Title: Classical Physics - Lecture 220919

Speakers: Meenu Kumari

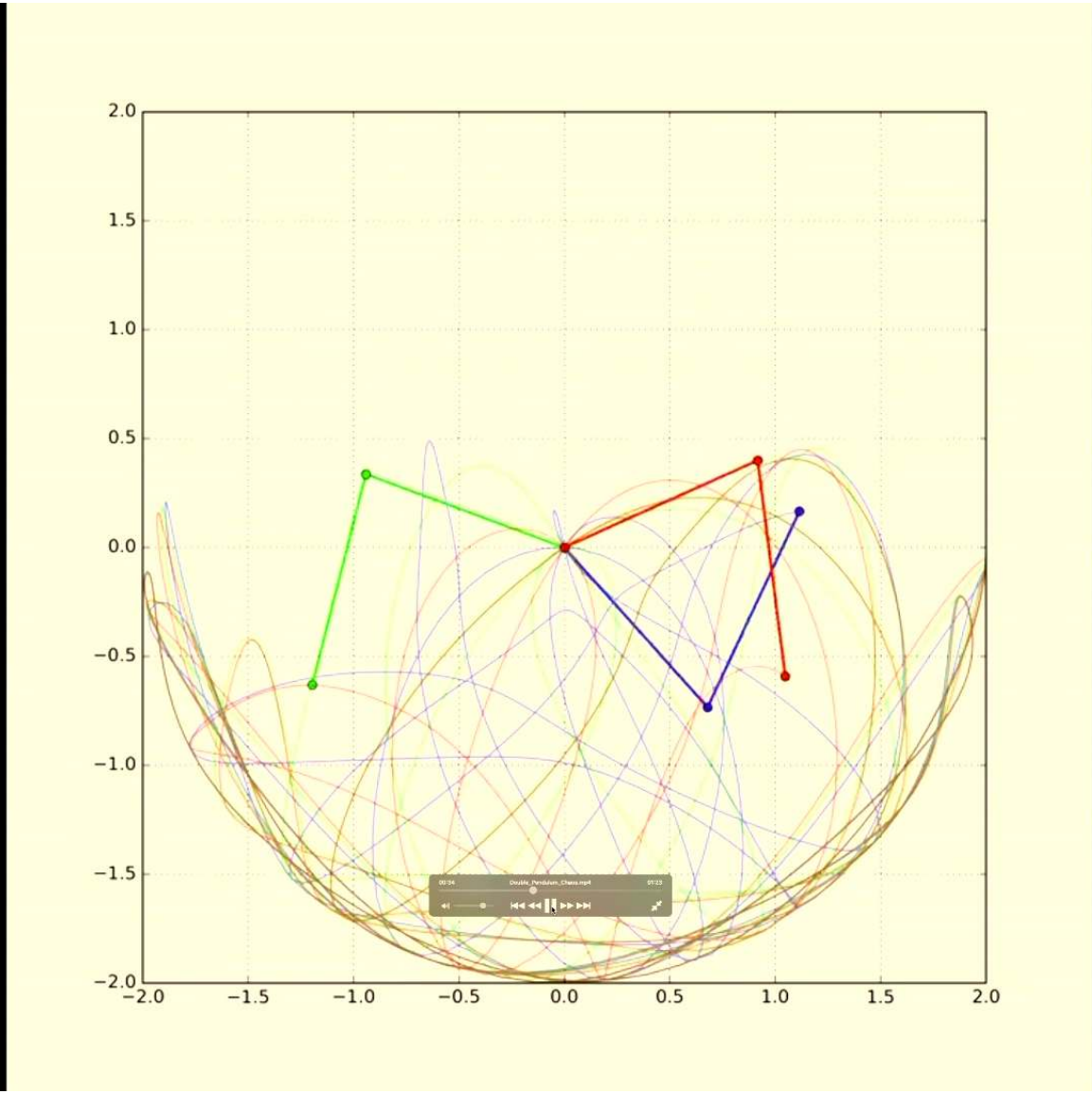
Collection: Classical Physics (2022/2023)

Date: September 19, 2022 - 9:00 AM

URL: https://pirsa.org/22090051

YES
ASK A FRIEND
TRY AGAIN
NO WAY
DEFINITELY
MAYBE

00:12
Pendulum_3D_two_lakes.mov
00:28


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

Classical Chaos

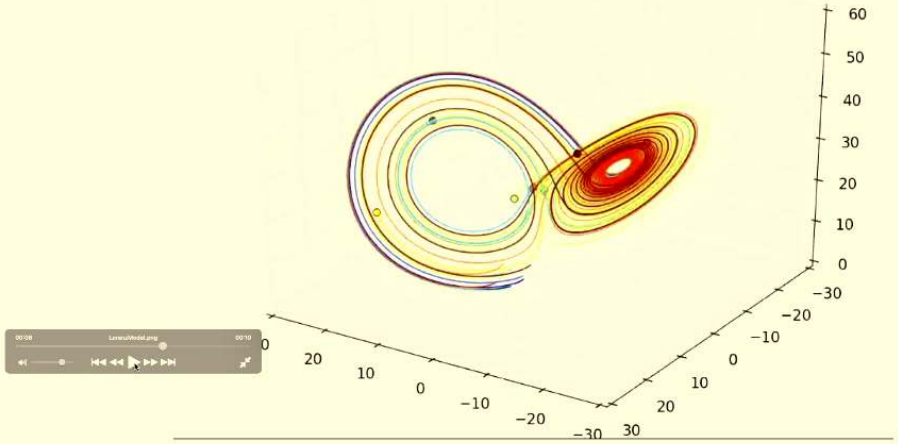
Lorenz oscillator : Classic example of chaotic behaviour

