Title: Dynamical frames in gauge theory and gravity

Speakers: Philipp Hoehn

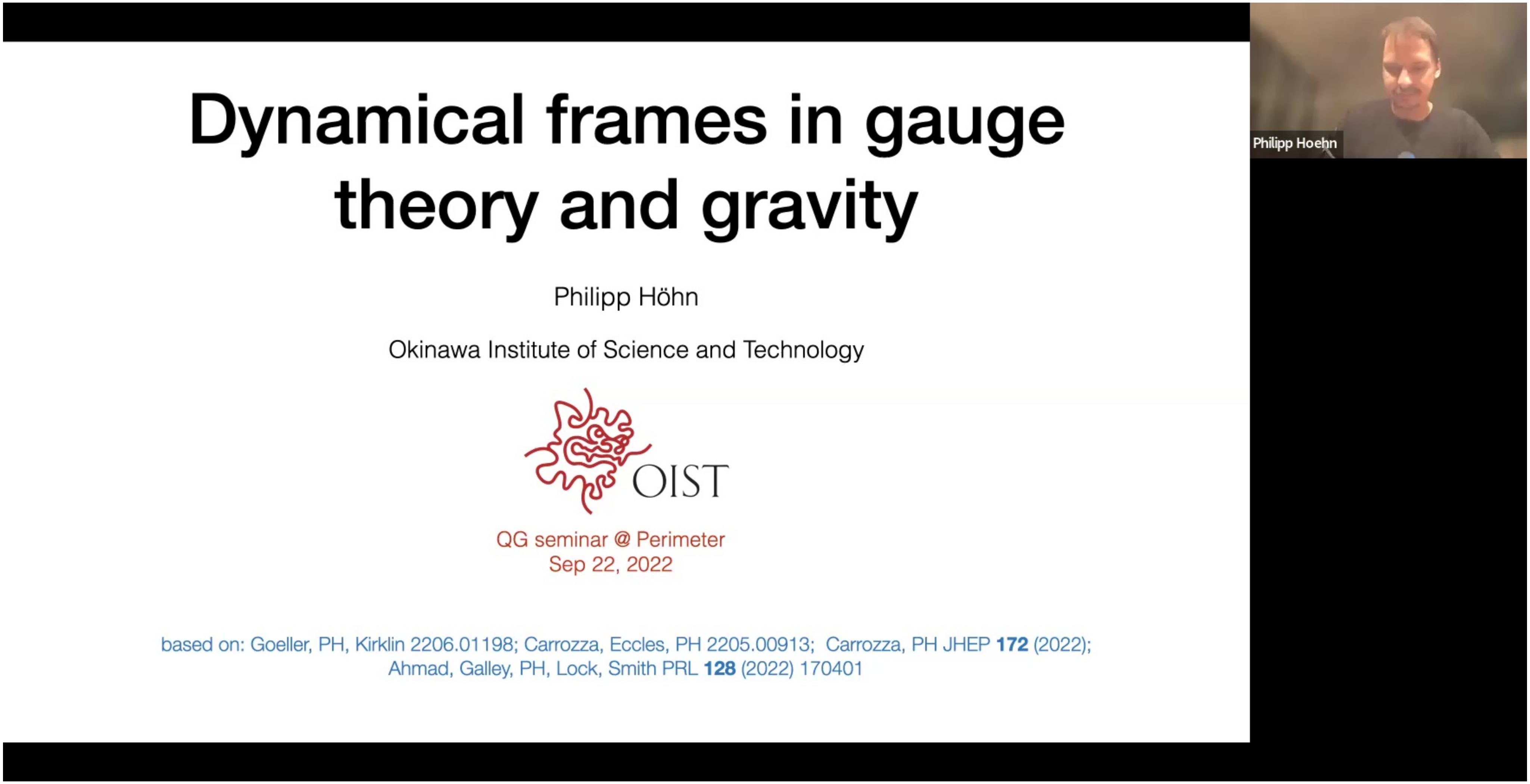
Series: Quantum Gravity

Date: September 22, 2022 - 9:30 AM

URL: https://pirsa.org/22090088

Abstract: Though often not spelled out explicitly, dynamical reference frames appear ubiquitously in gauge theory and gravity. They appear, for example, when constructing dressed/relational observables, describing physics relative to the frame in a gauge-invariant way. In this talk, I will sketch a general framework for constructing such frames and associated relational observables. It unifies previous approaches and encompasses the transformations relating different frame choices. In gravitational theories, this gives rise to an arguably more physical reformulation of general covariance in terms of dynamical rather than fixed frames. I will then discuss an ensuing relational form of locality, including bulk microcausality and local subsystems associated with subregions, both of which can be defined gauge-invariantly relative to a dynamical frame. In the latter case, the frame incarnates as an edge mode field, linking with recent work on finite subregions. In particular, the corresponding boundary charges and symmetries can be understood in terms of reorientations of the frame. Notably, the resulting notion of a subsystem is frame-dependent, as are therefore correlations, thermal properties and specifically entropies. I will conclude with an outlook on the quantum realm and connections with recent developments on quantum reference frames. [Based on 2206.01193, 2205.00913, JHEP 172 (2022), PRL 128 170401.]

Zoom link: https://pitp.zoom.us/j/97735460640?pwd=NThybFc3M3Z3cHhVRmRvczdrclhvZz09



Dynamical frames in gauge theory and gravity

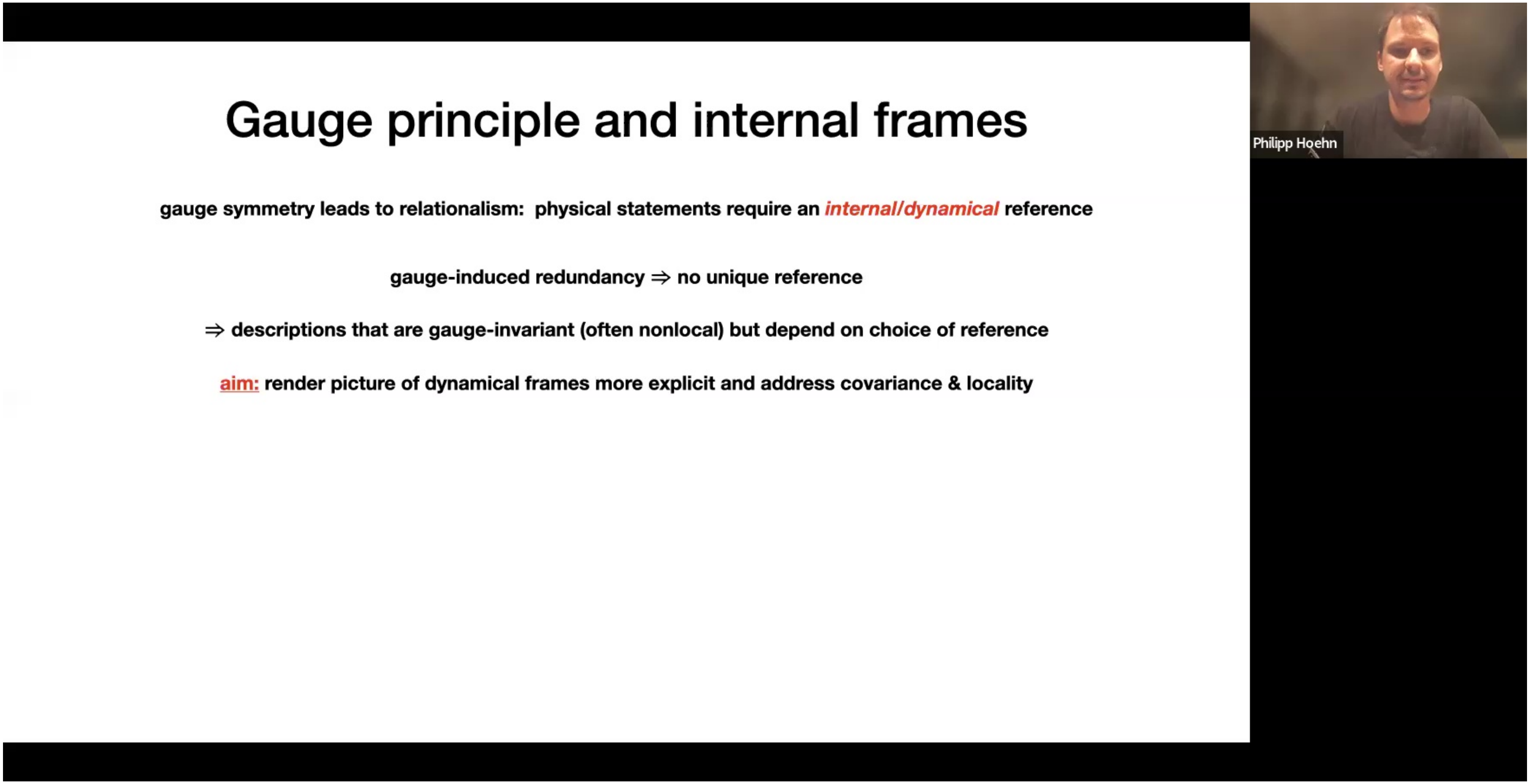
Philipp Höhn

Okinawa Institute of Science and Technology

OIST

QG seminar @ Perimeter
Sep 22, 2022

based on: Goeller, PH, Kirklin 2206.01198; Carrozza, Eccles, PH 2205.00913; Carrozza, PH JHEP **172** (2022); Ahmad, Galley, PH, Lock, Smith PRL **128** (2022) 170401



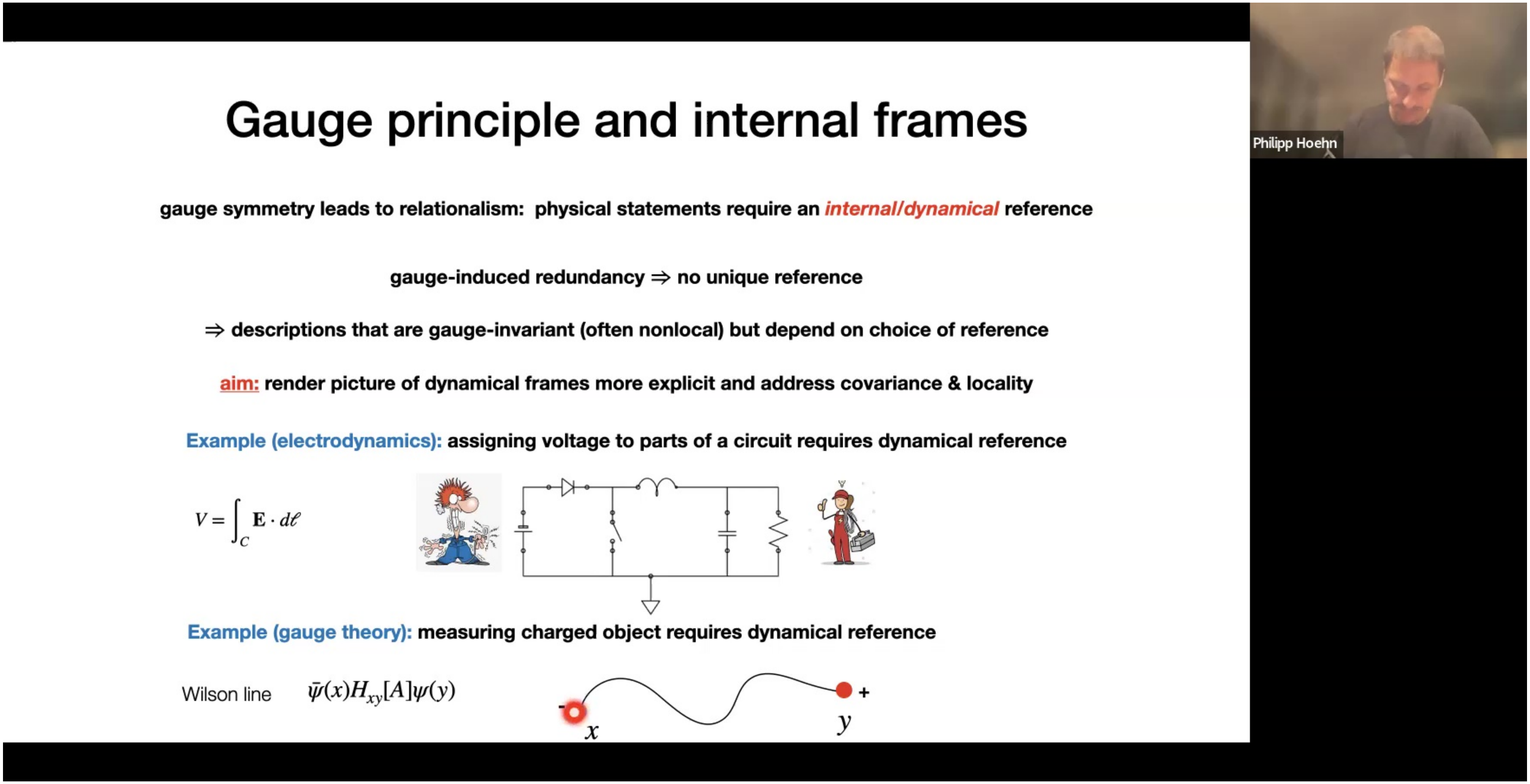
Gauge principle and internal frames

gauge symmetry leads to relationalism: physical statements require an *internal/dynamical* reference

gauge-induced redundancy ⇒ no unique reference

⇒ descriptions that are gauge-invariant (often nonlocal) but depend on choice of reference

aim: render picture of dynamical frames more explicit and address covariance & locality



Gauge principle and internal frames

gauge symmetry leads to relationalism: physical statements require an *internal/dynamical* reference

gauge-induced redundancy ⇒ no unique reference

⇒ descriptions that are gauge-invariant (often nonlocal) but depend on choice of reference

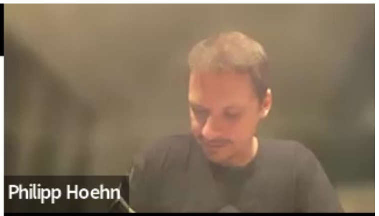
aim: render picture of dynamical frames more explicit and address covariance & locality

Example (electrodynamics): assigning voltage to parts of a circuit requires dynamical reference

$$V = \int_C \mathbf{E} \cdot d\ell$$

Example (gauge theory): measuring charged object requires dynamical reference

Wilson line $\bar{\psi}(x)H_{xy}[A]\psi(y)$



Gauge principle and internal frames

gauge symmetry leads to relationalism: physical statements require an *internal/dynamical* reference

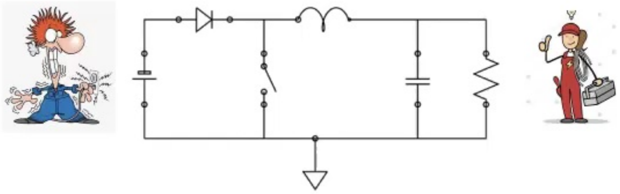
gauge-induced redundancy ⇒ no unique reference

⇒ descriptions that are gauge-invariant (often nonlocal) but depend on choice of reference

aim: render picture of dynamical frames more explicit and address covariance & locality

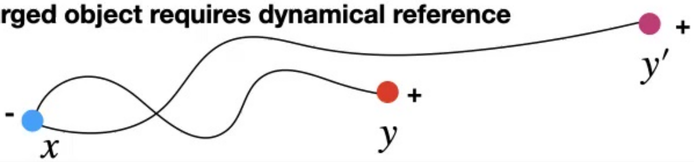
Example (electrodynamics): assigning voltage to parts of a circuit requires dynamical reference

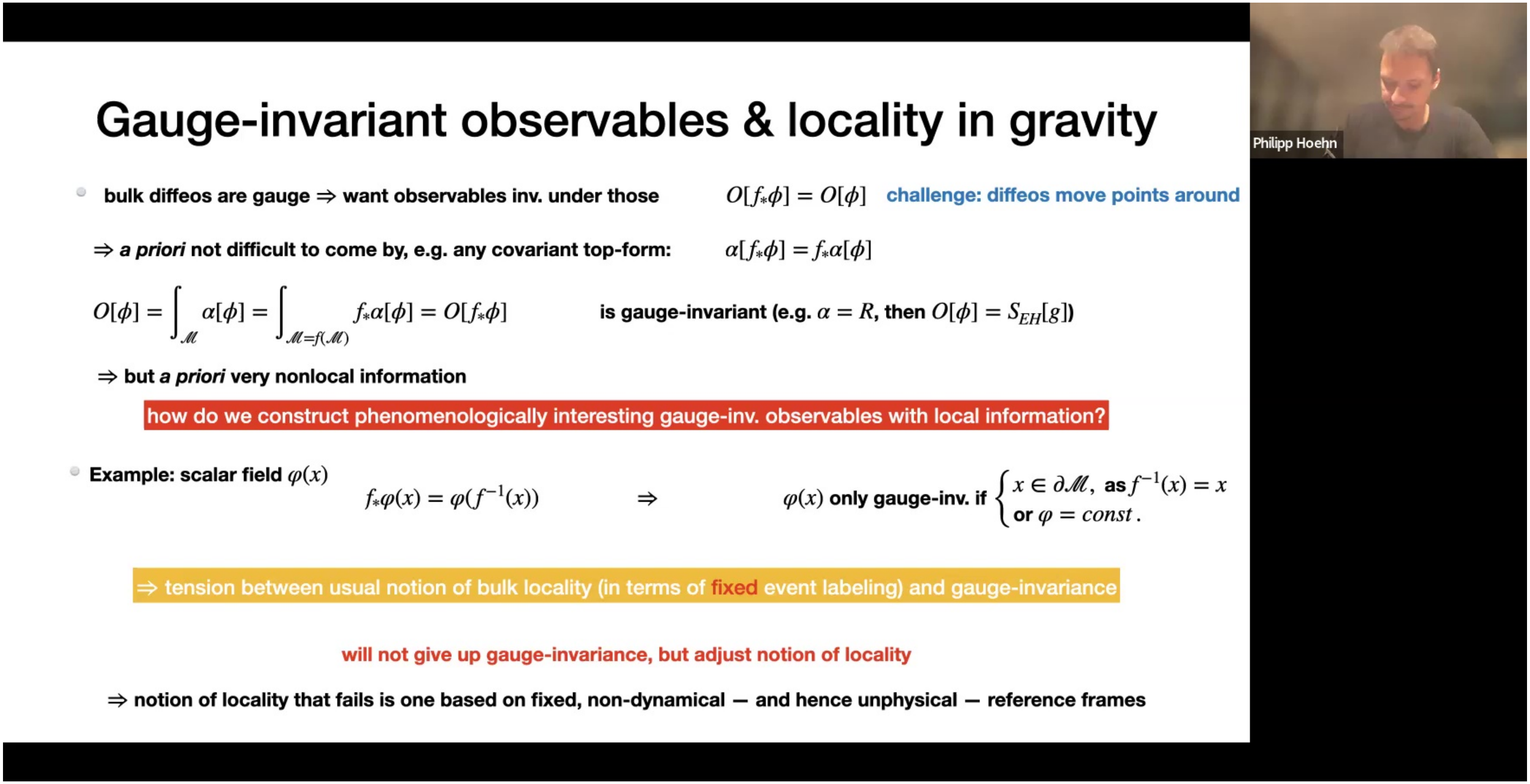
$$V = \int_C \mathbf{E} \cdot d\ell$$



Example (gauge theory): measuring charged object requires dynamical reference

Wilson line $\bar{\psi}(x) H_{xy}[A] \psi(y)$





Gauge-invariant observables & locality in gravity

- **bulk diffeos are gauge ⇒ want observables inv. under those** $O[f_*\phi] = O[\phi]$ **challenge: diffeos move points around**

⇒ *a priori* not difficult to come by, e.g. any covariant top-form: $\alpha[f_*\phi] = f_*\alpha[\phi]$

$$O[\phi] = \int_{\mathcal{M}} \alpha[\phi] = \int_{\mathcal{M}=f(\mathcal{M})} f_*\alpha[\phi] = O[f_*\phi]$$

is gauge-invariant (e.g. $\alpha = R$, then $O[\phi] = S_{EH}[g]$)

⇒ but *a priori* very nonlocal information

how do we construct phenomenologically interesting gauge-inv. observables with local information?

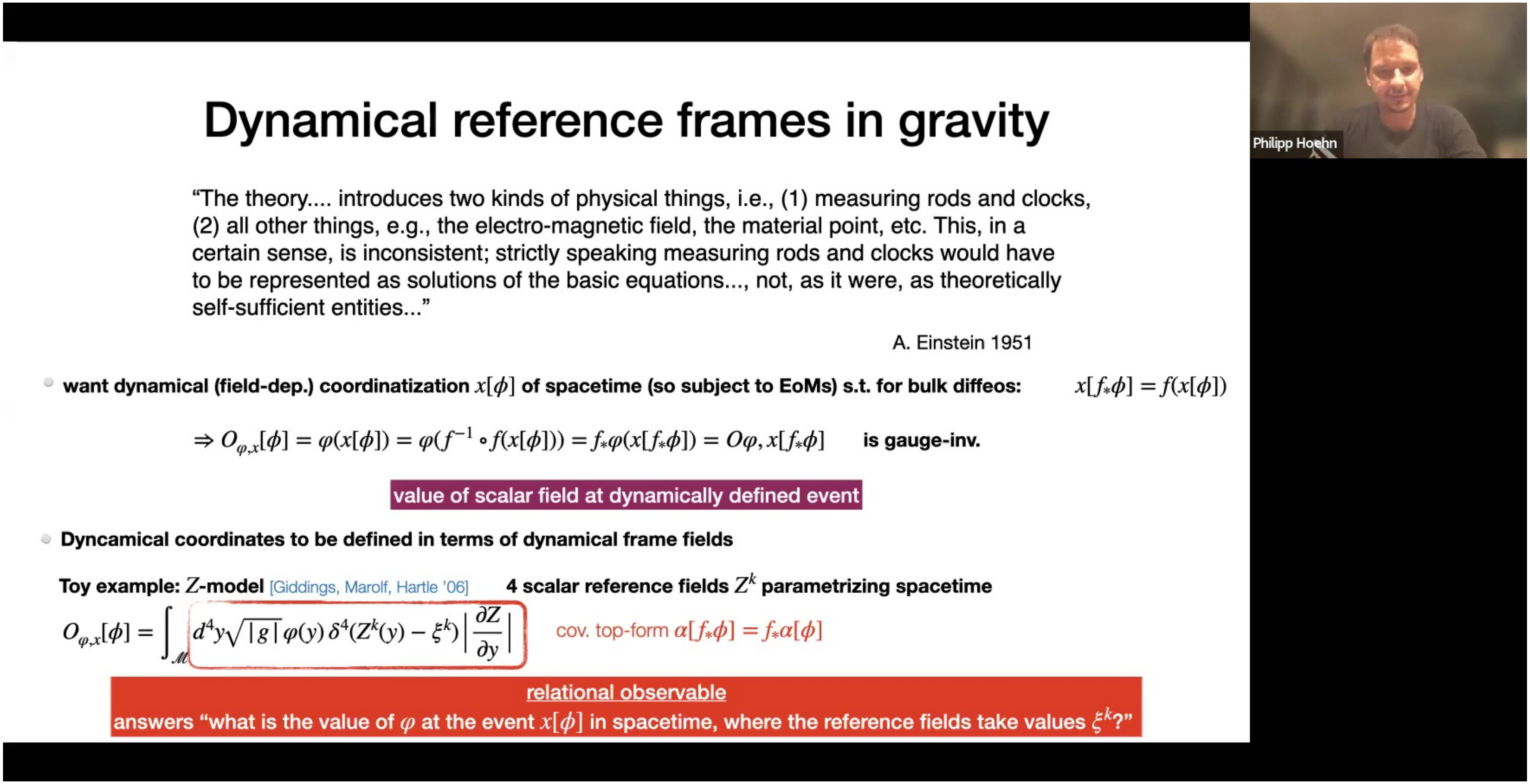
- **Example: scalar field $\varphi(x)$**

$$f_*\varphi(x) = \varphi(f^{-1}(x)) \quad \Rightarrow \quad \varphi(x) \text{ only gauge-inv. if } \begin{cases} x \in \partial\mathcal{M}, \text{ as } f^{-1}(x) = x \\ \text{or } \varphi = const\,. \end{cases}$$

⇒ tension between usual notion of bulk locality (in terms of fixed event labeling) and gauge-invariance

will not give up gauge-invariance, but adjust notion of locality

⇒ notion of locality that fails is one based on fixed, non-dynamical — and hence unphysical — reference frames



