Title: Classical Physics - Lecture 220919

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Collection: Classical Physics (2022/2023)

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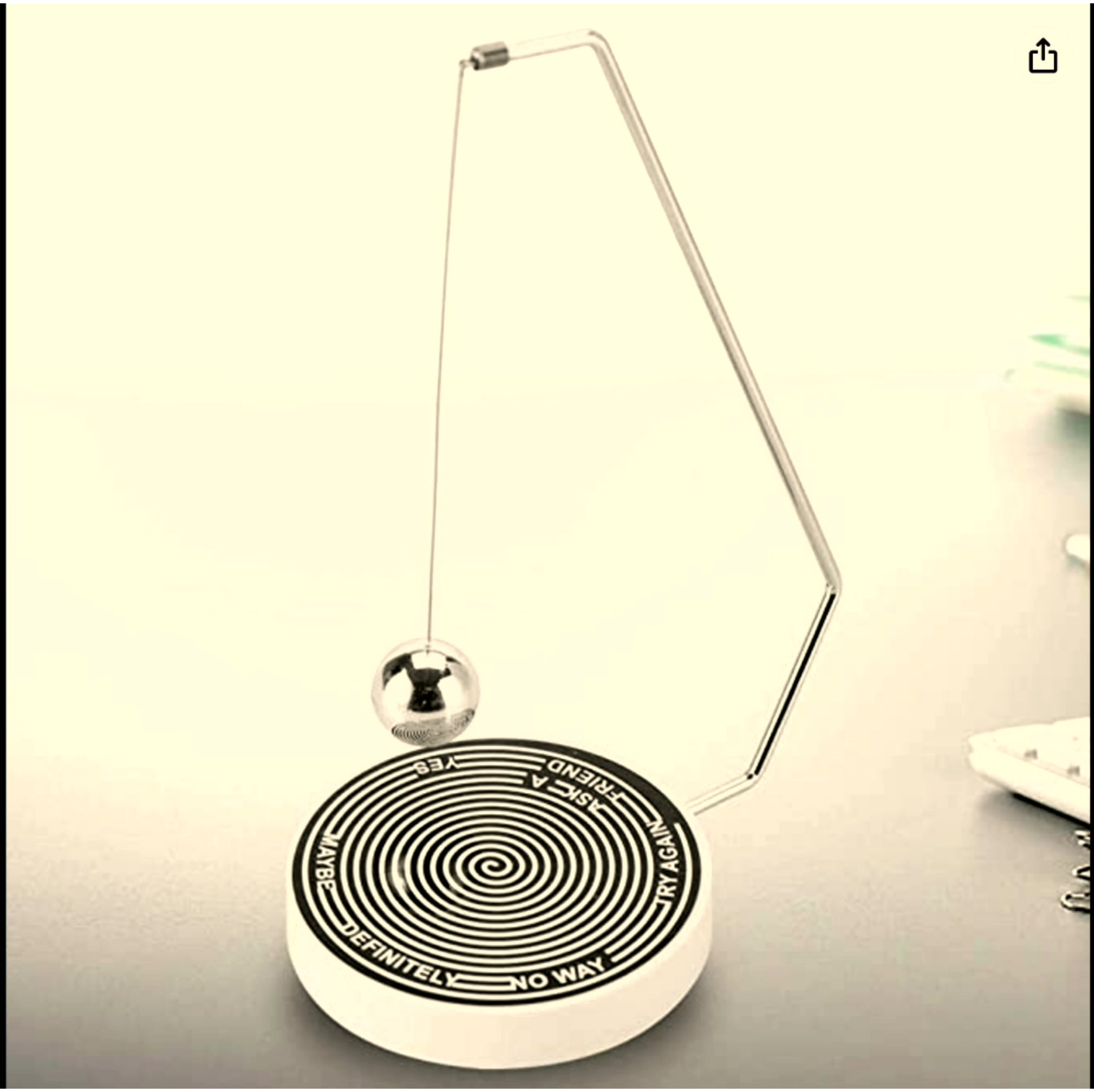
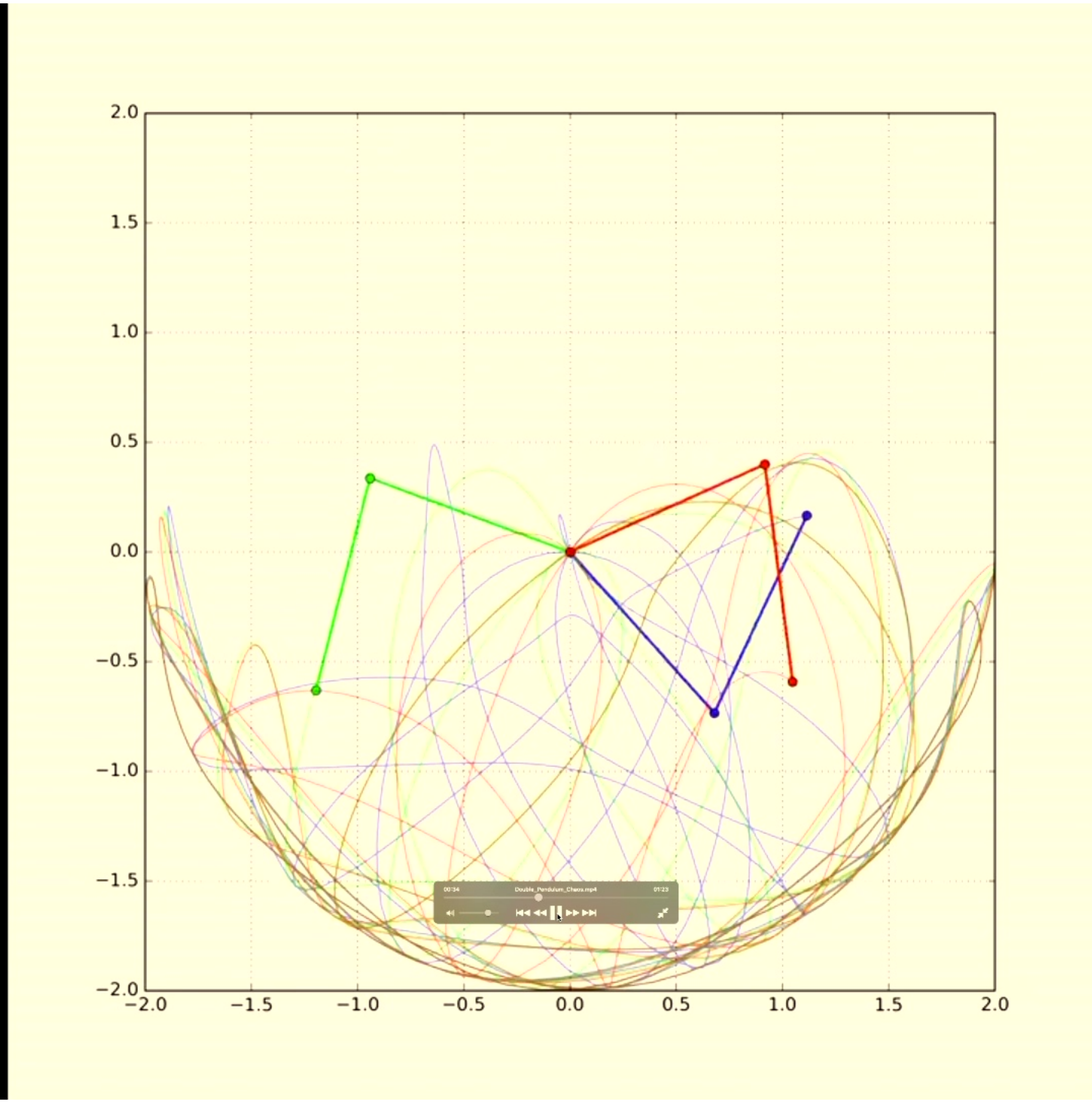

