Title: Generalized Wetterich equations on Lorentzian manifolds

Speakers: Katarzyna Rejzner

Series: Quantum Gravity

Date: September 08, 2022 - 2:30 PM

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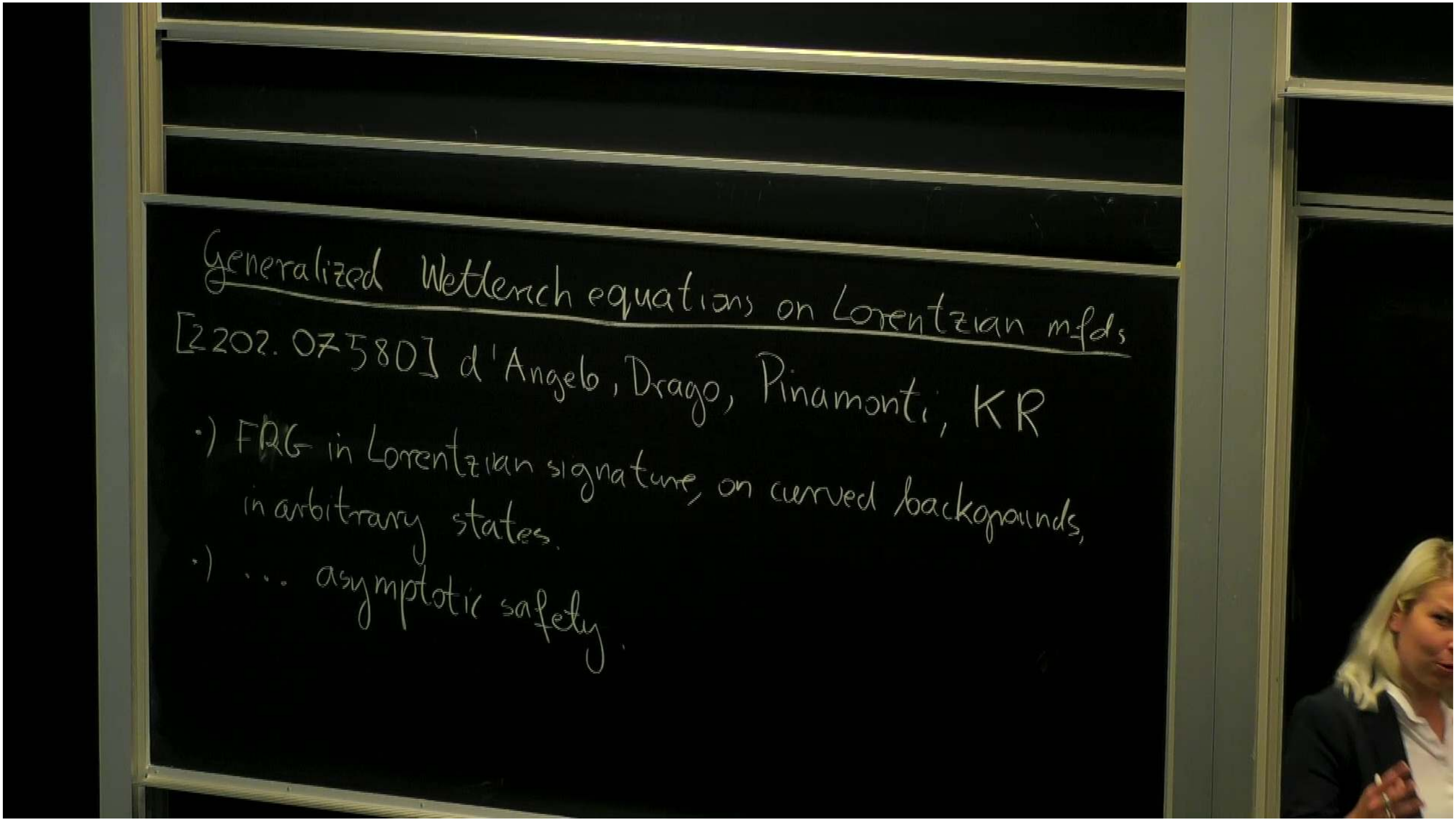
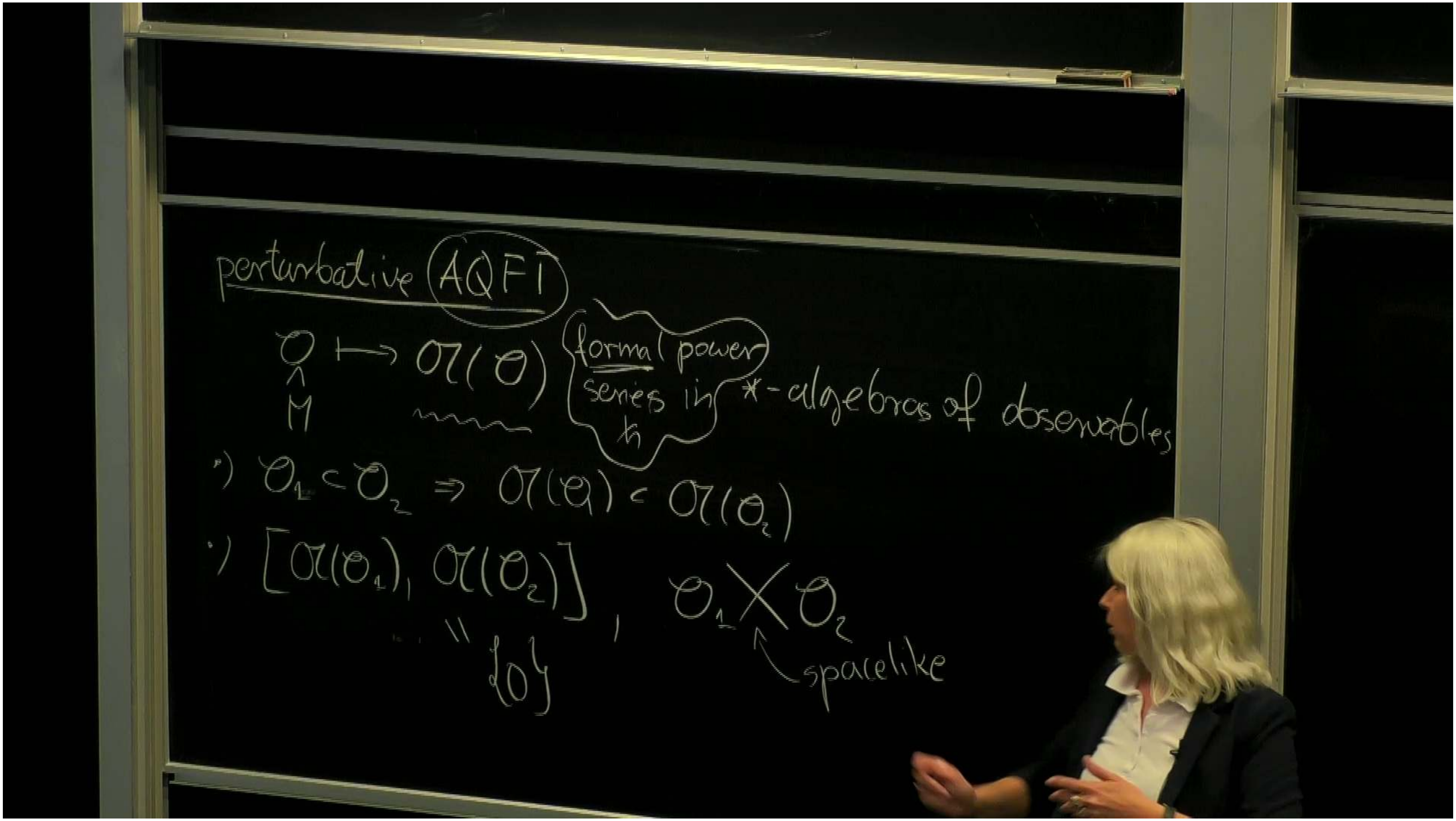
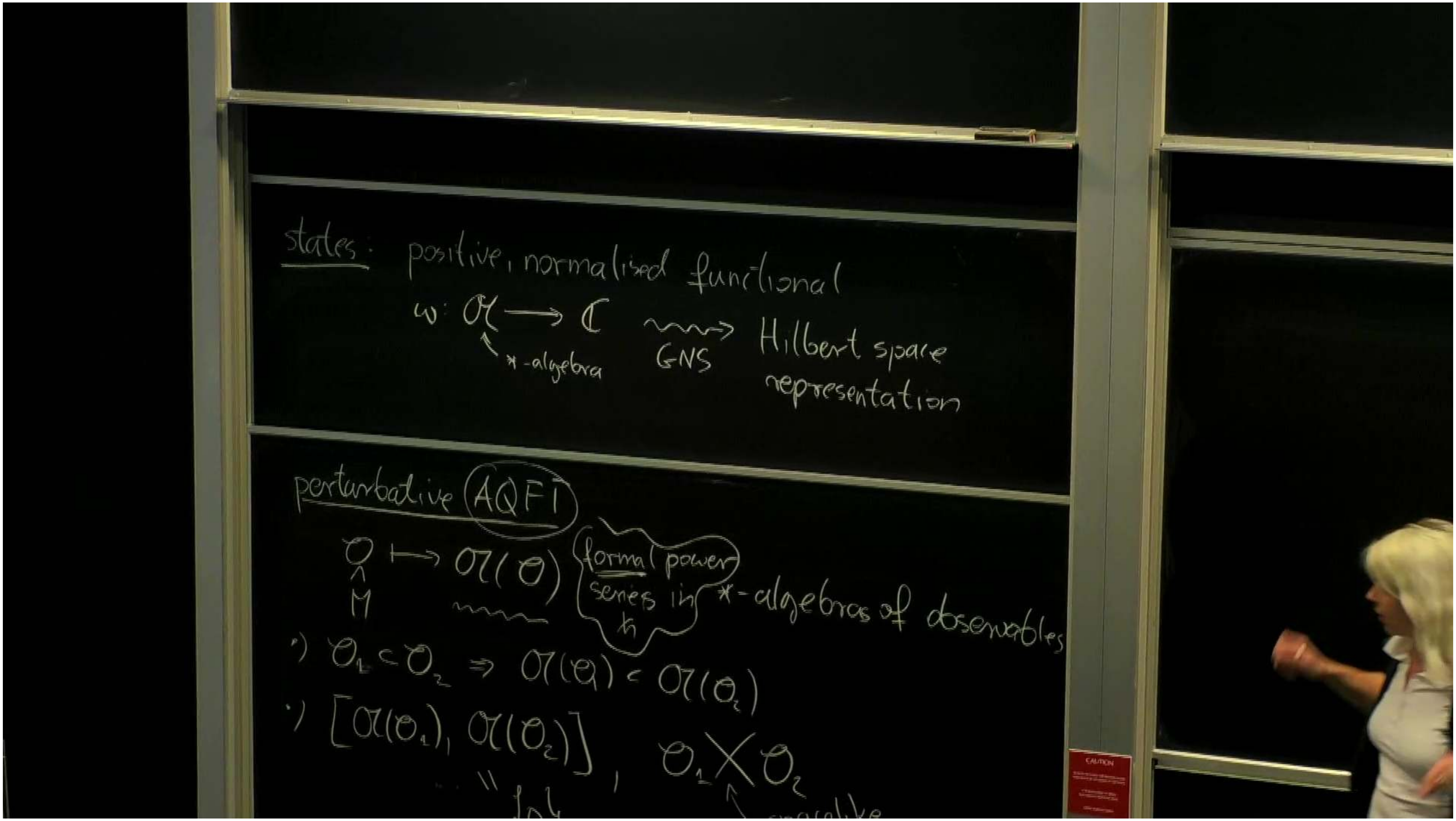
Abstract: In this talk I will present the recent results of my work with Edoardo d'Angelo, Nicolo Drago and Nicola Pinamonti. Using the methods of perturbative algebraic quantum field theory, we have formulated new flow equations on Lorentzian spacetimes that work for arbitrary states (not only the vacuum) and in Minkowski vaccum reduce to Wetterich equations. This has potential applications to the asymptotic safety approach to quantum gravity.

