Title: Celestial amplitudes from flat space limits of AdS Witten diagrams

Speakers:

Series: Quantum Fields and Strings

Date: September 06, 2022 - 2:00 PM

URL: https://pirsa.org/22090075

Abstract: The search for pragmatic observables of quantum gravity remains at the forefront of fundamental physics research. A large set of ideas collectively known as the gauge-gravity duality have proven fruitful in tackling this problem. While such a duality is believed to universally govern gravitational theories, its nature in theories of gravity that describe our universe to a good degree of approximation is still little understood.

In this talk I will discuss efforts in formulating a holographic correspondence for gravity in four-dimensional asymptotically flat spacetimes. The proposed dual theory lives on a two-dimensional celestial sphere at infinity and is constrained by a wide range of symmetries. I present recent evidence for this proposal by showing that it arises naturally in a flat space limit of AdS/CFT. I will illustrate this construction with two related examples: the propagation of a particle in a shockwave background and the high-energy scattering of 2 particles.

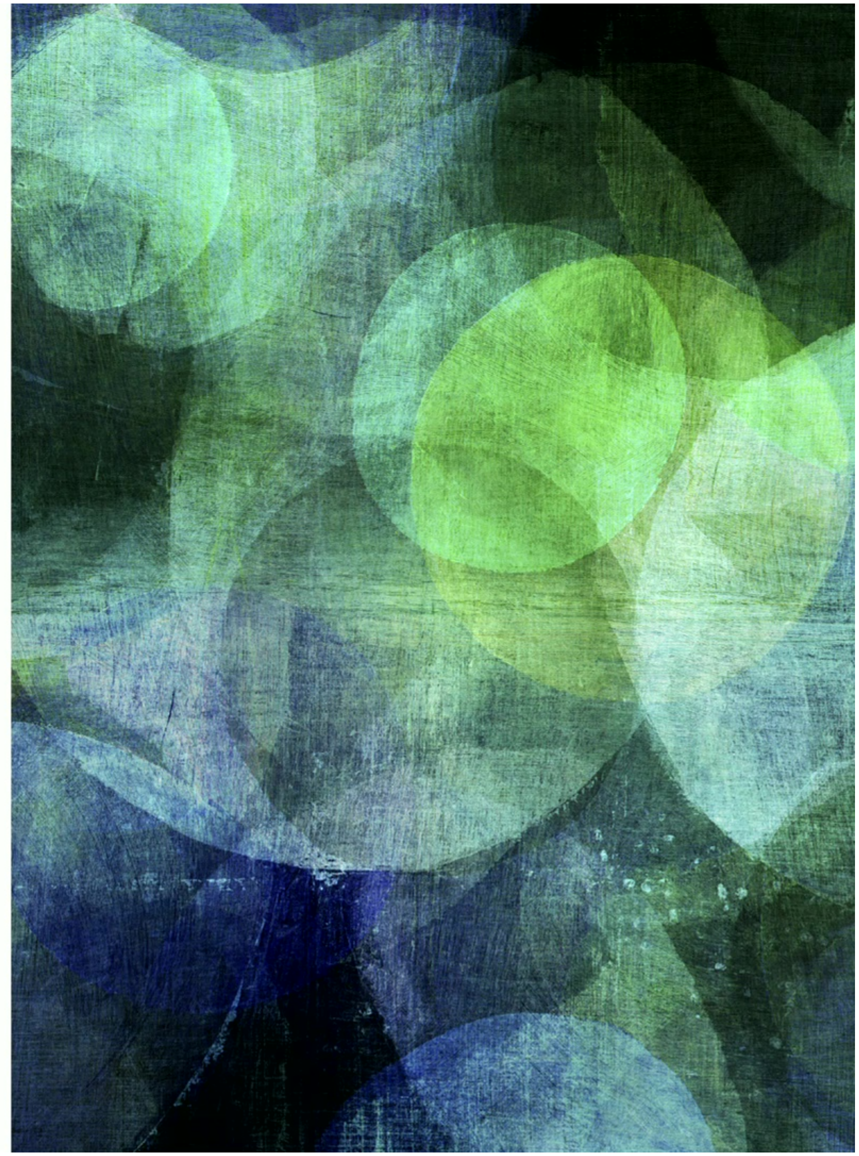
Zoom Link: https://pitp.zoom.us/j/97426266597?pwd=UmhncFR6NExFVzI2SlJwdE1LeUlIQT09

CELESTIAL AMPLITUDES FROM FLAT SPACE LIMITS OF ADS WITTEN DIAGRAMS

[based on 2206.10547 with L. de Gioia]

Ana Maria Raclariu
Perimeter Institute

September 6, 2022



THE PROBLEM

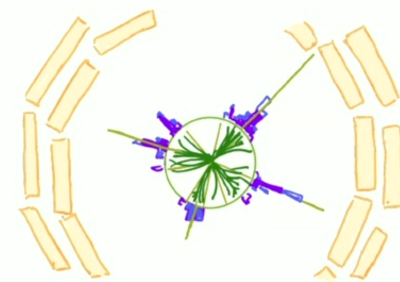
What is a complete set of observables in our universe?