YES
ASK A FRIEND
TRY AGAIN
NO WAY
DEFINITELY
MAYBE

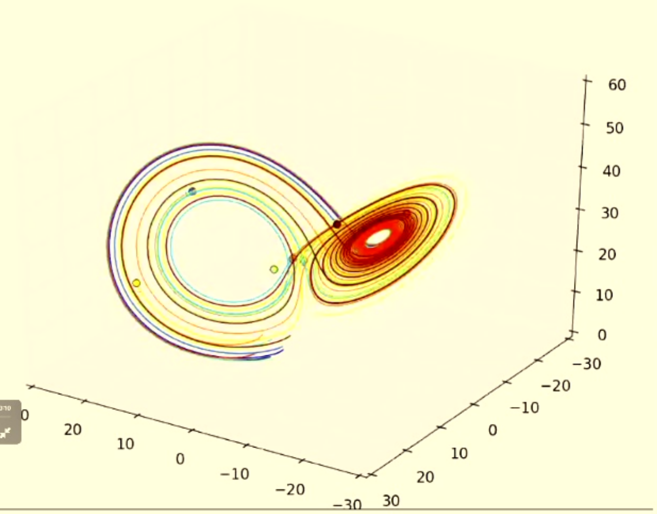

2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
00:34
Double_Pendulum_Chaos.mp4
01:23

Classical Chaos

Lorenz oscillator : Classic example of chaotic behaviour

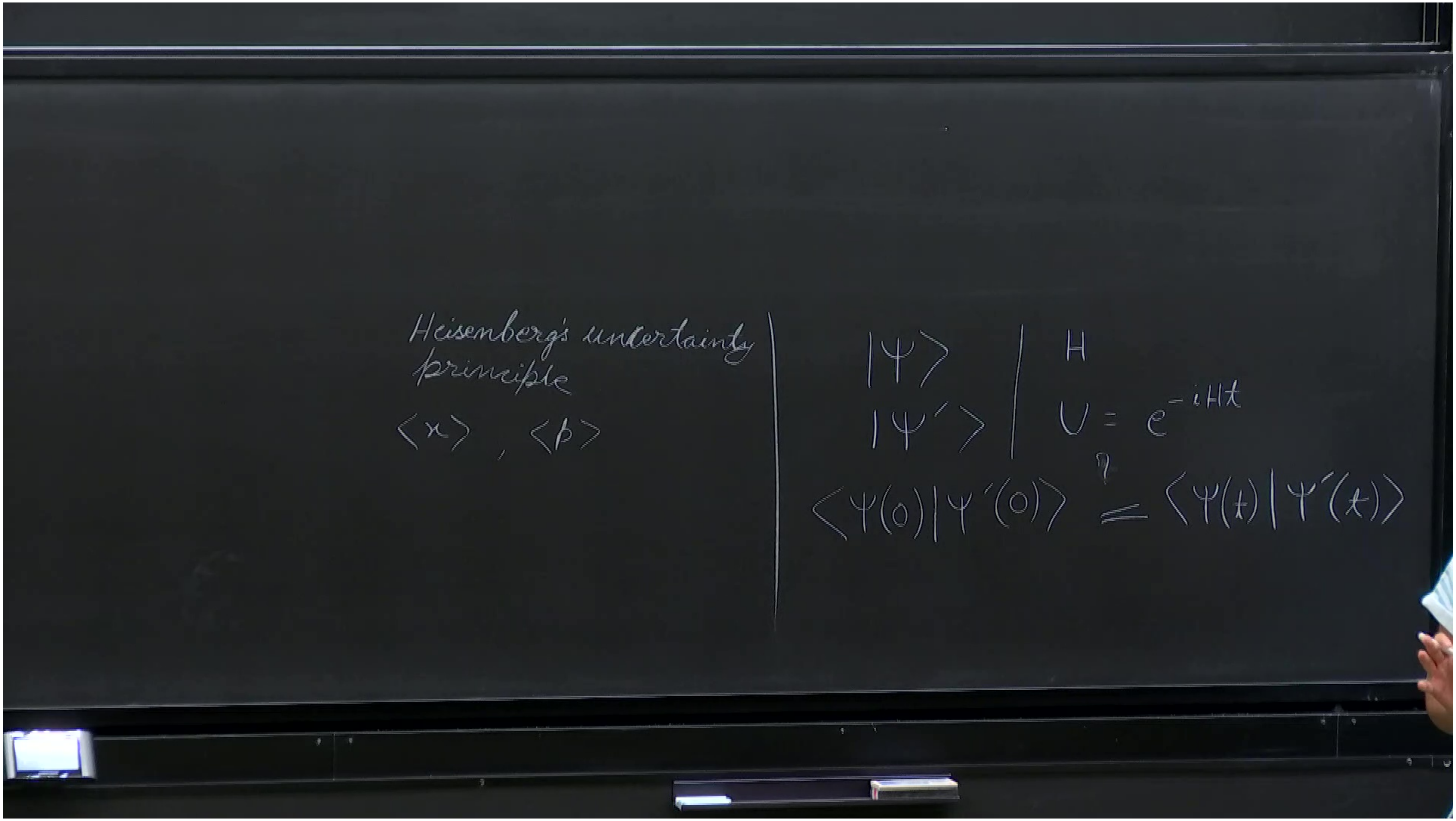
- Non-linear deterministic dynamical systems.

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$



Chaos

- Unpredictability due to extreme sensitivity to initial conditions
- Nonintegrability – a necessary condition in Hamiltonian system
- Nonlinear deterministic dynamical systems


Heisenberg's uncertainty principle
$\langle x \rangle$, $\langle p \rangle$
$|\psi\rangle$
$|\psi'\rangle$
H
$U = e^{-iHt}$
$\langle \psi(0) | \psi'(0) \rangle \overset{?}{=} \langle \psi(t) | \psi'(t) \rangle$

