Title: Quantum Theory - Lecture 220928

Speakers: Dan Wohns

Collection: Quantum Theory (2022-2023)

Date: September 28, 2022 - 10:45 AM

URL: https://pirsa.org/22090043

Feynman Diagrams

$$\langle f|S|i\rangle \xleftrightarrow{\text{LSZ}} \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \xleftrightarrow{\text{Dyson}} \langle 0|T\varphi_1\cdots\varphi_n|0\rangle \xleftrightarrow{\text{Wick}} \Delta_F$$

Feynman diagrams

lines

$\longleftrightarrow \Delta_F$

Develop: rules for which diagrams to draw

+ how to assign complex number to individual diagrams

Δ_F

l diagrams

$$\langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle = \frac{\langle 0 | T \varphi_1 \cdots \varphi_n \exp\left[i \int d^4 \mathcal{L}_{int} \right] | 0 \rangle}{\langle 0 | T \exp\left[i \int d^4 x \, \mathcal{L}_{int} \right] | 0 \rangle}$$

Δ_F

l diagrams

$$\langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle = \frac{\langle 0 | T \varphi_1 \cdots \varphi_n \exp[i \int d^4 \mathcal{L}_{int}] | 0 \rangle}{\langle 0 | T \exp[i \int d^4 x \, \mathcal{L}_{int}] | 0 \rangle}$$

coupling constant

$$\mathcal{L}_{int} = -\frac{\lambda}{4!} \varphi^4$$

ΔF

l diagrams

$$\langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle = \frac{\langle 0 | T \varphi_1 \cdots \varphi_n \exp[i \int d^4 \mathcal{L}_{int}] | 0 \rangle}{\langle 0 | T \exp[i \int d^4 x \, \mathcal{L}_{int}] | 0 \rangle}$$

coupling constant

$$\mathcal{L}_{int} = -\frac{\lambda}{4!} \varphi^4$$

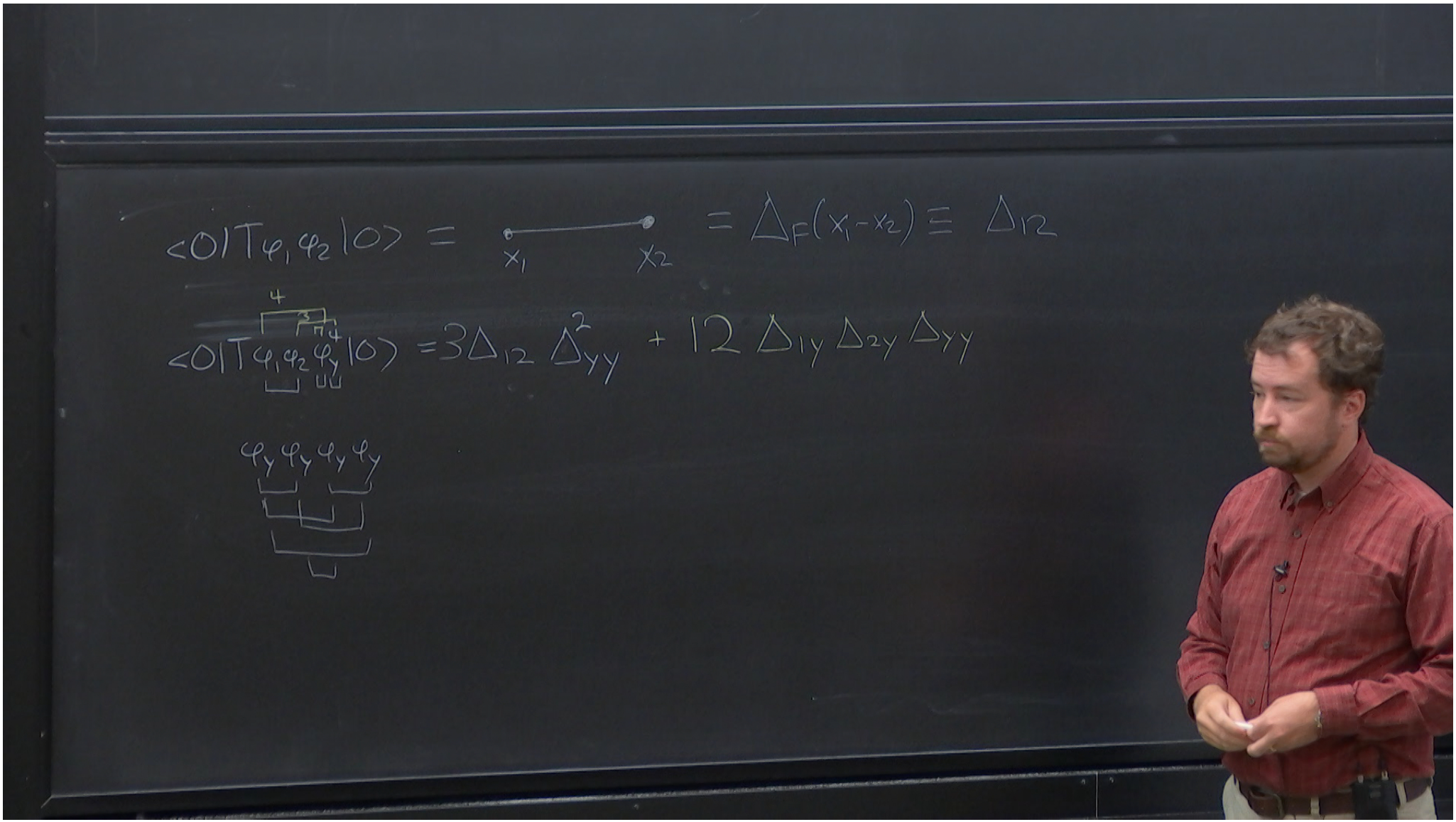
$$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle = \frac{\text{num}}{\text{den}}$$

