Title: Classical Physics - Lecture 220928

Speakers:

Collection: Classical Physics (2022/2023)

Date: September 28, 2022 - 9:00 AM

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FUN WITH LORENTZ TRANSFORMATIONS

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} \quad \text{S.T.} \quad \eta_{\alpha\beta} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} \eta_{\mu\nu}$$

THAT IS IT LEAVES THE SPACETIME INTERVAL INVARIANT

$$ds^2 = -dt^2 + dx^i dx^i$$

- WE ALSO CONSIDER THE INVERSE

$$x^{\mu} = \tilde{\Lambda}^{\mu}{}_{\nu} x^{\nu'} \quad : \quad \Lambda^{\mu}{}_{\nu} \tilde{\Lambda}^{\nu}{}_{\alpha} = \delta^{\mu}{}_{\alpha}, \quad \tilde{\Lambda}^{\nu}{}_{\alpha} \Lambda^{\alpha}{}_{\mu} = \delta^{\nu}_{\mu} \quad (\Lambda\tilde{\Lambda} = \mathbb{1} = \tilde{\Lambda}\Lambda)$$

THEN DEFINE: VECTORS $\quad V^{\mu'} = \Lambda^{\mu}{}_{\nu} V^{\nu}$

CO-VECTORS (FORMS) $\quad \omega'_{\mu} = \tilde{\Lambda}^{\nu}{}_{\mu} \omega_{\nu}$

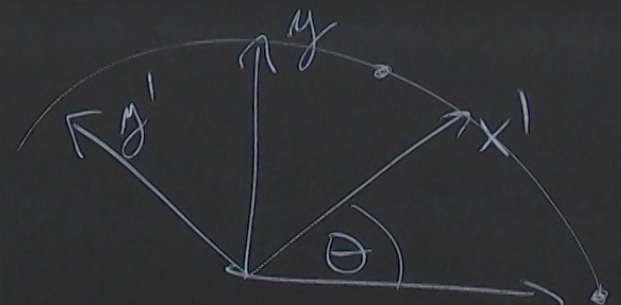
SCALAR $\quad \phi' = \phi$

TENSORS ... TRANSFORM AS VECTORS (FORMS) IN EACH INDEX.

LT = BOOSTS + ROTATIONS

ROTATION

$$\Lambda^{\alpha}{}_{\beta}(\mathrm{ROT}_{xy}) = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

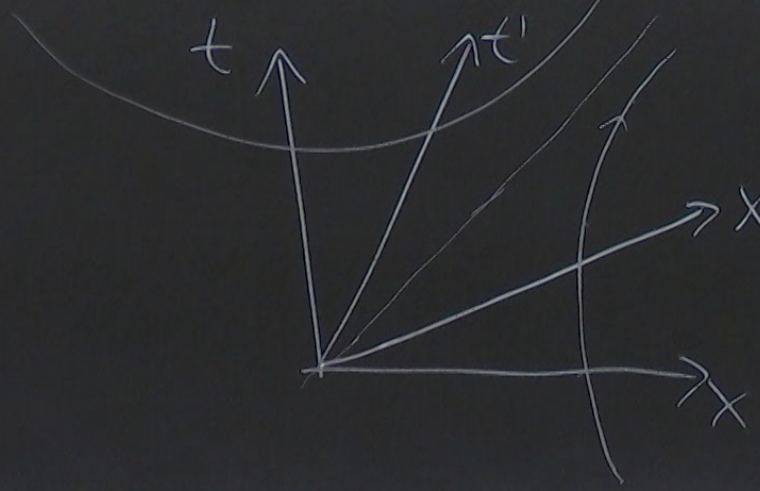


$$x^2 + y^2 = x'^2 + y'^2$$

TENSORS ... TRANSFORM AS VECTORS (FORMS) IN EACH INDEX.

BOOST (SQUEEZE t & x AXES)

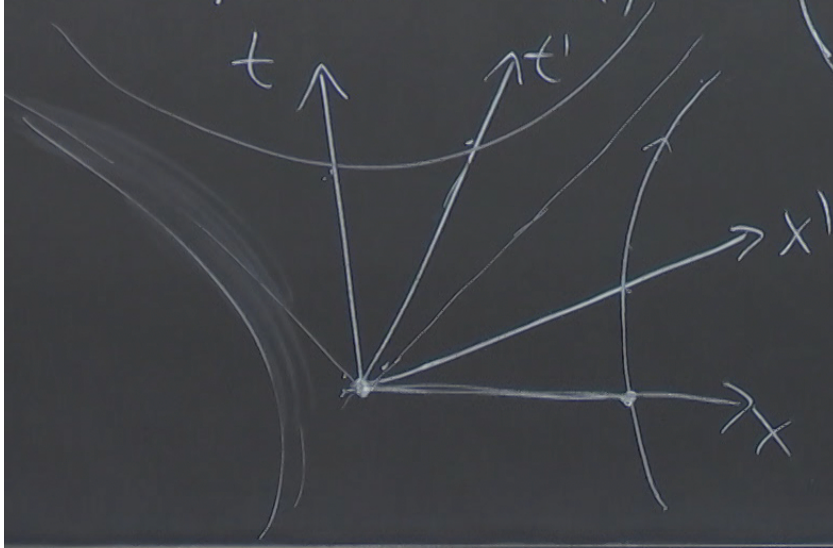
$$\Lambda^{\alpha}{}_{\beta}(\text{BOOST}_{tx}) = \begin{pmatrix} \cosh\theta & \sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$-t^2+x^2=-t'^2+x'^2$$

BOOST (SQUEEZE t & x AXES)

$$\Lambda^{\alpha}{}_{\beta}(\text{BOOST}_{tx}) = \begin{pmatrix} \cosh\theta & \sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



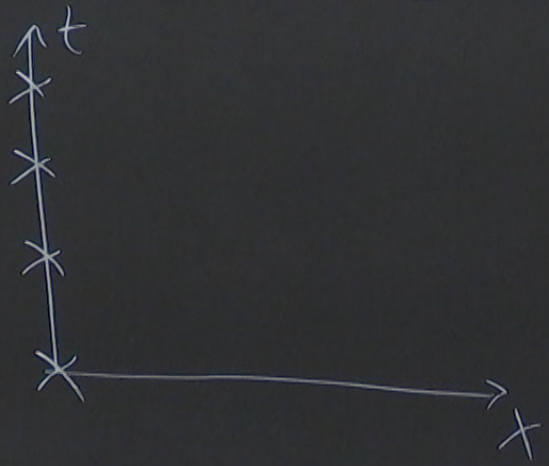
$$-t^2 + x^2 = -t'^2 + x'^2$$

$$\beta = \frac{v}{c} = v = \tanh\theta$$

$$\cosh\theta = \gamma = \frac{1}{\sqrt{1-v^2}}, \quad \sinh\theta = \beta\gamma$$

$$t' = t\cosh\theta + x\sinh\theta = \gamma(t - \beta x)$$
$$x' = -\sinh\theta\, t + \cosh\theta\, x = \gamma(x - \beta t)$$