Heisenberg's uncertainty principle

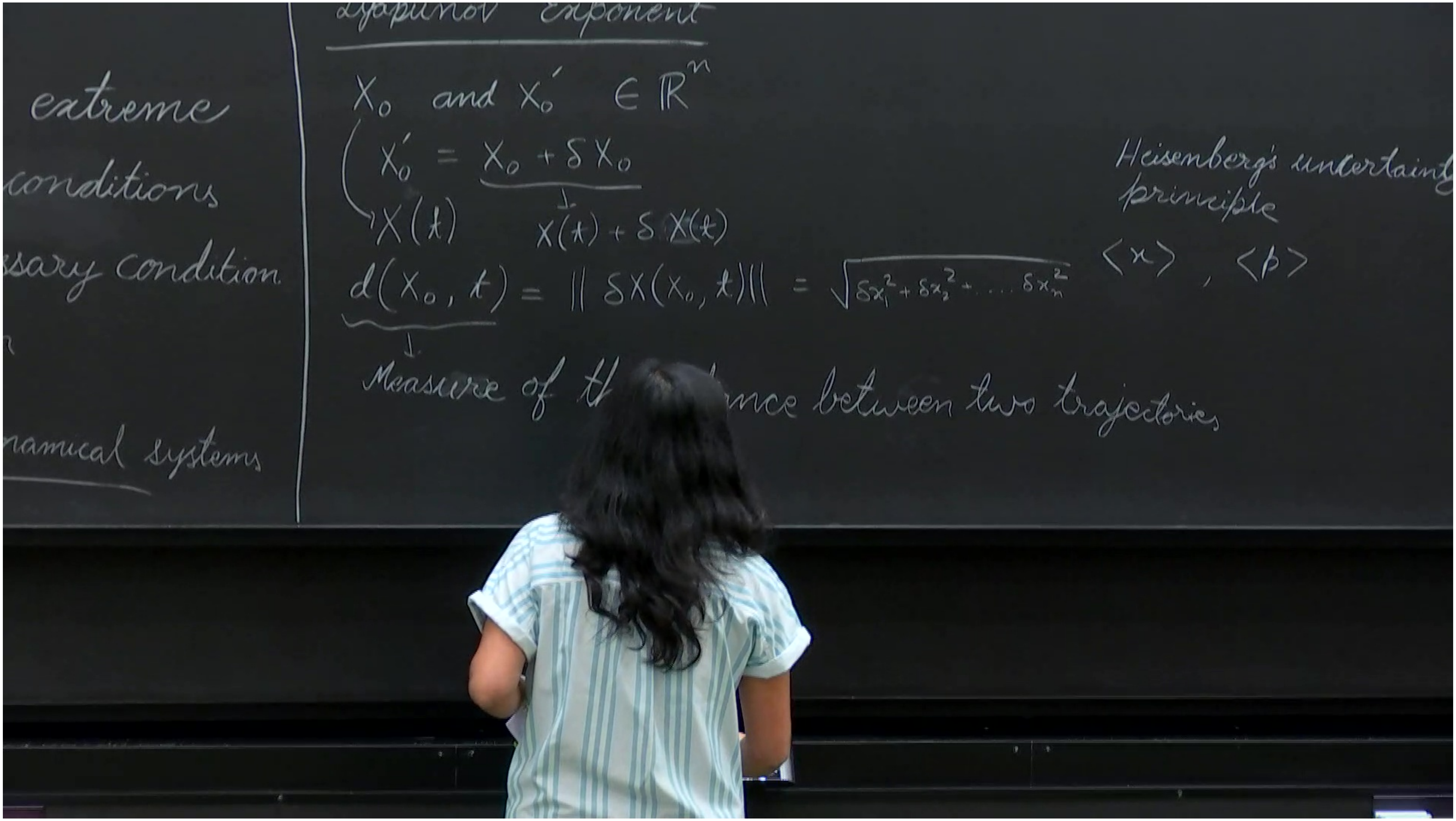
$\langle x \rangle \, , \, \langle p \rangle$

$|\psi(0)\rangle$ | H

$|\psi'(0)\rangle$ | $U = e^{-iHt}$

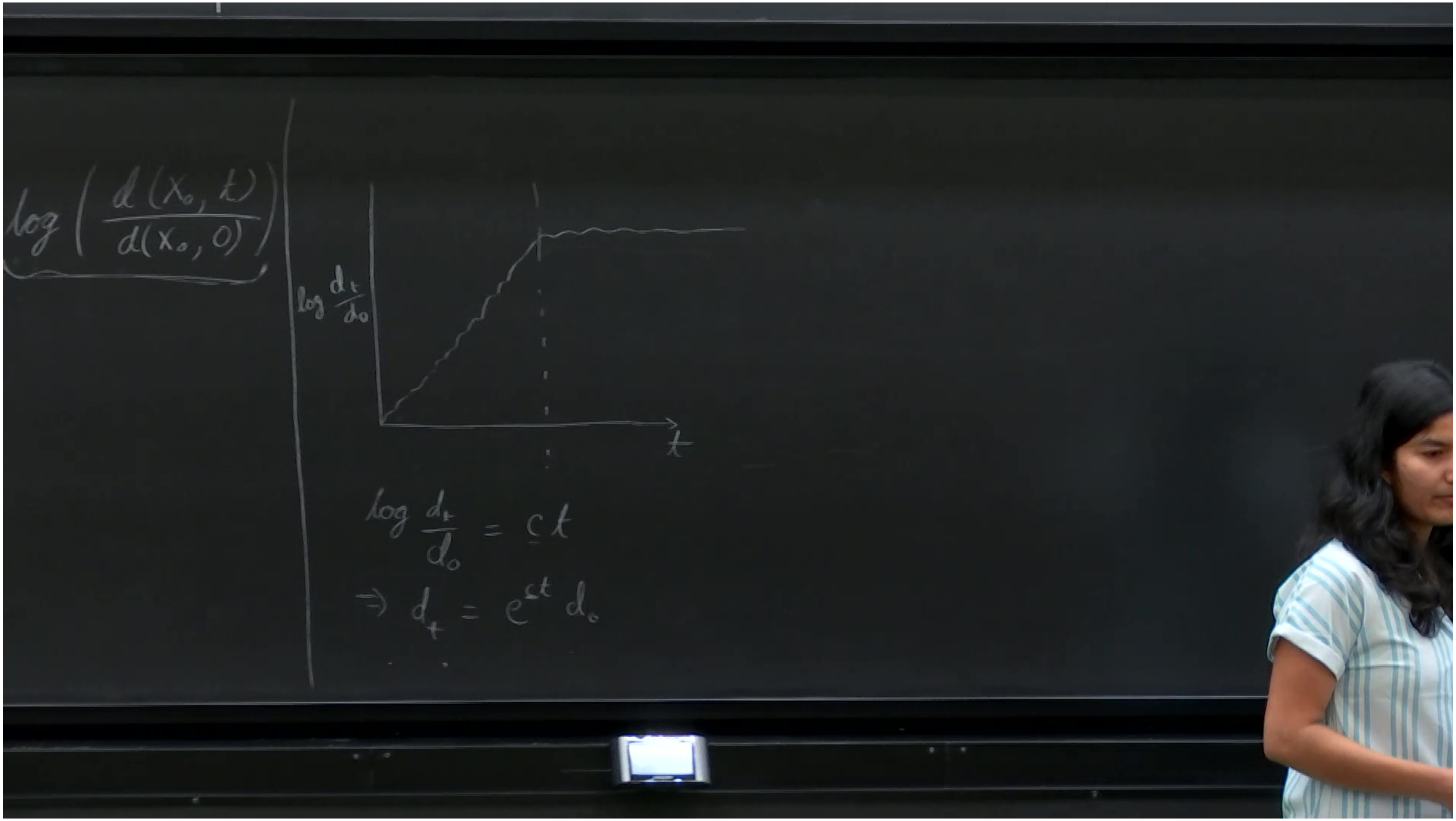
$$\langle \psi(0) | \psi'(0) \rangle \stackrel{?}{=} \langle \psi(t) | \psi'(t) \rangle$$

$$\underline{UU^{\dagger} = 1}$$


Lyapunov exponent
extreme
conditions
ssary condition
namical systems
X_0 and $X_0' \in \mathbb{R}^n$
$X_0' = X_0 + \delta X_0$
$X(t)$
$X(t) + \delta X(t)$
$d(X_0, t) = \| \delta X(X_0, t) \| = \sqrt{\delta x_1^2 + \delta x_2^2 + \ldots \delta x_n^2}$
Measure of th ance between two trajectories
Heisenberg's uncertainty principle
$\langle x \rangle$, $\langle p \rangle$