Zoom link: https://pitp.zoom.us/j/98616857260?pwd=ajZoS1lZc0cySlVmTVdVUnA5Ull6Zz09


Generalized Wetterich equations on Lorentzian mfds
[2202.07580] d'Angelo, Drago, Pinamonti, KR
·) FRG in Lorentzian signature, on curved backgrounds, in arbitrary states.
·) ... asymptotic safety.


perturbative AQFT
$\mathcal{O} \longmapsto \mathfrak{A}(\mathcal{O})$ formal power series in $\hbar$ *-algebras of observables
$\mathcal{O} \subset M$
•) $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$
•) $[\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = \{0\}$, $\mathcal{O}_1 \times \mathcal{O}_2$
spacelike


states: positive, normalised functional
$\omega: \mathfrak{A} \longrightarrow \mathbb{C}$
*-algebra
GNS
Hilbert space
representation
perturbative AQFT
$\mathcal{O} \longmapsto \mathfrak{A}(\mathcal{O})$
M
formal power series in $\hbar$
*-algebras of observables
$\cdot)\ \mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$
$\cdot)\ [\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)], \quad \mathcal{O}_1 \times \mathcal{O}_2$

...le

...ion

... observables

<u>Physical input</u>:

M – spacetime, glob. hyp. $M = \Sigma \times \mathbb{R}$

$\mathcal{E}$ – config. space, e.g. for the scalar field $\mathcal{E} = C^\infty(M, \mathbb{R})$

<u>dynamics</u>: using Lagrangians

<u>Free theory</u>: $I_0[\chi] = -\int_M \left(\frac{1}{2} \nabla_\mu \chi \nabla^\mu \chi + \frac{\xi}{2} R \chi^2 + \frac{m^2}{2} \chi^2\right) f \, dx$

$\chi \in \mathcal{E}$

$f \in C^\infty_c(M, \mathbb{R}) \equiv \mathcal{D}$

R

bounds,

observables

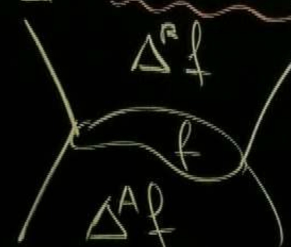
$f \in C_c^\infty(M, \mathbb{R}) \equiv \mathcal{D}$

EOM: $\langle dI_0(\chi), \psi \rangle \doteq \left\langle \frac{\delta I_0(f)}{\delta \chi}, \psi \right\rangle$, $f \equiv 1$ on supp ψ