- TIME DILATATION

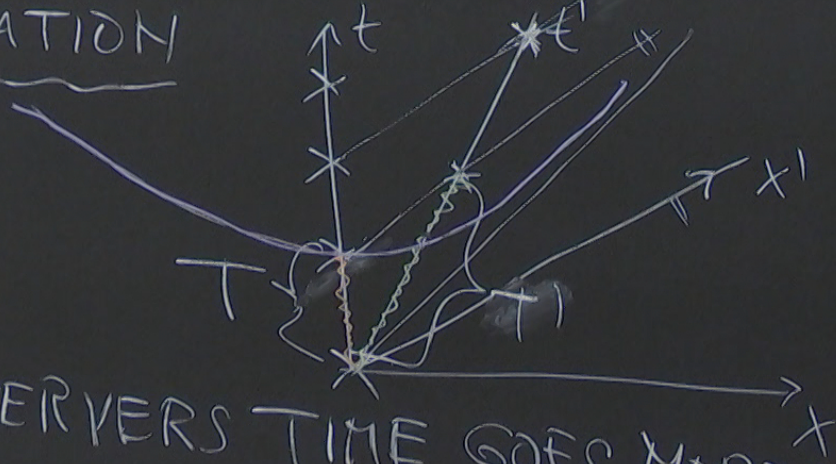


$$t' = t\cosh\theta + x\sinh\theta = \gamma(t - vx)$$
$$x' = -\sinh\theta\, t + \cosh\theta\, x = \gamma(x - vt)$$

$T' = \gamma T$

- TIME DILATATION

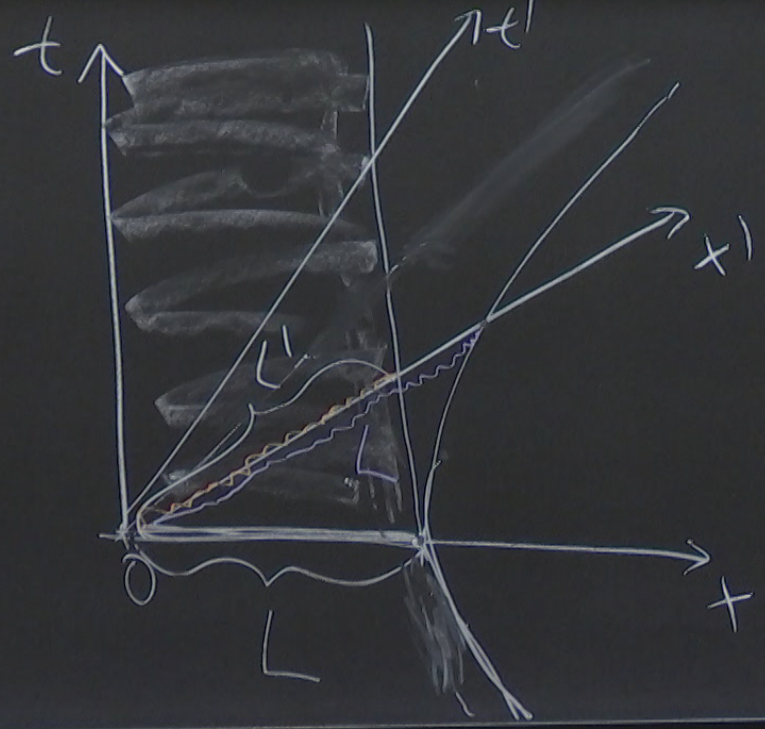
$T' > T$



FOR MOVING OBSERVERS TIME GOES MORE SLOWLY.

$\cosh\theta = \gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\sinh\theta = \beta\gamma$

- LORENTZ CONTRACTION

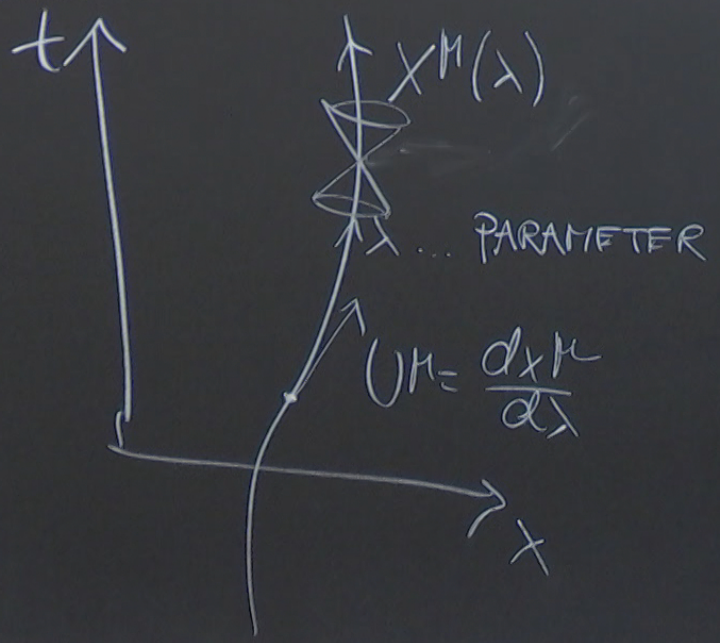


$L' < L$

FOR MOVING OBJECTS DISTANCES ARE SHORTER!

$L' = L/\gamma$

- MOTION OF RELATIVISTIC PARTICLES



MASSIVE PARTICLES
MOVE ON TIMELIKE TRAJECTORIES

- MASSLESS PARTICLES
  ... NULL (LIGHT-LIKE) TRAJ
- NICE PARAMETER FOR MASSIVE PARTICLES
  IS PROPER TIME $-d\tau^2 = -dt^2 + dx^i dx^i$

$$d\tau = \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$ ... REPARAM. INVARIANCE

CHOOSE $\lambda = t$.. COORDINATE TIME

$$d\tau = \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\, dt$$

TORIES

PARTICLES

$dt^2 + dx^i dx^i$

$$d\tau = \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$ ... REPARAM. INVARIANCE

