Title: Classical Physics - Lecture 220915

Speakers:

Collection: Classical Physics (2022/2023)

Date: September 15, 2022 - 11:15 AM

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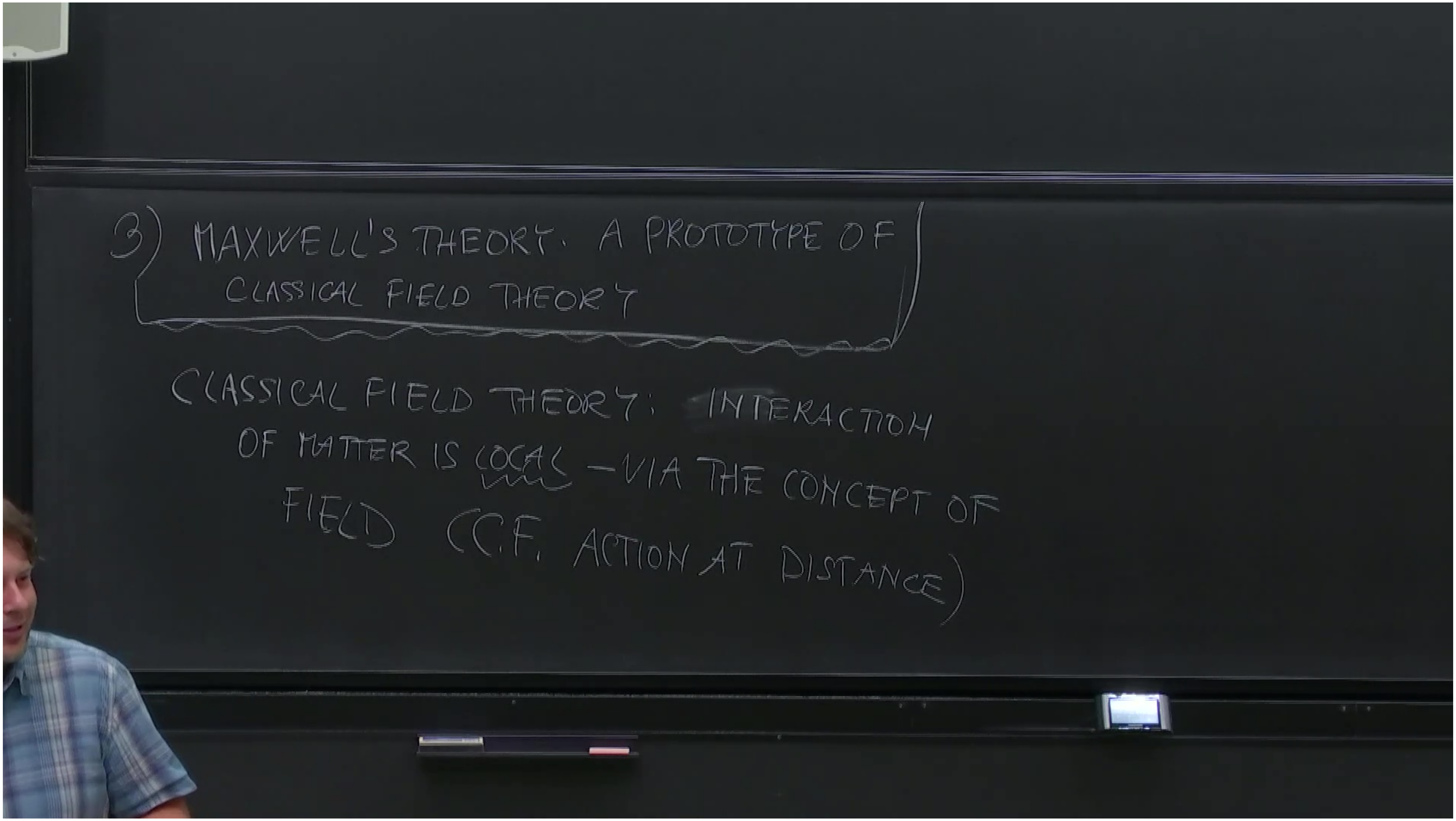
3) MAXWELL'S THEORY: A PROTOTYPE OF CLASSICAL FIELD THEORY


3) MAXWELL'S THEORY. A PROTOTYPE OF CLASSICAL FIELD THEORY
CLASSICAL FIELD THEORY: INTERACTION OF MATTER IS LOCAL - VIA THE CONCEPT OF FIELD (C.F. ACTION AT DISTANCE)

OF

ACTION

NCEPT OF

STANCE)

• FIELD ≈ PHYSICAL QUANTITY AT EACH POINT OF SPACE AND TIME

$$q(t) \longrightarrow \varphi(t, \vec{x})$$

CLASSICAL FIELD THEORY: MATTER (PARTICLES) & FIELD

LATER REPLACED BY FIELDS AS WELL.

EOM < FIELD EQS (EOM FOR FIELD PRODUCED BY SOURCES) ... MAXWELL EQS.

EOM FOR MATTER ... LORENTZ FORCE LAW.

FIELD (C.F. ACTION AT DISTANCE)

EOM < FIELD EQS (E...
EOM FOR MATTER

- POPULAR APPROXIMATION: "TEST MATTER"
  (MATTER DOES NOT PRODUCE ITS OWN FIELD
  JUST MOVES IN A FIXED FIELD)
- IN GENERAL BOTH SETS OF EQS.
  HAVE TO BE SOLVED SIMULTANEOUSLY.

FIELDS AS WELL.

(DISTANCE)

EOM: FIELD EQS (EOM FOR FIELD PRODUCED BY SOURCES)... MAXWELL EQS

EOM FOR MATTER ... LORENTZ FORCE LAW.

TER"

OWN FIELD

ELD)

EOUSLY.