Dynamical reference frames in gravity

> “The theory.... introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electro-magnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations..., not, as it were, as theoretically self-sufficient entities...”
>
> A. Einstein 1951

- **want dynamical (field-dep.) coordinatization $x[\phi]$ of spacetime (so subject to EoMs) s.t. for bulk diffeos:** $x[f_*\phi] = f(x[\phi])$

$$\Rightarrow O_{\varphi,x}[\phi] = \varphi(x[\phi]) = \varphi(f^{-1} \circ f(x[\phi])) = f_*\varphi(x[f_*\phi]) = O\varphi, x[f_*\phi]$$ **is gauge-inv.**

value of scalar field at dynamically defined event

- **Dyncamical coordinates to be defined in terms of dynamical frame fields**

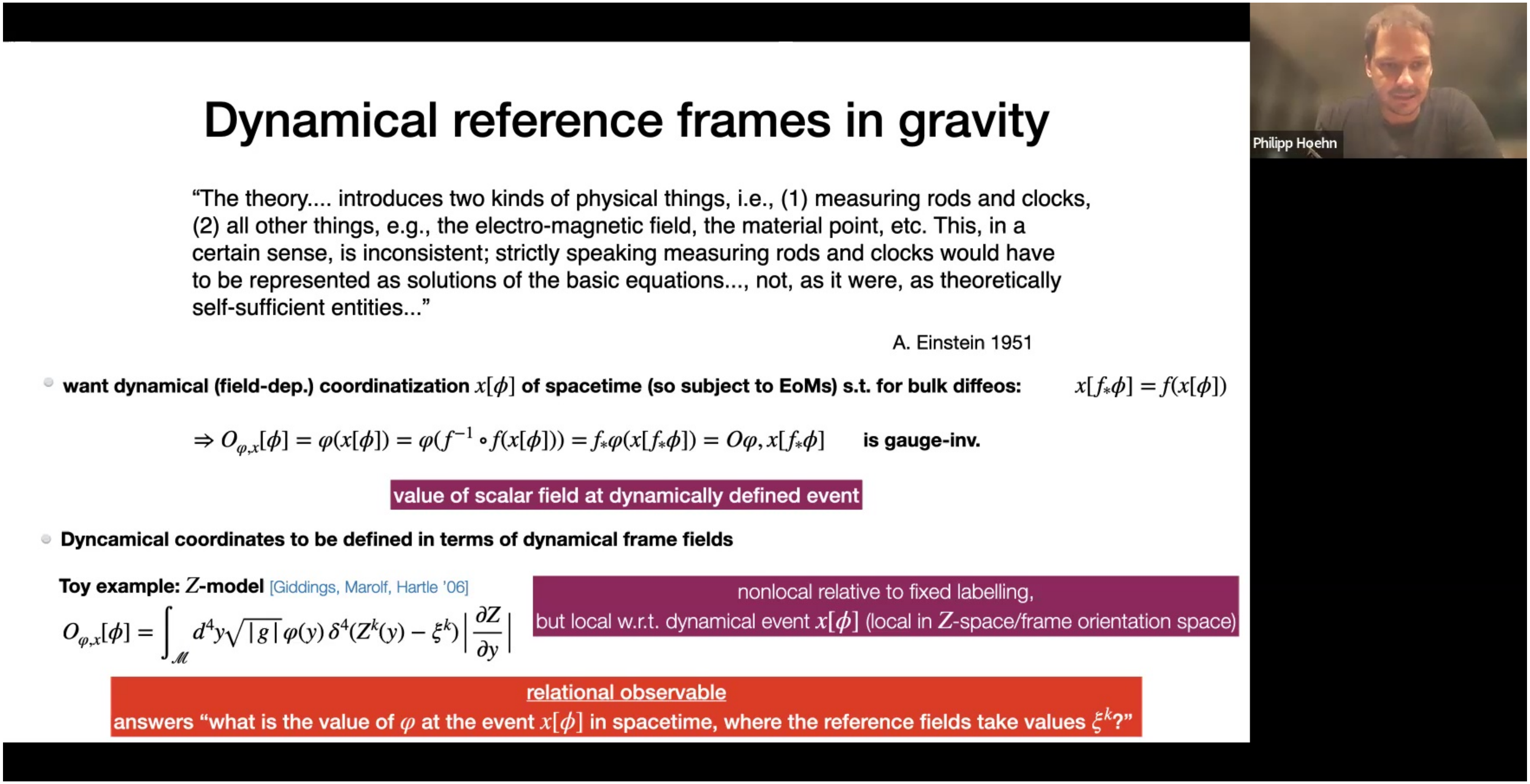
Toy example: Z-model [Giddings, Marolf, Hartle '06] **4 scalar reference fields Z^k parametrizing spacetime**

$$O_{\varphi,x}[\phi] = \int_{\mathscr{M}} \boxed{d^4y\sqrt{|g|}\,\varphi(y)\,\delta^4(Z^k(y) - \xi^k)\left|\frac{\partial Z}{\partial y}\right|}$$

cov. top-form $\alpha[f_*\phi] = f_*\alpha[\phi]$

relational observable

answers “what is the value of φ at the event $x[\phi]$ in spacetime, where the reference fields take values ξ^k?”



Dynamical reference frames in gravity

> "The theory.... introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electro-magnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations..., not, as it were, as theoretically self-sufficient entities..."
>
> A. Einstein 1951

- **want dynamical (field-dep.) coordinatization $x[\phi]$ of spacetime (so subject to EoMs) s.t. for bulk diffeos:** $x[f_*\phi] = f(x[\phi])$

$$\Rightarrow O_{\varphi,x}[\phi] = \varphi(x[\phi]) = \varphi(f^{-1}\circ f(x[\phi])) = f_*\varphi(x[f_*\phi]) = O\varphi, x[f_*\phi]$$ **is gauge-inv.**

value of scalar field at dynamically defined event

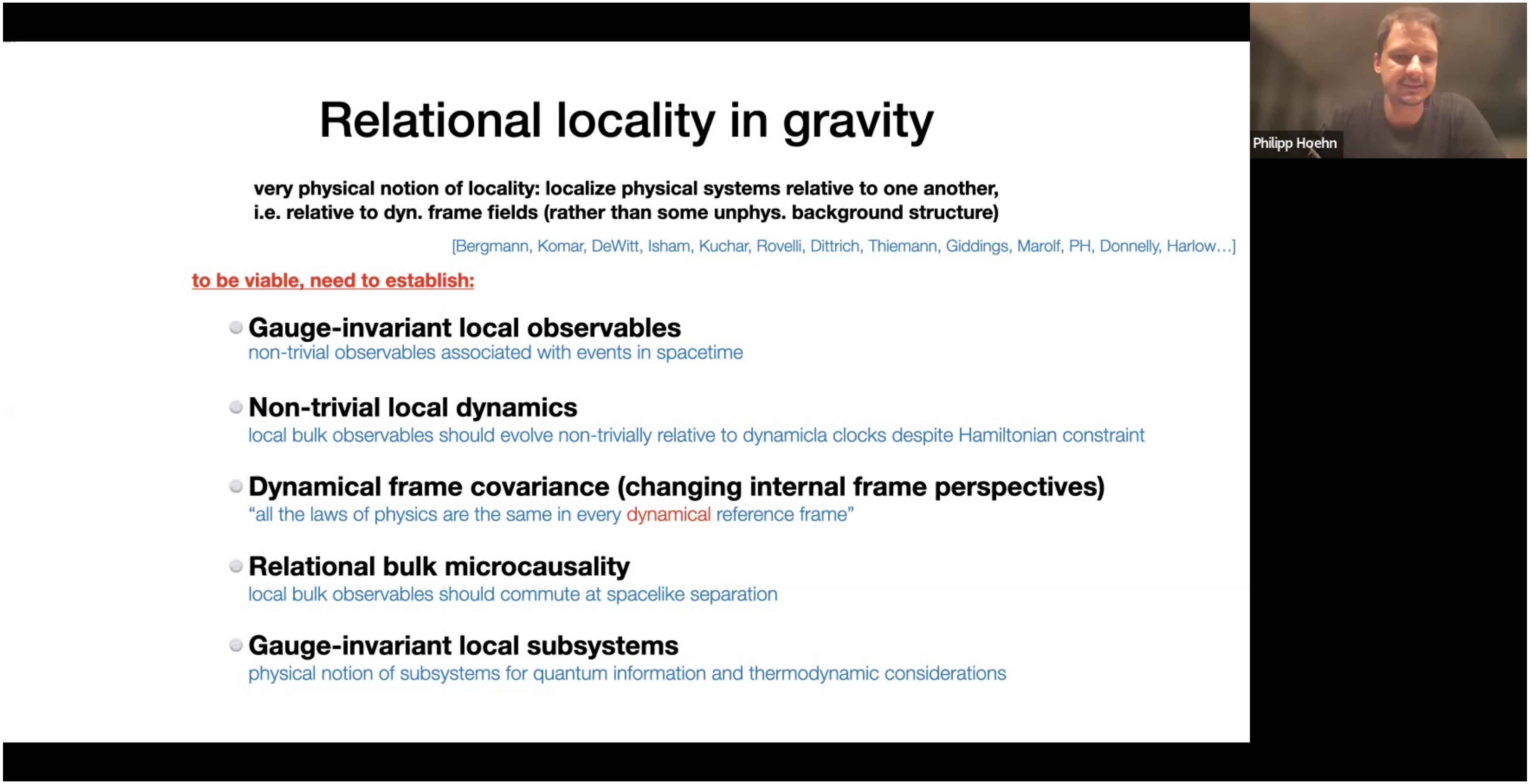
- **Dyncamical coordinates to be defined in terms of dynamical frame fields**

Toy example: Z-model [Giddings, Marolf, Hartle '06]

$$O_{\varphi,x}[\phi] = \int_{\mathcal{M}} d^4y\sqrt{|g|}\,\varphi(y)\,\delta^4(Z^k(y)-\xi^k)\left|\frac{\partial Z}{\partial y}\right|$$

nonlocal relative to fixed labelling,
but local w.r.t. dynamical event $x[\phi]$ (local in Z-space/frame orientation space)

relational observable
answers "what is the value of φ at the event $x[\phi]$ in spacetime, where the reference fields take values ξ^k?"



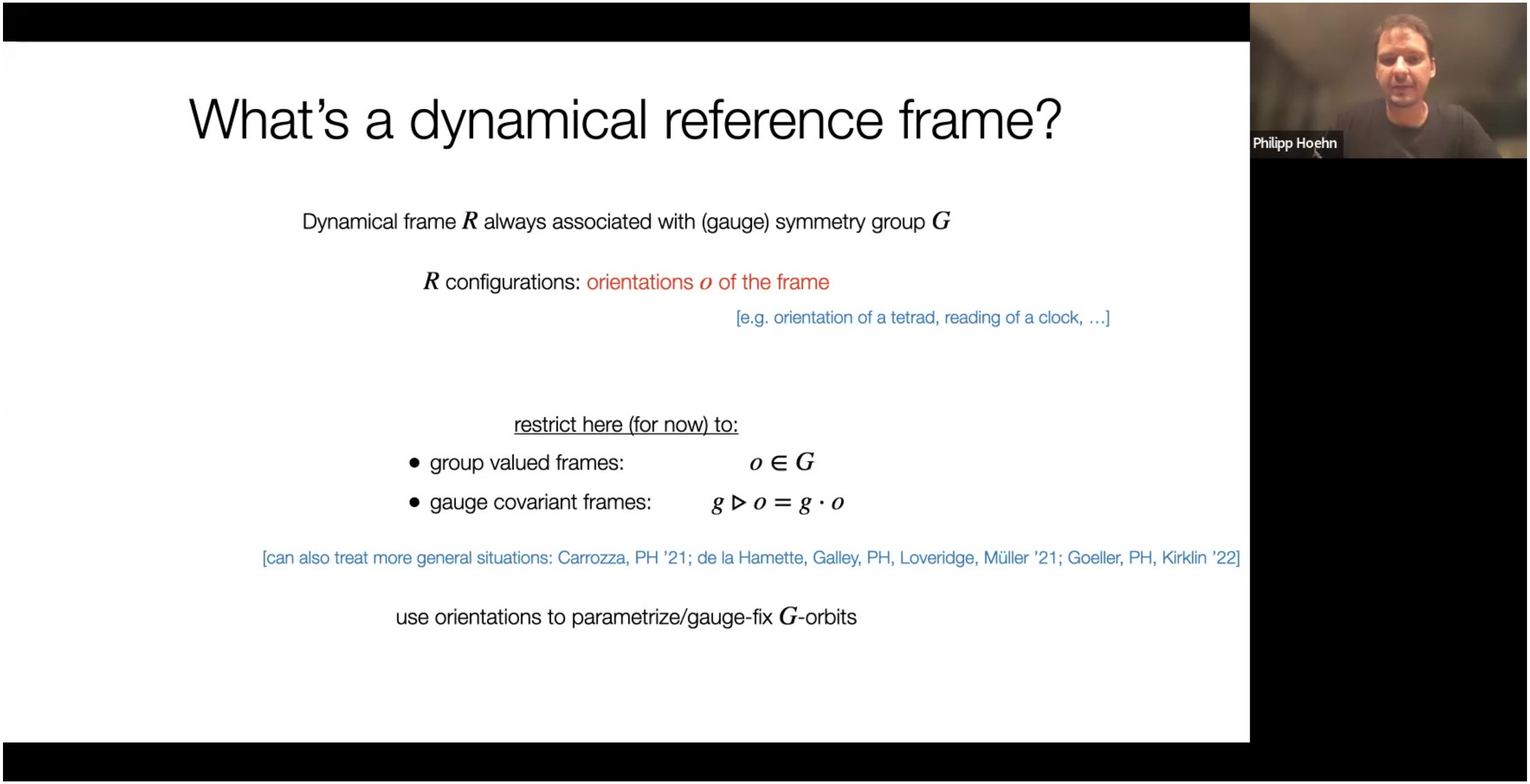
Relational locality in gravity

very physical notion of locality: localize physical systems relative to one another, i.e. relative to dyn. frame fields (rather than some unphys. background structure)

[Bergmann, Komar, DeWitt, Isham, Kuchar, Rovelli, Dittrich, Thiemann, Giddings, Marolf, PH, Donnelly, Harlow…]

to be viable, need to establish:

- **Gauge-invariant local observables**
  non-trivial observables associated with events in spacetime
- **Non-trivial local dynamics**
  local bulk observables should evolve non-trivially relative to dynamicla clocks despite Hamiltonian constraint
- **Dynamical frame covariance (changing internal frame perspectives)**
  “all the laws of physics are the same in every dynamical reference frame”
- **Relational bulk microcausality**
  local bulk observables should commute at spacelike separation
- **Gauge-invariant local subsystems**
  physical notion of subsystems for quantum information and thermodynamic considerations



What’s a dynamical reference frame?

Dynamical frame R always associated with (gauge) symmetry group G

R configurations: orientations o of the frame

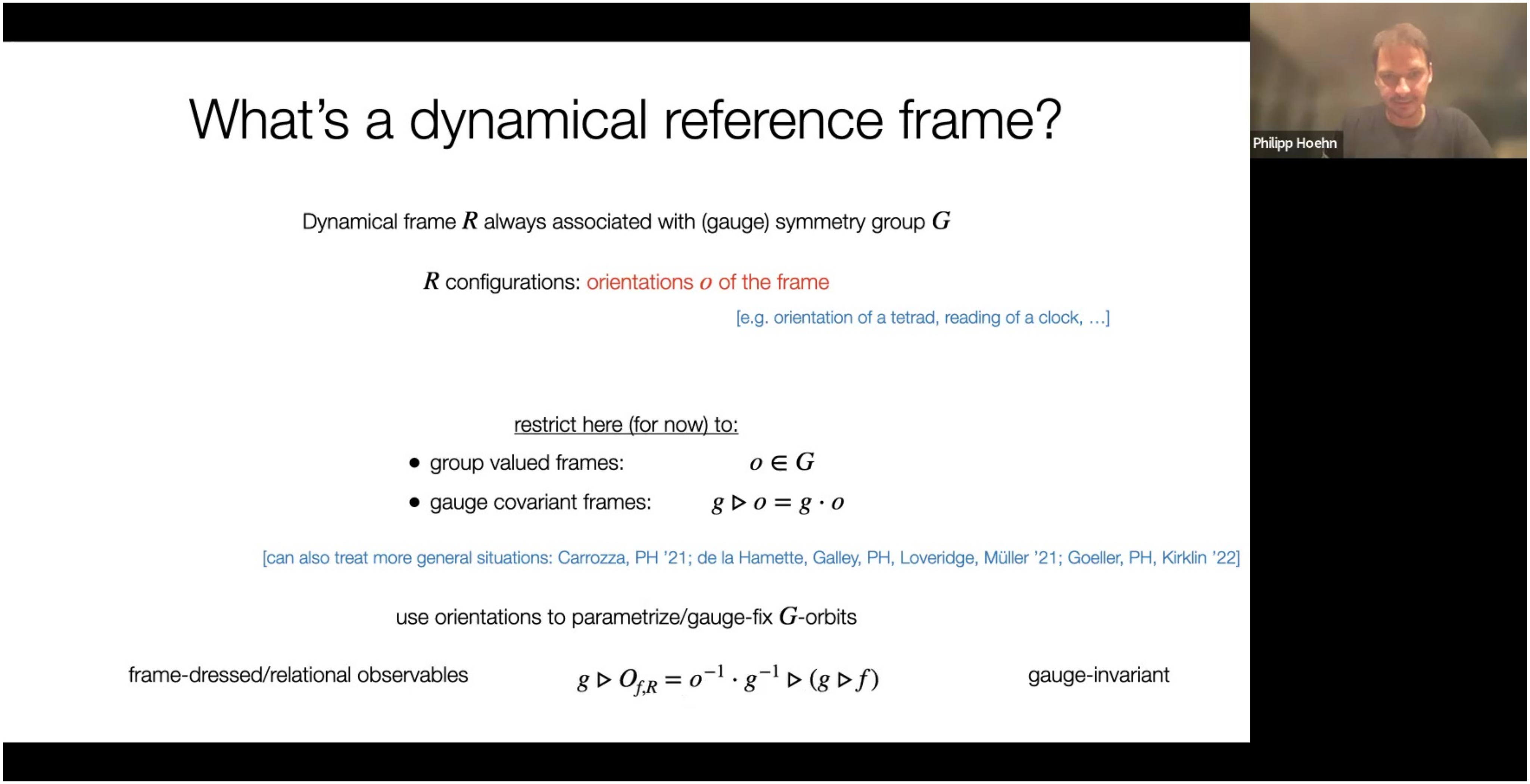
[e.g. orientation of a tetrad, reading of a clock, …]

restrict here (for now) to:

- group valued frames: $o \in G$
- gauge covariant frames: $g \triangleright o = g \cdot o$

[can also treat more general situations: Carrozza, PH ’21; de la Hamette, Galley, PH, Loveridge, Müller ’21; Goeller, PH, Kirklin ’22]

use orientations to parametrize/gauge-fix G-orbits



What’s a dynamical reference frame?

Dynamical frame R always associated with (gauge) symmetry group G

R configurations: orientations o of the frame

[e.g. orientation of a tetrad, reading of a clock, …]

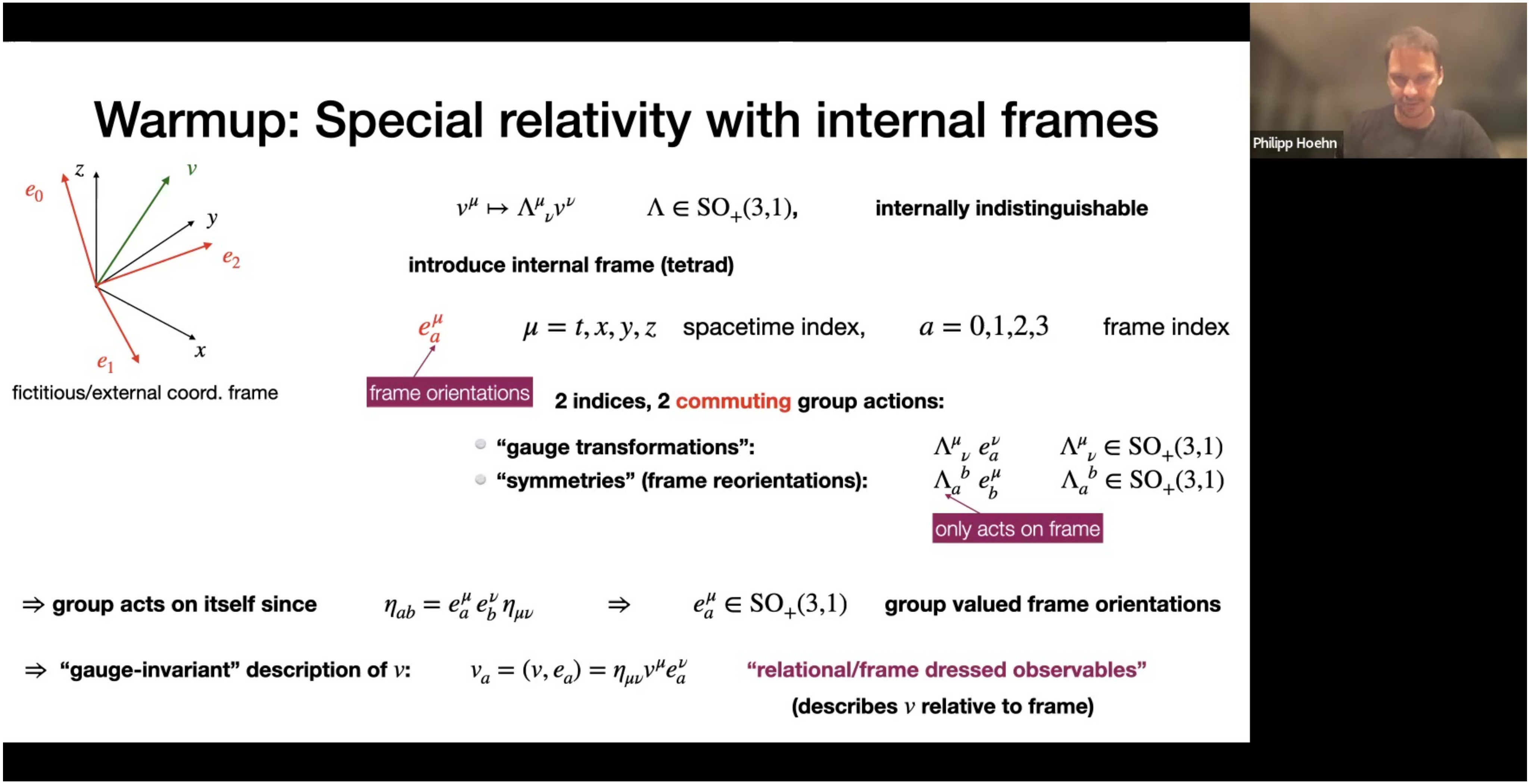
restrict here (for now) to:

- group valued frames: $o \in G$
- gauge covariant frames: $g \triangleright o = g \cdot o$

[can also treat more general situations: Carrozza, PH ’21; de la Hamette, Galley, PH, Loveridge, Müller ’21; Goeller, PH, Kirklin ’22]

use orientations to parametrize/gauge-fix G-orbits

frame-dressed/relational observables $g \triangleright O_{f,R} = o^{-1} \cdot g^{-1} \triangleright (g \triangleright f)$ gauge-invariant



Warmup: Special relativity with internal frames

fictitious/external coord. frame

$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in \mathrm{SO}_+(3{,}1)$, **internally indistinguishable**

introduce internal frame (tetrad)

$e_a^\mu \qquad \mu = t, x, y, z$ spacetime index, $\quad a = 0{,}1{,}2{,}3$ frame index

frame orientations

2 indices, 2 commuting group actions:

