Title: Quantum space-time and gravity from the IKKT matrix model

Speakers: Harold Steinacker

Series: Quantum Gravity

Date: September 07, 2022 - 11:00 AM

URL: https://pirsa.org/22090079

Abstract: We discuss how 3+1-dimensional physics including gravity can arise in the IKKT or IIB matrix model. The model is related to string theory, but it also provides a direct and accessible definition of 3+1-dimensional physics on suitable backgrounds. This leads to a higher-spin extension of gravity on covariant quantum space-time with manifest volume-preserving diffeos, which is UV finite at one loop.

Zoom Link: https://pitp.zoom.us/j/94588999721?pwd=VTdxQSt3MGhSMkt3UThCVjNRcXJBQT09

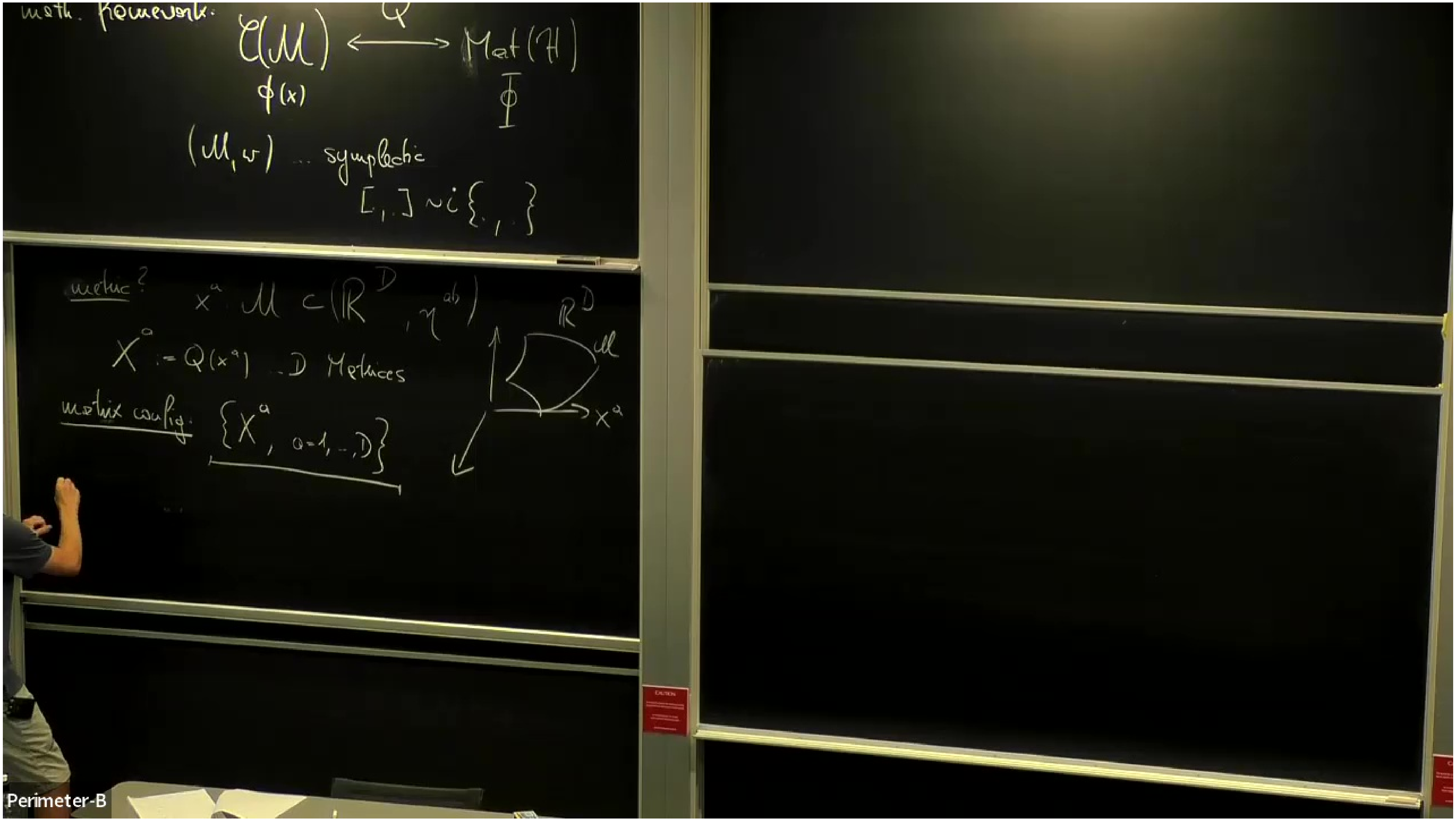
Emergent space-time & gravity from MM
in 3+1 d.

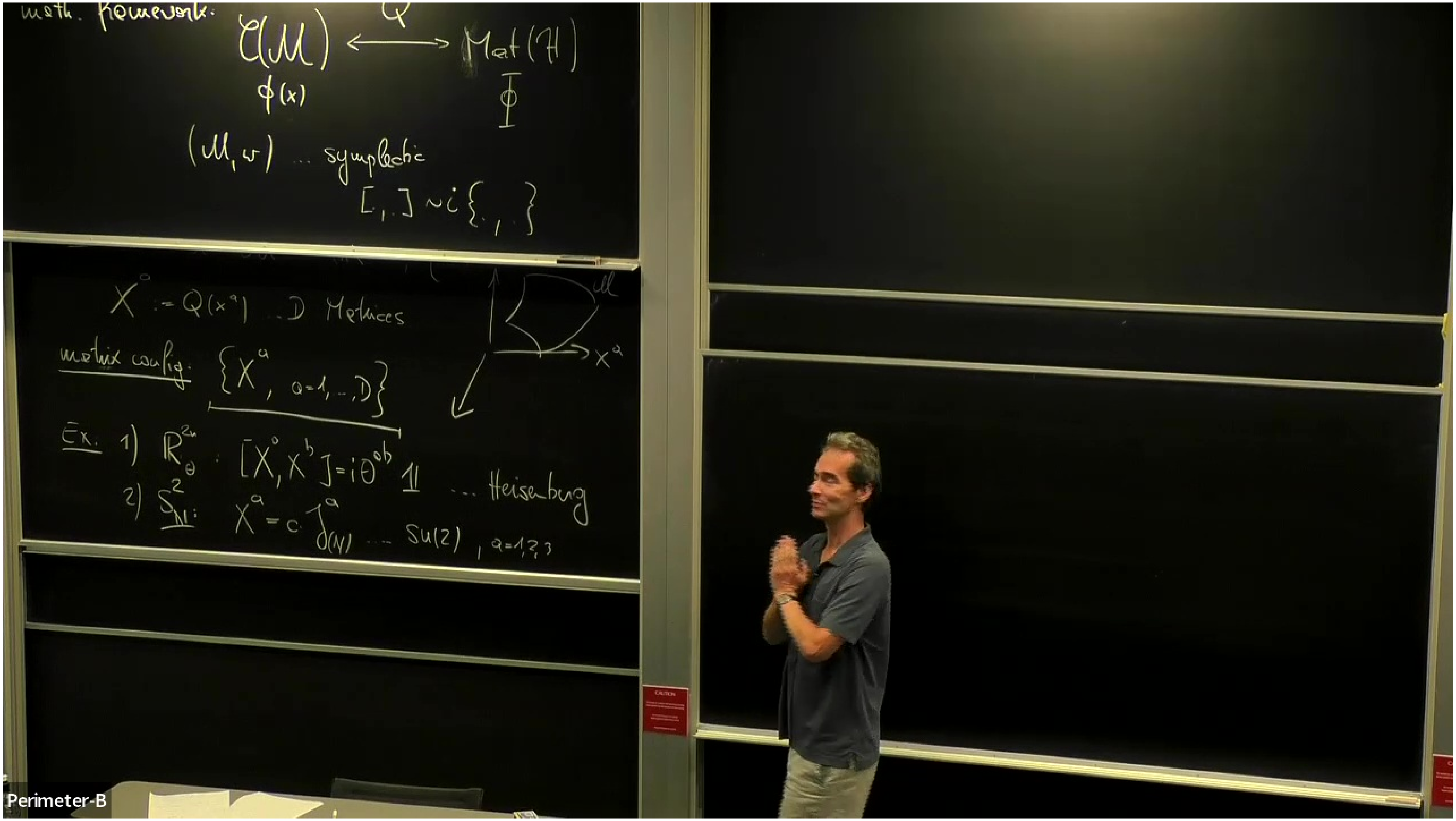
math. framework:

$$\begin{array}{ccc} \mathcal{C}(\mathcal{M}) & \xleftrightarrow{\;Q\;} & Mat(\mathcal{H}) \\ \phi(x) & & \Phi \end{array}$$