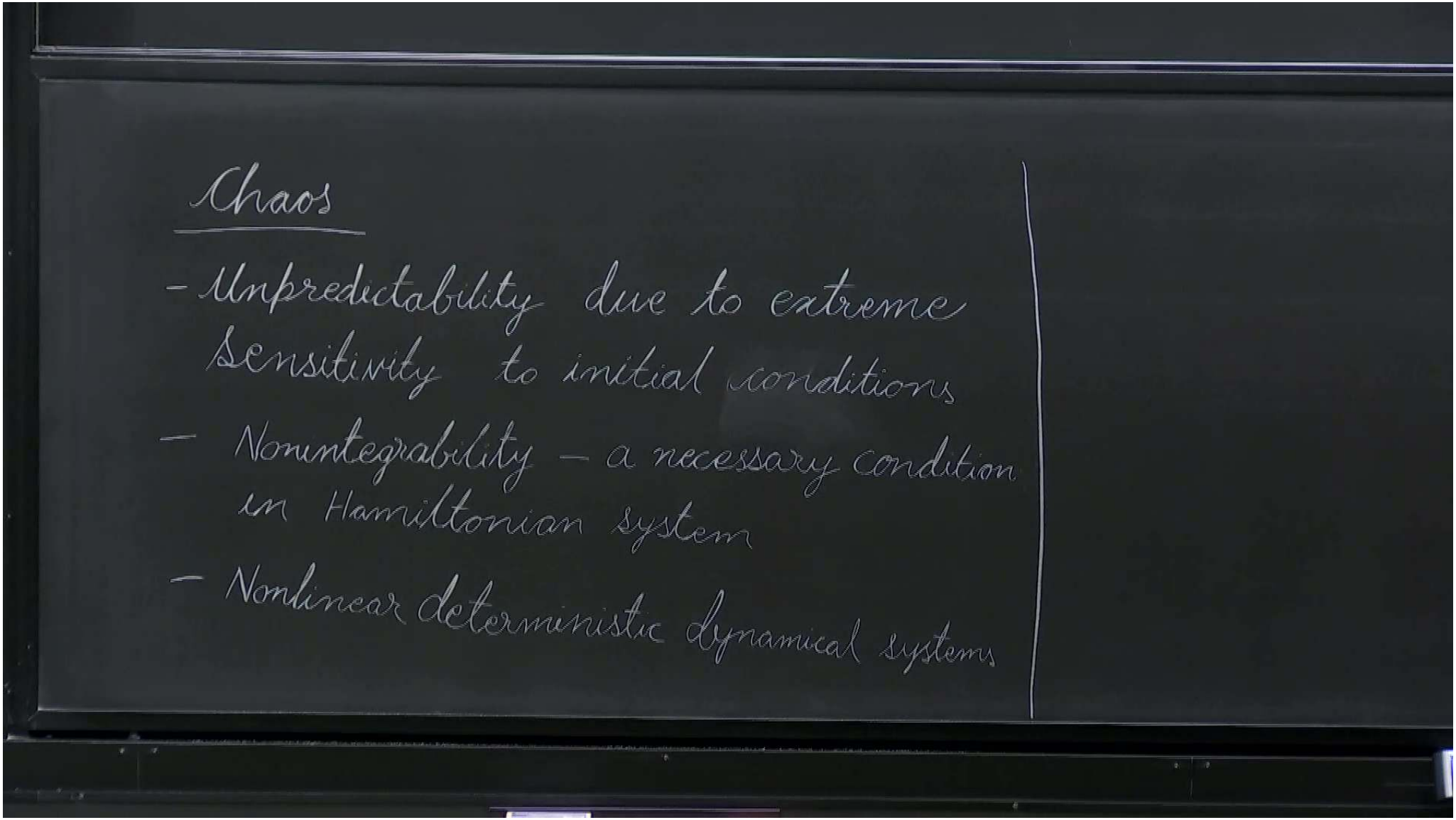
- Non-linear deterministic dynamical systems.