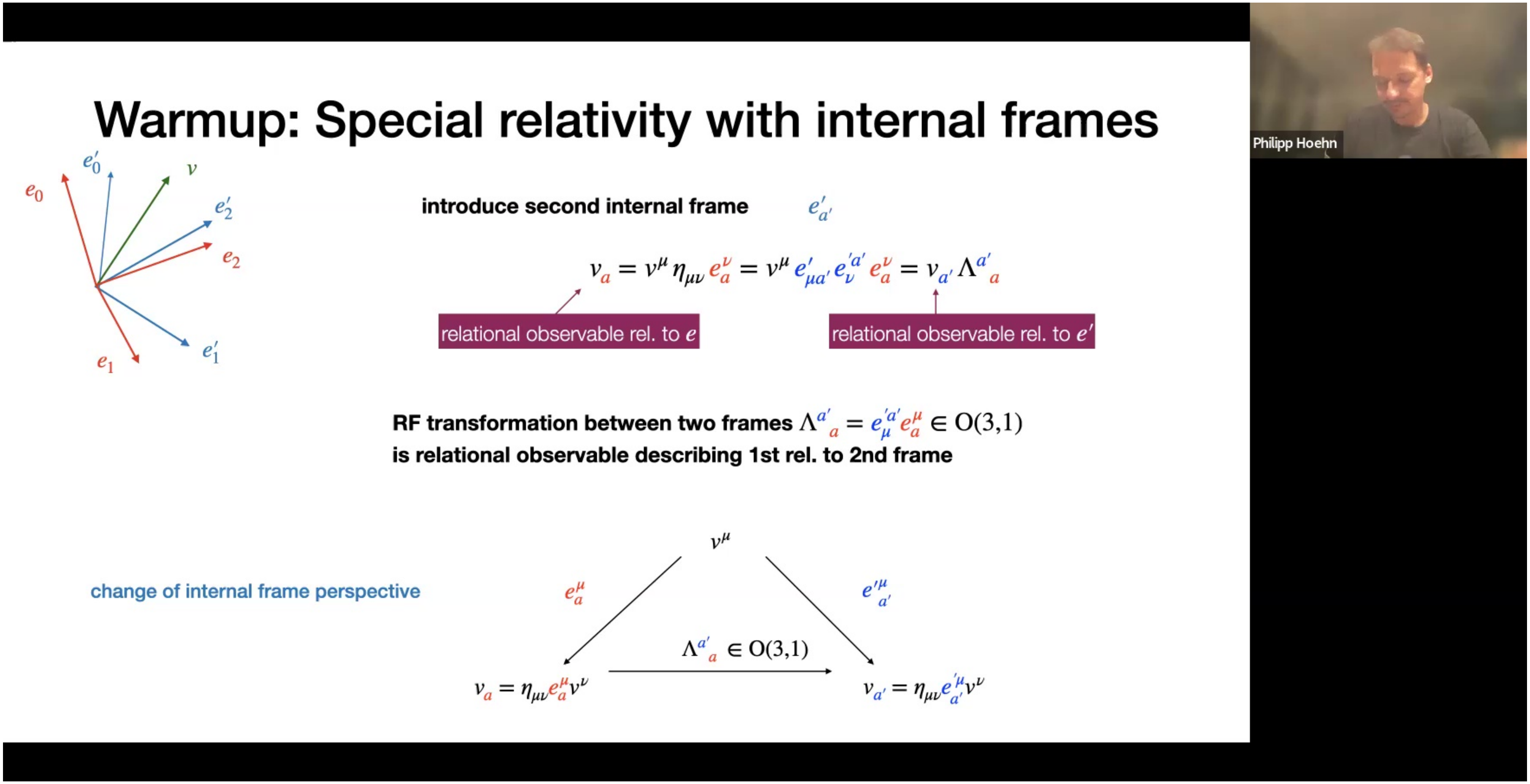
- **"gauge transformations":** $\Lambda^\mu{}_\nu \, e_a^\nu \qquad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3{,}1)$
- **"symmetries" (frame reorientations):** $\Lambda_a{}^b \, e_b^\mu \qquad \Lambda_a{}^b \in \mathrm{SO}_+(3{,}1)$

only acts on frame

⇒ **group acts on itself since** $\eta_{ab} = e_a^\mu e_b^\nu \eta_{\mu\nu} \quad \Rightarrow \quad e_a^\mu \in \mathrm{SO}_+(3{,}1)$ **group valued frame orientations**

⇒ **"gauge-invariant" description of v:** $v_a = (v, e_a) = \eta_{\mu\nu} v^\mu e_a^\nu$ **"relational/frame dressed observables"**

(describes v relative to frame)



Warmup: Special relativity with internal frames

introduce second internal frame $e'_{a'}$

$$v_a = v^\mu \eta_{\mu\nu} e_a^\nu = v^\mu e'_{\mu a'} e'^{a'}_\nu e_a^\nu = v_{a'} \Lambda^{a'}{}_a$$

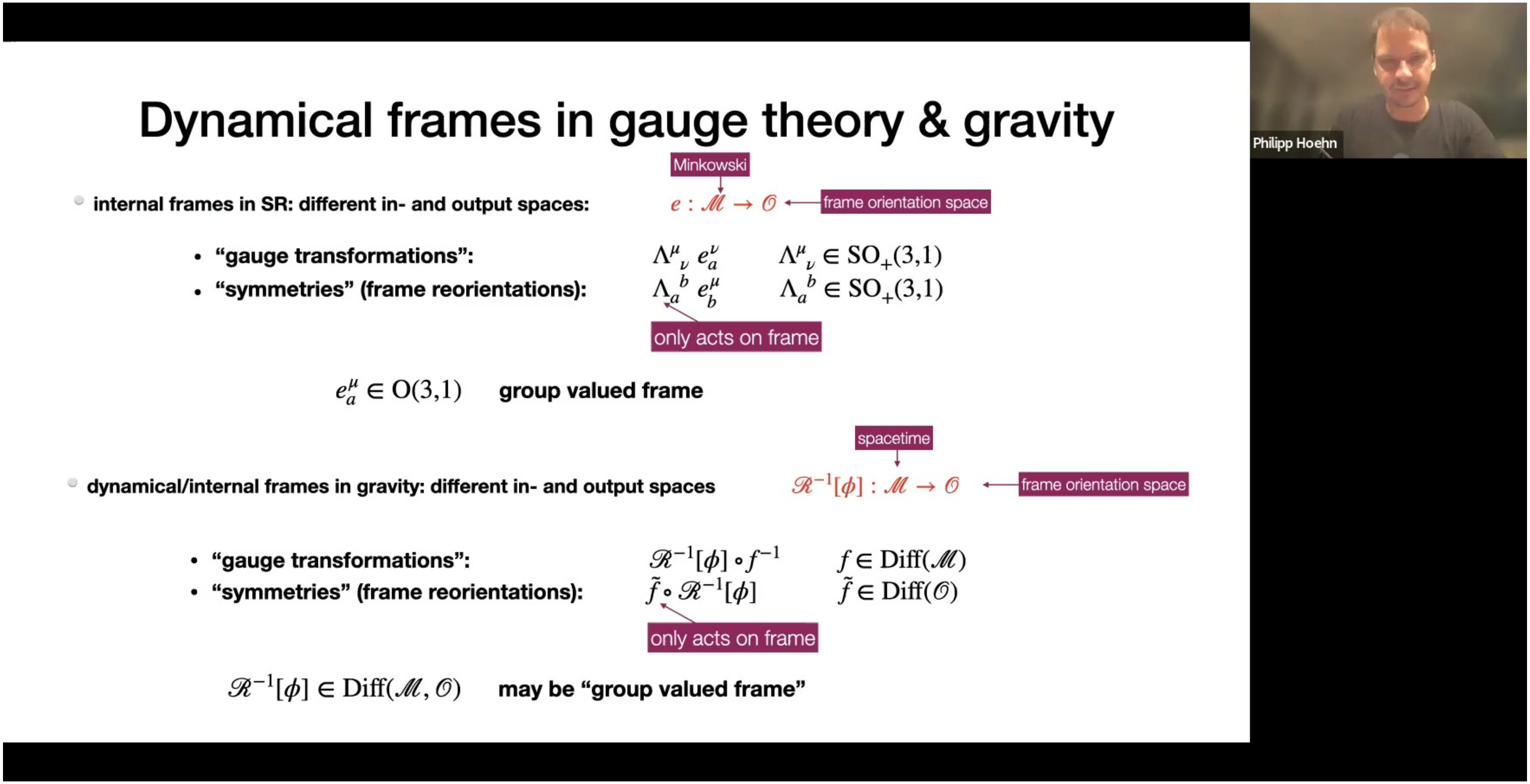
relational observable rel. to e

relational observable rel. to e'

RF transformation between two frames $\Lambda^{a'}{}_a = e'^{a'}_\mu e^\mu_a \in \mathrm{O}(3,1)$
is relational observable describing 1st rel. to 2nd frame

change of internal frame perspective

$$\begin{tikzcd}
 & v^\mu \arrow[dl, "e^\mu_a"'] \arrow[dr, "e'^\mu_{a'}"] & \\
v_a = \eta_{\mu\nu} e^\mu_a v^\nu \arrow[rr, "\Lambda^{a'}{}_a \in \mathrm{O}(3,1)"] & & v_{a'} = \eta_{\mu\nu} e'^\mu_{a'} v^\nu
\end{tikzcd}$$



Dynamical frames in gauge theory & gravity

Minkowski

- **internal frames in SR: different in- and output spaces:** $e : \mathscr{M} \to \mathscr{O}$ ← frame orientation space
  - **"gauge transformations":** $\Lambda^{\mu}{}_{\nu}\, e^{\nu}_{a}$ $\quad \Lambda^{\mu}{}_{\nu} \in \mathrm{SO}_{+}(3,1)$
  - **"symmetries" (frame reorientations):** $\Lambda_{a}{}^{b}\, e^{\mu}_{b}$ $\quad \Lambda_{a}{}^{b} \in \mathrm{SO}_{+}(3,1)$

only acts on frame

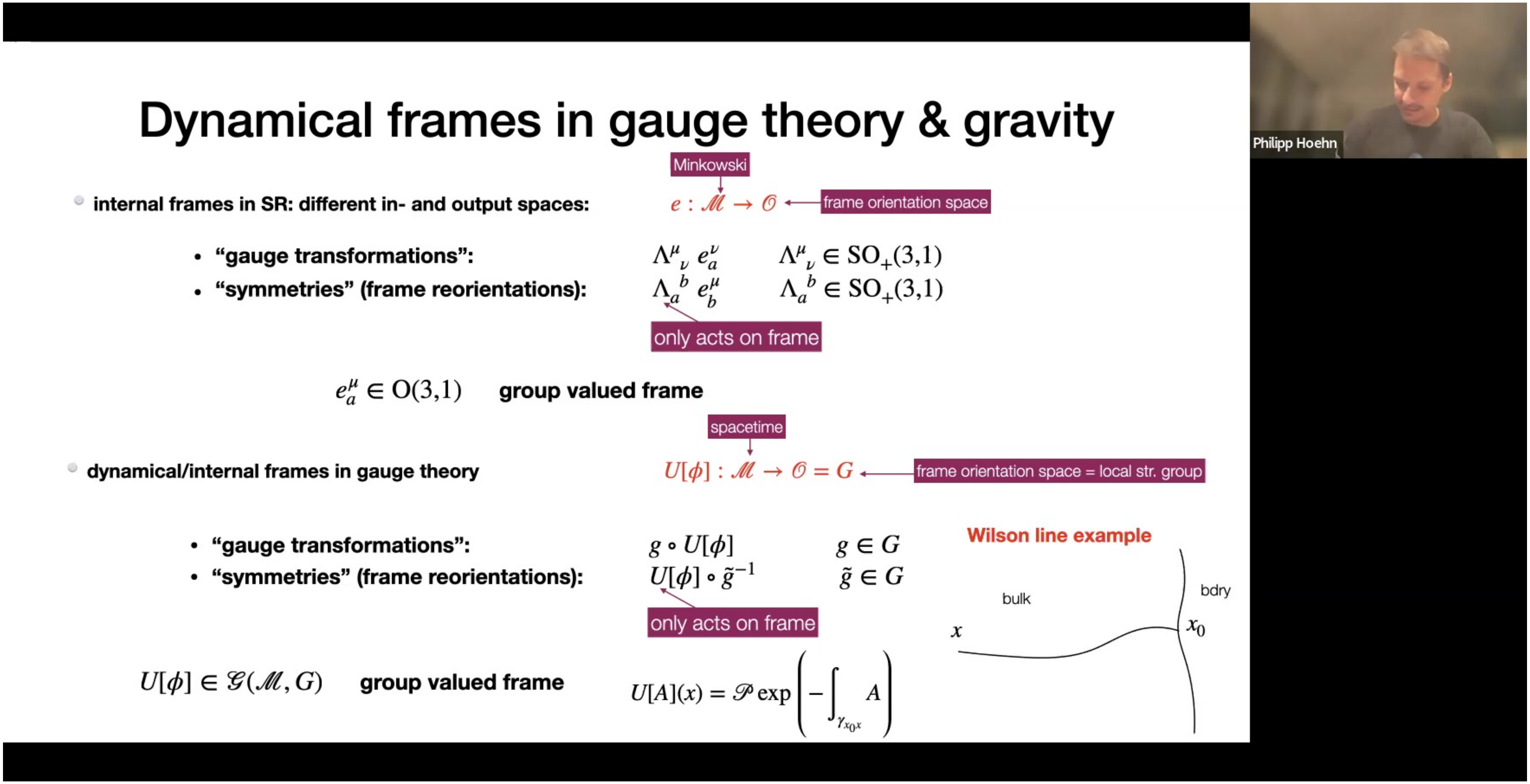
$e^{\mu}_{a} \in \mathrm{O}(3,1)$ **group valued frame**

spacetime

- **dynamical/internal frames in gravity: different in- and output spaces** $\mathscr{R}^{-1}[\phi] : \mathscr{M} \to \mathscr{O}$ ← frame orientation space
  - **"gauge transformations":** $\mathscr{R}^{-1}[\phi] \circ f^{-1}$ $\quad f \in \mathrm{Diff}(\mathscr{M})$
  - **"symmetries" (frame reorientations):** $\tilde{f} \circ \mathscr{R}^{-1}[\phi]$ $\quad \tilde{f} \in \mathrm{Diff}(\mathscr{O})$

only acts on frame

$\mathscr{R}^{-1}[\phi] \in \mathrm{Diff}(\mathscr{M}, \mathscr{O})$ **may be "group valued frame"**



Dynamical frames in gauge theory & gravity

- **internal frames in SR: different in- and output spaces:**

  Minkowski

  $e : \mathscr{M} \to \mathscr{O}$ ← frame orientation space

  - **"gauge transformations":** $\Lambda^\mu{}_\nu \, e^\nu_a$ $\quad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$
  - **"symmetries" (frame reorientations):** $\Lambda_a{}^b \, e^\mu_b$ $\quad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$

  only acts on frame

  $e^\mu_a \in \mathrm{O}(3,1)$ **group valued frame**

- **dynamical/internal frames in gauge theory**

  spacetime

  $U[\phi] : \mathscr{M} \to \mathscr{O} = G$ ← frame orientation space = local str. group

  - **"gauge transformations":** $g \circ U[\phi]$ $\quad g \in G$
  - **"symmetries" (frame reorientations):** $U[\phi] \circ \tilde{g}^{-1}$ $\quad \tilde{g} \in G$

  only acts on frame

  $U[\phi] \in \mathscr{G}(\mathscr{M}, G)$ **group valued frame**

Wilson line example

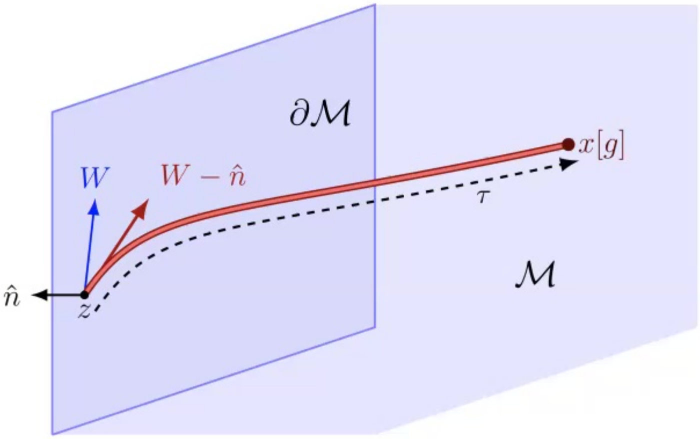
$$U[A](x) = \mathscr{P}\exp\left(-\int_{\gamma_{x_0 x}} A\right)$$

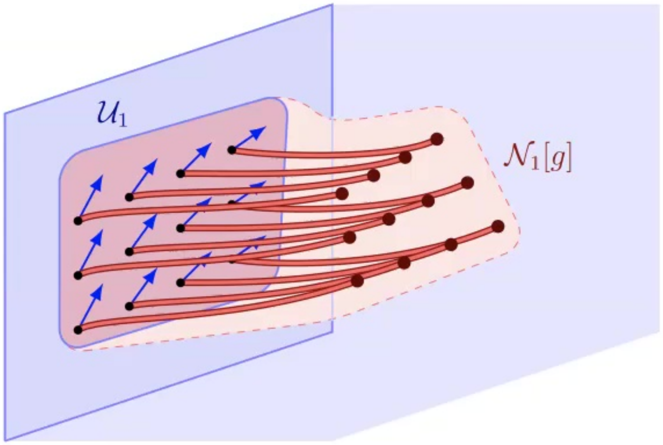
Generally covariant theories

… dynamical frames for the diffeo group

Example: boundary-anchored geodesic frames

[Goeller, PH, Kirklin '22]



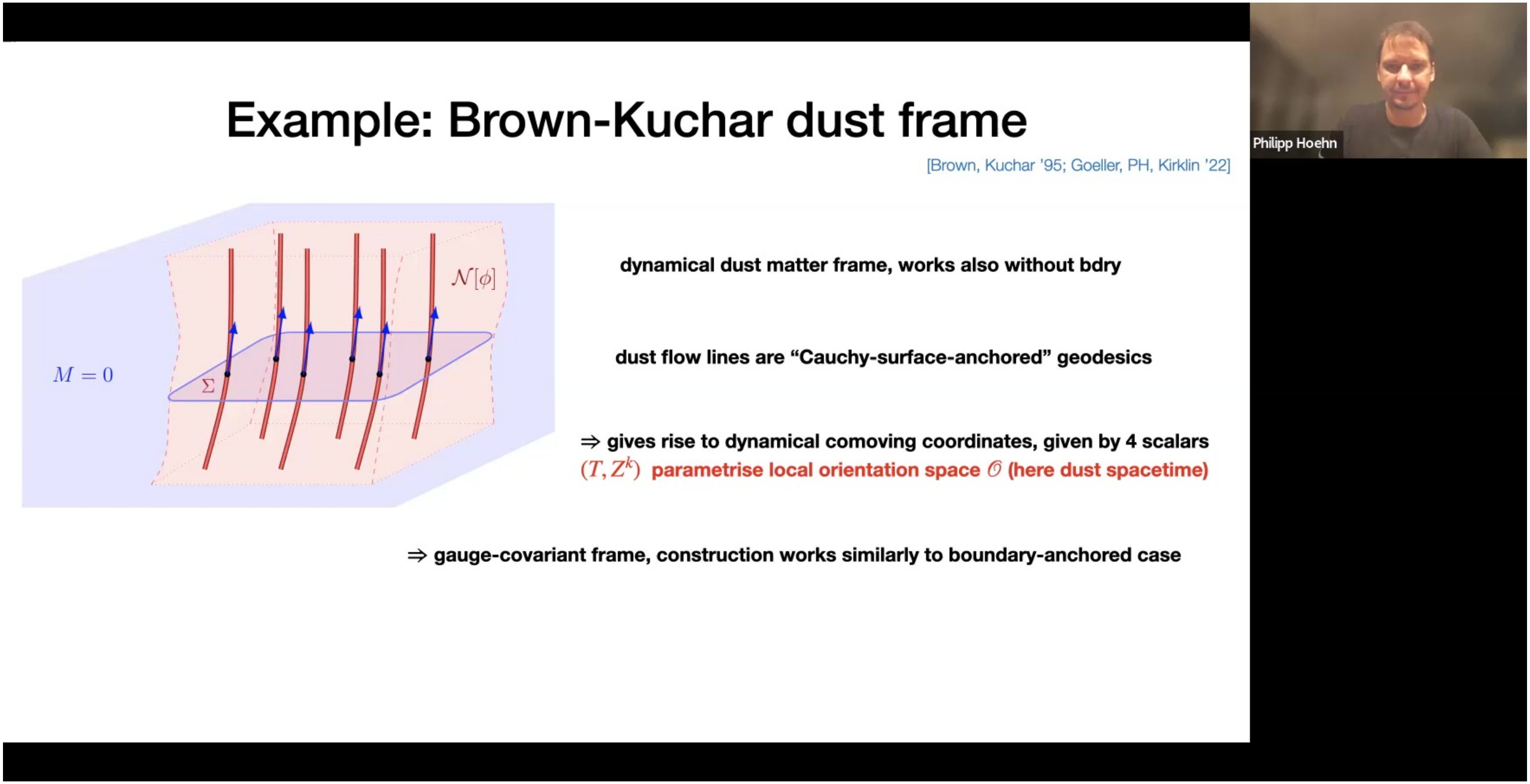


(τ, z, W) **parametrise local orientation space $\mathscr{O}$ of gauge cov. frame**

restrict to $\mathscr{O}_1 \subset \mathscr{O}$ s.t. injective (e.g. W)

⇒ get scalar frame field in some neighbourhood $\mathscr{R}_1^{-1}[g] : \mathscr{N}_1[g] \subset \mathscr{M} \to \mathscr{O}_1$

gauge-covariance under spacetime diffeos $\mathscr{R}_1^{-1}[f_* g] = \mathscr{R}_1^{-1}[g] \circ f^{-1}$



Example: Brown-Kuchar dust frame

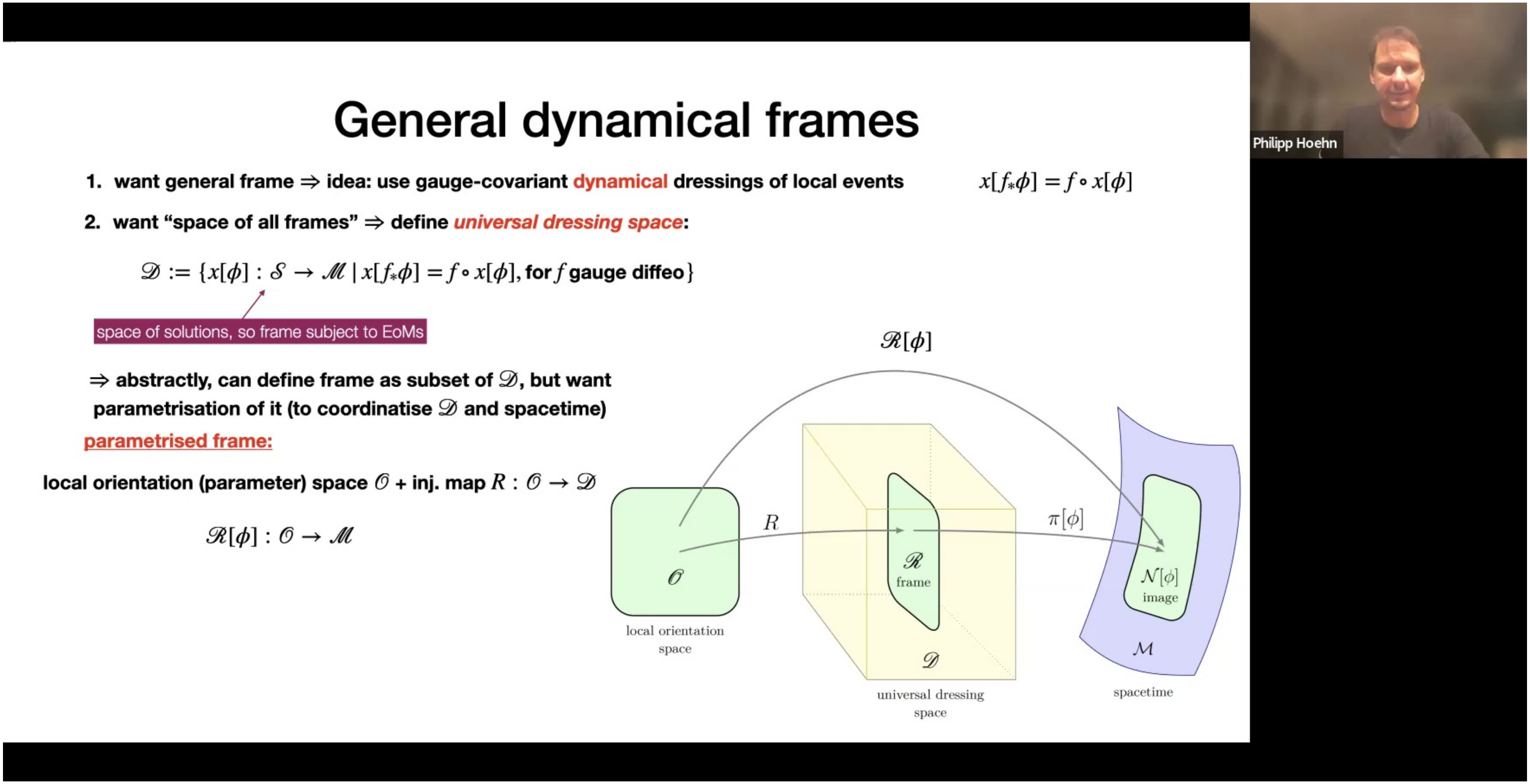
[Brown, Kuchar '95; Goeller, PH, Kirklin '22]

dynamical dust matter frame, works also without bdry

dust flow lines are "Cauchy-surface-anchored" geodesics

⇒ gives rise to dynamical comoving coordinates, given by 4 scalars
(T, Z^k) **parametrise local orientation space $\mathcal{O}$ (here dust spacetime)**