$(\mathcal{M}, \omega)$ .. symplectic

$[\,.\,,\,.\,] \sim i\{\,.\,,\,.\,\}$


math. framework:
C(M) ←Q→ Mat(H)
ϕ(x)
Φ
(M, ω) ... symplectic
[,] ∼ i{ , }
metric? x^a: M ⊂ (R^D, η^ab)
R^D
M
X^a := Q(x^a) .. D Matrices
matrix config: {X^a, a=1,..,D}
x^a
Perimeter-B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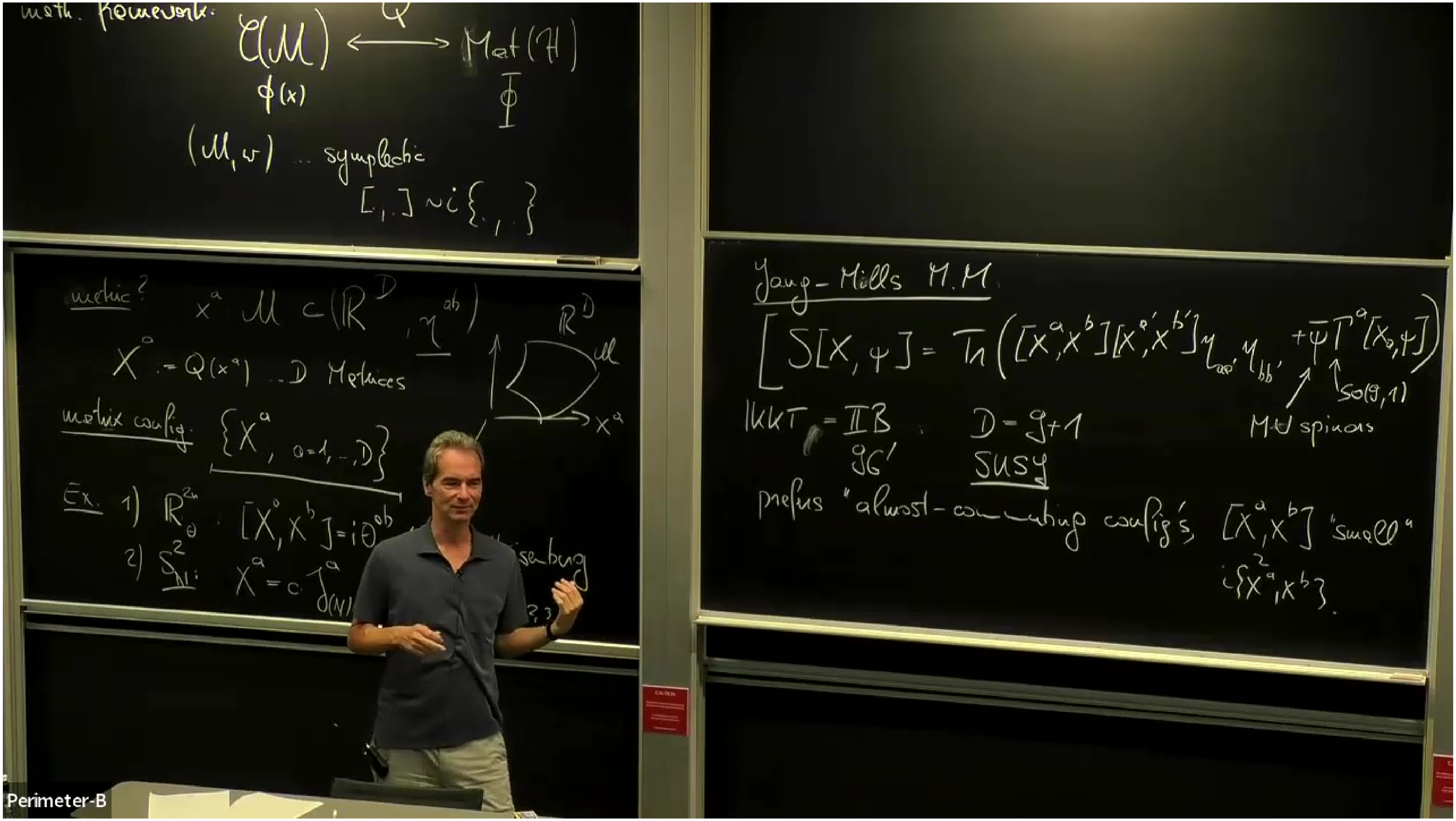
Ex. 1) R^2n_θ : [X^a, X^b] = iθ^ab 1 ... Heisenberg
2) S^2_N: X^a = c J^a_(N) ... su(2), a=1,2,3
Perimeter-B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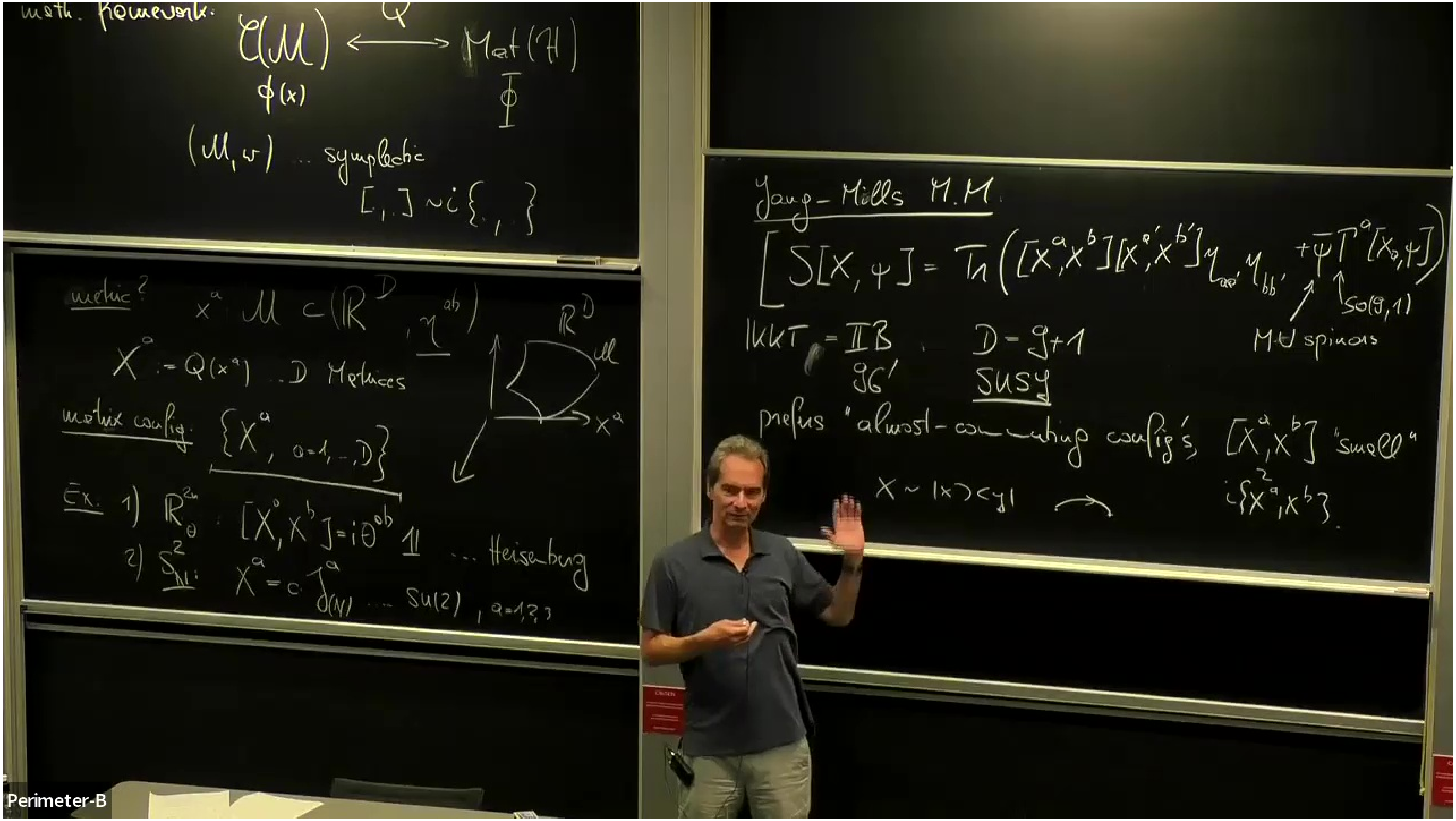
Yang-Mills M.M.
S[X, ψ] = Tr([X^a, X^b][X^a', X^b'] η_aa' η_bb' + ψ̄ Γ^a [X_a, ψ])
SO(9,1)
MW spinors
IKKT = IIB
D = 9+1
SUSY
prefers "almost-commuting config's, [X^a, X^b] "small"
i{X^a, X^b}.
Perimeter-B



