Title: Classical Physics - Lecture 220906

Speakers:

Collection: Classical Physics (2022/2023)

Date: September 06, 2022 - 9:00 AM

URL: https://pirsa.org/22090044

CLASSICAL PHYSICS

1) LAGRANGIAN MECHANICS

a) <u>HAMILTON'S PRINCIPLE OF LEAST ACTION</u>

- CHOOSE GENERALIZED COORDINATES q^i, $i = 1, \dots, n$

  ... AS MANY AS # DOF

CLASSICAL PHYSICS

1) LAGRANGIAN MECHANICS

a) HAMILTON'S PRINCIPLE OF LEAST ACTION

- CHOOSE GENERALIZED COORDINATES q^i, $i = 1, \ldots, n$

... AS MANY AS # DOF

CLASSICAL PHYSICS

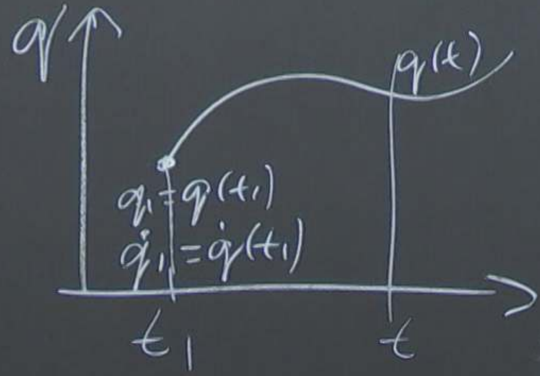
1) LAGRANGIAN MECHANICS

a) <u>HAMILTON'S PRINCIPLE OF LEAST ACTION</u>

- CHOOSE GENERALIZED COORDINATES q^i, $i = 1, \dots, n$

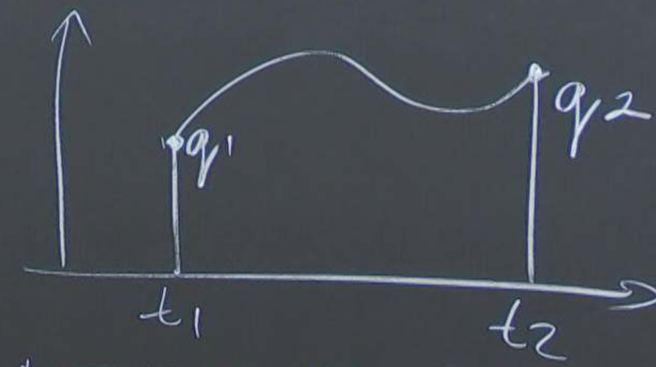
  ... AS MANY AS # DOF ... TO FULLY DESCRIBE "PICTURE" ... CONFIGURATION SPACE

INITIAL VALUE PR



$\Longleftrightarrow$

"BOUNDARY VALUE PR"



NATURAL IN QM.

$1, \dots, n$

ESCRIBE

TH: PRINCIPLE OF LEAST ACTION (HAMILTON)

MOTION OF MECH SYSTEM IN TIME INTERVAL $t \in (t_1, t_2)$ COINCIDES WITH THE EXTREMUM OF THE ACTION FUNCTIONAL

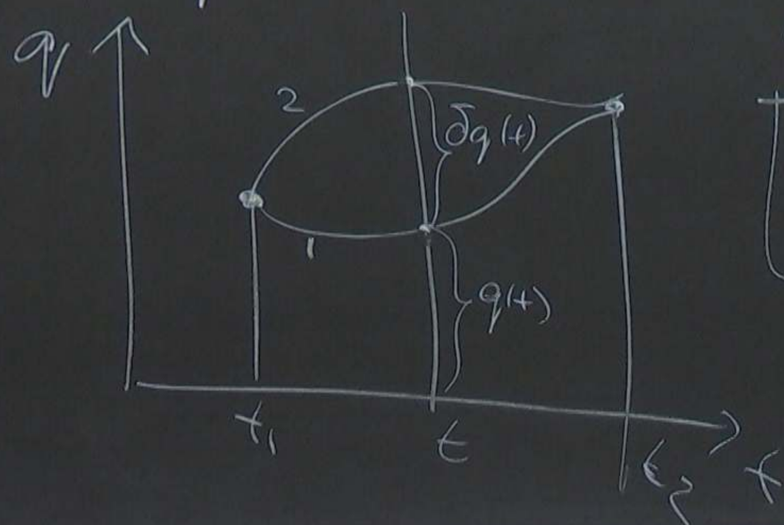
$$S = S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$$

LAGRANGIAN ($\simeq$ DESCR. OF SYSTEM)

- HOW DOES IT WORK?