⇒ gauge-covariant frame, construction works similarly to boundary-anchored case



General dynamical frames

1. want general frame ⇒ idea: use gauge-covariant dynamical dressings of local events $x[f_*\phi] = f \circ x[\phi]$
2. want "space of all frames" ⇒ define *universal dressing space*:

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \text{for } f \text{ gauge diffeo}\}$$

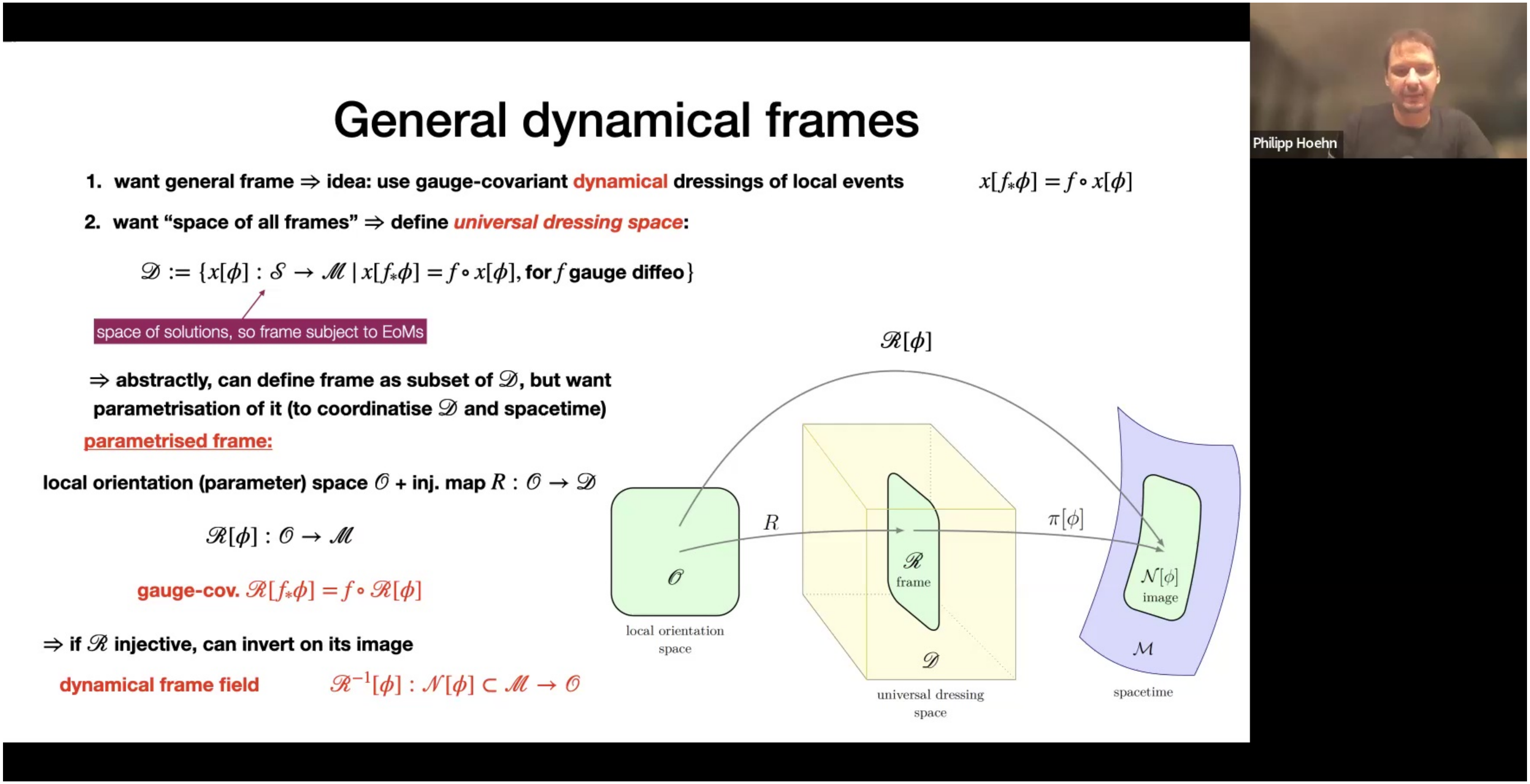
space of solutions, so frame subject to EoMs

⇒ abstractly, can define frame as subset of $\mathscr{D}$, but want parametrisation of it (to coordinatise $\mathscr{D}$ and spacetime)

parametrised frame:

local orientation (parameter) space $\mathscr{O}$ + inj. map $R : \mathscr{O} \to \mathscr{D}$

$$\mathscr{R}[\phi] : \mathscr{O} \to \mathscr{M}$$



General dynamical frames

1. **want general frame ⇒ idea: use gauge-covariant dynamical dressings of local events** $x[f_*\phi] = f \circ x[\phi]$
2. **want "space of all frames" ⇒ define *universal dressing space*:**

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \text{for } f \text{ gauge diffeo}\}$$

space of solutions, so frame subject to EoMs

⇒ abstractly, can define frame as subset of $\mathscr{D}$, but want parametrisation of it (to coordinatise $\mathscr{D}$ and spacetime)

parametrised frame:

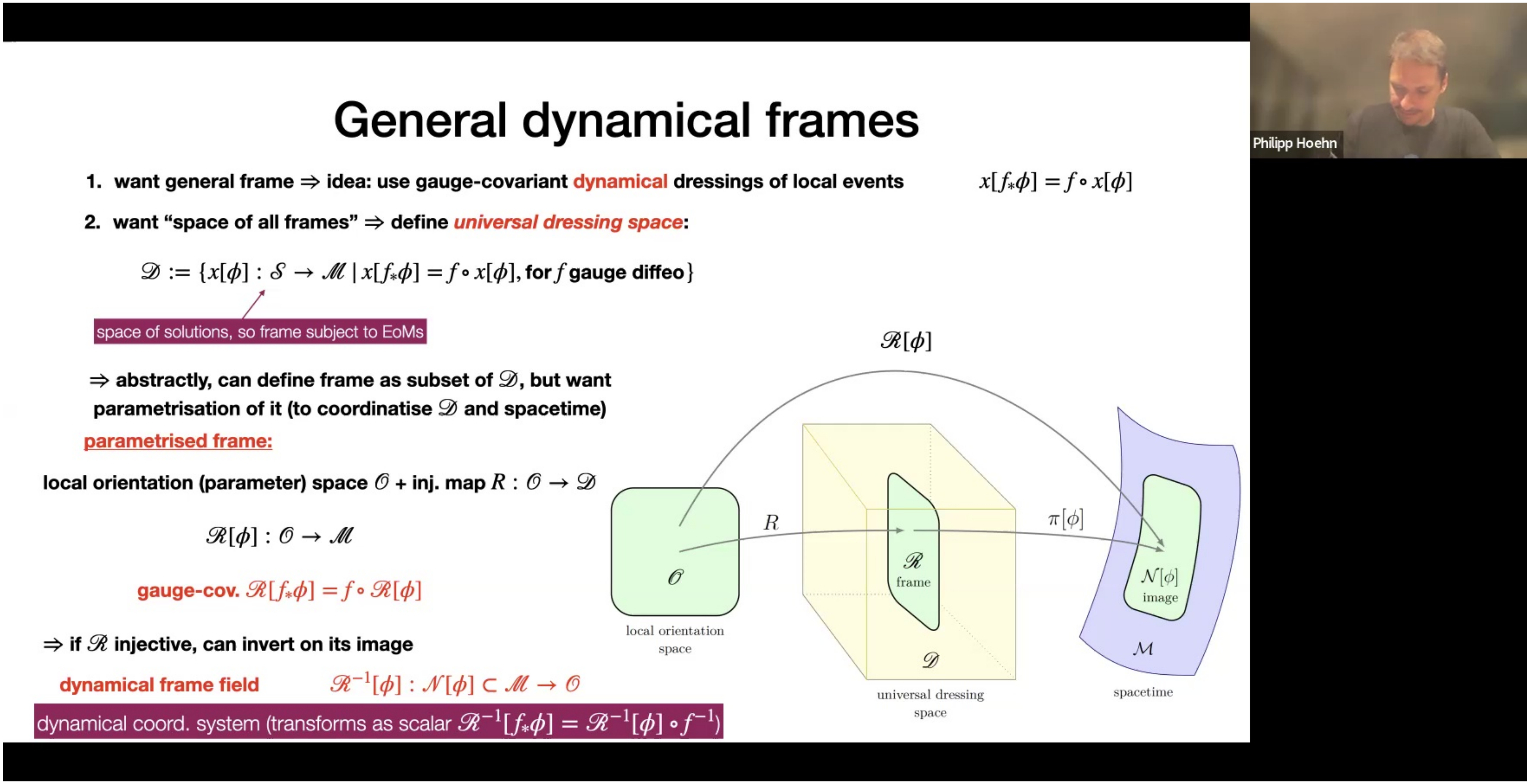
local orientation (parameter) space $\mathscr{O}$ + inj. map $R : \mathscr{O} \to \mathscr{D}$

$$\mathscr{R}[\phi] : \mathscr{O} \to \mathscr{M}$$

gauge-cov. $\mathscr{R}[f_*\phi] = f \circ \mathscr{R}[\phi]$

⇒ if $\mathscr{R}$ injective, can invert on its image

dynamical frame field $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \subset \mathscr{M} \to \mathscr{O}$



General dynamical frames

1. **want general frame ⇒ idea: use gauge-covariant dynamical dressings of local events** $x[f_*\phi] = f \circ x[\phi]$
2. **want "space of all frames" ⇒ define *universal dressing space*:**

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \text{for } f \text{ gauge diffeo}\}$$

space of solutions, so frame subject to EoMs

⇒ abstractly, can define frame as subset of $\mathscr{D}$, but want parametrisation of it (to coordinatise $\mathscr{D}$ and spacetime)

parametrised frame:

local orientation (parameter) space $\mathscr{O}$ + inj. map $R : \mathscr{O} \to \mathscr{D}$

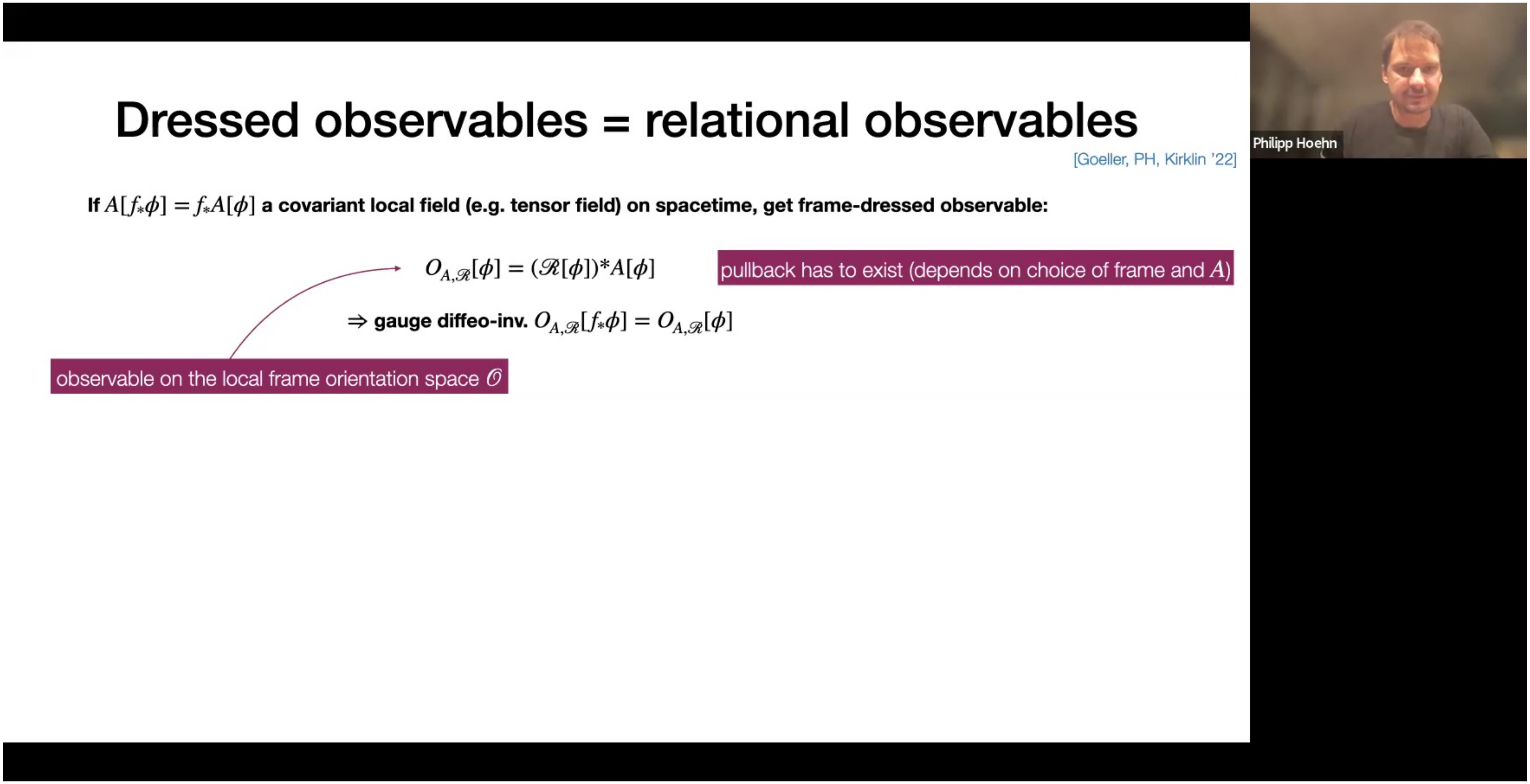
$$\mathscr{R}[\phi] : \mathscr{O} \to \mathscr{M}$$

gauge-cov. $\mathscr{R}[f_*\phi] = f \circ \mathscr{R}[\phi]$

⇒ if $\mathscr{R}$ injective, can invert on its image

dynamical frame field $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \subset \mathscr{M} \to \mathscr{O}$

dynamical coord. system (transforms as scalar $\mathscr{R}^{-1}[f_*\phi] = \mathscr{R}^{-1}[\phi] \circ f^{-1}$)



Dressed observables = relational observables

[Goeller, PH, Kirklin '22]

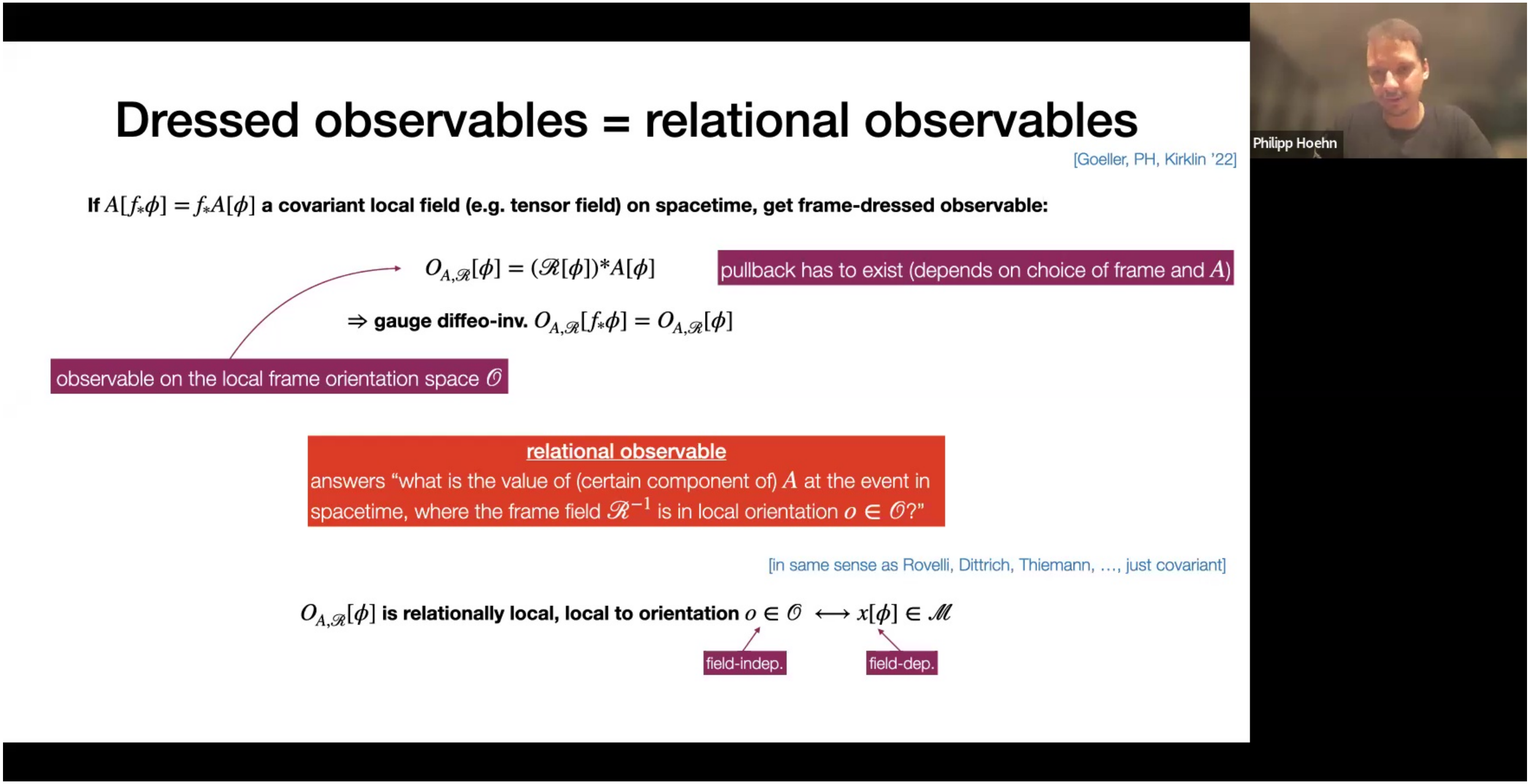
If $A[f_*\phi] = f_*A[\phi]$ **a covariant local field (e.g. tensor field) on spacetime, get frame-dressed observable:**

$$O_{A,\mathscr{R}}[\phi] = (\mathscr{R}[\phi])^*A[\phi]$$

pullback has to exist (depends on choice of frame and A)

$\Rightarrow$ **gauge diffeo-inv.** $O_{A,\mathscr{R}}[f_*\phi] = O_{A,\mathscr{R}}[\phi]$

observable on the local frame orientation space $\mathscr{O}$



Dressed observables = relational observables

[Goeller, PH, Kirklin '22]

If $A[f_*\phi] = f_*A[\phi]$ a covariant local field (e.g. tensor field) on spacetime, get frame-dressed observable:

$$O_{A,\mathscr{R}}[\phi] = (\mathscr{R}[\phi])^*A[\phi]$$

pullback has to exist (depends on choice of frame and A)

$\Rightarrow$ **gauge diffeo-inv.** $O_{A,\mathscr{R}}[f_*\phi] = O_{A,\mathscr{R}}[\phi]$

observable on the local frame orientation space $\mathscr{O}$

relational observable

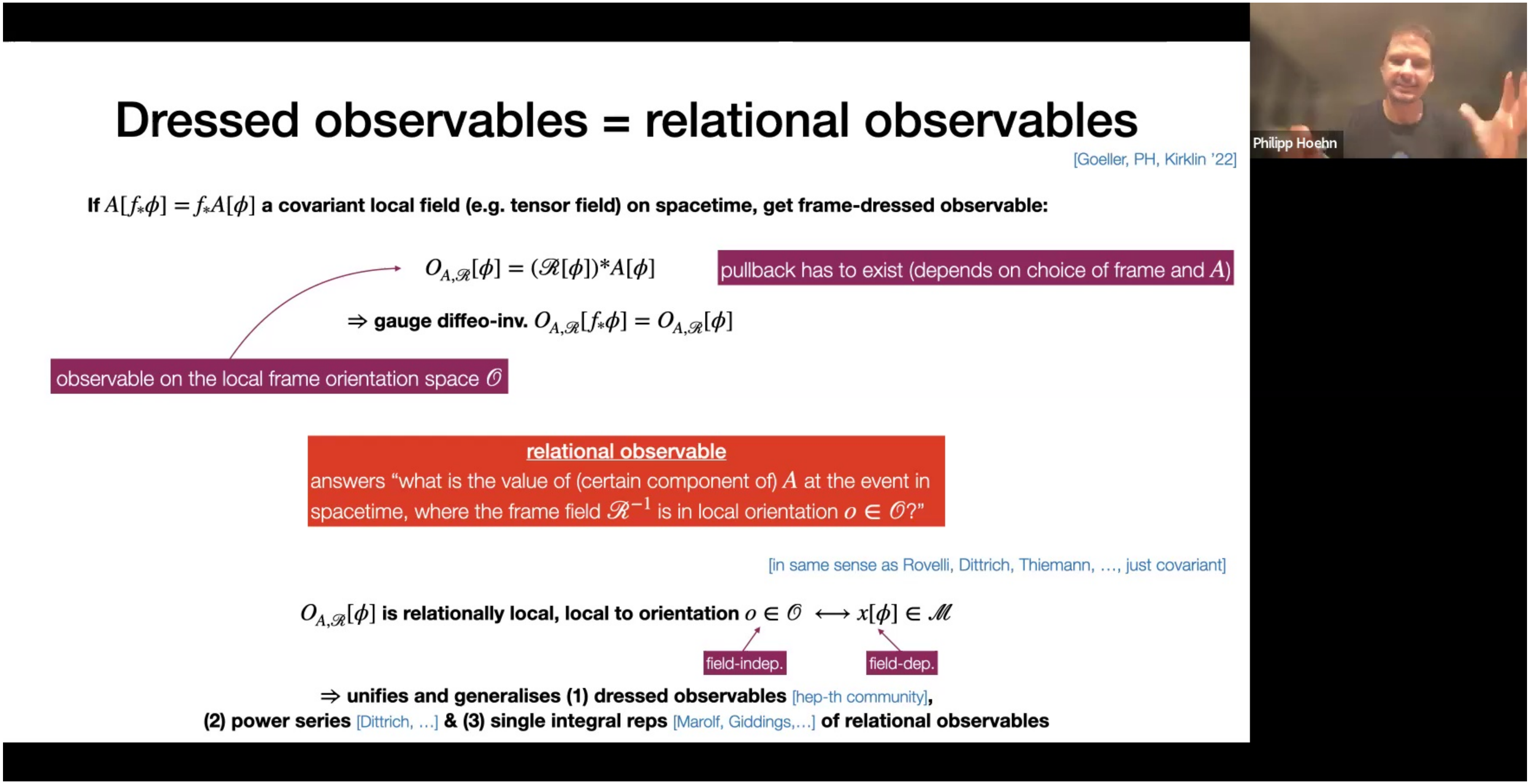
answers "what is the value of (certain component of) A at the event in spacetime, where the frame field $\mathscr{R}^{-1}$ is in local orientation $o \in \mathscr{O}$?"

