Title: Celestial Recursion

Speakers: Yangrui Hu

Series: Quantum Fields and Strings

Date: September 09, 2022 - 2:00 PM

URL: https://pirsa.org/22090082

Abstract: In this talk, I will begin with a brief introduction to celestial holography and setting up the celestial amplitudes, before diving into some recent results on examining the BCFW recursion relations for celestial amplitudes. We start by recasting the celestial incarnation of the BCFW shift as a generalization of the action of familiar asymptotic symmetries on hard particles, before focusing on two limits: large-z and infinitesimal-z. We then discuss how the celestial CFT data encodes the large-z behavior determining which shifts are allowed, while the infinitesimal limit is tied to the celestial bootstrap program via the BG equations that constrain the MHV sector.

Zoom link: https://pitp.zoom.us/j/99231185135?pwd=OWk4S2ZITmFGSmVsQUwwbHI5NW1rUT09

Celestial Recursion

2208.11635 w/ Sabrina Pasterski

Ref: [Arkani-Hamed, Kaplan, '08]

[Pate, Radariu, Strominger, Yuan, '19]

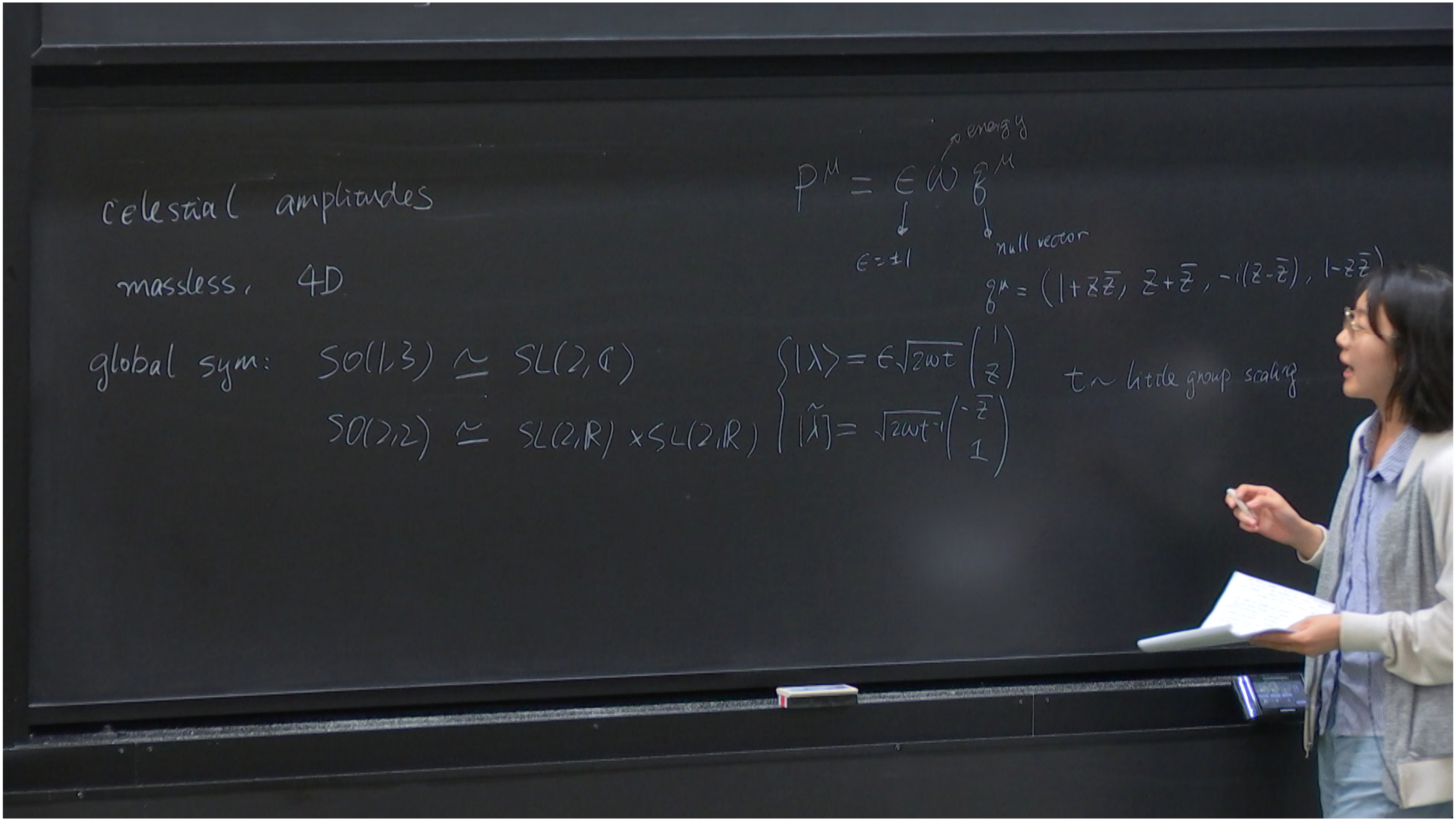
[Himwich, Pate, Singh, '21]

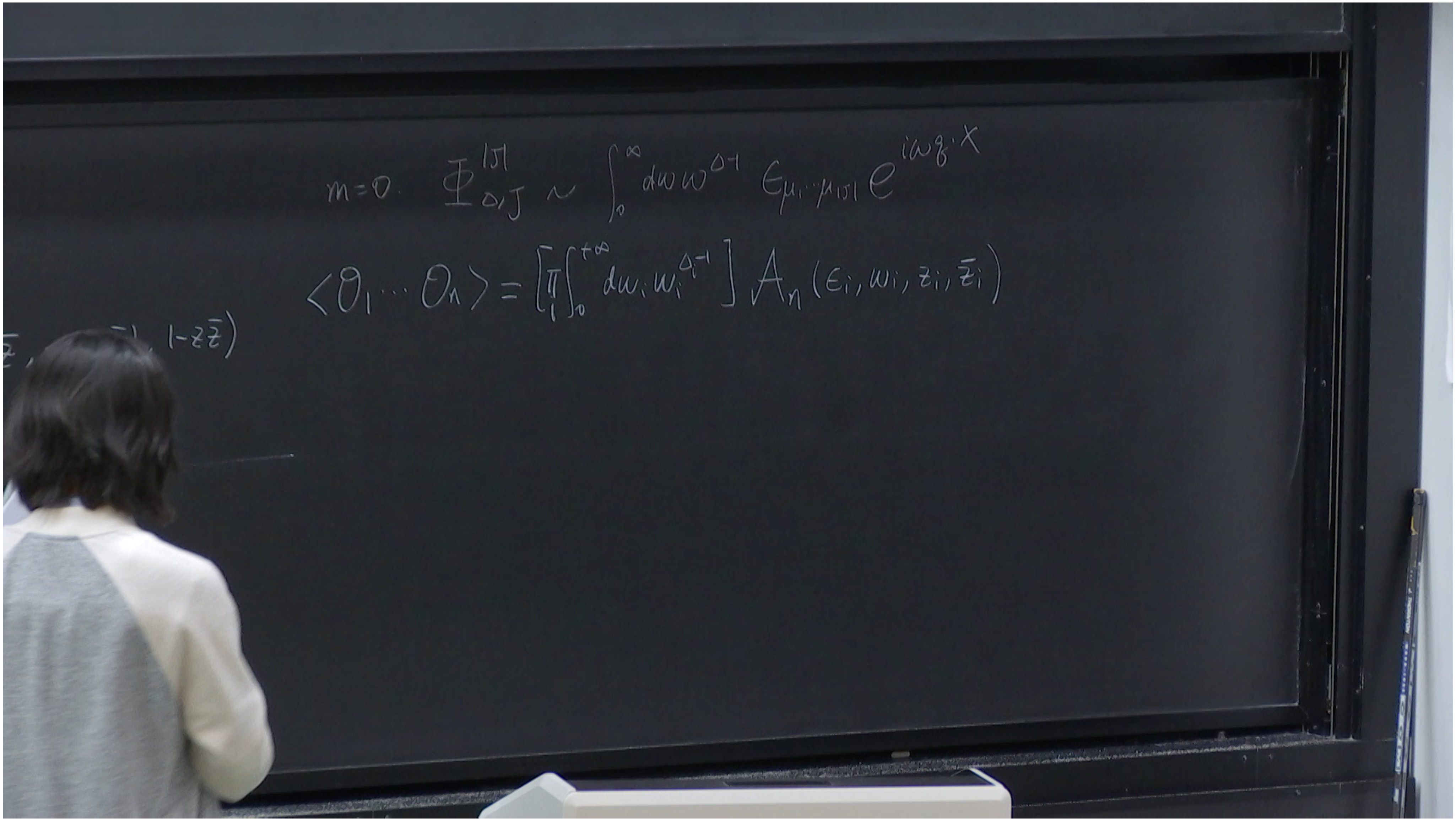
[Hu, Ren, Srikant, Volovich, '21]