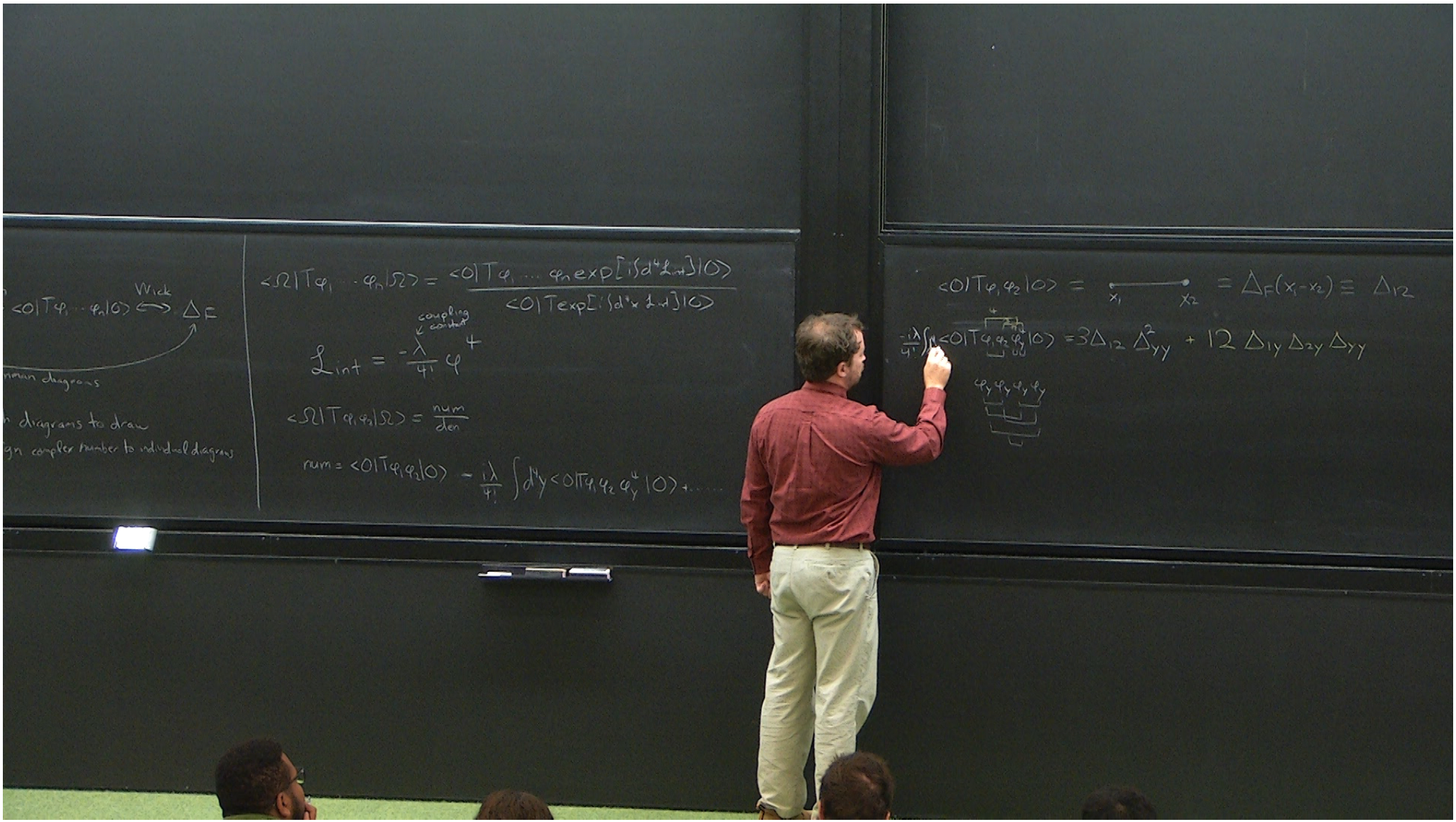
$$\text{num} = \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle + \frac{i\lambda}{4!} \int d^4 y \, \langle 0 | T \varphi_1 \varphi_2 \varphi_y^4 | 0 \rangle + \cdots\cdots$$

