Title: Bootstrapping string dynamics in the 6d N = (2, 0) theories

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Abstract: The 6d N = (2,0) theories are superconformal field theories believed to describe the low-energy dynamics of N coincident M5-branes. These theories don't have a known lagrangian description and remain largely mysterious, so it is an interesting question how one might calculate observables there. An exciting prospect is to use the analytical conformal bootstrap, which offers a way to systematically calculate 1/N corrections at large N. In this talk I will present the bootstrap approach to a case study, that of calculating the 2-point function of stress tensors in the presence of a surface defect. This setup turns out to be remarkably simple and helps us address some technical issues faced in similar calculations, notably we can derive a supersymmetric inversion formula and check crossing symmetry explicitly. I will also comment on the interpretation of our result in the context of holography, of the chiral algebra construction of Beem et al. and on what it can reveal about the interactions between M2 and M5-branes.

Zoom link: https://pitp.zoom.us/j/92631930165?pwd=Qm9MQzlNdHo0WGJINUZBUVNaOXZxZz09

Invitation

The 6d $\mathcal{N} = (2,0)$ theories are interacting SCFTs with

- no known lagrangian description
- no tunable coupling constant
- maximal supersymmetry in 6d

$\Rightarrow$ Do they really exist? string/M-theory constructions

$\Rightarrow$ How do we define them?

$\Rightarrow$ How do we calculate observables?

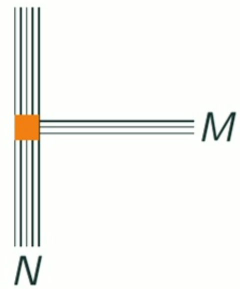
There are good reasons to study them.

From the point of view of QFT, they

- challenge the usual framework of perturbative QFT
- maximal dimension for any SCFT
- play a role in understanding dualities in lower dimensional theories

In string/M-theory,

- describe the low-energy dynamics of N M5-branes
- also interactions with M M2-branes



The holographic approach

In the past the 6d theories have been studied via holography.

At large N they are described by 11d supergravity on a $AdS_7 \times S^4$ background, with the radius of AdS (in Planck units)

$$\left(\frac{R_{AdS}}{l_P}\right)^3 = 8\pi N\,.$$

At subleading orders in N, calculating observables require M-theory corrections to 11d supergravity.

This is an open problem.

The bootstrap approach

The modern approach is to use bootstrap techniques at large N to calculate $1/N$ corrections.

In this talk, I discuss the 2-point function of stress tensor superprimaries $\mathcal{O}_2$ in the presence of a surface operator V

$$\langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)V\rangle \ .$$

Natural choice because

- Simple yet nontrivial setup
- Captures interesting physics
- The result overlaps with supergravity and chiral algebras

Plan of the talk

Review of anomaly coefficients

The bootstrap result is naturally expressed in terms c, d:

- c appears in $\langle TT \rangle$ and $\langle TTT \rangle$
- it depends only on N (in general, the choice of ADE group)
- d appears in $\langle TV \rangle$
- it depends on M, N (in general, the choice of rep. of ADE group)

They also appear in conformal anomalies and are known exactly

$$c = 4N^3 - 3N - 1\,, \qquad d = \frac{M(N-1)(M+2N)}{2N}\,.$$

At large N, $c \sim N^3$ and $d \sim MN$.

