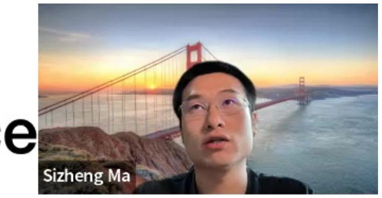
Incorporating the filter into Bayesian inference



Two additional properties:

- $\mathscr{F}_{lmn}(-f) = \left[\mathscr{F}_{lmn}(f)\right]^*$ → **The filtered signal is still real**

- $|\mathscr{F}_{lmn}(f)| = 1$, namely $\mathscr{F}_{lmn}(f) = e^{i\delta_{lmn}(f)}$ → **The filter has no impact on PSDs!**

Ma, et al., in preparation

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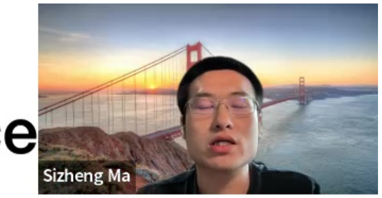
Incorporating the filter into Bayesian inference



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Ma, et al., in preparation

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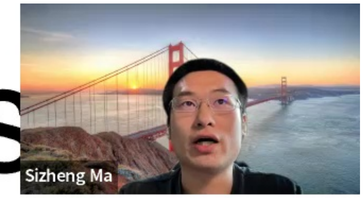
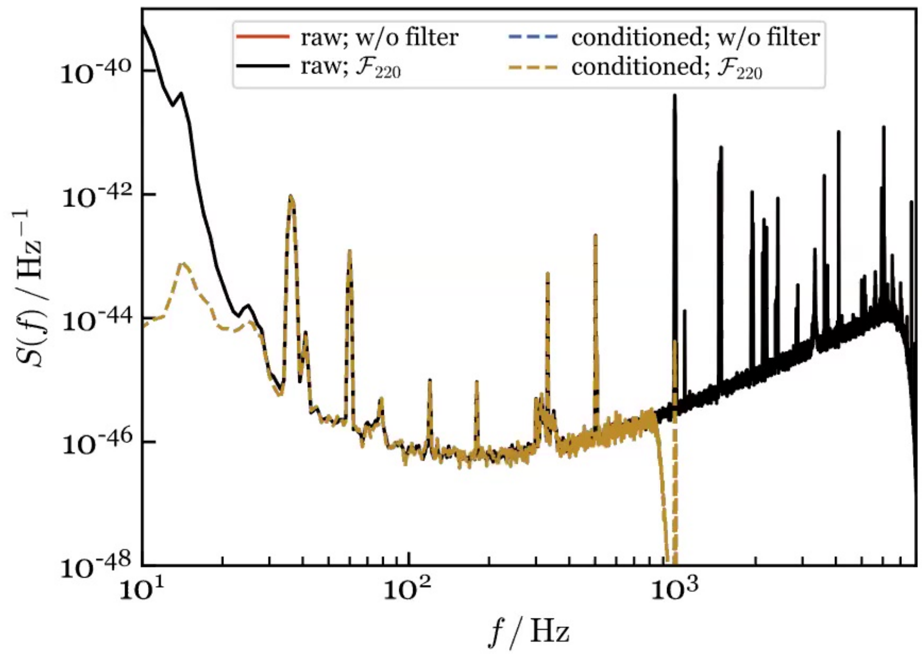
The filter has no impact on PS



$$\tilde{n}_f^F = e^{i\delta_{lmn}(f)}\tilde{n}_f$$

$$E[\tilde{n}_f^F \tilde{n}_{f'}^{F*}] = E[\tilde{n}_f \tilde{n}_{f'}^*] = \frac{1}{2}\delta(f - f')S(f)$$



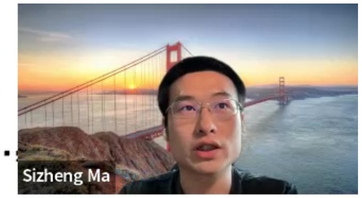
32 s data of GW150914 Hanford (Welch method)

Isi & Farr (2021), arXiv: 2107.05609 & https://github.com/maxisi/ringdown

Ma, et al., in preparation

A new likelihood function

Ma, et al.



Time Domain

Isi et al., arXiv: 2107.05609, 1905.00869

Traditional likelihood function

$$\ln P^{\mathrm{old}}(d_i \,|\, A_{lmn}, \phi_{lmn}, M_f, \chi_f, t_0, \mathrm{etc.}) = -\frac{1}{2}\sum_{ij}(d_i - h_i)C_{ij}^{-1}(d_j - h_j)$$

d_i : **GW data** $\in [t_0, t_0 + T]$ ← **0.2 s**

h_i : **ringdown template**

C_{ij}: **autocovariance function**

$$h_t = \sum_{lmn} A_{lmn} e^{-(t-t_0)/\tau_{lmn}} \cos\left[2\pi f_{lmn}(t - t_0) + \phi_{lmn}\right]$$

New likelihood function

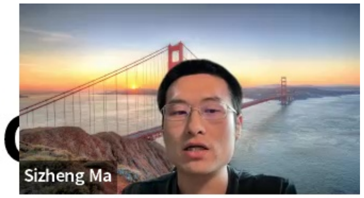
$$\ln P^{\mathrm{new}}(d_i \,|\, M_f, \chi_f, t_0) = -\frac{1}{2}\sum_{ij} d_i^F C_{ij}^{-1} d_j^F$$

d_i^F : **filtered GW data** $\in [t_0, t_0 + T]$

C_{ij}: **autocovariance function**

Need only remnant mass and spin

The new likelihood is merely a 2D func



The corresponding QNM can be removed regardless of its amplitude and phase, as well as inclination and polarization angles!

$$\ln P^{\text{old}}(d_i \,|\, A_{lmn}, \phi_{lmn}, M_f, \chi_f, t_0, \text{etc.}) = -\frac{1}{2}\sum_{ij}(d_i - h_i)C_{ij}^{-1}(d_j - h_j)$$

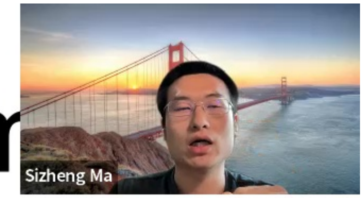
versus

$$\ln P^{\text{new}}(d_i \,|\, M_f, \chi_f, t_0) = -\frac{1}{2}\sum_{ij} d_i^F C_{ij}^{-1} d_j^F$$

- **There is no need to use fancy samplers (e.g. MCMC) to draw samplings**
- **We can directly plot the likelihood as a function of mass and spin (with t_0 being a hyperparameter)**

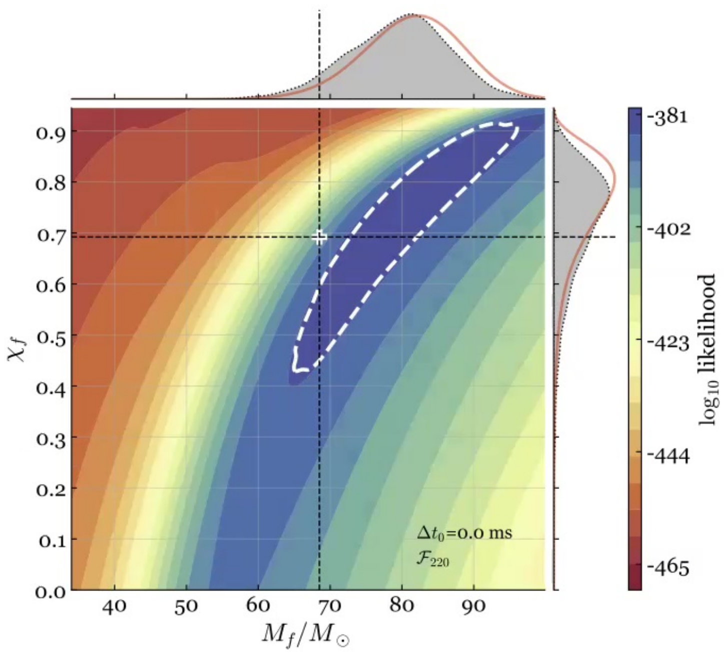
Ma, et al., in preparation

Taking GW150914 as an exam

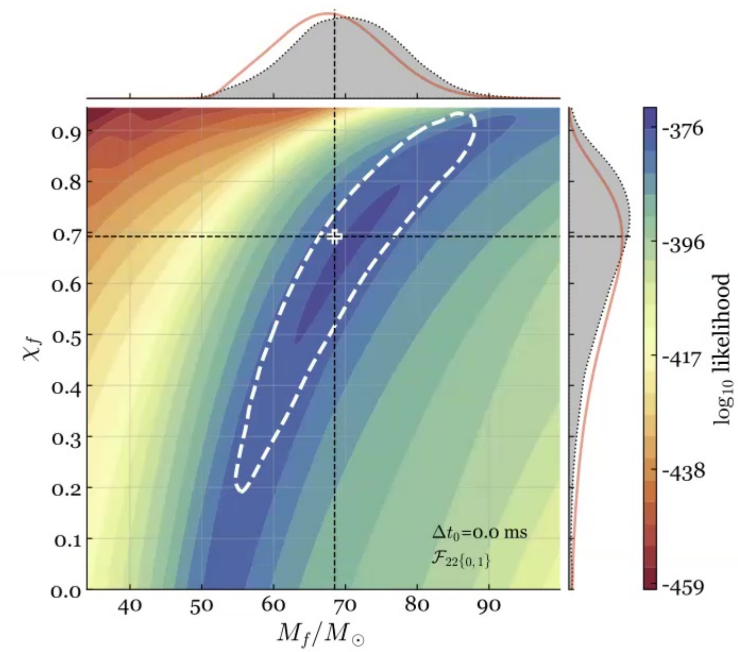


At $t_0 = t_{ref}$

Fundamental mode only



Fundamental mode + the first overtone



$t_{ref} = 1126259462.4083$ GPS at the geocenter
$T = 0.2$ s

- **The white dashed contour: the traditional MCMC method, with 90% credible region enclosed**
- **The plus sign: the IMR result**