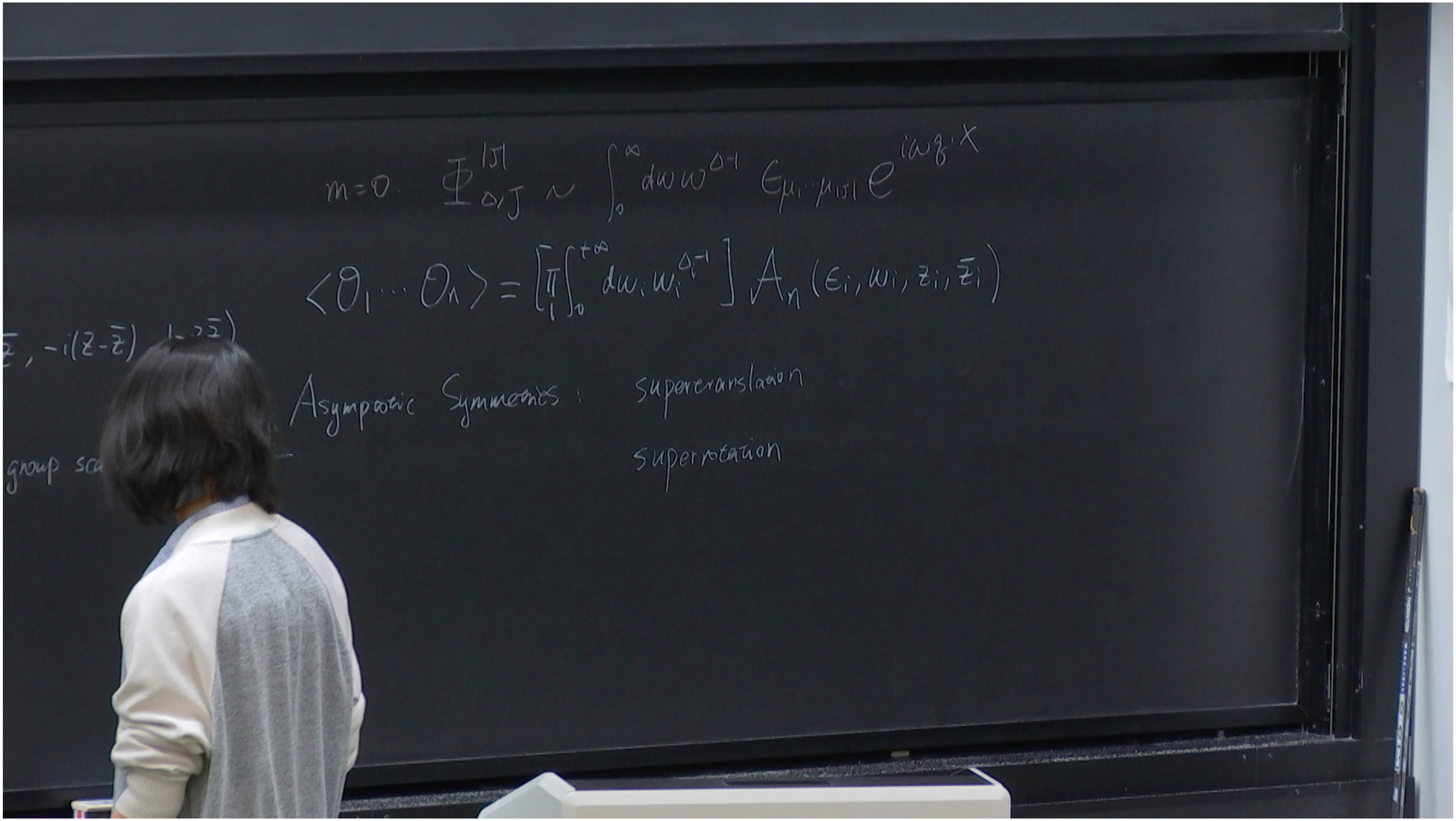
[Banerjee, Ghosh, '20]

Outline

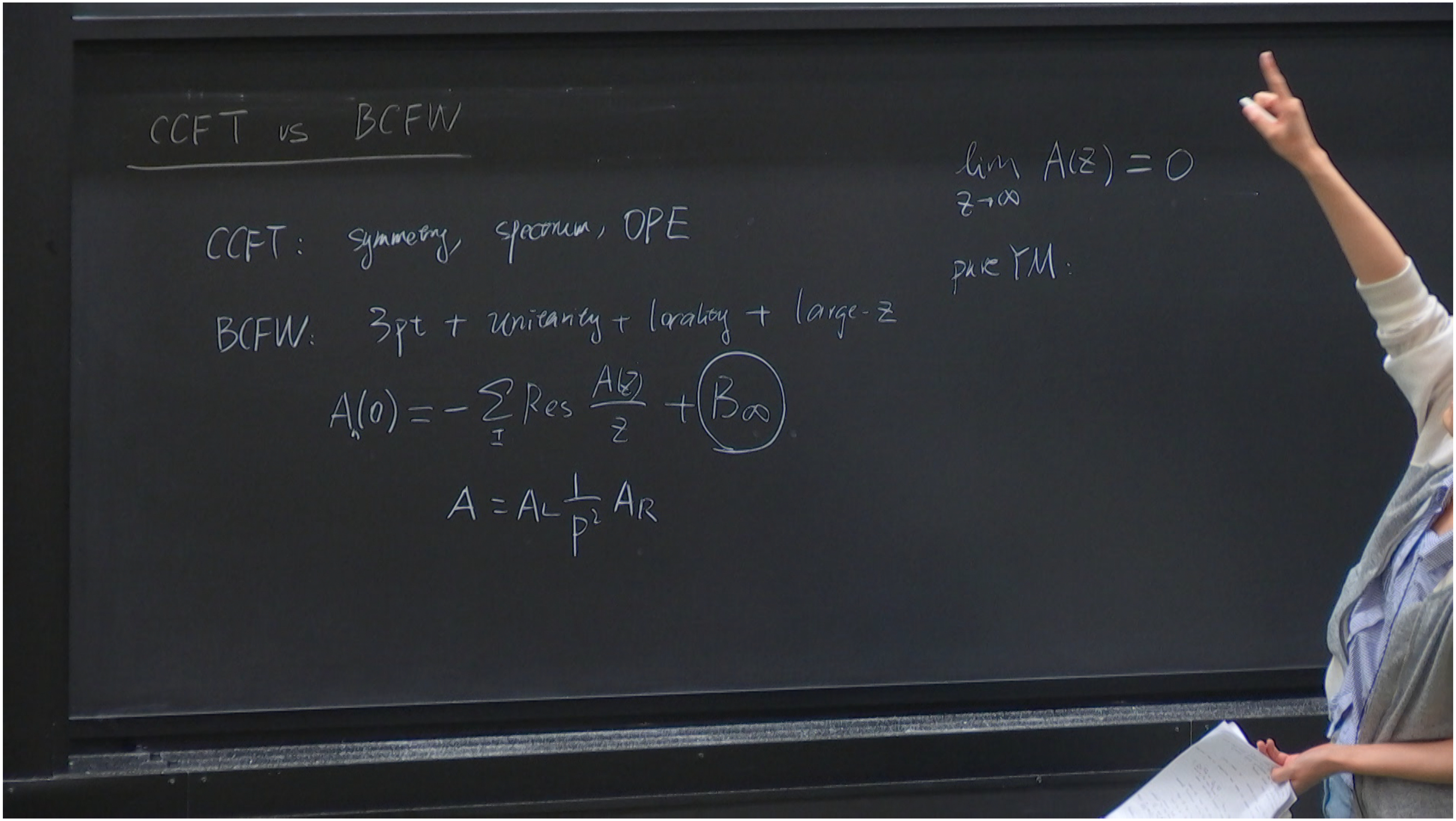
- Review of Celestial Holography
- Celestial CFT vs BCFW
- From celestial OPE to large-z behavior
- Summary of $z \to 0$ story


celestial amplitudes
massless, 4D
global sym: SO(1,3) ≃ SL(2,C)
SO(2,2) ≃ SL(2,R) × SL(2,R)
$p^\mu = \epsilon\,\omega\,q^\mu$
energy
$\epsilon = \pm 1$
null vector
$q^\mu = (1+z\bar{z},\ z+\bar{z},\ -i(z-\bar{z}),\ 1-z\bar{z})$
$|\lambda\rangle = \epsilon\sqrt{2\omega t}\begin{pmatrix}1\\ z\end{pmatrix}$
$|\tilde{\lambda}] = \sqrt{2\omega t^{-1}}\begin{pmatrix}-\bar{z}\\ 1\end{pmatrix}$
t ~ little group scaling