TO DERIVE EOM, CONSIDER "FIXED END POINTS"

$$\delta q(t_1) = 0 = \delta q(t_2)$$



$$\delta S = S[q(t) + \delta q(t)] - S[q(t)] \equiv 0$$

$$\boxed{\frac{d}{dt}\delta = \delta \frac{d}{dt}}$$

"ORTHOGONAL DIRECTIONS"

SYSTEM

$$\partial S = 0 = \int_{t_1}^{t_2} \delta L \, dt = \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot q} \delta \dot q \right) dt$$

BY PARTS

$$= \int \left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q} \right) \right] \delta q \, dt$$

$$+ \underbrace{\left[\frac{\partial L}{\partial \dot q} \delta q \right]_{t_1}^{t_2}}_{0}$$

$$\delta \frac{d}{dt} q = \frac{d}{dt} \delta q$$

$$\boxed{\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q} \right) = 0}$$

EULER-LAGRANGE EQ

DIRECTIONS"

REMARKS:

1) $L = L(q, \dot{q}, t) \Rightarrow n$ .. 2ND-ORDER EOM

2) $L = T - V$ (SEE L&L)

T: KINETIC

V: POTENTIAL

3) $L \to L' = L(q, \dot{q}, t) + \dfrac{d\Lambda(q,t)}{dt}$

IS THE "FREEDOM" IN DESCRIPTION

PROOF:

$$S' = \int L' \, dt = \underbrace{\int L \, dt}_{S} + \int \frac{d\Lambda}{dt} \, dt$$

$$\delta S' = \delta S + \underbrace{\int \delta \frac{d\Lambda}{dt} \, dt}_{\underbrace{\int \frac{d}{dt} \delta \Lambda \, dt}_{\left[\frac{\partial \Lambda}{\partial q} \delta q \right]_{t_1}^{t_2} = 0}} = \delta S$$

b) INTEGRALS OF MOTION

SOLVING EOM
- NUMERICALLY
- PERTURBAT
- ANALYTICALLY (INTEGRABLE SYSTEMS) (LOTS OF INTEGRALS OF MOTION)

DEF: INTEGRAL (CONSTANT) OF MOTION $I = I(q, \dot{q}, t)$

$$\boxed{\text{OBEYS } \frac{dI}{dt} = 0 \text{ FOR ANY } q(t) \text{ SOLVING EOM}}$$

b) INTEGRALS OF MOTION

SOLVING EOM <
- NUMERICALLY
- PERTURBAT
- ANALYTICALLY (INTEGRABLE SYSTEMS) (LOTS OF INTEGRALS OF MOTION)

DEF: INTEGRAL (CONSTANT) OF MOTION $I = I(q, \dot{q}, t)$

$$\boxed{\text{OBEYS} \quad \frac{dI}{dt} = 0 \quad \text{FOR ANY } q(t) \text{ SOLVING EOM}}$$

NOETHER'S THEOREM (VI - GENERAL)

FOR EVERY GLOBAL CONTINUOUS SYMMETRY OF SYSTEM, THERE IS A CORRESPONDING INTEGRAL OF MOTION

"GEOMETRIZES PHYSICS"

- TERMINOLOGY

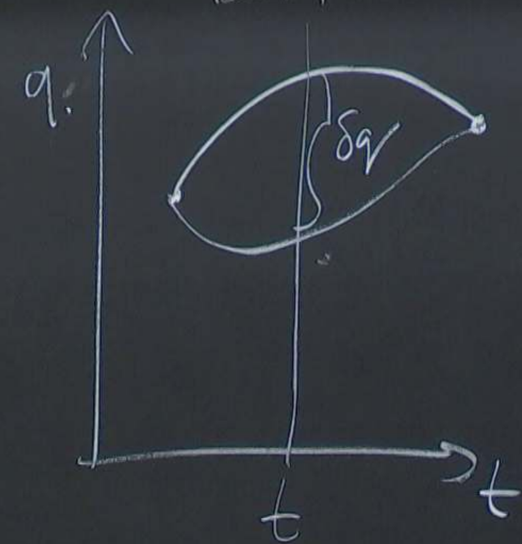
STATEMENT VALID
- ON-SHELL ... PROVIDED EOM ARE SATISFIED
- OFF-SHELL ... TRUE NO MATTER WHAT