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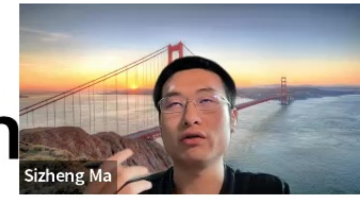
arXiv: 2107.05609, 1905.00869 & https://github.com/maxisi/ringdown

The performance of the new likelihood fun



- Fast and efficient: $5.5 \times 10^{-3}\,\mathrm{s}$ for a single evaluation; $7.8\,\mathrm{s}$ for a low-res 2D plot (personal laptop, not fully parallelized); 3 mins for a high-res 2D plot (24-core cluster, not fully parallelized).

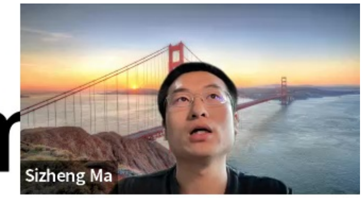
- Fully parallelizable: The evaluation of each pair of mass and spin is fully independent.

- The code doesn't slow down after increasing the number of modes because multiple filters can be imposed simultaneously.

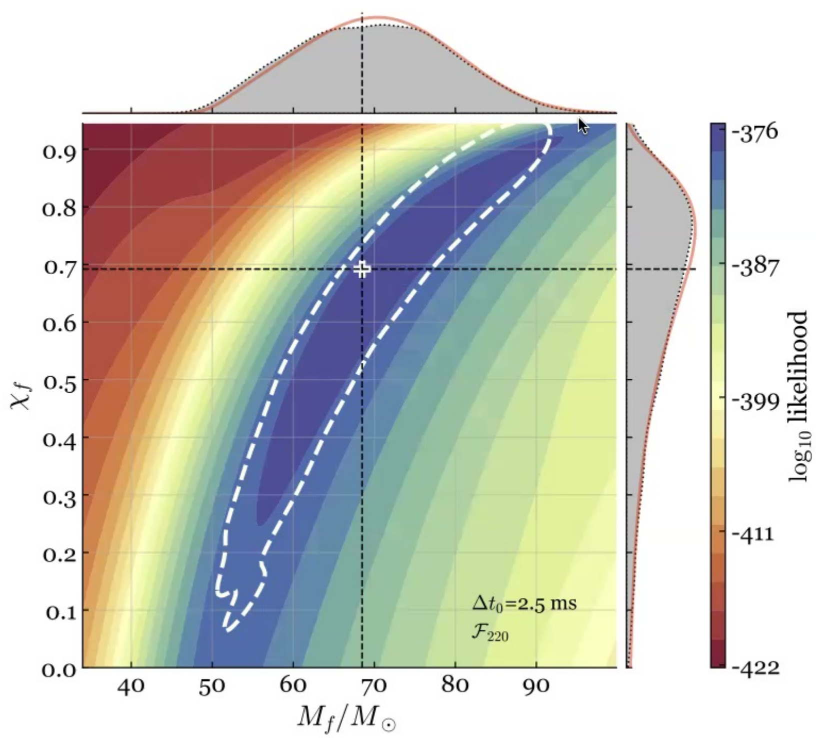
- The posteriors are consistent with the traditional MCMC analysis.

Ma, et al., in preparation

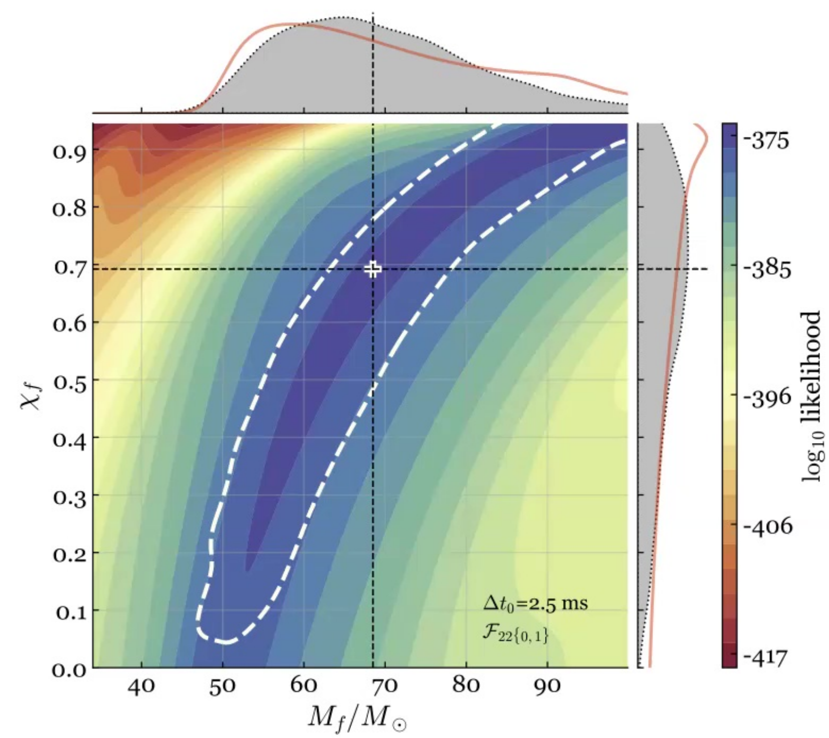
Taking GW150914 as an exam



At $t_0 = t_{\rm ref} + 2.5\,{\rm ms}$ **Fundamental mode only**



Fundamental mode + the first overtone



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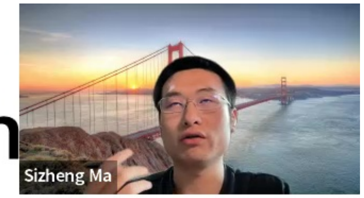
Ma, et al., in preparation

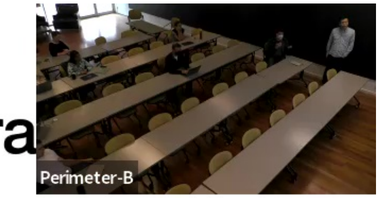
The performance of the new likelihood fun



- Fast and efficient: 5.5×10^{-3} s for a single evaluation; 7.8 s for a low-res 2D plot (personal laptop, not fully parallelized); 3 mins for a high-res 2D plot (24-core cluster, not fully parallelized).
- Fully parallelizable: The evaluation of each pair of mass and spin is fully independent.
- The code doesn't slow down after increasing the number of modes because multiple filters can be imposed simultaneously.
- The posteriors are consistent with the traditional MCMC analysis.

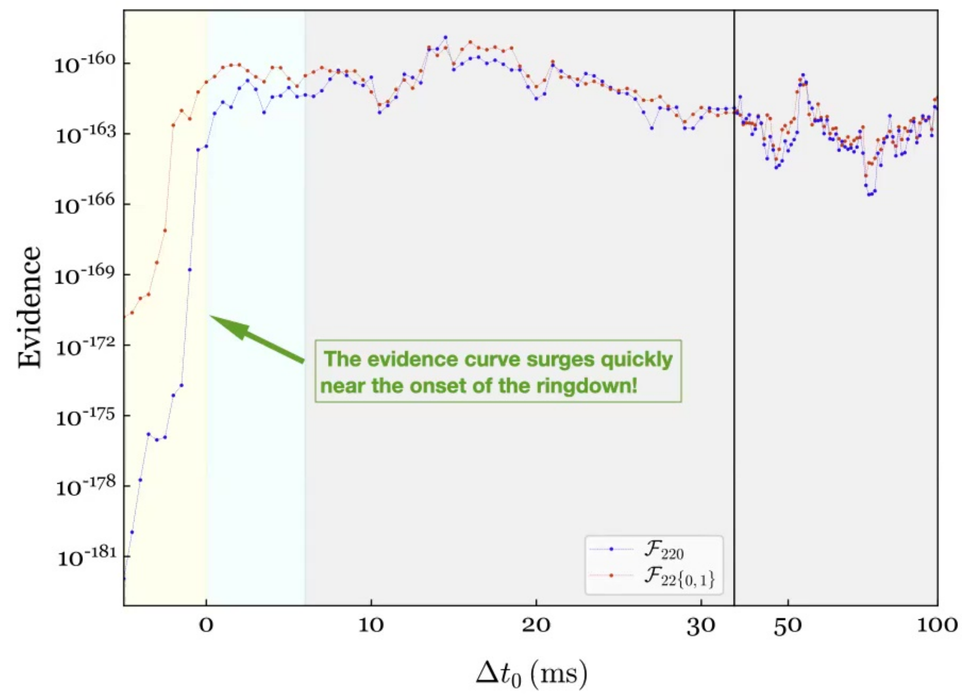
Ma, et al., in preparation

Determining the start time of ringdown (hyperpara



- The model evidence $P_{lmn}(t_0) = \iint dM_f d\chi_f\, p(d \,|\, M_f, \chi_f, t_0) \times \text{prior}$

Red: Fundamental mode + the first overtone
Blue: Fundamental mode



GW event trigger?