• MAXWELL'S THEORY (1860)

AMPERE & MAXWELL: $\nabla \times B - \frac{\partial E}{\partial t} = 4\pi j \quad (1)$

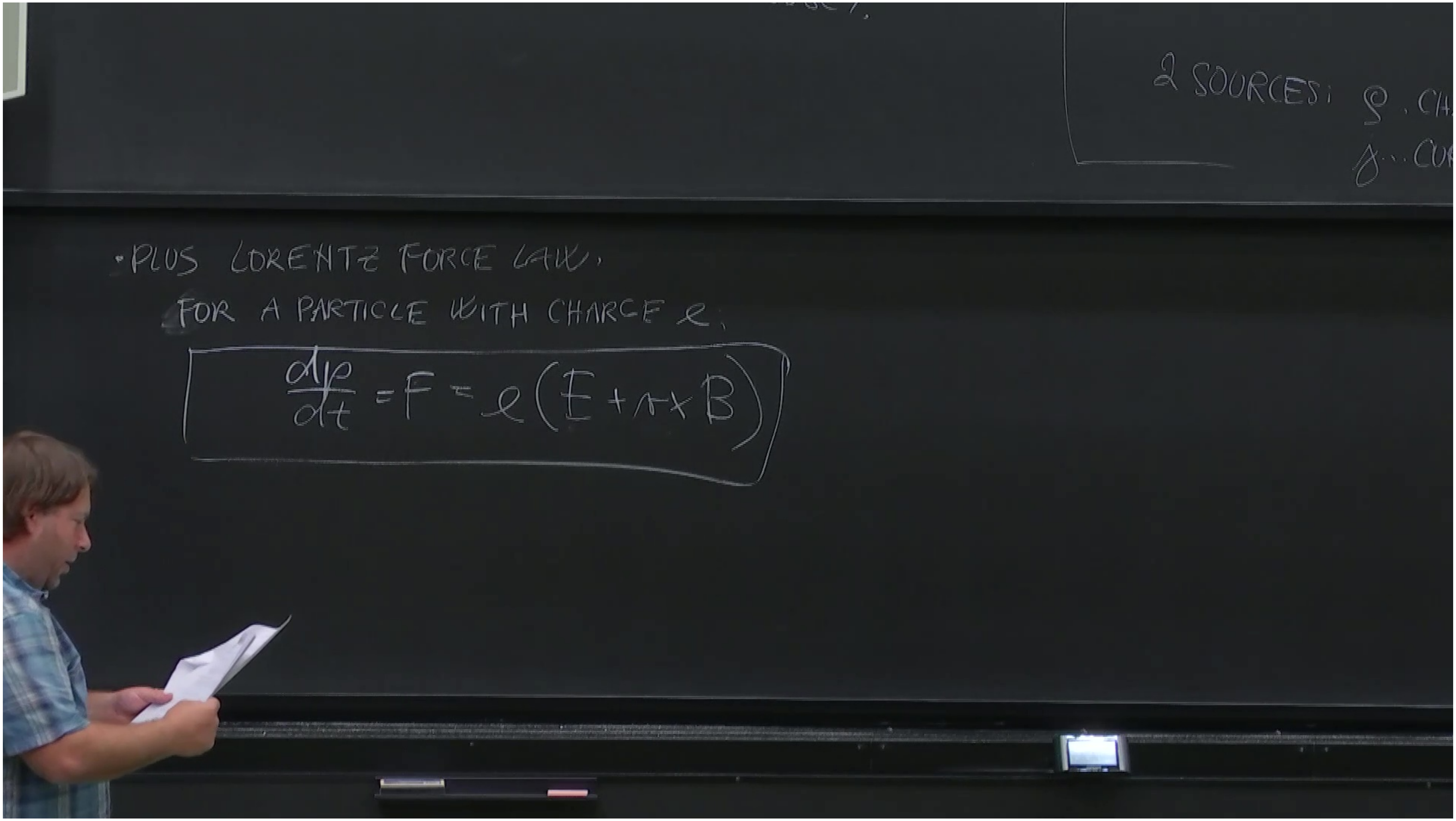
FARADAY: $\nabla \times E + \frac{\partial B}{\partial t} = 0 \quad (2)$

GAUSS: $\nabla \cdot E = 4\pi \rho \quad (3)$

$\nabla \cdot B = 0 \quad (4)$

2 SOURCES: ρ . CHARGE DENSITY

j .. CURRENT — " —


2 SOURCES:
• PLUS LORENTZ FORCE LAW.
FOR A PARTICLE WITH CHARGE e:
$\boxed{\frac{dp}{dt} = F = e(E + v \times B)}$

j ... CURRENT –"–

REMARKS. i) CHARGE CONSERVATION.

$$\nabla \cdot \nabla \times V = 0 \,, \quad \nabla \times \nabla f = 0 \qquad \text{IDENTITIES.}$$

$$\nabla \cdot (1) = \cancel{0} - \frac{\partial}{\partial t} \underbrace{\nabla \cdot E}_{(3)\; 4\pi \rho} = 4\pi \, \nabla \cdot j$$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0}$$

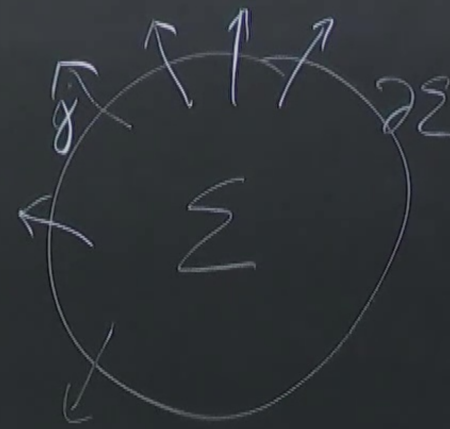
CONTINUITY EQ ... "CONSERVATION OF CHARGE"

• $Q = \int_{\Sigma_3} \rho \, dV$ ... CHARGE

$$-\frac{dQ}{dt} = -\int_{\Sigma} \frac{\partial \rho}{\partial t} dV = \int_{\Sigma} (\nabla \cdot j) \, dV$$

$$= \int_{\partial \Sigma} \vec{j} \cdot d\vec{S} = 0$$

IF NO FLUX THROUGH BOUNDARY



• $Q = \int_{\Sigma_3} \rho \, dV$ ... CHARGE

$\rho(t,x)\, d^3x$

$$-\frac{dQ}{dt} = -\int_{\Sigma} \frac{\partial \rho}{\partial t} dV = \int_{\Sigma} (\nabla \cdot j)\, dV$$

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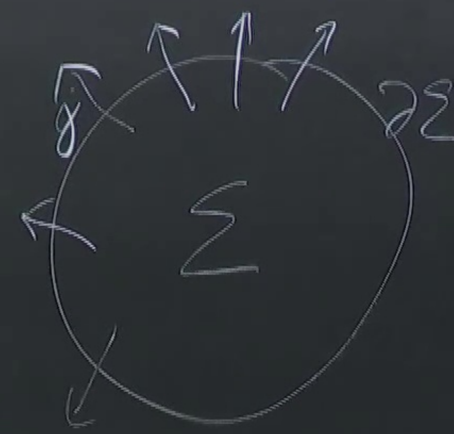