Kinematics

$\mathcal{O}_2$ is the superprimary of the stress tensor multiplet. It transforms in **14** of $\mathfrak{so}(5)$ R-symmetry. I use the notation

$$\mathcal{O}_2(x,u) \equiv \mathcal{O}^{I_1 I_2}(x) u_{I_1} u_{I_2}\,, \qquad u^2 = 0\,, \qquad I = 1,\ldots,5\,.$$

Surface operators V break

- conformal symmetry: $\mathfrak{so}(2,6) \to \mathfrak{so}(2,2)_{||} \oplus \mathfrak{so}(4)_{\perp}$
- R-symmetry: $\mathfrak{so}(5) \to \mathfrak{so}(4)$

Therefore $\mathcal{O}_2$ can acquire a vev

$$\langle \mathcal{O}_2(x,u) V \rangle = -\frac{d}{c^{1/2}} \frac{|u_\perp|^2}{|x_\perp|^4}\,.$$

The 2-point function takes the form

$$\langle \mathcal{O}_2(x_1, u_1)\mathcal{O}_2(x_2, u_2) V\rangle = \frac{|u_1^\perp|^2 |u_2^\perp|^2}{|x_1^\perp|^4 |x_2^\perp|^4} \sum_{j=0}^{2} F_j(z, \bar{z})\sigma^j$$

where

- $z, \bar{z}$ are cross-ratios
- $\sigma = \frac{u_1 \cdot u_2}{|u_1^\perp||u_2^\perp|}$ enumerates R-symmetry invariants

Our goal is to calculate

$$\mathcal{F}(z, \bar{z}, \omega) = \sum_{j=0}^{2} F_j(z, \bar{z})\sigma(\omega)^j \,, \qquad \sigma = \frac{(1-\omega)^2}{\omega} \,.$$

The cross-ratios admit a geometric interpretation

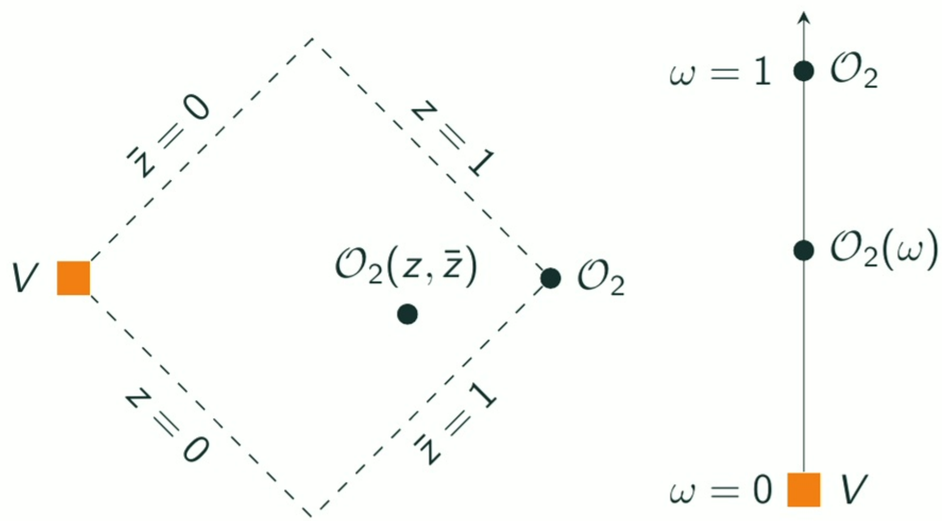


Figure 1: Cross-ratios in Lorentzian kinematics

Superconformal Ward identities

Supersymmetry imposes:

$$(\partial_z + \partial_\omega)\mathcal{F}(z,\bar{z},\omega)|_{z=\omega} = 0\,, \qquad (\partial_{\bar{z}} + \partial_\omega)\mathcal{F}(z,\bar{z},\omega)|_{\bar{z}=\omega} = 0\,.$$

$\mathcal{F}$ can be expressed in terms of F_0 and ζ as

$$\begin{aligned}\mathcal{F}(z,\bar{z},\omega) = & \frac{|z-\omega|^2|z-\omega^{-1}|^2}{|z-1|^4}F(z,\bar{z}) \\ & + \frac{\bar{z}(z-\omega)(z-\omega^{-1})(\omega-1)^2}{\omega(z-\bar{z})(z-\bar{z}^{-1})(\bar{z}-1)^2}\zeta(z) + \text{c.c.}\end{aligned}$$

$\zeta(z)$ appears because a sector of the theory is governed by a 2d chiral algebra [Beem et al., 2015].

