Title: Effective Field Theory of the waterfall phase in hybrid inflation

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Abstract: We examine the validity of the classical approximation of the waterfall phase transition in hybrid inflation from an effective field theory (EFT) point of view. The EFT is constructed by integrating out the waterfall field fluctuations, up to one-loop order in the perturbative expansion. Assuming slow-roll conditions are obeyed, right after the onset of the waterfall phase, we find the backreaction of the waterfall field fluctuations to the evolution of the system can be dominant. In this case the classical approximation is completely spoiled. We derive the necessary constraint that ensures the validity of the EFT.

Zoom Link: https://pitp.zoom.us/j/96209675696?pwd=ZkZ2NGpRVW9vVkdaY1QrWG5GRFltdz09

The EFT of Waterfall in Hybrid Inflation

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Motivation

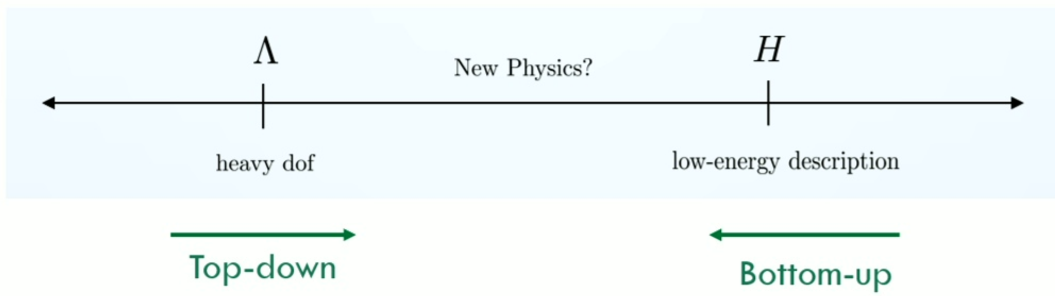
Several works exploit tachyonic instability

- Put the "classical approximation" to the test:
- Assumes negligible backreaction of tachyonic modes during the first stages of instability.
- We employ standard EFT methods to study the classical approx.

- Long-wave modes grow exponentially and become classical (Bellido et al, 2003)

- This justifies the use of real-time lattice simulations (Felder et al, 2001)
- As well older works on tachyonic preheating, use initial conditions borrowed from the classical approximation.

Why EFT?



EFT for Inflation is a powerful tool to study consistency of the classical approximation and look for new physics!

- Weinberg 2008
- Bottom-up: t-trans broken, spatial diffeo preserved (Cheung et al. 2008)
- Top-down: Heavy dof integrated out (Achucarro et al. 2011, 2012)

Hybrid Inflation

Lagrangian for Hybrid Inflation

$$S = \int \mathrm{d}^4 x a^3 \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\phi, \chi) \right],$$

$$V(\phi, \chi) = V_{inf}(\phi) + \frac{\lambda}{4} \left(\frac{M^2}{\lambda} - \chi^2 \right)^2 + \frac{1}{2} g^2 \phi^2 \chi^2,$$

$$V_{inf} = \frac{1}{2} m^2 \phi^2,$$

where $\phi(t, \vec{x})$ is the inflaton, V_{inf} is the inflaton potential, $\chi(t, \vec{x})$ is the waterfall field, M is the waterfall field mass, g and λ are coupling constants.

