Title: Classical Physics - Lecture 220927

Speakers:

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4) OVERVIEW OF SPECIAL RELATIVITY

a) FROM MAXWELL TO SR

MAXWELL THEORY - GAUGE THEORY

# 4) OVERVIEW OF SPECIAL RELATIVITY

## a) FROM MAXWELL TO SR

MAXWELL THEORY
- GAUGE THEORY
- CONFORMAL FIELD THEORY
- ELECTROMAGNETIC DUALITY

$(\rho = 0 = j, \; E \to B, \; B \to -E)$

HENNEAUX-TALK

- RELATIVISTIC

IN PARTICULAR PREDICTS $\exists \boxed{c}$ (SPEED OF LIGHT)

$\rightarrow$ EMERGENCE OF SPACETIME

$$x^\mu = (ct, x^i) \qquad (c=1)$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{c\partial t}, \partial_{x_i}\right)$$

MINKOWSKI METRIC

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \emptyset \\ \emptyset & & 1 & \\ & & & 1 \end{pmatrix}$$

$$(\eta^{-1})^{\alpha\beta} \equiv \eta^{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \emptyset \\ \emptyset & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\eta_{\alpha\beta}\, \eta^{\beta\gamma} = \delta^{\gamma}_{\alpha}$$

$V^{\mu}$ ... "VECTOR"

$\omega_{\mu}$ ... "CO-VECTOR"

S

- $\omega^{\mu} = \eta^{\mu\nu}\omega_{\nu}$

  $\underbrace{V_{\mu} = \eta_{\mu\nu}V^{\nu}}$

- $\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2 = \eta^{\alpha\beta}\partial_{\alpha}\partial_{\beta} = \eta_{\alpha\beta}\partial^{\alpha}\partial^{\beta}$

- GOAL: REWRITE MAXWELL'S THEORY IN THIS NEW LANG

$A^{\mu} = (\phi, \vec{A})$

- $\omega^\mu = \eta^{\mu\nu}\omega_\nu$

  $\underbrace{V_\mu = \eta_{\mu\nu}V^\nu}$

- $\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2 = \eta^{\alpha\beta}\partial_\alpha\partial_\beta = \eta_{\alpha\beta}\partial^\alpha\partial^\beta$

- <u>GOAL</u>: REWRITE MAXWELL'S THEORY IN THIS NEW LANGUAGE.

$$A^\mu = (\phi, \vec{A})$$

• LORENZ CONDITION

$$\partial_t \phi + \nabla \cdot A = 0$$

$$\boxed{\partial_\mu A^\mu = 0}$$

• $\phi \to \phi - \partial_t \Lambda$

$A \to A + \nabla \Lambda$

• $A_\mu = (-\phi, \vec{A})$

$$\boxed{A_\mu \to A_\mu + \partial_\mu \Lambda}$$

• $J^\mu = (\rho, \vec{j})$

$$\boxed{\Box A^\mu = -4\pi J^\mu}$$

$A^\mu_{RET}$

$$A^\mu = (\phi, \vec{A})$$

$$A^\mu_{RET} = \int d^3r' \, \frac{J^\mu(t - |r - r'|, r')}{|r - r'|}$$

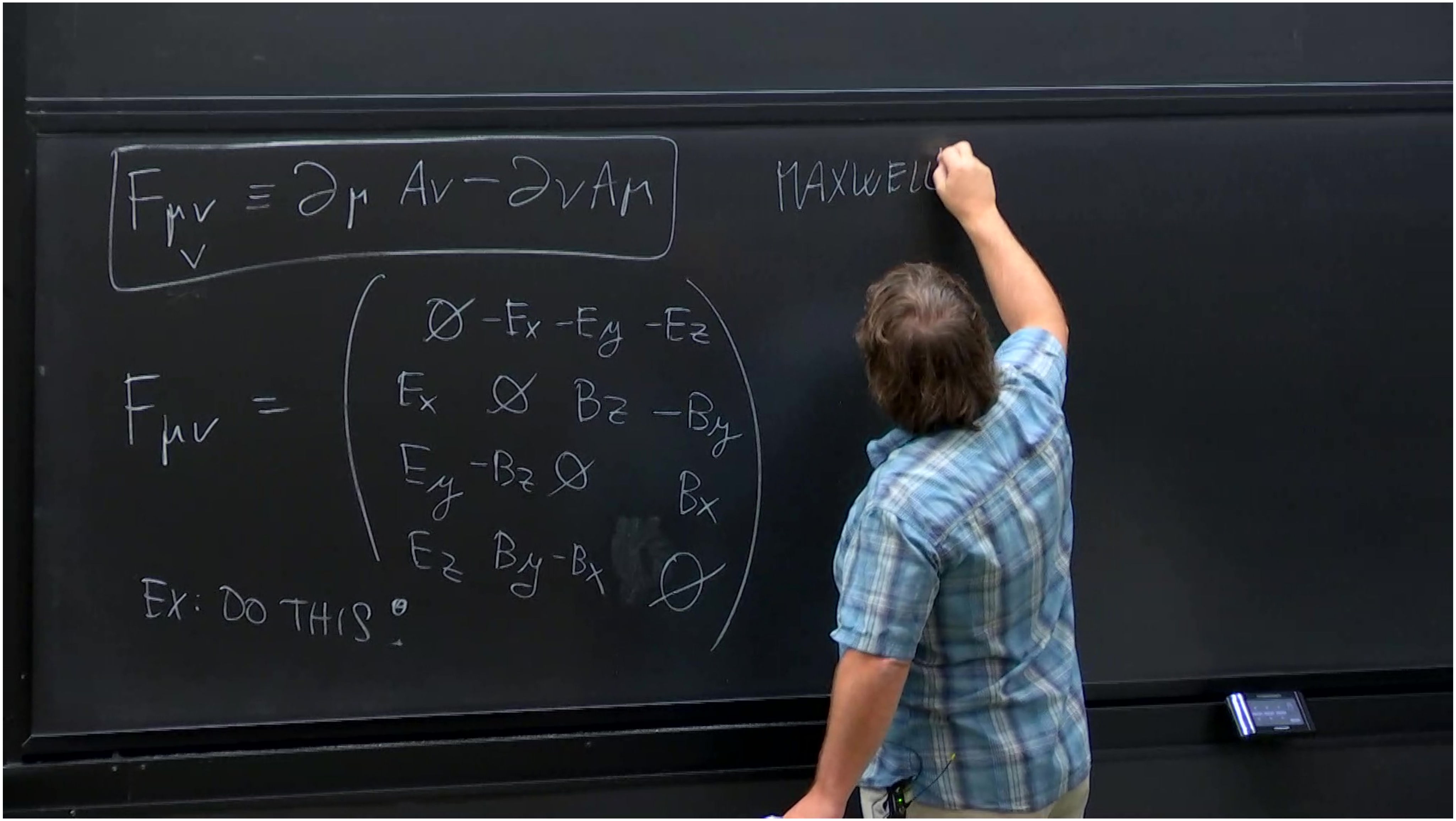
- $E = -\nabla\phi - \frac{\partial A}{\partial t}, \quad B = \nabla \times A$

$$(E, B) \Leftrightarrow \underbrace{\partial_\alpha A_\beta}_{16 \text{ COMPTS}} = T_{\alpha\beta}$$