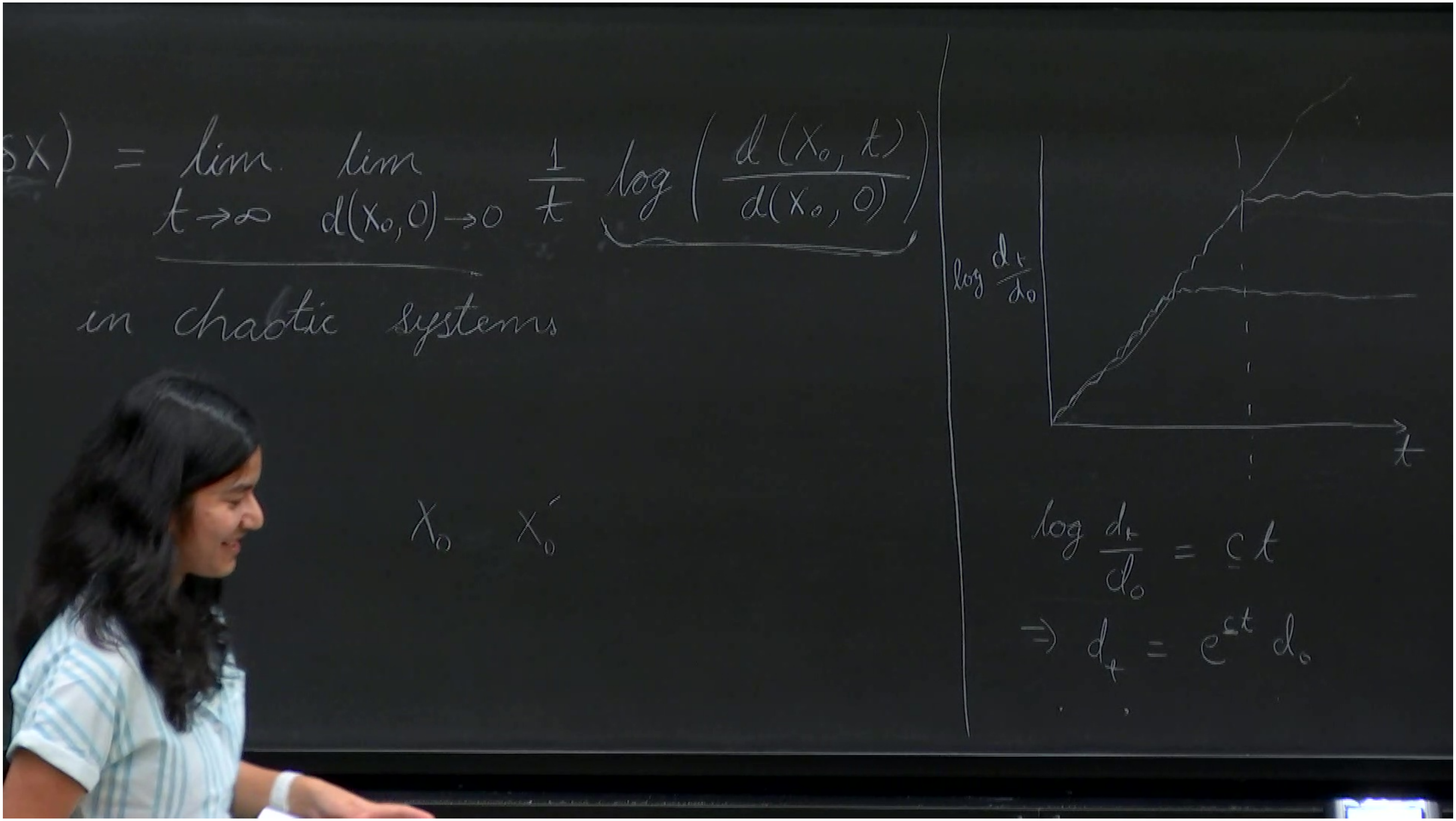
$$\lambda(X_0, \delta X) = \lim_{t \to \infty} \lim_{d(X_0,0) \to 0} \frac{1}{t} \log\left(\frac{d(X_0, t)}{d(X_0, 0)}\right)$$

$\lambda > 0$ in chaotic systems

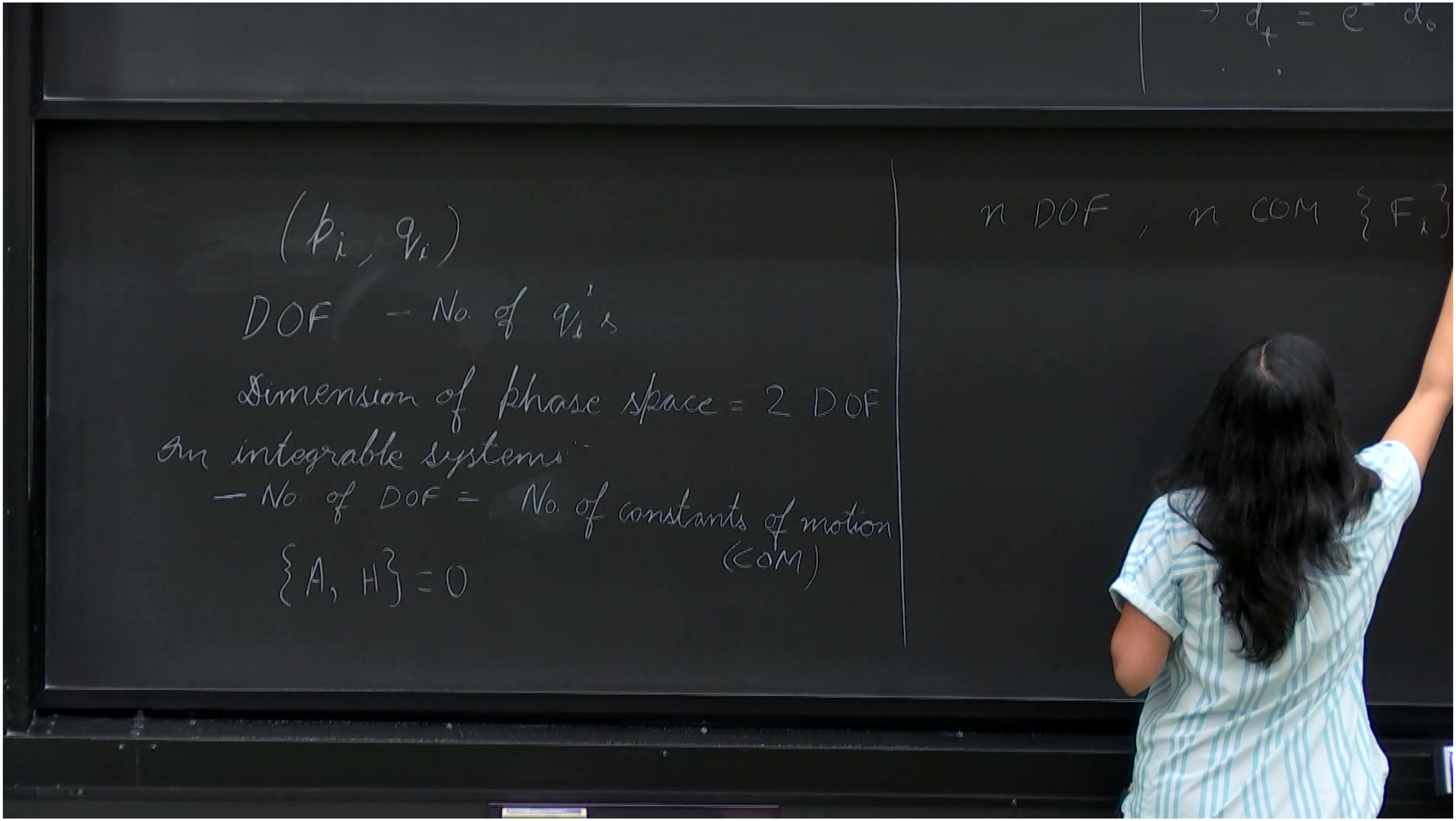

$\log\left(\frac{d(x_0,t)}{d(x_0,0)}\right)$
$\log \frac{d_t}{d_0}$
t
$\log \frac{d_t}{d_0} = ct$
$\Rightarrow d_t = e^{ct} d_0$