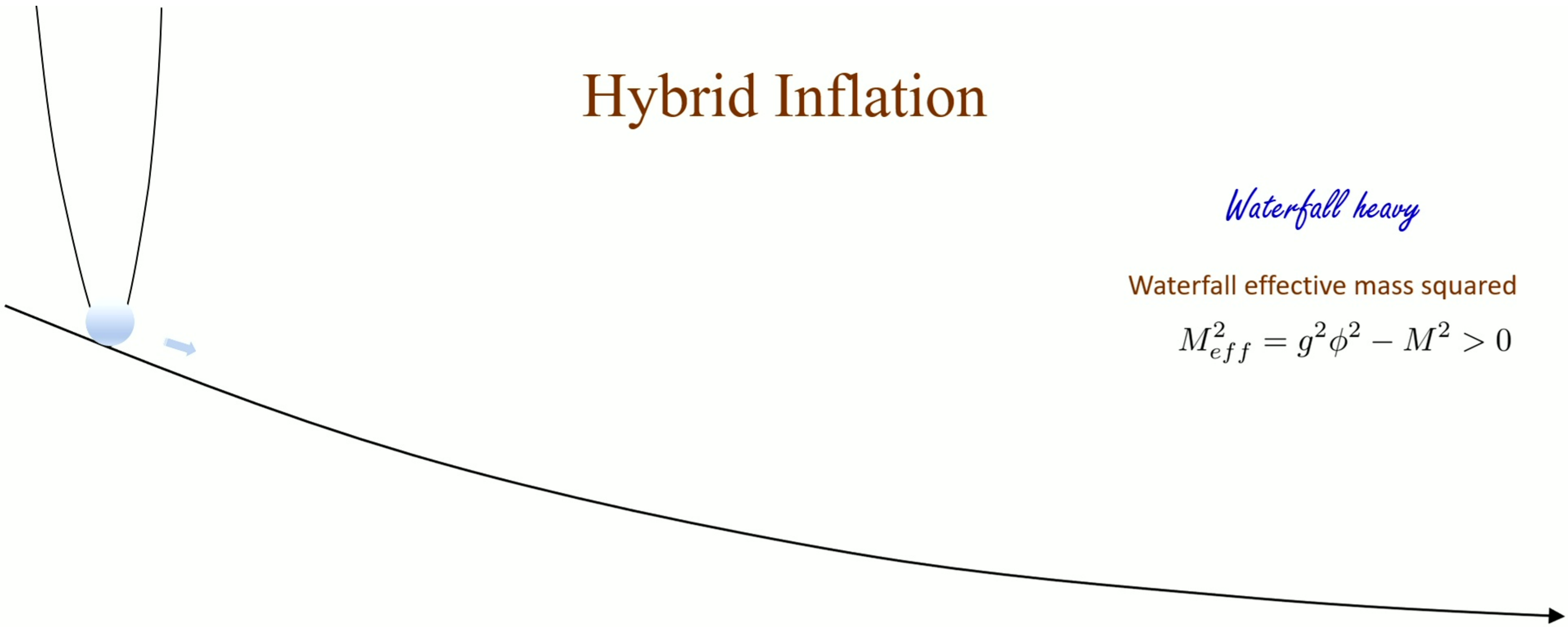
Linde (1994)

Hybrid Inflation

Waterfall heavy

Waterfall effective mass squared

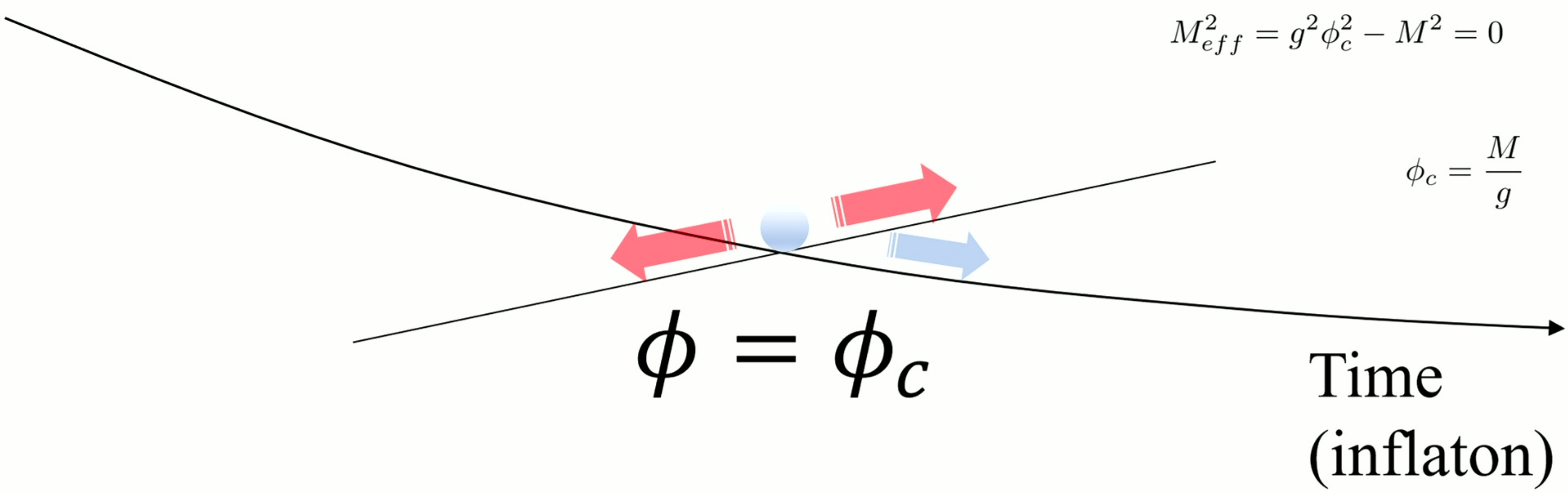
$$M_{eff}^2 = g^2\phi^2 - M^2 > 0$$



$$V_\phi = \frac{M^2}{4\lambda} + \frac{1}{2}m^2\phi^2, \quad V_\chi = \frac{1}{2}M_{eff}^2\chi^2 + \frac{\lambda}{4}\chi^4$$