CHOOSE $\lambda = t$.. COORDINATE TIME

TORIES

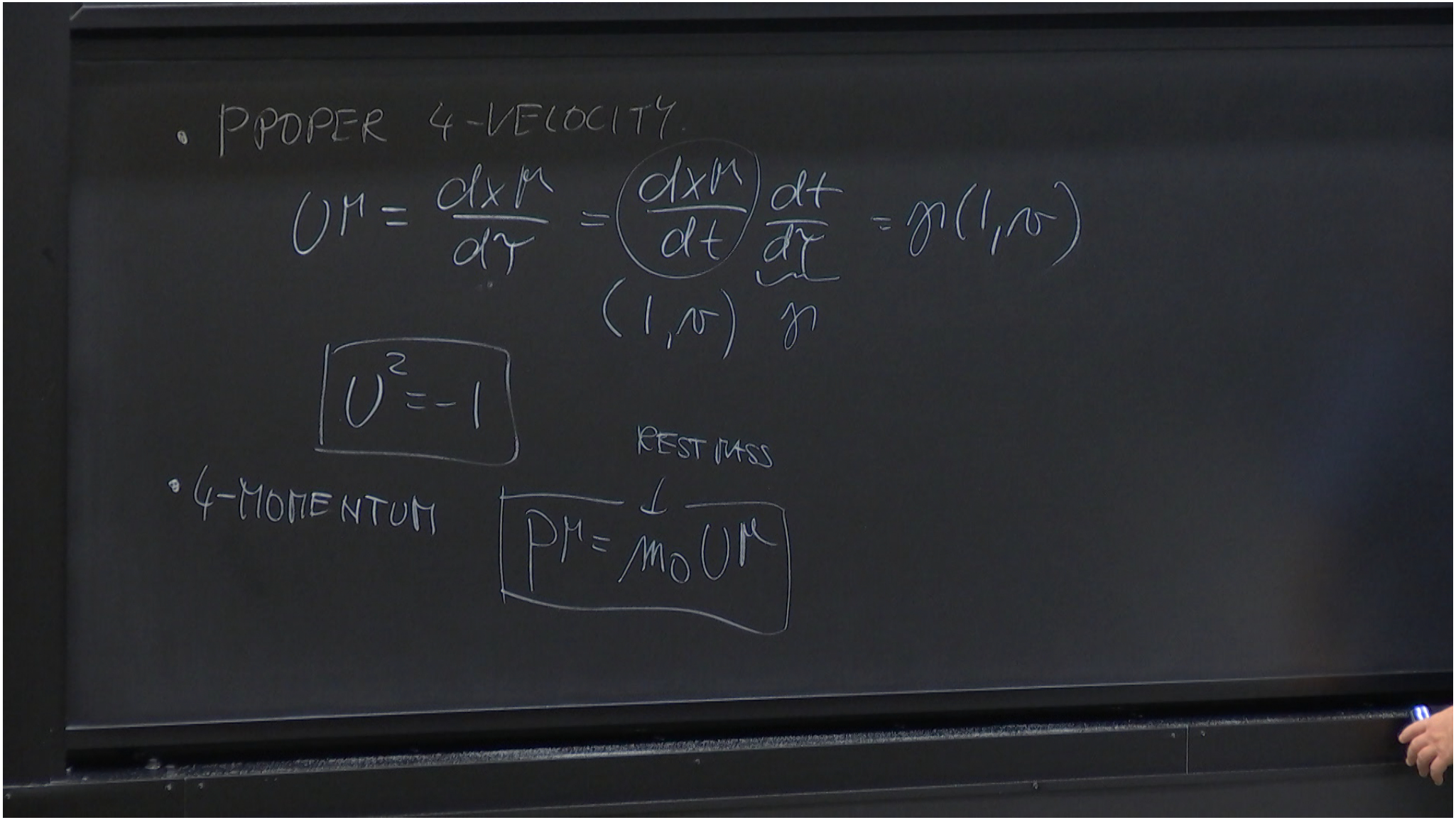
$$d\tau = \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\, dt = \sqrt{1 - \left|\frac{dx^i}{dt}\right|^2}\, dt$$

v

$$= \sqrt{1 - v^2}\, dt = \frac{dt}{\gamma}$$

PARTICLES

$dt^2 + dx^i dx^i$


• PROPER 4-VELOCITY
$U^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(1, v)$
$(1, v)$ γ
$U^2 = -1$
• 4-MOMENTUM
REST MASS
1
$P^\mu = m_0 U^\mu$

PROPER 4-VELOCITY

$$U^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(1, v)$$

$(1, v)$ γ

$$U^2 = -1$$

$$E = mc^2 = m$$

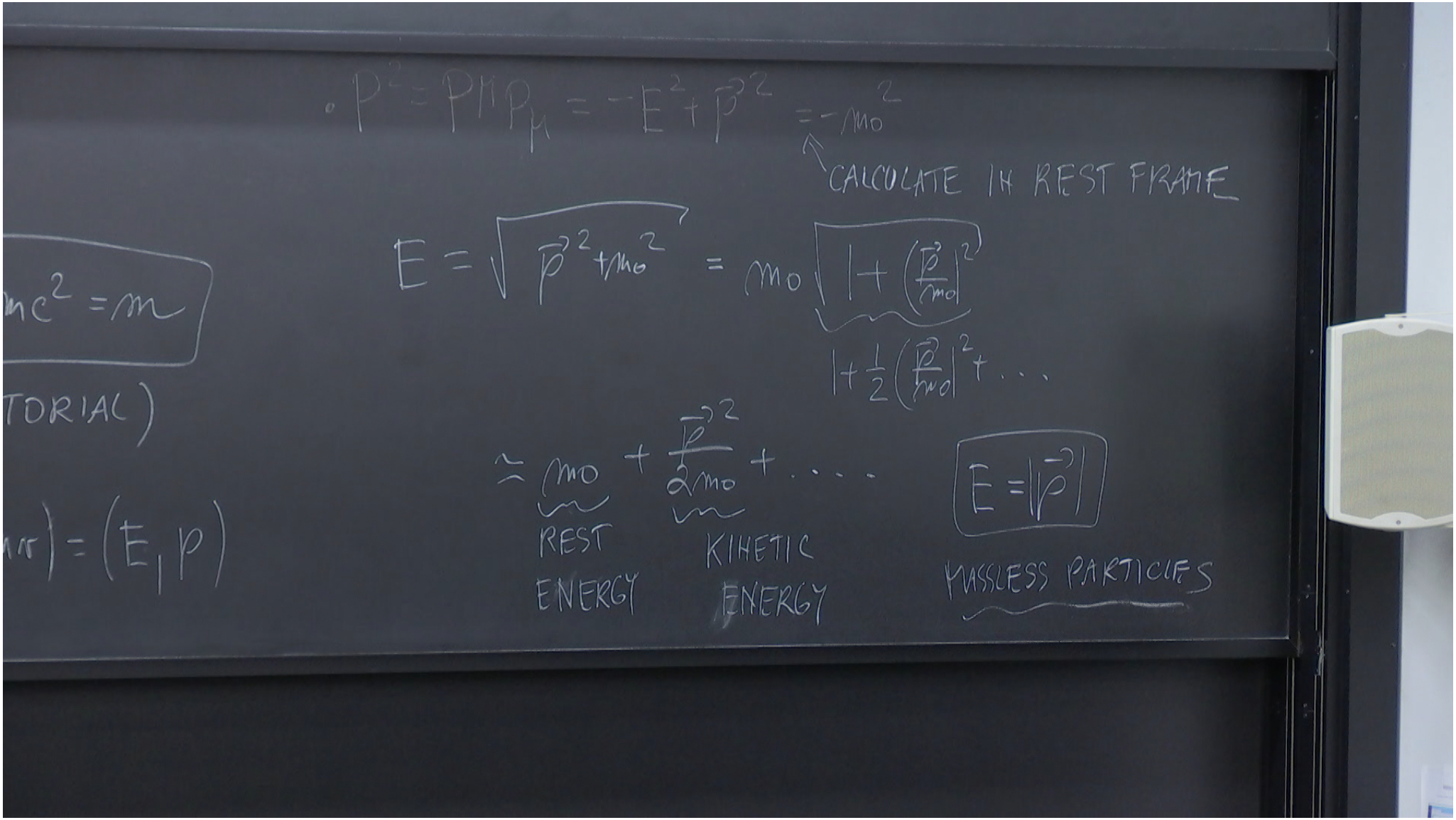
(SEE TUTORIAL)