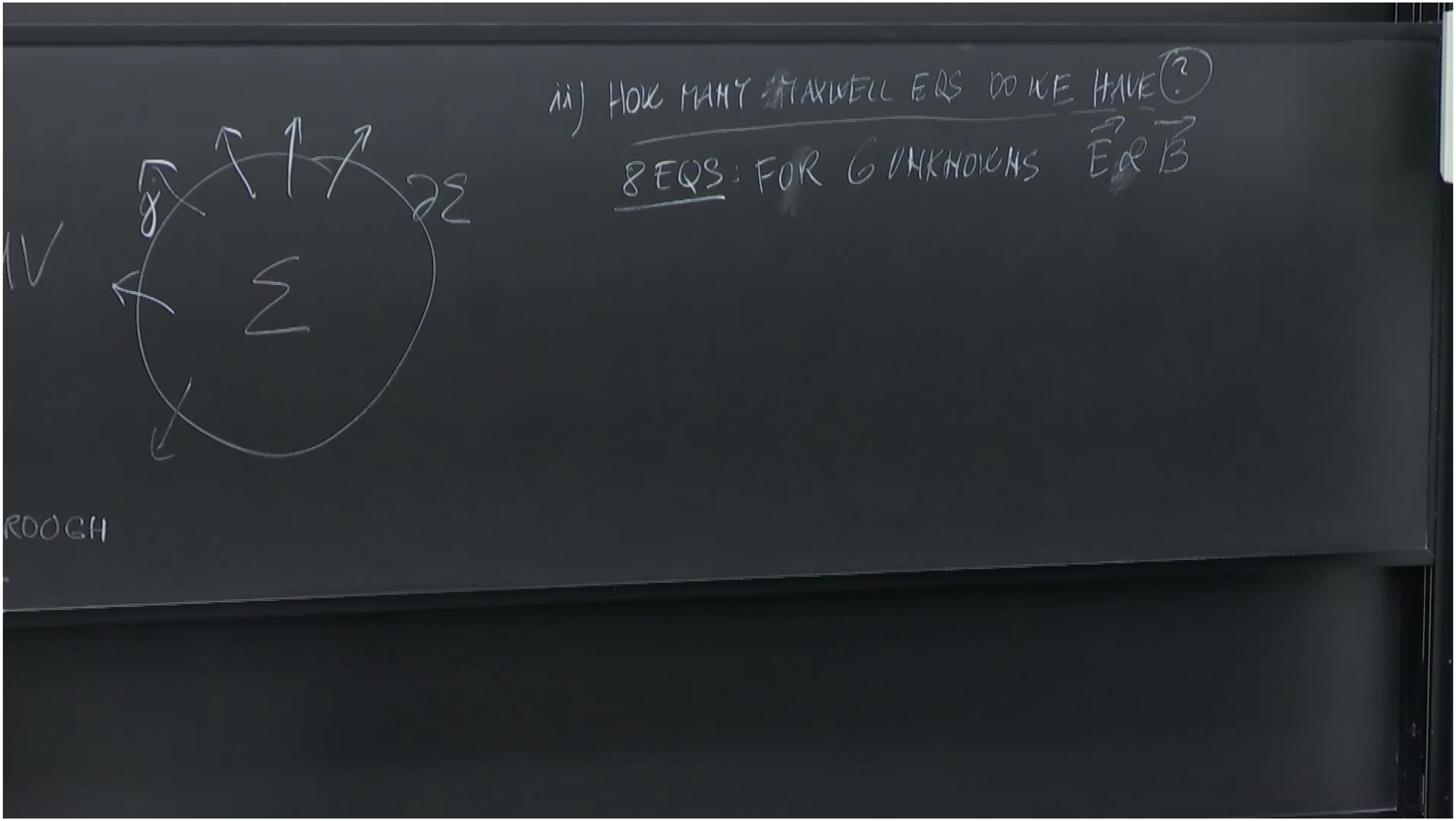
ii) HOW MANY MAXWELL EQS DO WE HAVE ?
8 EQS: FOR 6 UNKNOWNS E & B
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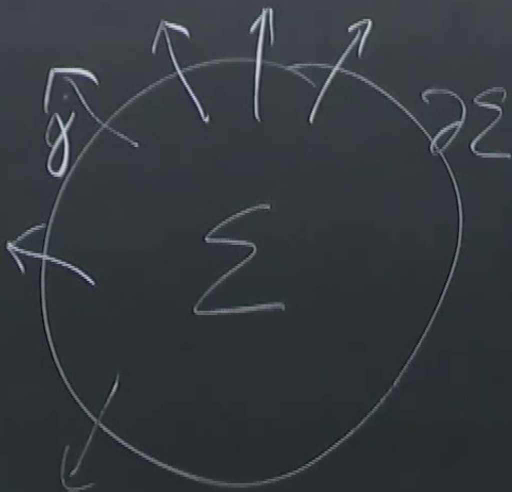
IV



ROUGH

ii) HOW MANY MAXWELL EQS DO WE HAVE ?

8 EQS: FOR 6 UNKNOWNS $\vec{E}$ & $\vec{B}$

• ONLY (1) & (2) ARE TRUE EVOLUTION EQS.

$$\nabla\cdot(2) = \emptyset + \frac{\partial}{\partial t}\nabla\cdot B = 0$$

CONT.

$$\nabla\cdot(1) = -\frac{\partial}{\partial t}\nabla\cdot E = 4\pi\nabla\cdot j = -4\pi\frac{\partial}{\partial t}\rho$$