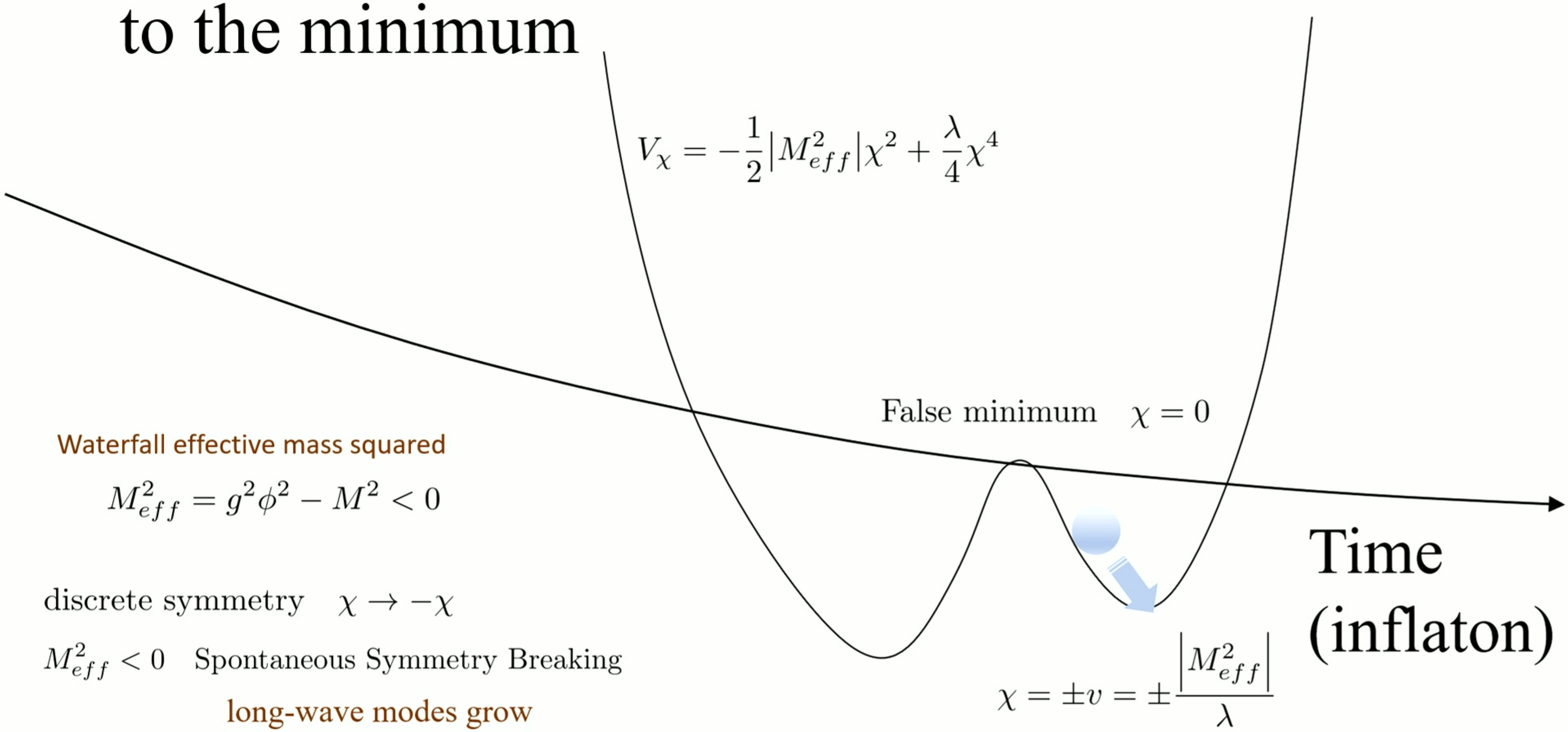
Inflation is occurring…

The direction orthogonal to the inflaton becomes massless



“Waterfall” phase transition

Very soon the waterfall field rolls to the minimum



Waterfall effective mass squared

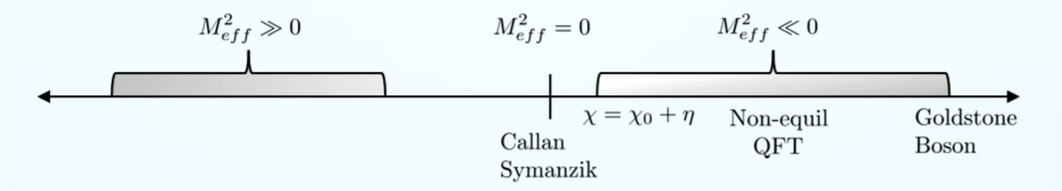
$$M_{eff}^2 = g^2\phi^2 - M^2 < 0$$

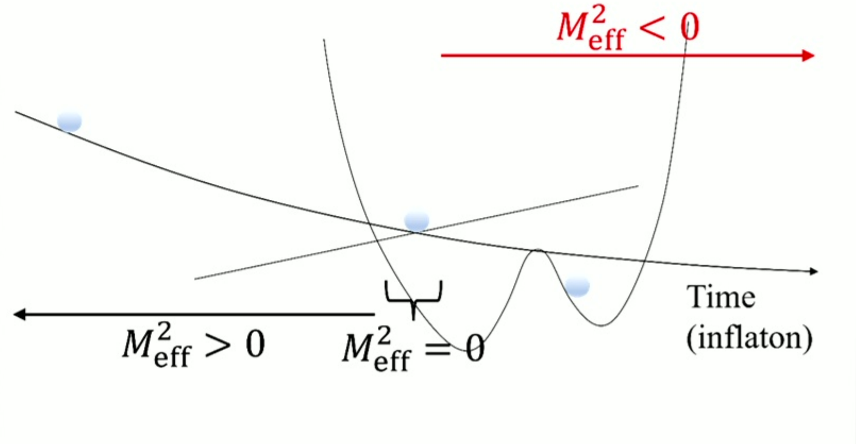
discrete symmetry $\chi \to -\chi$

$M_{eff}^2 < 0$ Spontaneous Symmetry Breaking

long-wave modes grow

EFT regimes of Hybrid Inflation





- $M^2_{eff} \gg 0$:

(Burgess, Cline, Holman - 2003)

- $M^2_{eff} = 0$:

Callan-Symanzik re-summation of large logs

- $M^2_{eff} < 0$:

- Perturb around the top of the double-well potential $\chi = \chi_0 + \eta$
- Goldstone description at the bottom of the double-well potential
- NE-QFT: Non-perturbative regime (Felder et al, 2000-2001)

Integrate out waterfall fluctuations

Perturb around the top of the double-well potential

- Expand around the classical value $\chi = \chi_0 + \eta$
- The generating functional for the waterfall field:

$$Z[J] = \exp\left(iW[J]\right) = \int \mathcal{D}\chi \exp\left\{ iS[\chi_0] + \frac{i}{2}\int \mathrm{d}^4 x \eta D \eta \right\}$$

- Klein Gordon operator (in the mostly plus metric convention)

$$D = \partial_\mu \partial^\mu - M_{eff}^2(\phi) - 3\lambda\chi_0^2 = \partial^2 - m^2, \quad m^2 = M_{eff}^2(\phi) + 3\lambda\chi_0^2$$