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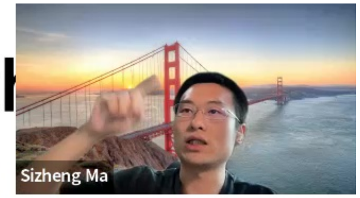
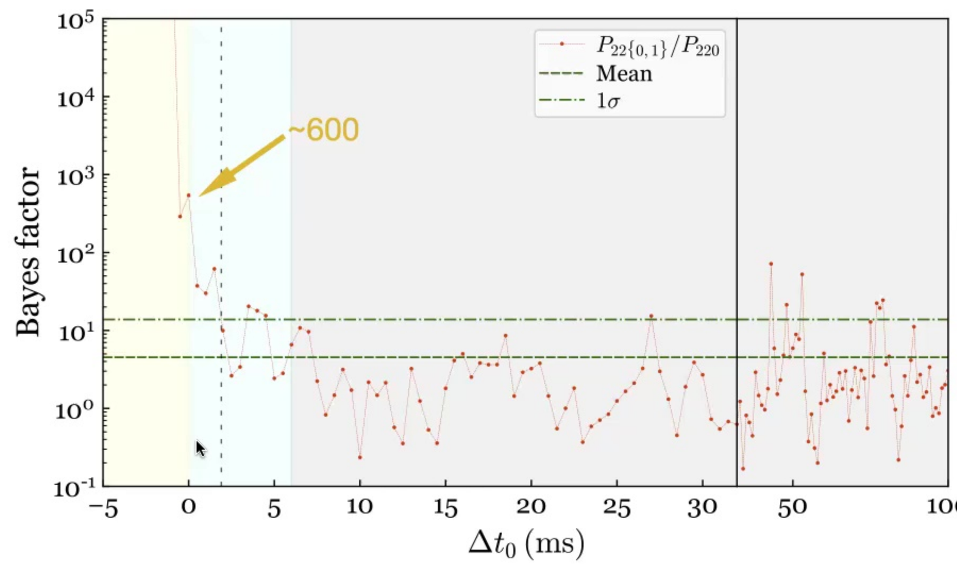
Ma, et al., in preparation

The Bayes factor of the first overtone in th… ringdown of GW150914



- The model evidence $P_{lmn}(t_0) = \int\int dM_f d\chi_f p(d \mid M_f, \chi_f, t_0) \times \text{prior}$

- The Bayes factor: $K_{221}(t_0) = \dfrac{P_{22\{0+1\}}}{P_{220}}$



Ma, et al., in preparation

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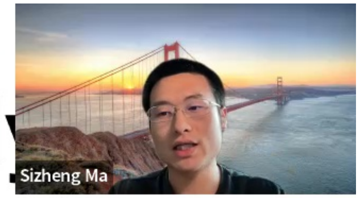
Three approaches to ringdown anal

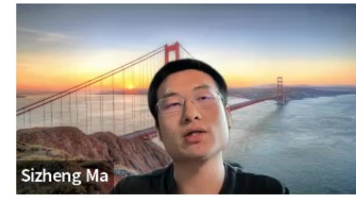


1. The traditional MCMC method (building a template)

2. The filter method (removing all ringdown signals)

3. <u>*A mixed approach!*</u>

The mixed approach



1. Remove a subset of QNM(s) via the filter(s)

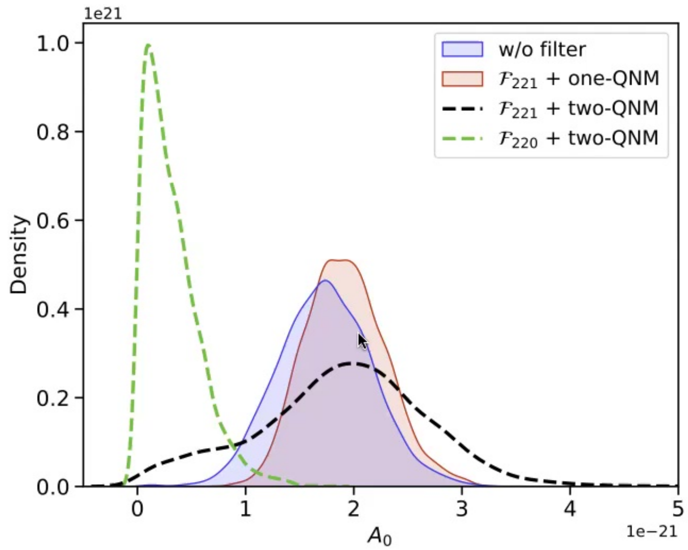
2. Fit the filtered data to the remaining QNM(s) model via MCMC

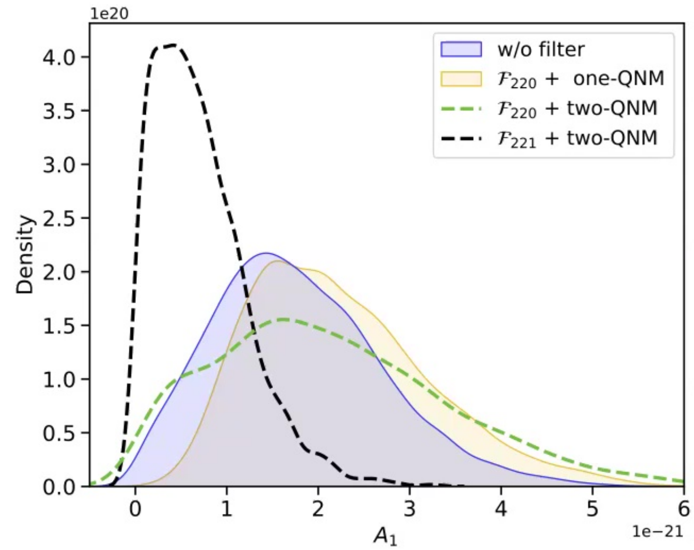
Obtain information of the remaining modes w/o the impact of other modes (cross-check)

Infer remnant mass and spin from each single QNM (test the no-hair theorem)

Taking GW150914 as an exam

Obtain information of the remaining modes
w/o the impact of other modes (cross-check)





The amplitude of the fundamental mode

The amplitude of the first overtone

Ma, et al., in preparation

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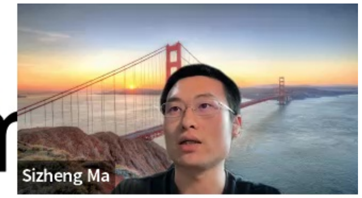
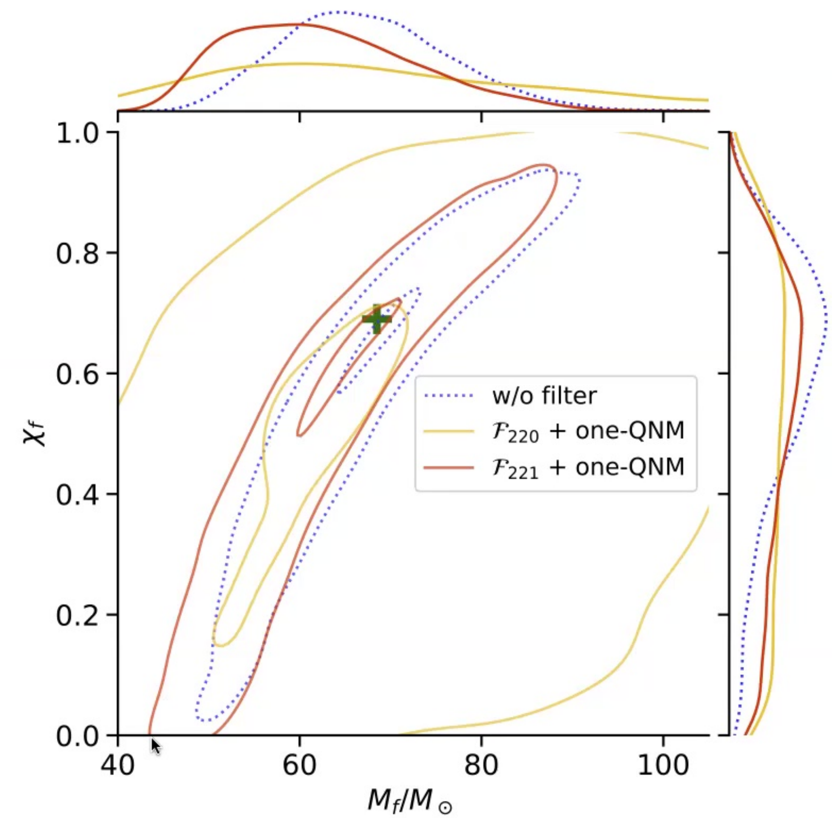
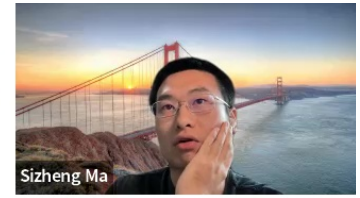
Taking GW150914 as an exam