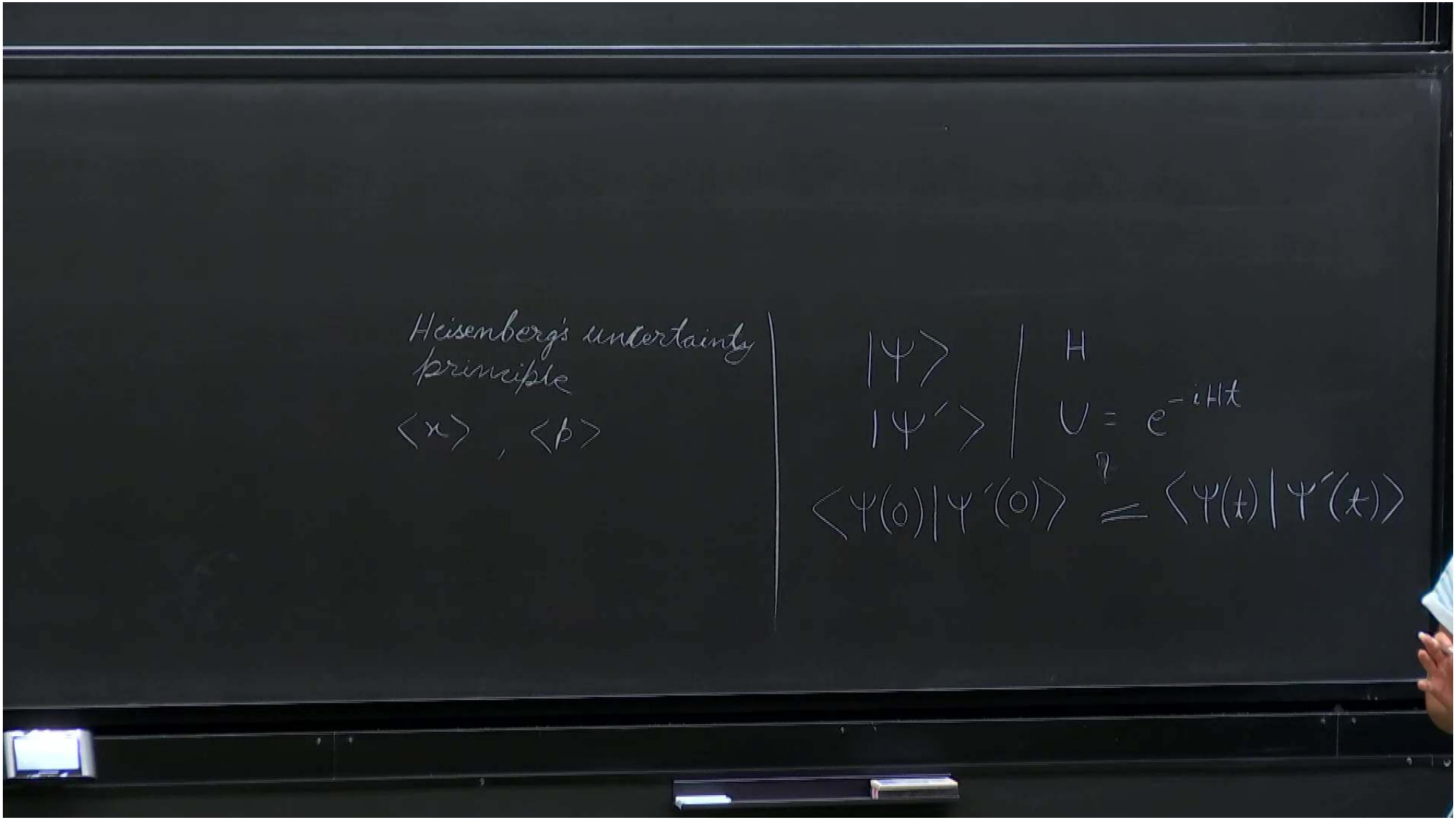
$$\frac{dx}{dt} = \sigma(y - x)$$

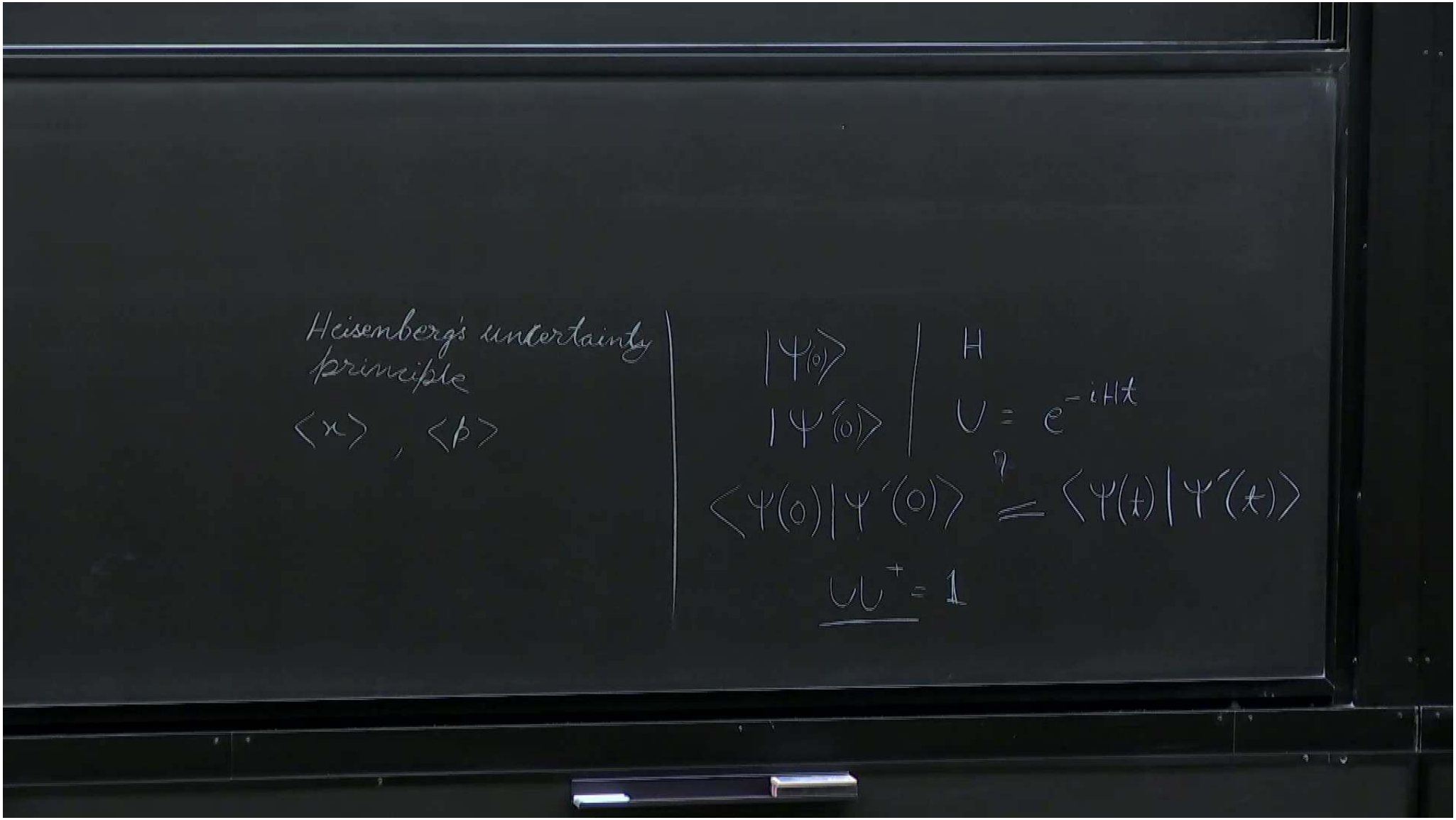
$$\frac{dy}{dt} = x(\rho - z) - y$$

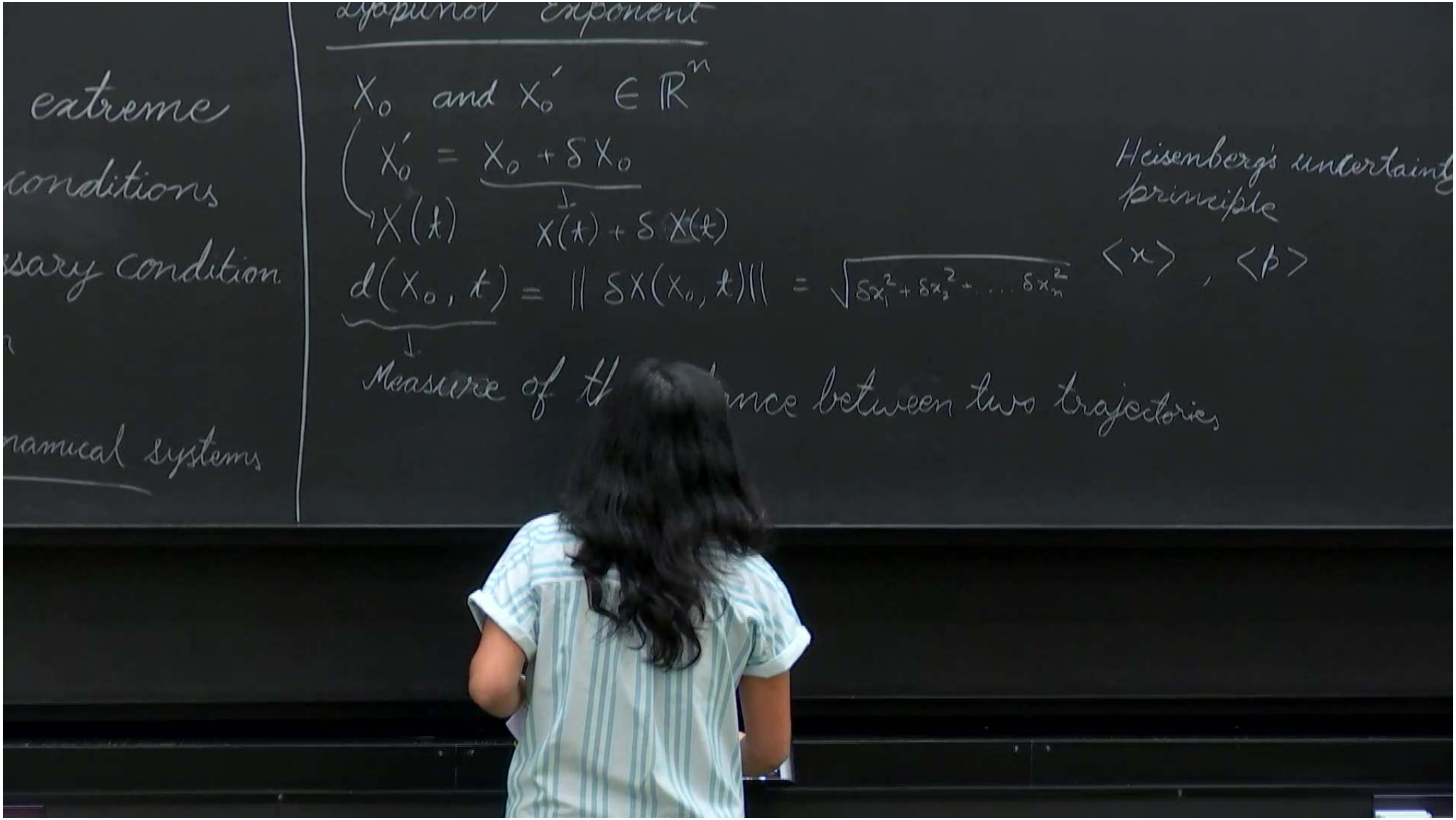
$$\frac{dz}{dt} = xy - \beta z$$




Chaos
- Unpredictability due to extreme sensitivity to initial conditions
- Nonintegrability – a necessary condition in Hamiltonian system
- Nonlinear deterministic dynamical systems


Heisenberg's uncertainty principle
⟨x⟩ , ⟨p⟩
|ψ⟩
|ψ'⟩
H
U = e^{-iHt}
⟨ψ(0)|ψ'(0)⟩ ? = ⟨ψ(t)|ψ'(t)⟩