$dI_0(\chi) = 0$ eq. of motion $\qquad \psi \in \mathcal{D}$

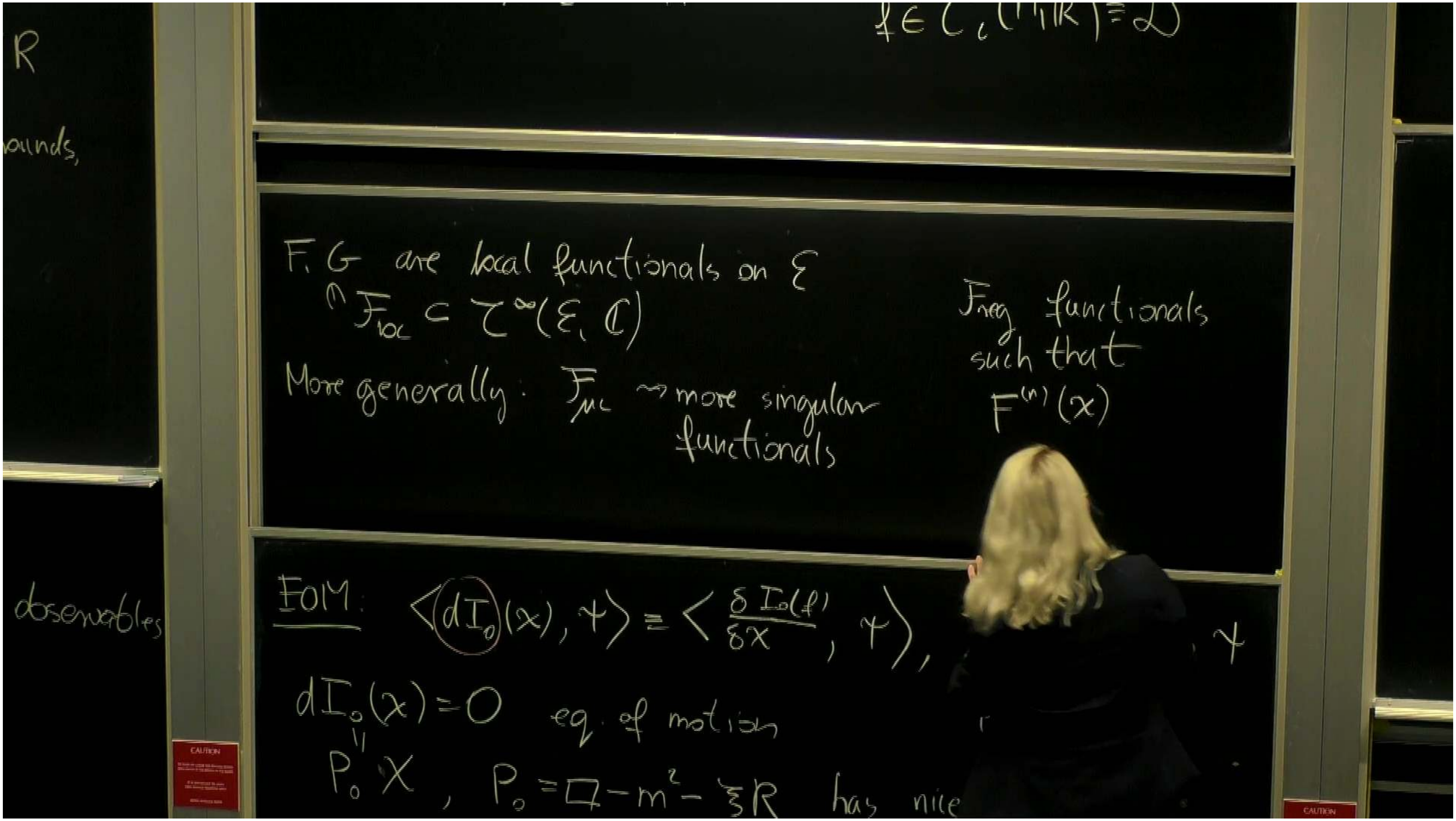
$\| \; P_0 \chi$, $P_0 = \Box - m^2 - \xi R$ has nice Green functions:

Δ^R, Δ^A retarded / advanced Green functions



Pauli-Jordan: $\Delta = \Delta^R - \Delta^A$

$\{F, G\} \doteq \langle F^{(1)}, \Delta G^{(1)} \rangle$


R
ounds,
observables
$f \in C_c^\infty(M, \mathbb{R}) \equiv \mathcal{D}$
F, G are local functionals on $\mathcal{E}$
$\mathcal{F}_{loc} \subset C^\infty(\mathcal{E}, \mathbb{C})$
More generally: $\mathcal{F}_{\mu c} \rightsquigarrow$ more singular functionals
$\mathcal{F}_{reg}$ functionals such that
$F^{(n)}(x)$
EOM: $\langle (dI_0)(x), \psi \rangle = \langle \frac{\delta I_0(f)}{\delta x}, \psi \rangle$, ψ
$dI_0(x) = 0$ eq. of motion
$P_0 X$, $P_0 = \square - m^2 - \xi R$ has nice
CAUTION

F, G are local functionals on $\mathcal{E}$

$\cap \mathcal{F}_{loc} \subset \mathcal{C}^\infty(\mathcal{E}, \mathbb{C})$

More generally: $\mathcal{F}_{\mu c}$ $\rightsquigarrow$ more singular functionals

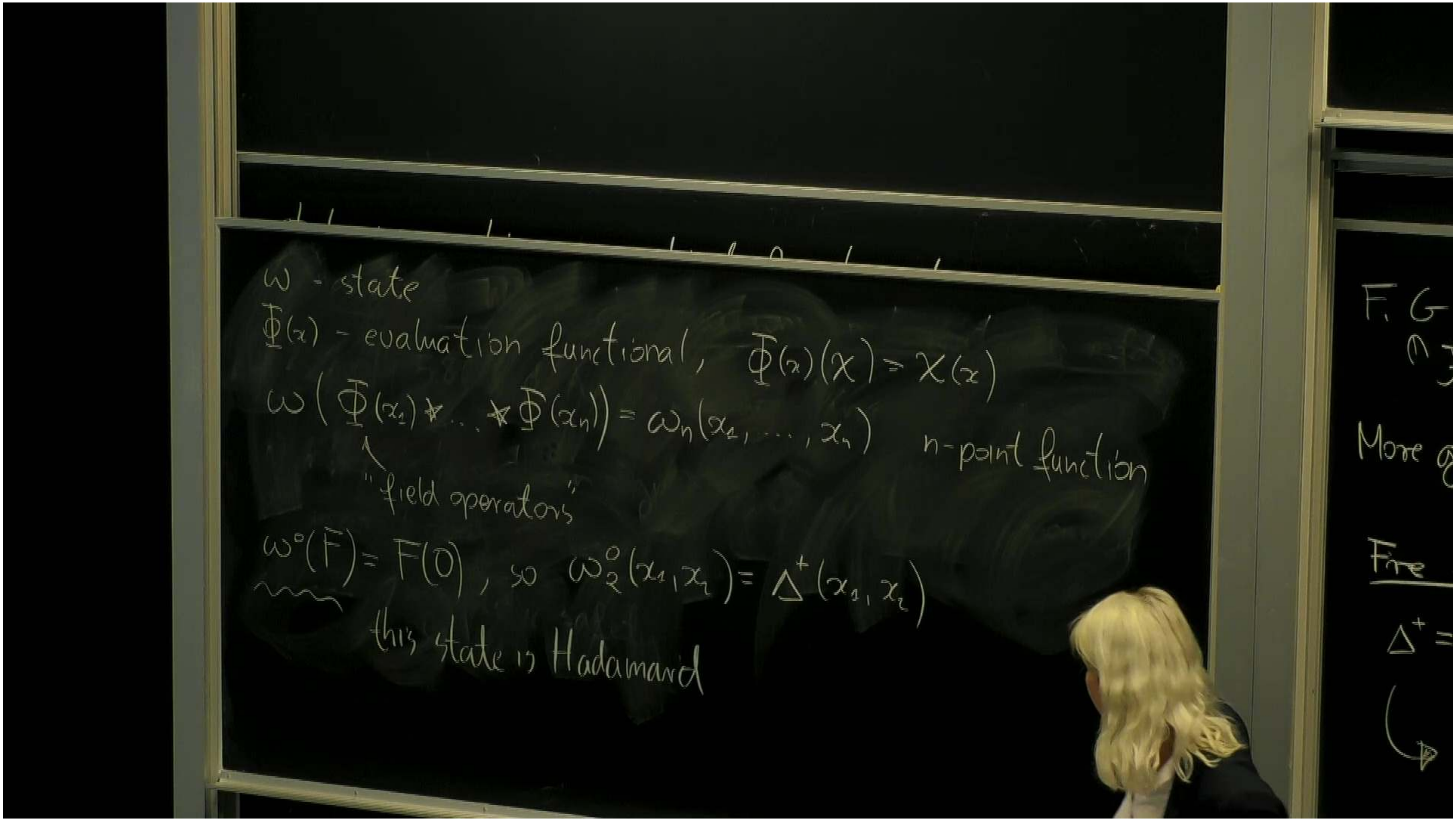
$\mathcal{F}_{reg}$ functionals such that $F^{(n)}(x)$ is smooth