[in same sense as Rovelli, Dittrich, Thiemann, …, just covariant]

$O_{A,\mathscr{R}}[\phi]$ **is relationally local, local to orientation** $o \in \mathscr{O} \;\longleftrightarrow\; x[\phi] \in \mathscr{M}$

field-indep.

field-dep.



Dressed observables = relational observables

[Goeller, PH, Kirklin '22]

If $A[f_*\phi] = f_*A[\phi]$ a covariant local field (e.g. tensor field) on spacetime, get frame-dressed observable:

$$O_{A,\mathscr{R}}[\phi] = (\mathscr{R}[\phi])^*A[\phi]$$

pullback has to exist (depends on choice of frame and A)

$\Rightarrow$ **gauge diffeo-inv.** $O_{A,\mathscr{R}}[f_*\phi] = O_{A,\mathscr{R}}[\phi]$

observable on the local frame orientation space $\mathscr{O}$

relational observable

answers "what is the value of (certain component of) A at the event in spacetime, where the frame field $\mathscr{R}^{-1}$ is in local orientation $o \in \mathscr{O}$?"

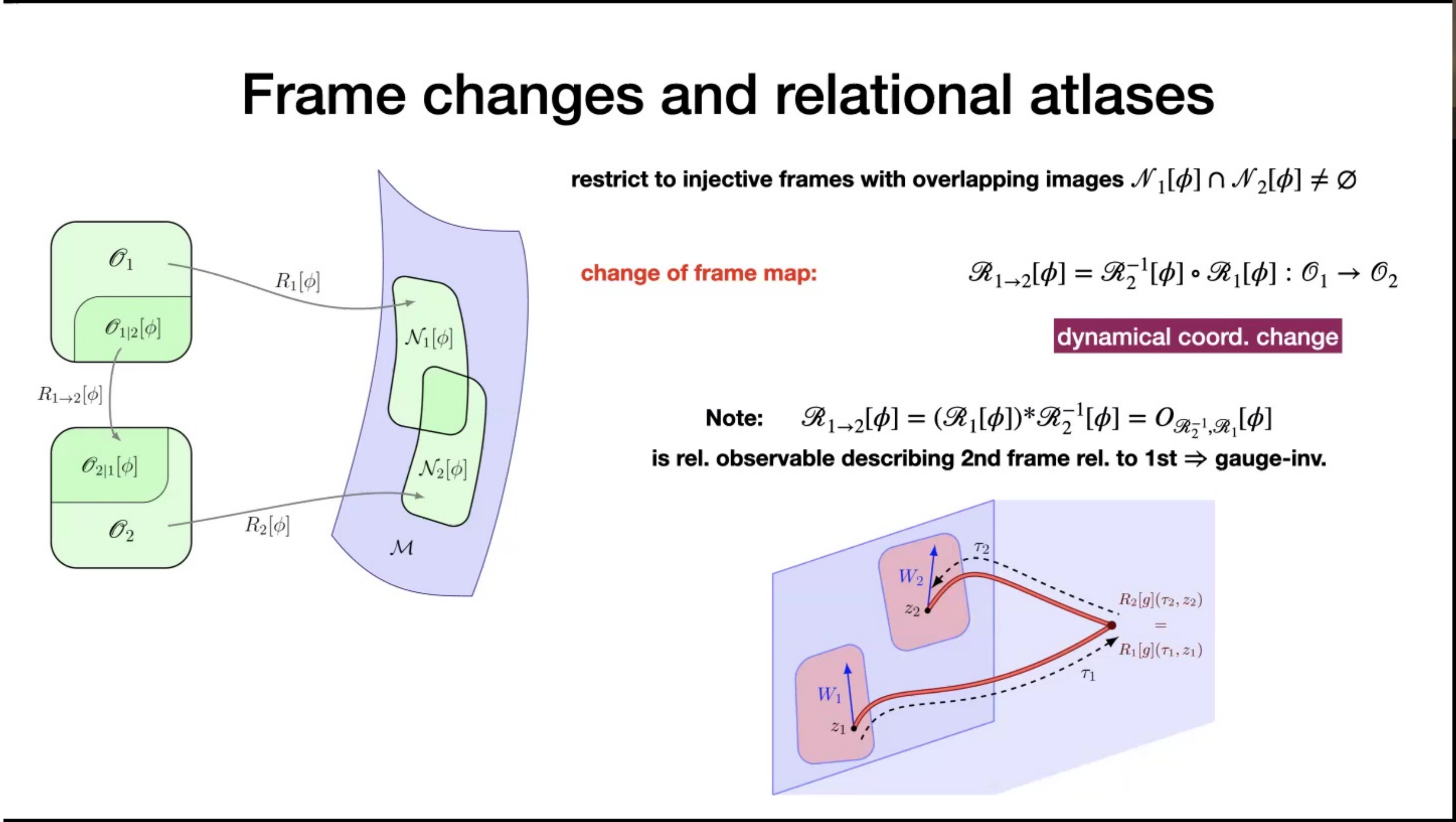
[in same sense as Rovelli, Dittrich, Thiemann, …, just covariant]

$O_{A,\mathscr{R}}[\phi]$ **is relationally local, local to orientation** $o \in \mathscr{O} \longleftrightarrow x[\phi] \in \mathscr{M}$

field-indep.

field-dep.

$\Rightarrow$ **unifies and generalises (1) dressed observables** [hep-th community]**,**
(2) power series [Dittrich, …] **& (3) single integral reps** [Marolf, Giddings,…] **of relational observables**



Frame changes and relational atlases

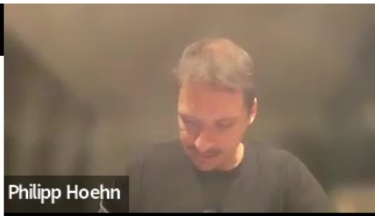
restrict to injective frames with overlapping images $\mathscr{N}_1[\phi] \cap \mathscr{N}_2[\phi] \neq \emptyset$

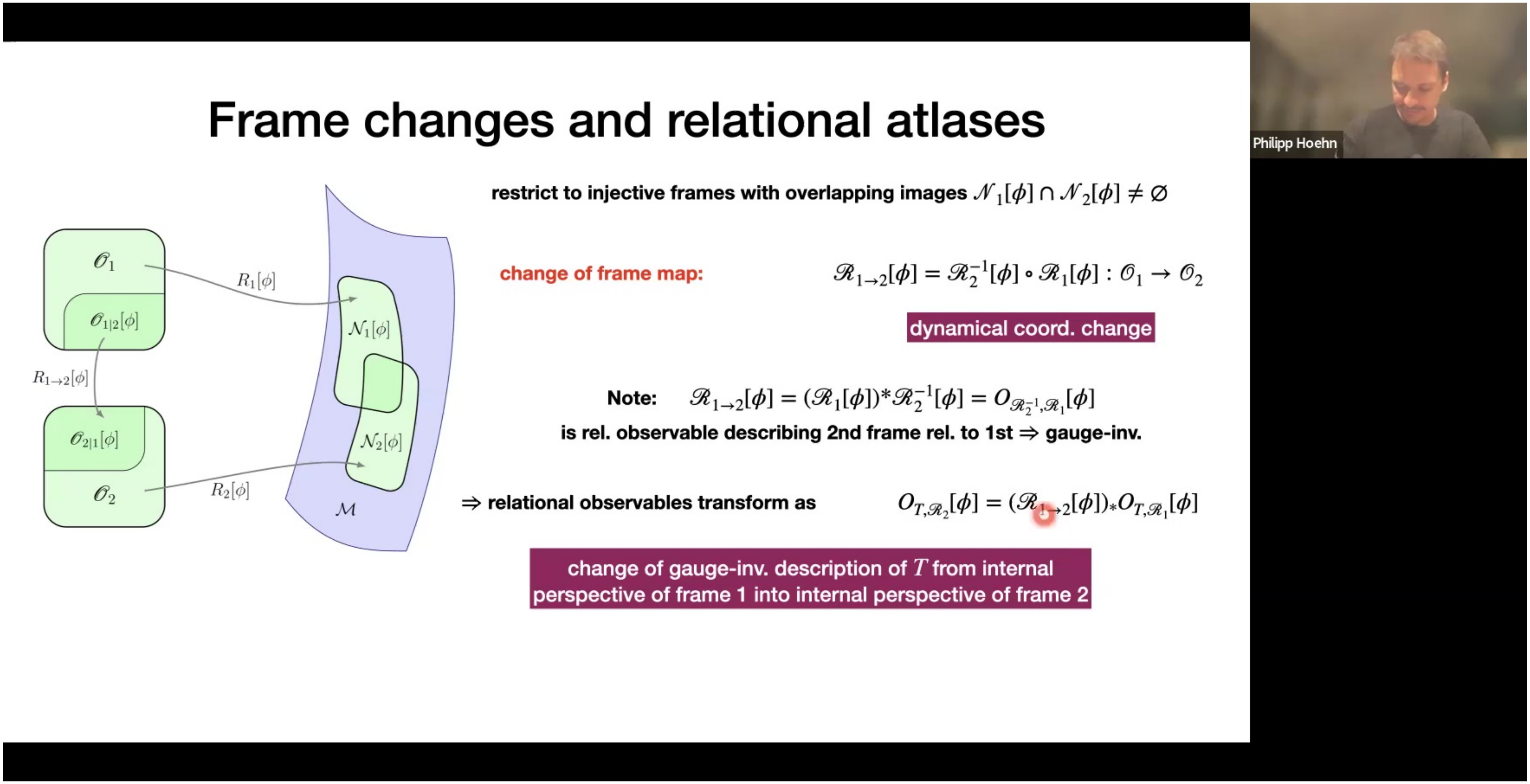
change of frame map: $\mathscr{R}_{1\to2}[\phi] = \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] : \mathscr{O}_1 \to \mathscr{O}_2$

dynamical coord. change

Note: $\mathscr{R}_{1\to2}[\phi] = (\mathscr{R}_1[\phi])^* \mathscr{R}_2^{-1}[\phi] = O_{\mathscr{R}_2^{-1},\mathscr{R}_1}[\phi]$

is rel. observable describing 2nd frame rel. to 1st ⇒ gauge-inv.





Frame changes and relational atlases

restrict to injective frames with overlapping images $\mathscr{N}_1[\phi] \cap \mathscr{N}_2[\phi] \neq \varnothing$

change of frame map: $\mathscr{R}_{1\to2}[\phi] = \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] : \mathscr{O}_1 \to \mathscr{O}_2$

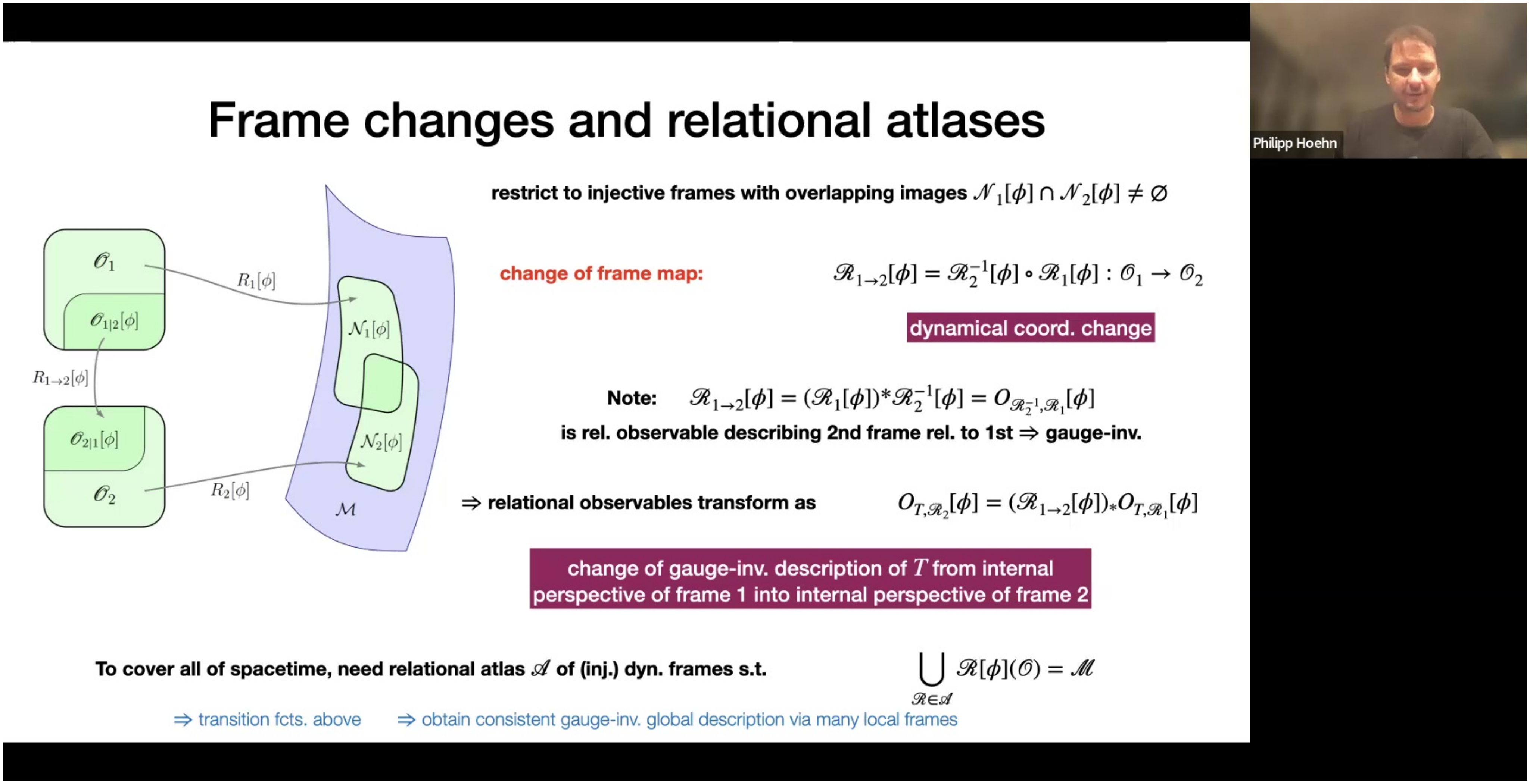
dynamical coord. change

Note: $\mathscr{R}_{1\to2}[\phi] = (\mathscr{R}_1[\phi])^* \mathscr{R}_2^{-1}[\phi] = O_{\mathscr{R}_2^{-1},\mathscr{R}_1}[\phi]$

is rel. observable describing 2nd frame rel. to 1st ⇒ gauge-inv.

⇒ relational observables transform as $O_{T,\mathscr{R}_2}[\phi] = (\mathscr{R}_{1\to2}[\phi])_* O_{T,\mathscr{R}_1}[\phi]$

change of gauge-inv. description of T from internal perspective of frame 1 into internal perspective of frame 2



Frame changes and relational atlases

restrict to injective frames with overlapping images $\mathcal{N}_1[\phi] \cap \mathcal{N}_2[\phi] \neq \emptyset$

change of frame map: $\mathscr{R}_{1\to 2}[\phi] = \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] : \mathscr{O}_1 \to \mathscr{O}_2$

dynamical coord. change

Note: $\mathscr{R}_{1\to 2}[\phi] = (\mathscr{R}_1[\phi])^* \mathscr{R}_2^{-1}[\phi] = O_{\mathscr{R}_2^{-1},\mathscr{R}_1}[\phi]$

is rel. observable describing 2nd frame rel. to 1st ⇒ gauge-inv.

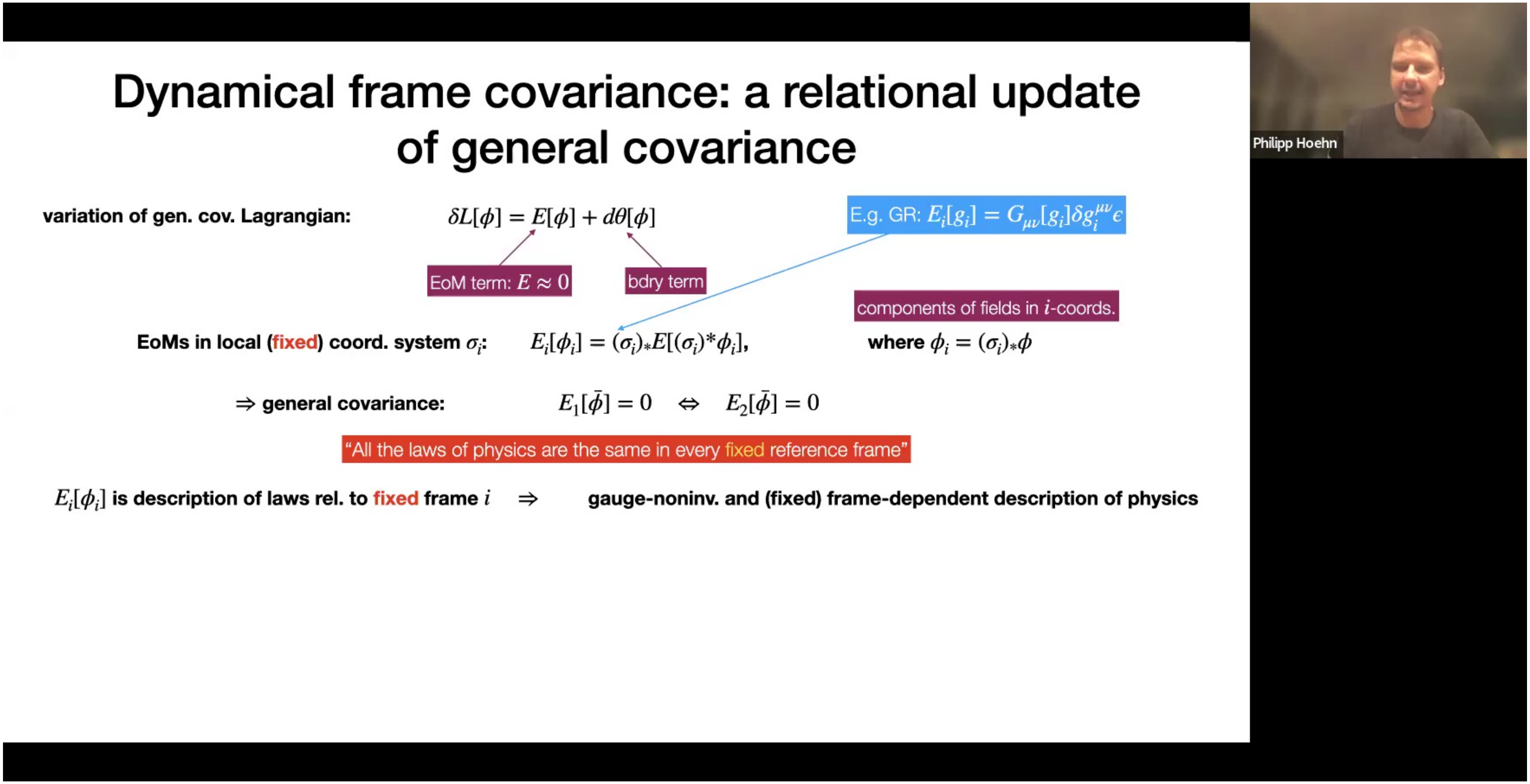
⇒ relational observables transform as $O_{T,\mathscr{R}_2}[\phi] = (\mathscr{R}_{1\to 2}[\phi])_* O_{T,\mathscr{R}_1}[\phi]$

change of gauge-inv. description of T from internal perspective of frame 1 into internal perspective of frame 2

To cover all of spacetime, need relational atlas $\mathscr{A}$ of (inj.) dyn. frames s.t.

$$\bigcup_{\mathscr{R}\in\mathscr{A}} \mathscr{R}[\phi](\mathscr{O}) = \mathscr{M}$$

⇒ transition fcts. above ⇒ obtain consistent gauge-inv. global description via many local frames



Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

EoM term: $E \approx 0$

bdry term

E.g. GR: $E_i[g_i] = G_{\mu\nu}[g_i]\delta g_i^{\mu\nu}\epsilon$

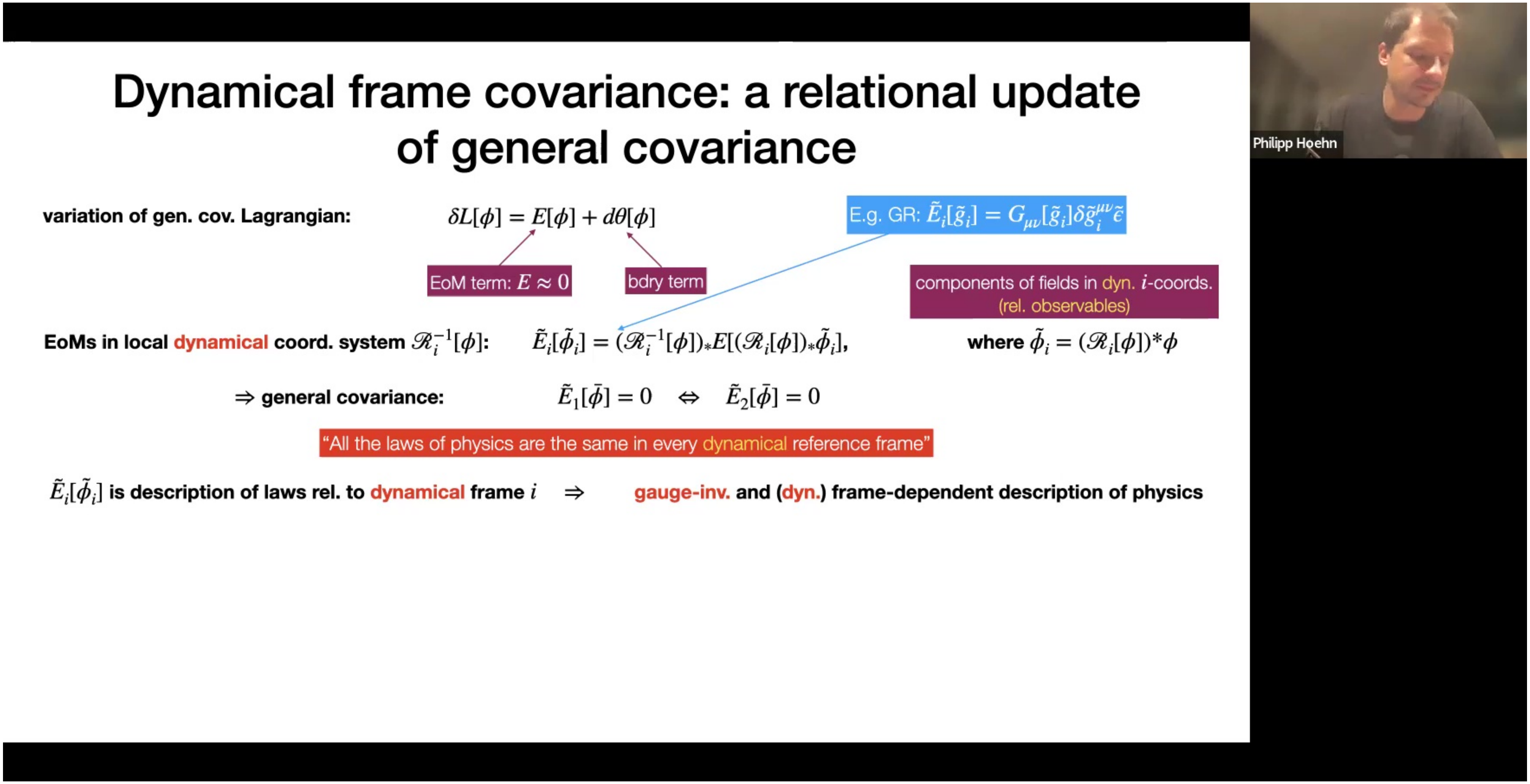
components of fields in i-coords.