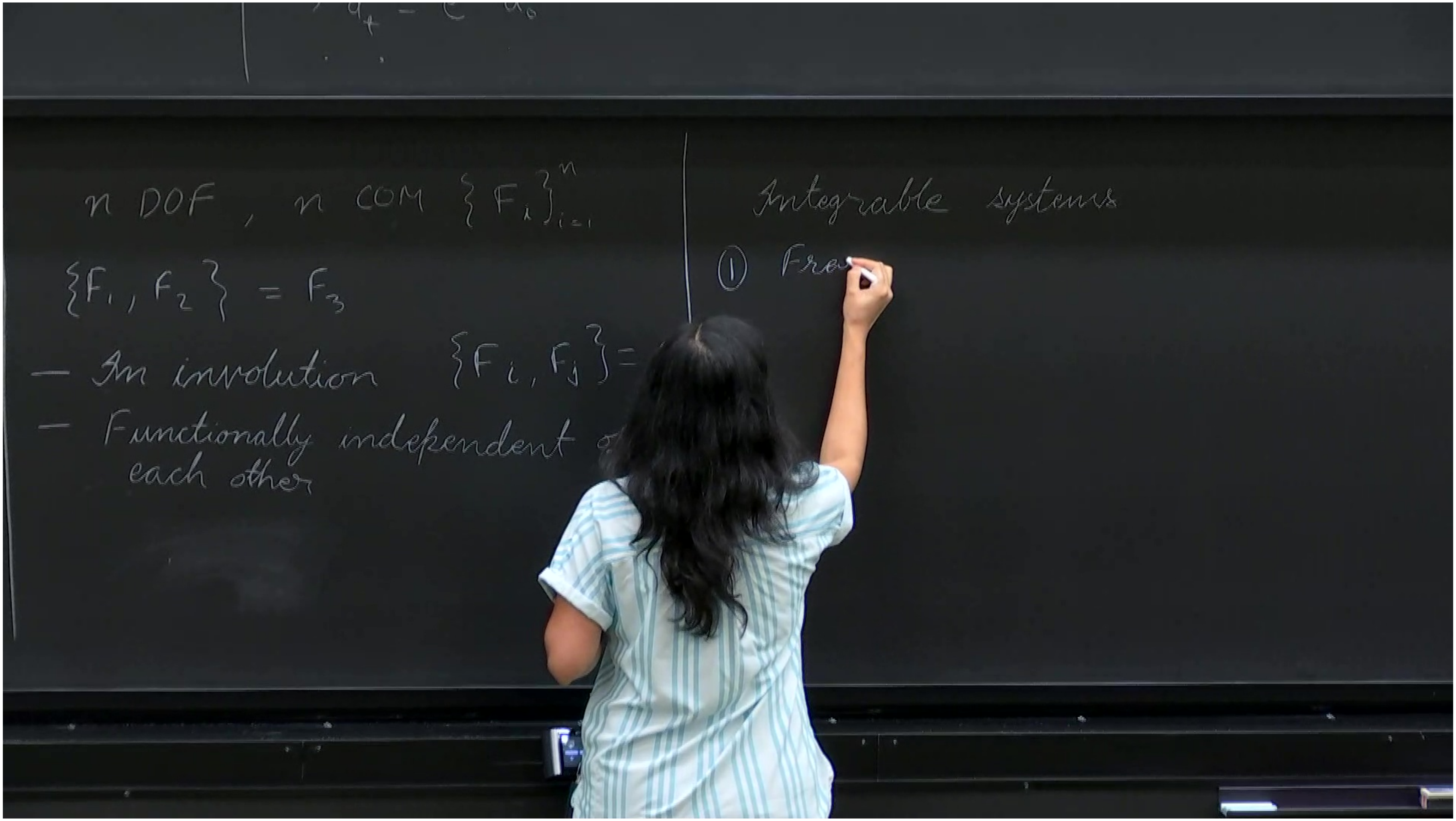
$\vec{F}$, $\vec{\nabla}\cdot\vec{F} < 0$ for dissipative system

$\vec{\nabla}\cdot\vec{F} = 0$ for Hamiltonian systems



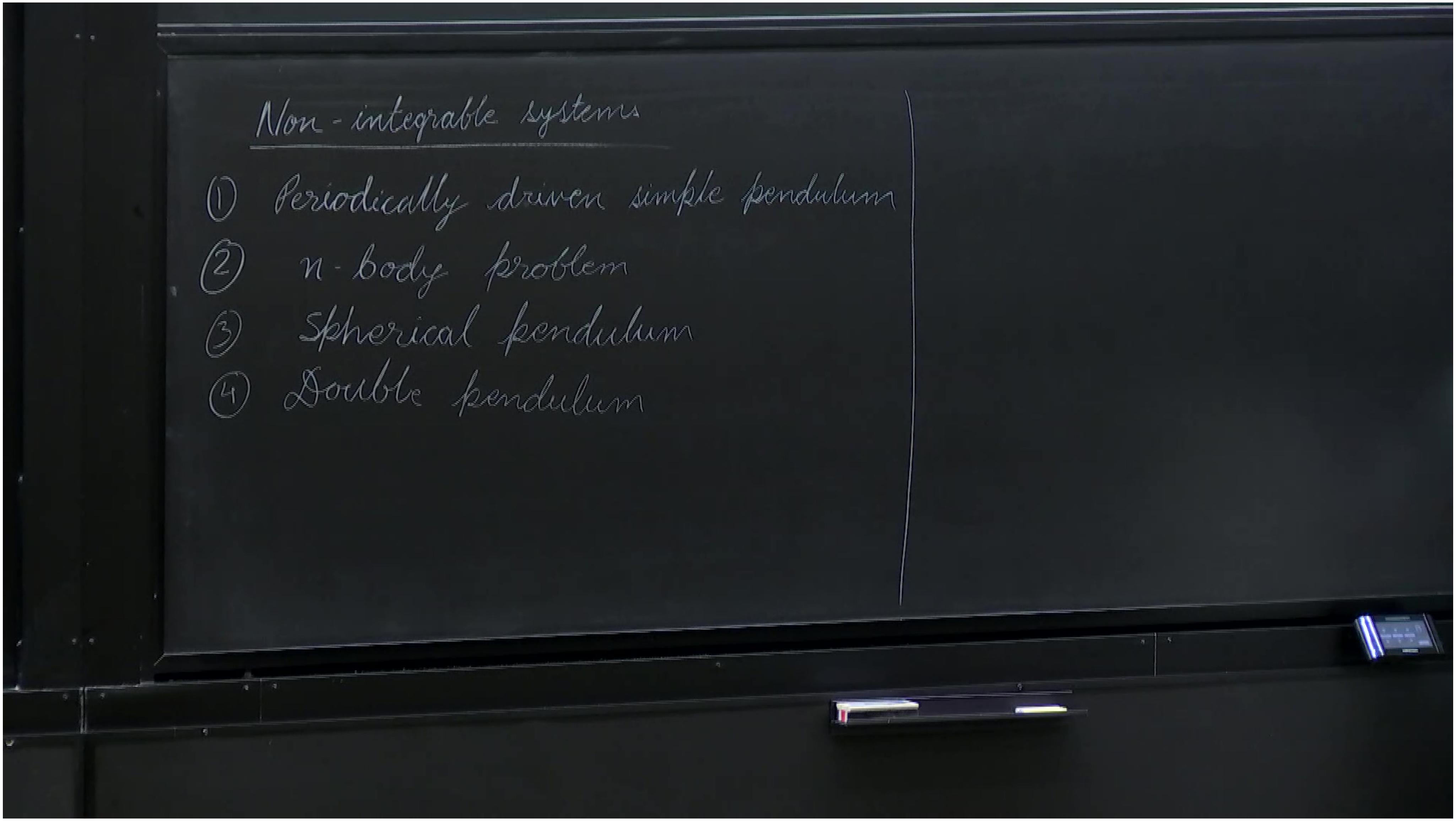
$\delta x) = \lim_{t\to\infty} \lim_{d(x_0,0)\to 0} \frac{1}{t} \log\left(\frac{d(x_0,t)}{d(x_0,0)}\right)$
in chaotic systems
x_0 x_0'
$\log \frac{d_t}{d_0}$
t
$\log \frac{d_t}{d_0} = c\,t$
$\Rightarrow d_t = e^{ct} d_0$