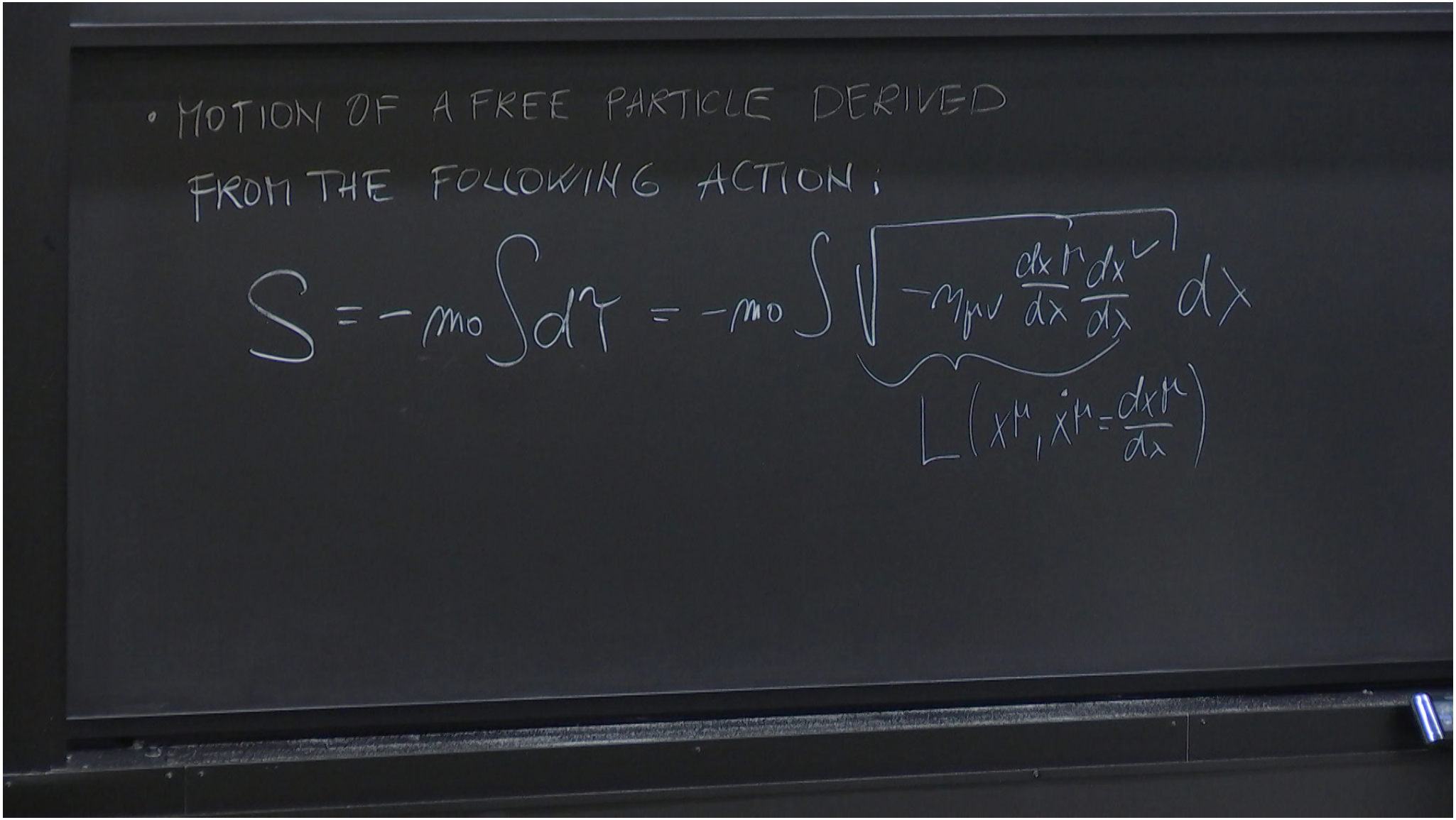
REST MASS

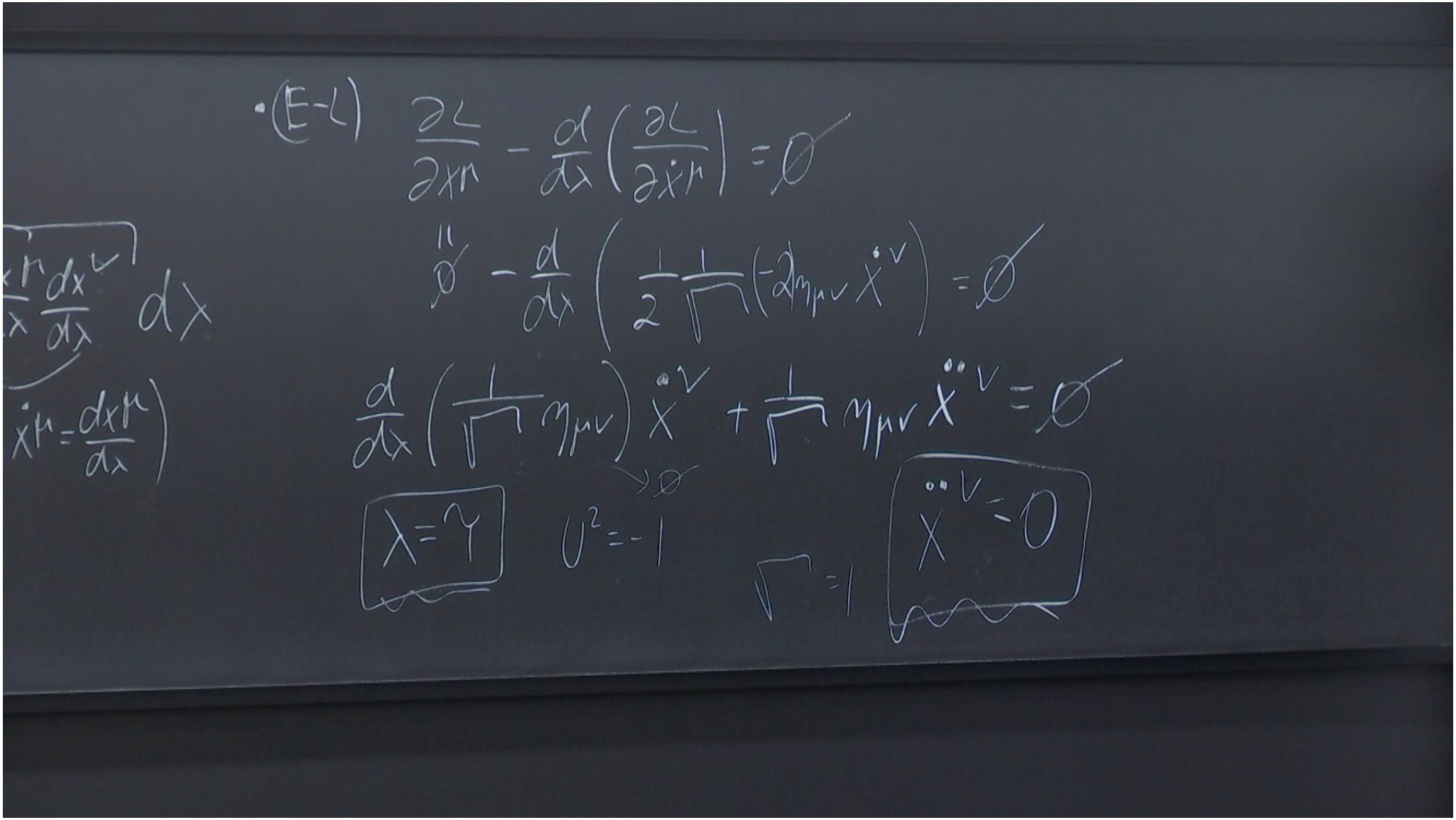
MOMENTUM