math. Framework:
$\mathcal{C}(\mathcal{M}) \xleftrightarrow{Q} Mat(\mathcal{H})$
$\phi(x)$ Φ
$(\mathcal{M}, \omega)$ ... symplectic
$[\,,\,] \sim i\{\,,\,\}$
metric? $x^a: \mathcal{M} \subset (\mathbb{R}^D, \eta^{ab})$
$X^a := Q(x^a)$ .. D Matrices
matrix config: $\{X^a, a=1,..,D\}$
Ex. 1) $\mathbb{R}^{2n}_\theta$: $[X^a,X^b] = i\theta^{ab}\mathbb{1}$ ... Heisenberg
2) S^2_N: $X^a = c\,J^a_{(N)}$ ... su(2), a=1,2,3
$\mathbb{R}^D$
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SO(9,1)
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IKKT = IIB
96'
D = 9+1
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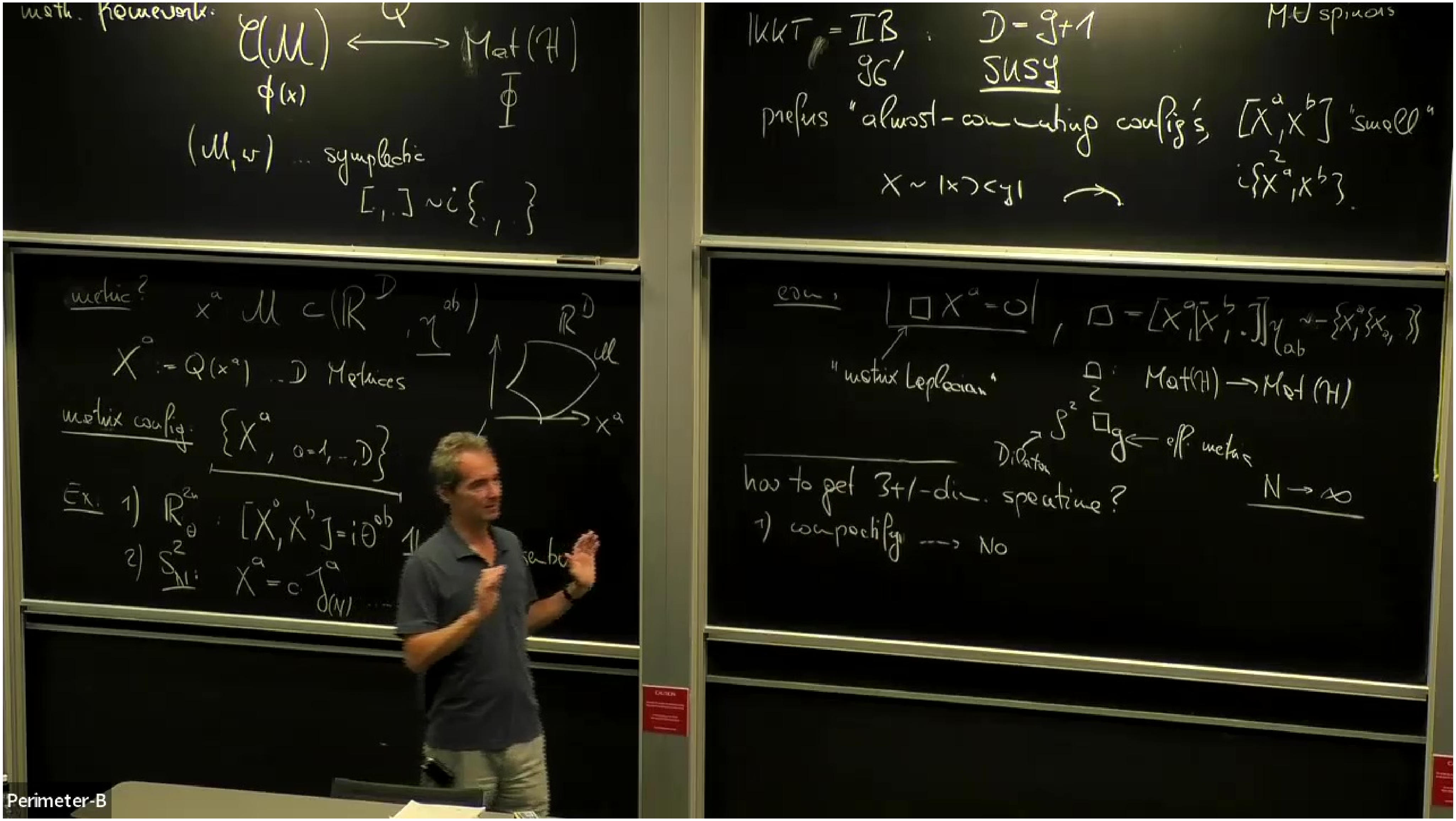
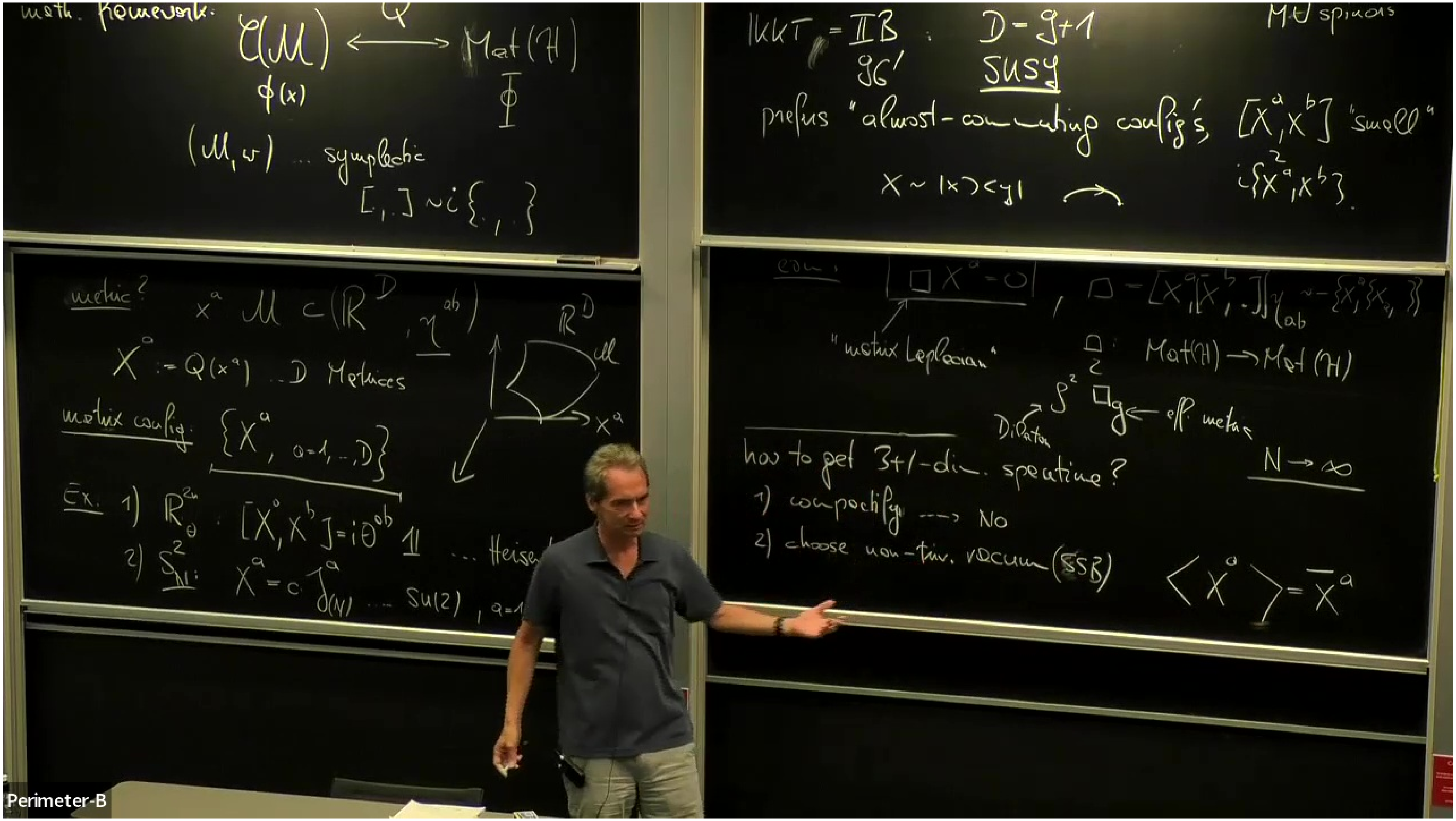
prefers "almost-commuting config's" $[X^a,X^b]$ "small"
$X \sim |x\rangle\langle y|$
$i\{X^a,X^b\}$
Perimeter-B