$$T_{\alpha\beta} = T_{(\alpha\beta)} + T_{[\alpha\beta]}$$

$$T_{(\alpha\beta)} = \frac{1}{2}(T_{\alpha\beta} + T_{\beta\alpha})$$

$$T_{[\alpha\beta]} = \frac{1}{2}(T_{\alpha\beta} - T_{\beta\alpha})$$

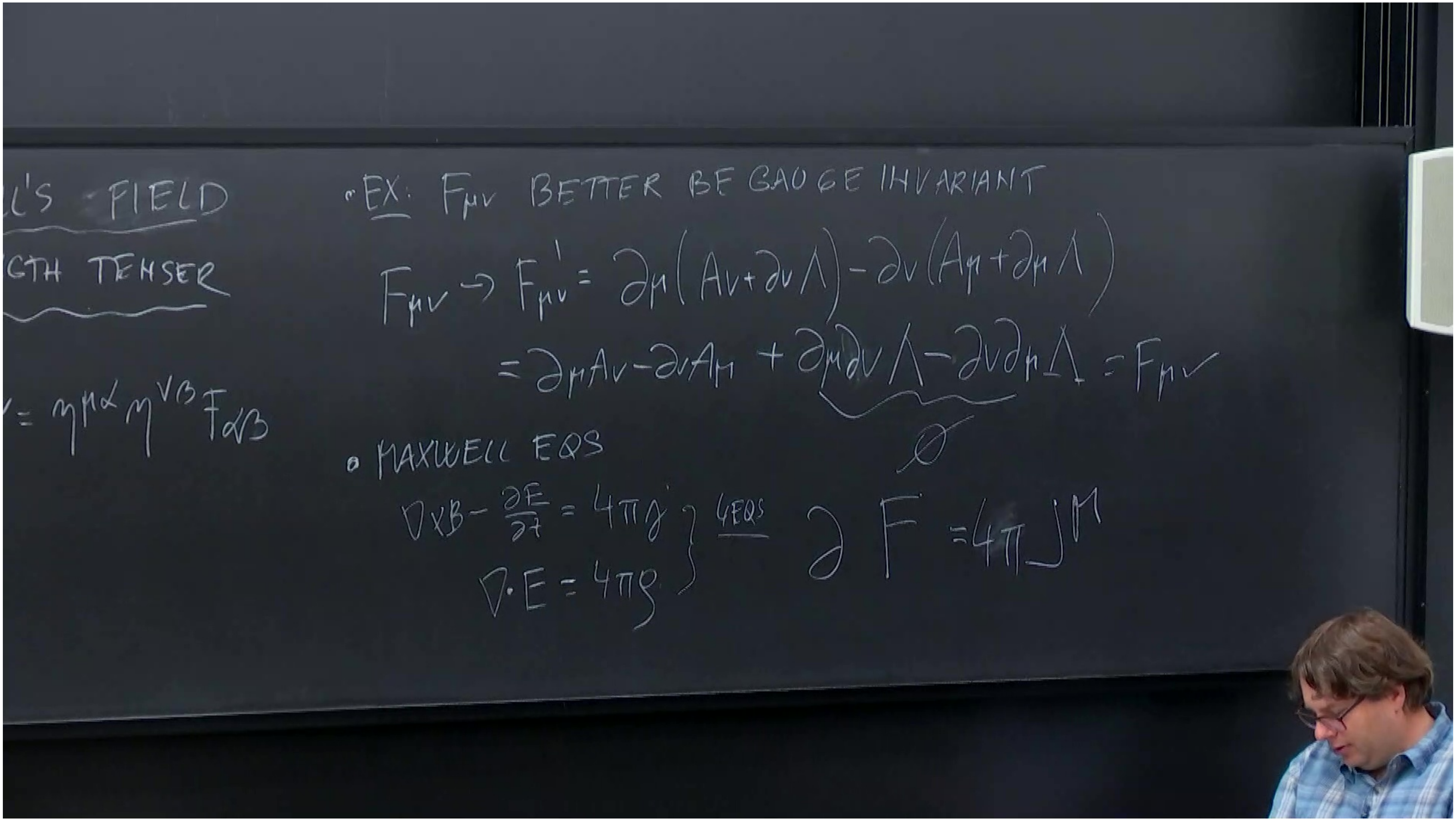

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$
$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$
EX: DO THIS!
MAXWELL

MAXWELL'S FIELD STRENGTH TENSER

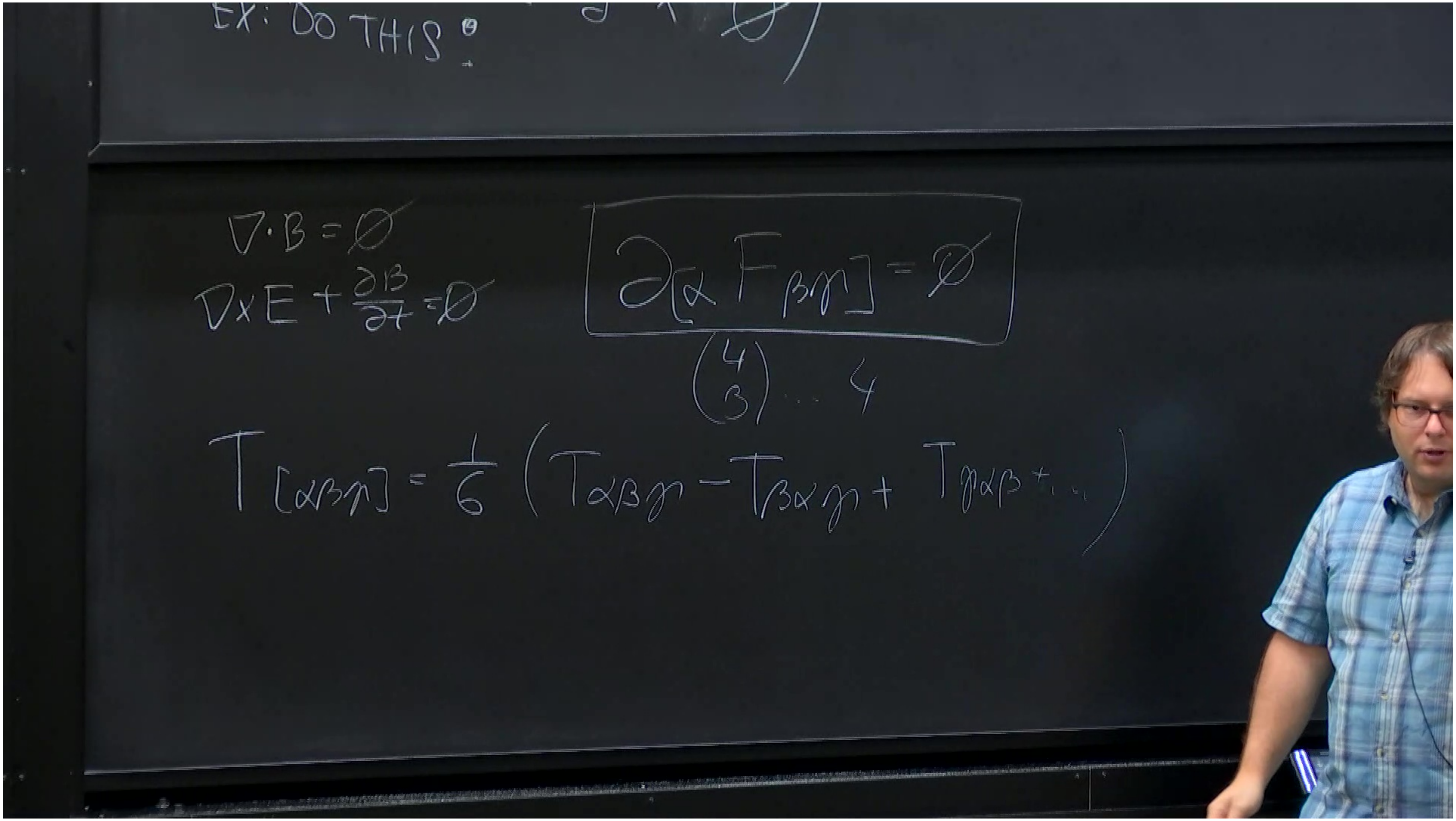
$$F^{\mu\nu} = \eta^{\mu\alpha}\eta^{\nu\beta}F_{\alpha\beta}$$

EX: $F_{\mu\nu}$ BETTER BE GAUGE INVARIANT

$$F_{\mu\nu} \to F'_{\mu\nu} = \partial_\mu(A_\nu + \partial_\nu \Lambda) - \partial_\nu(A_\mu + \partial_\mu \Lambda)$$



L'S FIELD
GTH TENSER
$= \eta^{\mu\alpha}\eta^{\nu\beta}F_{\alpha\beta}$
EX: $F_{\mu\nu}$ BETTER BE GAUGE INVARIANT
$F_{\mu\nu} \to F'_{\mu\nu} = \partial_\mu(A_\nu + \partial_\nu\Lambda) - \partial_\nu(A_\mu + \partial_\mu\Lambda)$
$= \partial_\mu A_\nu - \partial_\nu A_\mu + \underbrace{\partial_\mu\partial_\nu\Lambda - \partial_\nu\partial_\mu\Lambda}_{\emptyset} = F_{\mu\nu}$ ✓
MAXWELL EQS
$\nabla\times B - \frac{\partial E}{\partial t} = 4\pi j$
$\nabla\cdot E = 4\pi\rho$
4EQS
$\partial F = 4\pi J^\mu$