- Go to Euclidean space, $t = -i\tau$
- Integrate out with respect to η

The Effective Potential

Effective potential for the waterfall field

$$V_{eff}(\chi_0) = -\frac{1}{2}\left|M^2_{eff}(\phi)\right|\chi_0^2 + \frac{\lambda}{4}\chi_0^4 + \frac{1}{4}\frac{\left(-\left|M^2_{eff}(\phi)\right| + 3\lambda\chi_0^2\right)^2}{(4\pi)^2}\left[\ln\left(\frac{-\left|M^2_{eff}(\phi)\right| + 3\lambda\chi_0^2}{\Lambda^2}\right) - \frac{3}{2}\right]$$

Not valid for: $\chi_0 \to 0$ in the massless limit

Massless limit - Callan-Symanzik re-summation

$$V_{eff}(\chi) = \frac{1}{4}\chi_0^4\left[\bar{\lambda} + \frac{9\bar{\lambda}^2}{(4\pi)^2}\left(\ln\left(3\bar{\lambda}\right) - \frac{3}{2}\right)\right], \quad \bar{\lambda} = \frac{\lambda}{1 - \frac{\lambda}{8\pi^2}\ln\left(\frac{\chi_0}{\Lambda}\right)}$$

- Accurate for $\chi_0 \to 0$, as $\bar{\lambda} \to 0$
- Ensures that the V_{eff} has its minimum at $\chi_0 = 0$.

Ensure small logs

Quantum corrected Lagrangian

$$\mathcal{L}_{eff}(\chi_0) = \mathcal{L}_0(\chi_0) + \mathcal{L}_{qc}(\chi_0),$$

Need to assume small logs as part of our initial conditions ⇒ Set log initial argument to unity

No loss of generality as the CS expression ensures small Logs.

Demand that:

$$\ln\left(\frac{-\left|M^2_{eff}(\phi)\right| + 3\lambda\chi_0^2}{\Lambda^2}\right) \Rightarrow -\left|M^2_{eff}(\phi(t=0))\right| + 3\lambda\chi_0^2(t=0) = \Lambda^2$$