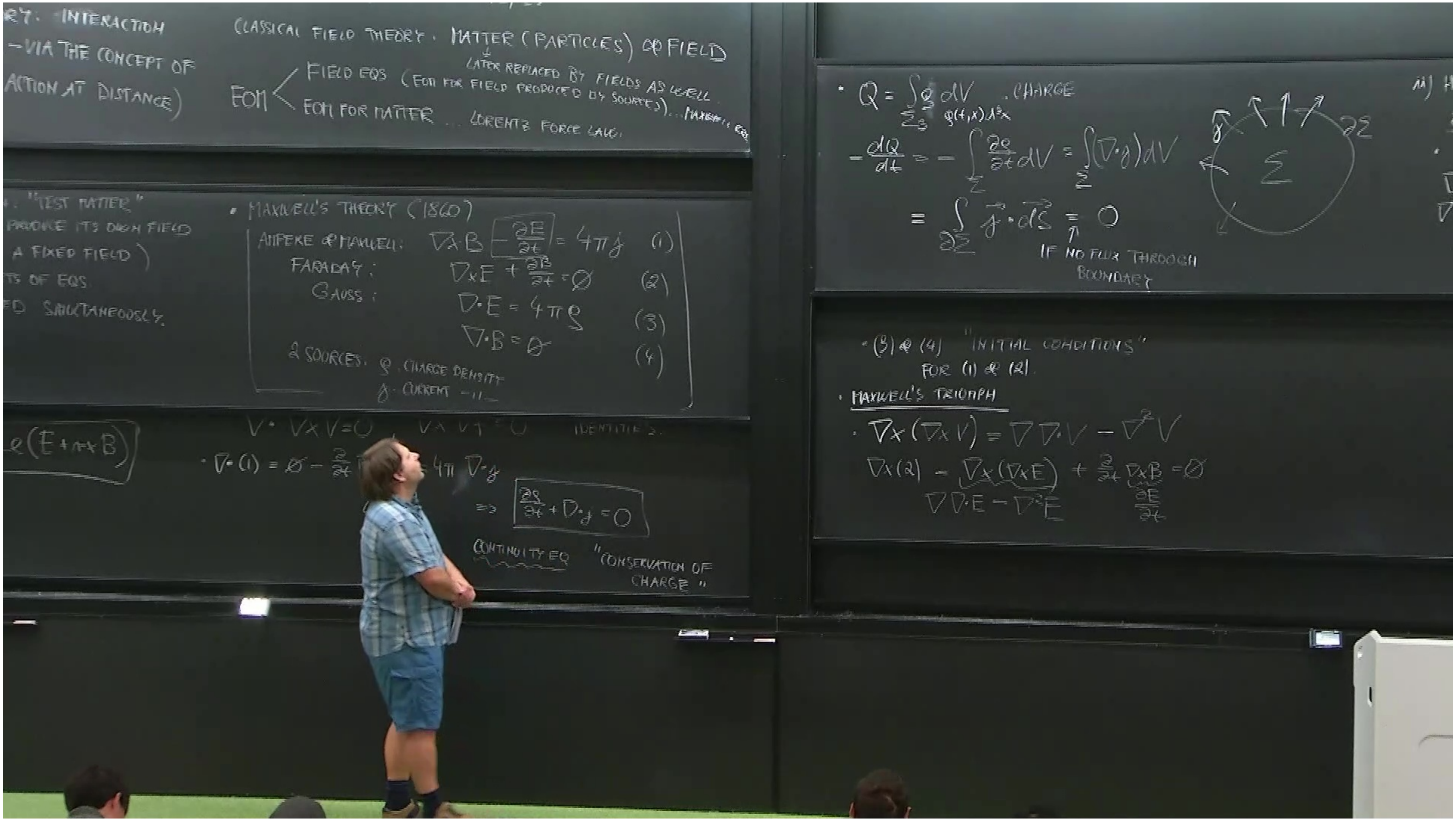
$$= -\frac{\partial}{\partial t}(\nabla\cdot E - 4\pi\rho) = 0$$

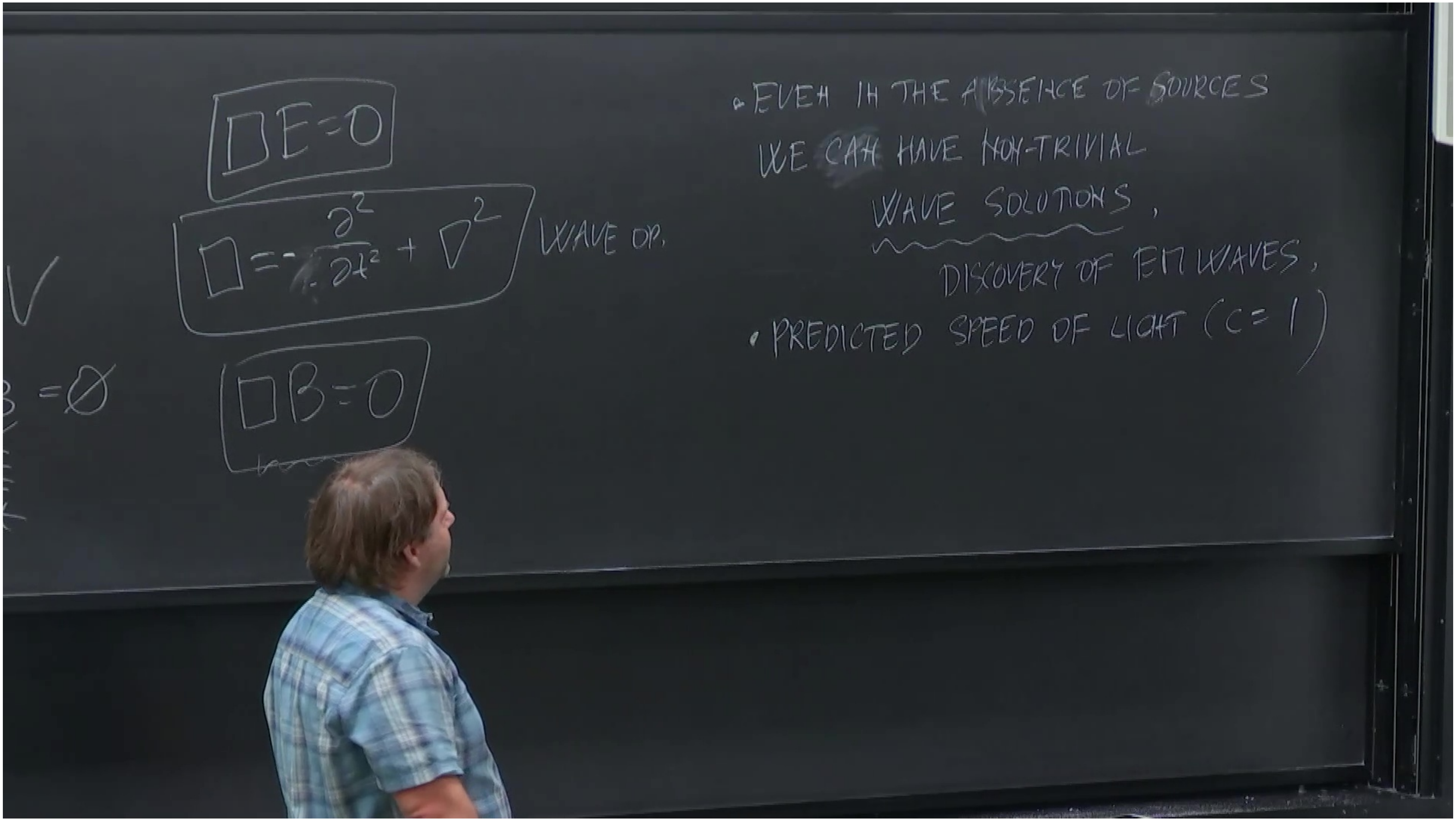
- (3) & (4) "INITIAL CONDITIONS" FOR (1) & (2).