math. framework:
Perimeter-B
IKKT = IIB
SUSY
"matrix Laplacian"
Dilaton
how to get 3+1-dim. spacetime?
1) compactify ---> No
N → ∞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□ : Mat(H) → Mat(H)
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1) compactify → No
2) choose non-triv. vacuum (SSB)
⟨X^a⟩ = X̄^a
Perimeter-B

math. framework: $\mathcal{C}(\mathcal{M}) \xleftrightarrow{Q} Mat(\mathcal{H})$

$\phi(x) \qquad \Phi$

$(\mathcal{M}, \omega)$ ... symplectic

$[\,,\,] \sim i\{\,,\,\}$

metric? x^a $\mathcal{M} \subset (\mathbb{R}^D, \eta^{ab})$

$X^a := Q(x^a)$ .. D Matrices

$\mathbb{R}^D$, $\mathcal{M}$, x^a

matrix config. $\{X^a, \; a=1,..,D\}$

Ex. 1) $\mathbb{R}^{2n}_\theta$: $[X^a, X^b] = i\theta^{ab}\mathbb{1}$

2) S^2_N: $X^a = c\, J^a_{(N)}$ ...

$\underbrace{\Box X = 0}_{\text{"brane"}}$, $\Box = [X, [X, .]]\eta_{ab} \sim \{x, \{x_a, .\}\}$

"matrix Laplacian" $\Box$: $Mat(\mathcal{H}) \to Mat(\mathcal{H})$

$\rho^2 \, \Box_g \leftarrow$ eff. metric

Dilaton

how to get 3+1-dim. spacetime? $N \to \infty$

1) compactify ---> No

2) choose non-triv. vacuum (SSB) $\langle X^a \rangle = \bar{X}^a$ 3+1-dim $\mathcal{M}$

irreducible

fluctuations, $\bar{X}^a + \mathcal{A}^a(x) = X^a$ $\quad X^a = \begin{pmatrix} \times & & 0 \\ & \ddots & \\ 0 & & \times \end{pmatrix}$

$\in Mat(\mathcal{H}) - \mathcal{C}(\mathcal{M})$, functions in 3+1-d

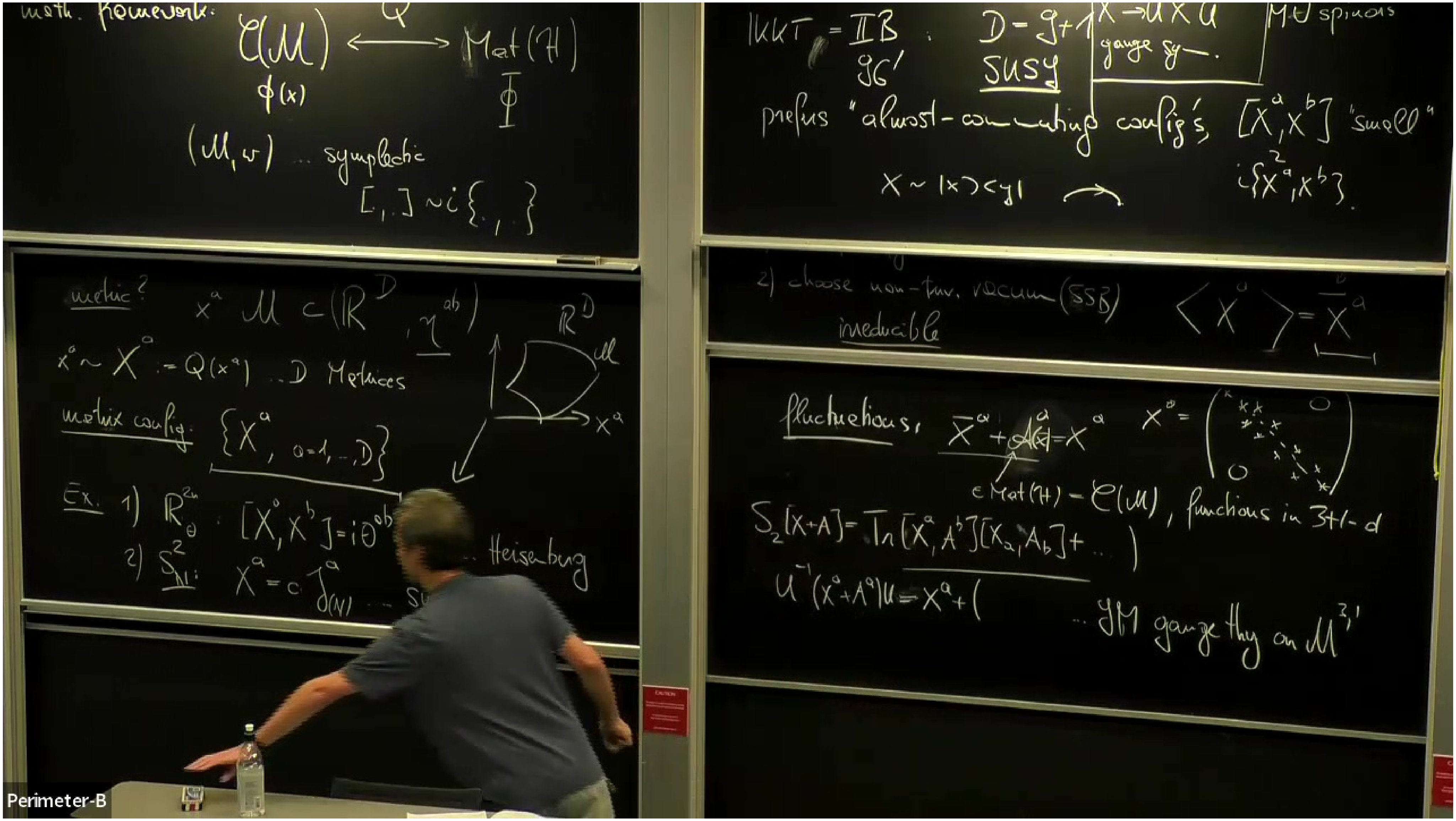
Yang-Mills M.M

$S[X, \psi] = Tr\left([X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} + \bar{\psi}\Gamma^a[X_a, \psi]\right)$