EoMs in local (fixed) coord. system σ_i: $E_i[\phi_i] = (\sigma_i)_* E[(\sigma_i)^* \phi_i]$, **where** $\phi_i = (\sigma_i)_* \phi$

⇒ **general covariance:** $E_1[\bar{\phi}] = 0 \quad \Leftrightarrow \quad E_2[\bar{\phi}] = 0$

“All the laws of physics are the same in every fixed reference frame”

$E_i[\phi_i]$ **is description of laws rel. to fixed frame** i ⇒ **gauge-noninv. and (fixed) frame-dependent description of physics**



Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

EoM term: $E \approx 0$

bdry term

E.g. GR: $\tilde{E}_i[\tilde{g}_i] = G_{\mu\nu}[\tilde{g}_i]\delta\tilde{g}_i^{\mu\nu}\tilde{\epsilon}$

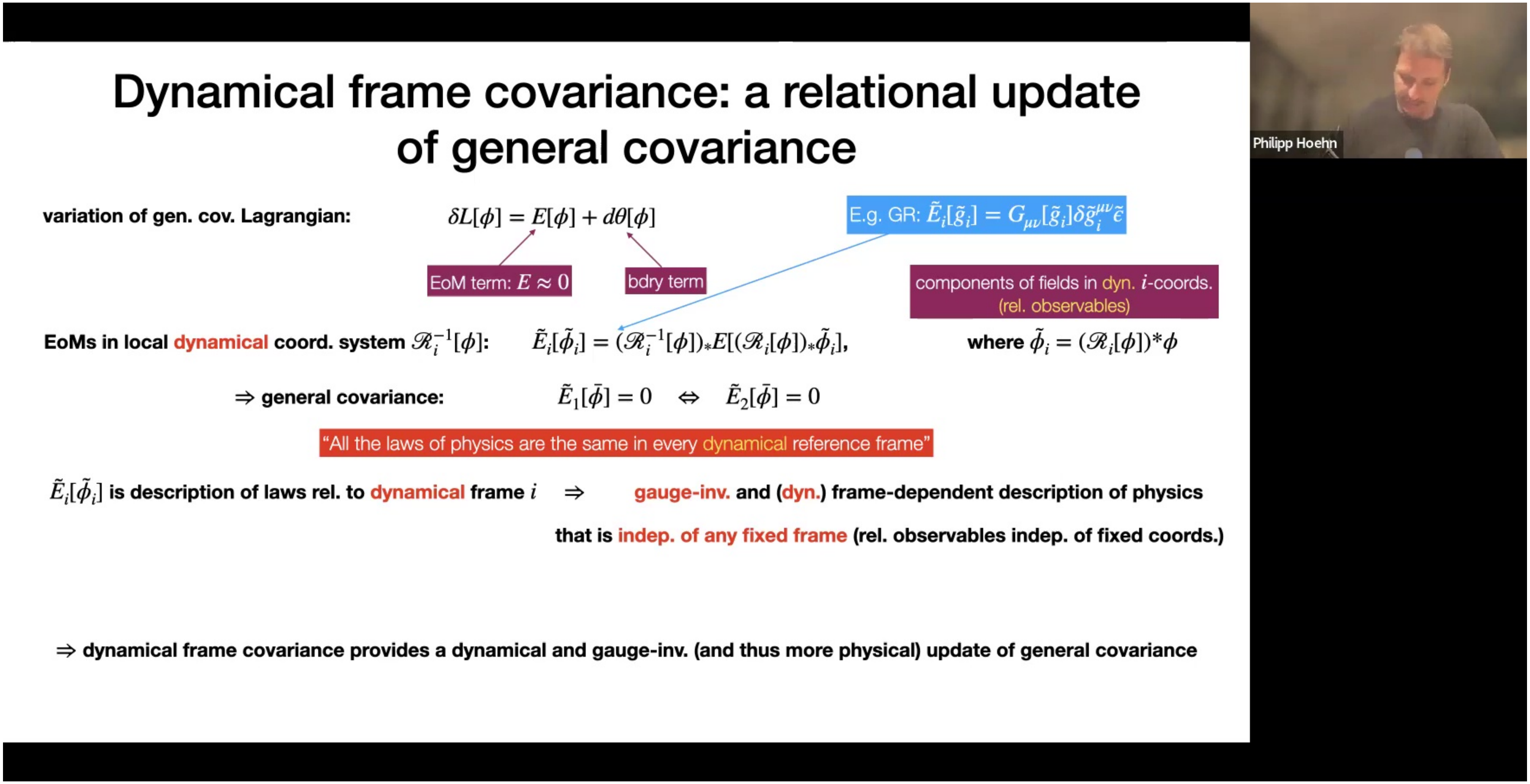
components of fields in dyn. i-coords. (rel. observables)

EoMs in local dynamical coord. system $\mathscr{R}_i^{-1}[\phi]$: $\tilde{E}_i[\tilde{\phi}_i] = (\mathscr{R}_i^{-1}[\phi])_* E[(\mathscr{R}_i[\phi])_* \tilde{\phi}_i]$, **where** $\tilde{\phi}_i = (\mathscr{R}_i[\phi])^* \phi$

⇒ **general covariance:** $\tilde{E}_1[\bar{\phi}] = 0 \quad \Leftrightarrow \quad \tilde{E}_2[\bar{\phi}] = 0$

"All the laws of physics are the same in every dynamical reference frame"

$\tilde{E}_i[\tilde{\phi}_i]$ **is description of laws rel. to dynamical frame** i ⇒ **gauge-inv. and (dyn.) frame-dependent description of physics**



Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

EoM term: $E \approx 0$

bdry term

E.g. GR: $\tilde{E}_i[\tilde{g}_i] = G_{\mu\nu}[\tilde{g}_i]\delta\tilde{g}_i^{\mu\nu}\tilde{\epsilon}$

components of fields in dyn. i-coords. (rel. observables)

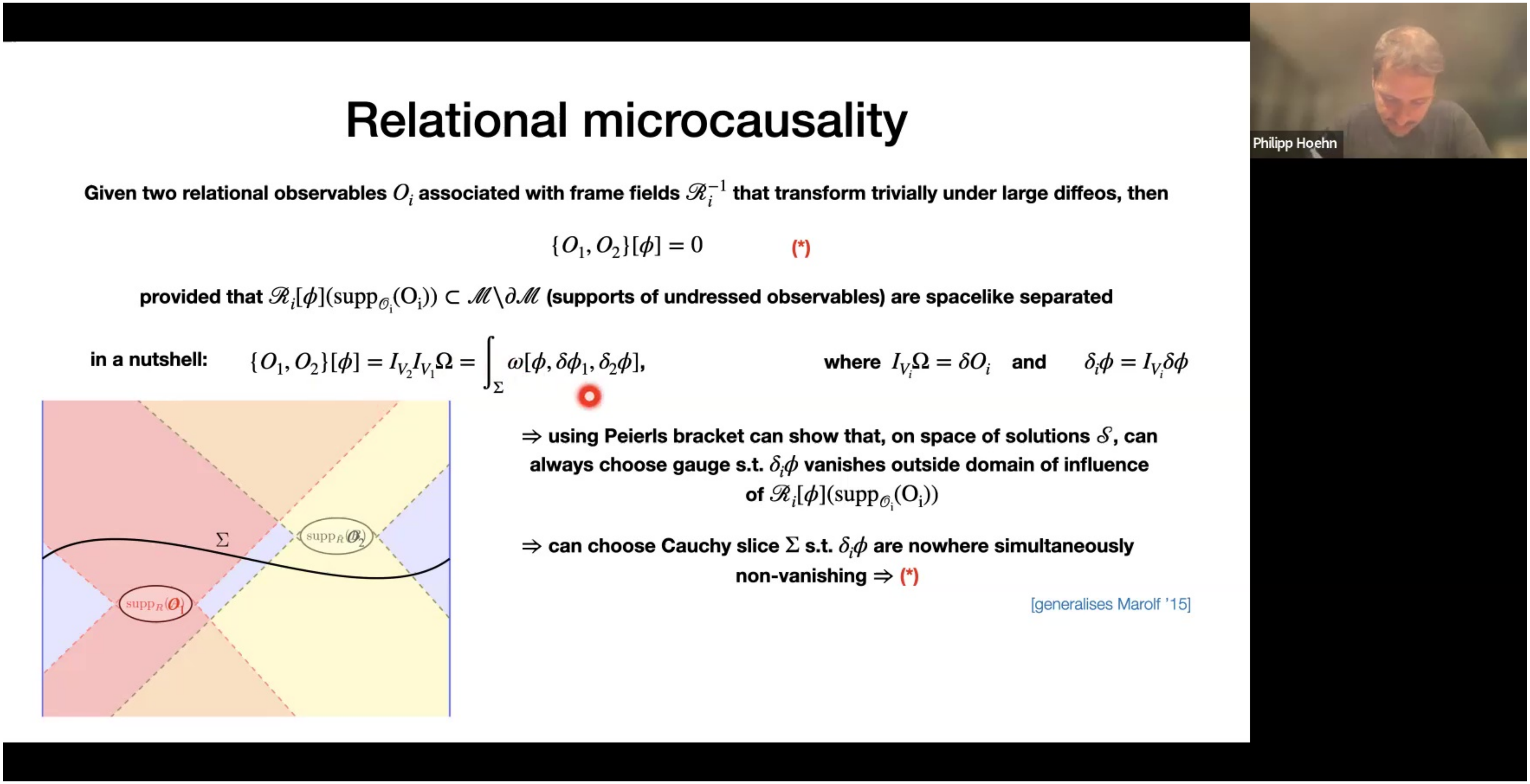
EoMs in local dynamical coord. system $\mathscr{R}_i^{-1}[\phi]$: $\tilde{E}_i[\tilde{\phi}_i] = (\mathscr{R}_i^{-1}[\phi])_* E[(\mathscr{R}_i[\phi])_* \tilde{\phi}_i]$, **where** $\tilde{\phi}_i = (\mathscr{R}_i[\phi])^* \phi$

⇒ **general covariance:** $\tilde{E}_1[\bar{\phi}] = 0 \quad \Leftrightarrow \quad \tilde{E}_2[\bar{\phi}] = 0$

"All the laws of physics are the same in every dynamical reference frame"

$\tilde{E}_i[\tilde{\phi}_i]$ **is description of laws rel. to dynamical frame** i ⇒ **gauge-inv. and (dyn.) frame-dependent description of physics that is indep. of any fixed frame (rel. observables indep. of fixed coords.)**

⇒ **dynamical frame covariance provides a dynamical and gauge-inv. (and thus more physical) update of general covariance**



Relational microcausality

Given two relational observables O_i associated with frame fields $\mathscr{R}_i^{-1}$ that transform trivially under large diffeos, then

$$\{O_1, O_2\}[\phi] = 0 \qquad \text{(*)}$$

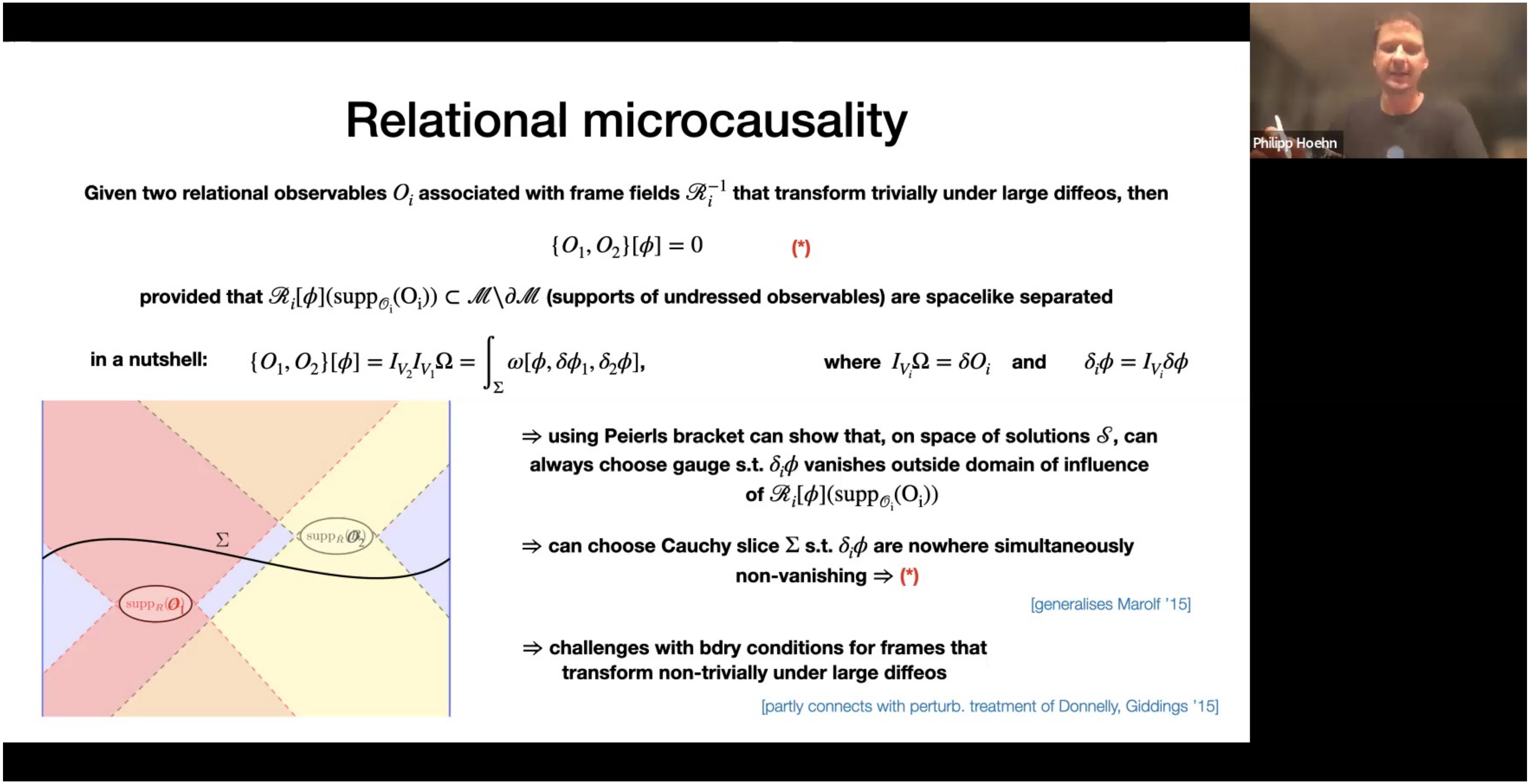
provided that $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathcal{O}_i}(\mathrm{O_i})) \subset \mathscr{M}\backslash\partial\mathscr{M}$ (supports of undressed observables) are spacelike separated

in a nutshell: $\{O_1, O_2\}[\phi] = I_{V_2} I_{V_1}\Omega = \int_\Sigma \omega[\phi, \delta\phi_1, \delta_2\phi]$, **where** $I_{V_i}\Omega = \delta O_i$ **and** $\delta_i\phi = I_{V_i}\delta\phi$

⇒ using Peierls bracket can show that, on space of solutions $\mathcal{S}$, can always choose gauge s.t. $\delta_i\phi$ vanishes outside domain of influence of $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathcal{O}_i}(\mathrm{O_i}))$

⇒ can choose Cauchy slice Σ s.t. $\delta_i\phi$ are nowhere simultaneously non-vanishing ⇒ (*)

[generalises Marolf '15]



Relational microcausality

Given two relational observables O_i associated with frame fields $\mathscr{R}_i^{-1}$ that transform trivially under large diffeos, then

$$\{O_1, O_2\}[\phi] = 0 \qquad \text{(*)}$$

provided that $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_i}(\mathrm{O_i})) \subset \mathscr{M}\backslash\partial\mathscr{M}$ (supports of undressed observables) are spacelike separated

in a nutshell: $\{O_1, O_2\}[\phi] = I_{V_2} I_{V_1}\Omega = \int_\Sigma \omega[\phi, \delta\phi_1, \delta_2\phi],$ **where** $I_{V_i}\Omega = \delta O_i$ **and** $\delta_i\phi = I_{V_i}\delta\phi$

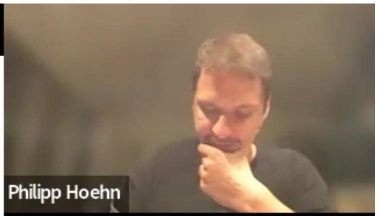
⇒ using Peierls bracket can show that, on space of solutions $\mathscr{S}$, can always choose gauge s.t. $\delta_i\phi$ vanishes outside domain of influence of $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_i}(\mathrm{O_i}))$

⇒ can choose Cauchy slice Σ s.t. $\delta_i\phi$ are nowhere simultaneously non-vanishing ⇒ (*)

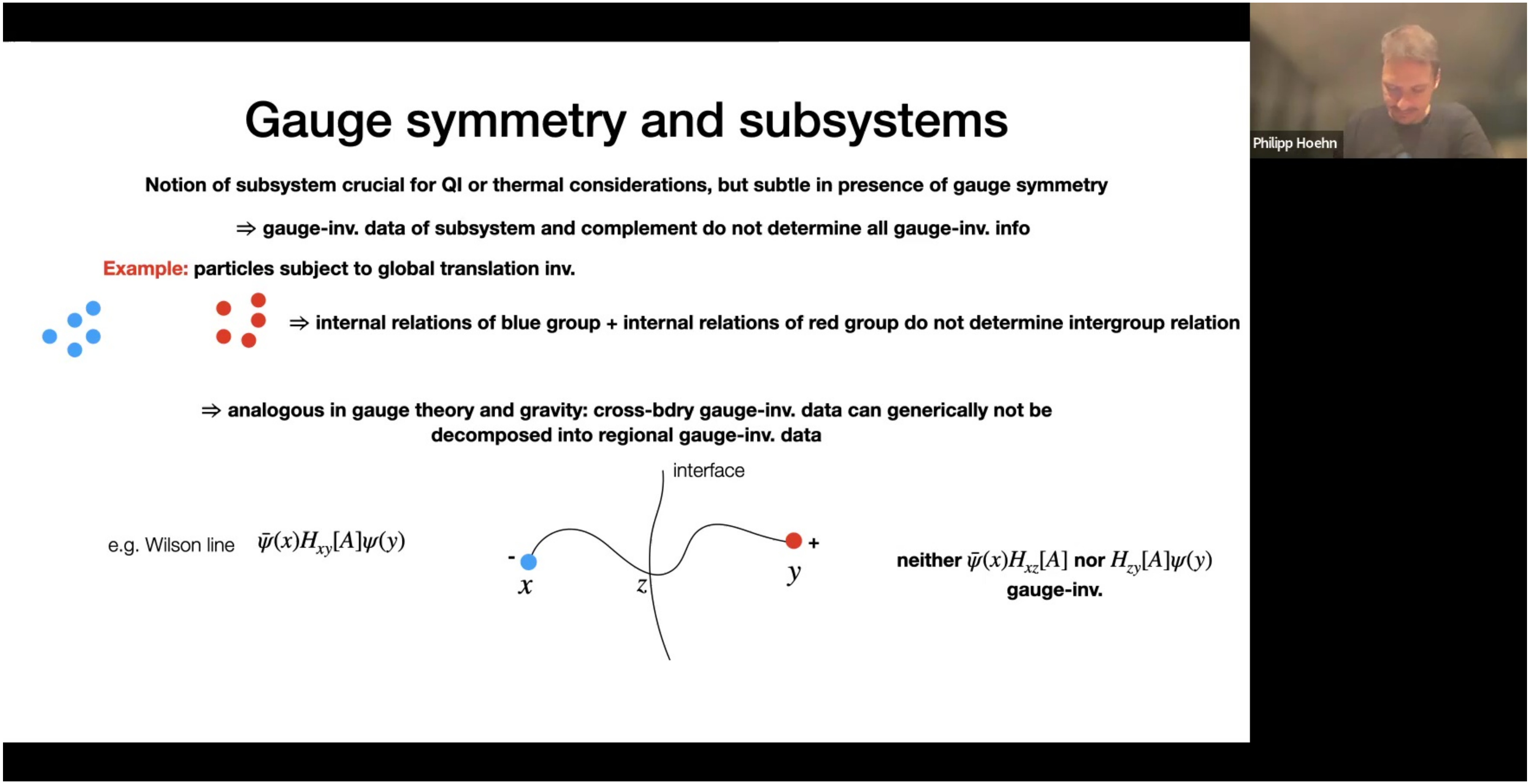
[generalises Marolf '15]

⇒ challenges with bdry conditions for frames that transform non-trivially under large diffeos

[partly connects with perturb. treatment of Donnelly, Giddings '15]



Local subsystems relative to a dynamical frame



Gauge symmetry and subsystems

Notion of subsystem crucial for QI or thermal considerations, but subtle in presence of gauge symmetry

⇒ gauge-inv. data of subsystem and complement do not determine all gauge-inv. info

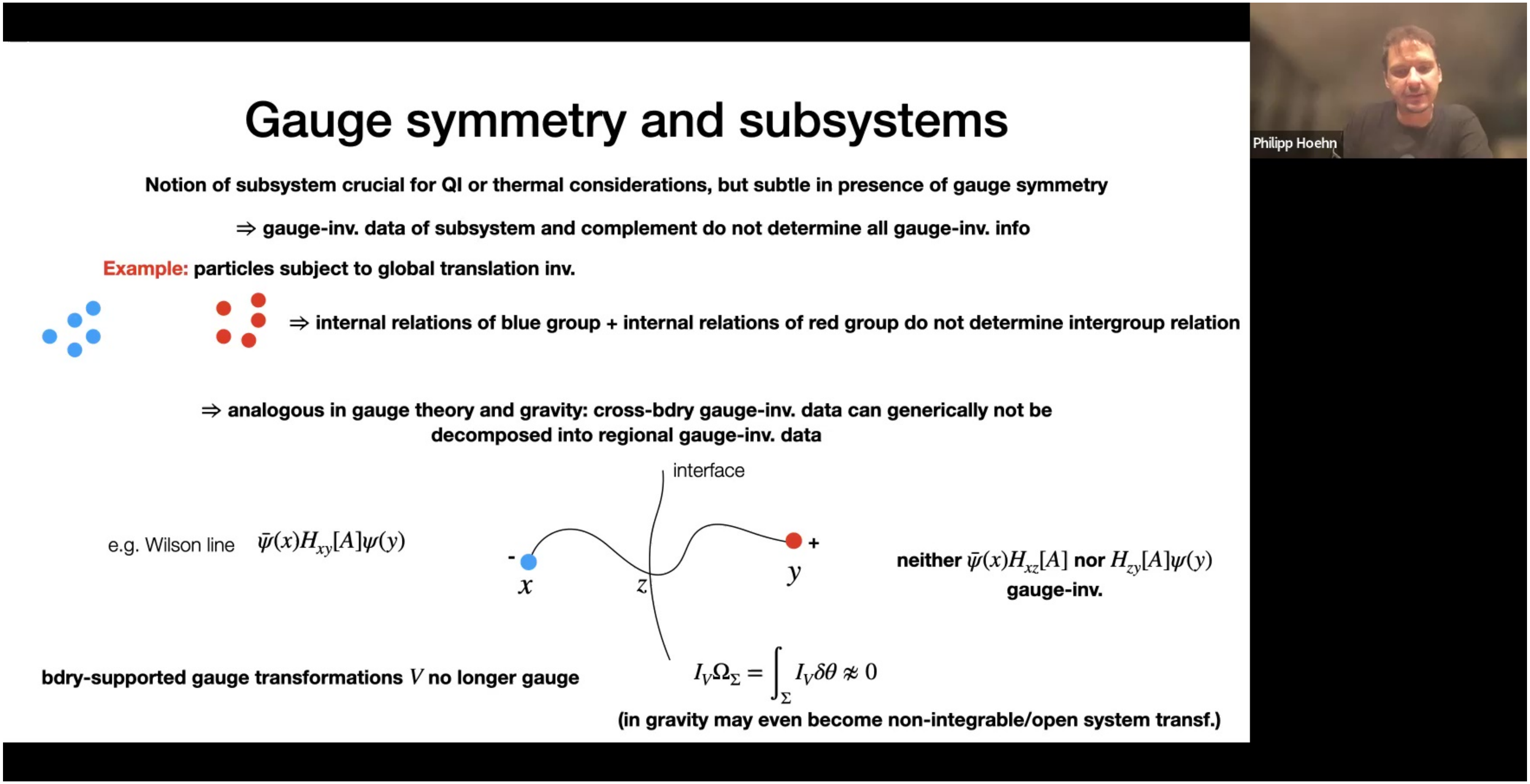
Example: particles subject to global translation inv.

⇒ internal relations of blue group + internal relations of red group do not determine intergroup relation

⇒ analogous in gauge theory and gravity: cross-bdry gauge-inv. data can generically not be decomposed into regional gauge-inv. data

e.g. Wilson line $\bar{\psi}(x)H_{xy}[A]\psi(y)$

neither $\bar{\psi}(x)H_{xz}[A]$ nor $H_{zy}[A]\psi(y)$ gauge-inv.