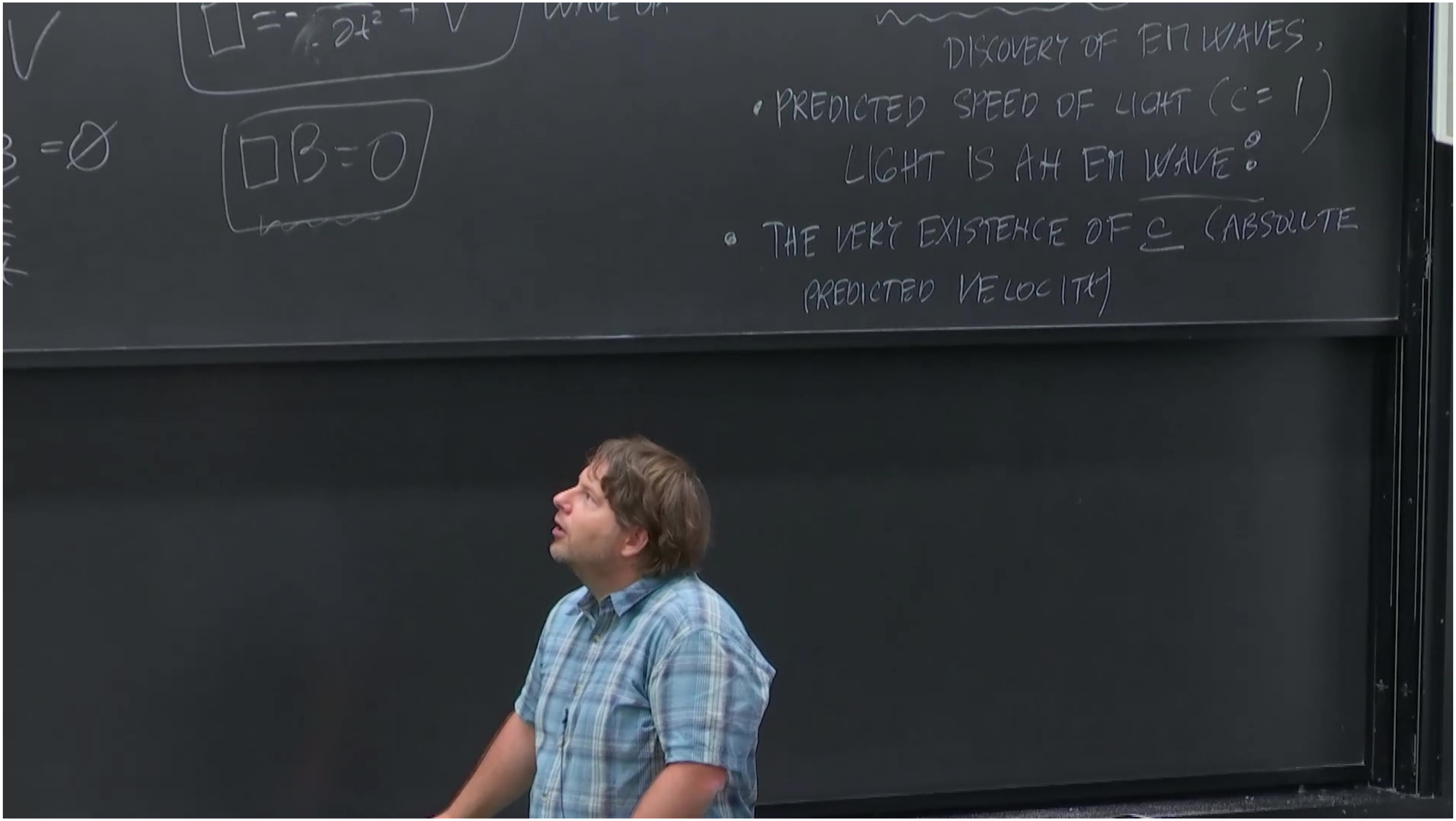
- <u>MAXWELL'S TRIUMPH</u>

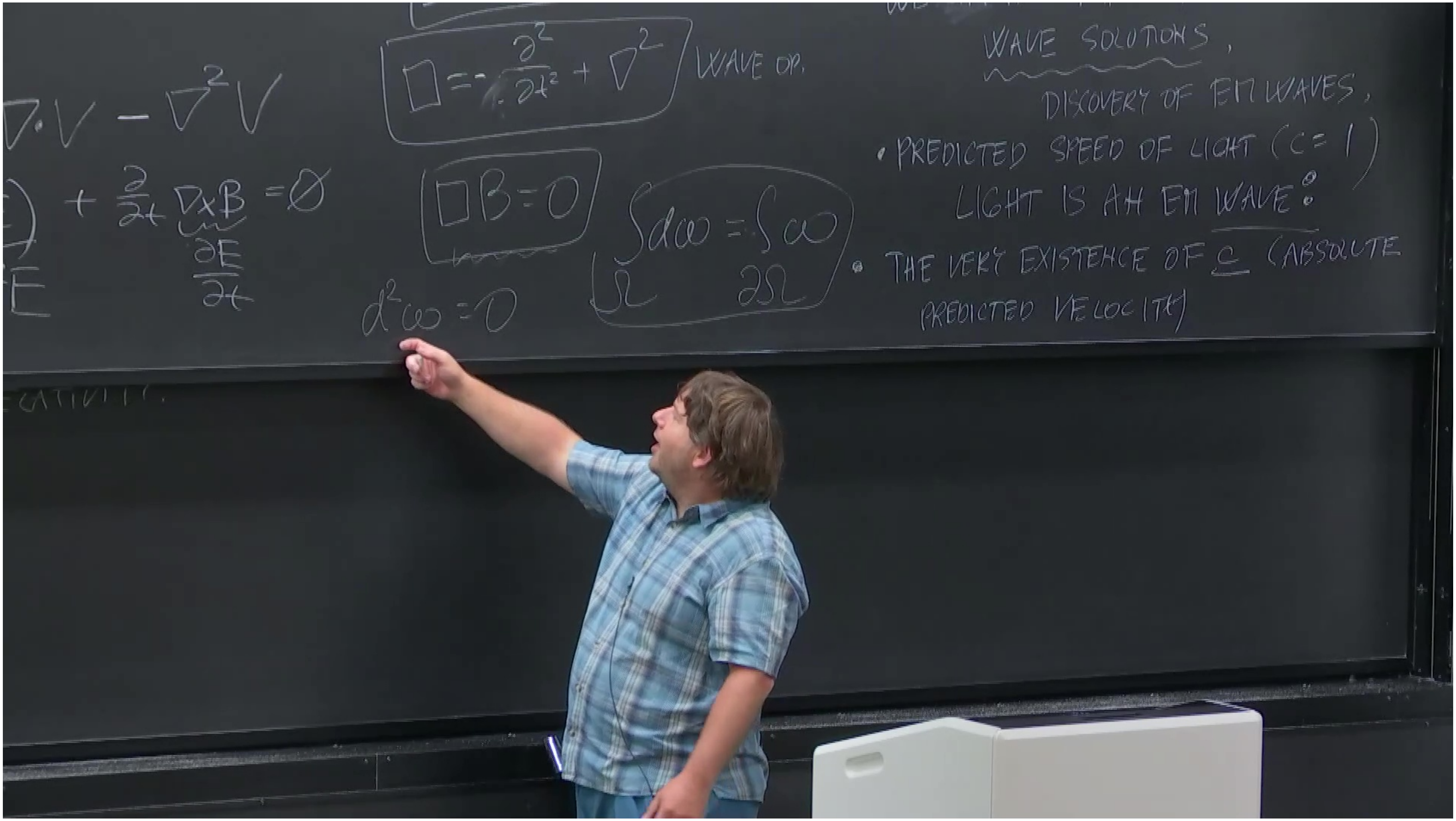
- $$\nabla \times (\nabla \times V) = \nabla \nabla \cdot V - \nabla^2 V$$