IKKT $= IIB$, $D = 9+1$

$SO(9,1)$

MW spinors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∈ Mat(H) = C(M), functions in 3+1-d
S_2[X+A] = Tr([X^a, A^b][X_a, A_b] + ...)
U^-1(X^a + A^a)U = X^a + (
... YM gauge thy on M^3,1
Perimeter-B

Emergent space-time & gravity from MM in 3+1 d.

math. framework

$$\mathcal{C}(\mathcal{M}) \xleftrightarrow{\;Q\;} Mat(\mathcal{H})$$

$\phi(x)$ $\qquad\qquad$ Φ

$(\mathcal{M}, \omega)$ symplectic

$[\,,\,] \sim i\{\,,\,\}$

$\subset$ $\mathfrak{g}(N)$ $\quad$ $SU(2)$, $a=1,2,3$

IKKT = IIB : $D=9+1$ $\quad X \to U X U$ gauge sym. $\quad$ MW spinors

g_6' $\quad$ SUSY

prefers "almost-commuting config's, $[X^a, X^b]$ "small"

$X \sim |x\rangle\langle y|$ $\quad \to \quad$ $i\{X^a, X^b\}$

2) choose non-triv. vacuum (SSB) $\quad \langle X^a \rangle = \bar{X}^a$

irreducible

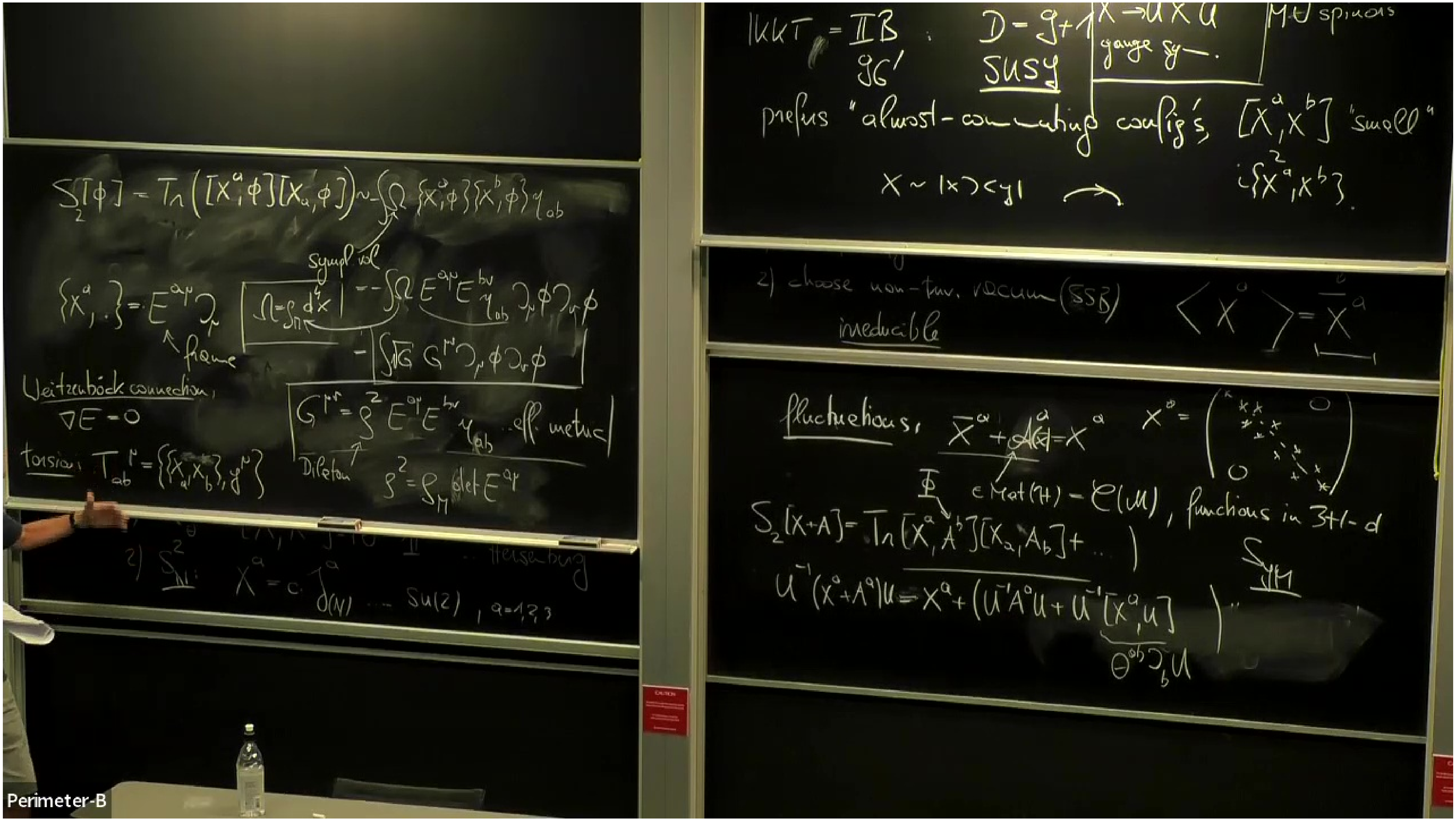
fluctuations, $\bar{X}^a + A^a(x) = X^a$ $\quad X^a = \begin{pmatrix} * & & 0 \\ & \ddots & \\ 0 & & * \end{pmatrix}$

$\in Mat(\mathcal{H}) - \mathcal{C}(\mathcal{M})$, functions in 3+1-d

$S_2[X+A] = Tr\left([X^a, A^b][X_a, A_b] + \ldots\right)$ $\quad S_{YM}$

$U^{-1}(X^a + A^a)U = X^a + \left(U^{-1}A^a U + \underbrace{U^{-1}[X^a, U]}_{\Theta^{ab}\partial_b U}\right)$