$$\langle 0|T\varphi_1\varphi_2|0\rangle = \underset{x_1}{\bullet}\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\underset{x_2}{\bullet} = \Delta_F(x_1-x_2) \equiv \Delta_{12}$$

$$\langle 0|T\varphi_1\varphi_2\varphi_y\varphi_y|0\rangle = 3\Delta_{12}\Delta_{yy}^2 + 12\Delta_{1y}\Delta_{2y}\Delta_{yy}$$

$$\varphi_y\ \varphi_y\ \varphi_y\ \varphi_y$$




Wick
diagrams to draw
coupling constant
$\langle\Omega|T\varphi_1 \cdots \varphi_n|\Omega\rangle = \frac{\langle 0|T\varphi_1 \cdots \varphi_n \exp[i\int d^4x\, \mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\, \mathcal{L}_{int}]|0\rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$
$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{num}{den}$
$num = \langle 0|T\varphi_1\varphi_2|0\rangle - \frac{i\lambda}{4!}\int d^4y \langle 0|T\varphi_1\varphi_2\varphi_y^4|0\rangle + \cdots$
$\langle 0|T\varphi_1\varphi_2|0\rangle = \Delta_F(x_1-x_2) \equiv \Delta_{12}$
x_1
x_2
$-\frac{i\lambda}{4!}\int d^4y \langle 0|T\varphi_1\varphi_2\varphi_y^4|0\rangle = 3\Delta_{12}\Delta_{yy}^2 + 12\Delta_{1y}\Delta_{2y}\Delta_{yy}$

Δ_F

diagrams

$$\langle\Omega|T\varphi_1 \cdots \varphi_n|\Omega\rangle = \frac{\langle 0|T\varphi_1 \cdots \varphi_n \exp[i\int d^4 \mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\, \mathcal{L}_{int}]|0\rangle}$$

coupling constant

$$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$$

$$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\text{num}}{\text{den}}$$

$$\text{num} = \langle 0|T\varphi_1\varphi_2|0\rangle - \frac{i\lambda}{4!}\int d^4y \langle 0|T\varphi_1\varphi_2\varphi_y^4|0\rangle + \cdots\cdots$$