EX: DO THIS!
$\nabla \cdot B = 0$
$\nabla \times E + \frac{\partial B}{\partial t} = 0$
$\boxed{\partial_{[\alpha} F_{\beta\gamma]} = 0}$
$\binom{4}{3} \dots 4$
$T_{[\alpha\beta\gamma]} = \frac{1}{6}\left(T_{\alpha\beta\gamma} - T_{\beta\alpha\gamma} + T_{\gamma\alpha\beta} + \dots\right)$

$\nabla \cdot B = \emptyset$

$\nabla \times E + \frac{\partial B}{\partial t} = \emptyset$

$$\boxed{\partial_{[\alpha} F_{\beta\gamma]} = \emptyset}$$

$\binom{4}{3} \ldots 4$

* 4D LEVI-CIVITA TEN

$\varepsilon^{\alpha\beta\gamma\delta} = \varepsilon^{[}$

$(*F)^{\mu\nu} = \frac{1}{2}$

$$T_{[\alpha\beta\gamma]} = \frac{1}{6}\left(T_{\alpha\beta\gamma} - T_{\beta\alpha\gamma} + T_{\gamma\alpha\beta} + \ldots\right)$$

$\partial_\alpha\left(\varepsilon^{\mu\alpha\beta\gamma} F_{\beta\gamma}\right) = \emptyset$

$\varepsilon^{\mu\alpha\beta\gamma}\, \partial_\alpha F_{\beta\gamma} = \emptyset$

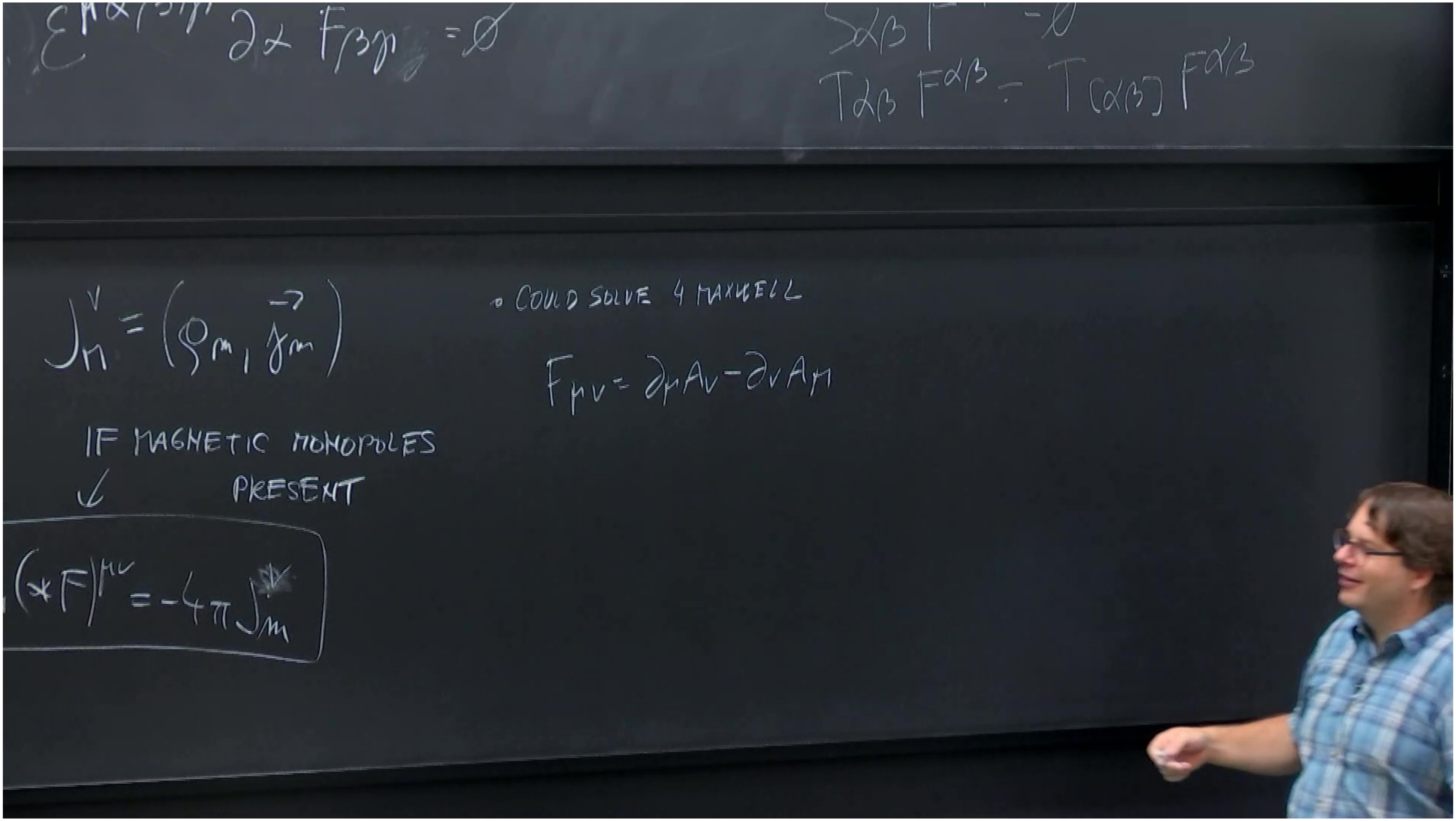
2ND SET $\boxed{\partial_\alpha (*F)^{\alpha\beta} = 0}$

$$J_M^\nu = (\rho_m, \vec{j}_m)$$

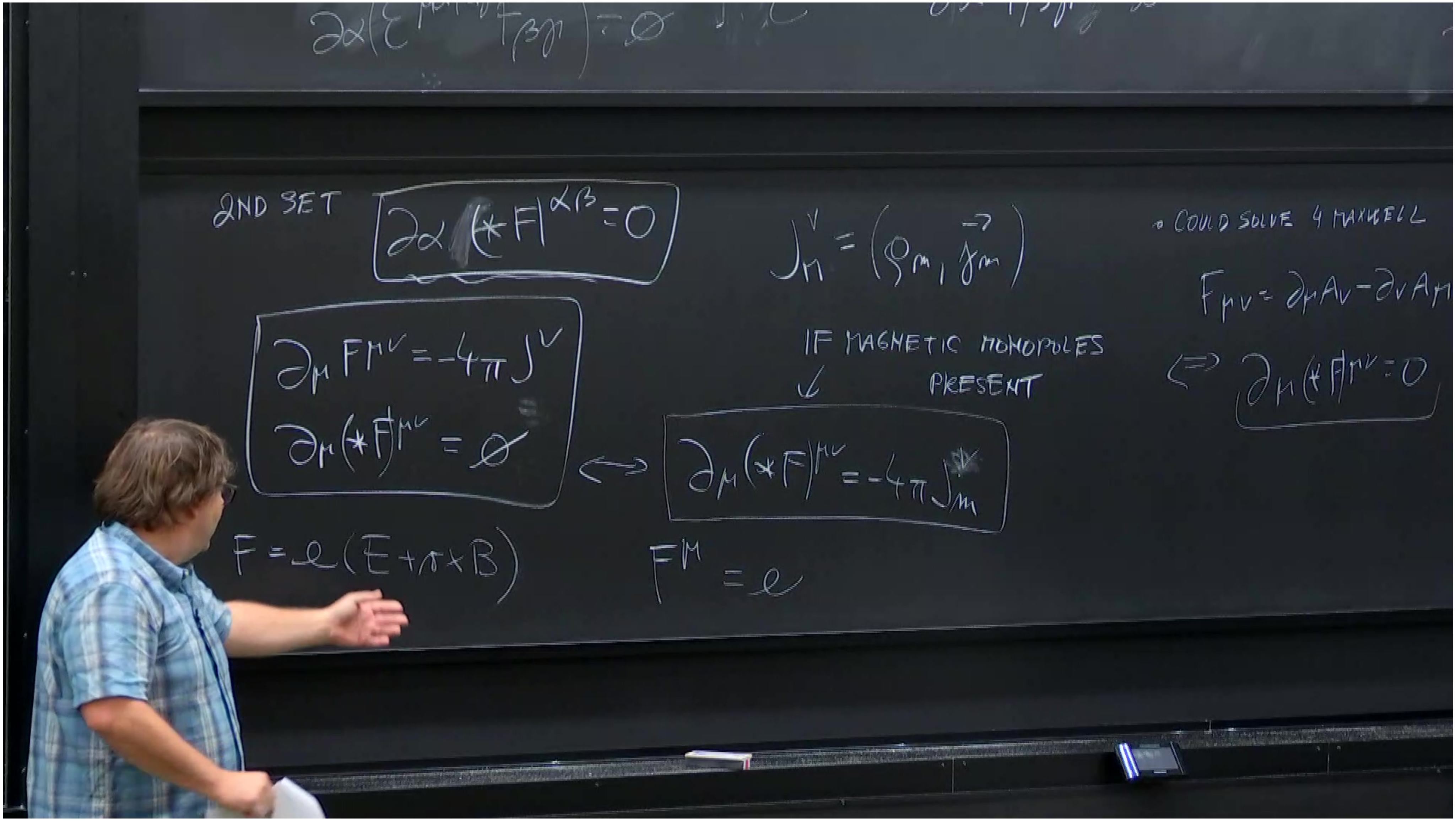
$$\boxed{\begin{aligned} \partial_\mu F^{\mu\nu} &= -4\pi J^\nu \\ \partial_\mu (*F)^{\mu\nu} &= \emptyset \end{aligned}}$$