Perimeter-B


IKKT = IIB
susy
D = 9+1
X → U X U
gauge sy.
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X ~ |x⟩⟨y|
i{X^a, X^b}
symp. vol
frame
Weitzenböck connection,
∇E = 0
torsion: T_ab^μ = {{X_a, X_b}, y^μ}
Dilaton
eff. metric
2) choose non-triv. vacuum (SSB)
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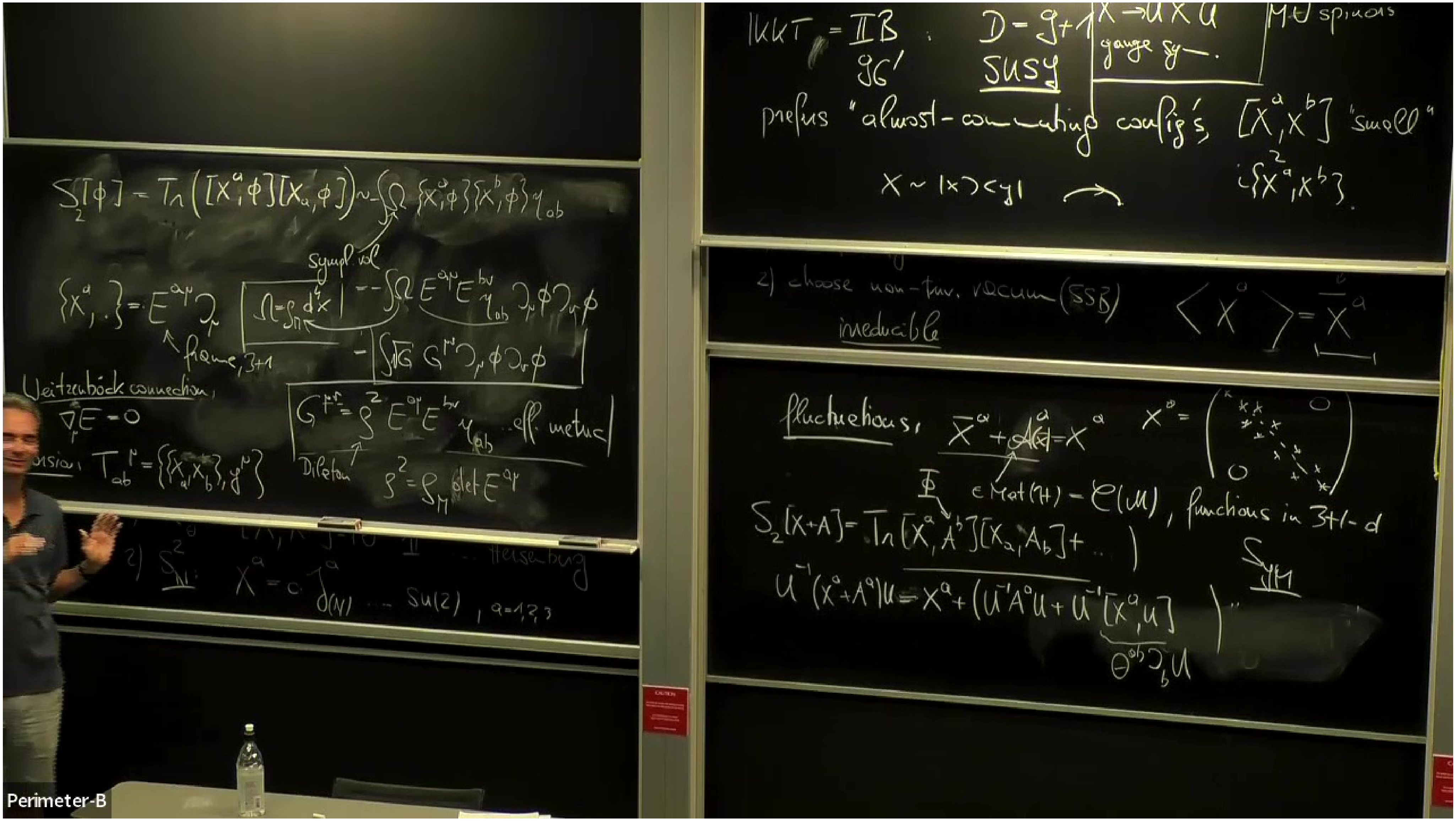
S_2[X+A] = Tr([X^a, A^b][X_a, A_b] + ...)
SYM
U^{-1}(X^a + A^a)U = X^a + (U^{-1}A^aU + U^{-1}[X^a, U])
θ^{ab}∂_b U
2) S_N^2: X^a = c J^a_(N) ... su(2), a=1,2,3
Perimeter-B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Sympl. vol
frame, 3+1
Weitzenböck connection,
eff. metric
Dilaton
Heisenberg
Perimeter-B

$\{X, .\} =: E^{\cdot\mu}\partial_\mu$ frame, 3+1

$S[\phi] = \int d^4x \ldots - \int \sqrt{G}\, G^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$

Weitzenböck connection, $\tilde\nabla_\mu E = 0$

$G^{\mu\nu} = \rho^2 E^{a\mu} E^{b\nu} \eta_{ab}$ eff. metric

torsion: $T_{ab}{}^{\mu} = \{\{X_a, X_b\}, y^\mu\}$

Dilaton $\rho^2 = \rho_M \det E^{a\mu}$

$IKKT = IIB$: $D = 9+1$ SUSY

GS'

$X \to U X U$ gauge sy.

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$X \sim |x\rangle\langle y| \rightarrow$ $i\{X^a, X^b\}$

"pre-gravity" from class. MM: - Ricci-flat at the level

Quantize model at 1-loop $\int DX^a D\psi\, e^{iS[X,\psi]}$

$\Gamma_{1\text{-}l} \supset S_{E\text{-}H} = \int \frac{1}{G_N} T^{\mu\nu\rho} T_{\mu\nu\rho} + \ldots \quad \ldots T_{eff}(x)$

$+ \int \Omega \rho^2 \Lambda \quad \sim \int R + \partial\tilde\rho\,\partial\tilde\rho$ gravit. axion

$\mathcal{M}^{3,1} \times K \subset \mathbb{R}^{9,1}$, Λ_{KK}

metric? $x^a : \mathcal{M} \subset (\mathbb{R}^D, \eta^{ab})$ $\mathbb{R}^D$ $\mathcal{M}$ x^a

$x^a \sim X^a = Q(x^a)$ D Matrices

matrix config. $\{X^a, \; a = 1, \ldots, D\}$

Ex. 1) $\mathbb{R}^{2n}$ ...

2) choose non-triv. vacuum (SSB) irreducible $\langle X^a \rangle = \bar{X}^a$

fluctuations, $\bar{X}^a + A^a(x) = X^a$ $\quad X^a = \begin{pmatrix} \times\times & & 0 \\ & \ddots & \\ 0 & & \times \end{pmatrix}$

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$S_2[X+A] = \mathrm{Tr}\,([X^a, A^b][X_a, A_b] + \ldots)$ S_{YM}

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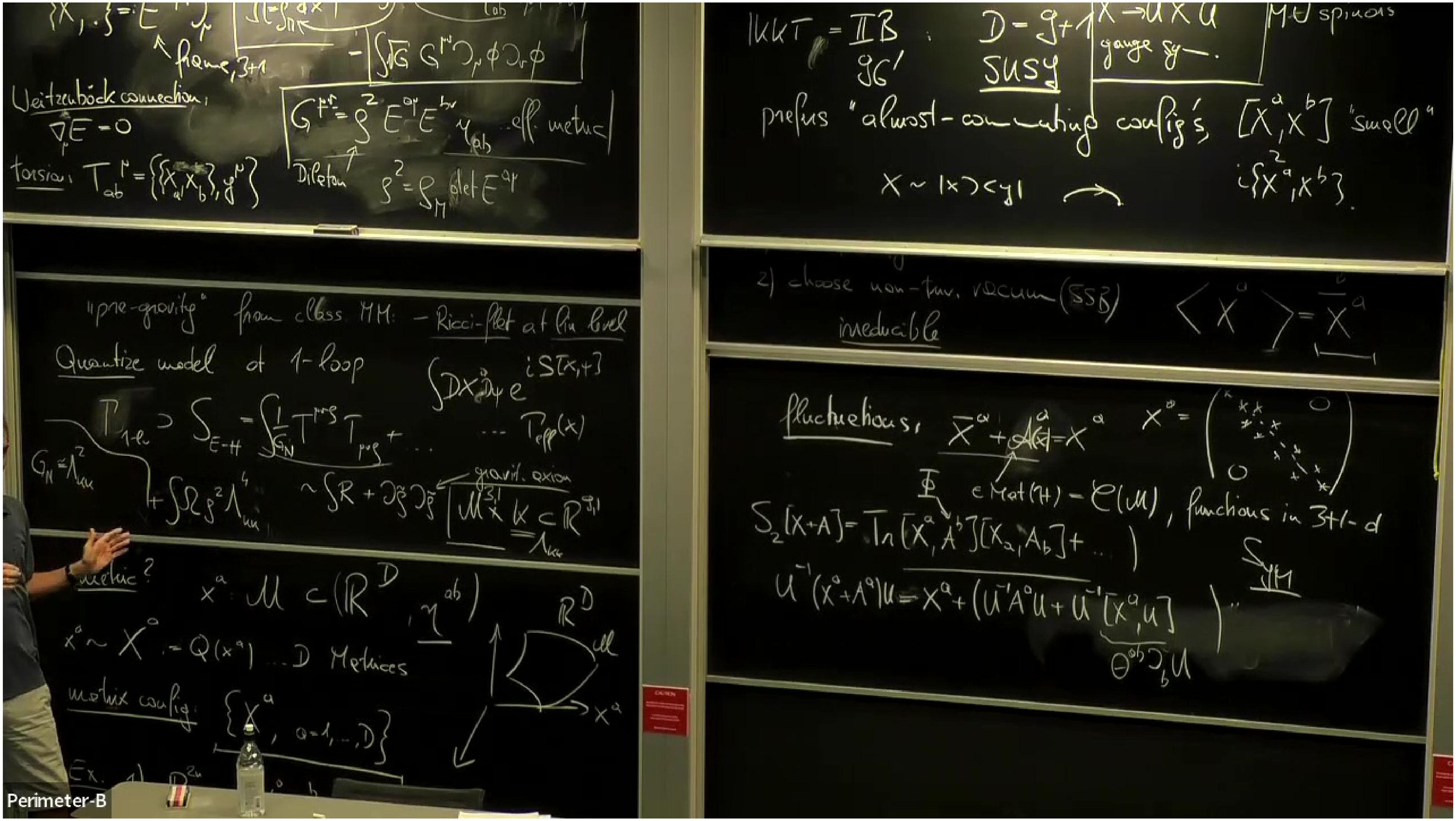
$\theta^{ab}\partial_b U$