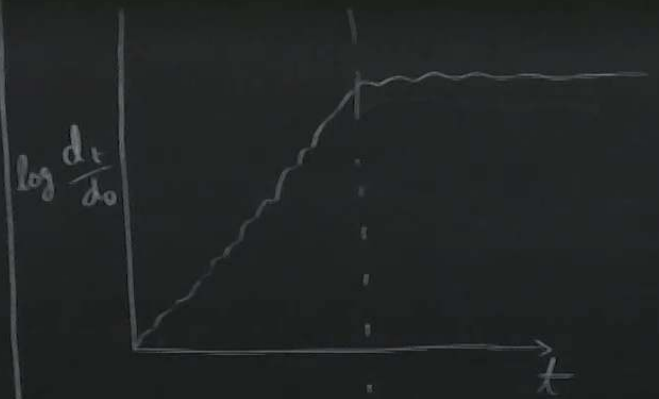

Heisenberg's uncertainty principle
⟨x⟩ , ⟨p⟩
|ψ(0)⟩ | H
|ψ'(0)⟩ | U = e^{-iHt}
⟨ψ(0)|ψ'(0)⟩ ? = ⟨ψ(t)|ψ'(t)⟩
UU† = 𝟙


extreme
conditions
ssary condition
namical systems
Lyapunov exponent
X_0 and $X_0' \in \mathbb{R}^n$
$X_0' = X_0 + \delta X_0$
$X(t)$ $X(t) + \delta X(t)$
$d(X_0, t) = \| \delta X(X_0, t) \| = \sqrt{\delta x_1^2 + \delta x_2^2 + \ldots \delta x_n^2}$
Measure of th nce between two trajectories
Heisenberg's uncertainty principle
$\langle x \rangle$, $\langle p \rangle$

$$\lambda(X_0, \delta X) = \lim_{t\to\infty} \lim_{d(X_0,0)\to 0} \frac{1}{t} \log\left(\frac{d(X_0, t)}{d(X_0, 0)}\right)$$

$\lambda > 0$ in chaotic systems

$$\log\left(\frac{d(x_0, t)}{d(x_0, 0)}\right)$$



$$\log \frac{d_t}{d_0} = \lambda t$$

$$\Rightarrow d_t = e^{\lambda t} d_0$$

$\vec{F}$, $\vec{\nabla} \cdot \vec{F} < 0$ for dissipative system

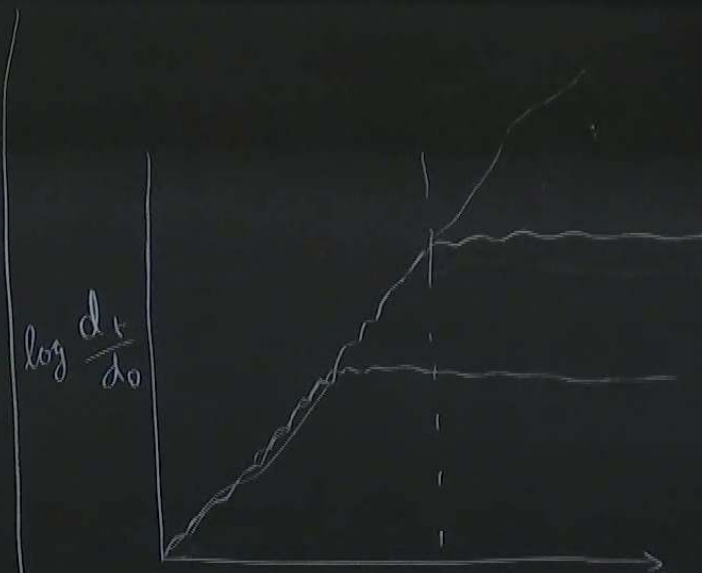
$\vec{\nabla} \cdot \vec{F} = 0$ for Hamiltonian systems

t

$$\lambda(x) = \lim_{t\to\infty} \lim_{d(x_0,0)\to 0} \frac{1}{t} \log\left(\frac{d(x_0,t)}{d(x_0,0)}\right)$$

in chaotic systems

$x_0 \quad x_0'$



$$\log \frac{d_t}{d_0} = \lambda t$$

$$\Rightarrow d_t = e^{\lambda t} d_0$$

(p_i, q_i)