$m=0.$ $\Phi^{|J|}_{\Delta,J} \sim \int_0^\infty d\omega\, \omega^{\Delta-1}\, \epsilon_{\mu_1\cdots\mu_{|J|}}\, e^{i\omega q\cdot X}$
$\langle O_1 \cdots O_n \rangle = \left[\prod_i \int_0^{+\infty} d\omega_i\, \omega_i^{\Delta_i-1}\right] A_n(\epsilon_i, \omega_i, z_i, \bar{z}_i)$



$m=0.$ $\Phi^{|J|}_{\Delta,J} \sim \int_0^\infty d\omega\, \omega^{\Delta-1}\, \epsilon_{\mu_1\cdots\mu_{|J|}}\, e^{i\omega q\cdot X}$
$\langle O_1 \cdots O_n \rangle = \left[\prod_i \int_0^{+\infty} d\omega_i\, \omega_i^{\Delta_i-1}\right] A_n(\epsilon_i, \omega_i, z_i, \bar{z}_i)$
Asymptotic Symmetries: supertranslation
superrotation


CCFT vs BCFW
CCFT: symmetry, spectrum, OPE
BCFW: 3pt + unitarity + locality + large-z
$A_n(0) = -\sum_I \text{Res} \frac{A(z)}{z} + B_\infty$
$A = A_L \frac{1}{P^2} A_R$
$\lim_{z\to\infty} A(z) = 0$
pure YM:

CCFT vs BCFW

CCFT: symmetry, spectrum, OPE

BCFW: 3pt + unitarity + locality + large-z

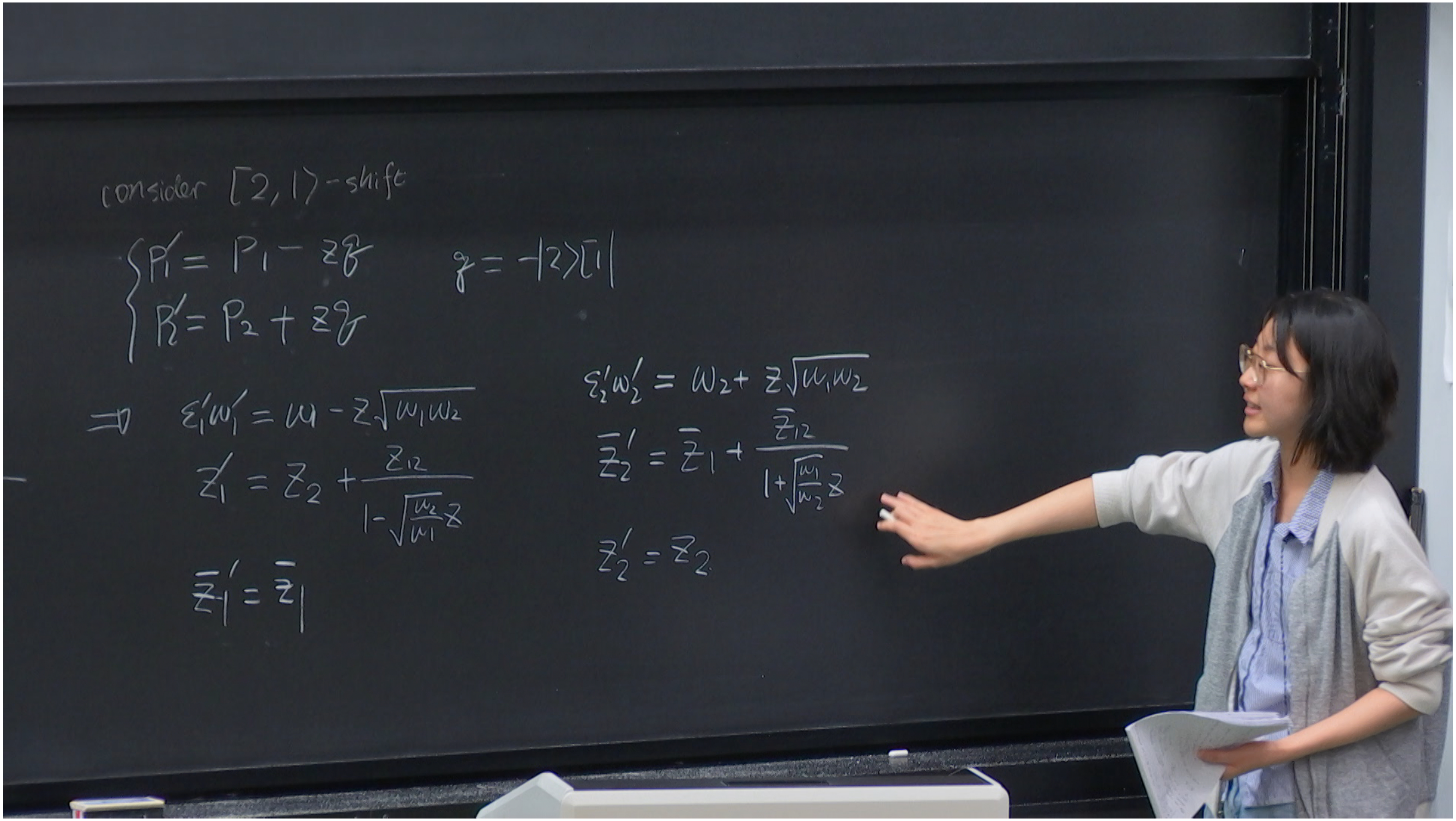
$$A_n(0) = -\sum_I \mathrm{Res} \frac{A(z)}{z} + B_\infty$$

$$A = A_L \frac{1}{p^2} A_R$$

$$\lim_{z\to\infty} A(z) = 0$$

pure YM: $\langle i,j]$

$\langle i,j]$	$\langle ++]$	$\langle +-]$	$\langle --]$	$\langle -+]$
$A(z)$	$\frac{1}{z}$	$\frac{1}{z}$	$\frac{1}{z}$	z^3


consider $[2,1\rangle$-shift
$P_1' = P_1 - z g$
$P_2' = P_2 + z g$
$g = -|2\rangle[1|$
$\Rightarrow \varepsilon_1' w_1' = w_1 - z\sqrt{w_1 w_2}$
$z_1' = z_2 + \frac{z_{12}}{1-\sqrt{\frac{w_2}{w_1}}z}$
$\bar{z}_1' = \bar{z}_1$
$\varepsilon_2' w_2' = w_2 + z\sqrt{w_1 w_2}$
$\bar{z}_2' = \bar{z}_1 + \frac{\bar{z}_{12}}{1+\sqrt{\frac{w_1}{w_2}}z}$
$z_2' = z_2$

consider $[2,1\rangle$-shift

$$\begin{cases} P_1' = P_1 - zq \\ P_2' = P_2 + zq \end{cases} \qquad q = -|2\rangle[1|$$

$$\Rightarrow \quad \varepsilon_1' w_1' = w_1 - z\sqrt{w_1 w_2}$$

$$z_1' = z_2 + \frac{z_{12}}{1-\sqrt{\frac{w_2}{w_1}}\,z}$$

$$\bar{z}_1' = \bar{z}_1$$

$$\varepsilon_2' w_2' = w_2 + z\sqrt{w_1 w_2}$$

$$\bar{z}_2' = \bar{z}_1 + \frac{\bar{z}_{12}}{1+\sqrt{\frac{w_1}{w_2}}\,z}$$

$$z_2' = z_2$$

$z \to \infty$

$$z_1' \to z_2 = z_2'$$

$$\bar{z}_2' \to \bar{z}_1 = \bar{z}_1'$$

$\Rightarrow$ coincident limit on CS

Plan: $\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle \xrightarrow{[2,1\rangle\text{-shift}} \Lambda_1 \tilde{\Lambda}_2 \langle \cdots \rangle \longrightarrow z \to \infty$ $\qquad \Lambda_1 |\lambda_1\rangle =$

$\longrightarrow$ look at how OPE transf

$\longrightarrow$ extract large-z behavior.