IF MAGNETIC MONOPOLES PRESENT

$$\Longleftrightarrow \quad \boxed{\partial_\mu (*F)^{\mu\nu} = -4\pi J_m^\nu}$$



$\varepsilon^{\mu\alpha\beta\gamma} \partial_\alpha F_{\beta\gamma} = 0$
$T_{\alpha\beta} F^{\alpha\beta} = T_{[\alpha\beta]} F^{\alpha\beta}$
$J_m^\nu = (\rho_m, \vec{j}_m)$
COULD SOLVE 4 MAXWELL
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
IF MAGNETIC MONOPOLES
PRESENT
$(*F)^{\mu\nu} = -4\pi J_m$


2ND SET
$\partial_\alpha (*F)^{\alpha\beta} = 0$
$J_m^\nu = (\rho_m, \vec{j}_m)$
• COULD SOLVE 4 MAXWELL
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\partial_\mu F^{\mu\nu} = -4\pi J^\nu$
$\partial_\mu (*F)^{\mu\nu} = \emptyset$
IF MAGNETIC MONOPOLES PRESENT
$\partial_\mu (*F)^{\mu\nu} = -4\pi J_m^\nu$
$\Longleftrightarrow \partial_\mu (*F)^{\mu\nu} = 0$
$F = e(E + v \times B)$
$F^M = e$

2ND SET $\boxed{\partial_\alpha (*F)^{\alpha\beta} = 0}$

$J_m^\nu = (\rho_m, \vec{j}_m)$

• COULD SOLVE 4 MAXWELL

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\Longleftrightarrow \boxed{\partial_\mu (*F)^{\mu\nu} = 0}$

$$\boxed{\begin{aligned} \partial_\mu F^{\mu\nu} &= -4\pi J^\nu \\ \partial_\mu (*F)^{\mu\nu} &= \cancel{0} \end{aligned}}$$