• TYPES OF SYMMETRIES

CONSIDER A TRANSFORMATION

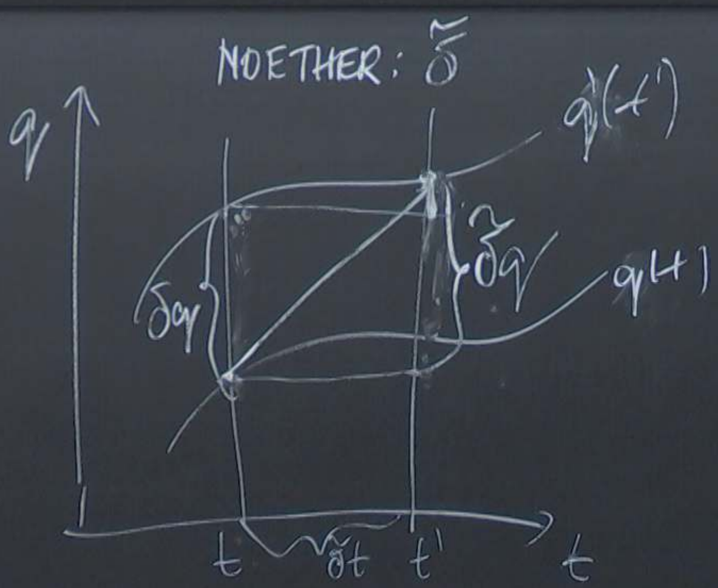
$$\boxed{\begin{aligned} & t \to t' = t + \tilde{\delta} t \\ & q \to q'(t') = q(t) + \tilde{\delta} q(t) \end{aligned}}$$