Free quantum:

$\Delta^+ = \frac{i}{2}\Delta + \Delta^S$ is a bisolution for P_0, has some nice singularity structure (WF-set condition)

Δ^S ← symmetric

↳ Hadamard function


ω - state
$\Phi(x)$ - evaluation functional, $\Phi(x)(\chi) = \chi(x)$
$\omega(\Phi(x_1) \star \ldots \star \Phi(x_n)) = \omega_n(x_1, \ldots, x_n)$ n-point function
"field operators"
$\omega^0(F) = F(0)$, so $\omega^0_2(x_1, x_2) = \Delta^+(x_1, x_2)$
this state is Hadamard
F, G
More
Fre
$\Delta^+ =$

this state is Hadamard

<u>Interaction</u>: $I = I_0 + V$, $V(x) = -\lambda \int \frac{x^n}{n!} f \, dx$

Feynman propagator: $\Delta^F = \frac{i}{2}(\Delta^R + \Delta^A) + \Delta^S$

Naive time-ordered product:

F. G

More

Fre

$\Delta^+ =$

$\Delta^{A}F$

$\int_M \frac{x^n}{n!} f\, dx$

F, G are local functionals on $\mathcal{E}$

$\cap\ \mathcal{F}_{loc} \subset C^\infty(\mathcal{E}, \mathbb{C})$

More generally: $\mathcal{F}_{\mu c} \rightsquigarrow$ more singular functionals

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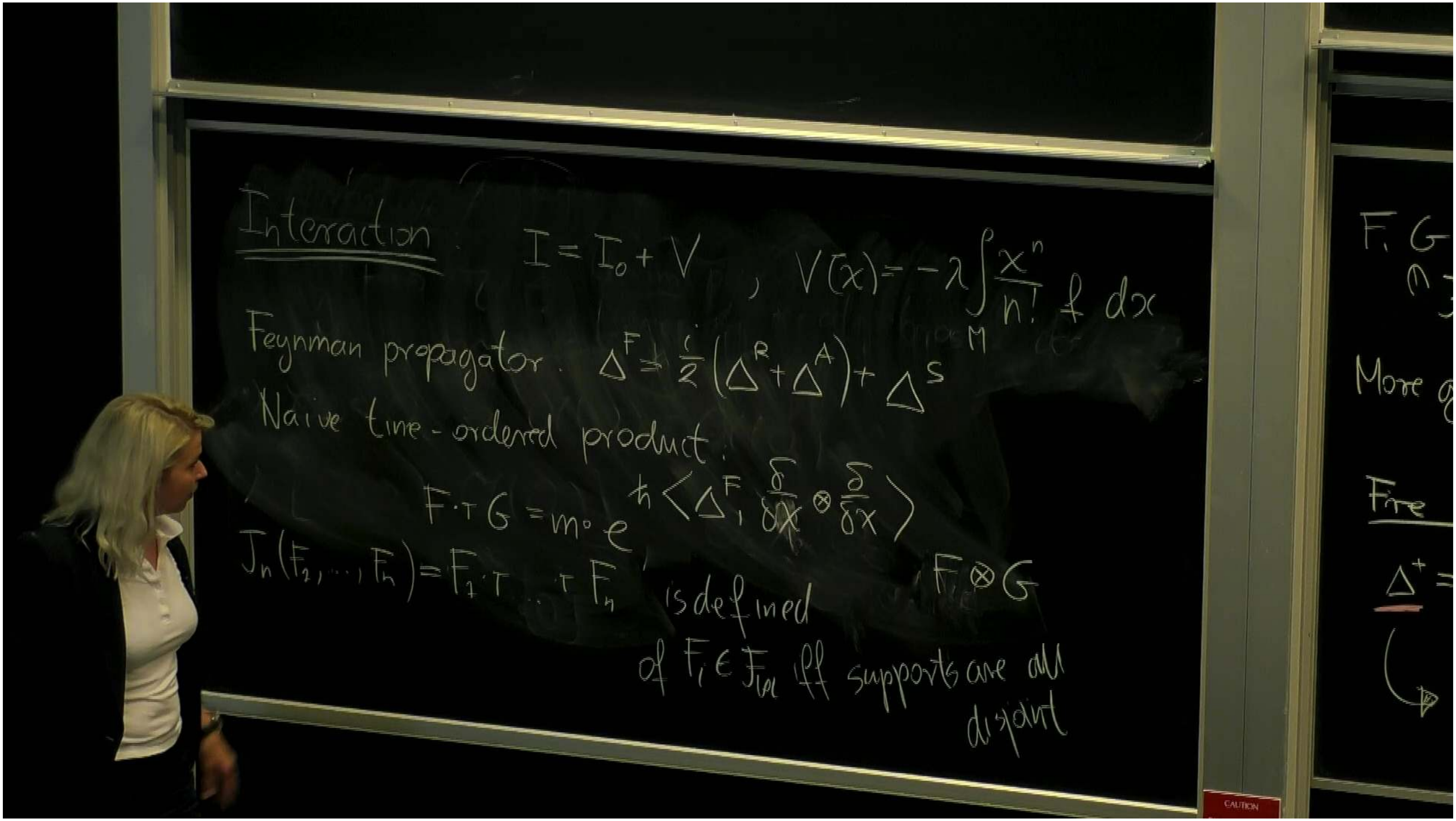
Free quantum:

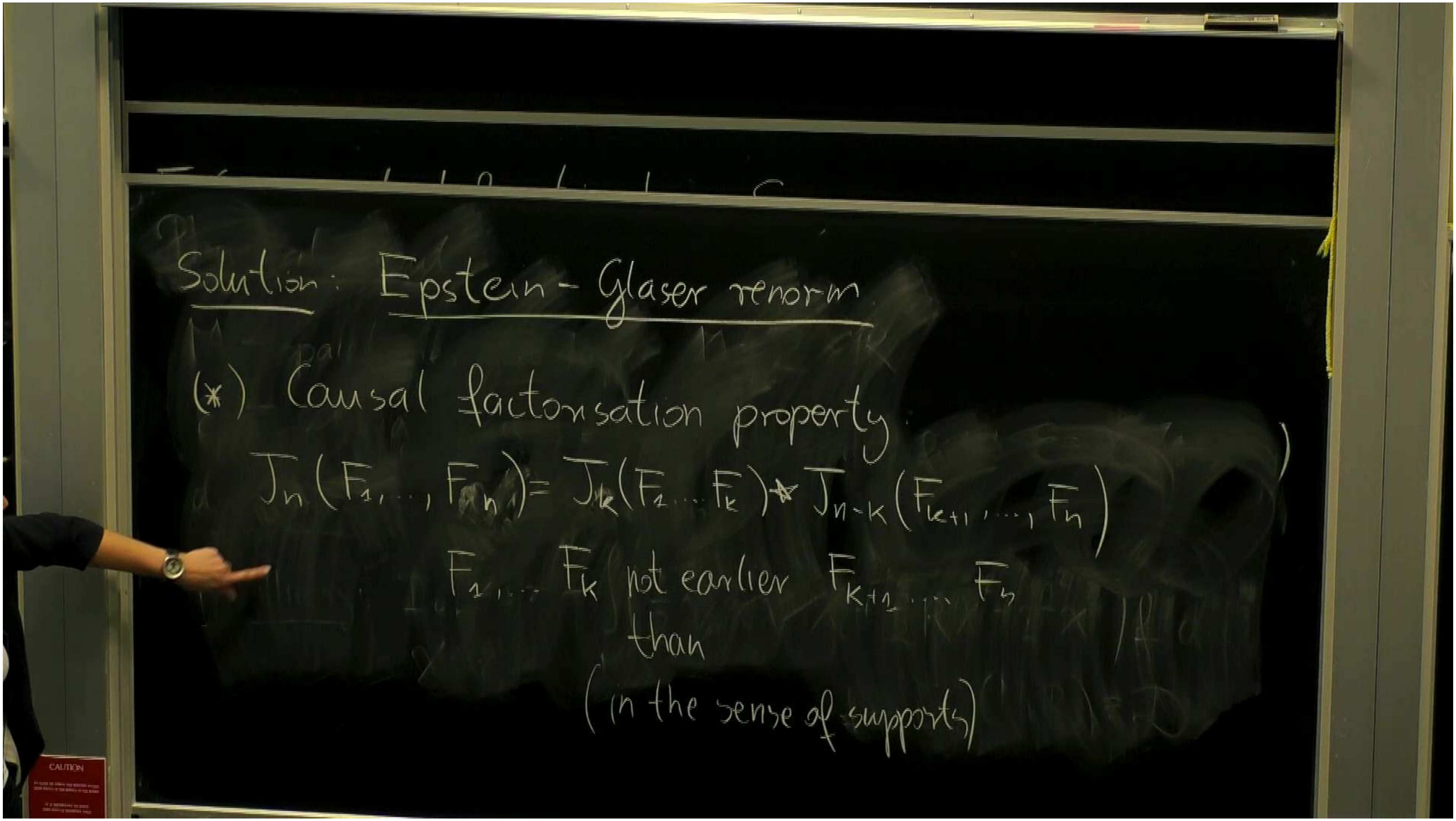
$\Delta^+ = \frac{i}{2}\Delta + \Delta^S$ is a bisolution for P_0, has some nice singularity structure (WF-set condition)

Δ^S ← symmetric

Δ^+ → Hadamard function

$$\boxed{F \star G = m \circ e^{\hbar \langle \Delta^+, \frac{\delta}{\delta x} \otimes \frac{\delta}{\delta x} \rangle} F \otimes G}$$


Interaction: $I = I_0 + V$, $V(x) = -\lambda \int \frac{x^n}{n!} f \, dx$
Feynman propagator: $\Delta^F = \frac{i}{2}(\Delta^R + \Delta^A) + \Delta^S$
Naive time-ordered product:
$F \cdot_T G = m \circ e^{\hbar \langle \Delta^F, \frac{\delta}{\delta x} \otimes \frac{\delta}{\delta x} \rangle} F \otimes G$
$J_n(F_1, \ldots, F_n) = F_1 \cdot_T \ldots \cdot_T F_n$ is defined
of $F_i \in \mathcal{F}_{loc}$ iff supports are all disjoint


Solution: Epstein-Glaser renorm
(*) Causal factorisation property:
$T_n(F_1,\dots,F_n) = T_k(F_1 \dots F_k) \star T_{n-k}(F_{k+1},\dots,F_n)$
$F_1,\dots F_k$ not earlier than $F_{k+1},\dots,F_n$
(in the sense of supports)

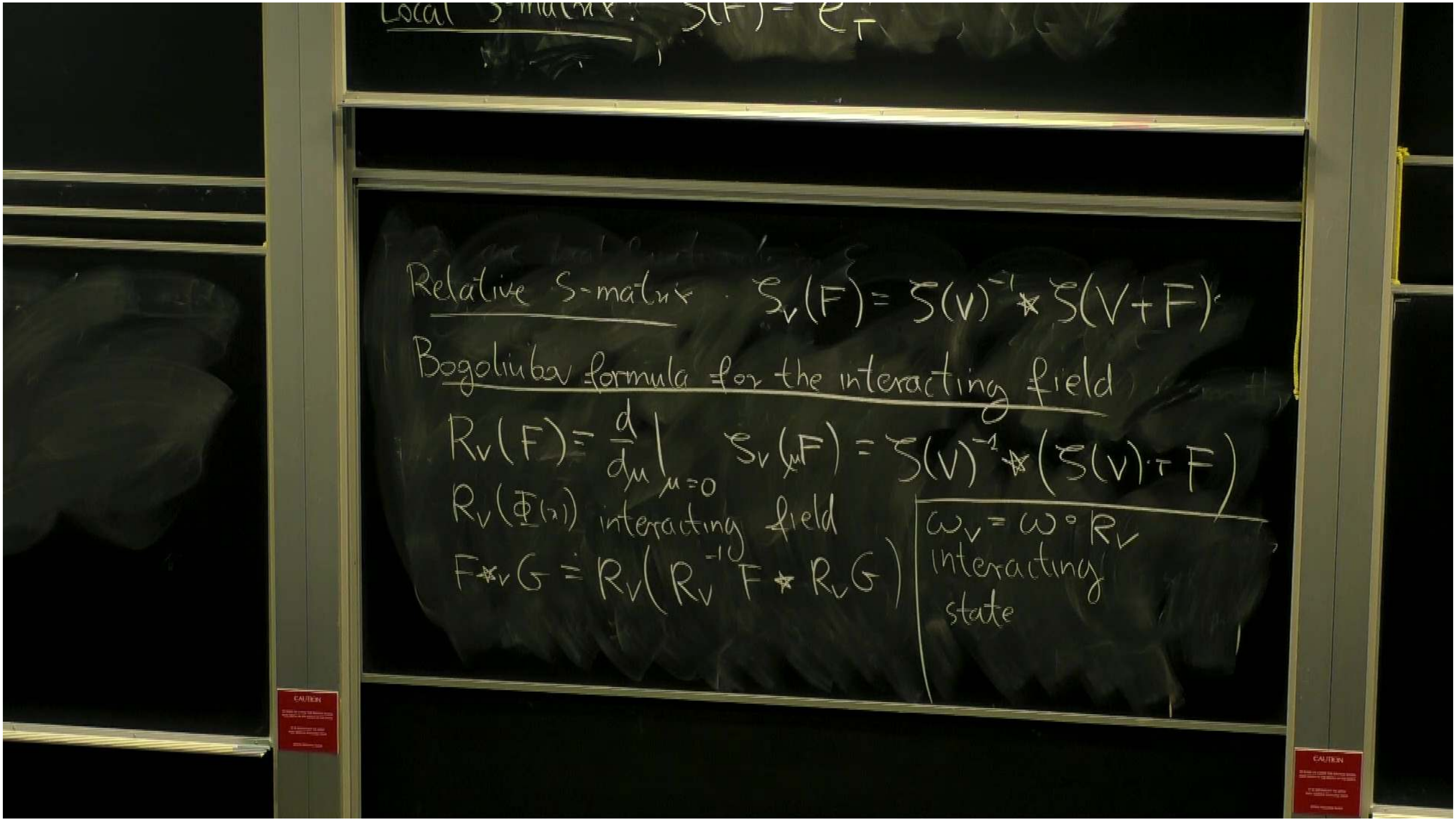
Construct: $\cdot_T$ and $\mathcal{T}$ such that $F \cdot_T G = \mathcal{T}(\mathcal{T}^{-1}F \cdot \mathcal{T}^{-1}G)$