Step 1: $\begin{cases} |\lambda_i'\rangle = \Lambda_i |\lambda_i\rangle \\ |\tilde{\lambda}_i] = \tilde{\Lambda}_i |\tilde{\lambda}_i] \end{cases}$ $\qquad \Lambda_i = \begin{pmatrix} d_i & c_i \\ b_i & a_i \end{pmatrix} \quad \tilde{\Lambda}_i = \begin{pmatrix} \bar{a}_i & -\bar{b}_i \\ -\bar{c}_i & \bar{d}_i \end{pmatrix}$

$[2,1\rangle$: $\Lambda_i = \mathbb{1}_2 \quad (i \neq 1)$

$\tilde{\Lambda}_j = \mathbb{1}_2 \quad (j \neq 2)$

$$\Lambda_1 |\lambda_1\rangle = \sqrt{|c_1 z_1 + d_1|}\; \varepsilon_1' \sqrt{2\omega_1'} \begin{pmatrix} 1 \\ z_1 \end{pmatrix}$$

$$\tilde{\Lambda}_2 |\tilde{\lambda}_2] = \sqrt{|\bar{c}_2 \bar{z}_2 + \bar{d}_2|}\; \sqrt{2\omega_2'} \begin{pmatrix} -\bar{z}_2 \\ 1 \end{pmatrix}$$

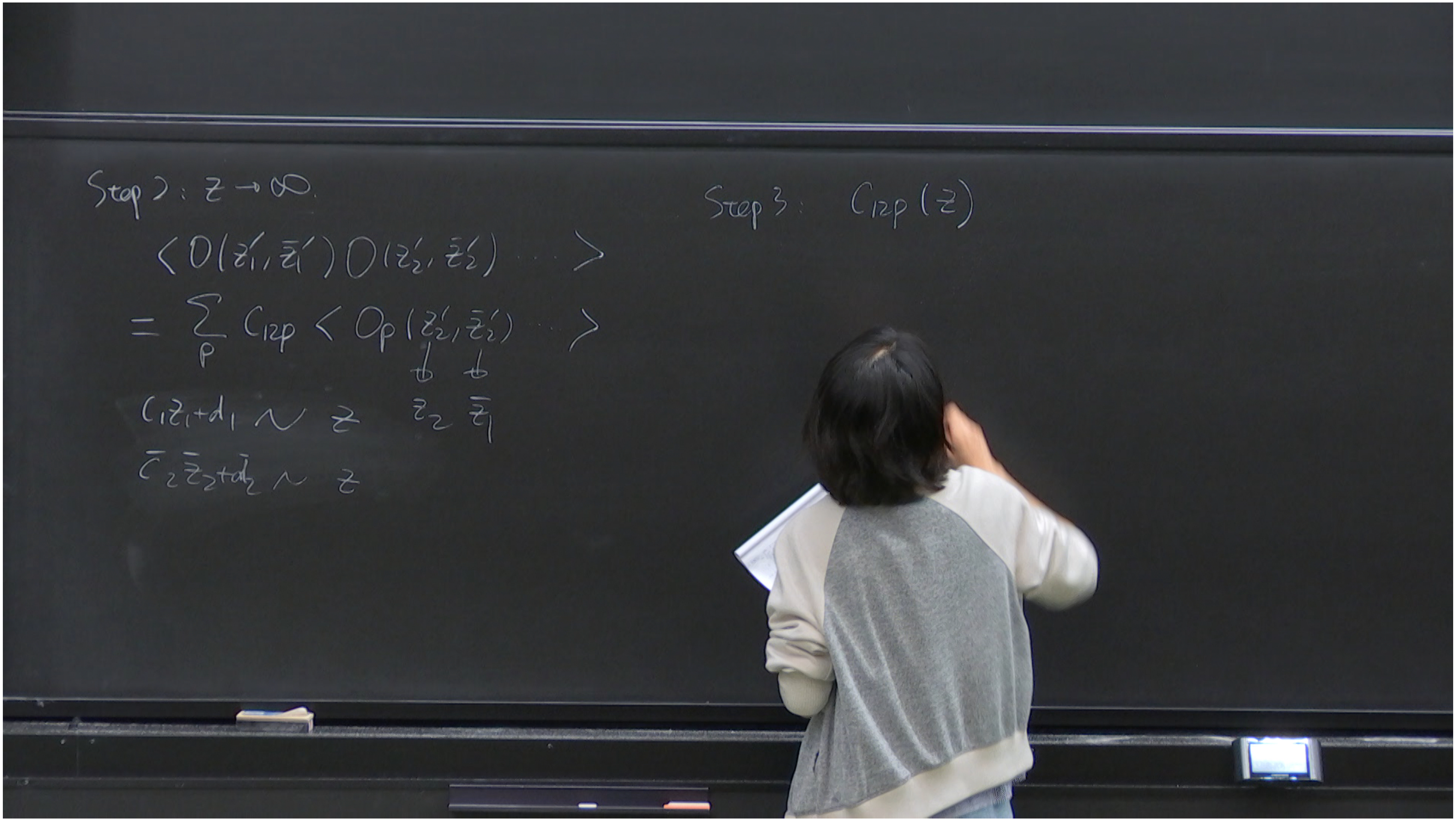
$$z_1' = \frac{a_1 z_1 + b_1}{c_1 z_1 + d_1} \qquad \bar{z}_2' = \frac{\bar{a}_2 \bar{z}_2 + \bar{b}_2}{\bar{c}_2 \bar{z}_2 + \bar{d}_2}$$

$$\omega_1' = \omega_1 |c_1 z_1 + d_1|$$

$$\omega_2' = \omega_2 |\bar{c}_2 \bar{z}_2 + \bar{d}_2|$$

$$\begin{cases} h = \frac{\Delta + J}{2} \\ \bar{h} = \frac{\Delta - J}{2} \end{cases}$$

$$\Rightarrow \Lambda_1 \tilde{\Lambda}_2 \langle O_1 \cdots O_n \rangle = |c_1 z_1 + d_1|^{-2h_1} |\bar{c}_2 \bar{z}_2 + \bar{d}_2|^{-2\bar{h}_2} \langle O_1(z_1', \bar{z}_1') O_2(z_2', \bar{z}_2') \cdots \rangle$$


Step 2: $z \to \infty$
$\langle O(z_1', \bar{z}_1') O(z_2', \bar{z}_2') \cdots \rangle$
$= \sum_p C_{12p} \langle O_p(z_2', \bar{z}_2') \cdots \rangle$
$\bar{z}_2$ $\bar{z}_1$
$c_1 z_1 + d_1 \sim z$
$\bar{c}_2 \bar{z}_2 + \bar{d}_2 \sim z$
Step 3: $C_{12p}(z)$

Step 2: $z \to \infty$

$$\langle O(z_1', \bar{z}_1') \, O(z_2', \bar{z}_2') \cdots \rangle$$

$$= \sum_p C_{12p} \langle O_p(z_2', \bar{z}_2') \cdots \rangle$$

$$\qquad\qquad\qquad\qquad \downarrow \quad \downarrow$$

$$c_1 z_1 + d_1 \sim z \qquad z_2 \quad \bar{z}_1$$

$$\bar{c}_2 \bar{z}_2 + \bar{d}_2 \sim z$$

Step 3: $C_{12p}(z)$

$$O_{\Delta_1, J_1}(z_1', \bar{z}_1') \, O_{\Delta_2, J_2}(z_2', \bar{z}_2') \sim \sum_{J_p} g_{12p} \frac{\bar{z}_{12}'^{\,J_1 + J_2 - J_p - 1}}{z_{12}'} B(\Delta_1$$

Step 3: $C_{12p}(z)$

$$O_{\Delta_1,J_1}(z_1',\bar{z}_1')\,O_{\Delta_2,J_2}(z_2',\bar{z}_2') \sim \sum_{J_p} g_{12p}\,\frac{\bar{z}_{12}'^{\,J_1+J_2-J_p-1}}{z_{12}'}\,B(\Delta_1-1+J_2-J_p,\ \Delta_2-1+J_1-J_p)\,O_{\Delta_p,J_p}$$

$$+\sum_{J_p} \underbrace{g_{12p}}_{\text{3pt coupling const}}\,\frac{z_{12}'^{\,J_p-J_1-J_2-1}}{\bar{z}_{12}'}\,B(\Delta_1-1-J_2+J_p,\ \Delta_2-1-J_1+J_p)\,O_{\Delta_p,J_p}$$