Infer remnant mass and spin from each single QNM
(test the no-hair theorem)



Ma, et al., in preparation

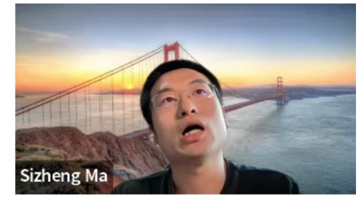


Outline

- Properties of the rational filter in terms of numerical-relativity waveforms (arXiv: *2207.10870*)
- Incorporating the rational filter into Bayesian inference (Ma, et al., in prep.)
- The full filter (briefly, arXiv: *2207.10870*, 2203.03174)

QNM filter = rational filter + full filter

- Cauchy-Characteristic Matching (Ma, et al., in prep.)



The full filter

$$R^{\text{up}}_{lm} \sim \begin{cases} r^3 e^{i\omega r_*}, & r_* \to +\infty, \\ D^{\text{out}}_{lm} e^{i\omega r_*} + \Delta^2 D^{\text{in}}_{lm} e^{-i\omega r_*}, & r_* \to -\infty, \end{cases}$$

↓

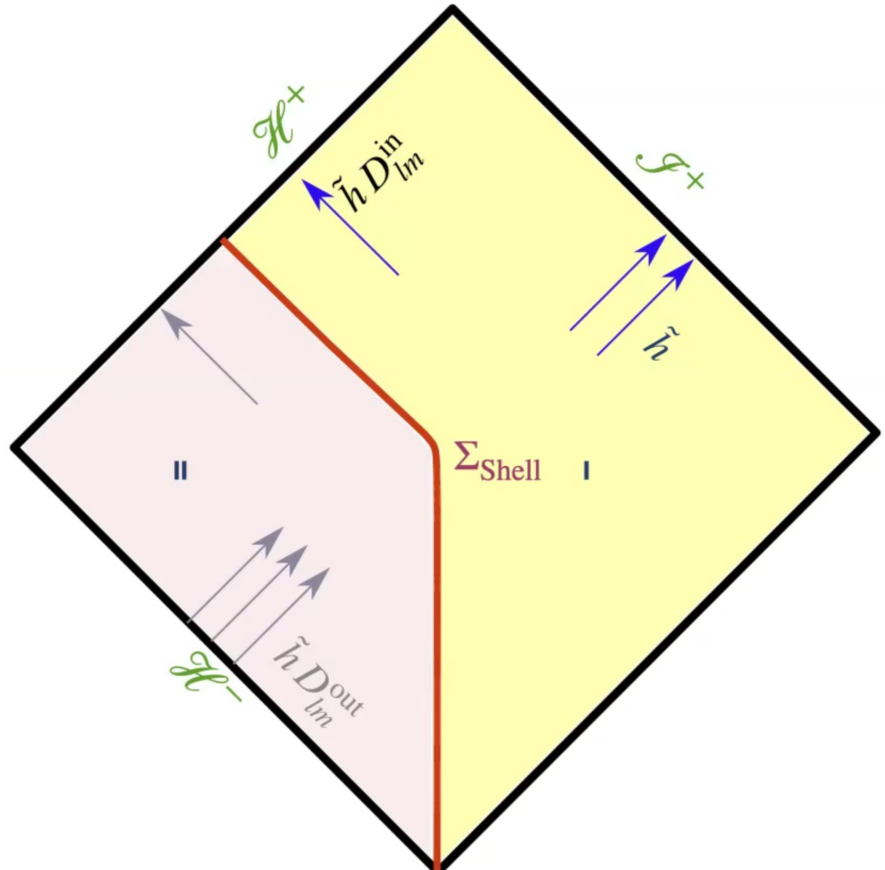
$$D^{\text{out}}_{lm}(\omega_{lmn}) = 0$$

↓

$$D^{\text{out}}_{lm} \sim \prod_n (\omega - \omega_{lmn})$$

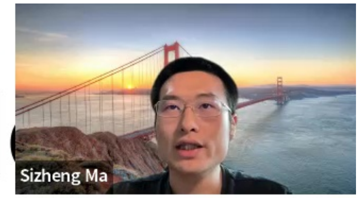
↓

$$\mathcal{F}^D_{lm} = \frac{D^{\text{out}}_{lm}}{D^{\text{out}*}_{lm}}$$



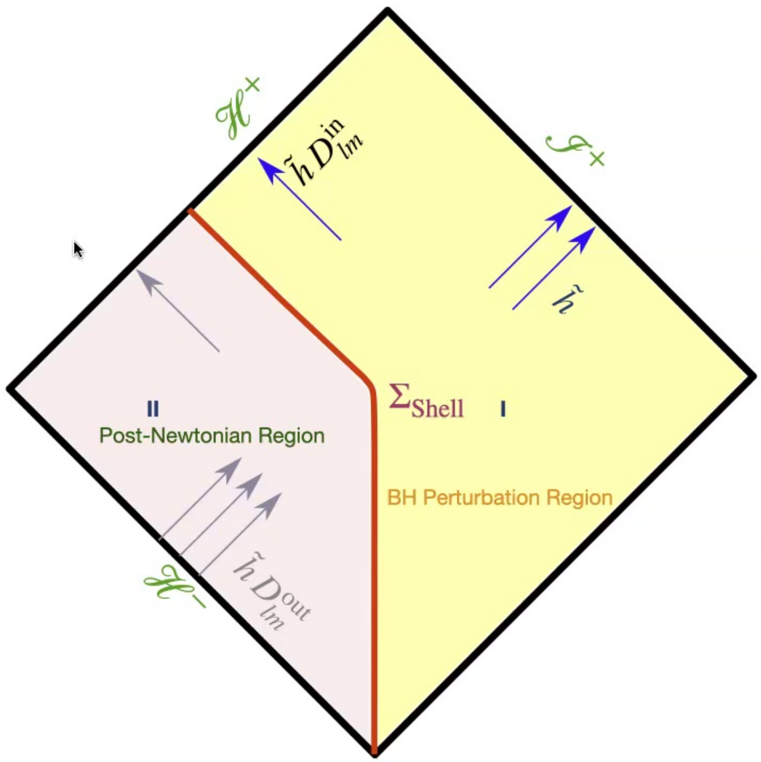
arXiv: 2207.10870, 2203.03174

The hybrid waveform mode



Nichols and Chen (2010) & (2012)

arXiv: 1007.2024, 1109.0081



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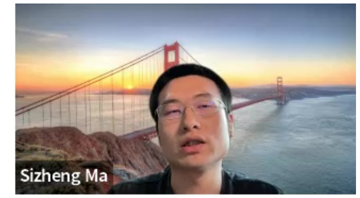
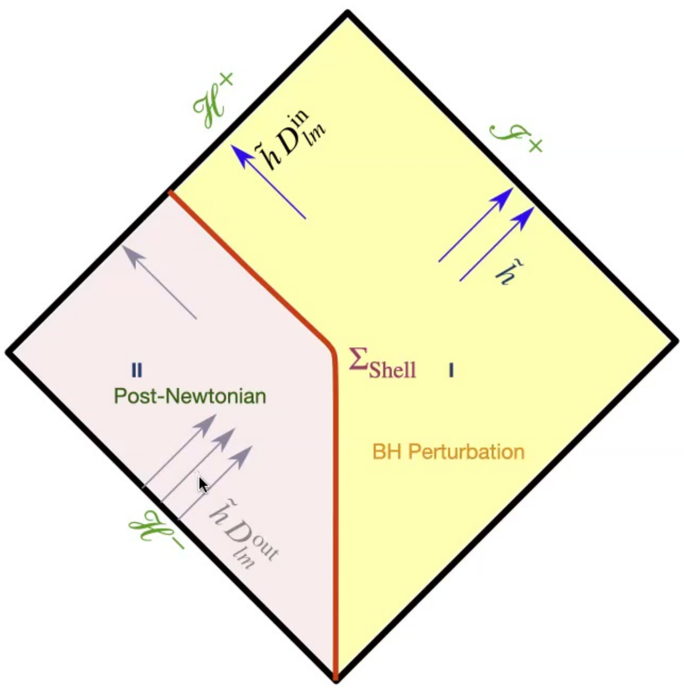
Ma et al., arXiv: 2203.03174