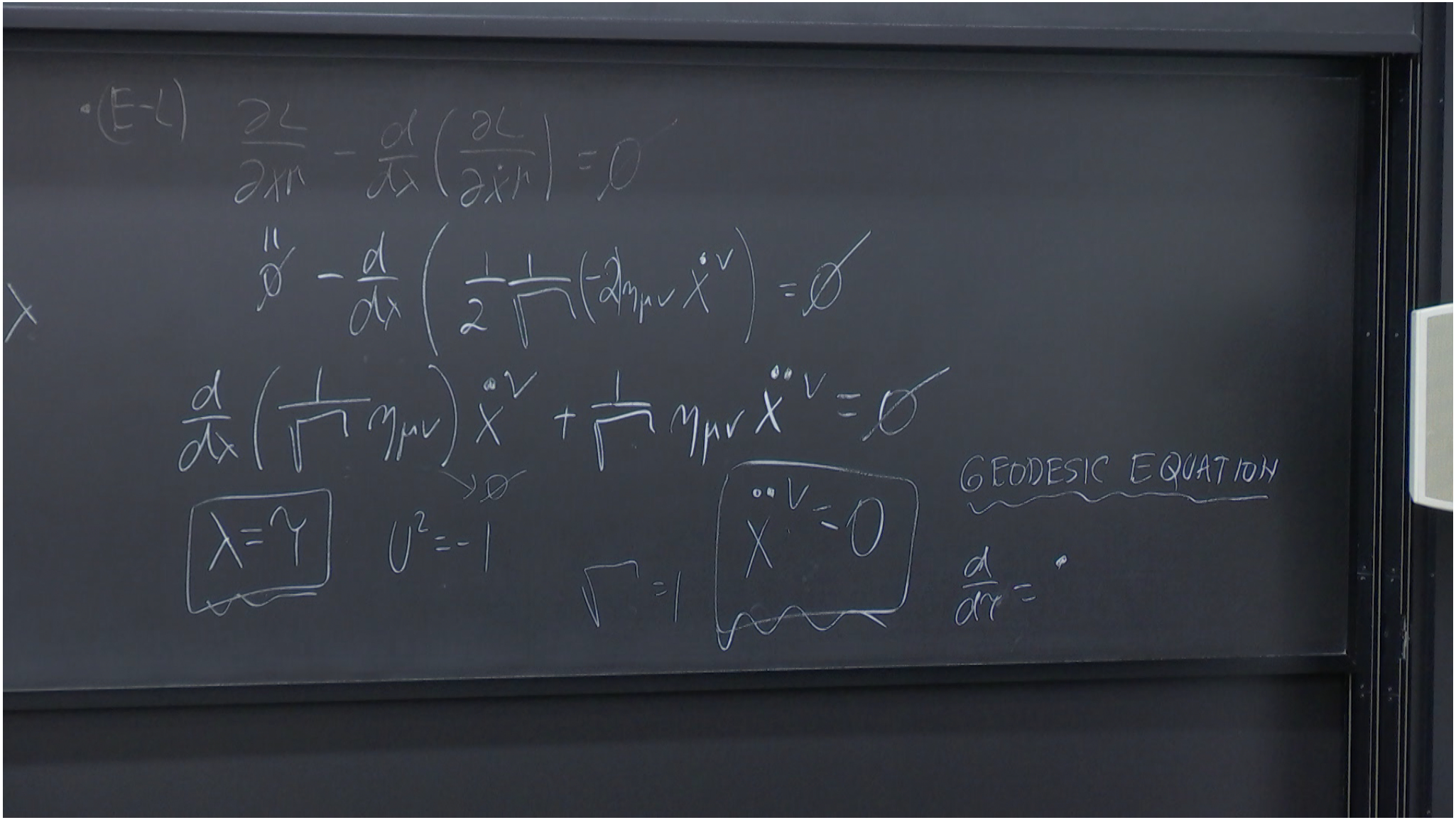
$$p^\mu = m_0 U^\mu = (m_0\gamma, m_0\gamma v) = (m, mv) = (E, p)$$