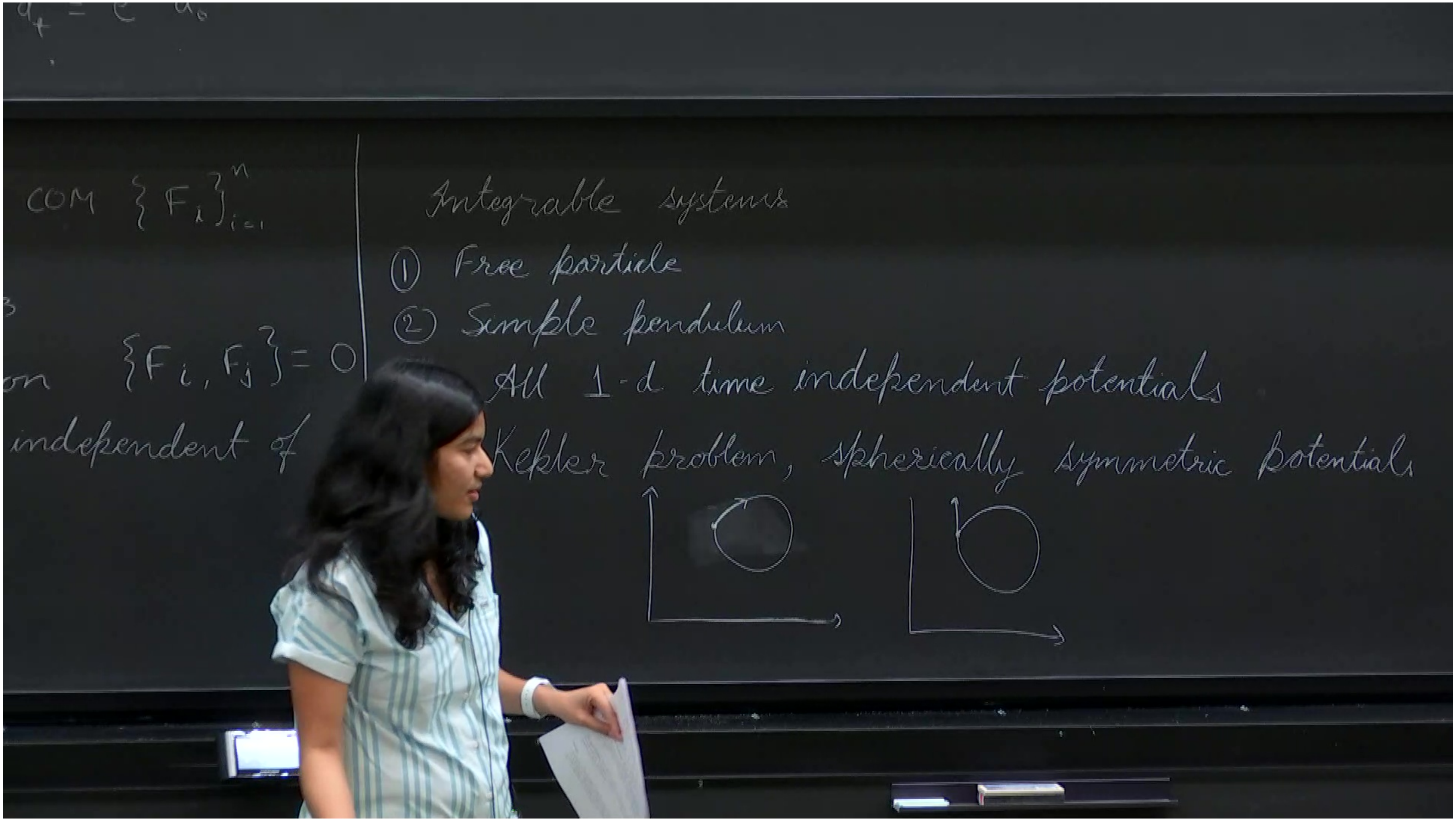

(p_i, q_i)
DOF – No. of q_i's
Dimension of phase space = 2 DOF
In integrable systems
– No of DOF = No. of constants of motion (COM)
$\{A, H\} = 0$
n DOF, n COM $\{F_i\}$