Gauge symmetry and subsystems

Notion of subsystem crucial for QI or thermal considerations, but subtle in presence of gauge symmetry

⇒ gauge-inv. data of subsystem and complement do not determine all gauge-inv. info

Example: **particles subject to global translation inv.**

⇒ internal relations of blue group + internal relations of red group do not determine intergroup relation

⇒ analogous in gauge theory and gravity: cross-bdry gauge-inv. data can generically not be decomposed into regional gauge-inv. data

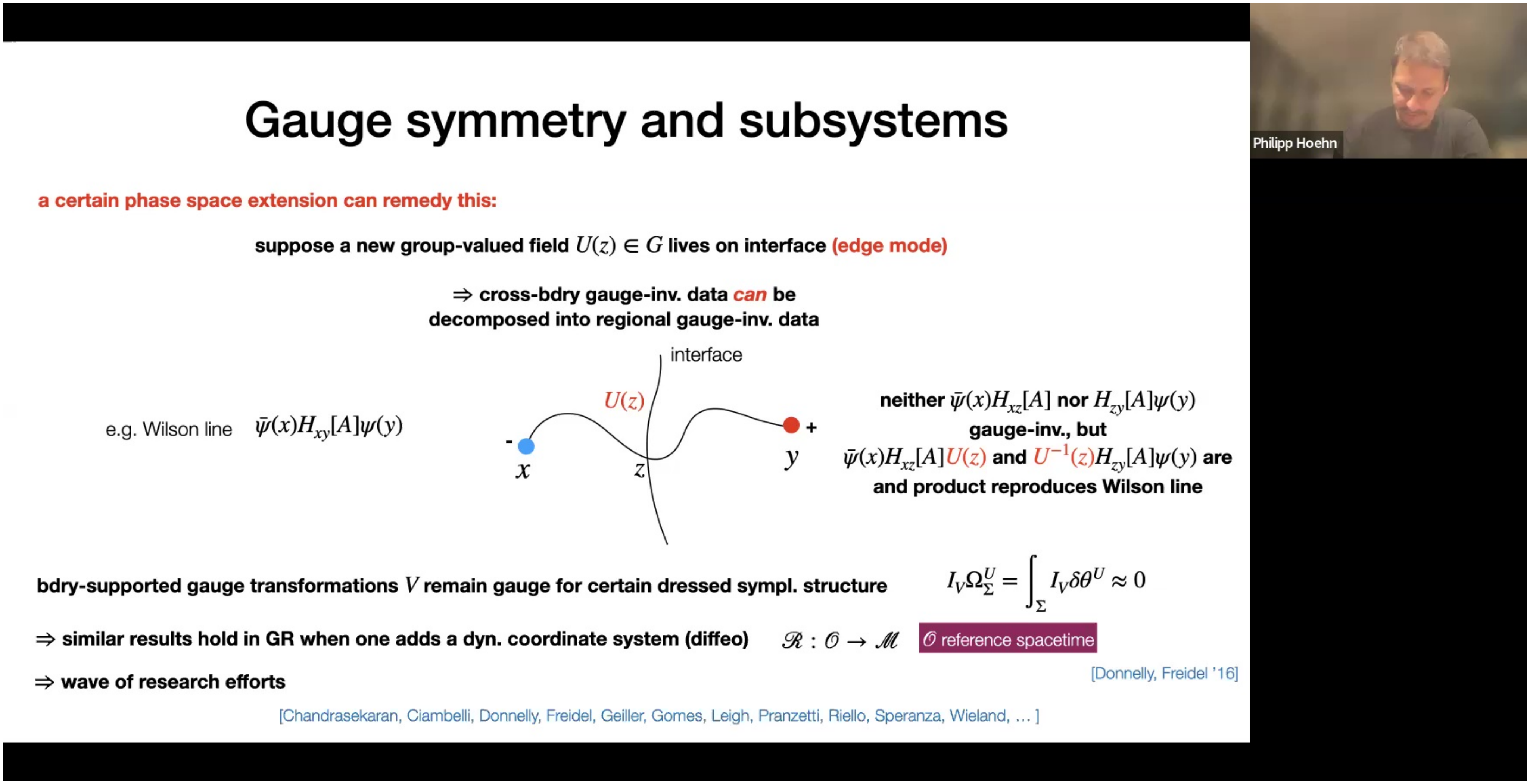
e.g. Wilson line $\bar{\psi}(x)H_{xy}[A]\psi(y)$

neither $\bar{\psi}(x)H_{xz}[A]$ nor $H_{zy}[A]\psi(y)$ gauge-inv.

bdry-supported gauge transformations V no longer gauge

$$I_V\Omega_\Sigma = \int_\Sigma I_V\delta\theta \not\approx 0$$

(in gravity may even become non-integrable/open system transf.)



Gauge symmetry and subsystems

a certain phase space extension can remedy this:

suppose a new group-valued field $U(z) \in G$ lives on interface (edge mode)

$\Rightarrow$ cross-bdry gauge-inv. data *can* be decomposed into regional gauge-inv. data

e.g. Wilson line $\bar{\psi}(x)H_{xy}[A]\psi(y)$

neither $\bar{\psi}(x)H_{xz}[A]$ nor $H_{zy}[A]\psi(y)$ gauge-inv., but $\bar{\psi}(x)H_{xz}[A]U(z)$ and $U^{-1}(z)H_{zy}[A]\psi(y)$ are and product reproduces Wilson line

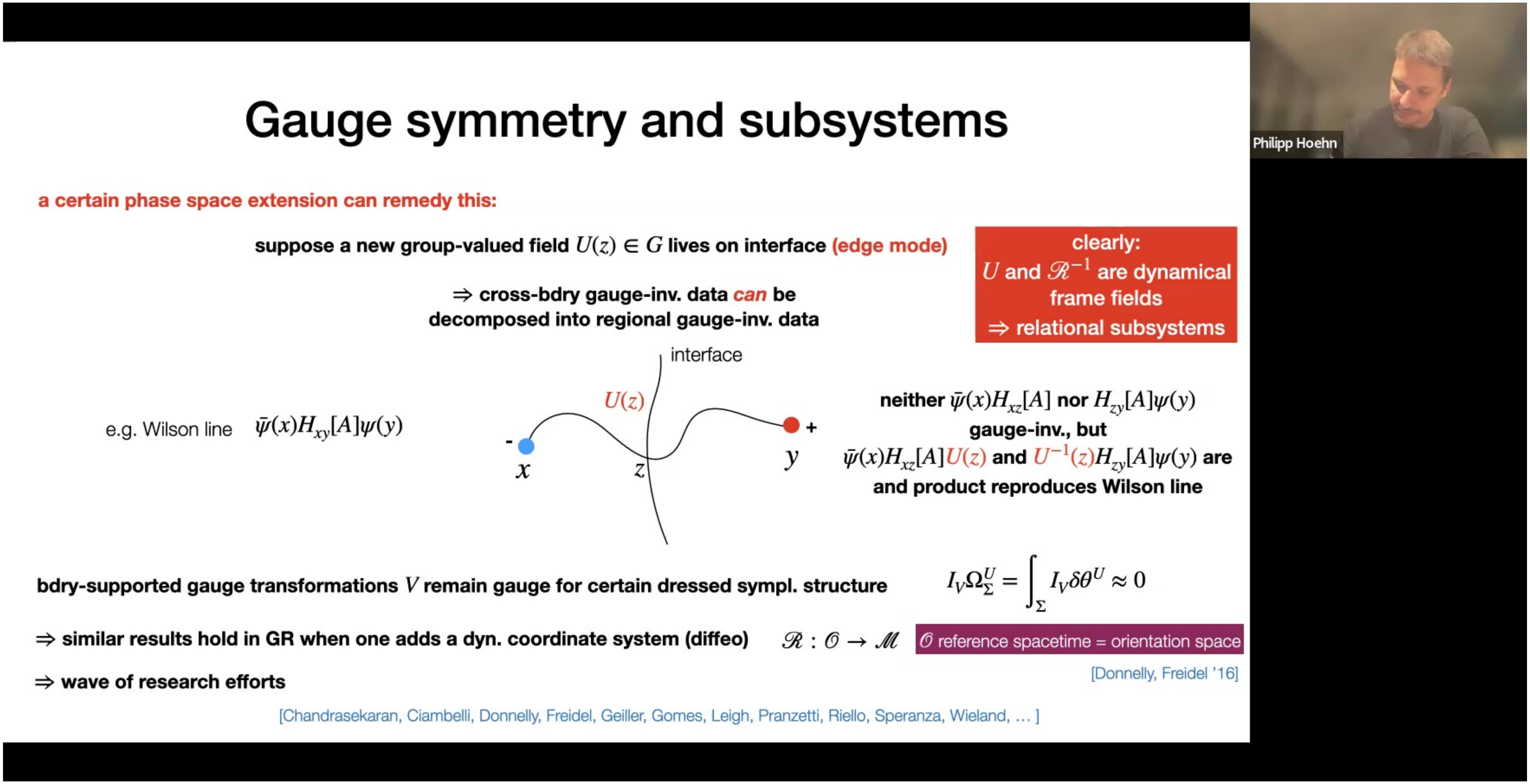
bdry-supported gauge transformations V remain gauge for certain dressed sympl. structure $I_V\Omega^U_\Sigma = \int_\Sigma I_V\delta\theta^U \approx 0$

$\Rightarrow$ similar results hold in GR when one adds a dyn. coordinate system (diffeo) $\mathscr{R} : \mathscr{O} \to \mathscr{M}$ $\mathscr{O}$ reference spacetime

[Donnelly, Freidel '16]

$\Rightarrow$ wave of research efforts

[Chandrasekaran, Ciambelli, Donnelly, Freidel, Geiller, Gomes, Leigh, Pranzetti, Riello, Speranza, Wieland, …]



Gauge symmetry and subsystems

a certain phase space extension can remedy this:

suppose a new group-valued field $U(z) \in G$ lives on interface (edge mode)

$\Rightarrow$ cross-bdry gauge-inv. data *can* be decomposed into regional gauge-inv. data

clearly:
U and $\mathscr{R}^{-1}$ are dynamical frame fields
$\Rightarrow$ relational subsystems

e.g. Wilson line $\bar{\psi}(x)H_{xy}[A]\psi(y)$

neither $\bar{\psi}(x)H_{xz}[A]$ nor $H_{zy}[A]\psi(y)$ gauge-inv., but $\bar{\psi}(x)H_{xz}[A]U(z)$ and $U^{-1}(z)H_{zy}[A]\psi(y)$ are and product reproduces Wilson line

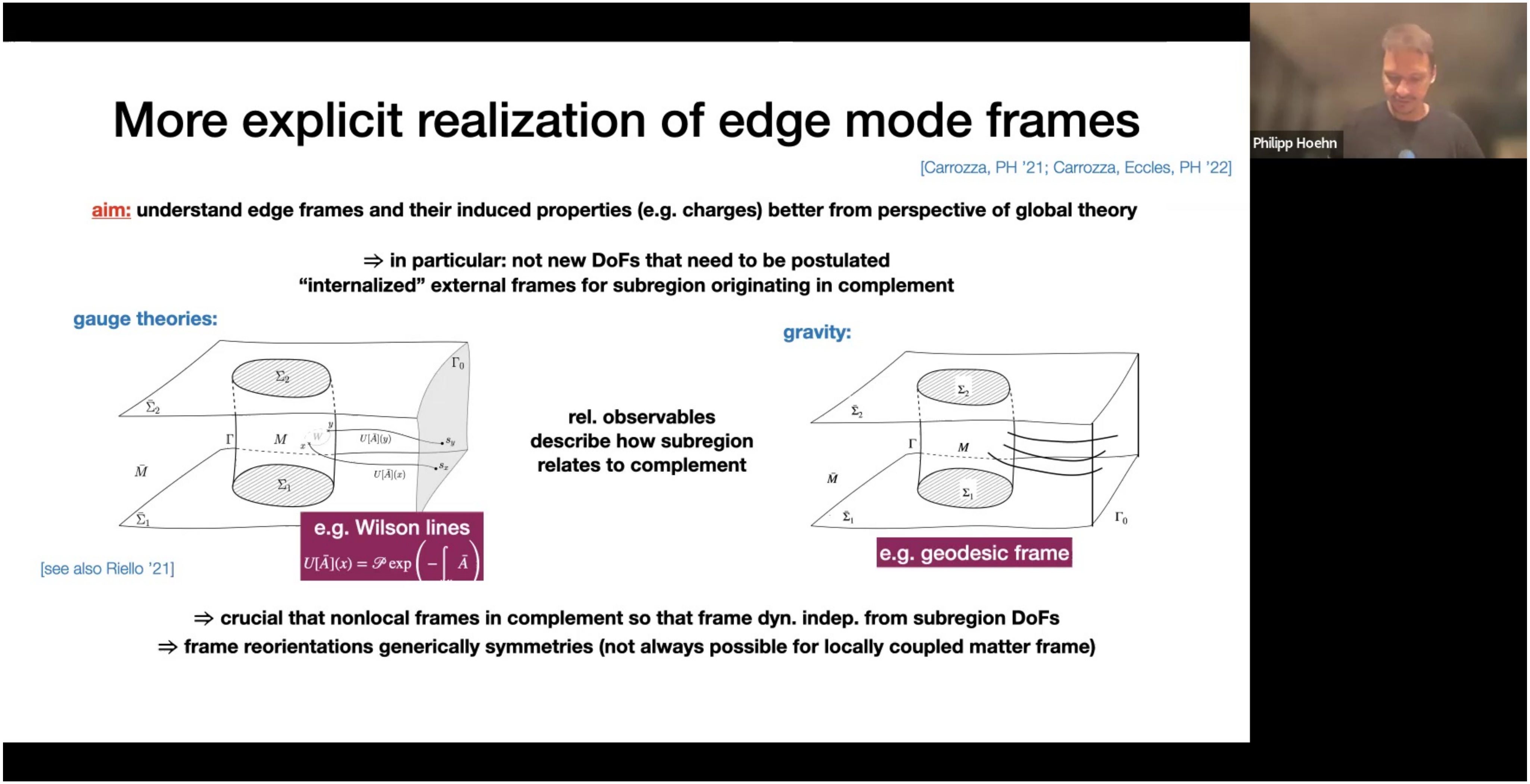
bdry-supported gauge transformations V remain gauge for certain dressed sympl. structure $I_V\Omega^U_\Sigma = \int_\Sigma I_V\delta\theta^U \approx 0$

$\Rightarrow$ similar results hold in GR when one adds a dyn. coordinate system (diffeo) $\mathscr{R} : \mathscr{O} \to \mathscr{M}$ $\mathscr{O}$ reference spacetime = orientation space

[Donnelly, Freidel '16]

$\Rightarrow$ wave of research efforts

[Chandrasekaran, Ciambelli, Donnelly, Freidel, Geiller, Gomes, Leigh, Pranzetti, Riello, Speranza, Wieland, …]



More explicit realization of edge mode frames

[Carrozza, PH '21; Carrozza, Eccles, PH '22]

aim: understand edge frames and their induced properties (e.g. charges) better from perspective of global theory

⇒ in particular: not new DoFs that need to be postulated
"internalized" external frames for subregion originating in complement

gauge theories:

e.g. Wilson lines

$U[\bar{A}](x) = \mathscr{P}\exp\left(-\int \bar{A}\right)$

[see also Riello '21]

rel. observables describe how subregion relates to complement

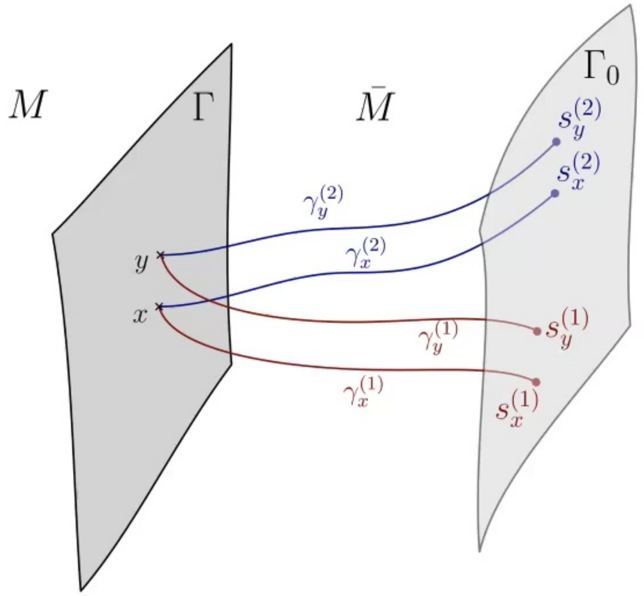
gravity:

e.g. geodesic frame

⇒ crucial that nonlocal frames in complement so that frame dyn. indep. from subregion DoFs

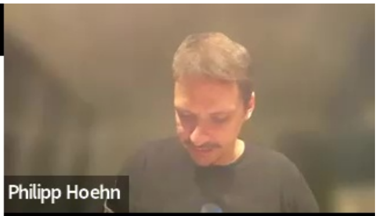
⇒ frame reorientations generically symmetries (not always possible for locally coupled matter frame)

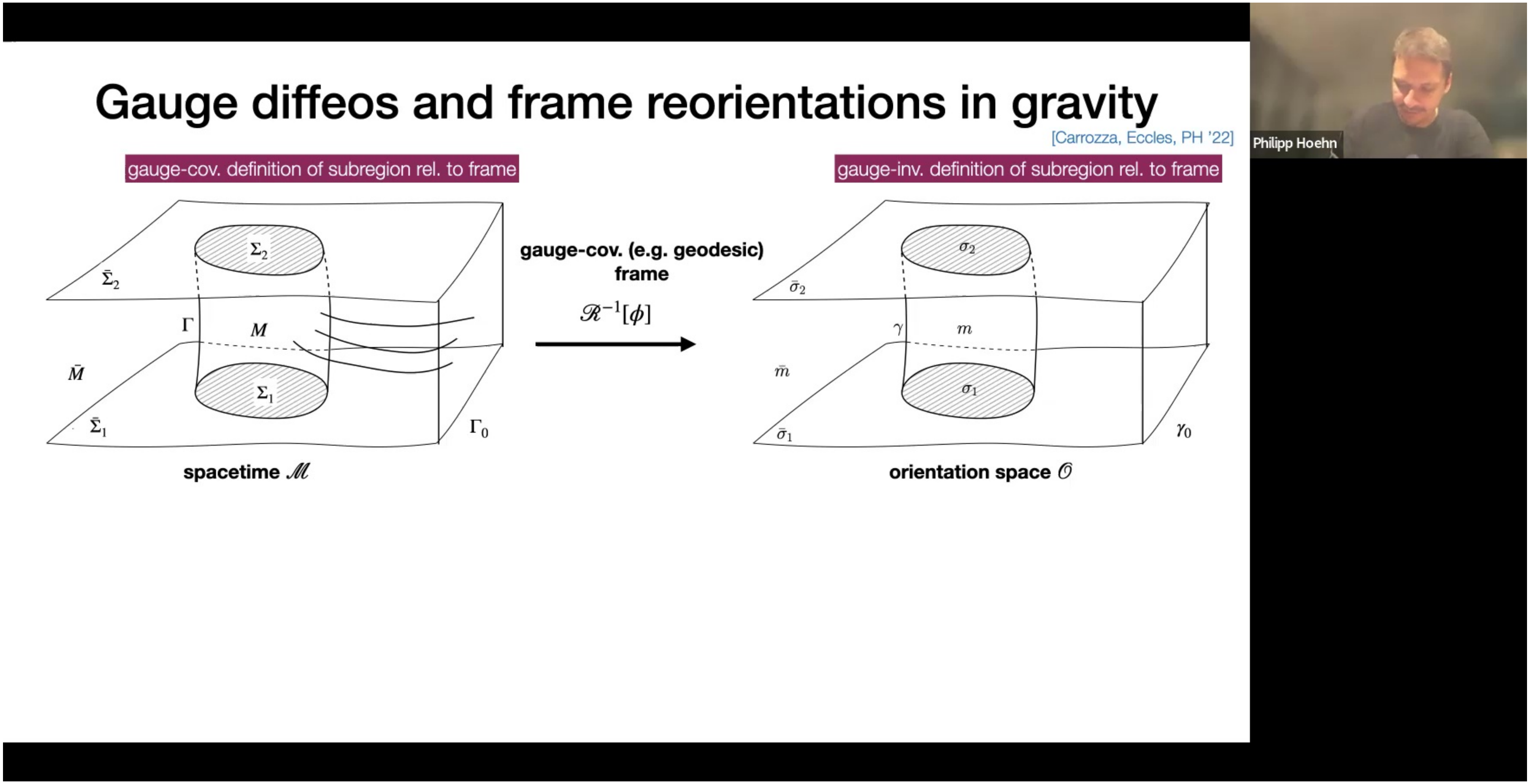
No unique edge mode frame field



⇒ different systems of Wilson lines or geodesics ⇒ different frame fields

⇒ frame covariance





Gauge diffeos and frame reorientations in gravity

[Carrozza, Eccles, PH '22]

gauge-cov. definition of subregion rel. to frame

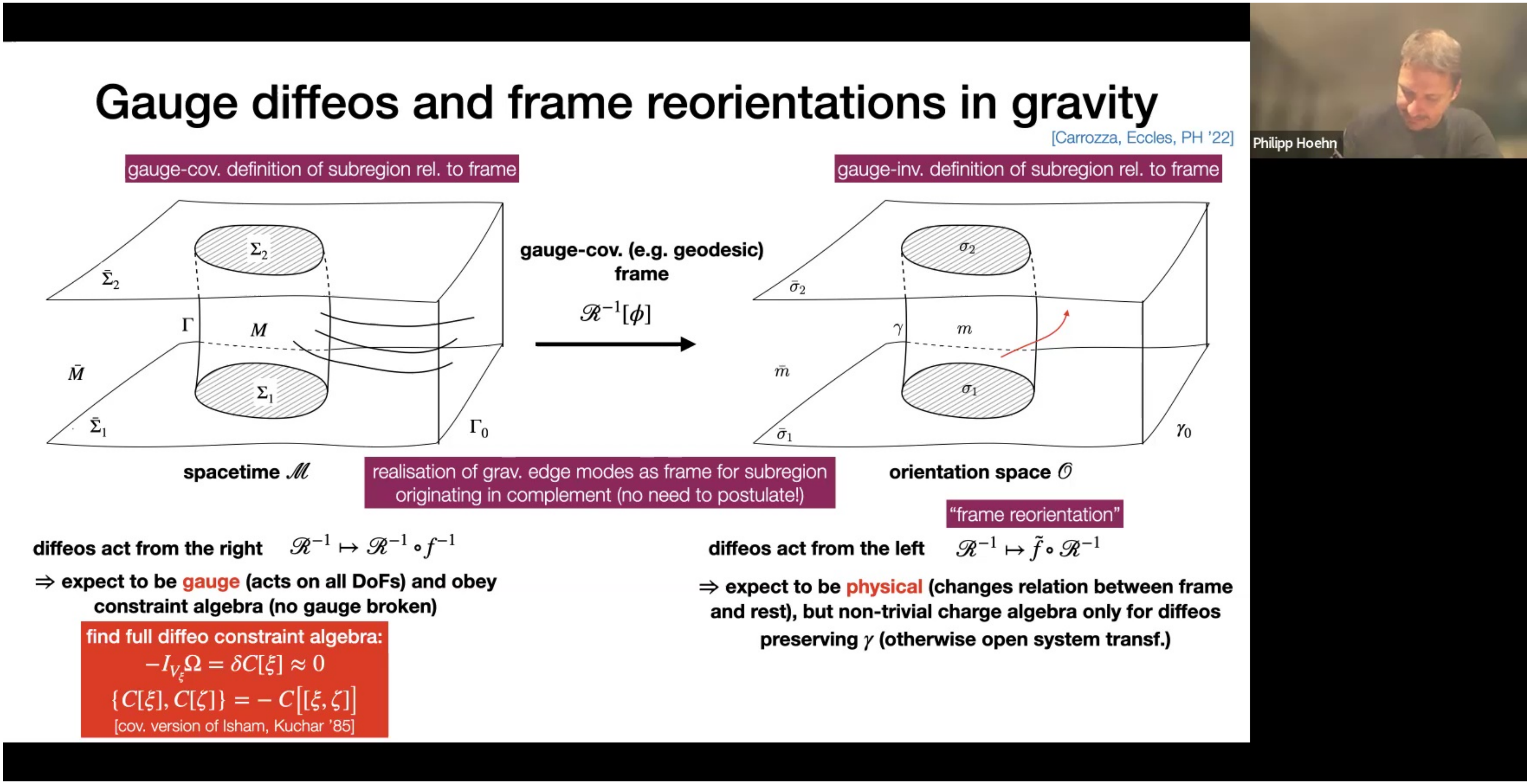
gauge-inv. definition of subregion rel. to frame

spacetime $\mathscr{M}$

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$

orientation space $\mathscr{O}$



Gauge diffeos and frame reorientations in gravity

[Carrozza, Eccles, PH '22]

gauge-cov. definition of subregion rel. to frame

gauge-inv. definition of subregion rel. to frame

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$

spacetime $\mathscr{M}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

orientation space $\mathscr{O}$

"frame reorientation"