$$\nabla \times (2) = \nabla \times (\nabla \times E) + \frac{\partial}{\partial t} \nabla \times B = 0$$


CLASSICAL FIELD THEORY. MATTER (PARTICLES) & FIELD
LATER REPLACED BY FIELDS AS WELL
FIELD EQS (EOM FOR FIELD PRODUCED BY SOURCES)... MAXWELL'S EQ
EOM FOR MATTER ... LORENTZ FORCE LAW
MAXWELL'S THEORY (1860)
AMPERE & MAXWELL:
FARADAY:
GAUSS:
2 SOURCES. CHARGE DENSITY
CURRENT
IDENTITIES
CONTINUITY EQ "CONSERVATION OF CHARGE"
CHARGE
IF NO FLUX THROUGH BOUNDARY
(3) & (4) "INITIAL CONDITIONS" FOR (1) & (2).
MAXWELL'S TRIUMPH


□E = 0
□ = - ∂²/∂t² + ∇²
WAVE OP.
□B = 0
• EVEN IN THE ABSENCE OF SOURCES
WE CAN HAVE NON-TRIVIAL
WAVE SOLUTIONS,
DISCOVERY OF EM WAVES,
• PREDICTED SPEED OF LIGHT (C = 1)


DISCOVERY OF EM WAVES,
• PREDICTED SPEED OF LIGHT (c = 1)
LIGHT IS AN EM WAVE!
• THE VERY EXISTENCE OF c (ABSOLUTE
PREDICTED VELOCITY)
□B = 0


WAVE OP.
WAVE SOLUTIONS,
DISCOVERY OF EM WAVES,
• PREDICTED SPEED OF LIGHT (c = 1)
LIGHT IS AN EM WAVE!
• THE VERY EXISTENCE OF c (ABSOLUTE PREDICTED VELOCITY)

- ELECTROMAGNETIC POTENTIALS

(IN CLASSICAL THEORY... CONVENIENT TOOL)

- (4) AUTOMATICALLY SATISFIED

IF $\boxed{B = \nabla \times A}$

- (2)

$$\nabla \times E + \frac{\partial}{\partial t} \nabla \times A = \emptyset = \nabla \times \Big(\overbrace{E + \frac{\partial A}{\partial t}}^{-\nabla \phi} \Big) = \emptyset$$

$$\boxed{E = -\frac{\partial A}{\partial t} - \nabla \phi}$$

(2) . AUTOMA

CURRENT

$$\boxed{E = -\frac{\partial A}{\partial t} - \nabla\phi}$$

(2). AUTOMATICALLY SATISFIED.

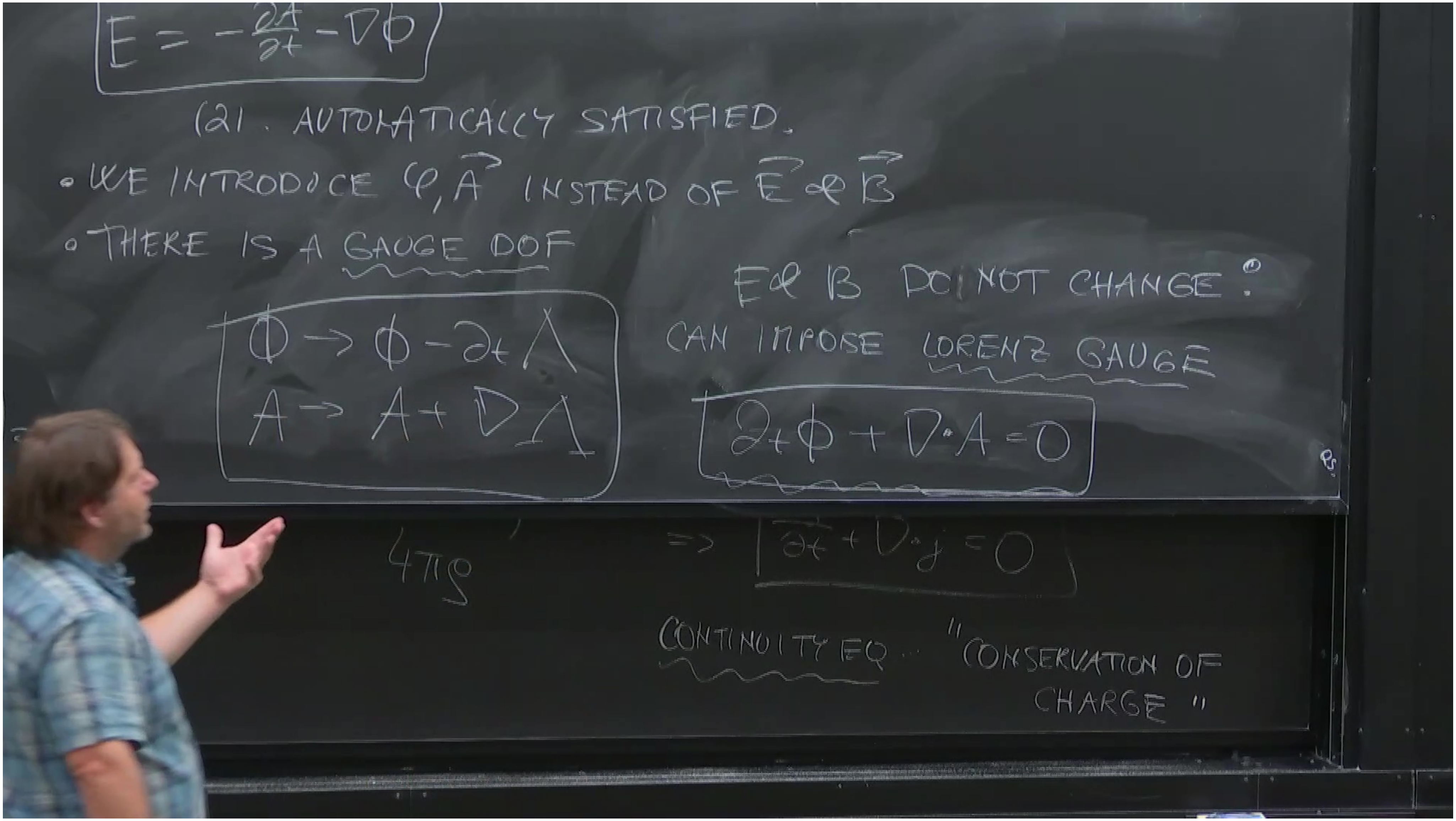
- WE INTRODUCE $\varphi, \vec{A}$ INSTEAD OF $\vec{E}$ & $\vec{B}$
- THERE IS A GAUGE DOF