The image wave

$$R^{\rm up}_{lm} \sim \begin{cases} r^3 e^{i\omega r_*}, & r_* \to +\infty, \\ D^{\rm out}_{lm} e^{i\omega r_*} + \Delta^2 D^{\rm in}_{lm} e^{-i\omega r_*}, & r_* \to -\infty, \end{cases}$$

$$\mathscr{F}^D_{lm} = \frac{D^{\rm out}_{lm}}{D^{\rm out*}_{lm}}$$



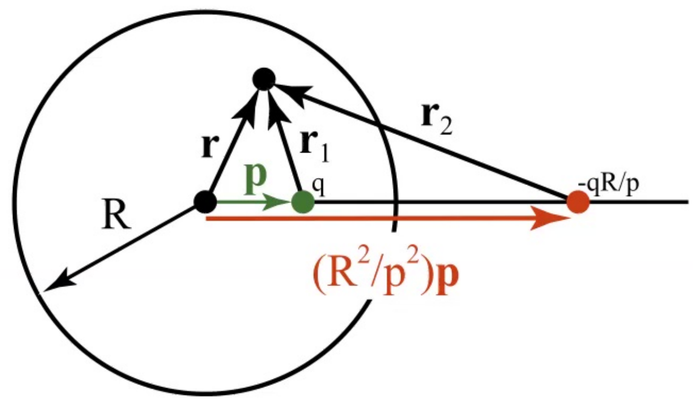
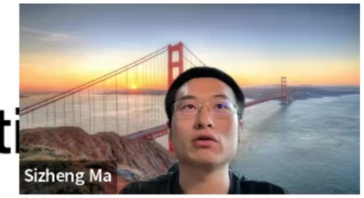
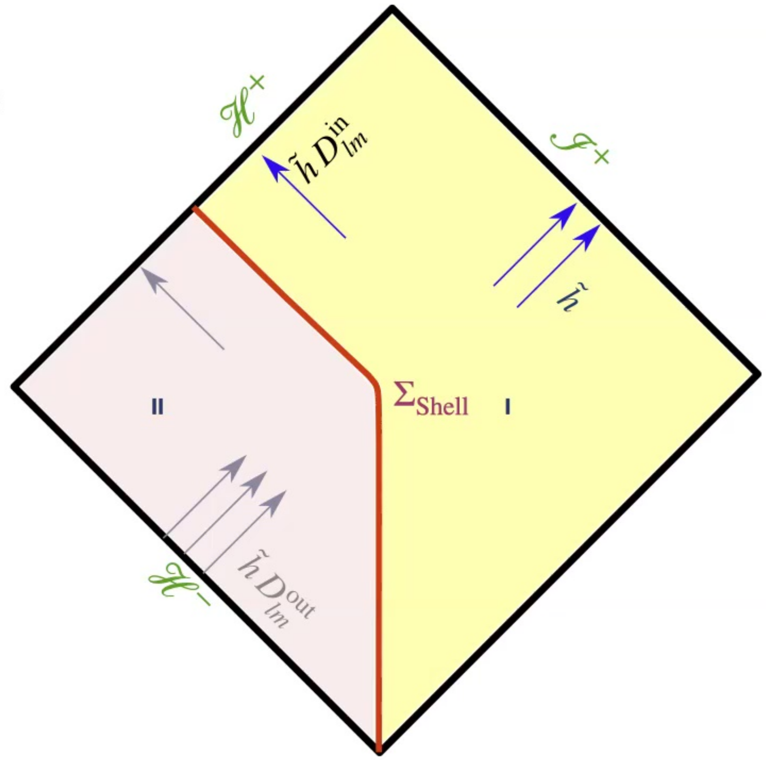


Image Credit: Wikipedia

Metric reconstruction near BBHs and its applicati



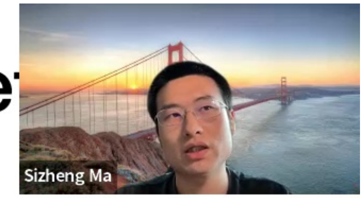
- **Computing gravitational-wave echoes.** Ma et al., arXiv: 2203.03174
- **Second-order BH perturbations?**