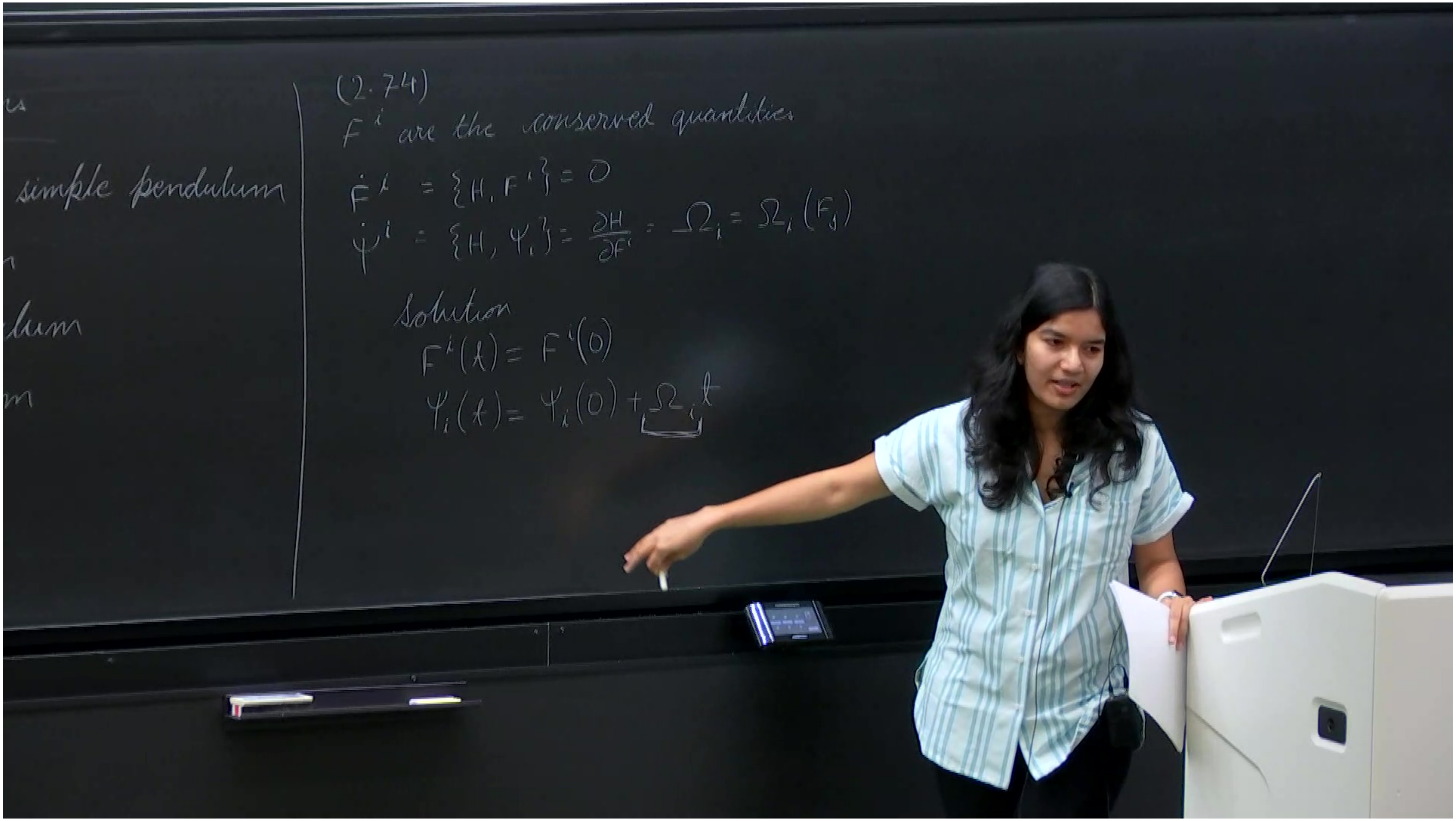

n DOF , n COM $\{F_i\}_{i=1}^{n}$
$\{F_1, F_2\} = F_3$
— In involution $\{F_i, F_j\} =$
— Functionally independent of
each other
Integrable systems
① Fre

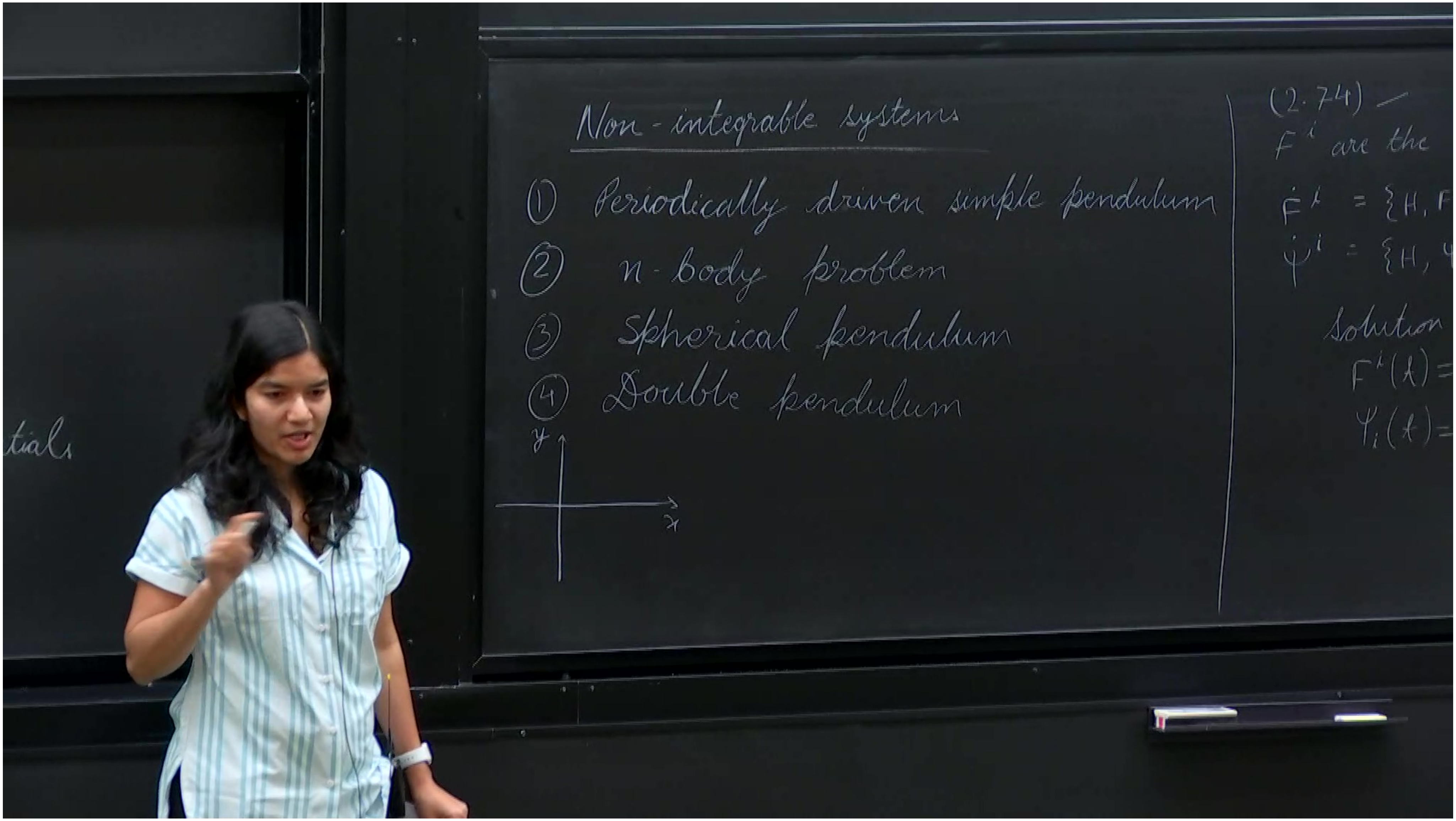
Integrable systems

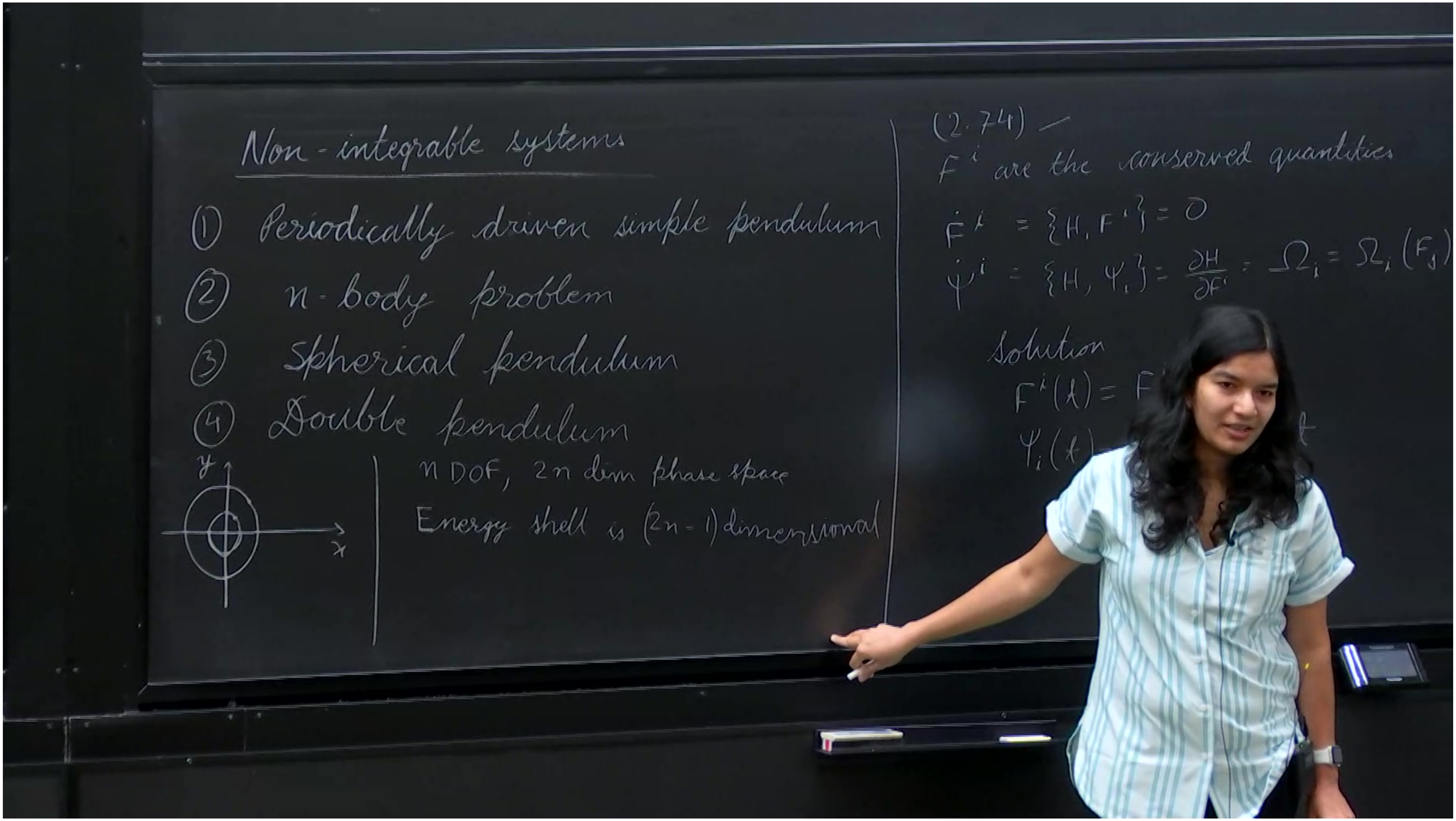
1. Free particle
2. Simple pendulum
3. All 1-d time independent potentials
4. Kepler problem, spherically symmetric potentials