- $\Lambda \ll 1 \implies$ accelerated expansion irrelevant on astrophysical timescales
- Gravitational radiation, strong gravity/black hole physics: $\Lambda = 0$

Candidate observables: scattering amplitudes

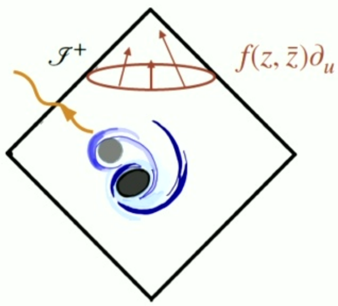
✓ low energy observables, eg. scattering angles, source properties from radiative spectra

? IR divergences $\implies$ classically ill-defined ?



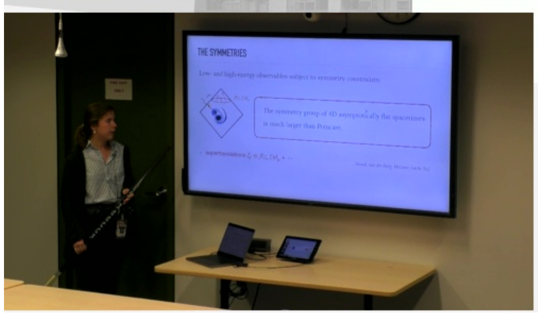
THE SYMMETRIES

Low- and high-energy observables subject to symmetry constraints:



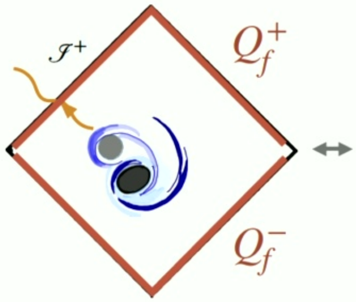
The symmetry group of 4D asymptotically flat spacetimes is much larger than Poincare.

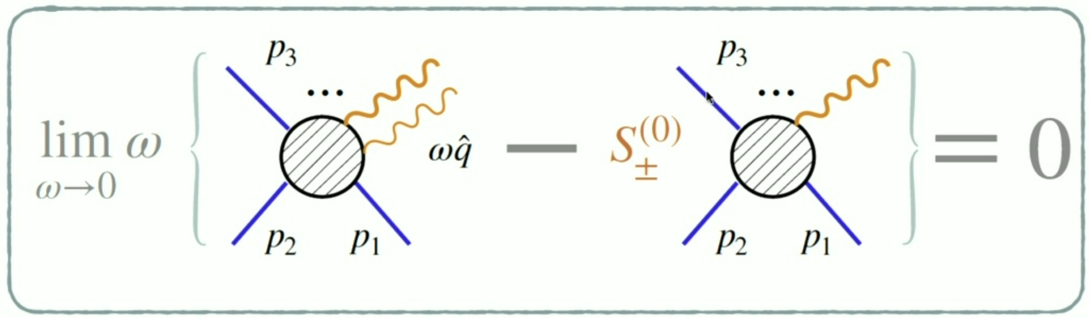
- supertranslations $\xi_f \equiv f(z, \bar{z})\partial_u + \cdots$ *[Bondi, van der Burg, Metzner, Sachs '62]*



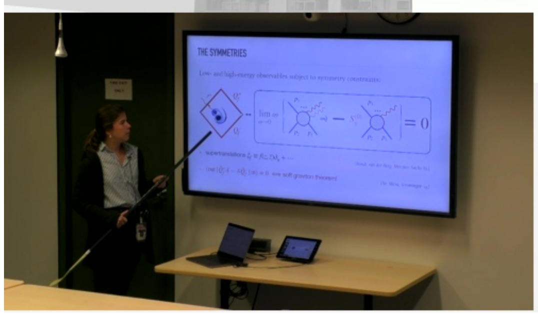
THE SYMMETRIES

Low- and high-energy observables subject to symmetry constraints:



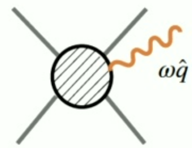
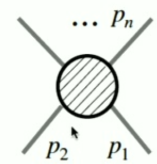


- supertranslations $\xi_f \equiv f(z,\bar{z})\partial_u + \cdots$ *[Bondi, van der Burg, Metzner, Sachs '62]*
- $\langle \mathrm{out}\,|\,\hat{Q}_f^+ \mathcal{S} - \mathcal{S}\hat{Q}_f^-\,|\,\mathrm{in}\rangle = 0 \iff$ soft graviton theorem! *[He, Mitra, Strominger '14]*