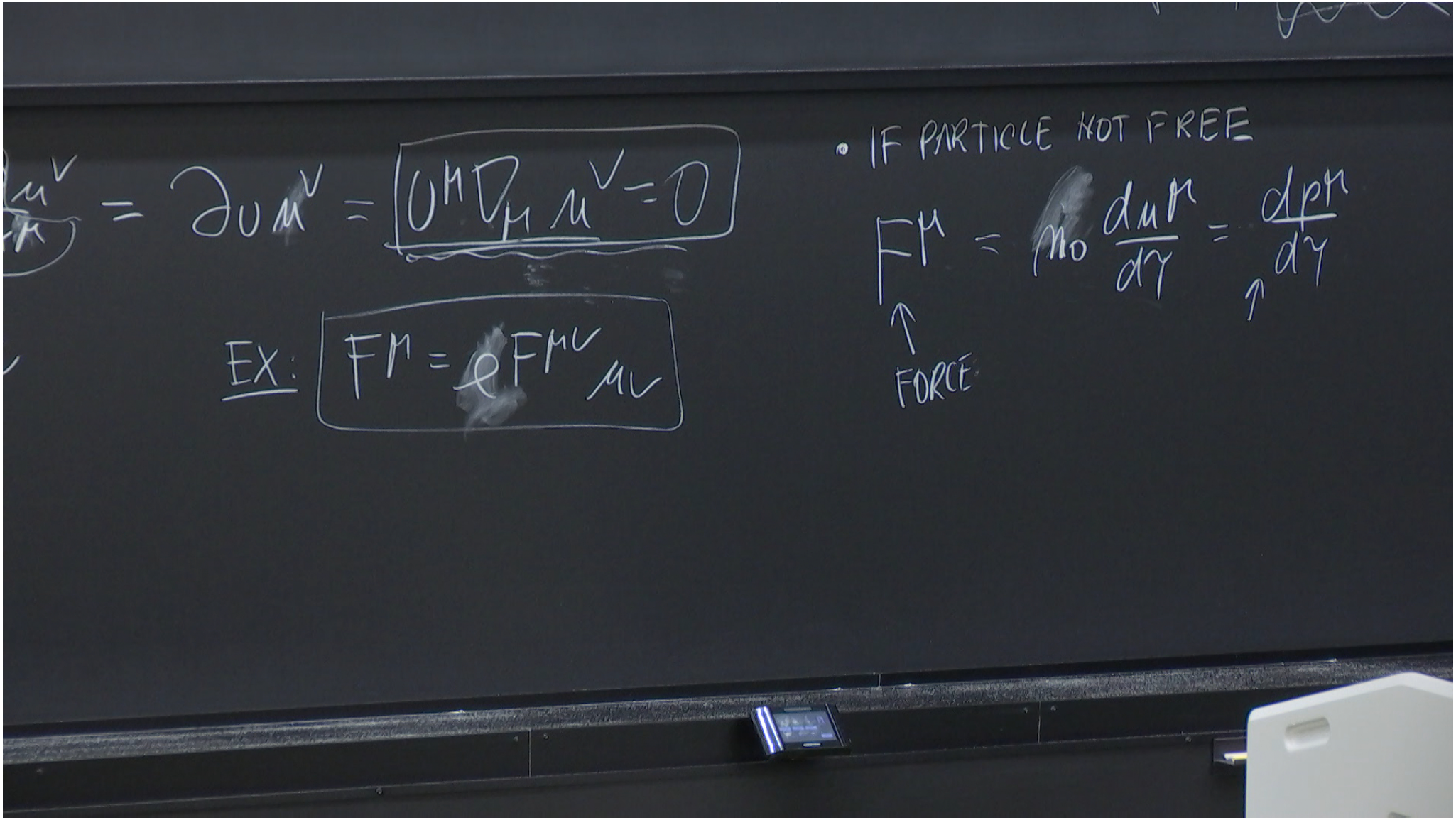
m : MASS


$P^2 = P^\mu P_\mu = -E^2 + \vec{p}^2 = -m_0^2$
CALCULATE IN REST FRAME
$E = \sqrt{\vec{p}^2 + m_0^2} = m_0 \sqrt{1 + \left(\frac{\vec{p}}{m_0}\right)^2}$
$1 + \frac{1}{2}\left(\frac{\vec{p}}{m_0}\right)^2 + \ldots$
$\simeq m_0 + \frac{\vec{p}^2}{2m_0} + \ldots$
REST ENERGY
KINETIC ENERGY
$E = |\vec{p}|$
MASSLESS PARTICLES
$mc^2 = m$
TORIAL)
(E, p)


• MOTION OF A FREE PARTICLE DERIVED FROM THE FOLLOWING ACTION;
$S = -m_0 \int d\tau = -m_0 \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, d\lambda$
$L\left(x^\mu, \dot{x}^\mu = \frac{dx^\mu}{d\lambda}\right)$

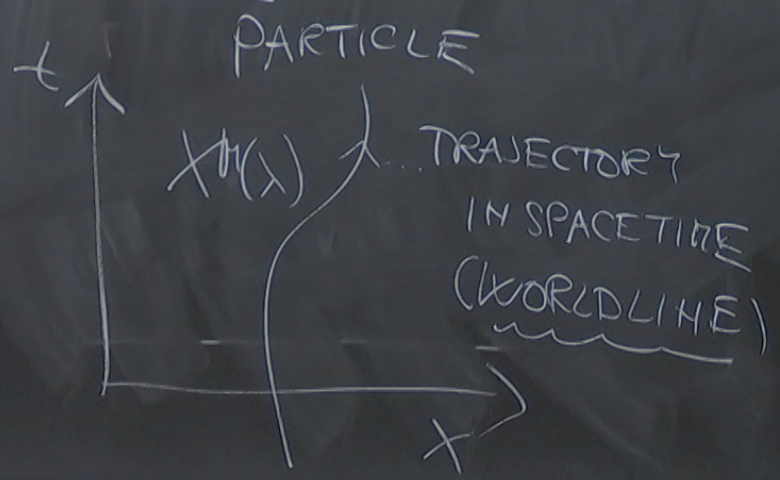

(E-L) $\frac{\partial \mathcal{L}}{\partial x^\mu} - \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}\right) = 0$
$\ddot{x}^\mu - \frac{d}{d\lambda}\left(\frac{1}{2}\frac{1}{\Gamma}(-2)\eta_{\mu\nu}\dot{x}^\nu\right) = 0$
$\frac{d}{d\lambda}\left(\frac{1}{\Gamma}\eta_{\mu\nu}\right)\ddot{x}^\nu + \frac{1}{\Gamma}\eta_{\mu\nu}\ddot{x}^\nu = 0$
$\lambda = \tau$
$u^2 = -1$
$\Gamma = 1$
$\ddot{x}^\nu = 0$
$\dot{x}^\mu = \frac{dx^\mu}{d\lambda}$