DOF — No. of q_i's

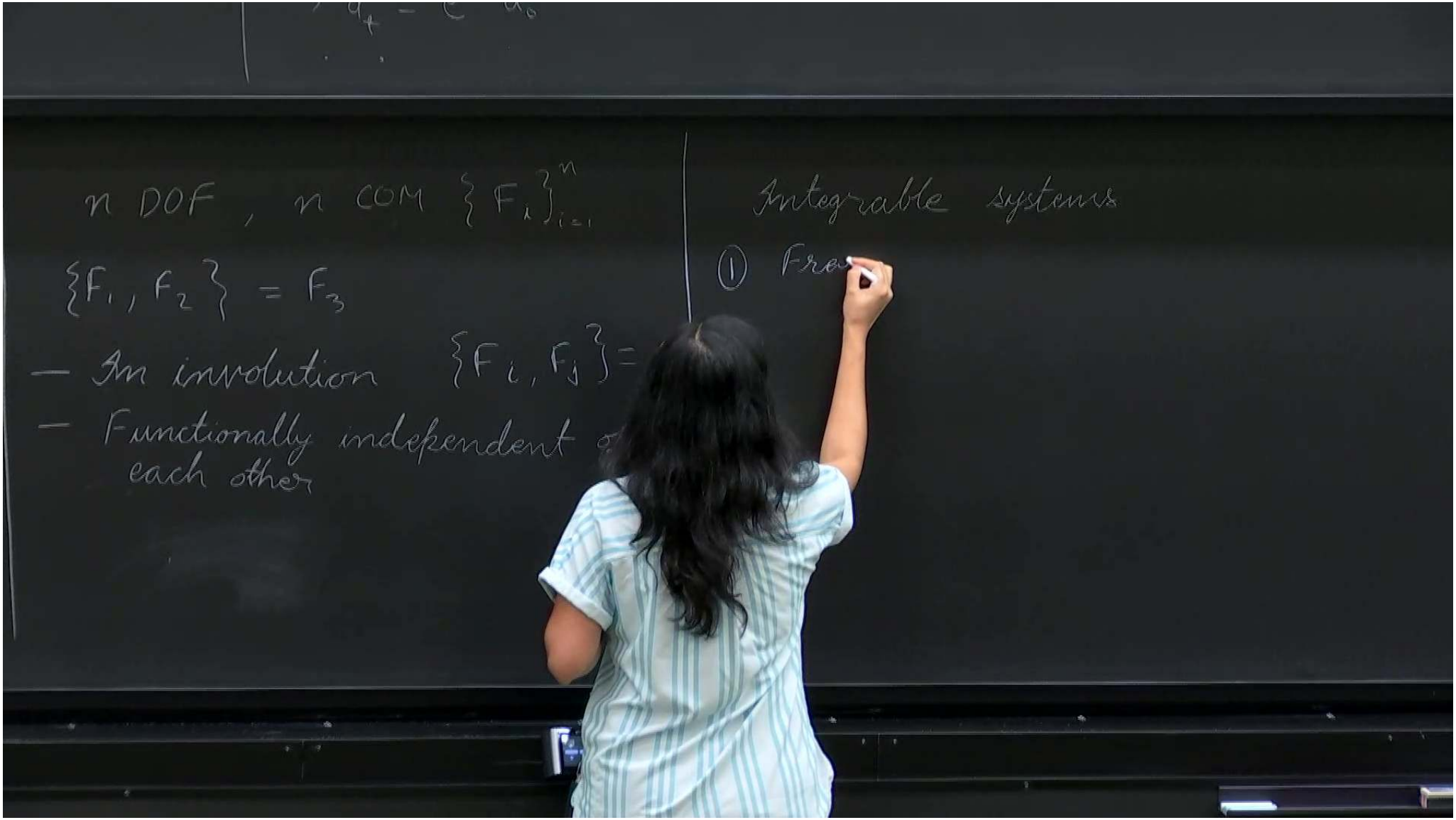
Dimension of phase space = 2 DOF

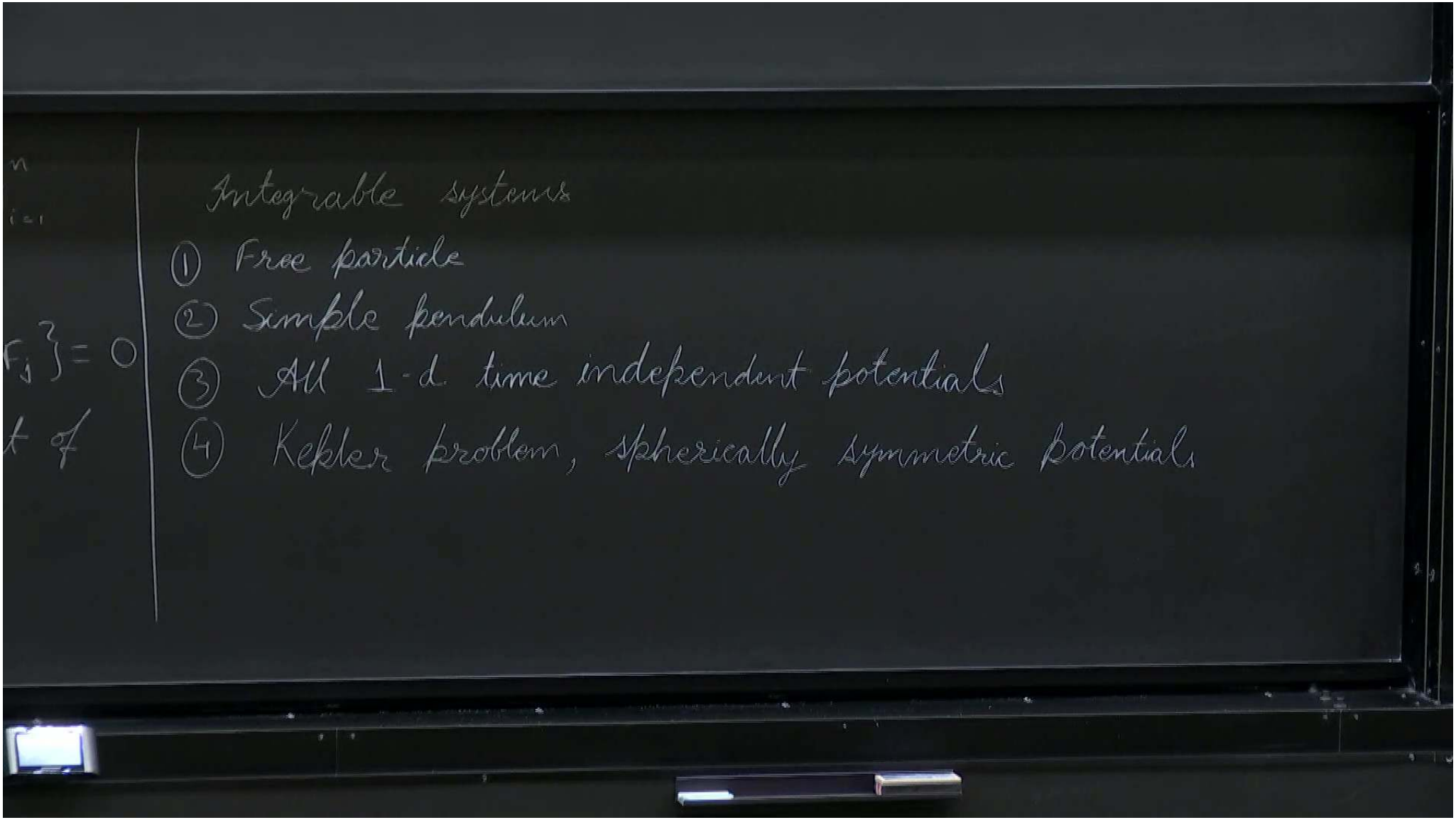
An integrable systems

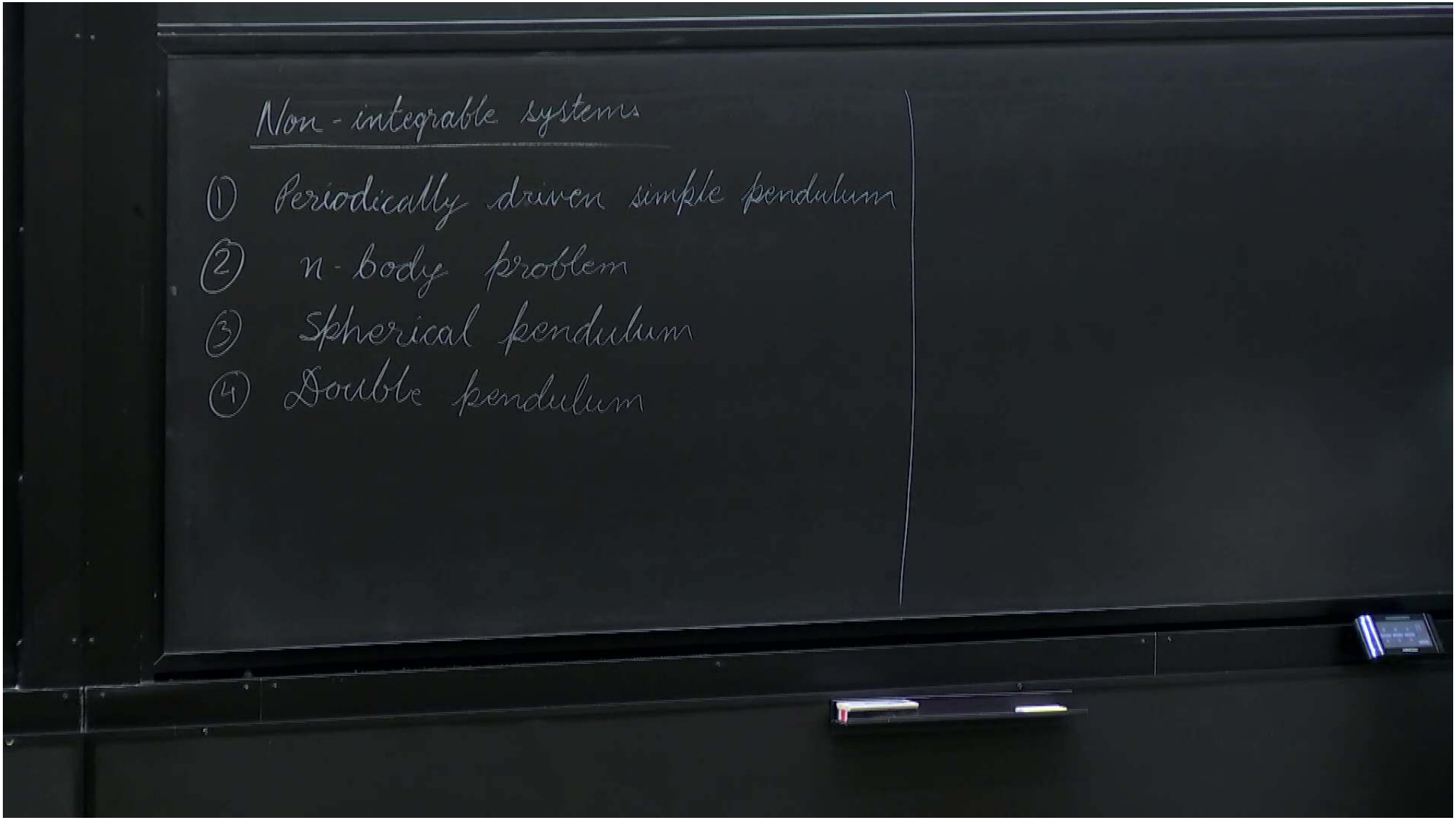
— No. of DOF = No. of constants of motion (COM)