Large N expansion

Finally, at large N $\mathcal{F}$ admits an expansion of the form

$$\mathcal{F}(z,\bar{z},\omega) = \frac{(z\bar{z})^2(1-\omega)^4}{(1-z)^4(1-\bar{z})^4\omega^2} + \frac{d^2}{c} - \frac{d}{c}\mathcal{F}^{(1)}(z,\bar{z},\omega) + \dots$$

which can be understood in terms of Witten diagrams

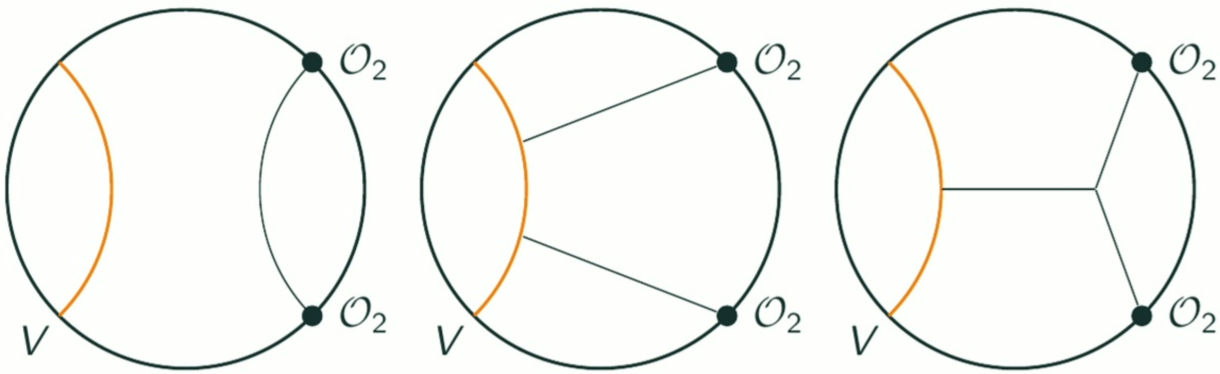


Figure 2: Witten diagrams at large N

The result

We will obtain

$$\zeta^{(1)}(z) = \frac{z}{(1-z)^2}\,,$$

$$F^{(1)}(z,\bar z) = \frac{z\bar z\left(1+z\bar z+(z\bar z)^2\right)}{(1-z\bar z)^6}\left[2\left(1+18z\bar z+(z\bar z)^2\right)-(z+\bar z)\left(1+z\bar z\right)\right]$$
$$-6\,\frac{(z\bar z)^2\log z\bar z}{(1-z\bar z)^7}\left[(1+z\bar z)\left(3+4z\bar z+3(z\bar z)^2\right)\right.$$
$$\left.+2\left(z+\bar z\right)\left(1+3z\bar z+(z\bar z)^2\right)\right]\,.$$

Superblock decomposition

We impose associativity of the OPE

$$\mathcal{F}(z,\bar{z},\omega) = \sum_l B_{2l}^2 \hat{\mathcal{G}}_l(z,\bar{z},\omega)$$
$$= \frac{(z\bar{z})^2(1-\omega)^4}{(1-z)^4(1-\bar{z})^4\omega^2}\left[1 + \sum_k C_{22k} A_k \mathcal{G}_k(z,\bar{z},\omega)\right].$$

$\mathcal{G}, \hat{\mathcal{G}}$ are resp. bulk and defect channel superconformal blocks.

B appears in

$$\mathcal{O}_k V \sim \sum_l B_{kl}\,(\ldots \mathrm{kinematics} \ldots)\, V[\hat{\mathcal{O}}_l]\,.$$

Solving these equations is difficult.