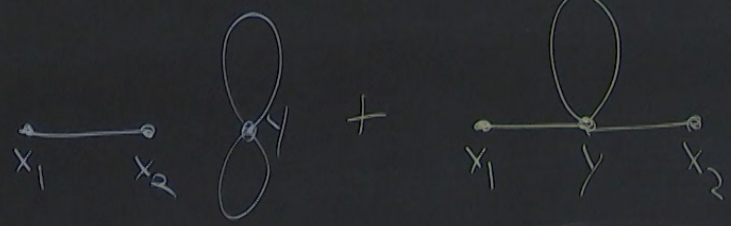
$$\langle 0|T\varphi_1\varphi_2|0\rangle =$$

$$= \Delta_F(x_1-x_2) \equiv \Delta_{12}$$

$$\frac{-i\lambda}{4!}\int d^4y \langle 0|T\varphi_1\varphi_2\varphi_y\varphi_y\varphi_y\varphi_y|0\rangle = \int\left(3\Delta_{12}\Delta_{yy}^2 + 12\Delta_{1y}\Delta_{2y}\Delta_{yy}\right)d^4y \cdot \frac{-i\lambda}{4!}$$

$$\varphi_y\varphi_y\varphi_y\varphi_y$$

$=$

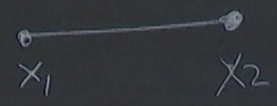


Feynma

Feynman rules for numerator

$$\langle 0 | \varphi_1 \cdots \varphi_n \exp\left[i \int d^4x \, \mathcal{L}_{int} \right] | 0 \rangle = \text{sum}$$

$$\langle 0|T\varphi_1\varphi_2|0\rangle =$$

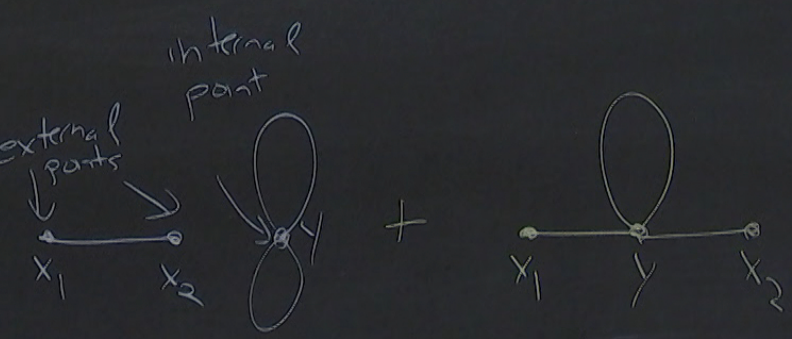


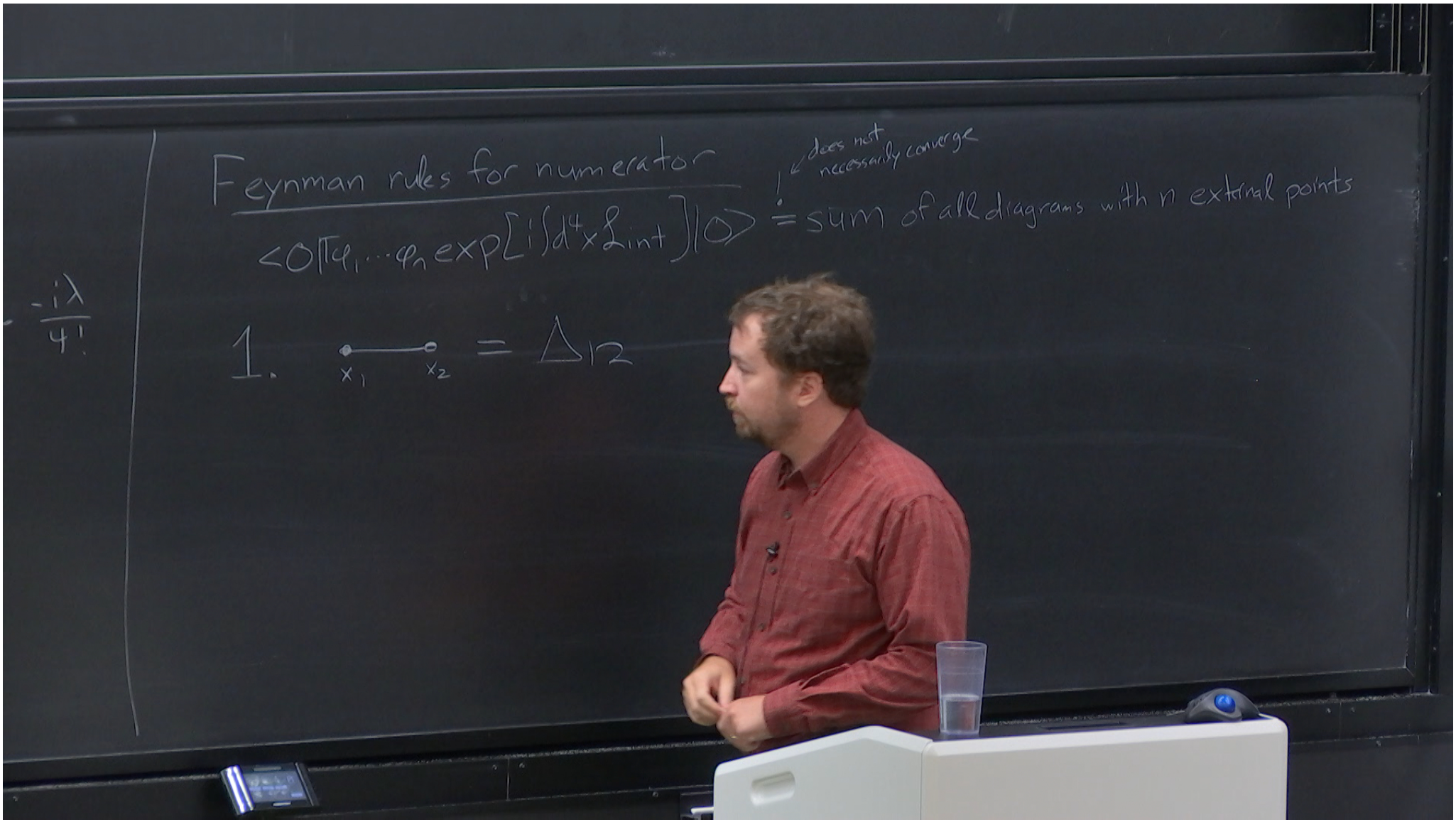
$$= \Delta_F(x_1 - x_2) \equiv \Delta_{12}$$