$\frac{1}{z}$

$\frac{1}{\ }$

Step 3: $C_{12p}(z)$

$$O_{\Delta_1,J_1}(z_1',\bar{z}_1')\, O_{\Delta_2,J_2}(z_2',\bar{z}_2') \sim \sum_{J_p} g_{12p} \frac{\bar{z}_{12}'^{\,J_1+J_2-J_p-1}}{z_{12}'} B\left(\Delta_1-1+J_2-J_p,\ \Delta_2-1+J_1-J_p\right) O_{\Delta_p,J_p}$$

$$+ \sum_{J_p} g_{12p} \frac{z_{12}'^{\,J_p-J_1-J_2-1}}{\bar{z}_{12}'} B\left(\Delta_1-1-J_2+J_p,\ \Delta_2-1-J_1+J_p\right) O_{\Delta_p,J_p}$$

g_{12p}: 3pt coupling const

$$z_{12}' \sim \frac{1}{z}$$

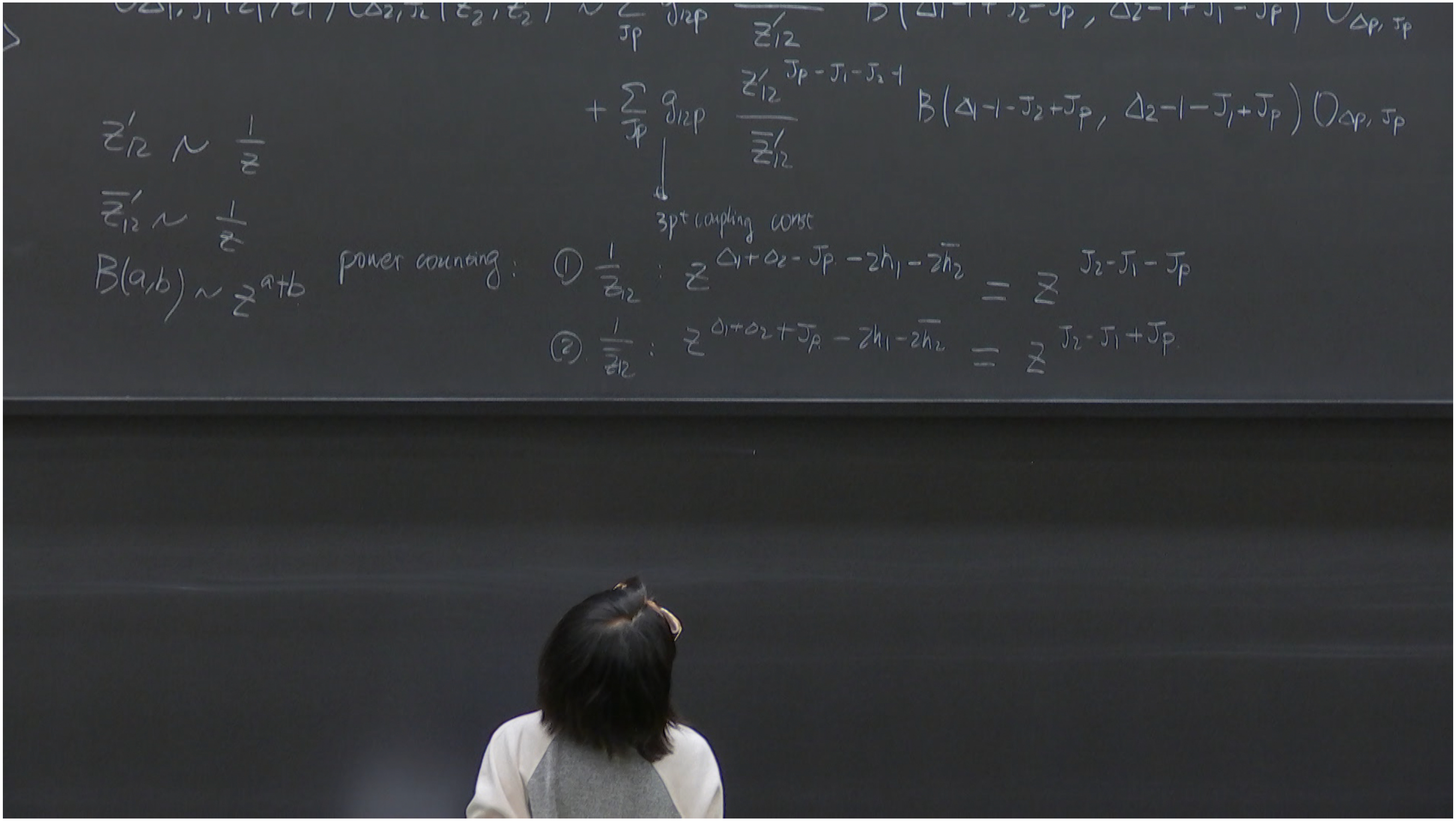
$$\bar{z}_{12}' \sim \frac{1}{z}$$

$$B(a,b) \sim z^{a+b}$$

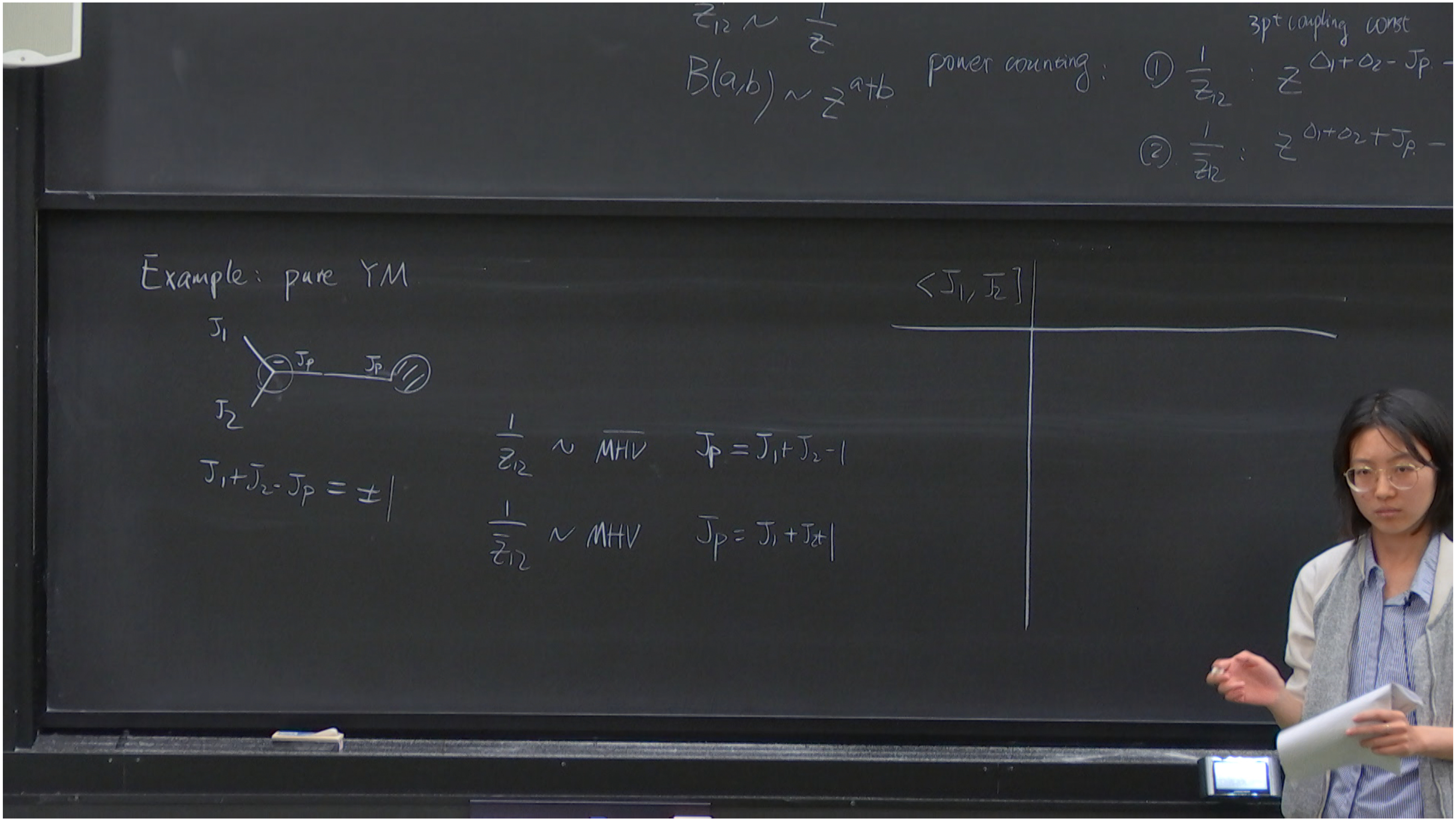
power counting:

① $\frac{1}{z_{12}}$: $z^{\Delta_1+\Delta_2-J_p-2h_1-2\bar{h}_2} = z^{J_2-J_1-J_p}$