At large N $\mathcal{F}^{(1)}$ can be reconstructed from a few blocks: [Barrat et al., 2021]

1. A correlator is fixed in term of its discontinuity
2. The discontinuity can be calculated from the bulk channel OPE. It receives contributions from blocks with
   - Anomalous dimensions
   - Low enough twist
3. At large N, blocks with anomalous dimensions are suppressed by c^{-1}, so $\mathrm{Disc}\,\mathcal{F}^{(1)}$ only receives contributions from blocks with low twists.

There are 3 problems:

1. R-symmetry?
2. not manifestly supersymmetric
3. misses some low spin contributions to the correlator

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A supersymmetric inversion formula

We can solve these issues by deriving a supersymmetric inversion formula. Start with the usual inversion formula [Lemos et al., 2018]

$$b(\hat{\Delta}, s) \sim \int \mathrm{d}z\mathrm{d}\bar{z}\mu(z, \bar{z})g_{\hat{\Delta},s}(z, \bar{z})\,\mathrm{Disc}\,H(z, \bar{z}).$$

R-symmetry

Defect operators $\hat{\mathcal{O}}$ transform under $\mathfrak{so}(4)$ R-symmetry with spin r, and the R-symmetry block is

$$U_r\left(\frac{\omega+\omega^{-1}}{2}\right).$$

Using orthogonality of Chebyshev polynomials, we can decompose $\mathcal{F}$ in R-symmetry channels.

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Supersymmetry

The superblocks that can appear in the OPE are

Multiplet	$\hat{\Delta}$
$L[0]_{\hat{\Delta},s}$	$\hat{\Delta} > 2 + s$
$A[r]_s$	$\hat{\Delta} = 2 + s + 2r$
$B[r]$	$\hat{\Delta} = 2r$

Table 1: Defect supermultiplets and their dimensions

Each block with $r = 2$ appears in a single superblock, and B is proportional to its coefficient b

$$\hat{\mathcal{G}}_{A[0,1]} = \sum \hat{g}_{\hat{\Delta},s,r} = \hat{g}_{4,0,1} + \hat{g}_{5,1,0} + \hat{g}_{5,1,2} + \hat{g}_{6,0,1} + \hat{g}_{6,2,1} + \hat{g}_{7,1,0} \, .$$

So we can calculate B from the $r = 2$ channel only.

We obtain

$$B(\hat{\Delta}, s) \sim \int \mathrm{d}z\mathrm{d}\bar{z}\mu'(z,\bar{z})\,\mathrm{Disc}(F(z,\bar{z}),\zeta(z)) = \sum \frac{B_{m,s}^2}{\hat{\Delta}_{m,s} - \hat{\Delta}}\,.$$

Low-spins

Not completely fixed, but as we will see this formula only misses two superblocks!

(In constrast, the "bosonic" inversion formula requires adding an infinite number of blocks to recover the correlator)

Supersymmetry helps because the analyticity in spins constrains the block with highest spin of any superconformal block.

Calculating the discontinuity

We're ready to calculate the discontinuity and reconstruct the correlator. The bulk operators which contribute to the OPE are [Heslop, 2004]

$$
\begin{aligned}
D[2,0] \times D[2,0] =& \mathbb{1} + D[2,0] + D[4,0] + D[0,4] \\
&+ \sum_{l=0,2,\ldots} B[2,0]_l + B[0,2]_{l+1} \\
&+ \sum L[0,0]_{\Delta,l} \,.
\end{aligned}
$$

Multiplet	Δ	Comment
$\mathbb{1}$	0	identity
$D[2,0]$	4	stress tensor
$D[4,0]$	8	
$B[2,0]_l$	$8+l$	
$L[0,0]_{\Delta,l}$	$\Delta > 6+l$	long multiplets