$$\frac{-i\lambda}{4!}\int d^4y \langle 0|T\varphi_1\varphi_2\varphi_y\varphi_y|0\rangle = \int\left(3\Delta_{12}\Delta_{yy}^2 + 12\Delta_{1y}\Delta_{2y}\Delta_{yy}\right)d^4y \cdot \frac{-i\lambda}{4!}$$

$\varphi_y\,\varphi_y\,\varphi_y\,\varphi_y$

$=$



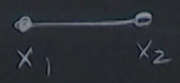

Feynman rules for numerator
does not necessarily converge
$\langle 0 | T\varphi_1 \cdots \varphi_n \exp[i\int d^4x \mathcal{L}_{int}] | 0 \rangle \doteq$ sum of all diagrams with n external points
$-\frac{i\lambda}{4!}$
1.
x_1
x_2
$= \Delta_{12}$

$-\frac{i\lambda}{4!}$

Feynman rules for numerator

$$\langle 0|T\varphi_1 \cdots \varphi_n \exp\left[i\int d^4x\, \mathcal{L}_{int}\right]|0\rangle \overset{!}{=} \text{sum of all diagrams with } n \text{ external points}$$

does not necessarily converge

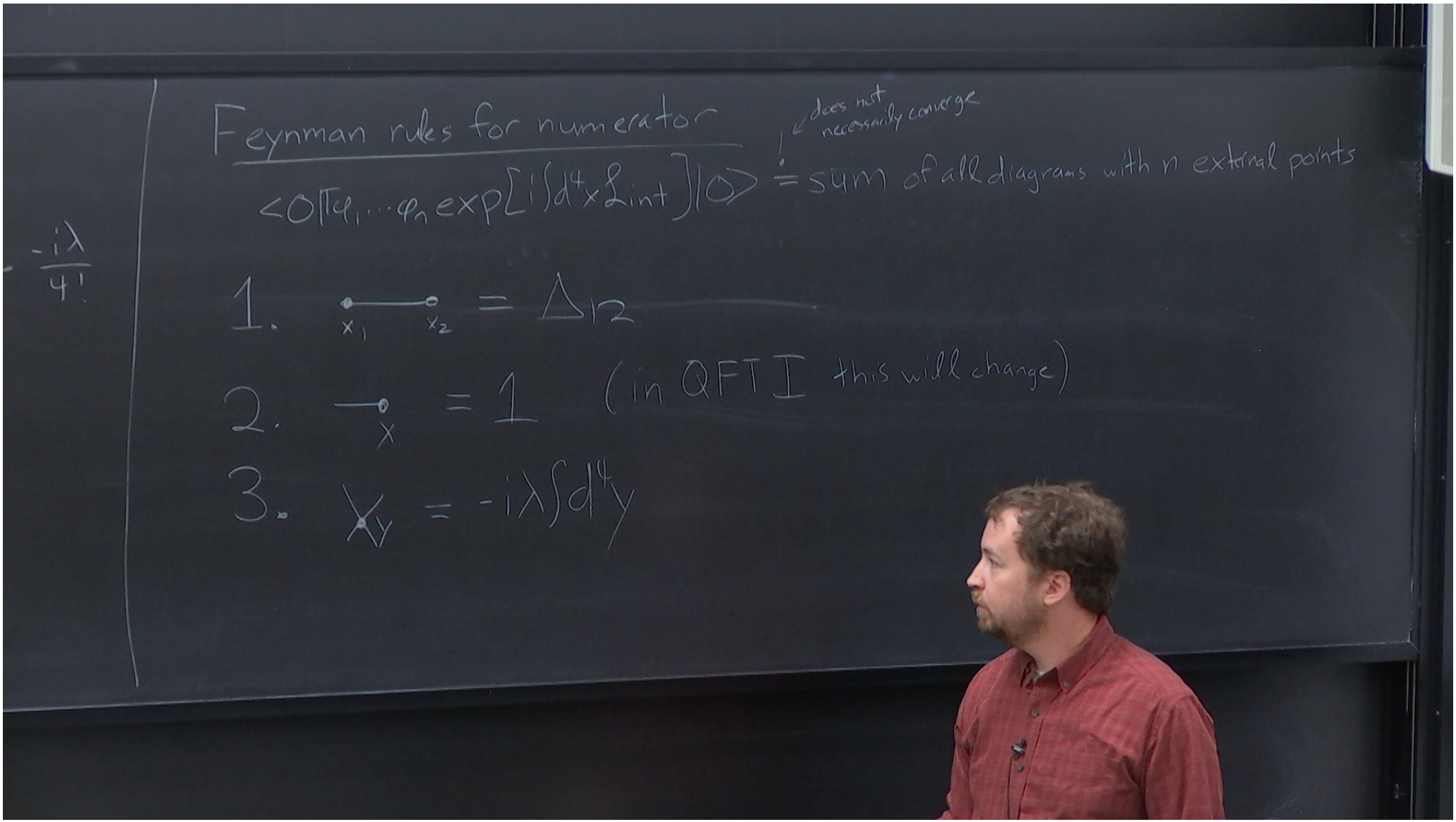
1. 

$= \Delta_{12}$

2.

$= 1$ (in QFT I this will change)

3

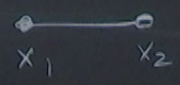

Feynman rules for numerator
does not necessarily converge
$\langle 0 | T\varphi_1 \cdots \varphi_n \exp[i\int d^4x \mathcal{L}_{int}] | 0 \rangle \overset{!}{=}$ sum of all diagrams with n external points
$-\frac{i\lambda}{4!}$
1. x_1 x_2 $= \Delta_{12}$
2. x $= 1$ (in QFT I this will change)
3. x y $= -i\lambda\int d^4y$

$-\frac{i\lambda}{4!}$

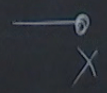
Feynman rules for numerator

$$\langle 0 | T\varphi_1 \cdots \varphi_n \exp\left[i\int d^4x\, \mathcal{L}_{int}\right] | 0 \rangle \overset{!}{=} \text{sum of all diagrams with } n \text{ external points}$$

does not necessarily converge

1. 