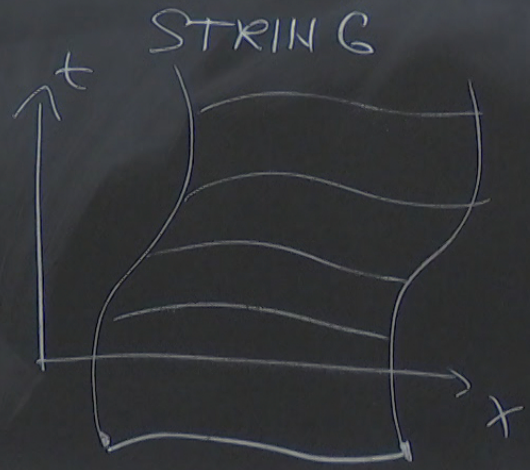

(E-L) $\frac{\partial \mathcal{L}}{\partial x^\mu} - \frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}\right) = 0$
$\ddot{x}^\mu - \frac{d}{d\lambda}\left(\frac{1}{2}\frac{1}{\sqrt{\Gamma}}(-2\eta_{\mu\nu}\dot{x}^\nu)\right) = 0$
$\frac{d}{d\lambda}\left(\frac{1}{\sqrt{\Gamma}}\eta_{\mu\nu}\right)\dot{x}^\nu + \frac{1}{\sqrt{\Gamma}}\eta_{\mu\nu}\ddot{x}^\nu = 0$
$\lambda = \tau$
$U^2 = -1$
$\sqrt{\Gamma} = 1$
$\ddot{x}^\nu = 0$
GEODESIC EQUATION
$\frac{d}{d\tau} =$


$= \partial_\mu u^\nu = \boxed{u^\mu \nabla_\mu u^\nu = 0}$
Ex: $\boxed{F^\mu = e F^{\mu\nu} u_\nu}$
• IF PARTICLE NOT FREE
$F^\mu = m_0 \frac{du^\mu}{d\tau} = \frac{dp^\mu}{d\tau}$
FORCE

- IN STRINGTHEORY WE HAVE HIGHER DIM OBJECTS (STRINGS, ...)

  p-BRANES (PARTICLE 0-BRANE, STRING - 1-BRANE)



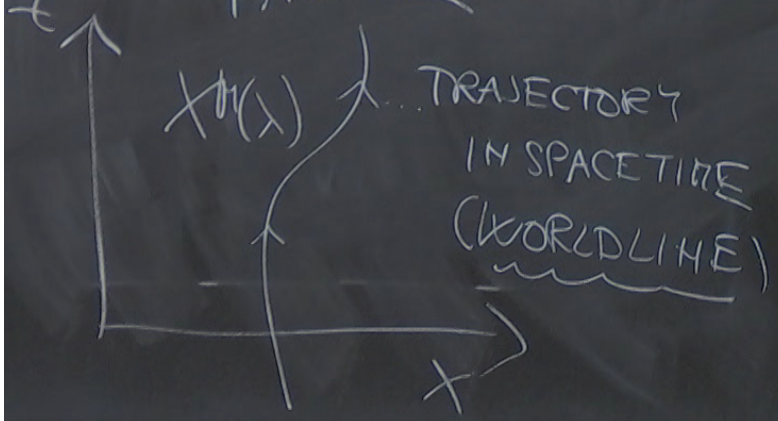


INGTHEORY WE HAVE HIGHER DIM OBJECTS

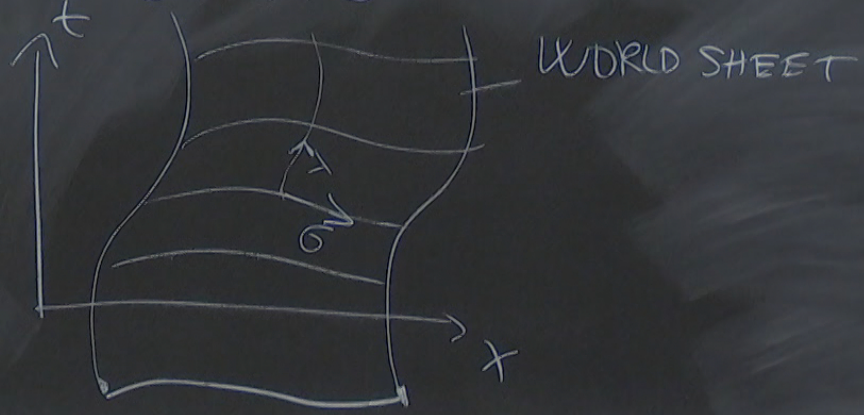
RINGS, ...)

-BRANES (PARTICLE 0-BRANE, STRING - 1-BRANE)

PARTICLE



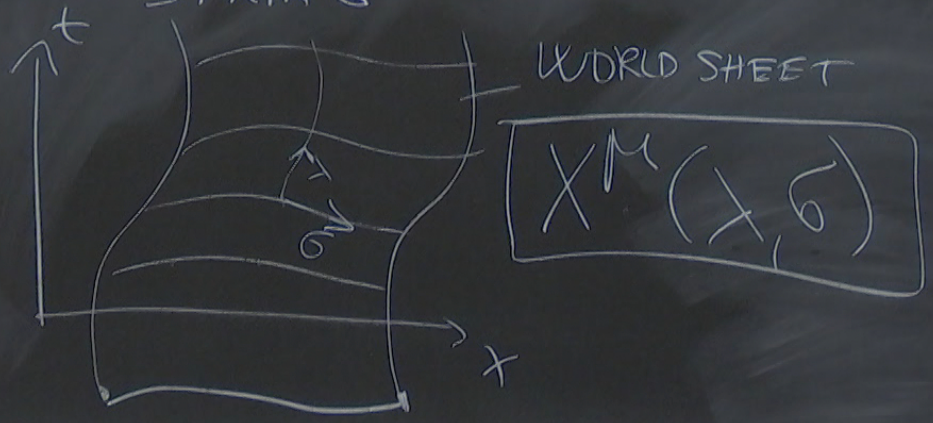
STRING



WE HAVE HIGHER DIM OBJECTS

(PARTICLE 0-BRANE, STRING - 1-BRANE)

STRING



WORLD SHEET

$X^M(\tau, \sigma)$

TRAJECTORY IN SPACETIME (WORLDLINE)

THEORY WE HAVE HIGHER DIM OBJECTS

S, ...)