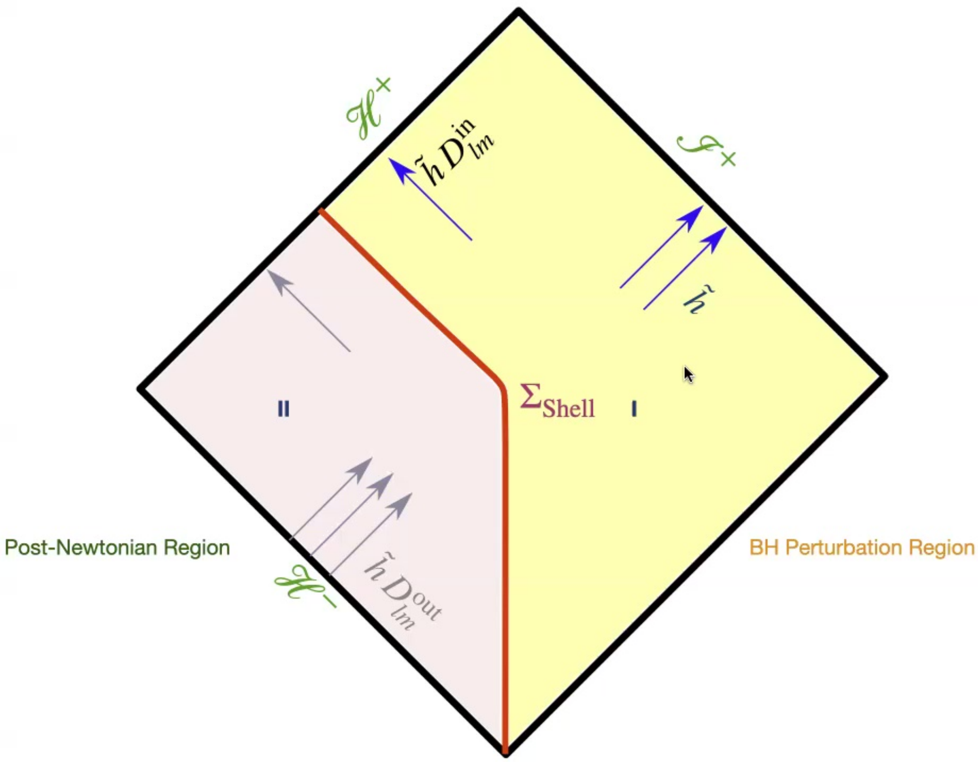


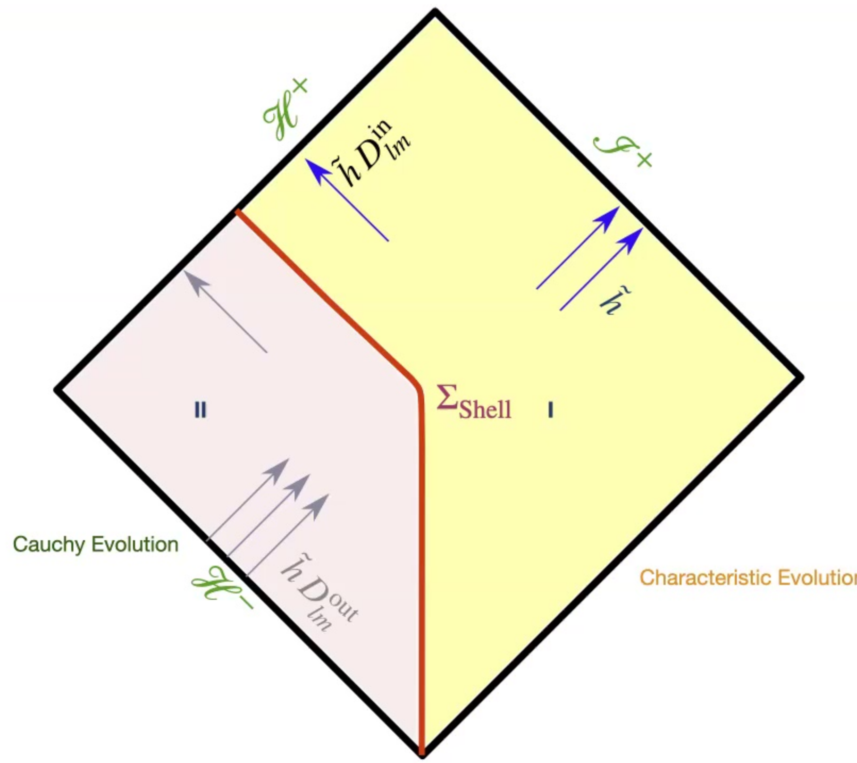
27

Numerical-relativity version of the hybrid me

Cauchy-Characteristic Matching

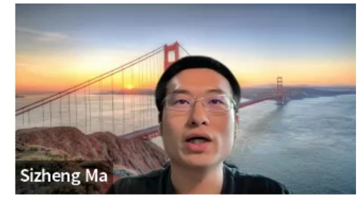






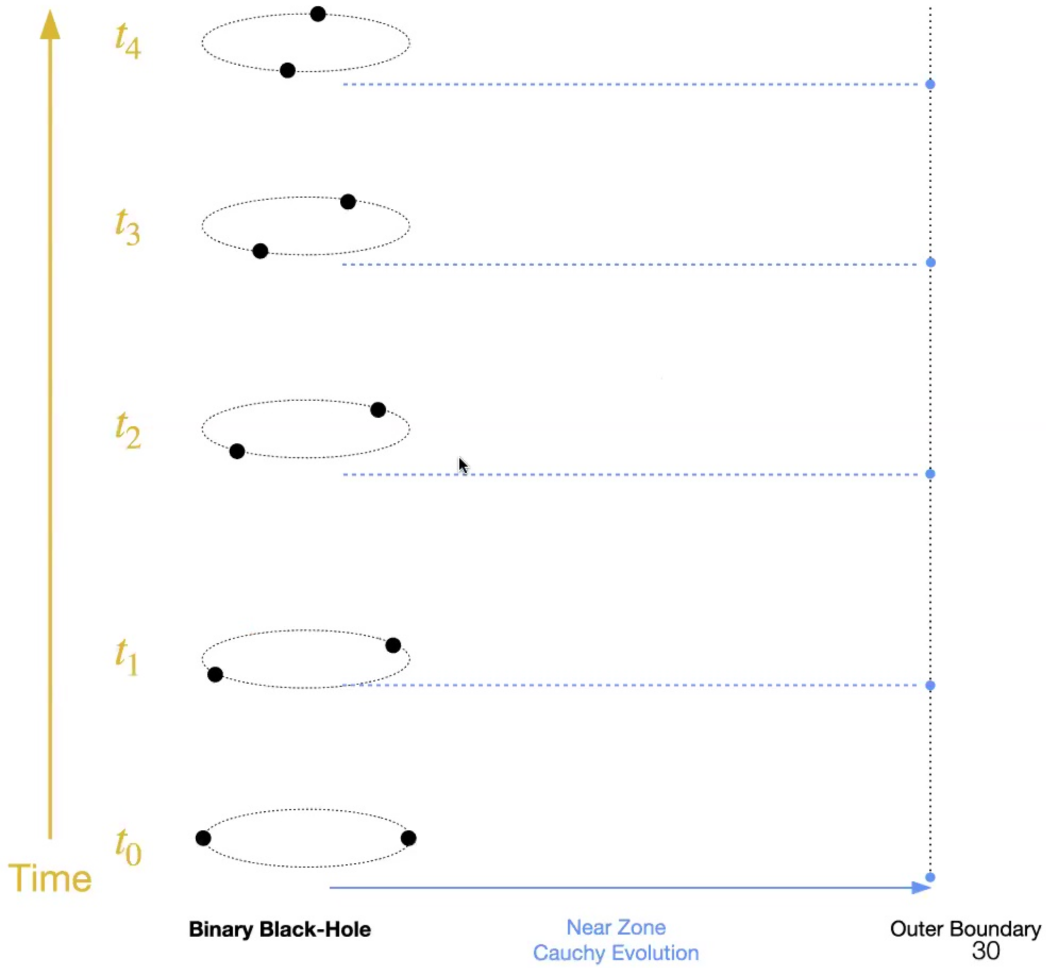
Fully relativistic, 3D CCM in SpECTRE, Ma et al., in preparation

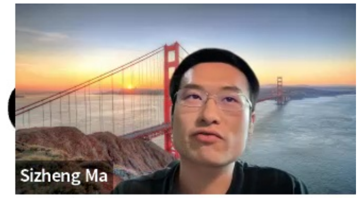
Outline



- Properties of the rational filter in terms of numerical-relativity waveforms (arXiv: *2207.10870*)