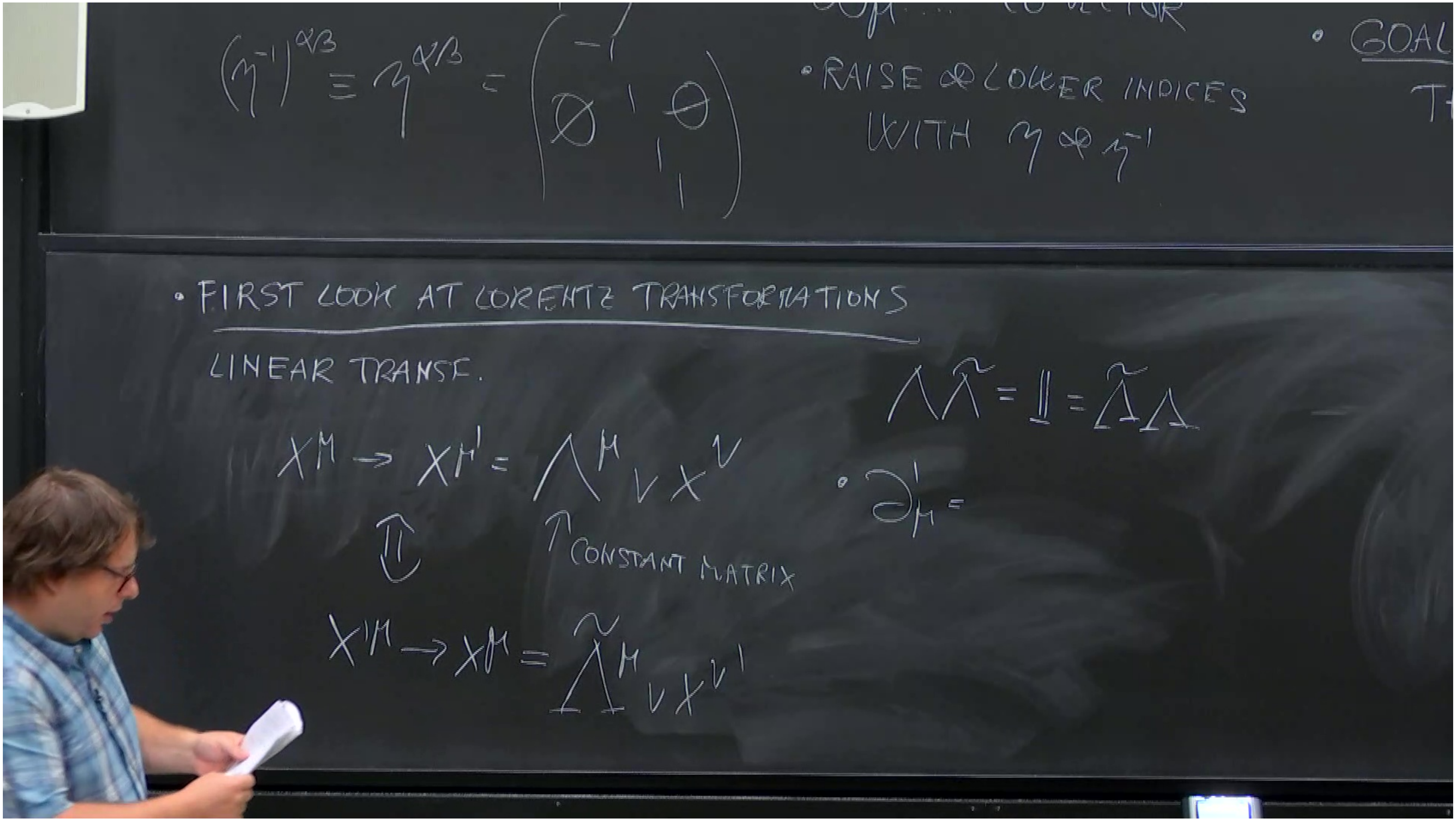
IF MAGNETIC MONOPOLES PRESENT

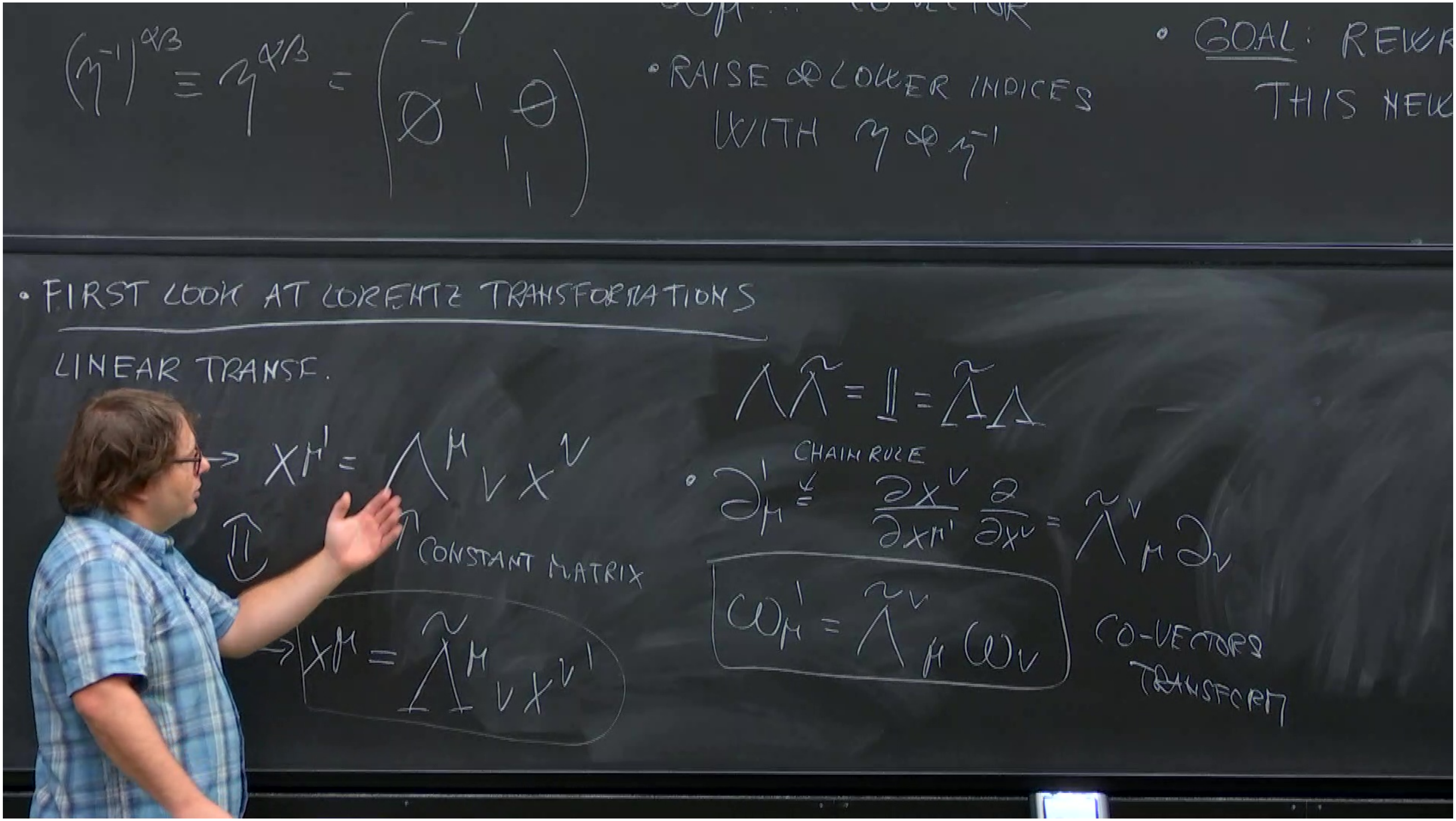
$$\Longleftrightarrow \boxed{\partial_\mu (*F)^{\mu\nu} = -4\pi J_m^\nu}$$