THE QUEST FOR A SOLUTION

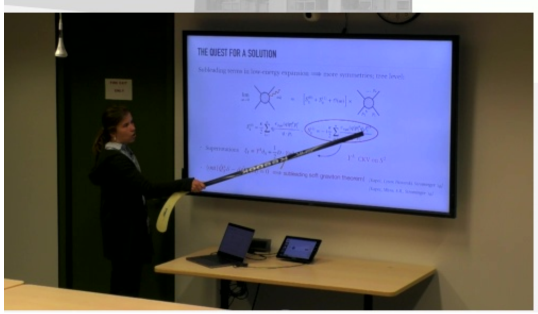
Subleading terms in low-energy expansion $\Longrightarrow$ more symmetries; tree level:

$\lim\limits_{\omega\to 0}$  $= \left[S_{\pm}^{(0)} + S_{\pm}^{(1)} + \mathcal{O}(\omega)\right] \times$ 

$$S_{\pm}^{(0)} = \frac{\kappa}{2}\sum_{i=1}^{n} \eta_i \frac{\epsilon_{\pm\mu\nu}(q) p_i^\mu p_i^\nu}{q\cdot p_i}\ , \quad S_{\pm}^{(1)} = -i\frac{\kappa}{2}\sum_{i=1}^{n} \frac{\epsilon_{\pm\mu\nu}(q) p_i^\mu q_\lambda J_i^{\nu\lambda}}{q\cdot p_i}$$

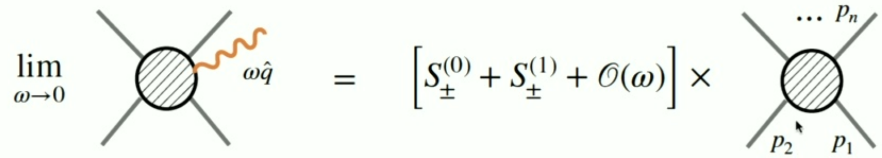
- Superrotations $\xi_Y = Y^A\partial_A + \frac{1}{2}D\cdot Y u \partial_u$ $\qquad$ Y^A CKV on S^2

- $\langle \text{out} \,|\, \hat{Q}_Y^+ \mathcal{S} - \mathcal{S}\hat{Q}_Y^- \,|\, \text{in}\rangle = 0 \iff$ subleading soft graviton theorem! *[Kapec, Lysov, Pasterski, Strominger '14]* *[Kapec, Mitra, A.R., Strominger '14]*



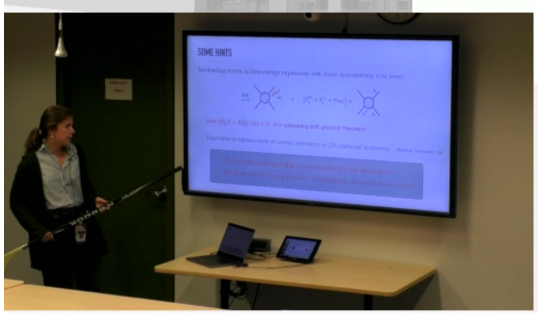
SOME HINTS

Subleading terms in low-energy expansion $\Longrightarrow$ more symmetries; tree level:

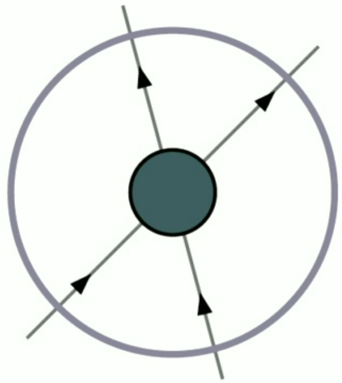


- $\langle \text{out} \,|\, \hat{Q}_Y^{+} \mathcal{S} - \mathcal{S} \hat{Q}_Y^{-} \,|\, \text{in} \rangle = 0 \iff$ subleading soft graviton theorem!
- Equivalent to enhancement of Lorentz symmetry to 2D conformal symmetry *[Barnich, Troessaert '09]*

> - Holographic correspondence in asymptotically flat spacetimes?
> - Implications for the organization of asymptotic data and observables?