formally the convolution with the Gaussian measure of the free theory

$$(\mathcal{T}F)(0) = \int F(X)\, e^{\frac{i}{\hbar} I_0(X)}\, \mathcal{D}X$$

Green functions: $G_n(x_1, \dots, x_n) = \omega\left(\Phi(x_1) \cdot_T \dots \cdot_T \Phi(x_n)\right)$

Local S-matrix: $\mathcal{S}(F) = e_T^{\frac{i}{\hbar} F}$


Relative S-matrix $S_V(F) = S(V)^{-1} \star S(V+F)$
Bogoliubov formula for the interacting field
$R_V(F) = \frac{d}{d\mu}\Big|_{\mu=0} S_V(\mu F) = S(V)^{-1} \star (S(V) \cdot_T F)$
$R_V(\Phi(x))$ interacting field
$F \star_V G \doteq R_V(R_V^{-1} F \star R_V G)$
$\omega_V = \omega \circ R_V$
interacting state

$$\omega_V\left(\Phi(x_1) \star_V \cdots \star_V \Phi(x_n)\right) = \omega\left(R_V(\Phi(x_1)) \star \cdots \star R_V(\Phi(x_n))\right)$$

$\cdot_T$ time-ordered version

Green functions: $\omega \circ R_V\left(\Phi(x_1) \cdot_T \cdots \cdot_T \Phi(x_n)\right)$

interacting Green function.

$$\omega_V\left(\Phi(x_1) \star_V \cdots \star_V \Phi(x_n)\right) = \omega\left(R_V(\Phi(x_1)) \star \cdots \star R_V(\Phi(x_n))\right)$$

$\cdot_T$ time-ordered version

Green functions: $\omega \circ R_V\left(\Phi(x_1) \cdot_T \cdots \cdot_T \Phi(x_n)\right)$

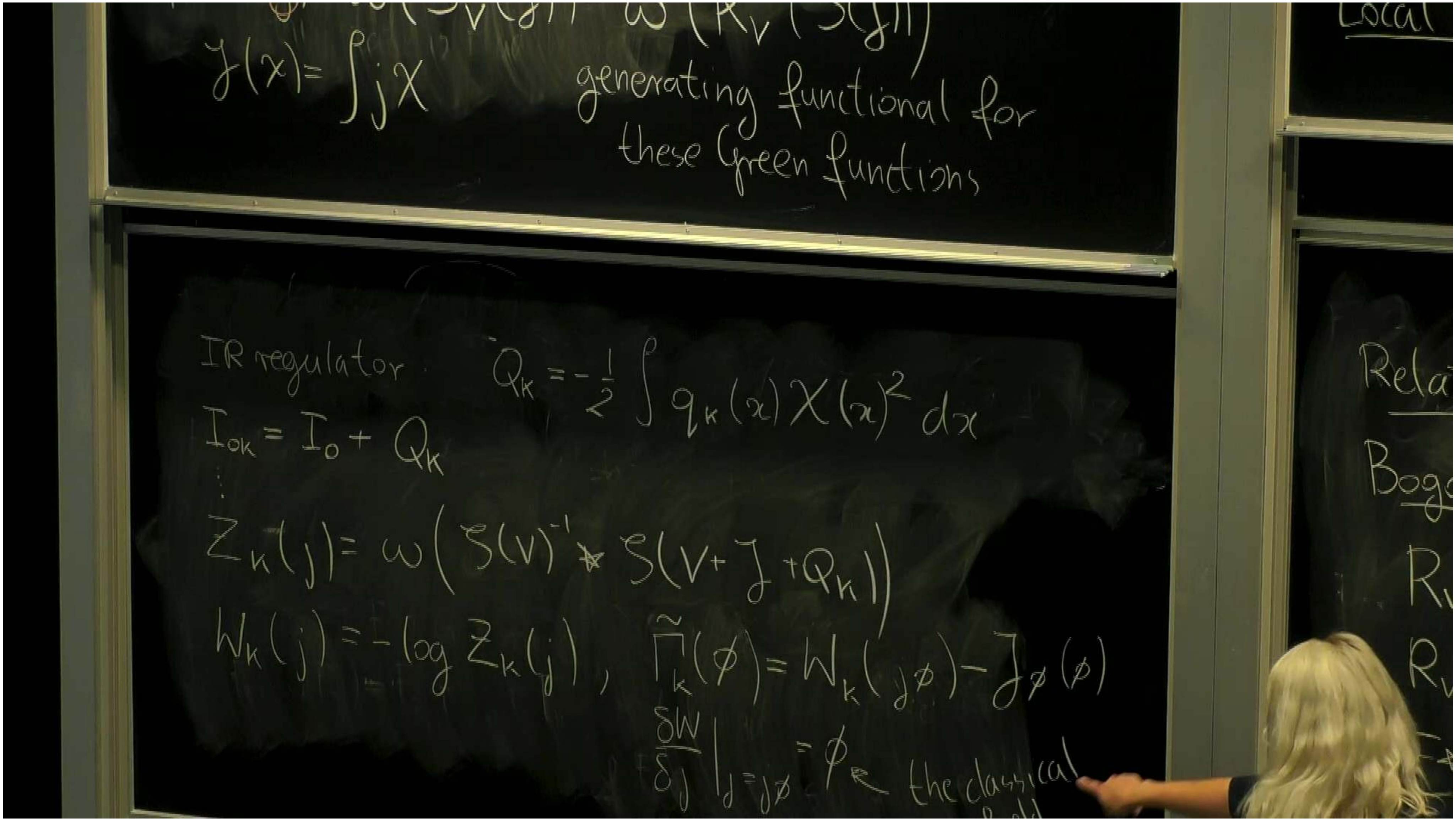
interacting Green function.

source

Def. $Z(j) = \omega(S_V(J)) = \omega(R_V(S(J)))$

$J(x) = \int j X$

generating functional for these Green functions


$J(x) = \int jX$
generating functional for these Green functions
IR regulator: $Q_k = -\frac{1}{2} \int q_k(x) X(x)^2 dx$
$I_{0k} = I_0 + Q_k$
$Z_k(j) = \omega\left(S(V)^{-1} \star S(V + J + Q_k) \right)$
$W_k(j) = -\log Z_k(j)$, $\tilde{\Gamma}_k(\phi) = W_k(j_\phi) - J_\phi(\phi)$
$\frac{\delta W}{\delta j}\Big|_{j=j_\phi} = \phi$ ← the classical