Perimeter-B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frame, 3+1
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Quantize model at 1-loop
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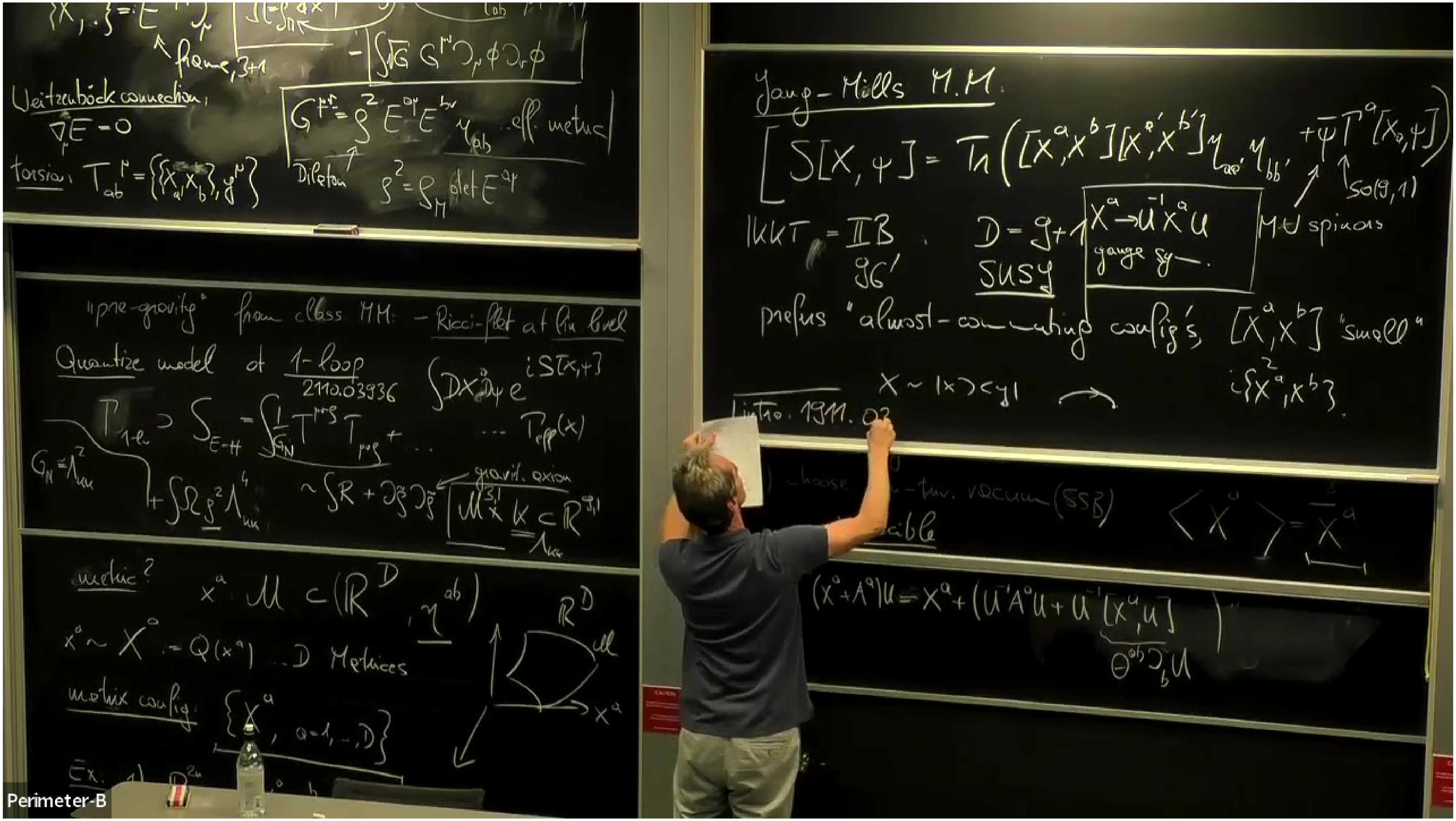
CAUTION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Ex.
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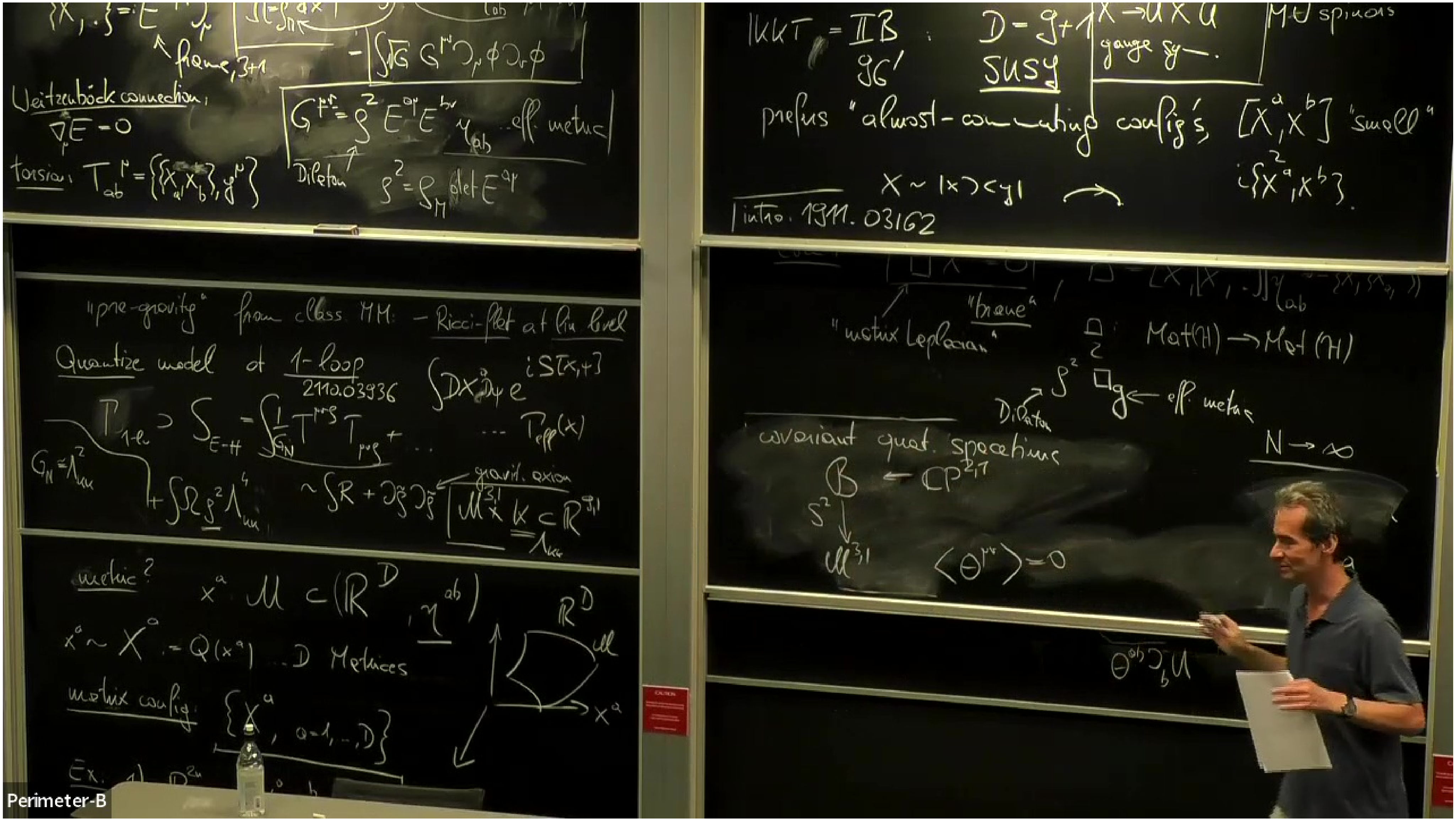