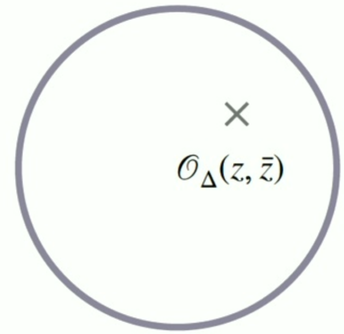
CELESTIAL/FLAT SPACE HOLOGRAPHY



$$p = \omega\,(1,\,\hat{x})$$

$$|p\rangle \rightarrow |\Delta, z, \bar{z}\rangle = \int_0^\infty d\omega\omega^{\Delta-1}\,|\omega, z, \bar{z}\rangle$$

$\mathcal{S}$-matrix in boost basis = Celestial amplitude

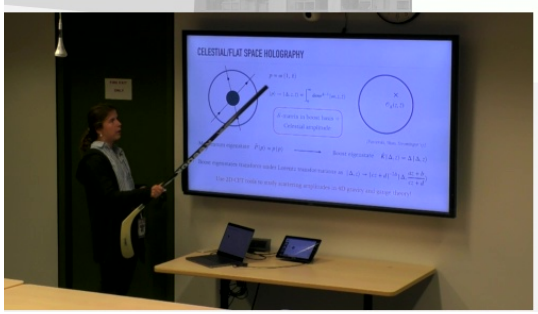


[Pasterski, Shao, Strominger '17]

Momentum eigenstate $\hat{P}\,|\,p\rangle = p\,|\,p\rangle$ ⟶ Boost eigenstate $\hat{K}\,|\,\Delta, z\rangle = \Delta\,|\,\Delta, z\rangle$

- Boost eigenstates transform under Lorentz transformations as $|\,\Delta, z\rangle \rightarrow |\,cz+d\,|^{-2\Delta}\,|\,\Delta, \dfrac{az+b}{cz+d}\rangle$

Use 2D CFT tools to study scattering amplitudes in 4D gravity and gauge theory!



CELESTIAL HOLOGRAPHY: SUCCESSES

2D CFT structure in 4D gravity

✓ Observables transform like primary correlators

✓ Stress tensor

✓ OPE

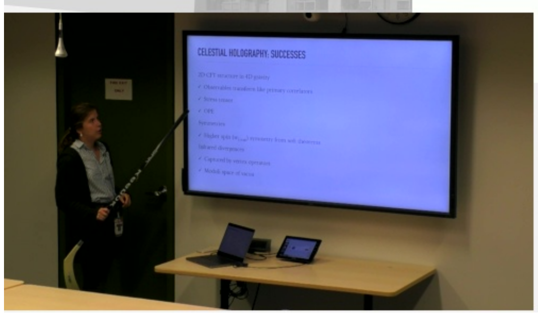
Symmetries

✓ Higher spin ($w_{1+\infty}$) symmetry from soft theorems

Infrared divergences

✓ Captured by vertex operators

✓ Moduli space of vacua



CELESTIAL HOLOGRAPHY: SUCCESSES AND MORE PROBLEMS

2D CFT structure in 4D gravity

- ✓ Observables transform like primary correlators — Unitarity obscured, bootstrap?
- ✓ Stress tensor — Central charge?
- ✓ OPE — More structure in scattering amplitudes?

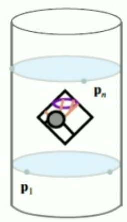
Symmetries

- ✓ Higher spin ($w_{1+\infty}$) symmetry from soft theorems — Beyond tree-level, mixed helicities…?

Infrared divergences

- ✓ Captured by vertex operators — Infrared finite observables?
- ✓ Moduli space of vacua

LESSONS FROM ADS

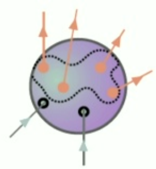


S-matrix from flat space limit of AdS/CFT observables:

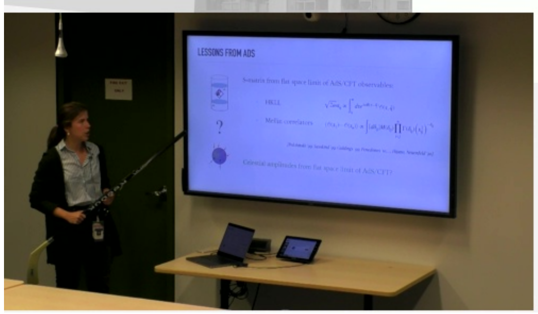
- HKLL $$\sqrt{2\omega}a_q \propto \int_0^{\pi} d\tau e^{i\omega R(\tau-\frac{\pi}{2})}\mathcal{O}(\tau,\hat{q})$$
- Mellin correlators $$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle \propto \int [d\delta_{ij}]M(\delta_{ij})\prod_{i<j}^{n}\Gamma(\delta_{ij})\left(x_{ij}^2\right)^{-\delta_{ij}}$$

[Polchinski '99; Susskind '99; Giddings '99; Penedones '10;...; Hijano, Neuenfeld '20]

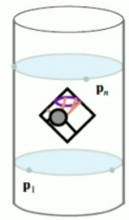
?



Celestial amplitudes from flat space limit of AdS/CFT?