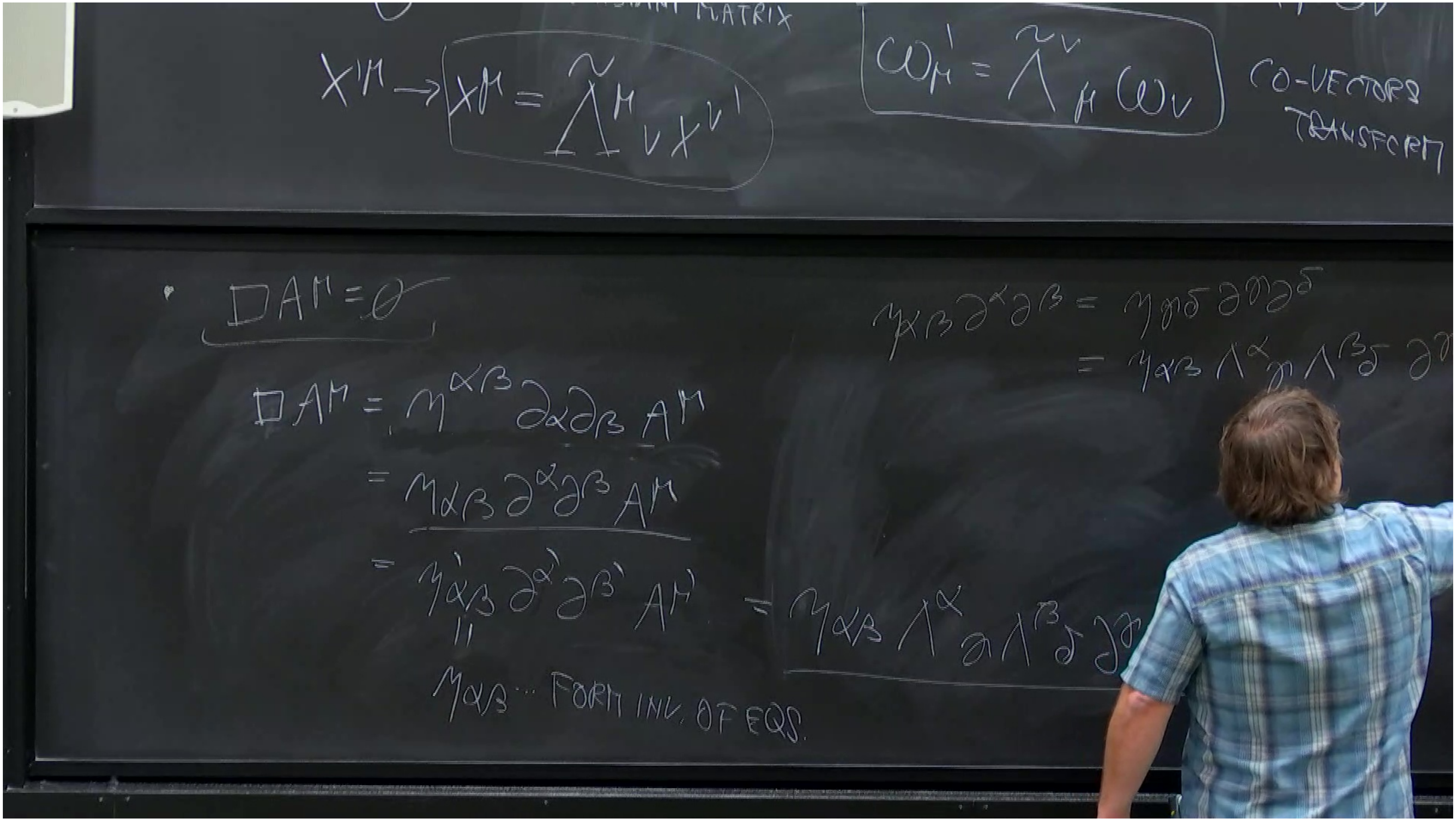
• $F = e(E + v \times B)$

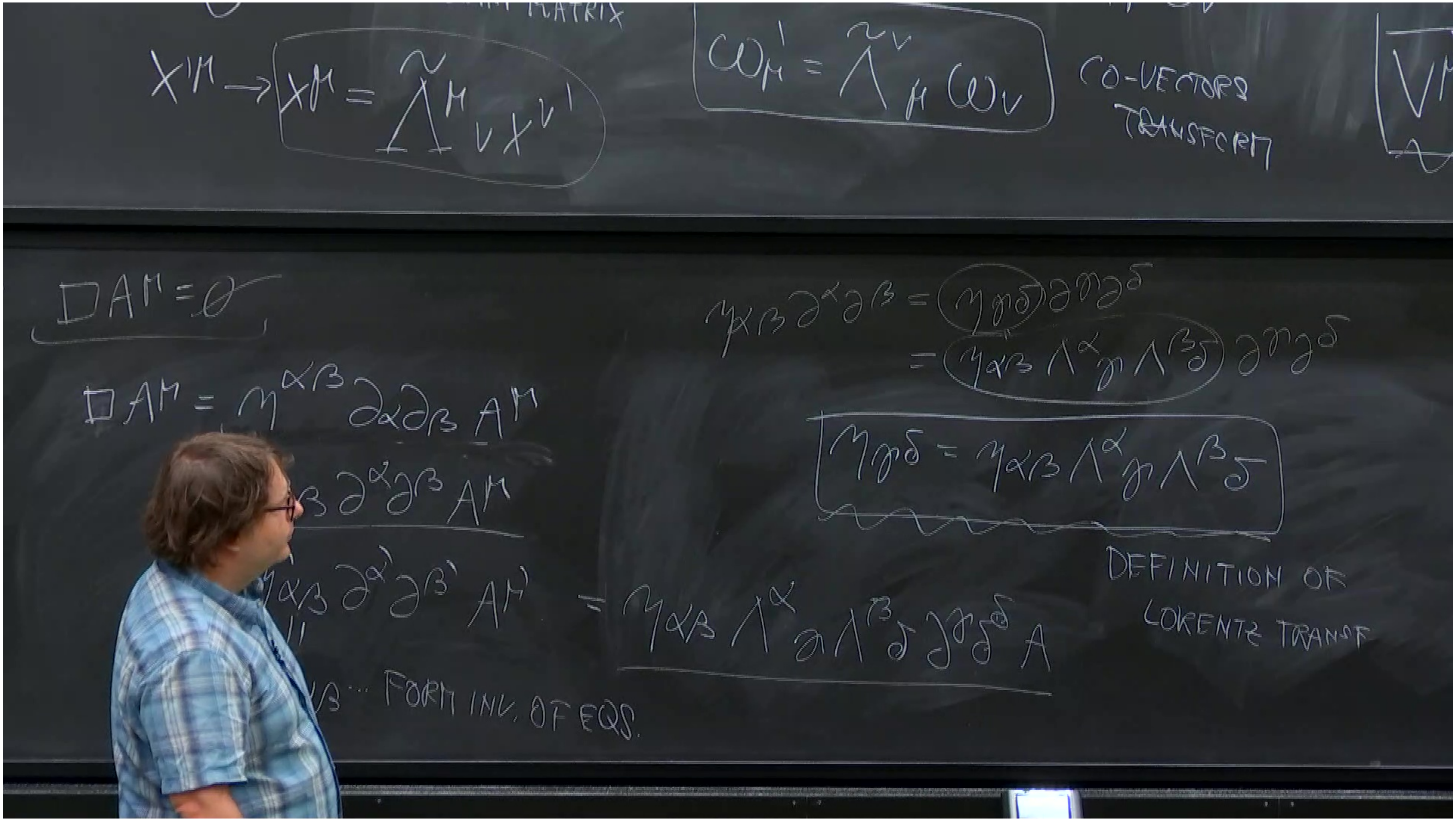
$$\boxed{F^\mu = e\, F^{\mu\nu} u_\nu}$$

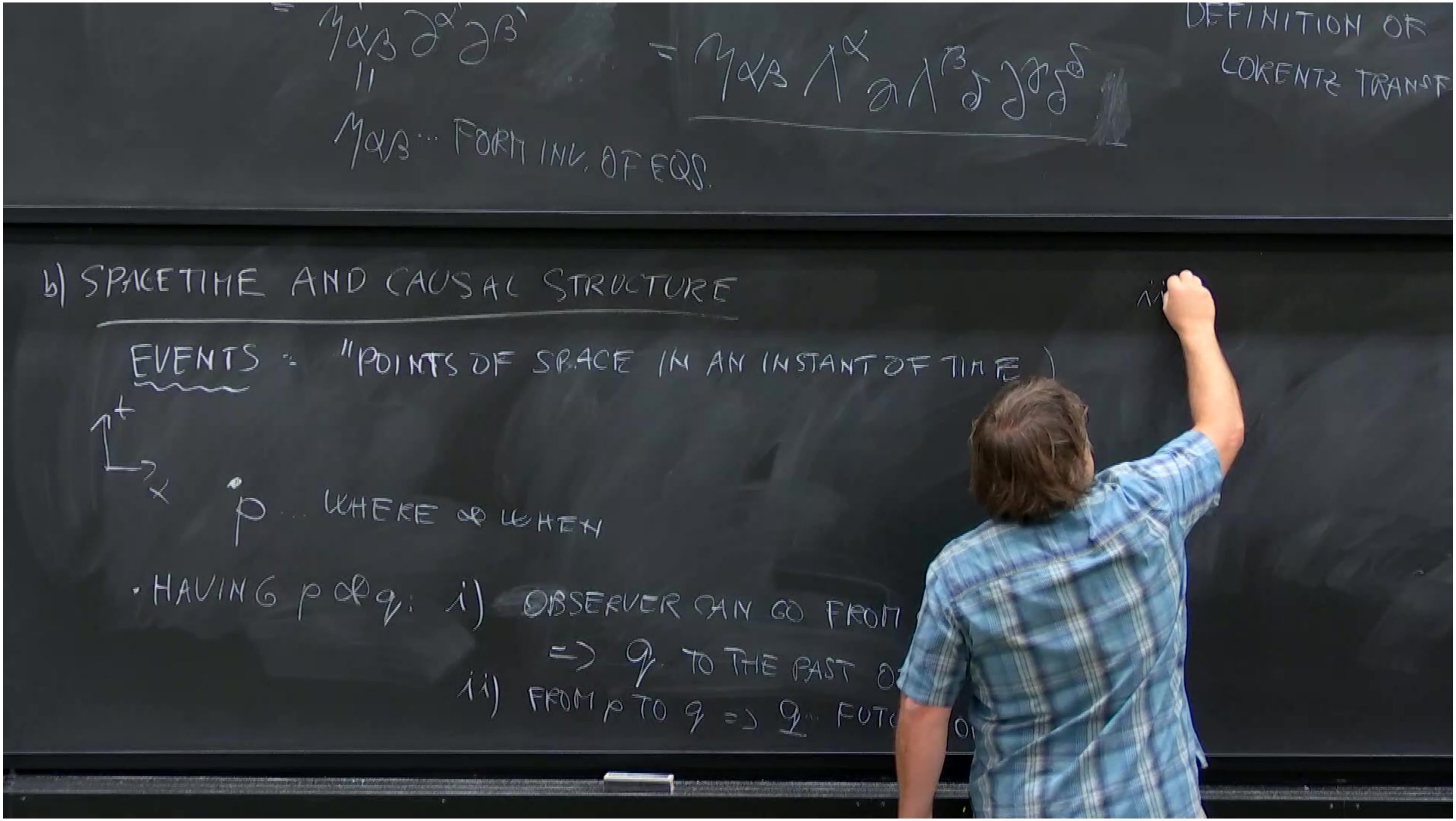
LORENTZ FORCE


$(\eta^{-1})^{\alpha\beta} \equiv \eta^{\alpha\beta} =$
• RAISE & LOWER INDICES WITH $\eta$ & $\eta^{-1}$
• GOAL
• FIRST LOOK AT LORENTZ TRANSFORMATIONS
LINEAR TRANSF.
$X^\mu \to X^{\mu'} = \Lambda^\mu{}_\nu X^\nu$
CONSTANT MATRIX
$X'^\mu \to X^\mu = \tilde{\Lambda}^\mu{}_\nu X^{\nu'}$
$\Lambda\tilde{\Lambda} = \mathbb{1} = \tilde{\Lambda}\Lambda$
• $\partial'_\mu =$


$(\eta^{-1})^{\alpha\beta} \equiv \eta^{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$
• RAISE & LOWER INDICES WITH $\eta$ & $\eta^{-1}$
• GOAL: REWR
THIS NEW
• FIRST LOOK AT LORENTZ TRANSFORMATIONS
LINEAR TRANSF.
$x^{\mu'} = \Lambda^{\mu}{}_{\nu} x^{\nu}$
CONSTANT MATRIX
$x^{\mu} = \tilde{\Lambda}^{\mu}{}_{\nu} x^{\nu'}$
$\Lambda \tilde{\Lambda} = \mathbb{1} = \tilde{\Lambda} \Lambda$
CHAIN RULE
• $\partial'_{\mu} = \frac{\partial x^{\nu}}{\partial x^{\mu'}} \frac{\partial}{\partial x^{\nu}} = \tilde{\Lambda}^{\nu}{}_{\mu} \partial_{\nu}$
$\omega'_{\mu} = \tilde{\Lambda}^{\nu}{}_{\mu} \omega_{\nu}$
CO-VECTORS TRANSFORM