E & B DO NOT CHANGE!

$$\boxed{\begin{aligned} \phi &\to \phi - \partial_t \Lambda \\ A &\to A + \nabla \Lambda \end{aligned}}$$

$= \emptyset$

CONSERVATION OF CHARGE"


$E = -\frac{\partial A}{\partial t} - \nabla \phi$
(2) . AUTOMATICALLY SATISFIED.
• WE INTRODUCE $\varphi, \vec{A}$ INSTEAD OF $\vec{E}$ & $\vec{B}$
• THERE IS A GAUGE DOF
$\phi \to \phi - \partial_t \Lambda$
$A \to A + \nabla \Lambda$
E & B DO NOT CHANGE!
CAN IMPOSE LORENZ GAUGE
$\partial_t \phi + \nabla \cdot A = 0$
$4\pi\rho$
$\Rightarrow \partial_t + \nabla \cdot j = 0$
CONTINUITY EQ. "CONSERVATION OF CHARGE"

$\partial_t \nabla_x A = \emptyset = \nabla_x \left(E + \frac{\partial A}{\partial t} \right) = \emptyset$

$A \to A$

• IF IN VACUUM ($\rho = 0 = j$) THERE REMAINS STILL SOME GAUGE FREEDOM

→ KILL ONE MORE "DOF"

ENDING UP WITH 2 TRUE DOF

(2 POLARIZATIONS OF EM WAVES)