EOM: δ



FIXED END POINTS

NOETHER: $\tilde{\delta}$

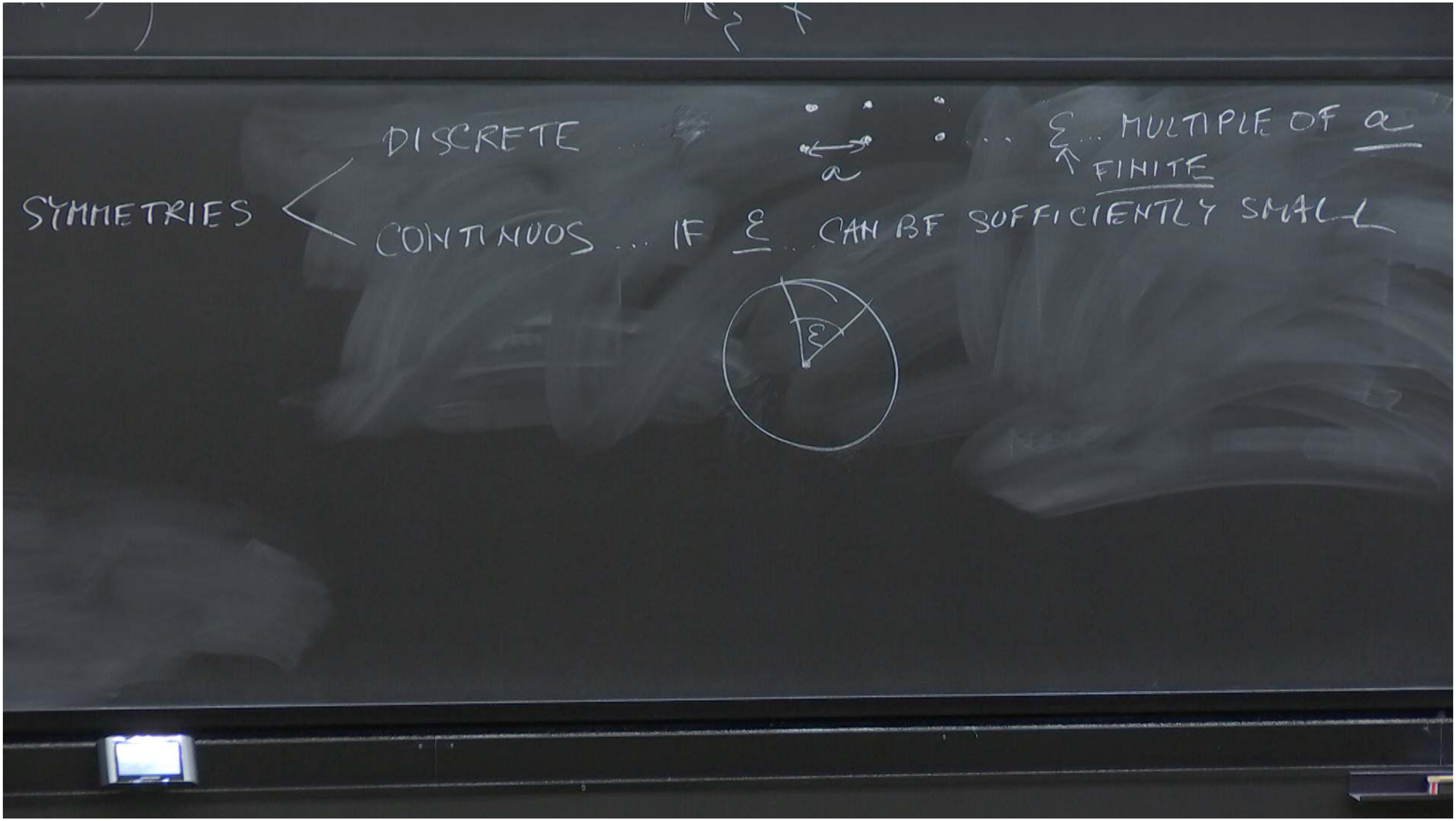


NO FIXED ENDPOINTS

$$\boxed{\tilde{\delta} q = \delta q + \frac{dq}{dt}\tilde{\delta} t}$$

MORE GENERALLY ($q \to f$)

$$\boxed{\tilde{\delta} = \delta + \tilde{\delta} t \frac{d}{dt}}$$


DISCRETE
ε ... MULTIPLE OF a
a
FINITE
SYMMETRIES
CONTINUOS ... IF ε CAN BE SUFFICIENTLY SMALL
ε

NEXT, WRITE

$$\boxed{\begin{aligned} \tilde{\delta} q &= \varepsilon \, \Delta q \\ \tilde{\delta} t &= \varepsilon \, \Delta t \end{aligned}}$$

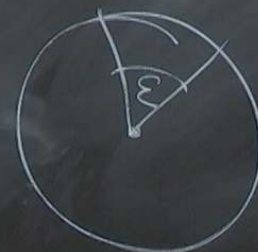
"GENERATORS"
(WHAT I AM DOING)

PARAMETER
(HOW MUCH YOU MOVE)

SYMMETRIES <

SYMMETRIES

- DISCRETE ... ε ... MULTIPLE OF a — FINITE
- CONTINUOS ... IF ε CAN BE SUFFICIENTLY SMALL

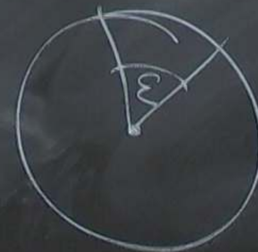


SYMMETRIES

- GLOBAL $\varepsilon = \text{CONST}$
- LOCAL ... $\varepsilon(t)$ ... GAUGE SYMMETRIES

SYMMETRIES

- DISCRETE ... ε ... MULTIPLE OF a ($\leftrightarrow$ a); ε FINITE
- CONTINUOS ... IF ε CAN BE SUFFICIENTLY SMALL



SYMMETRIES

- GLOBAL $\varepsilon = \text{CONST}$
- LOCAL ... $\varepsilon(t)$ ... GAUGE SYMMETRIES

(2ND NOETHER'S TH. → BIANCHI IDENTITIES)

NOETHER'S TH (V2 - EXPLICIT)

LET $\tilde{\delta}$ BE A GLOBAL CONT. SYMMETRY, THAT IS, OFF-SHELL WE FIND $\tilde{\delta} q$ & $\tilde{\delta} t$ SUCH THAT

$$\tilde{\delta} S = 0 \Rightarrow I = \frac{\partial L}{\partial \dot{q}} \tilde{\delta} q + \left(L - \dot{q} \frac{\partial L}{\partial \dot{q}} \right) \tilde{\delta} t$$

IS AN (ON-SHELL) INTEGRAL OF MOTION

EXAMPLES: 1) TIME TRANSL. SYMMETRY ... $L \neq L(t)$

THAT IS: $\tilde{\delta} t = \varepsilon = \text{CONST}, \quad \tilde{\delta} q = 0$

$\Rightarrow \tilde{\delta} S = 0$ .. A SYMMETRY,

$$\boxed{I = E = \frac{\partial L}{\partial \dot{q}^i}\dot{q}^i - L}$$

GENERALIZED ENERGY

IDENTITIES)