LESSONS FROM ADS



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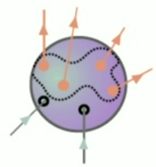
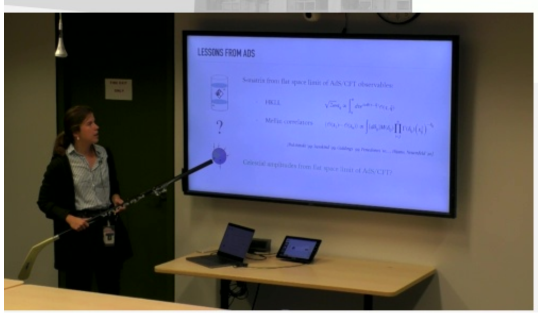
?

[Polchinski '99; Susskind '99; Giddings '99; Penedones '10;...; Hijano, Neuenfeld '20]

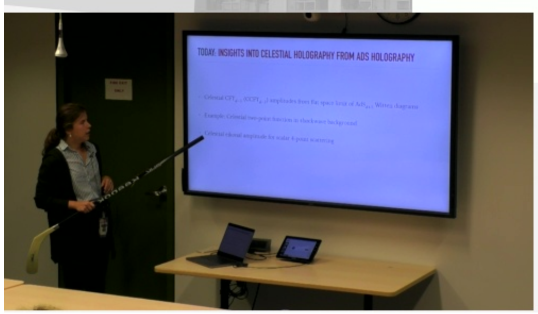


Celestial amplitudes from flat space limit of AdS/CFT?



TODAY: INSIGHTS INTO CELESTIAL HOLOGRAPHY FROM ADS HOLOGRAPHY

- Celestial CFT$_{d-1}$ (CCFT$_{d-1}$) amplitudes from flat space limit of AdS$_{d+1}$ Witten diagrams
- Example: Celestial two-point function in shockwave background
- Celestial eikonal amplitude for scalar 4-point scattering



PART I: FROM ADS TO CELESTIAL

CELESTIAL AMPLITUDES

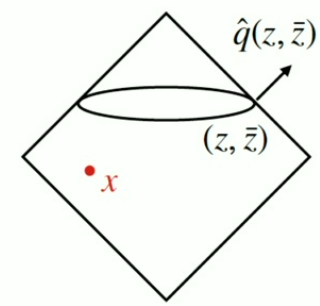
In 4D, scalar wave equation $\nabla^2\Psi = 0$ admits conformal primary solutions

with respect to $SO(1,3) \simeq SL(2,\mathbb{C})$: *[Pasterski, Shao, Strominger '17]*

$$(L_0 + \bar{L}_0)\varphi_\Delta = \Delta\varphi_\Delta, \quad (L_0 - \bar{L}_0)\varphi_\Delta = 0$$

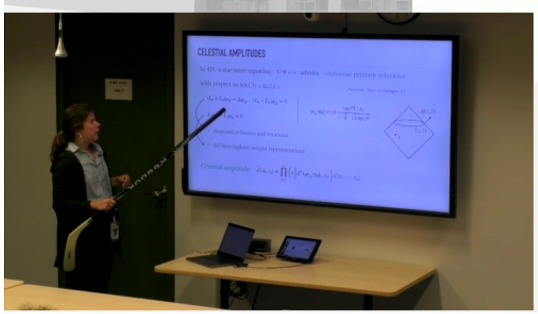
$$L_1\varphi_\Delta = \bar{L}_1\varphi_\Delta = 0$$

$$\varphi_\Delta(\eta\hat{q}; x) \equiv \frac{(i\eta)^\Delta\Gamma(\Delta)}{(-\hat{q}\cdot x \pm i\eta\epsilon)^\Delta}$$

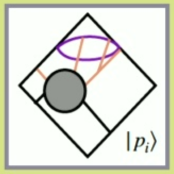


- diagonalize boosts and rotations
- fall into highest-weight representations

Celestial amplitude: $\widetilde{\mathscr{A}}(\Delta_j, z_j) \equiv \prod_{i=1}^{n}\left(i\int d^4x_i\varphi_{\Delta_i}(\eta_i\hat{q}_i; x_i)\right)C(x_1,\cdots,x_n)$

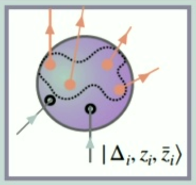


CELESTIAL AMPLITUDES



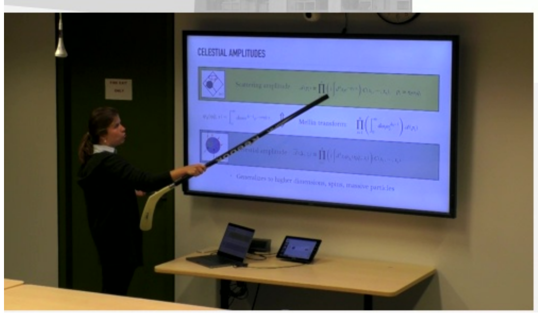
Scattering amplitude $\mathscr{A}(p_i) \equiv \prod_{i=1}^{n} \left(i \int d^4 x_i e^{-ip_i \cdot x_i} \right) C(x_1, \cdots, x_n), \quad p_i = \eta_i \omega_i \hat{q}_i$