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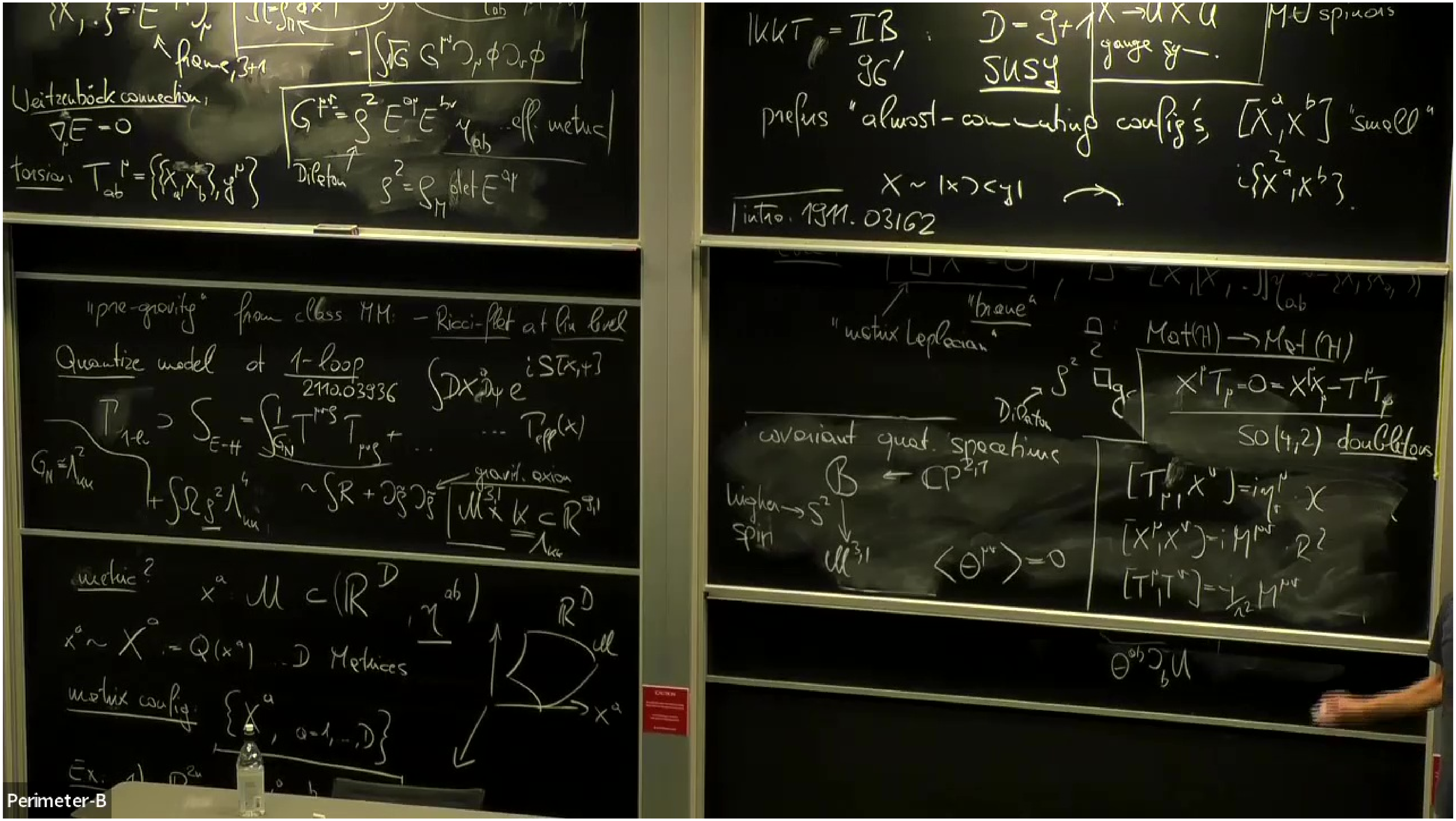
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higher spin
SO(4,2) doubletons
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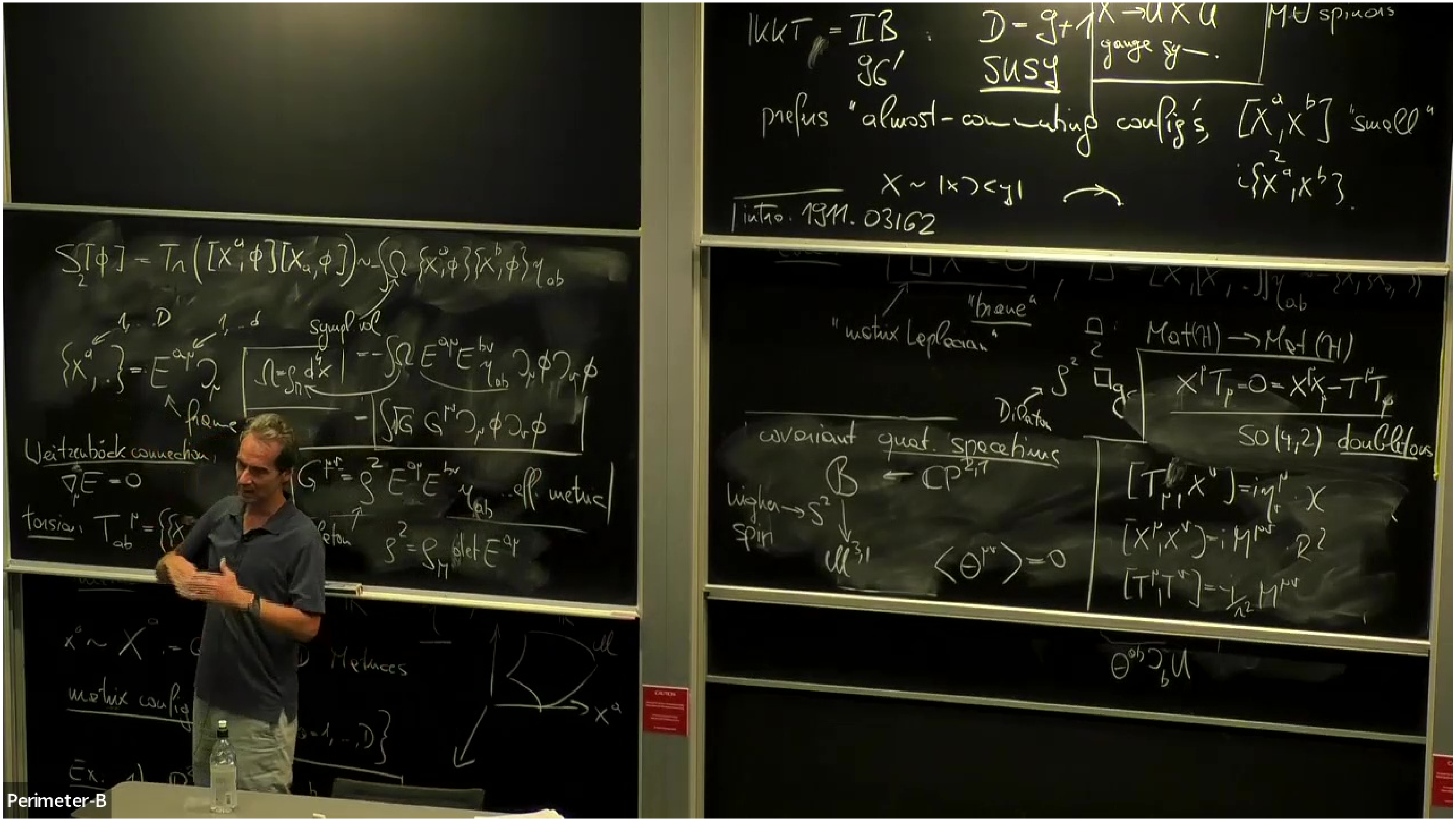
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