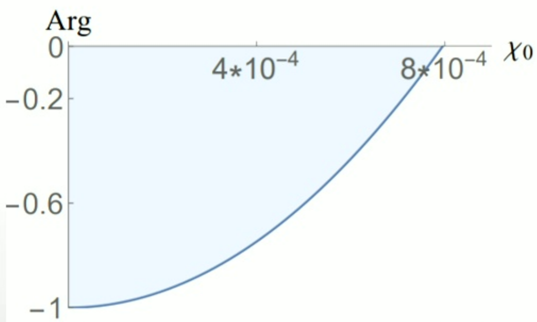
Validity of perturbative expansion

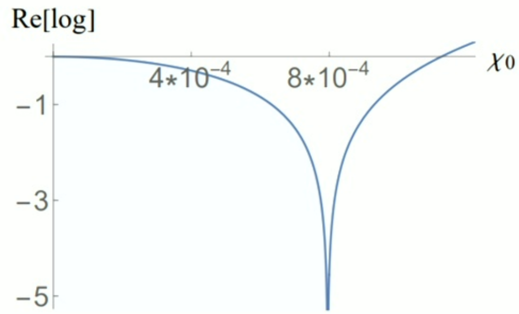
Logarithmic corrections

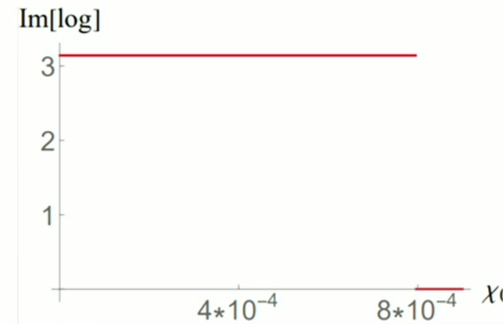
$$\ln\left(\frac{-\left|M_{eff}^2(\phi)\right| + 3\lambda\chi_0^2}{\Lambda^2}\right)$$

Constraint

$$|\chi_0| \leq \left|\frac{M_{eff}}{\sqrt{3\lambda}}\right|$$







Quantum Instability

$$V_{eff}(\chi_0) = -\frac{1}{2}|M_{eff}|^2\chi_0^2 + \frac{\lambda}{4}\chi_0^4 + \frac{1}{4}\frac{\left(-|M_{eff}|^2 + 3\lambda\chi_0^2\right)^2}{(4\pi)^2}\left[i\pi + \ln\left(\frac{|M_{eff}|^2 - 3\lambda\chi_0^2}{\Lambda^2}\right) - \frac{3}{2}\right]$$

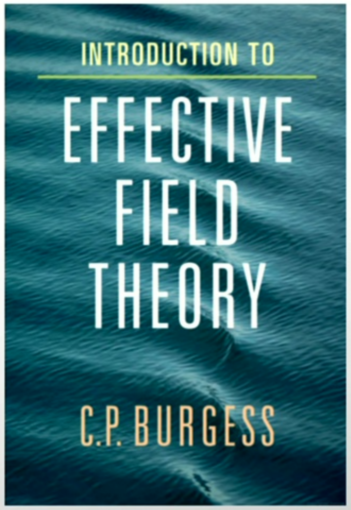
Imaginary potentials

Quantum Instability

$$V_{eff}(\chi_0) = -\frac{1}{2}|M_{eff}|^2\chi_0^2 + \frac{\lambda}{4}\chi_0^4 + \frac{1}{4}\frac{\left(-|M_{eff}|^2 + 3\lambda\chi_0^2\right)^2}{(4\pi)^2}\left[i\pi + \ln\left(\frac{|M_{eff}|^2 - 3\lambda\chi_0^2}{\Lambda^2}\right) - \frac{3}{2}\right]$$

Physical significance

Imaginary part is related to vacuum decay rate
(Weinberg, Wu, 1987)



Loss of unitarity ⟶ Loss of probability

Can trust EFT only if imaginary part is much small

We can use the classical approximation if the backreaction is small

System to solve – classical equations

Standard Hybrid Inflation

$$\ddot{\phi} + 3H\dot{\phi} + (m^2 + g^2\chi^2)\phi = 0,$$

$$\ddot{\chi} + 3H\dot{\chi} + M_{eff}^2(\phi)\chi + \lambda\chi^3 = 0,$$

$$H^2 = \frac{1}{3M_P^2}\left(\frac{(\dot{\phi})^2}{2} + \frac{(\dot{\chi})^2}{2} + V(\phi,\chi)\right),$$

$$M_{eff}^2 = g^2\phi^2 - M^2 < 0$$

System to solve – Waterfall regime

$$\ddot{\phi} + 3H\dot{\phi} + (m^2 + g^2\chi_0^2)\phi + \frac{g^2\phi}{32\pi^2}(-|M_{eff}^2| + 3\lambda\chi_0^2)$$
$$+ \frac{g^2\phi}{16\pi^2}(-|M_{eff}^2| + 3\lambda\chi_0^2)\left[\ln\left(\frac{\left|M_{eff}^2\right| - 3\lambda\chi_0^2}{\Lambda^2}\right) + i\pi - \frac{3}{2}\right]$$