MATRIX
X'^μ → X^μ = Λ̃^μ_ν X^ν'
ω'_μ = Λ̃^ν_μ ω_ν
CO-VECTORS TRANSFORM
□A^μ = 0
□A^μ = η^{αβ} ∂_α ∂_β A^μ
= η_{αβ} ∂^α ∂^β A^μ
= η'_{αβ} ∂^α' ∂^β' A^μ' = η_{αβ} Λ^α_γ Λ^β_δ ∂^γ ∂^δ
η_{αβ} … FORM INV. OF EQS.
η_{αβ} ∂^α ∂^β = η_{γδ} ∂^γ ∂^δ
= η_{αβ} Λ^α_γ Λ^β_δ ∂^γ ∂^δ


MATRIX
$x'^\mu \to x^\mu = \tilde{\Lambda}^\mu{}_\nu x^{\nu'}$
$\omega'_\mu = \tilde{\Lambda}^\nu{}_\mu \omega_\nu$
CO-VECTORS TRANSFORM
$\Box A^\mu = 0$
$\Box A^\mu = \eta^{\alpha\beta} \partial_\alpha \partial_\beta A^\mu$
$\eta_{\alpha\beta} \partial^\alpha \partial^\beta = \eta_{\gamma\delta} \partial^\gamma \partial^\delta$
$= \eta_{\alpha\beta} \Lambda^\alpha{}_\gamma \Lambda^\beta{}_\delta \partial^\gamma \partial^\delta$
$\eta_{\gamma\delta} = \eta_{\alpha\beta} \Lambda^\alpha{}_\gamma \Lambda^\beta{}_\delta$
DEFINITION OF LORENTZ TRANSF
$= \eta_{\alpha\beta} \Lambda^\alpha{}_\gamma \Lambda^\beta{}_\delta \partial^\gamma \partial^\delta A$
FORM INV. OF EQS.


$= \eta_{\alpha\beta}\partial^\alpha\partial^\beta$
$= \eta_{\alpha\beta}\Lambda^\alpha{}_\gamma\Lambda^\beta{}_\delta\partial^\gamma\partial^\delta$
DEFINITION OF
LORENTZ TRANSF
$\eta_{\alpha\beta}$ ... FORM INV. OF EQS.
b) SPACETIME AND CAUSAL STRUCTURE
EVENTS : "POINTS OF SPACE IN AN INSTANT OF TIME
t
x
p ... WHERE & WHEN
· HAVING p & q: i) OBSERVER CAN GO FROM
=> q TO THE PAST OF
ii) FROM p TO q => q ... FUT

$\Lambda^{\alpha}{}_{\gamma}\Lambda^{\beta}{}_{\delta}\,\eta^{\gamma\delta}$

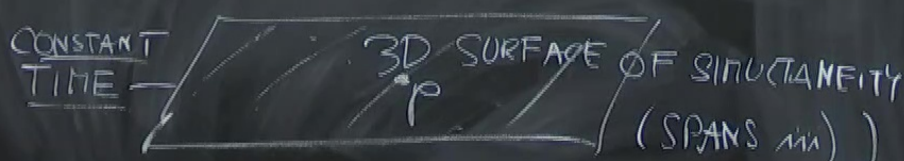
DEFINITION OF
LORENTZ TRANSF

iii) OBSERVER CAN NEVER SEE BOTH $q$ & $p$

NEWTON

TANT OF TIME )