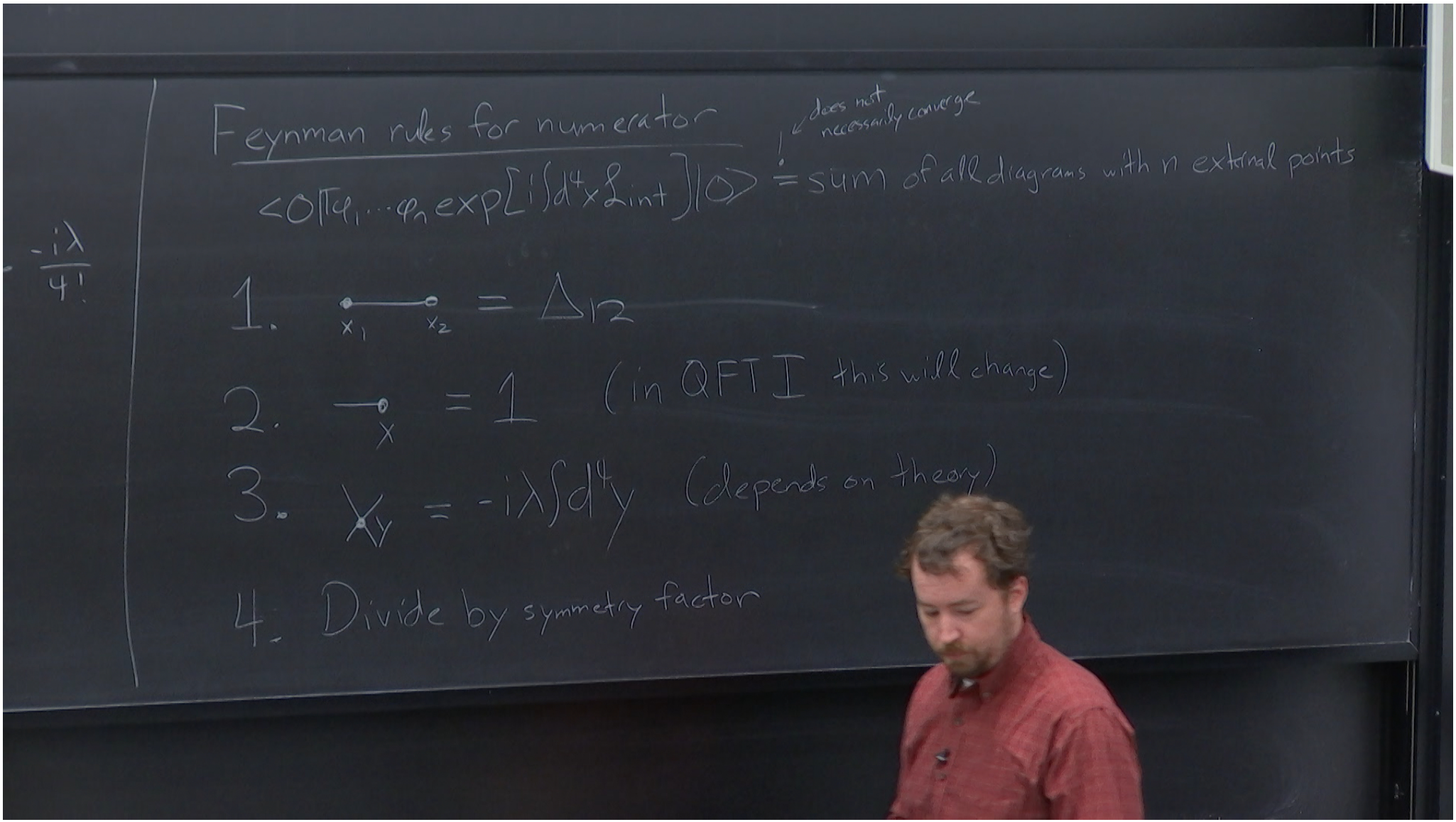
$= \Delta_{12}$

2. 

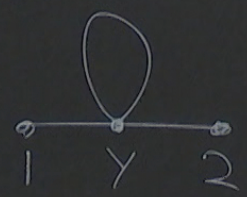
$= 1$ (in QFT I this will change)

3.