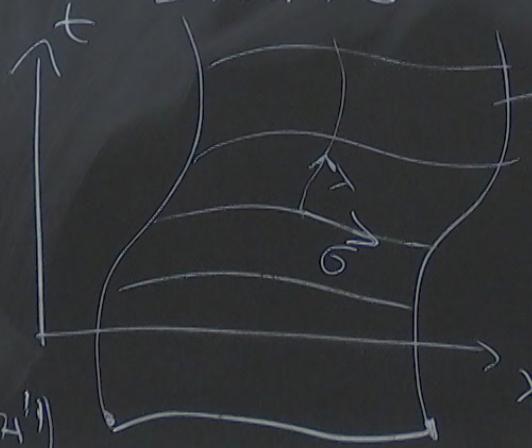
NES (PARTICLE 0-BRANE, STRING - 1-BRANE)

PARTICLE

$X^M(\lambda)$ ... TRAJECTORY IN SPACETIME (WORLDLINE)

'EXTREME "LENGTH"'

STRING

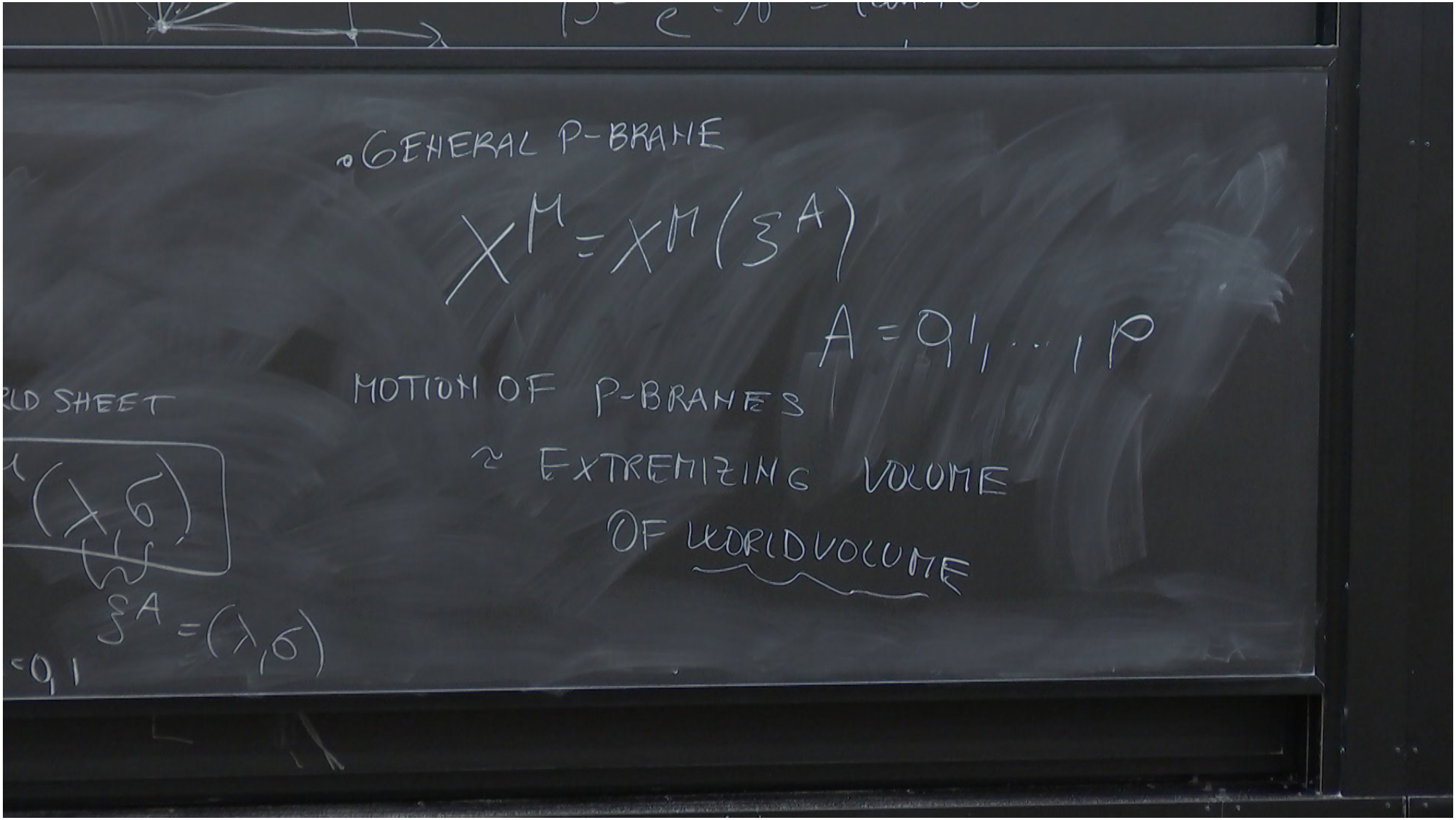


WORLD SHEET

$X^M(\lambda, \sigma)$

$\xi^A = (\lambda, \sigma)$

A = 0, 1


GENERAL P-BRANE
$X^M = X^M(\xi^A)$
$A = 0, 1, \ldots, p$
MOTION OF P-BRANES
≃ EXTREMIZING VOLUME
OF WORLDVOLUME
RLD SHEET
$\xi^A = (\lambda, \sigma)$
$<0, 1$

THIS IS GIVEN BY NAMBU-GOTO ACTION

$$S_{NG} = \int \sqrt{-\det(\gamma_{AB})}\, d^{p+1}\xi$$

$$\gamma_{AB} = \frac{\partial X^\mu}{\partial \xi^A} \frac{\partial X^\nu}{\partial \xi^B} \eta_{\mu\nu}$$

INDUCED METRIC

• THESE

- THESE ACTIONS ARE SIMPLE BUT NOT "NICE"
  - PROBLEMS WITH $\sqrt{}$?
  - PROBLEMS WITH $m_0 \to \emptyset$

INSTEAD CONSIDER POLYAKOV ACTION

$$S_P = \frac{1}{2}\int\left(\frac{1}{e}\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} - m_0 e^2\right)d\lambda$$

FIELDS $(X^\mu(\lambda), e(\lambda))$

$A = 0,1$ $\quad J^\mu = (\lambda, \sigma)$

- THESE ACTIONS ARE SIMPLE BUT NOT "NICE"
  - PROBLEMS WITH $\int$
  - PROBLEMS WITH $m_0 \to \emptyset$

INSTEAD CONSIDER POLYAKOV ACTION

$$S_P = \frac{1}{2} \int \left(\frac{1}{e} \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} - m_0 e^2 \right) d\lambda$$

FIELDS $(X^\mu(\lambda), e(\lambda))$

- LORENTZ CONTRACTION

FIELDS $(X^\mu(\tau), e(\tau))$

CAN TAKE $m_0 \to 0$

$$S_{\text{MASSLESS}} = \int \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} d\tau \qquad (m_0 = 0)$$

$$S_P = \int d^2\xi \sqrt{-\det \gamma}\, e^{AB} \gamma_{AB} \partial_A X^\mu \partial_B X_\mu \qquad \text{STRINGS}$$

$(DX^\mu)^2 \quad (D\varphi)^2$