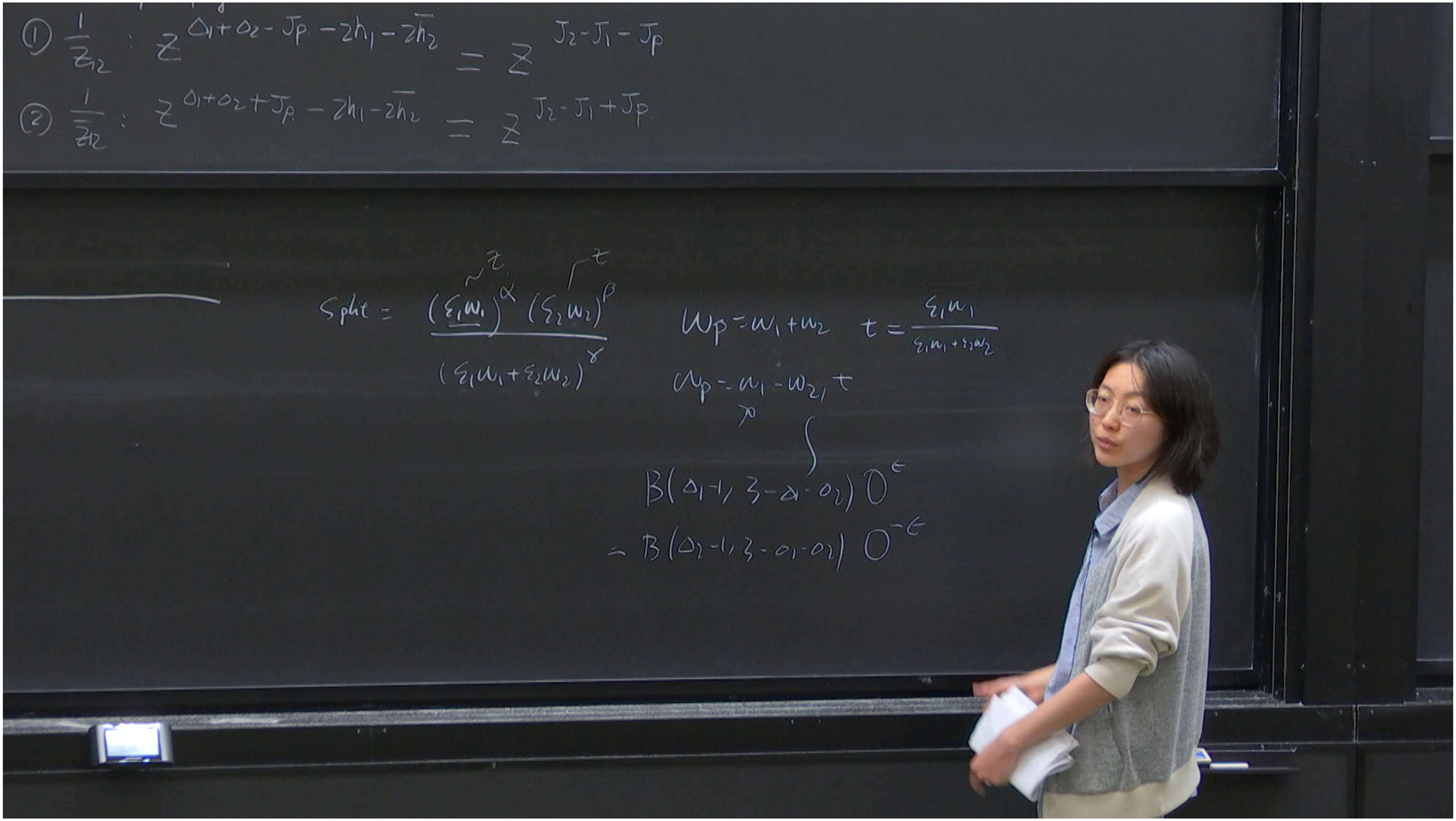
② $\frac{1}{\bar{z}_{12}}$: $z^{\Delta_1+\Delta_2+J_p-2h_1-2\bar{h}_2} = z^{J_2-J_1+}$

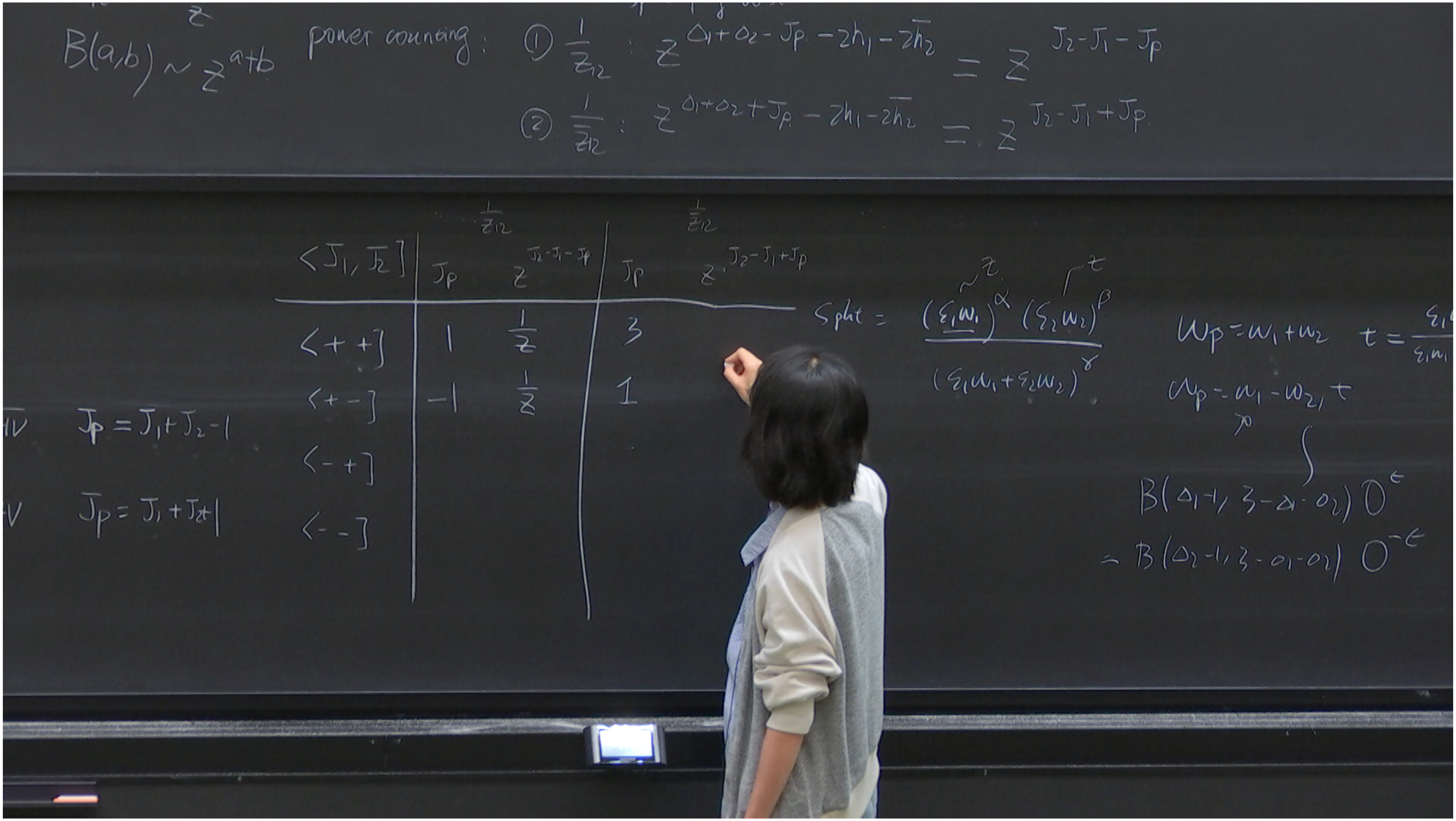


+ Σ g12p
3pt coupling const
power counting:



$z_{12} \sim \frac{1}{z}$
$B(a,b) \sim z^{a+b}$
3pt coupling const
power counting:
① $\frac{1}{z_{12}}$: $z^{\Delta_1+\Delta_2-J_p-}$
② $\frac{1}{\bar{z}_{12}}$: $z^{\Delta_1+\Delta_2+J_p-}$
Example: pure YM
J_1
J_2
J_p
J_p
$J_1+J_2-J_p=\pm 1$
$\frac{1}{z_{12}} \sim \overline{MHV}$
$J_p = J_1+J_2-1$
$\frac{1}{\bar{z}_{12}} \sim MHV$
$J_p = J_1+J_2+1$
$\langle J_1, J_2]$


① 1/z12 : z^(Δ1+Δ2−Jp−2h1−2h̄2) = z^(J2−J1−Jp)
② 1/z12 : z^(Δ1+Δ2+Jp−2h1−2h̄2) = z^(J2−J1+Jp)
Split = (ε1ω1)^α (ε2ω2)^β / (ε1ω1+ε2ω2)^γ
ωp = ω1+ω2
t = ε1ω1 / (ε1ω1+ε2ω2)
B(Δ1−1, 3−Δ1−Δ2) O^ε
= B(Δ2−1, 3−Δ1−Δ2) O^−ε


B(a,b) ~ z^(a+b)
power counting: ① 1/z12 : z^(Δ1+Δ2−Jp−2h1−2h̄2) = z^(J2−J1−Jp)
② 1/z12 : z^(Δ1+Δ2+Jp−2h1−2h̄2) = z^(J2−J1+Jp)
<J1, J2]   Jp   z^(J2−J1−Jp)   Jp   z^(J2−J1+Jp)
<++]   1   1/z   3
<+−]   −1   1/z   1
<−+]
<−−]
Jp = J1 + J2 − 1
Jp = J1 + J2 + 1
Split = (ε1ω1)^α (ε2ω2)^β / (ε1ω1 + ε2ω2)^γ
ωp = ω1 + ω2
ωp − ω1 − ω2, t
B(Δ1−1, 3−Δ1−Δ2) O^ε
= B(Δ2−1, 3−Δ1−Δ2) O^−ε