$$\ddot{\chi}_0 + 3H\dot{\chi}_0 - |M_{eff}^2|\chi_0 + \lambda\chi_0^3 + \frac{3\lambda\chi_0}{32\pi^2}(-|M_{eff}^2| + 3\lambda\chi_0^2)$$
$$+ \frac{3\lambda\chi_0}{16\pi^2}(-|M_{eff}^2| + 3\lambda\chi_0^2)\left[\ln\left(\frac{\left|M_{eff}^2\right| - 3\lambda\chi_0^2}{\Lambda^2}\right) + i\pi - \frac{3}{2}\right]$$

$$H^2 = \frac{1}{3M_p^2}\left\{\frac{(\dot{\phi})^2}{2} + \frac{(\dot{\chi}_0)^2}{2} + \frac{M^4}{4\lambda} + \frac{m^2\phi^2}{2} - \frac{1}{2}|M_{eff}|^2\chi_0^2 + \frac{\lambda}{4}\chi_0^4\right.$$
$$\left. + \frac{1}{4}\frac{\left(-|M_{eff}|^2 + 3\lambda\chi_0^2\right)^2}{(4\pi)^2}\left[i\pi + \ln\left(\frac{|M_{eff}|^2 - 3\lambda\chi_0^2}{\Lambda^2}\right) - \frac{3}{2}\right]\right\}$$

Slow-roll initial conditions

Slow-roll parameter

$$\epsilon = \frac{(\dot{\phi})^2 + (\dot{\chi}_0)^2}{2M_P^2 H^2} = \frac{V_{,\phi}^2 + V_{,\chi_0}^2}{18 M_p^2 H^4}$$

⇒

Demand that:

$$\mathrm{Re}\,\epsilon \ll 1, \quad \mathrm{Im}\,\epsilon \ll \mathrm{Re}\,\epsilon$$

Constraint inflaton velocity (initial conditions)

$$\dot{\phi} = \frac{1}{3H}\left[m^2\phi + \frac{g^2 M^2 \phi}{16\pi^2} - \frac{g^4\phi^3}{16\pi^2} + i\left(-\frac{g^2 M^2\phi}{16\pi} + \frac{g^4\phi^3}{16\pi}\right)\right]$$

$$\chi_0 \to 0, \quad \ln(\chi_0, \phi)|_{t=0} \to 0$$

Constraint

$$\frac{m}{g} > \frac{\sqrt{M^2 - g^2\phi^2}}{4\sqrt{\pi}} = \frac{M_{eff}}{4\sqrt{\pi}} = \frac{M}{4\sqrt{\pi}}, \quad \phi < M/g$$

Small inflaton regime

$m < g$

- *m sufficiently large*
- *g sufficiently small*

Taylor expand Log

Impose small logs – easier control over the perturbative regime

$$\ln(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n} = (x-1) + \mathcal{O}(x^2), \quad x \ll 1$$

$$V_{eff}(\chi_0) = -\frac{1}{2}|M_{eff}|^2\chi_0^2 + \frac{\lambda}{4}\chi_0^4 + \frac{1}{4}\frac{\left(-|M_{eff}|^2 + 3\lambda\chi_0^2\right)^2}{(4\pi)^2}\left[i\pi + \left|\frac{(|M_{eff}|)^2 - 3\lambda\chi_0^2}{\Lambda^2}\right| - \frac{5}{2}\right]$$

Much easier to compute EOMs and derive constraints

Full agreement with previous calculations

Backreaction to gravity?

De Moivre's theorem

$$H = \frac{1}{\sqrt{3}M_P}|z|^{\frac{1}{2}}\left[\cos\left(\frac{1}{2}\arg(z)\right) + i\sin\left(\frac{1}{2}\arg(z)\right)\right]$$