EXAMPLES: 1) TIME TRANSL. SYMMETRY ... $L \neq L(t)$

THAT IS: $\tilde{\delta} t = \varepsilon = \text{CONST}$, $\tilde{\delta} q = 0$

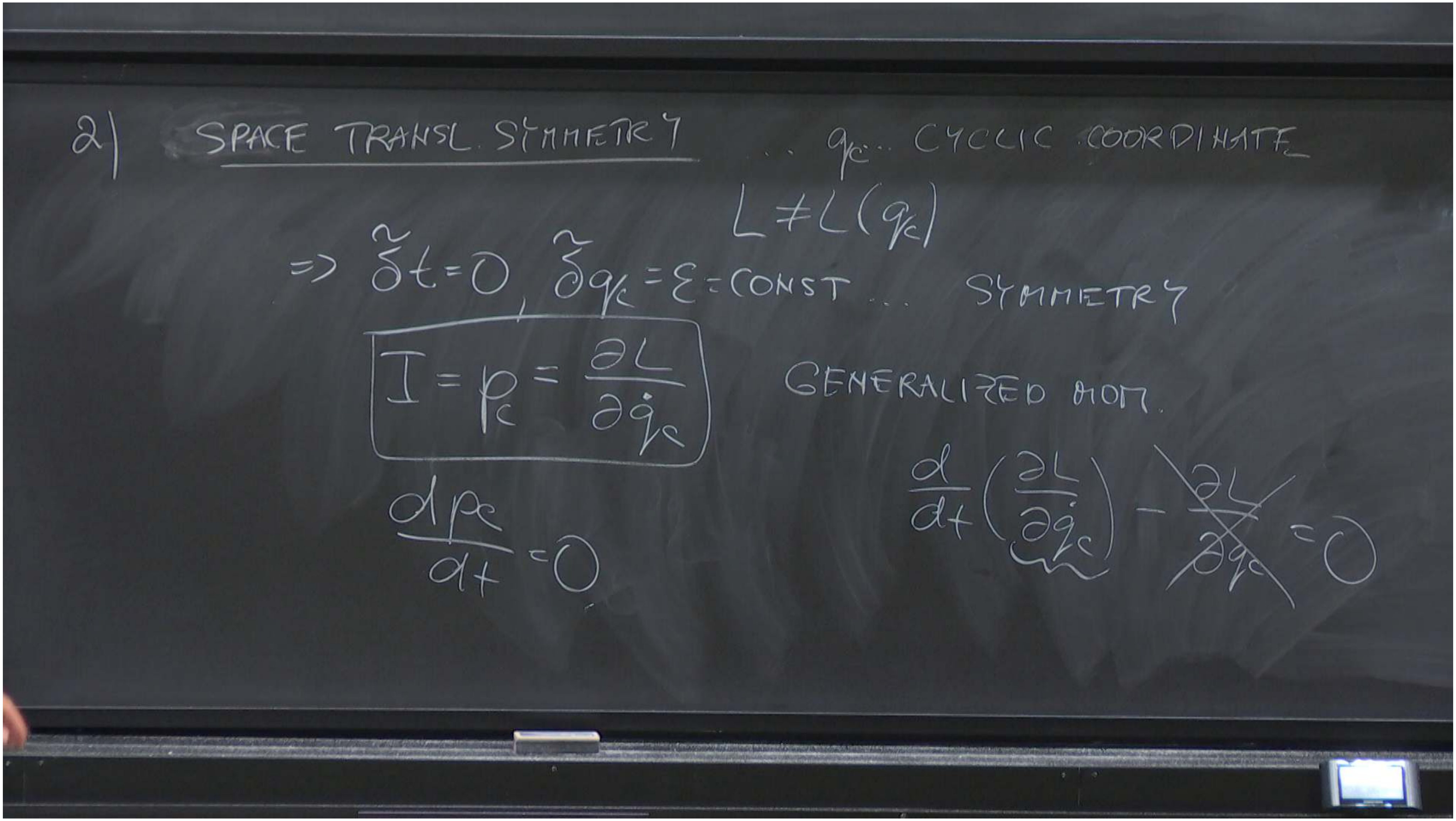
$\Rightarrow \tilde{\delta} S = 0$ ... A SYMMETRY.

$$\boxed{I = E = \frac{\partial L}{\partial \dot{q}^i} \dot{q}^i - L}$$

GENERALIZED ENERGY

EX: $\frac{dE}{dt} = 0$

REMARKS:


2) SPACE TRANSL. SYMMETRY
... q_c ... CYCLIC COORDINATE
$L \neq L(q_c)$
$\Rightarrow \tilde{\delta} t = 0, \; \tilde{\delta} q_c = \varepsilon = \mathrm{CONST}$ ... SYMMETRY
$\boxed{I = p_c = \frac{\partial L}{\partial \dot{q}_c}}$
GENERALIZED MOM.
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_c}\right) - \cancel{\frac{\partial L}{\partial q_c}} = 0$
$\frac{dp_c}{dt} = 0$