Average effective action: $\Gamma_k(\phi) = \tilde{\Gamma}(\phi) - Q_k(\phi)$

Generalized Wetterich equation

$$\partial_k \Gamma_k(\phi) = -\frac{1}{2}\int dx\; \partial_k q_k(x) \left(\frac{1}{Z_k(j_\phi)}\, \omega\Big(R_V\big(S(J_\phi + Q_k)\cdot \tau : X^2(x)\big) - \phi^2(x)\Big)\right)$$

$$\frac{\delta^2 W_k(j)}{\delta j(x)\,\delta j(y)} + W^{(1)}(x)\, W_k^{(1)}(y)$$

evaluated at $j = j_\phi$ give ϕ

Average effective action: $\Gamma_k(\phi) = \tilde{\Gamma}(\phi) - Q_k(\phi)$

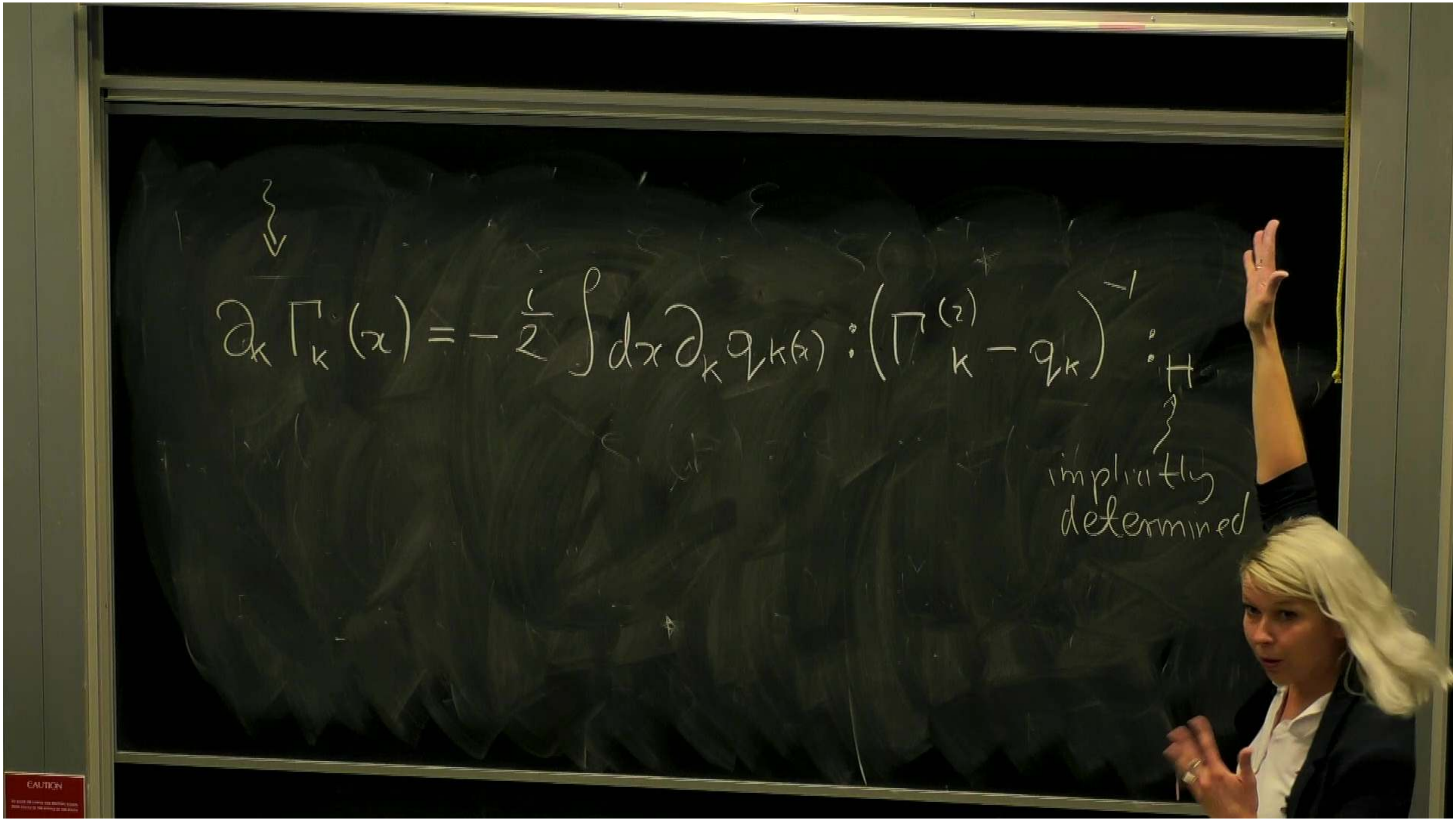
Generalized Wetterich equation

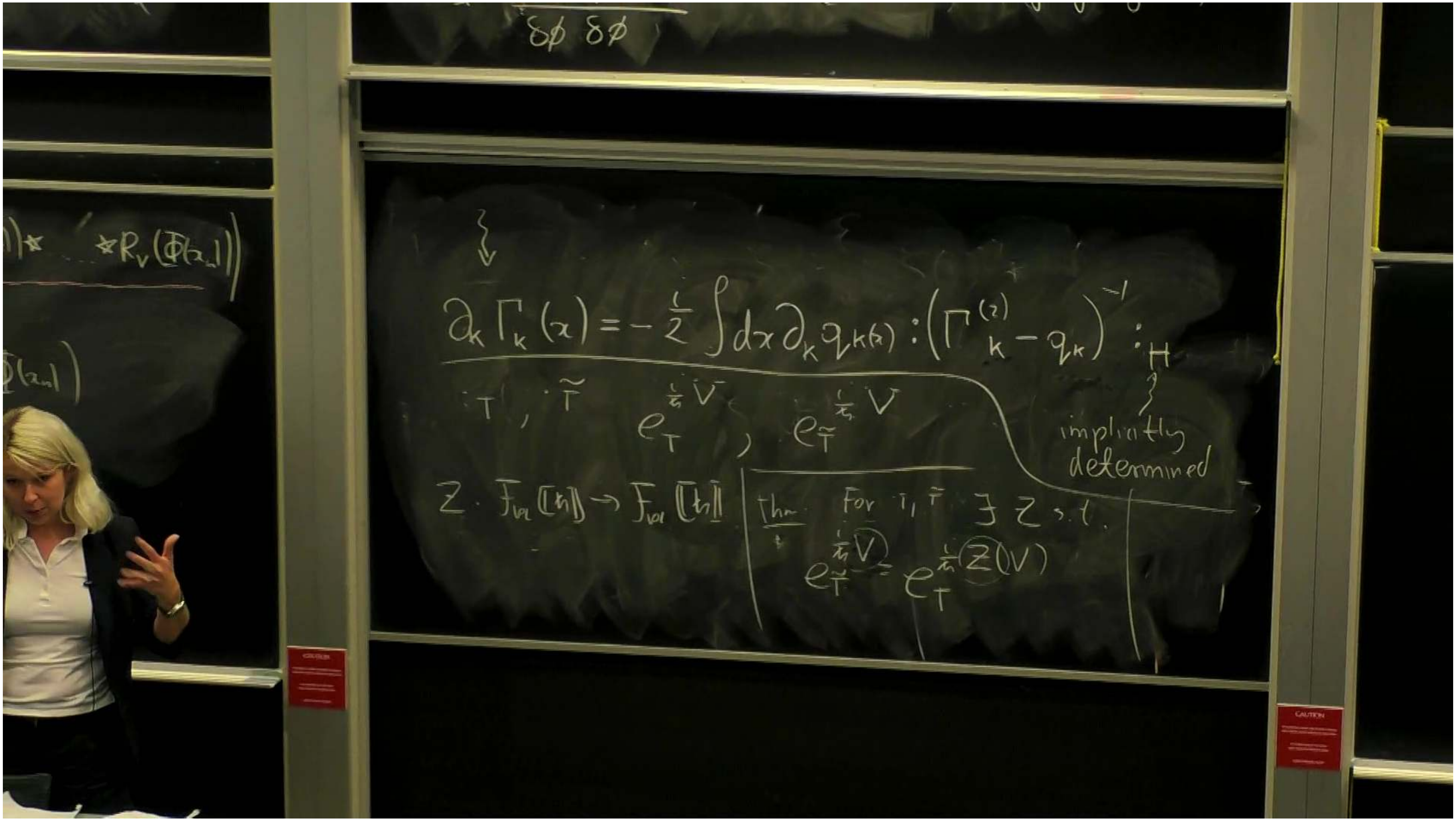
$$\partial_k \Gamma_k(\phi) = -\frac{1}{2}\int dx\, \partial_k q_k(x) \left(\frac{1}{Z_k(j_\phi)}\, \omega\left(R_V\left(S(j_\phi + Q_k)\cdot \tau : \chi^2(x)\right)\right) - \phi^2(x) \right)$$