Table 2: Bulk supermultiplets and their dimension

The only superblocks that contribute to the discontinuity are $\mathbb{1}$ and $D[2,0]$. The bulk identity gives rise to the leading term in the correlator

$$\frac{(z\bar{z})^2(1-\omega)^4}{(1-z)^4(1-\bar{z})^4\omega^2}$$

$D[2,0]$ gives rise to $\mathcal{F}^{(1)}$. Its superblock is

$$\zeta_{D[2,0]}(z) = -\frac{12z^2}{(1-z)^4}\left[1+\frac{1}{2}\frac{1+z}{1-z}\log z\right], \qquad F_{D[2,0]}(z,\bar{z}) = 0\,.$$

Calculating the discontinuity and using the supersymmetric inversion formula, we find

$$\mathcal{F}(z,\bar{z},\omega) = \frac{d^2}{c} - \frac{d}{c}\hat{\mathcal{G}}_{B[1]} + B^2_{0,0}\hat{\mathcal{G}}_{B[2]} + \sum_{s\geq 1} B^2_{0,s}\hat{\mathcal{G}}_{A[1]_{s-1}} + \sum_{\substack{m\geq 1\\ s\geq 0}} B^2_{m,s}\hat{\mathcal{G}}_{L[0]_{\hat{\Delta},s}}\,,$$

where dimensions of long operators are

$$\hat{\Delta}_{m,s} = 2 + s + 2m - \frac{d}{c}\gamma^{(1)}_{m,s} + \ldots \qquad \gamma_{m,s} = \frac{12m(m+1)(m+2)}{(s+1)(s+2m+3)}\,,$$

and the OPE coefficients are

$$\left\langle B^2_{m,s}\right\rangle = 2^s\frac{(m+1)(s+1)(m+s+2)(2m+s+3)}{6} - \frac{d}{c}2^{s-1}(m+1)(5m^3 + m^2(19+4s) + m(20+8s) + 2(2+s)) + \ldots$$

Conclusion

Crossing symmetry

We can check that the result satisfies crossing symmetry and admits a bulk channel decomposition

$$\mathcal{F} = \left(\frac{z\bar{z}(1-\omega)^2}{(1-z)^2(1-\bar{z})^2\omega} \right)^2 \left[1 - \frac{d}{c}\mathcal{G}_{D[2,0]} + (CA)_{0,0}\mathcal{G}_{D[4,0]} \right.$$
$$\left. -\frac{1}{4}\sum_{l\geq 2}(CA)_{0,l}\mathcal{G}_{B[2,0]_{l-2}} + \sum_{n,l\geq 2}(CA)_{n,l}\mathcal{G}_{L[0,0]_{\Delta-4,l}} \right] ,$$

where l is even, $\Delta = 8 + 2n + l$ and

$$\langle CA \rangle_{n,l} = 2^l \frac{(n+1)!(n+l+2)!(n+l+3)!}{(2n+1)!(2n+2l+5)!} \times$$
$$\left[\frac{d^2}{c}\delta_{n,even}2^{-n}(l+2)(2n+l+5) - \frac{d}{c}\frac{(n-1)(n+4)!}{16(n+3)} + \ldots \right]$$

Conclusion

- We used bootstrap techniques to calculate $\mathcal{F}^{(1)}$. To do so we obtained the relevant blocks and calculated the dCFT data $\{B, (CA), \hat{\Delta}\}$.
- Simpler setup than the Wilson line, which means we could derive a supersymmetric inversion formula and check crossing symmetry.
- We conjecture that ζ is exact. To check.

$$\zeta(z) = \frac{z^2}{(1-z)^4} + \frac{d^2}{c} - \frac{d}{c}\frac{z}{(1-z)^2} \, .$$

- Result for any representation with $d \ll c^{1/2}$

Outlook

- What happens when $d \sim c^{1/2}$? Can we see the bubbling geometries?
- Go to higher orders in N. Mellin space might be useful
- For 4-point functions, Mellin amplitudes have a flat space limit which calculates scattering in M-theory. Does it work for defects?
- More general bulk operators?
- Bootstrapping other setups?

Thank you for your attention