$\varphi_\Delta(\eta \hat{q}; x) = \int_0^\infty d\omega \omega^{\Delta - 1} e^{-i\omega\eta\hat{q}\cdot x}$ $\Updownarrow$ Mellin transform: $\prod_{i=1}^{n} \left(\int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} \right) \mathscr{A}(p_i)$

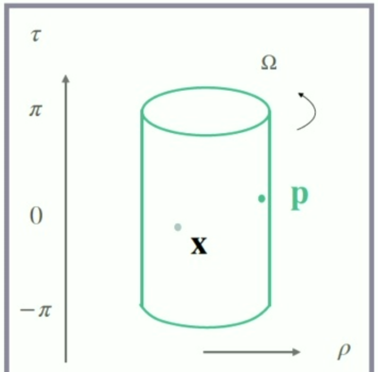


Celestial amplitude $\widetilde{\mathscr{A}}(\Delta_j, z_j) \equiv \prod_{i=1}^{n} \left(i \int d^4 x_i \varphi_{\Delta_i}(\eta_i \hat{q}_i; x_i) \right) C(x_1, \cdots, x_n)$

- Generalizes to higher dimensions, spins, massive particles



WITTEN DIAGRAMS



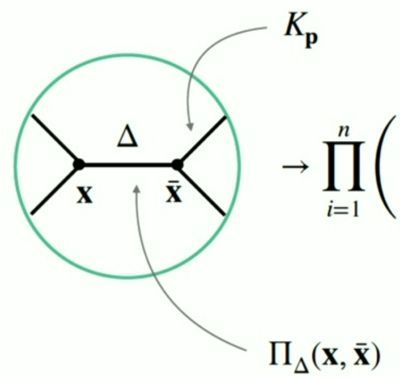
$$X^+ = -R\frac{\cos\tau - \sin\rho\,\Omega_{d+1}}{\cos\rho},$$
$$X^- = -R\frac{\cos\tau + \sin\rho\,\Omega_{d+1}}{\cos\rho},$$
$$X^1 = -R\frac{\sin\tau}{\cos\rho},$$
$$X^i = R\tan\rho\,\Omega_i, \quad \Omega_i \in S^{d-1}$$

Embedding space

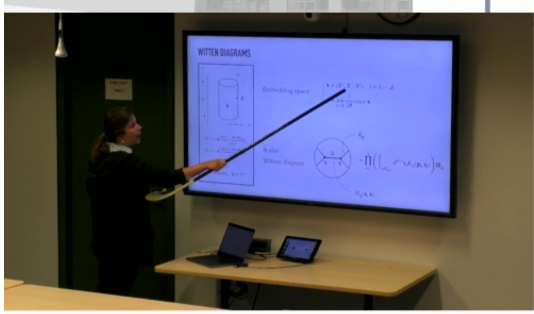
$$\begin{cases} \mathbf{x} = (X^+, X^-, X^i), \quad i = 1, \cdots, d \\ \mathbf{p} = \lim_{\rho\to\frac{\pi}{2}} \frac{1}{2R}\cos\rho\ \mathbf{x} \end{cases}$$

Scalar

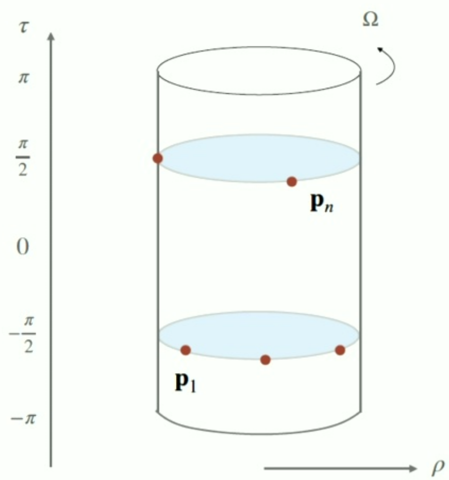
Witten diagram



$$\to \prod_{i=1}^{n}\left(\int_{AdS_{d+1}} d^{d+1}\mathbf{x}_i K_{\Delta_i}(\mathbf{p}_i, \mathbf{x}_i)\right)\mathbf{\Pi}_\Delta$$