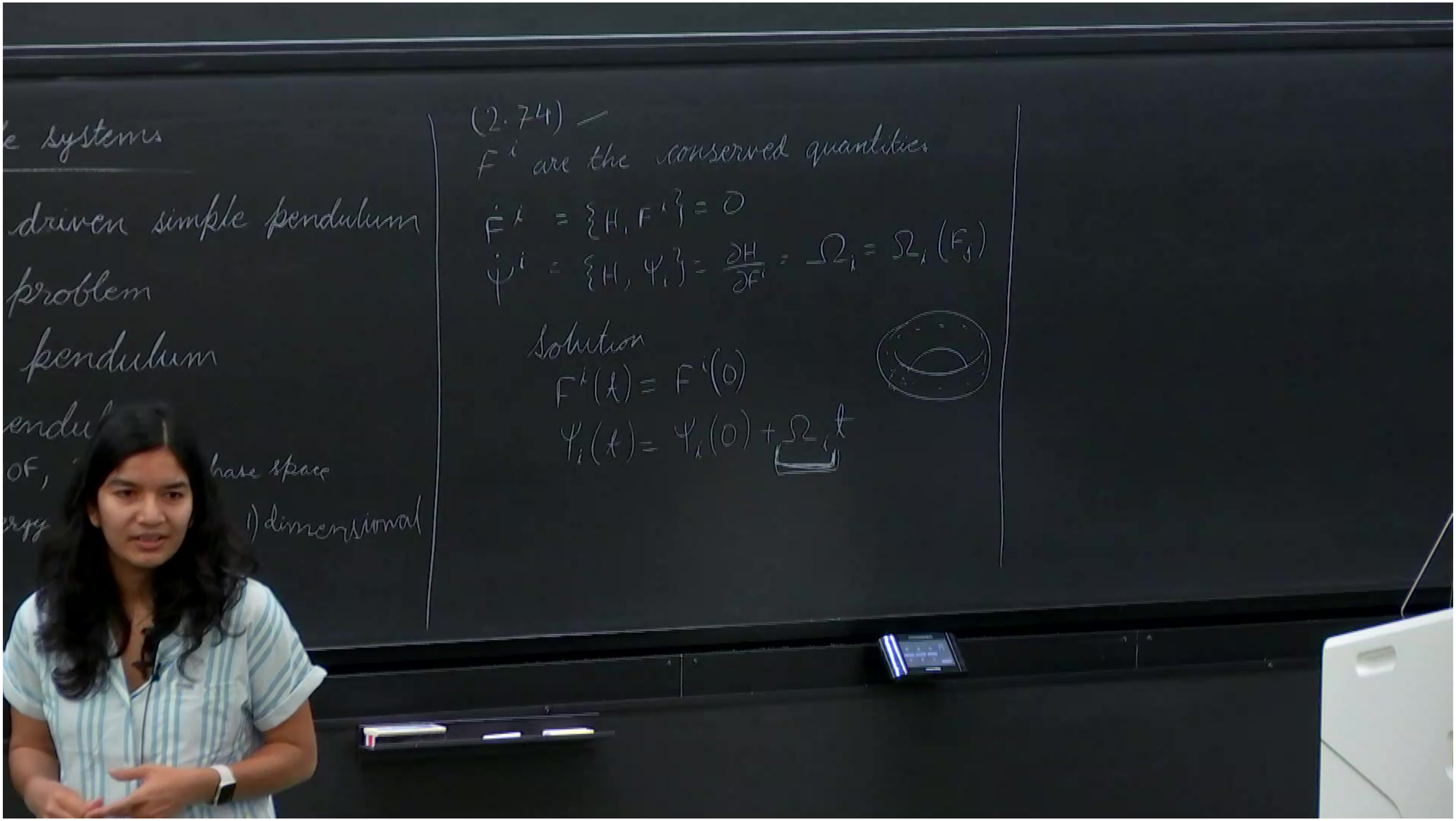

Non-integrable systems
① Periodically driven simple pendulum
② n-body problem
③ Spherical pendulum
④ Double pendulum


COM {F_i}_{i=1}^n
on {F_i, F_j} = 0
independent of
Integrable systems
① Free particle
② Simple pendulum
All 1-d time independent potentials
Kepler problem, spherically symmetric potentials


(2.74)
F^i are the conserved quantities
simple pendulum
Ḟ^i = {H, F^i} = 0
ψ̇^i = {H, ψ_i} = ∂H/∂F^i = Ω_i = Ω_i(F_j)
Solution
F^i(t) = F^i(0)
ψ_i(t) = ψ_i(0) + Ω_i t
lum


Non-integrable systems
1 Periodically driven simple pendulum
2 n-body problem
3 Spherical pendulum
4 Double pendulum
y
x
(2.74)
are the
Solution


Non-integrable systems
(1) Periodically driven simple pendulum
(2) n-body problem
(3) Spherical pendulum
(4) Double pendulum
y
x
n DOF, 2n dim phase space
Energy shell is (2n-1) dimensional
(2.74)
F^i are the conserved quantities
$\dot{F}^i = \{H, F^i\} = 0$
$\dot{\psi}^i = \{H, \psi_i\} = \frac{\partial H}{\partial F^i} = \Omega_i = \Omega_i(F_j)$
Solution
$F^i(t) = F$
$\psi_i(t)$
t


e systems
driven simple pendulum
problem
pendulum
(2.74)
F^i are the conserved quantities
Ḟ^i = {H, F^i} = 0
ψ̇^i = {H, ψ_i} = ∂H/∂F^i = Ω_i = Ω_i(F_j)
Solution
F^i(t) = F^i(0)
ψ_i(t) = ψ_i(0) + Ω_i t
hase space
1) dimensional