$$\arg z = \arg\left(a_1 - a_2\chi_0^2 + a_3\chi_0^4\right)$$

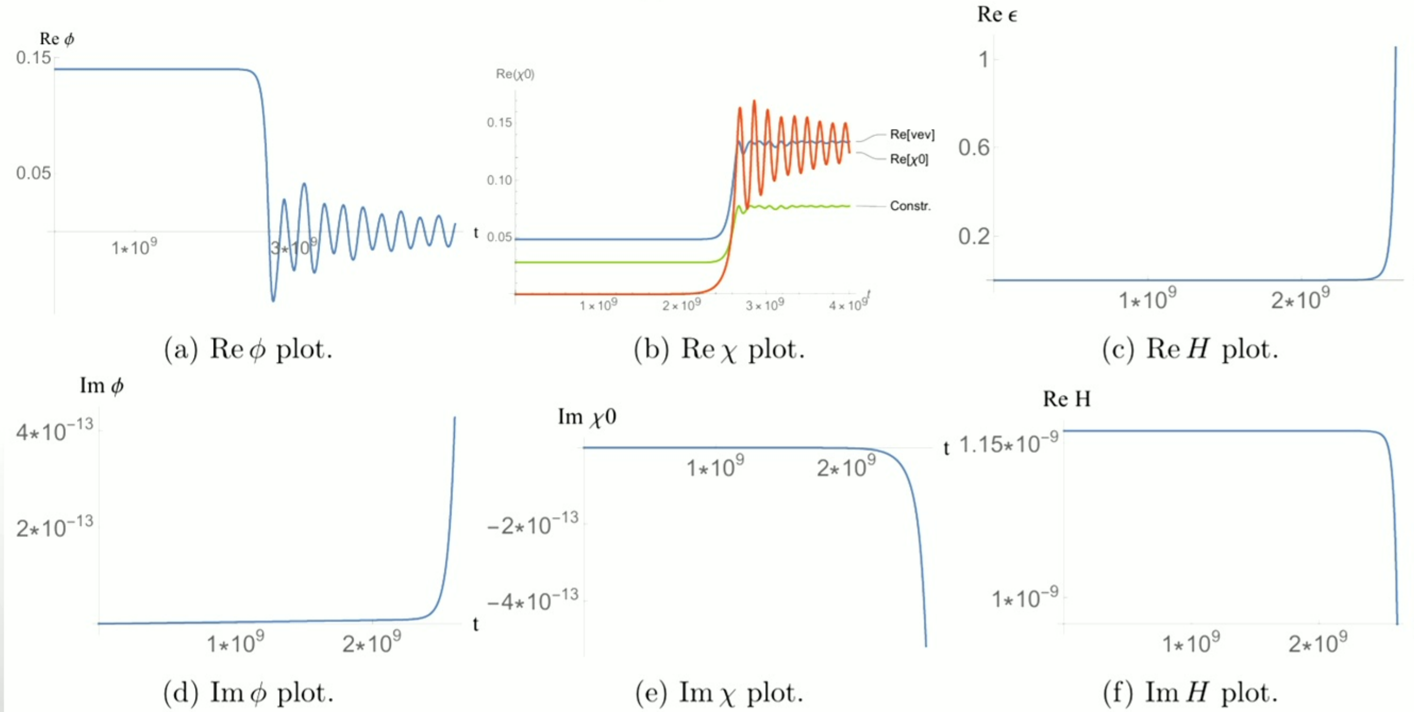
Assuming
Slow-roll inflation
& small log approx.

Backreaction to gravity is controlled – no evidence of violating the EFT
as long as a_1 dominates

$$M_P = 1 \rightarrow \arg z \ll 1,$$
$$\cos(\arg z) \sim 1, \quad \sin(\arg z) \ll 1$$

Regime with two stages of inflation

Classical approximation is valid



(a) Re ϕ plot. (b) Re χ plot. (c) Re H plot.

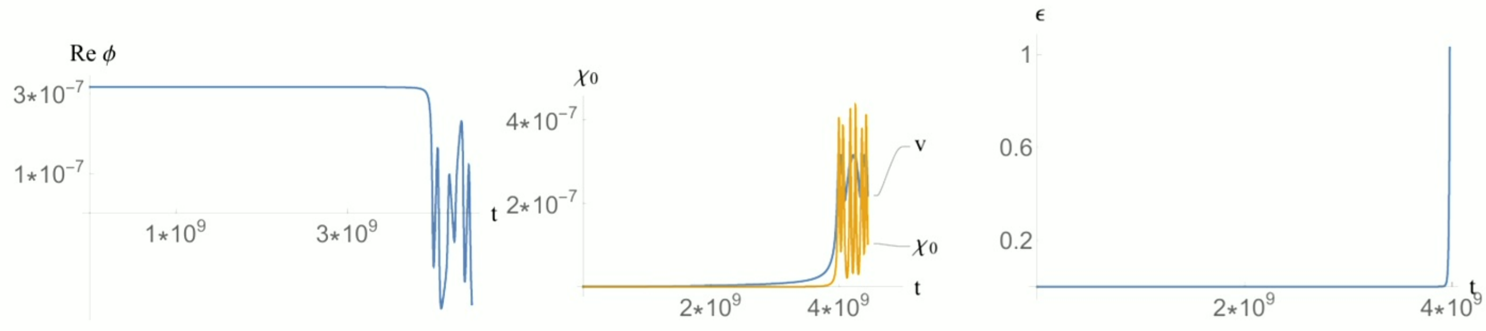
(d) Im ϕ plot. (e) Im χ plot. (f) Im H plot.

$$\lambda = 5\times10^{-14},\quad g = 2\times10^{-7},\quad M = 3\times10^{-8},\quad m = 8\times10^{-12},\quad M_p = 1,$$
$$\chi_0(t=0) = 10^{-12},\quad \phi(t=0) = 1.4\times10^{-1},\quad \Lambda \sim 1.1\times10^{-8}$$

[Clesse, 2011]