FLAT SPACE FROM ADS OBSERVABLES

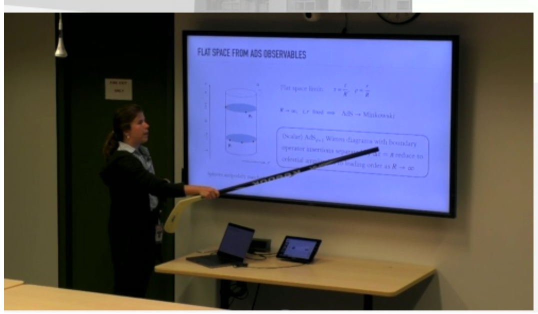


Spheres antipodally matched

Flat space limit: $\tau = \frac{t}{R}, \quad \rho = \frac{r}{R}$

$R \to \infty, \quad t, r \text{ fixed} \implies \text{AdS} \to \text{Minkowski}$

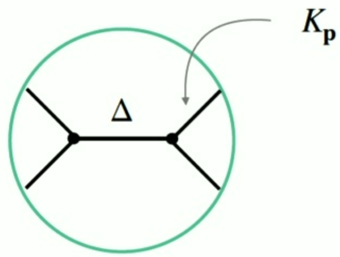
(Scalar) AdS$_{d+1}$ Witten diagrams with boundary operator insertions separated by $\Delta\tau = \pi$ reduce to celestial amplitudes to leading order as $R \to \infty$



FLAT SPACE FROM ADS OBSERVABLES

- Bulk-to-boundary propagators: $$K_\Delta(\mathbf{p};\mathbf{x}) = \frac{C^d_\Delta}{(-2\mathbf{x}\cdot\mathbf{p} + i\epsilon)^\Delta}$$

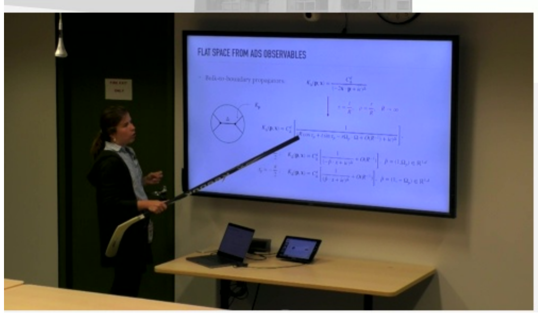
$$\Big\downarrow \quad \tau = \frac{t}{R}, \quad \rho = \frac{r}{R}, \quad R \to \infty$$



$$K_\Delta(\mathbf{p},\mathbf{x}) = C^d_\Delta \left[\frac{1}{(R\cos\tau_p + t\sin\tau_p - r\Omega_p\cdot\Omega + O(R^{-1}) + i\epsilon)^\Delta}\right],$$

$$\tau_p = \frac{\pi}{2}: \quad K_\Delta(\mathbf{p},\mathbf{x}) = C^d_\Delta\left[\frac{1}{(-\hat{p}\cdot x + i\epsilon)^\Delta} + O(R^{-1})\right], \ \hat{p} = (1,\Omega_p) \in \mathbb{R}^{1,d}$$

$$\tau_p = -\frac{\pi}{2}: \quad K_\Delta(\mathbf{p},\mathbf{x}) = C^d_\Delta\left[\frac{1}{(\hat{p}\cdot x + i\epsilon)^\Delta} + O(R^{-1})\right], \ \hat{p} = (1,-\Omega_p) \in \mathbb{R}^{1,d}$$



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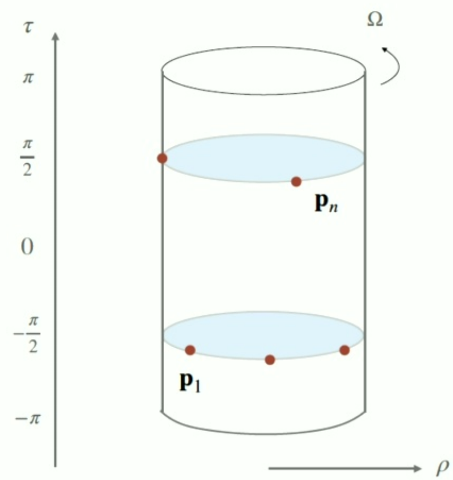
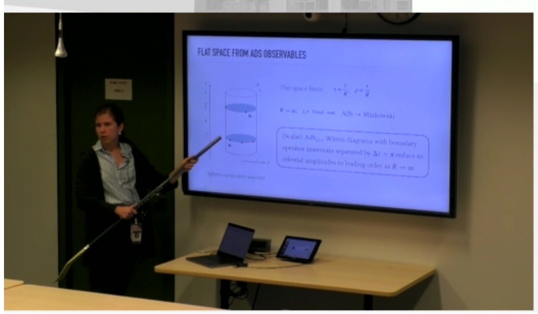