- Incorporating the rational filter into Bayesian inference (Ma, et al., in prep.)
- The full filter (briefly, arXiv: *2207.10870*, 2203.03174)
- Cauchy-Characteristic Matching (briefly, Ma, et al., in prep)

Cauchy-Characteristic Extracti





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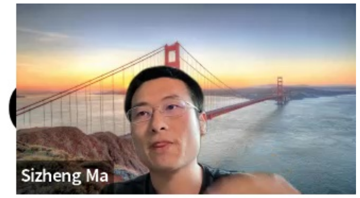
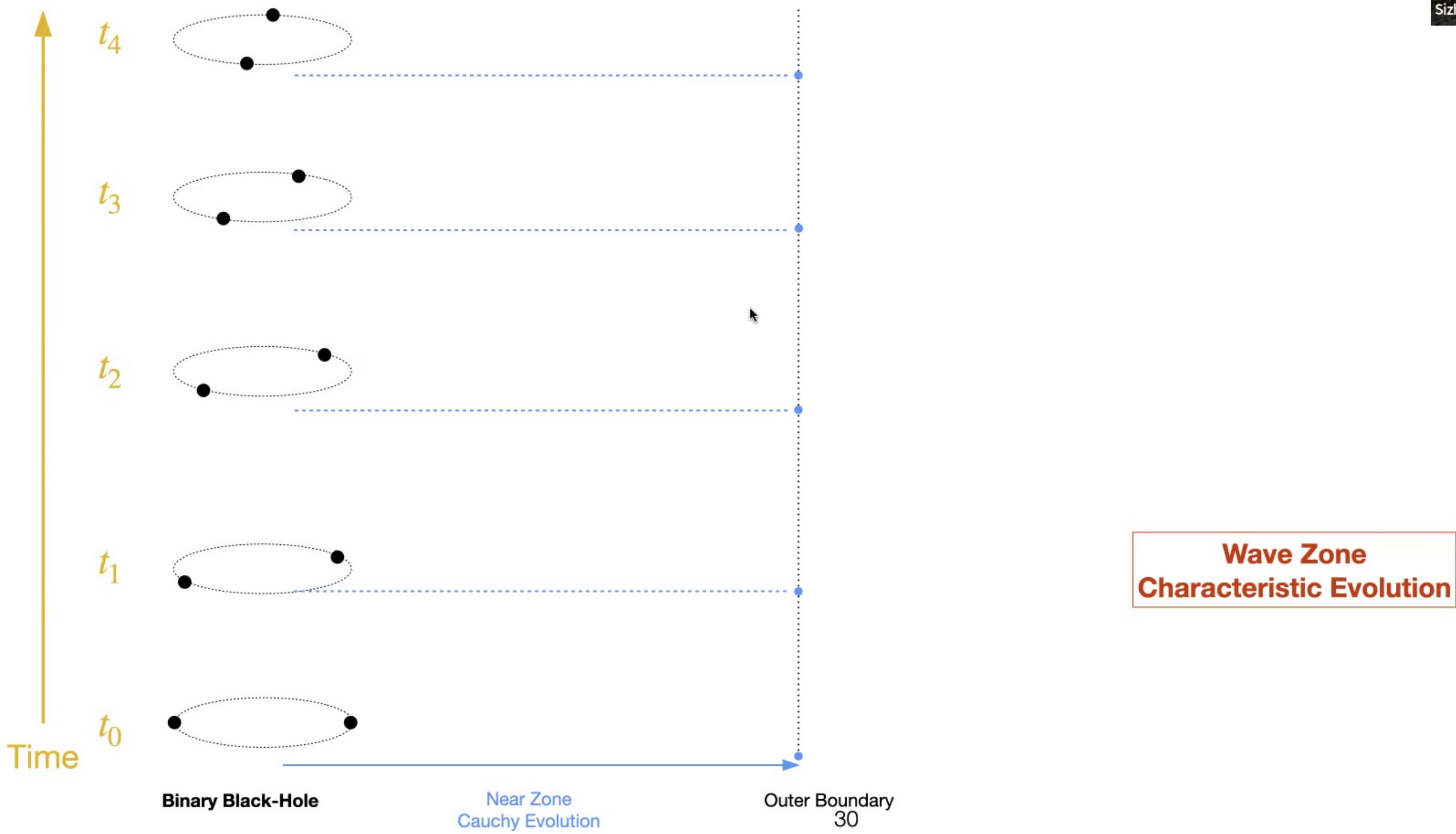
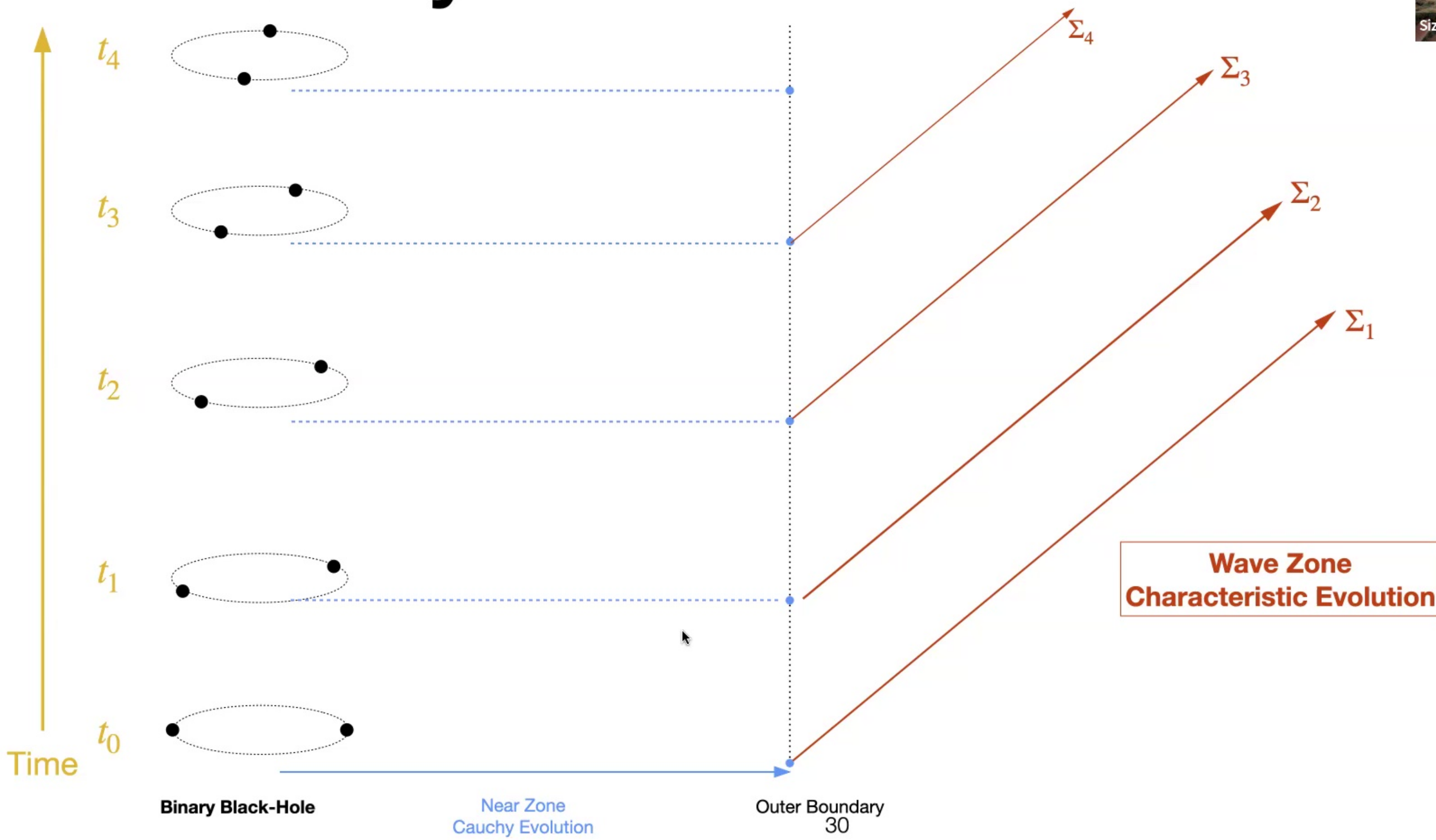
Cauchy-Characteristic Extracti