$B(a,b) \sim z^{a+b}$

power counting: ① $\frac{1}{z_{12}}$: $z^{\Delta_1+\Delta_2-J_p-2h_1-2\bar{h}_2} = z^{J_2-J_1-J_p}$

② $\frac{1}{\bar{z}_{12}}$: $z^{\Delta_1+\Delta_2+J_p-2h_1-2\bar{h}_2} = z^{J_2-J_1+J_p}$

		$\frac{1}{z_{12}}$		$\frac{1}{\bar{z}_{12}}$	
	$\langle J_1, J_2]$	J_p	$z^{J_2-J_1-J_p}$	J_p	$z^{J_2-J_1+J_p}$
$\frac{1}{z}$	$\langle ++]$	1	$\frac{1}{z}$	3	
$\frac{1}{z}$	$\langle +-]$	-1	$\frac{1}{z}$	1	$\frac{1}{z}$
z^3	$\langle -+]$	-1	z^3	1	z^3
$\frac{1}{z}$	$\langle --]$	$\cancel{-3}$		-1	$\frac{1}{z}$

$J_p = J_1 + J_2 - 1$

$J_p = J_1 + J_2 + 1$

$$\text{Split} = \frac{(\varepsilon_1\omega_1)^\alpha (\varepsilon_2\omega_2)^\beta}{(\varepsilon_1\omega_1+\varepsilon_2\omega_2)^\gamma}$$

$\omega_p = \omega_1 + \omega_2 \quad t = \frac{\varepsilon_1}{\varepsilon_1\omega_1}$

$\omega_p - \omega_1 - \omega_2, t$

$\int$

$B(\Delta_1 - 1, 3 - \Delta_1 - \Delta_2)\, O^{\epsilon}$

$= B(\Delta_2 - 1, 3 - \Delta_1 - \Delta_2)\, O^{-\epsilon}$

$B(a,b) \sim z^{a+b}$

power counting: ① $\frac{1}{z_{12}}$: $z^{\Delta_1+\Delta_2-J_p-2h_1-2\bar{h}_2} = z^{J_2-J_1-J_p}$

② $\frac{1}{z_{12}}$: $z^{\Delta_1+\Delta_2+J_p-2h_1-2\bar{h}_2} = z^{J_2-J_1+J_p}$

	$\langle J_1, \bar{J}_2]$	J_p	$\frac{1}{z_{12}}$ $z^{J_2-J_1-J_p}$	J_p	$\frac{1}{z_{12}}$ $z^{J_2-J_1+J_p}$
$\frac{1}{z}$	$\langle ++]$	1	$\frac{1}{z}$	3	
$\frac{1}{z}$	$\langle +-]$	-1	$\frac{1}{z}$	1	$\frac{1}{z}$
z^3	$\langle -+]$	-1	z^3	1	z^3
$\frac{1}{z}$	$\langle --]$	$\cancel{-3}$		-1	$\frac{1}{z}$

IV $J_p = J_1 + J_2 - 1$

V $J_p = J_1 + J_2 + 1$

$\text{Split} = \frac{(\varepsilon_1\omega_1)^{\alpha}(\varepsilon_2\omega_2)^{\beta}}{(\varepsilon_1\omega_1+\varepsilon_2\omega_2)^{\gamma}}$ ($\sim z$, $\sim t$)

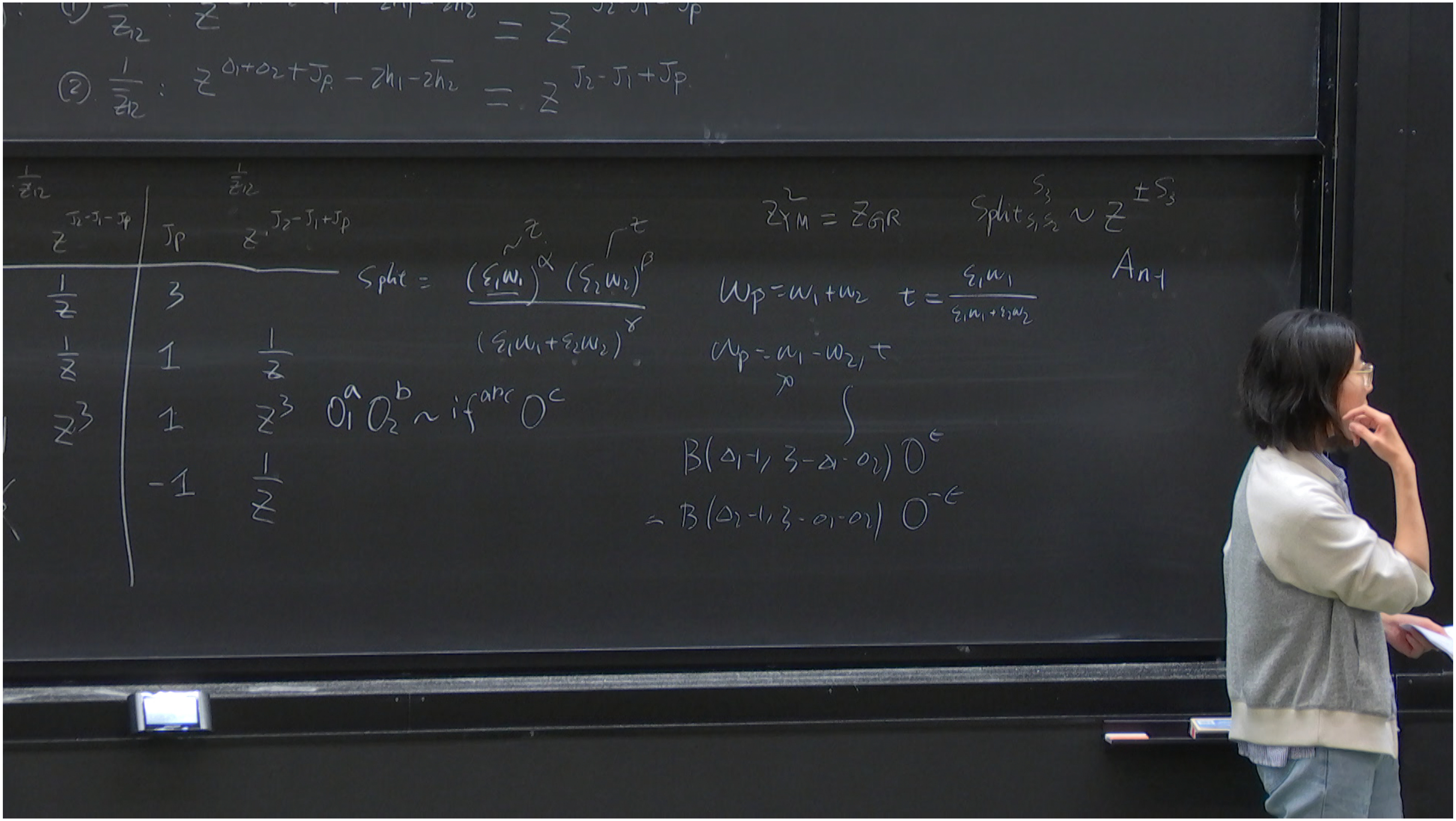
$\omega_p = \omega_1 + \omega_2 \quad t = \frac{\varepsilon_1 \ldots}{\varepsilon_1\omega_1 \ldots}$