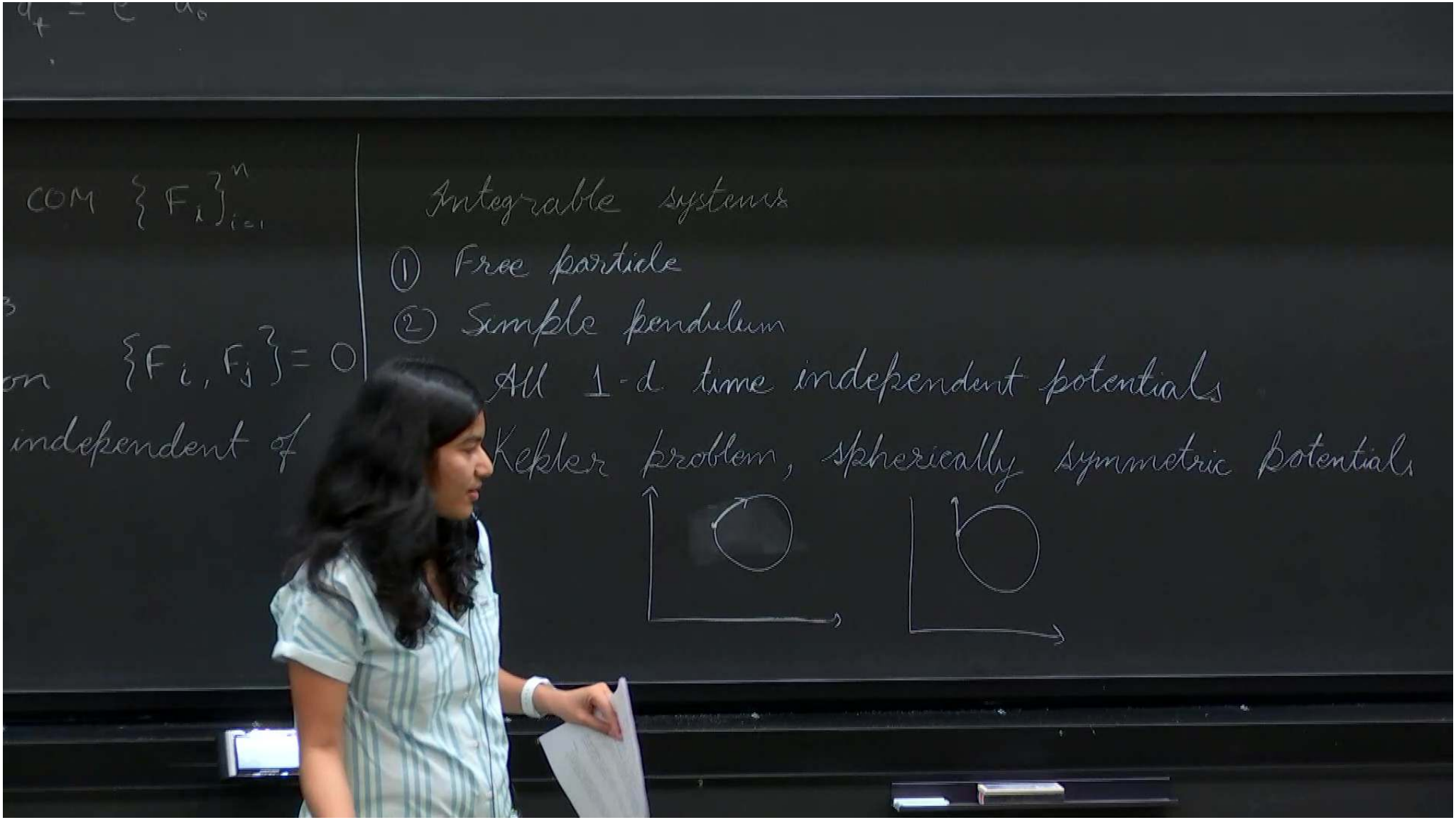
$\{A, H\} = 0$

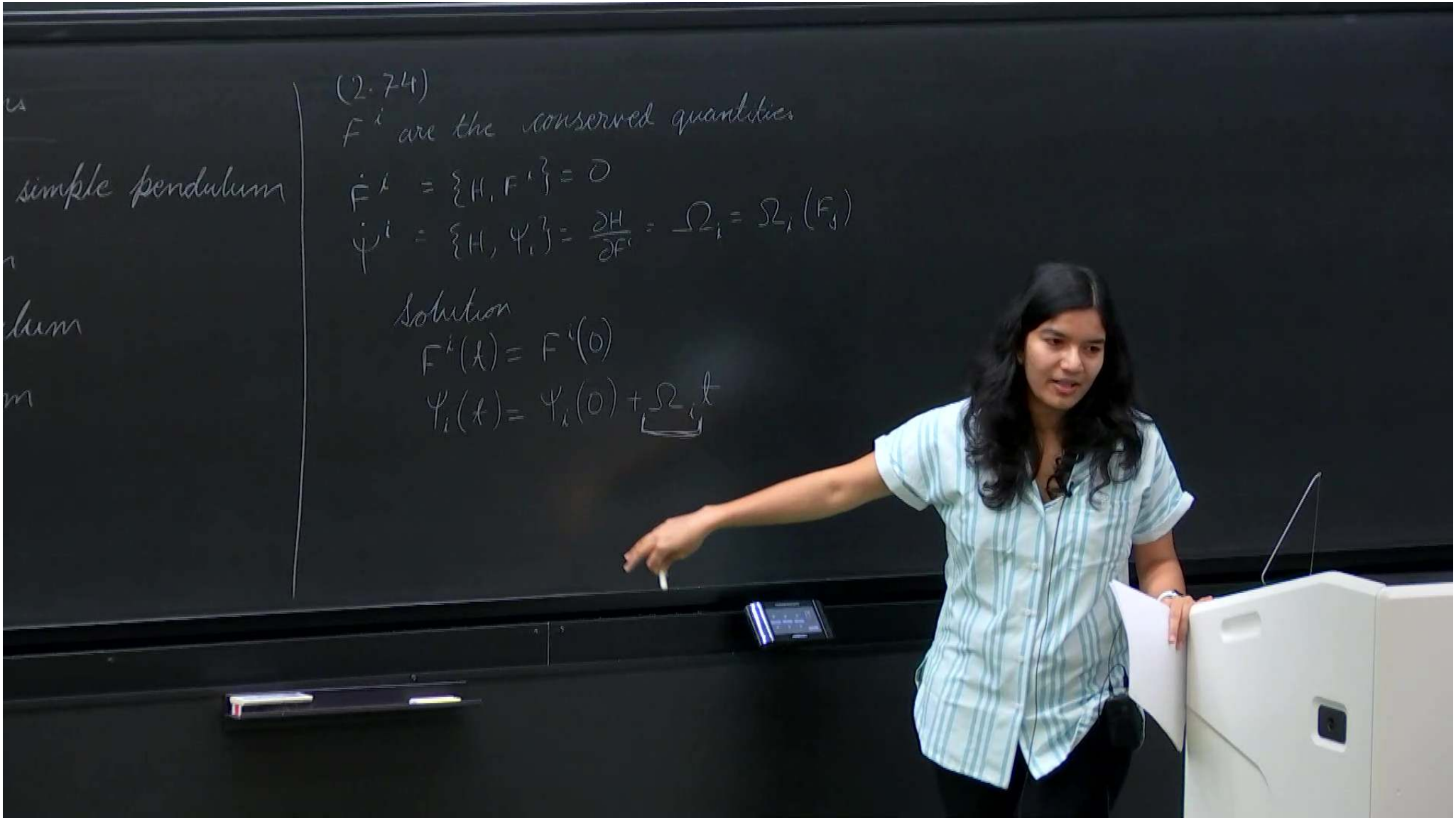
n DOF, n COM $\{F_i\}$

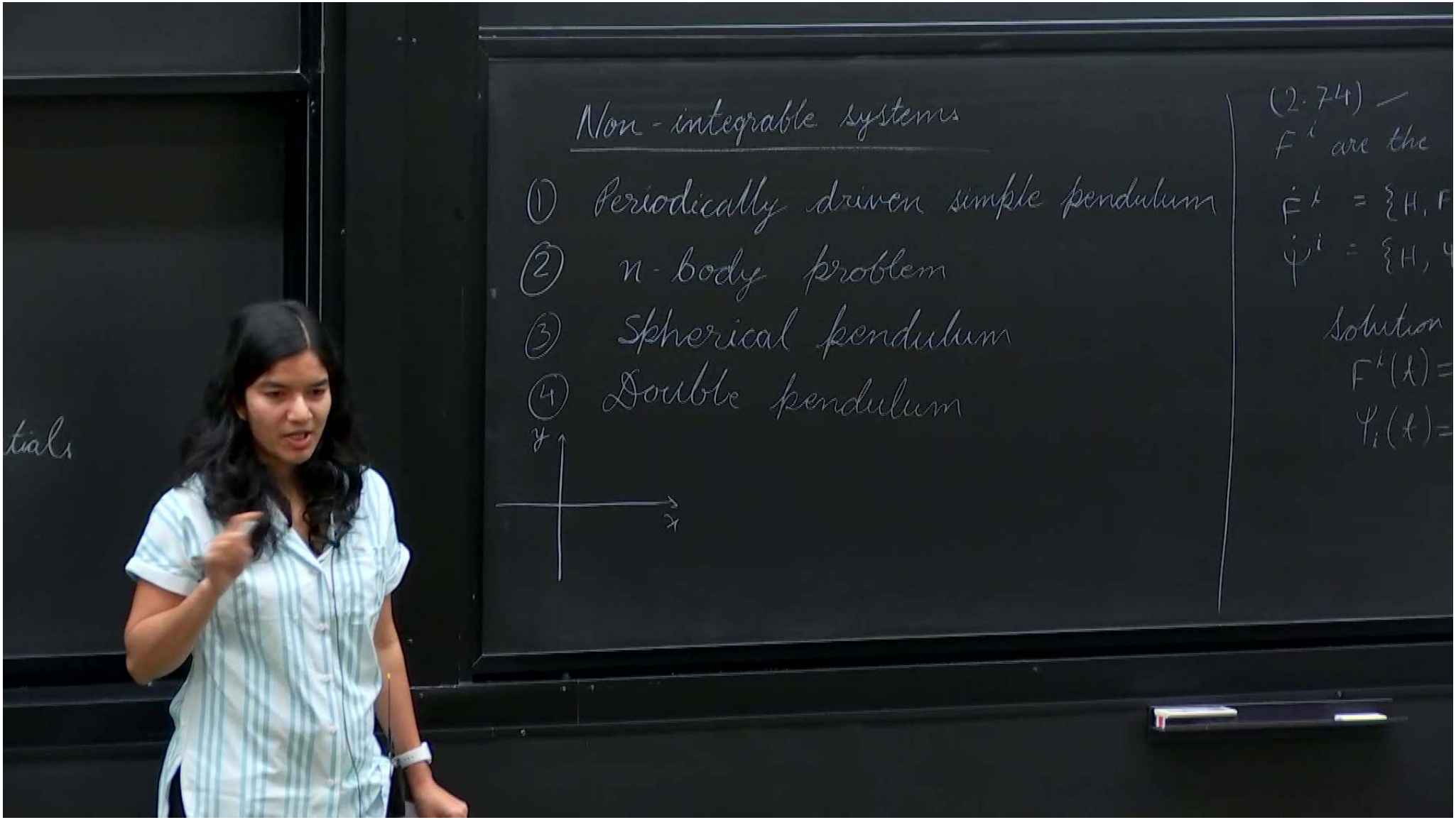

n DOF, n COM $\{F_i\}_{i=1}^{n}$
$\{F_1, F_2\} = F_3$
— An involution $\{F_i, F_j\} =$
— Functionally independent of
each other
Integrable systems
① Fro