Spheres antipodally matched

Flat space limit: $\tau = \frac{t}{R}, \quad \rho = \frac{r}{R}$

$R \to \infty, \quad t, r \text{ fixed} \implies$ AdS $\to$ Minkowski

(Scalar) AdS$_{d+1}$ Witten diagrams with boundary operator insertions separated by $\Delta\tau = \pi$ reduce to celestial amplitudes to leading order as $R \to \infty$



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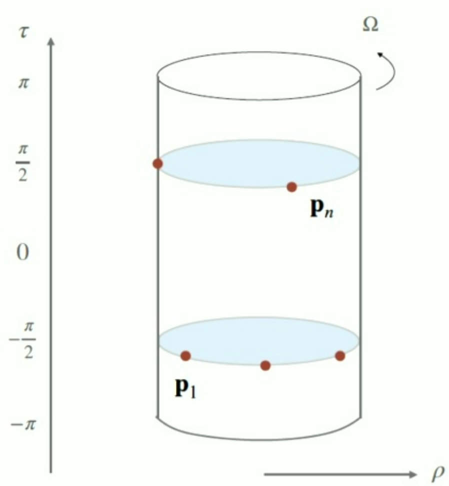

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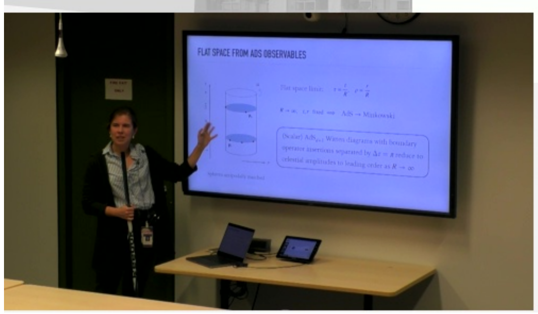


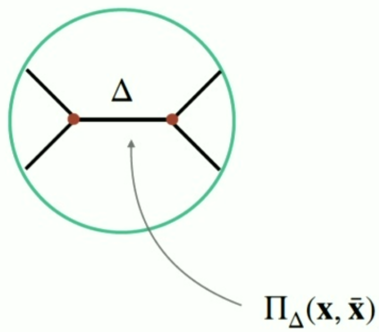
FLAT SPACE FROM ADS OBSERVABLES

- Bulk-to-bulk propagators:

$$\left(\Box_{AdS_{d+1}} - \frac{\Delta(\Delta-d)}{R^2}\right)\mathbf{\Pi}_\Delta(\mathbf{x},\bar{\mathbf{x}}) = i\delta_{AdS_{d+1}}(\mathbf{x},\bar{\mathbf{x}})$$

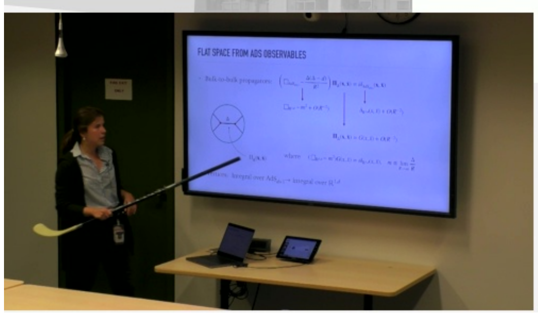
$$\Box_{\mathbb{R}^{1,d}} - m^2 + O(R^{-2}) \qquad \delta_{\mathbb{R}^{1,d}}(x,\bar{x}) + O(R^{-2})$$

$$\mathbf{\Pi}_\Delta(\mathbf{x},\bar{\mathbf{x}}) = G(x,\bar{x}) + O(R^{-2})$$

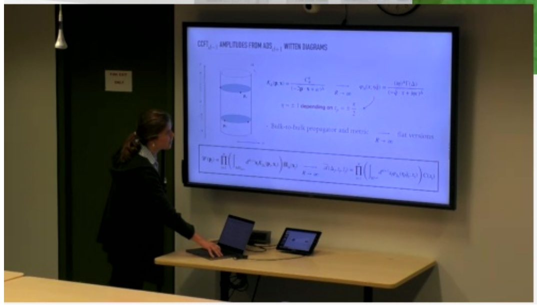


where $\quad (\Box_{\mathbb{R}^{1,d}} - m^2)G(x,\bar{x}) = i\delta_{\mathbb{R}^{1,d}}(x,\bar{x}), \quad m \equiv \lim_{R\to\infty}\frac{\Delta}{R}$

- Vertices: Integral over $\mathrm{AdS}_{d+1} \rightarrow$ integral over $\mathbb{R}^{1,d}$



PART II: APPLICATIONS



OUTLOOK

- CCFT_{d-1} from CFT_d? Top down CCFT constructions?
- BMS symmetries & more from AdS_4 flat space limit; matching condition?
- Scattering in other backgrounds, chaos?
- Causality signatures in CCFT, relation to memory effects?
- Leading eikonal amplitude vs. Weinberg IR-divergent phase?

Thank you!