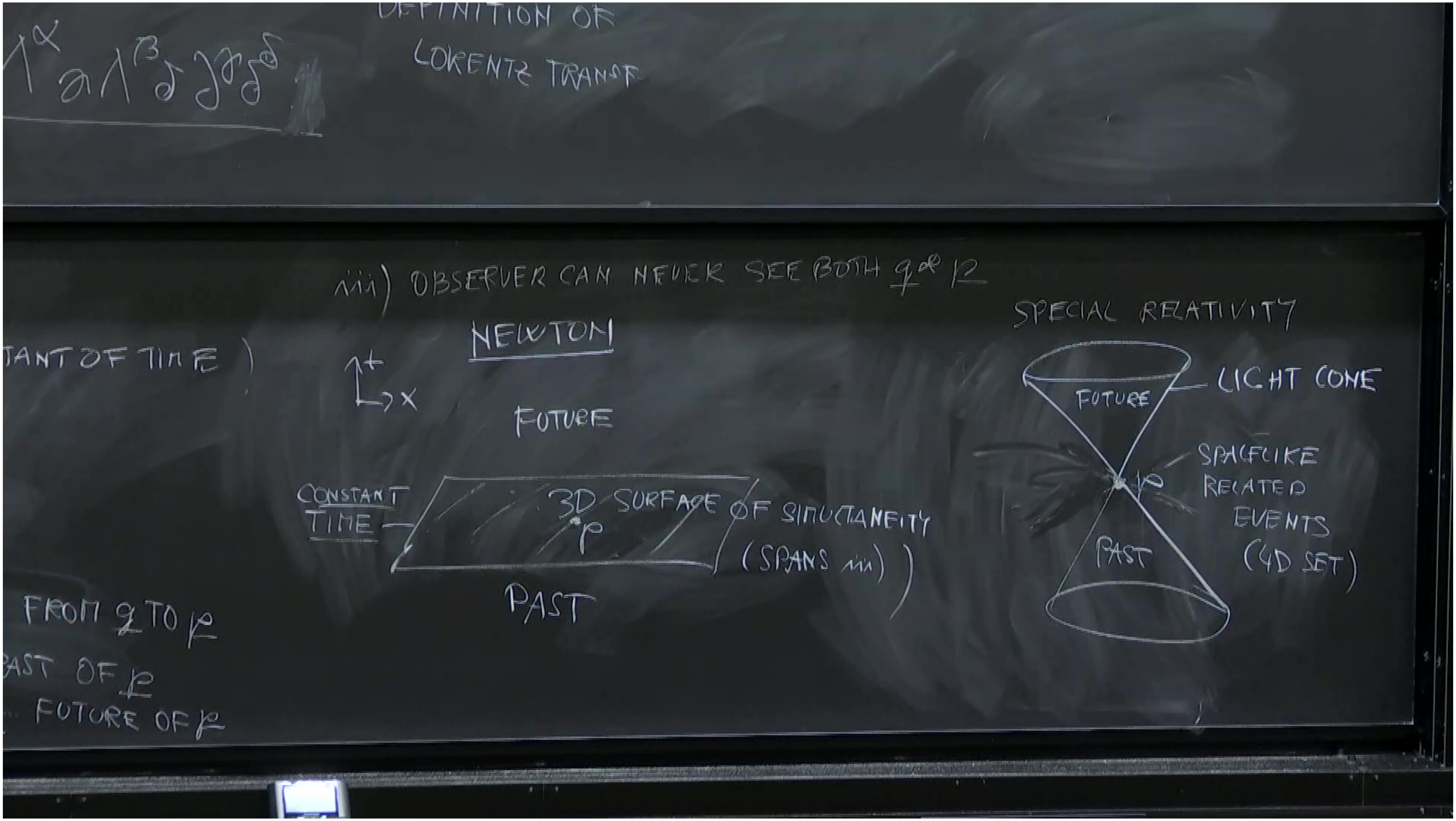
$t$, $x$

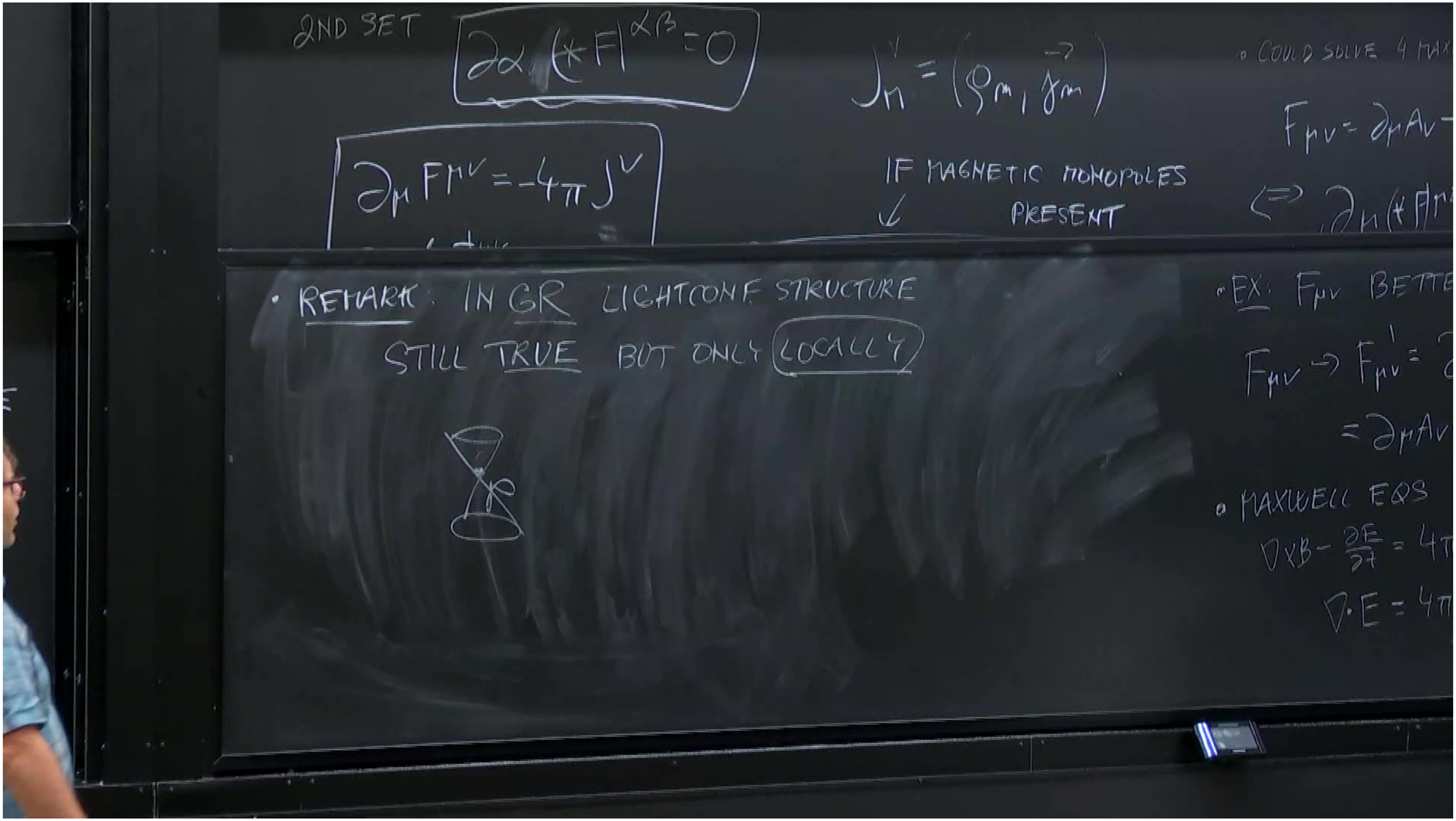


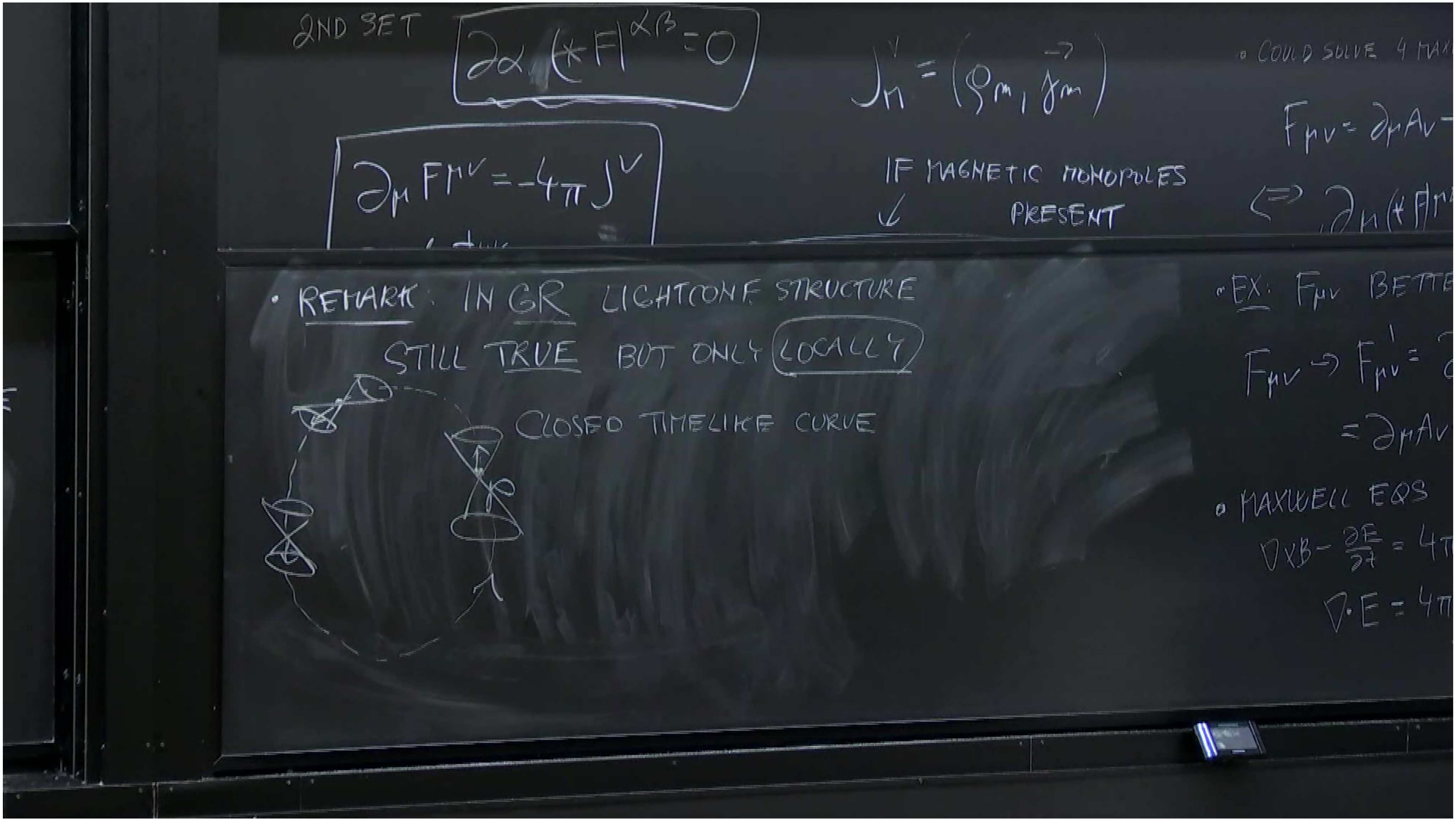

FROM $q$ TO $p$

AST OF $p$

... FUTURE OF $p$


DEFINITION OF
LORENTZ TRANSF
iii) OBSERVER CAN NEVER SEE BOTH q & p
NEWTON
TANT OF TIME )
FUTURE
CONSTANT TIME
3D SURFACE OF SIMULTANEITY
(SPANS iii)
PAST
SPECIAL RELATIVITY
FUTURE
LIGHT CONE
SPACELIKE RELATED EVENTS (4D SET)
PAST
FROM q TO p
PAST OF p
FUTURE OF p


2ND SET
∂α (*F)^αβ = 0
J_m^ν = (ρ_m, j_m)
∂μ F^μν = -4π J^ν
IF MAGNETIC MONOPOLES PRESENT
COULD SOLVE 4 MAX
Fμν = ∂μ Aν
• REMARK: IN GR LIGHTCONE STRUCTURE
STILL TRUE BUT ONLY LOCALLY
• EX: Fμν BETTE
Fμν → F'μν =
= ∂μ Aν
• MAXWELL EQS
∇×B − ∂E/∂t = 4π
∇·E = 4π



2ND SET
∂α (*F)^αβ = 0
J_m^ν = (ρ_m, j_m)
• COULD SOLVE 4
F_μν = ∂_μ A_ν
∂_μ F^μν = −4π J^ν
IF MAGNETIC MONOPOLES PRESENT
• REMARK: IN GR LIGHTCONE STRUCTURE
STILL TRUE BUT ONLY LOCALLY
CLOSED TIMELIKE CURVE
• EX: F_μν
F_μν → F'_μν =
= ∂_μ A_ν
• MAXWELL EQS
∇×B − ∂E/∂t = 4π
∇·E = 4π