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

⇒ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

find full diffeo constraint algebra:

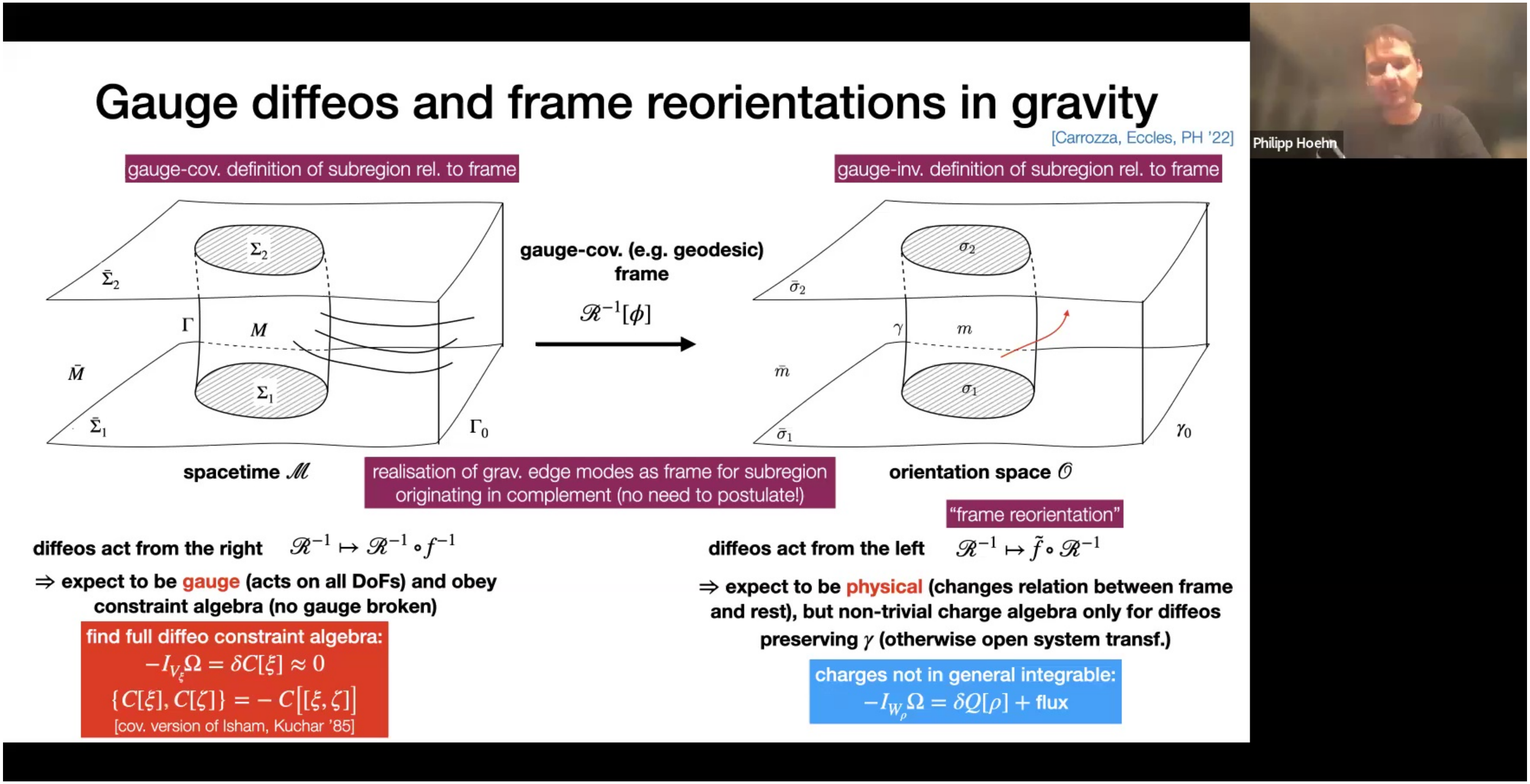
$$-I_{V_\xi}\Omega = \delta C[\xi] \approx 0$$

$$\{C[\xi], C[\zeta]\} = -C\big[[\xi,\zeta]\big]$$

[cov. version of Isham, Kuchar '85]

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

⇒ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos preserving γ (otherwise open system transf.)



Gauge diffeos and frame reorientations in gravity

[Carrozza, Eccles, PH '22]

gauge-cov. definition of subregion rel. to frame

gauge-inv. definition of subregion rel. to frame

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$

spacetime $\mathscr{M}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

orientation space $\mathscr{O}$

"frame reorientation"

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

⇒ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

find full diffeo constraint algebra:

$$-I_{V_\xi}\Omega = \delta C[\xi] \approx 0$$

$$\{C[\xi], C[\zeta]\} = -C\big[[\xi,\zeta]\big]$$

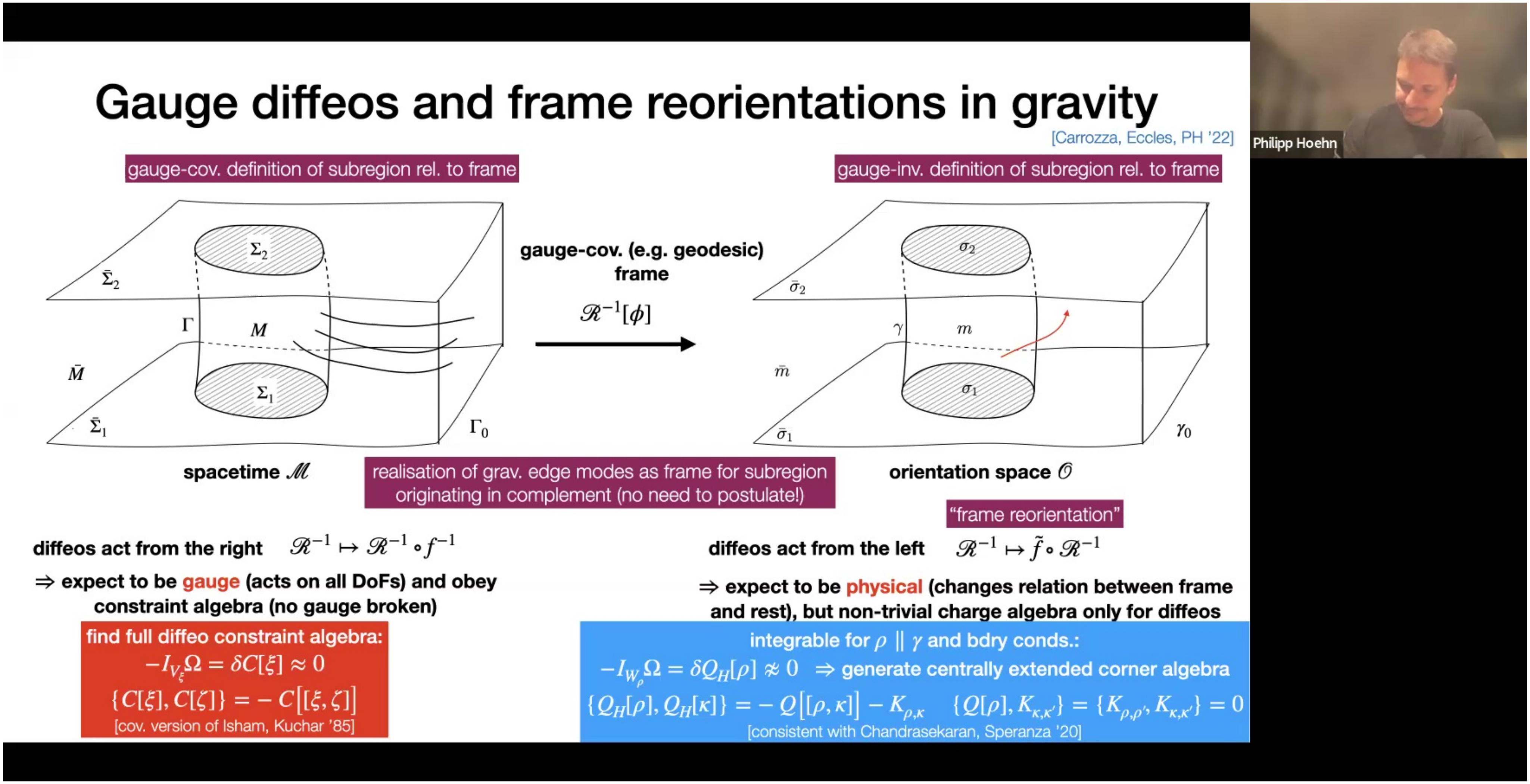
[cov. version of Isham, Kuchar '85]

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

⇒ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos preserving γ (otherwise open system transf.)

charges not in general integrable:

$$-I_{W_\rho}\Omega = \delta Q[\rho] + \text{flux}$$



Gauge diffeos and frame reorientations in gravity

[Carrozza, Eccles, PH '22]

gauge-cov. definition of subregion rel. to frame

gauge-inv. definition of subregion rel. to frame

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$

spacetime $\mathscr{M}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

orientation space $\mathscr{O}$

"frame reorientation"

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

⇒ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

find full diffeo constraint algebra:

$$-I_{V_\xi}\Omega = \delta C[\xi] \approx 0$$

$$\{C[\xi], C[\zeta]\} = -\,C\big[[\xi,\zeta]\big]$$

[cov. version of Isham, Kuchar '85]

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

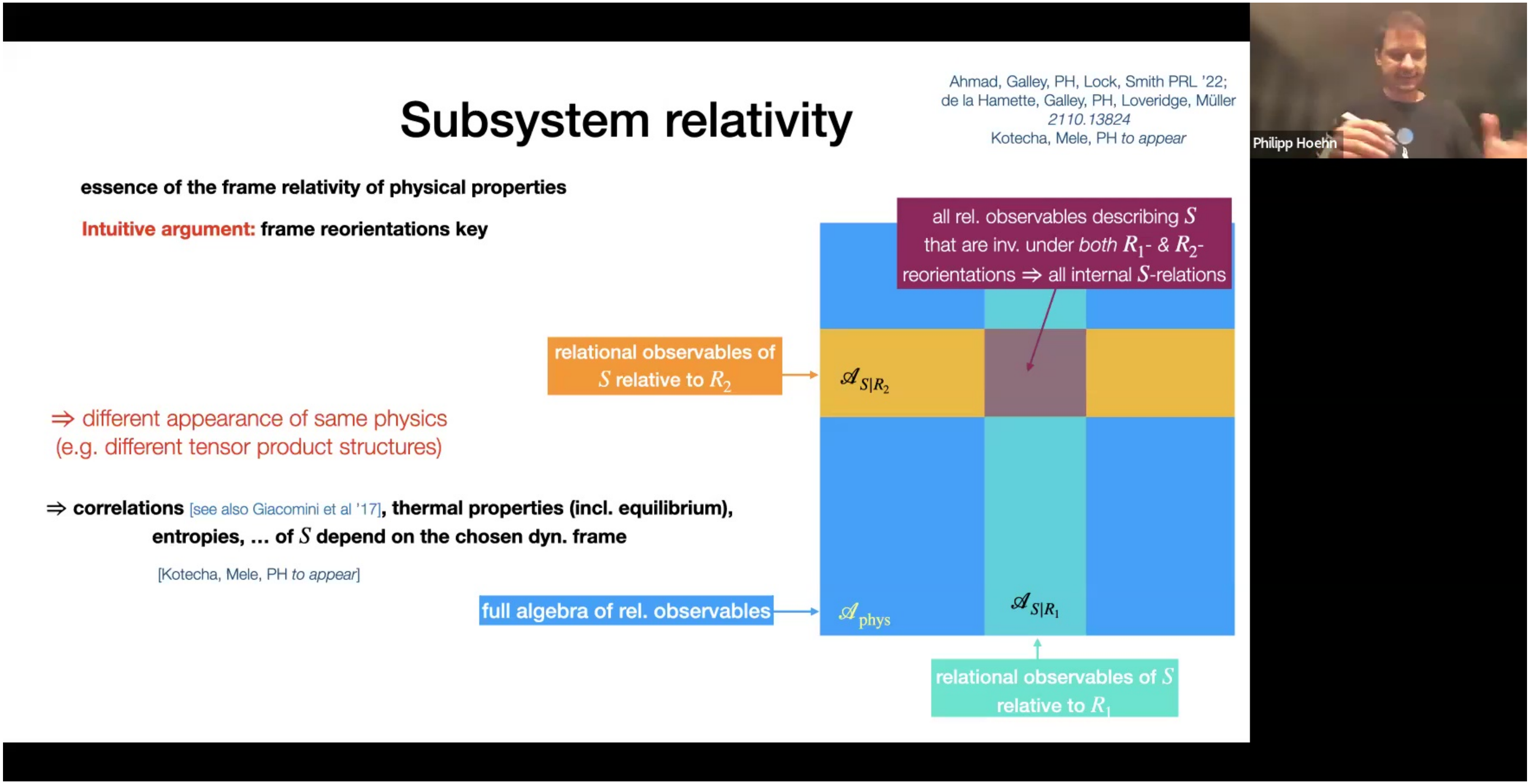
⇒ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos

integrable for $\rho \parallel \gamma$ and bdry conds.:

$-I_{W_\rho}\Omega = \delta Q_H[\rho] \not\approx 0$ **⇒ generate centrally extended corner algebra**

$$\{Q_H[\rho], Q_H[\kappa]\} = -\,Q\big[[\rho,\kappa]\big] - K_{\rho,\kappa} \quad \{Q[\rho], K_{\kappa,\kappa'}\} = \{K_{\rho,\rho'}, K_{\kappa,\kappa'}\} = 0$$

[consistent with Chandrasekaran, Speranza '20]



Subsystem relativity

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller
2110.13824
Kotecha, Mele, PH *to appear*

essence of the frame relativity of physical properties

Intuitive argument: frame reorientations key

all rel. observables describing S that are inv. under *both* R_1- & R_2-reorientations ⇒ all internal S-relations

relational observables of S relative to R_2

$\mathscr{A}_{S|R_2}$

⇒ different appearance of same physics
(e.g. different tensor product structures)

⇒ **correlations** [see also Giacomini et al '17], **thermal properties (incl. equilibrium), entropies, … of S depend on the chosen dyn. frame**

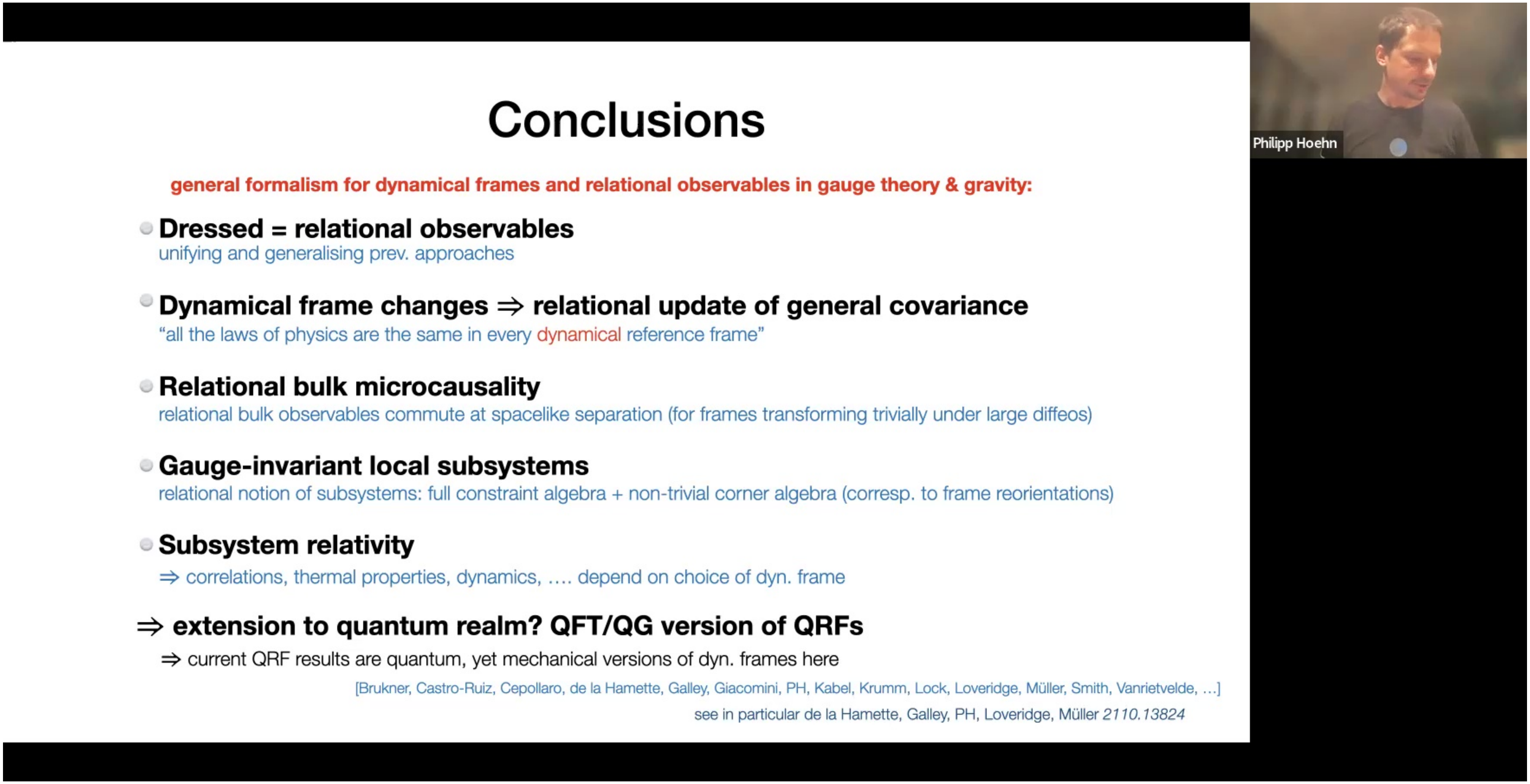
[Kotecha, Mele, PH *to appear*]

full algebra of rel. observables

$\mathscr{A}_{\text{phys}}$

$\mathscr{A}_{S|R_1}$

relational observables of S relative to R_1



Conclusions

general formalism for dynamical frames and relational observables in gauge theory & gravity:

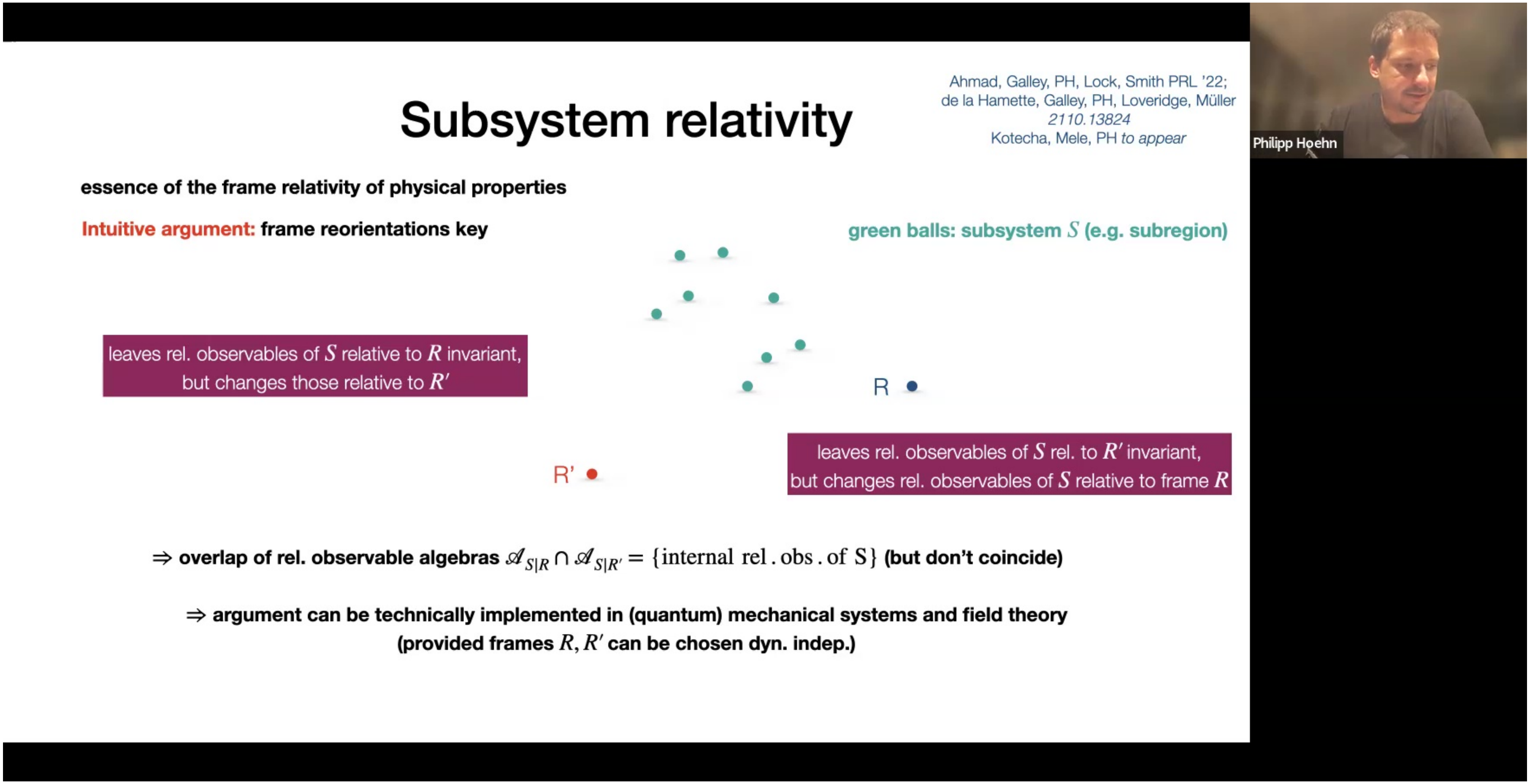
- **Dressed = relational observables**
  unifying and generalising prev. approaches
- **Dynamical frame changes ⇒ relational update of general covariance**
  "all the laws of physics are the same in every dynamical reference frame"
- **Relational bulk microcausality**
  relational bulk observables commute at spacelike separation (for frames transforming trivially under large diffeos)
- **Gauge-invariant local subsystems**
  relational notion of subsystems: full constraint algebra + non-trivial corner algebra (corresp. to frame reorientations)
- **Subsystem relativity**
  ⇒ correlations, thermal properties, dynamics, …. depend on choice of dyn. frame

⇒ extension to quantum realm? QFT/QG version of QRFs

⇒ current QRF results are quantum, yet mechanical versions of dyn. frames here

[Brukner, Castro-Ruiz, Cepollaro, de la Hamette, Galley, Giacomini, PH, Kabel, Krumm, Lock, Loveridge, Müller, Smith, Vanrietvelde, …]

see in particular de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*



Subsystem relativity

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller
2110.13824
Kotecha, Mele, PH *to appear*

essence of the frame relativity of physical properties

Intuitive argument: frame reorientations key

green balls: subsystem S (e.g. subregion)

leaves rel. observables of S relative to R invariant, but changes those relative to R'

leaves rel. observables of S rel. to R' invariant, but changes rel. observables of S relative to frame R

⇒ **overlap of rel. observable algebras** $\mathscr{A}_{S|R} \cap \mathscr{A}_{S|R'} = \{\text{internal rel . obs . of S}\}$ **(but don't coincide)**

⇒ **argument can be technically implemented in (quantum) mechanical systems and field theory (provided frames R, R' can be chosen dyn. indep.)**