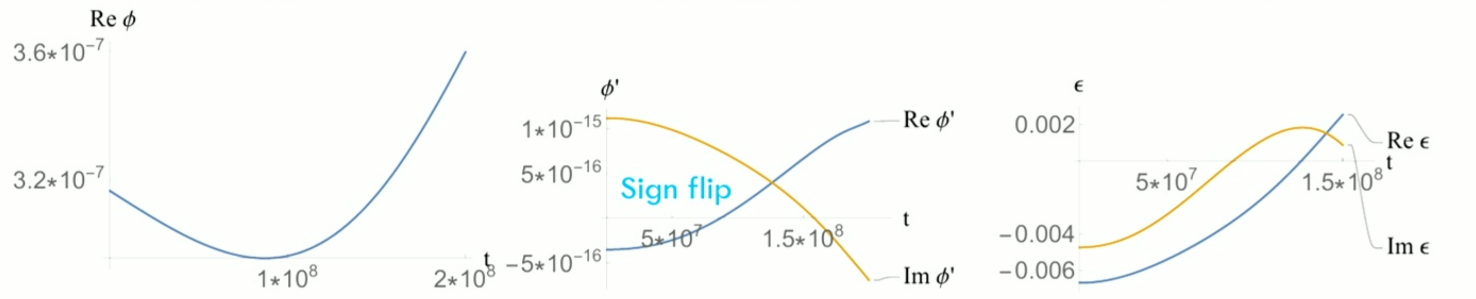
Ordinary hybrid inflation

Classical approximation is not valid



(a) Re ϕ plot. (b) Re χ_0 plot. (c) Re ϵ plot.

standard hybrid



(d) Re ϕ plot. (e) $\dot{\phi}$ plot. (f) ϵ plot.

with quantum corrections

You get nonsense!

$$\lambda = 10^{-1}, \quad g = 10^{-\frac{1}{2}}, \quad M = 10^{-7}, \quad m = 10^{-16}, \quad M_p = 1,$$
$$\chi_0(t=0) = 10^{-14}, \quad \phi(t=0) = 3.15 \times 10^{-7}, \quad \Lambda \sim 2.2 \times 10^{-10}$$

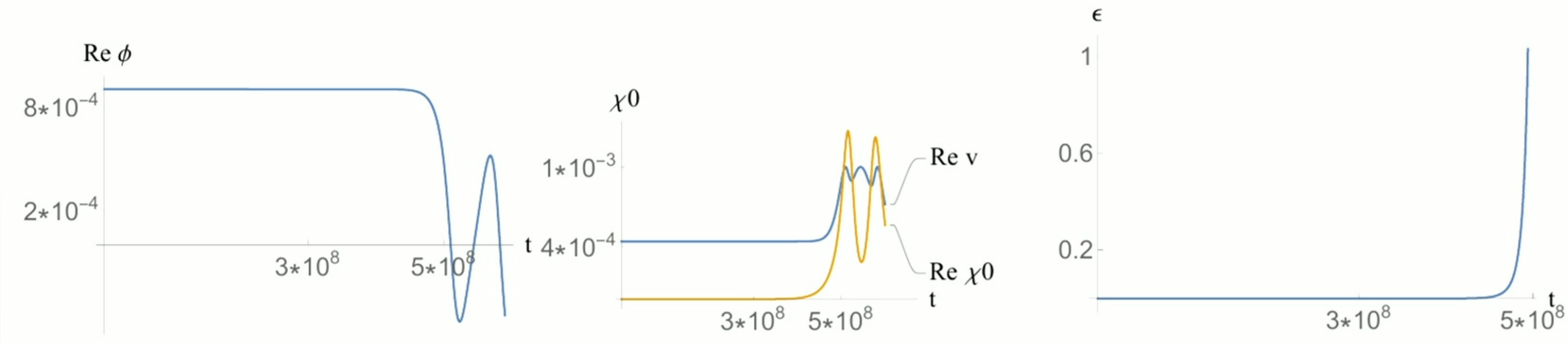
[Linde, 1994]
EW SB, GUT

Enforce constraint

$$\frac{m}{g} > \frac{M}{4\sqrt{\pi}}, \quad \phi < M/g$$

Value of lambda also important for graceful exit

Classical approximation is valid



(a) Re ϕ plot. (b) Re χ plot. (c) Re ϵ plot.

$$\lambda = 10^{-8}, g = 10^{-4}, M = 10^{-7}, m = 9.9 \times 10^{-12}, M_p = 1,$$
$$\chi_0(t=0) = 10^{-14}, \phi(t=0) = 9 \times 10^{-4}, \Lambda \sim 4.35 \times 10^{-8}$$

[Linde, 1994]

Conclusions

Take home message:

Care must be taken when using the classical approximation.

Further applications:

Opportunity to study multifield theories with curved field dynamics during SSB.

Thank you for your attention!