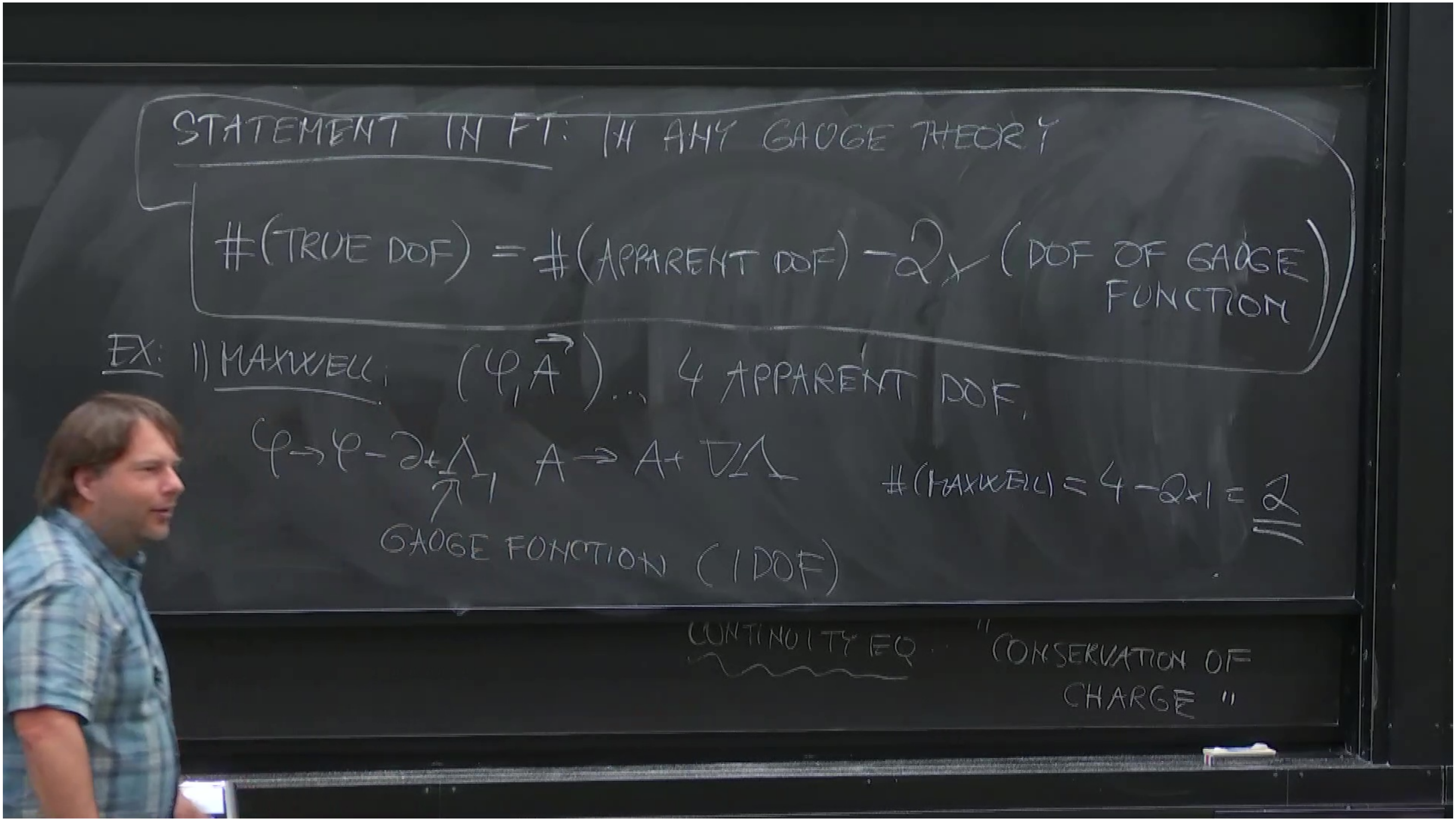
• MAXWELL'S THE

AMPERE & MAX

FARADAY

GAUSS

2 SOURCES:

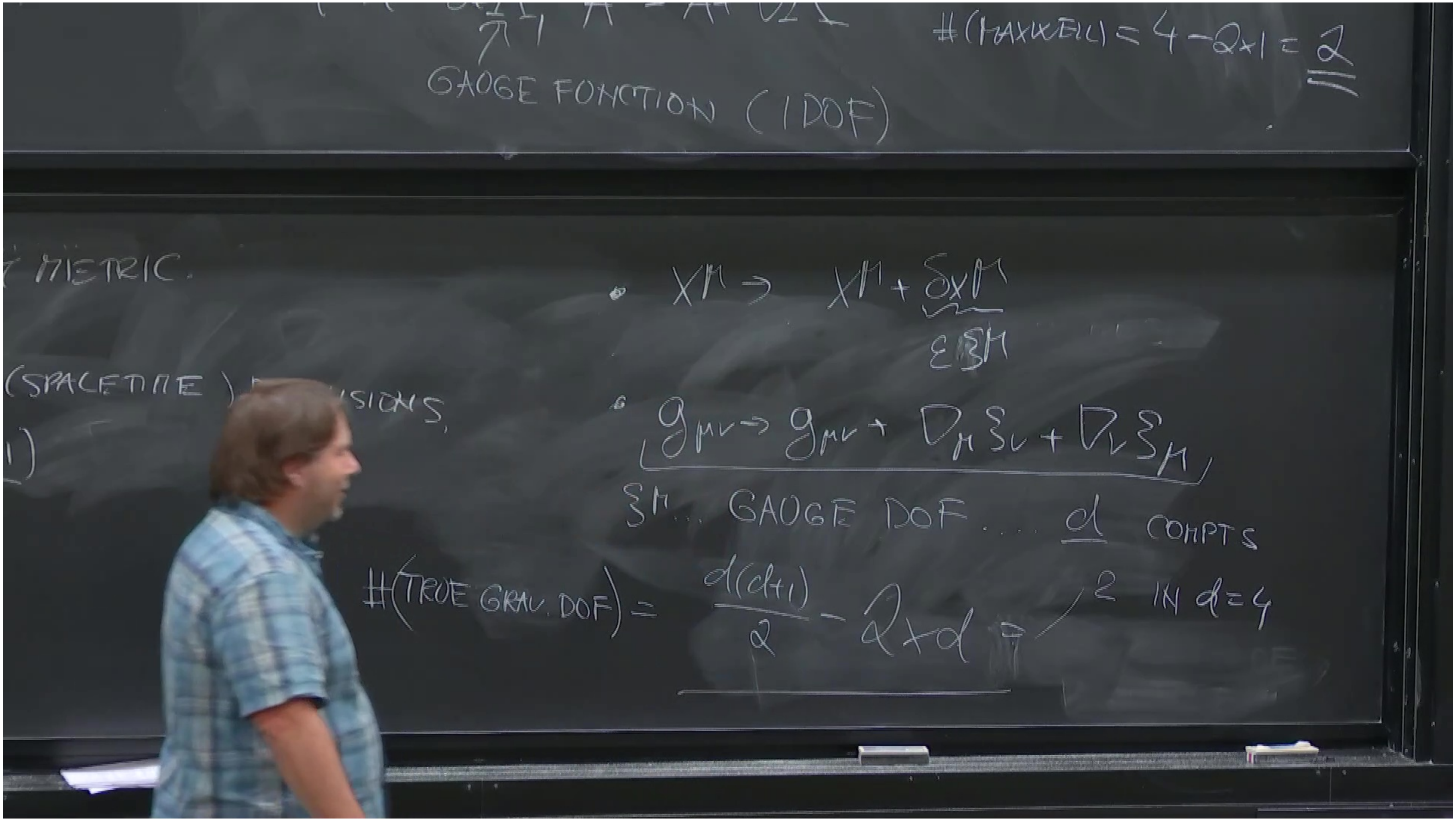

STATEMENT IN FT: IN ANY GAUGE THEORY
#(TRUE DOF) = #(APPARENT DOF) − 2 × (DOF OF GAUGE FUNCTION)
EX: 1) MAXWELL: (φ, A) ... 4 APPARENT DOF.
φ → φ − ∂tΛ, A → A + ∇Λ
GAUGE FUNCTION (1 DOF)
#(MAXWELL) = 4 − 2×1 = 2
CONTINUITY EQ
"CONSERVATION OF CHARGE"

GAUGE FUNC

ii) GRAVITATIONAL FIELD: DESCRIBED BY METRIC.

- $g_{\mu\nu}$ ... IN d - NUMBER OF (SPACETIME) DIMENSIONS.

  # APPARENT DOF $= \frac{d(d+1)}{2}$


#(MAXWELL) = 4 − 2×1 = 2
GAUGE FUNCTION (1 DOF)
METRIC.
(SPACETIME)
SIONS.
x^μ → x^μ + δx^μ
εξ^μ
g_μν → g_μν + ∇_μ ξ_ν + ∇_ν ξ_μ
ξ^μ ... GAUGE DOF .... d COMPTS
#(TRUE GRAV. DOF) = d(d+1)/2 − 2×d = 2 IN d=4

GAUGE FUNCTION (1 DOF)

$\#(\text{MAXWELL}) = 4 - 2\times 1 = \underline{\underline{2}}$

METRIC.

(SPACETIME) DIMENSIONS.

- $x^\mu \to x^\mu + \underbrace{\delta x^\mu}_{\epsilon\,\xi^\mu}$
- $\underline{g_{\mu\nu} \to g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu}$

ξ^μ ... GAUGE DOF ... $\underline{d}$ COMPTS

$\#(\text{TRUE GRAV. DOF}) = \frac{d(d+1)}{2} - 2\times d = \begin{cases} 2 & \text{IN } d=4 \\ 5 & \text{IN } d=5 \\ 0 & \text{IN } d=3 \end{cases}$