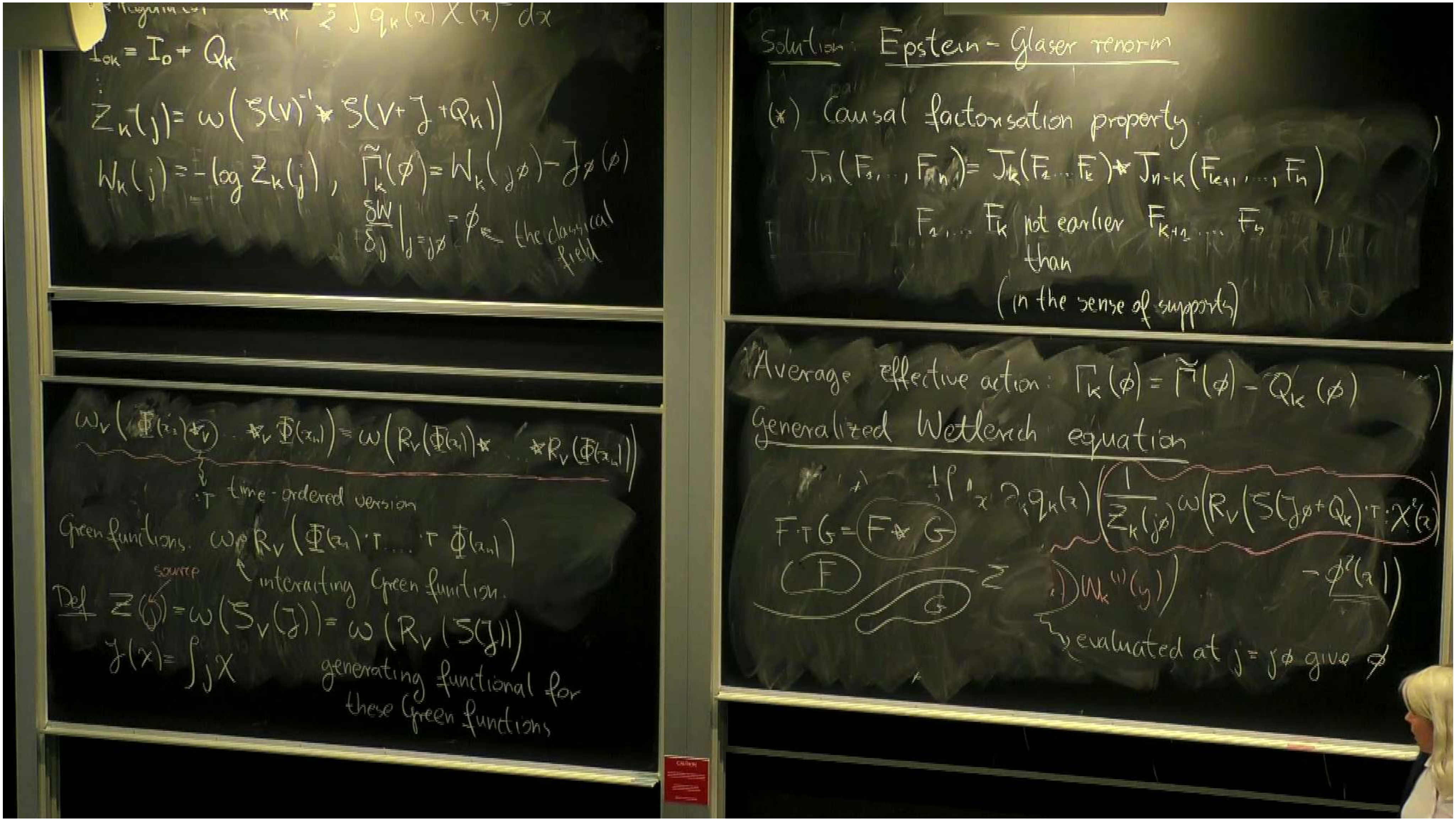
$$-i\left(\frac{\delta^2 W_k(j)}{\delta j(x)\,\delta j(y)} + W^{(1)}(x)\, W_k^{(1)}(y)\right)$$

inverse of $\dfrac{\delta^2(\Gamma_k + Q_k)}{\delta\phi\,\delta\phi}$

evaluated at $j = j_\phi$ give ϕ


$\partial_k \Gamma_k(x) = -\frac{1}{2}\int dx\, \partial_k q_{k(x)} :\left(\Gamma^{(2)}_k - q_k\right)^{-1}:_H$
implicitly
determined
CAUTION


$\delta\phi \; \delta\phi$
$\partial_k \Gamma_k(x) = -\frac{i}{2} \int dx \, \partial_k q_{k}(x) :\left(\Gamma^{(2)}_k - q_k\right)^{-1}:_H$
implicitly determined
$T, \tilde{T} \quad e_T^{\frac{i}{\hbar}V}, \; e_{\tilde{T}}^{\frac{i}{\hbar}V}$
$Z : \mathcal{F}_{loc}[[\hbar]] \to \mathcal{F}_{loc}[[\hbar]]$
thm For $T, \tilde{T}$ $\exists$ Z s.t.
$e_{\tilde{T}}^{\frac{i}{\hbar}V} = e_T^{\frac{i}{\hbar}Z(V)}$
$\star R_V(\Phi(x_n))$



$I_{0k} = I_0 + Q_k$
$Z_k(j) = \omega\left(S(V)^{-1} \star S(V + J + Q_k)\right)$
$W_k(j) = -\log Z_k(j)$, $\tilde{\Gamma}_k(\phi) = W_k(j_\phi) - J_\phi(\phi)$
$\frac{\delta W}{\delta j}\Big|_{j=j_\phi} = \phi$ the classical field
$\omega_V(\Phi(x_1) \star_V \dots \star_V \Phi(x_n)) = \omega(R_V(\Phi(x_1)) \star \dots \star R_V(\Phi(x_n)))$
$\cdot_T$ time-ordered version
Green functions. $\omega(R_V(\Phi(x_1) \cdot_T \dots \cdot_T \Phi(x_n)))$
interacting Green function.
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Average effective action: $\Gamma_k(\phi) = \tilde{\Gamma}(\phi) - Q_k(\phi)$
Generalized Wetterich equation
$F \cdot_T G = F \star G$
$\frac{1}{Z_k(j_\phi)}\omega(R_V(S(J_\phi + Q_k) \cdot_T X^2(x)$
$W_k^{(1)}(y)$
$-\phi^2(x)$
evaluated at $j = j_\phi$ give ϕ