Cauchy-Characteristic Extracti

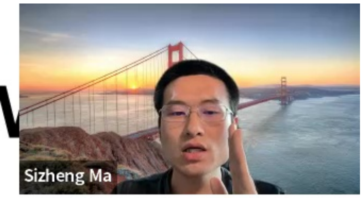


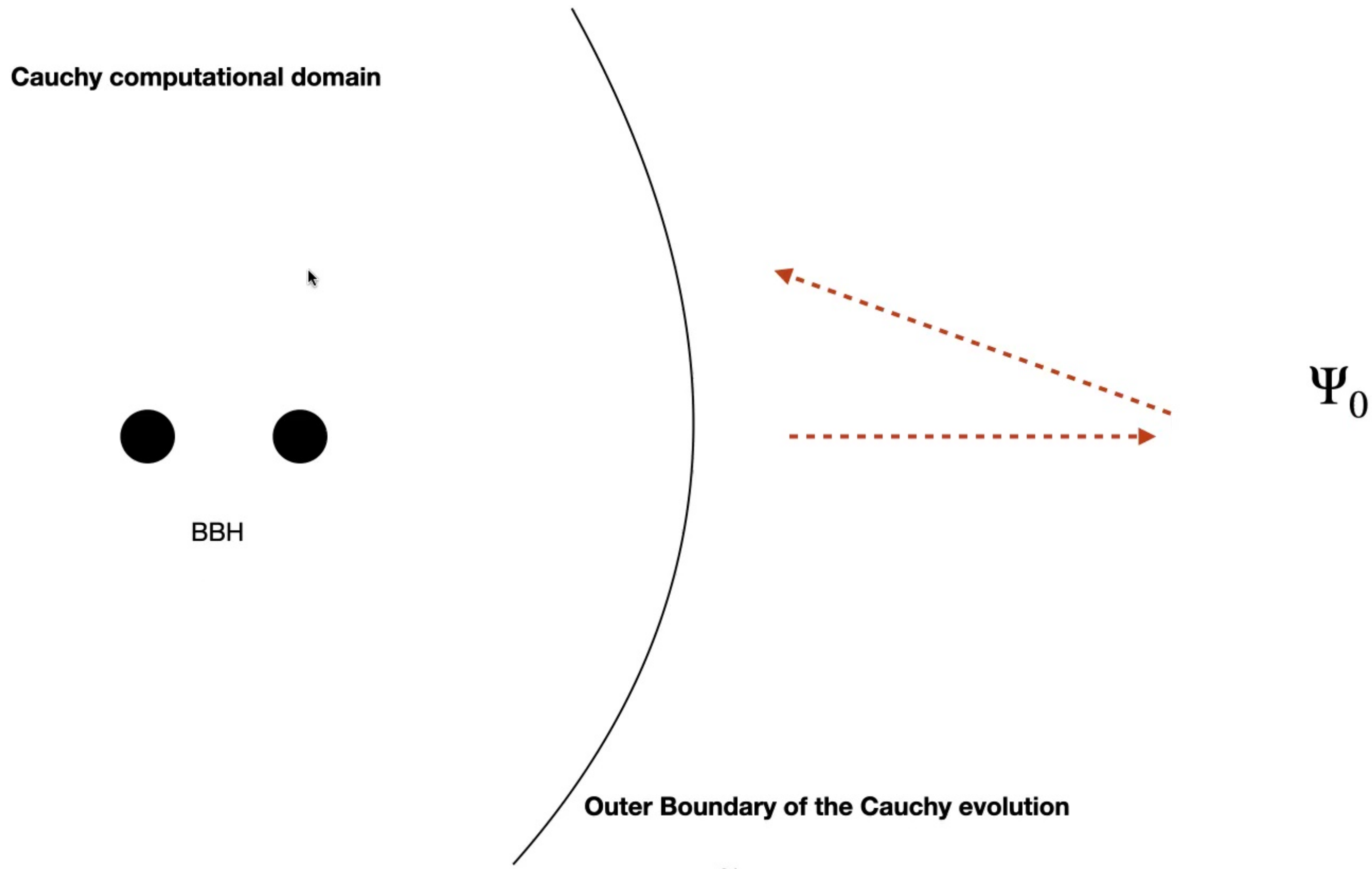
30

Sizheng Ma

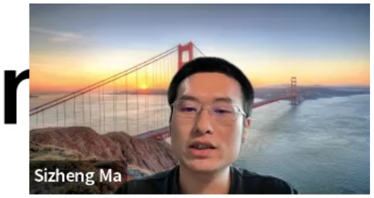
LIGO

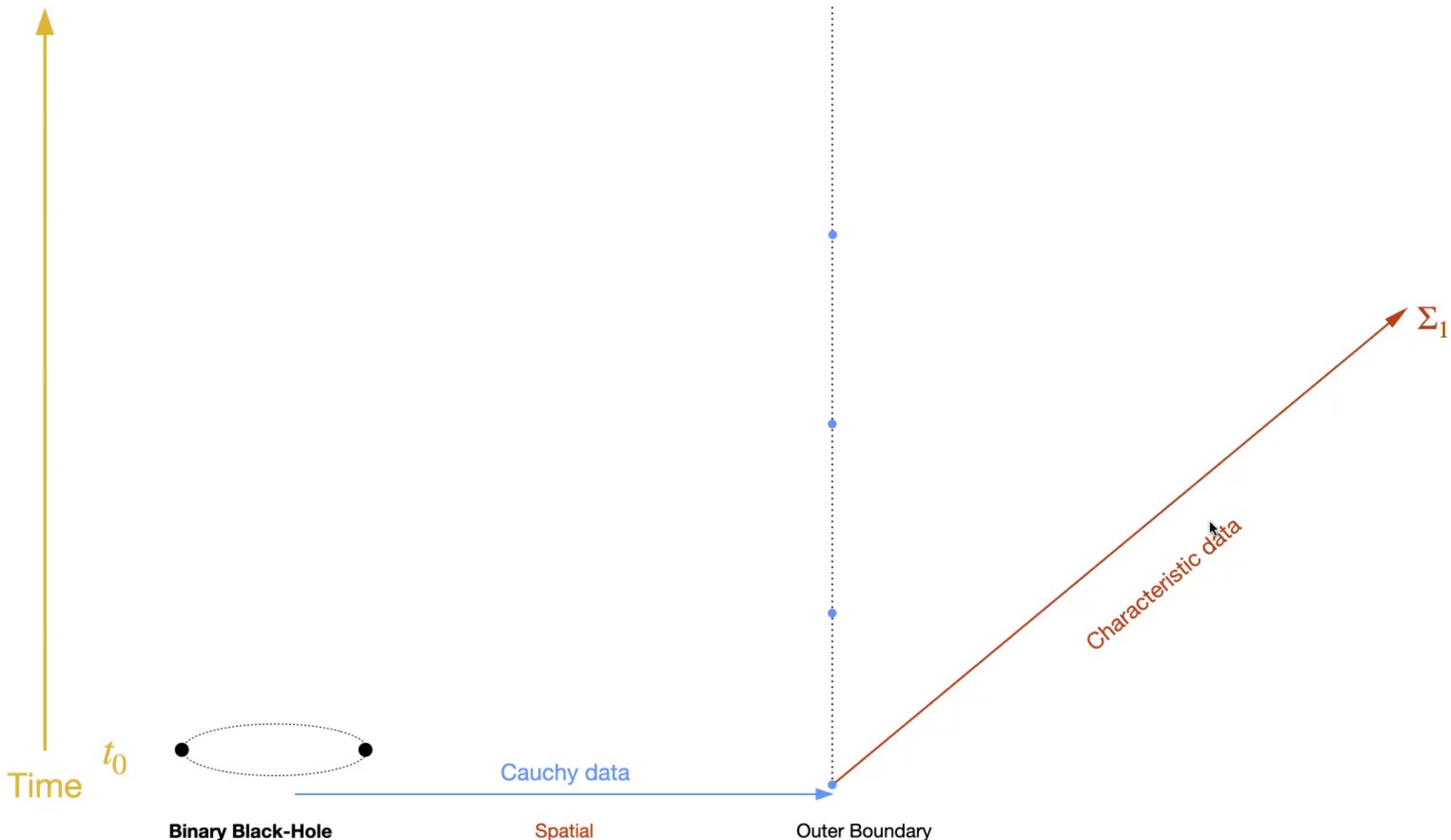
The backscattered gravitational w





Cauchy-Characteristic Matchin





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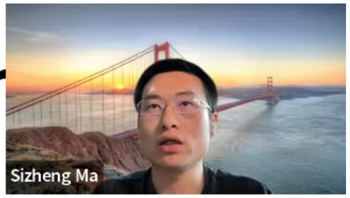
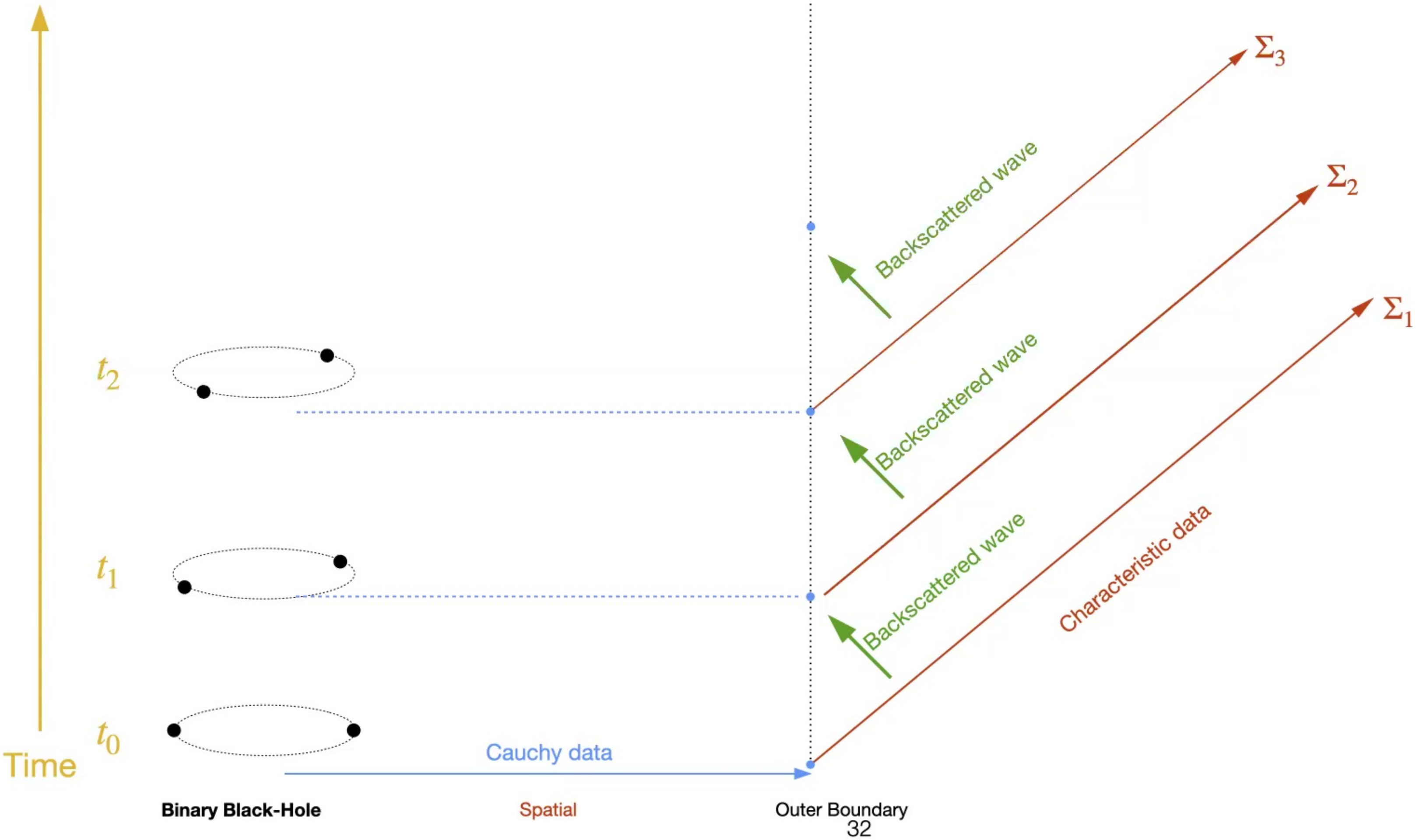
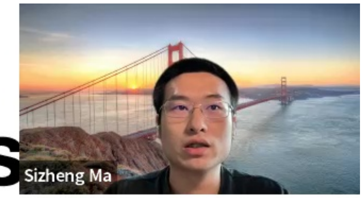
Cauchy-Characteristic Matchi





The Bjorhus boundary conditions



$$\partial_t u^\alpha + A^{k\alpha}_\beta \partial_k u^\beta = F^\alpha$$

Generalized Harmonic
(SpECTRE)

$$d_t u^{1-}_{ab} =$$

Incoming characteristic field
on boundary

- 4 DOF: Contstraint-preserving
- 4 DOF: Gauge
- 2 DOF: incoming GWs

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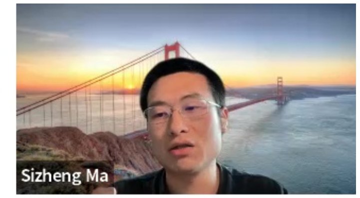
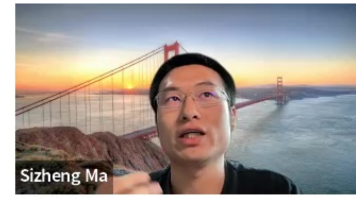
The performance of the fully relativistic 3D



- Spurious numerical reflections at the outer boundary are reduced
- The interface between the Cauchy and characteristic system is transparent to physical GWs

Ma et al., in preparation



Summary

1. The rational filter and the corresponding Bayesian framework
    - This novel framework is a powerful tool to do BH spectroscopy (theoretically and observationally)
    - A LIGO O4 pipeline?
2. The full filter; metric reconstruction; echo
3. Cauchy-characteristic matching

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