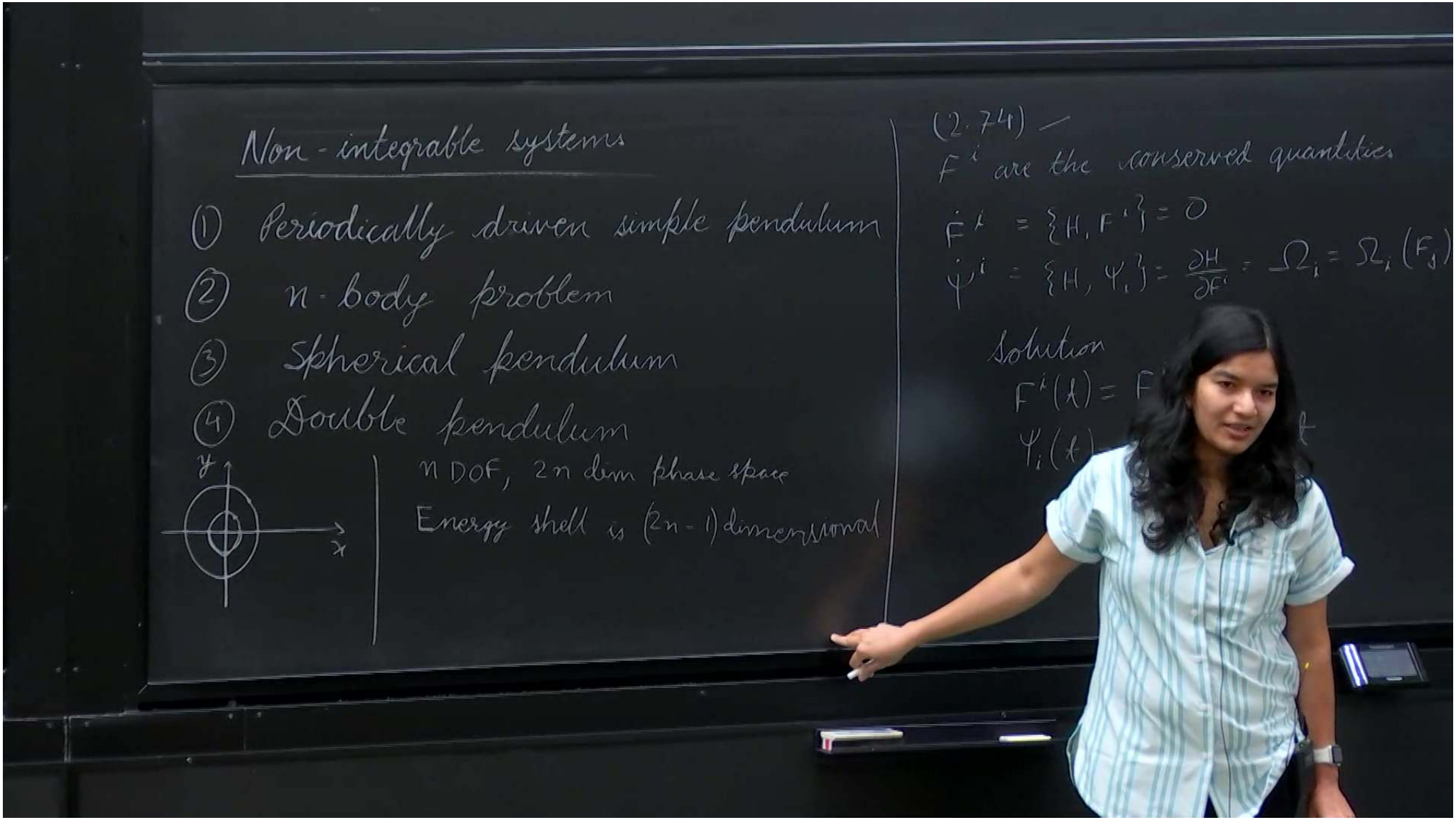

Integrable systems
(1) Free particle
(2) Simple pendulum
(3) All 1-d time independent potentials
(4) Kepler problem, spherically symmetric potentials

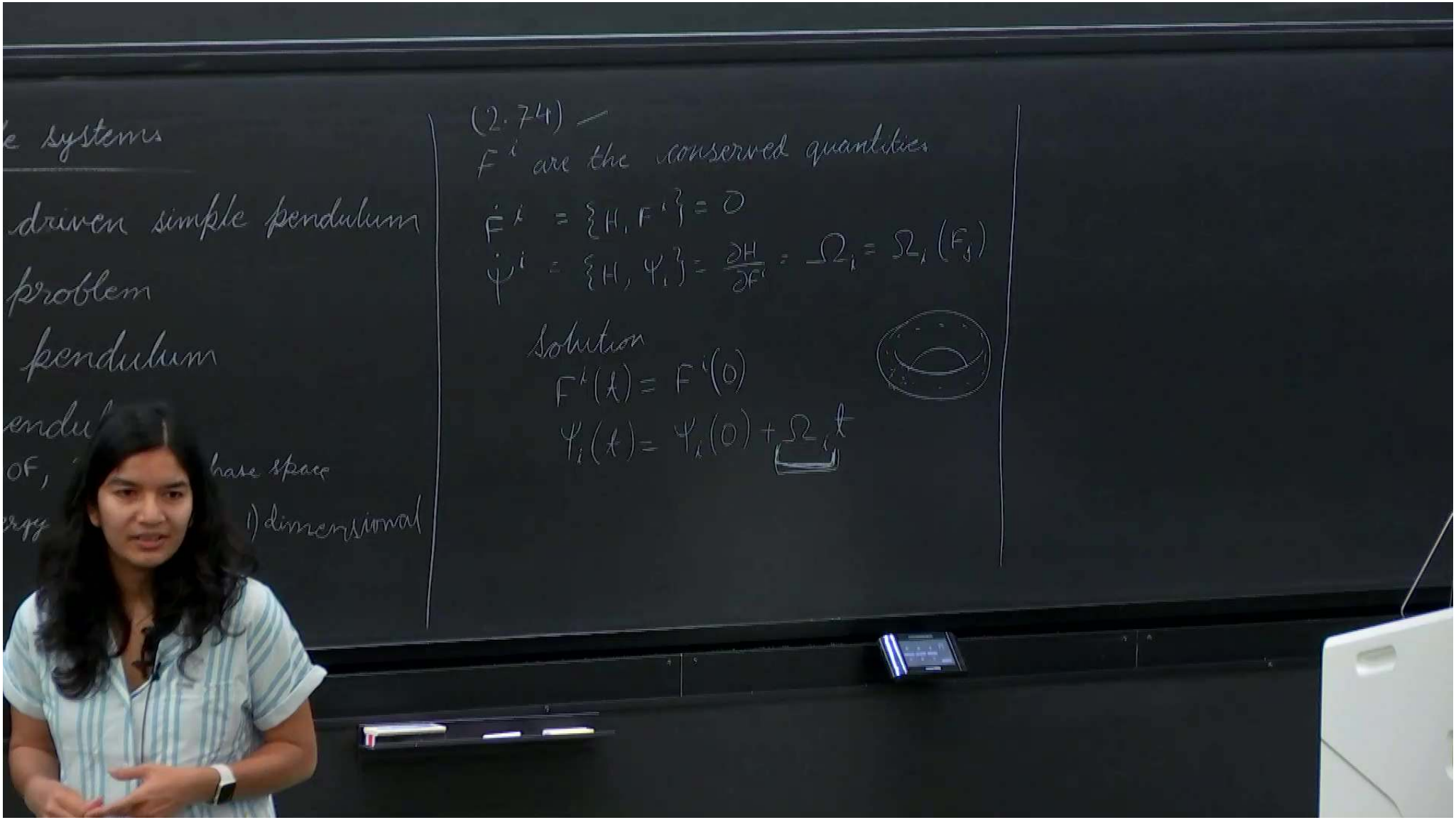

Non - integrable systems
(1) Periodically driven simple pendulum
(2) n - body problem
(3) Spherical pendulum
(4) Double pendulum


COM $\{F_i\}_{i=1}^{n}$
$\{F_i, F_j\} = 0$
independent of
Integrable systems
① Free particle
② Simple pendulum
All 1-d time independent potentials
Kepler problem, spherically symmetric potentials


simple pendulum
(2.74)
F^i are the conserved quantities
$\dot{F}^i = \{H, F^i\} = 0$
$\dot{\psi}^i = \{H, \psi_i\} = \frac{\partial H}{\partial F^i} = \Omega_i = \Omega_i(F_j)$
Solution
$F^i(t) = F^i(0)$
$\psi_i(t) = \psi_i(0) + \Omega_i t$


Non - integrable systems
(1) Periodically driven simple pendulum
(2) n - body problem
(3) Spherical pendulum
(4) Double pendulum
y
x
(2.74) —
F^i are the
Solution
tials


Non-integrable systems
① Periodically driven simple pendulum
② n-body problem
③ Spherical pendulum
④ Double pendulum
y
x
n DOF, 2n dim phase space
Energy shell is (2n-1) dimensional
(2.74)
F^i are the conserved quantities
$\dot{F}^i = \{H, F^i\} = 0$
$\dot{\psi}^i = \{H, \psi_i\} = \frac{\partial H}{\partial F^i} = \Omega_i = \Omega_i(F_j)$
Solution
$F^i(t) = F$
$\psi_i(t)$
t


e systems
driven simple pendulum
problem
pendulum
endul
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) dimensional
(2.74)
F^i are the conserved quantities
$\dot{F}^i = \{H, F^i\} = 0$
$\dot{\psi}^i = \{H, \psi_i\} = \frac{\partial H}{\partial F^i} = \Omega_i = \Omega_i(F_j)$
Solution
$F^i(t) = F^i(0)$
$\psi_i(t) = \psi_i(0) + \Omega_i t$