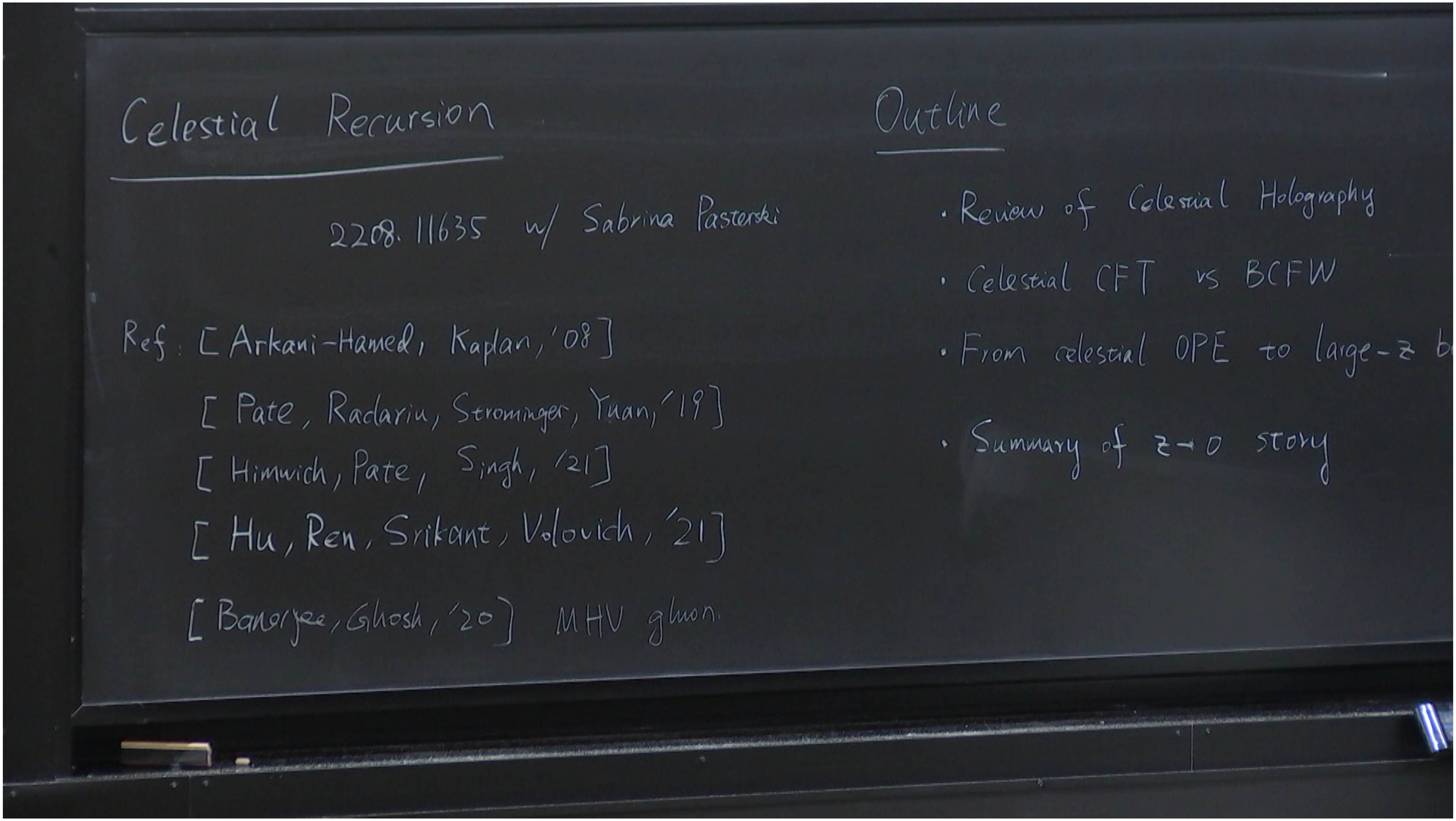
$\omega_p = \omega_1 - \omega_2, \; t$

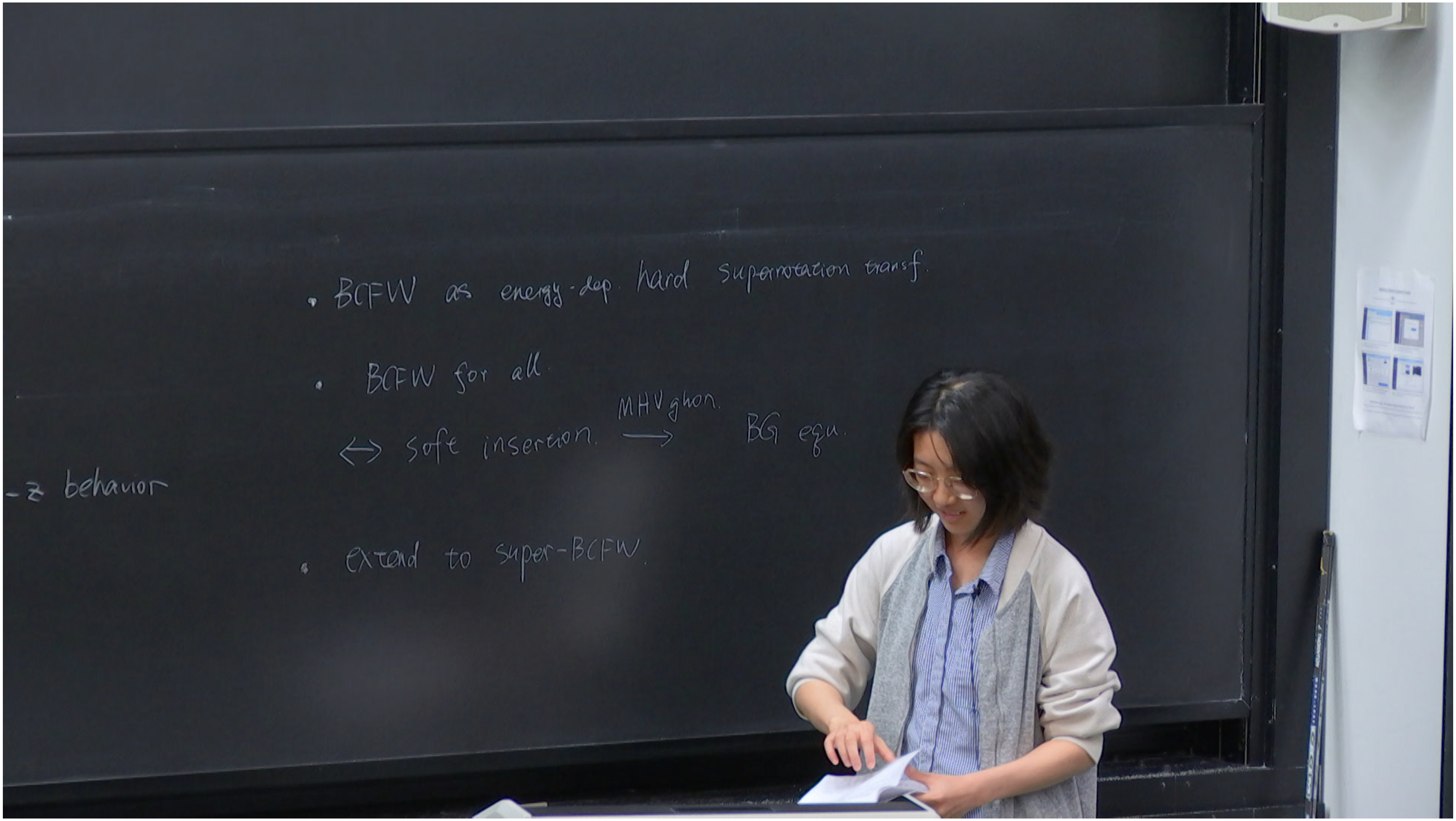
$O_1^a O_2^b \sim i f^{abc} O^c$

$\int B(\Delta_1 - 1, 3 - \Delta_1 - \Delta_2)\, O^{\epsilon}$

$= B(\Delta_2 - 1, 3 - \Delta_1 - \Delta_2)\, O^{-\epsilon}$


② 1/z̄12 : z^(Δ1+Δ2+Jp−2h1−2h̄2) = z^(J2−J1+Jp)
z^(J2−J1−Jp) | Jp | z^(J2−J1+Jp)
1/z | 3 |
1/z | 1 | 1/z
z^3 | 1 | z^3
| −1 | 1/z
Split = (ε1ω1)^α (ε2ω2)^β / (ε1ω1 + ε2ω2)^γ
O1^a O2^b ~ i f^(abc) O^c
Z_YM^2 = Z_GR
Split_(s1,s2) ~ z^(±S3)
A_(n−1)
ωp = ω1 + ω2
t = ε1ω1 / (ε1ω1 + ε2ω2)
B(Δ1−1, 3−Δ1−Δ2) O^ε
− B(Δ2−1, 3−Δ1−Δ2) O^(−ε)


Celestial Recursion
2208.11635 w/ Sabrina Pasterski
Ref: [Arkani-Hamed, Kaplan, '08]
[Pate, Radariu, Strominger, Yuan, '19]
[Himwich, Pate, Singh, '21]
[Hu, Ren, Srikant, Volovich, '21]
[Banerjee, Ghosh, '20] MHV gluon.
Outline
· Review of Celestial Holography
· Celestial CFT vs BCFW
· From celestial OPE to large-z b
· Summary of z→0 story


• BCFW as energy-dep. hard superrotation transf.
• BCFW for all
⟷ soft insertion
MHV gluon.
BG equ.
-z behavior
• extend to super-BCFW

- BCFW as energy-dep. hard superrotation transf.

- BCFW for all

$\Leftrightarrow$ soft insertion $\xrightarrow{\text{MHV gluon}}$ BG equ.

- extend to super-BCFW

$$\lambda_1 \to \lambda_1 - z\lambda_2$$
$$\tilde{\lambda}_2 \to \tilde{\lambda}_2 + z\tilde{\lambda}_1$$
$$\eta_i^A \to \eta_i^A - z\eta_j^A$$