$= -i\lambda \int d^4y$ (depends on theory)


Feynman rules for numerator
does not necessarily converge
$\langle 0|T\varphi_1 \cdots \varphi_n \exp[i\int d^4x \mathcal{L}_{int}]|0\rangle \overset{!}{=}$ sum of all diagrams with n external points
$-\frac{i\lambda}{4!}$
1. x_1 x_2 $= \Delta_{12}$
2. x $= 1$ (in QFT I this will change)
3. x y $= -i\lambda\int d^4y$ (depends on theory)
4. Divide by symmetry factor

S = number of ways to map diagram to itself with external points held fixed

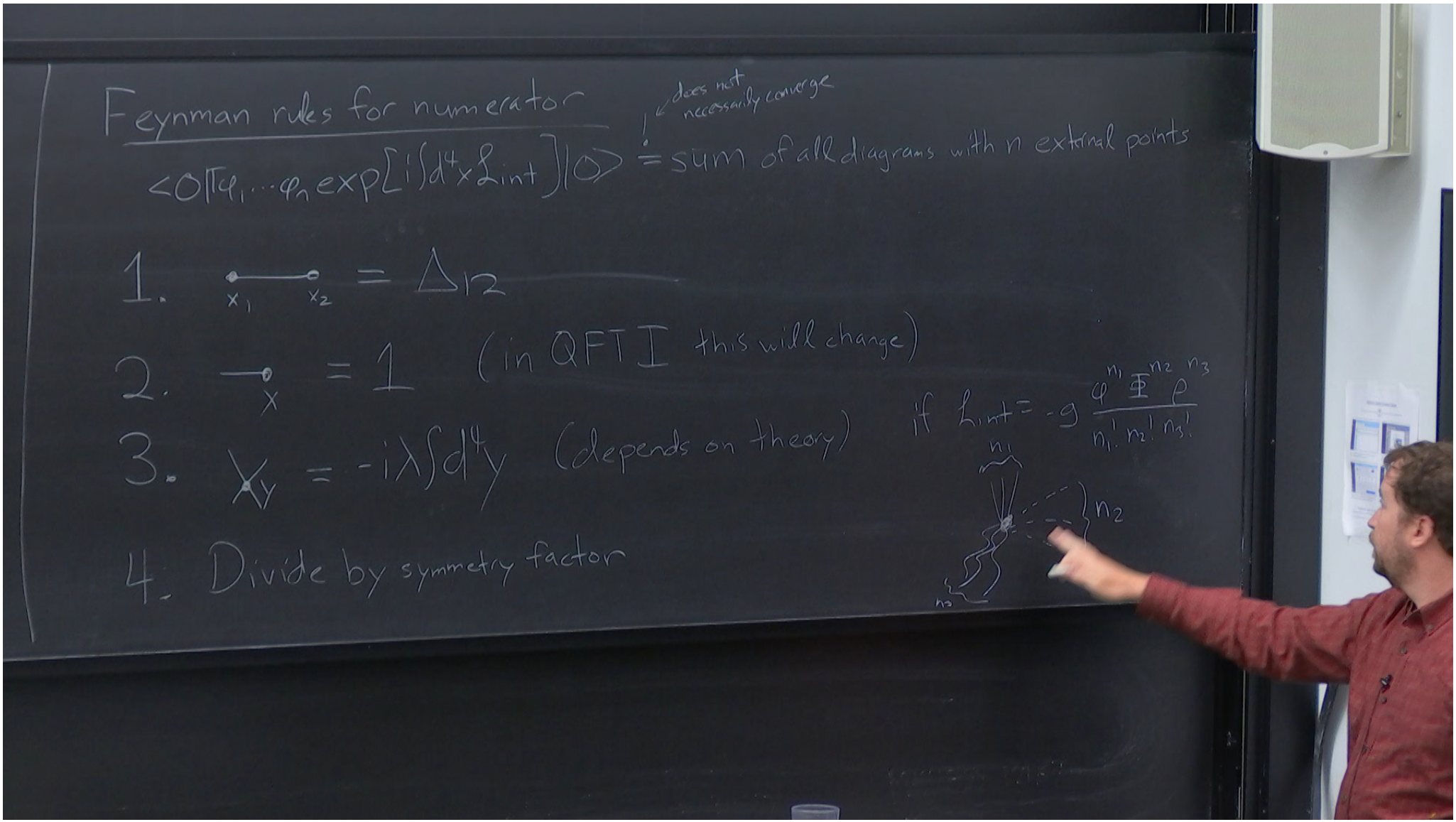


$S = 2$ swapping ends of line from y to y

$$\frac{1}{S} = \frac{1}{2} = \frac{12}{4!}$$

number of Wick contractions

comes from normalization of $\mathcal{L}_{int}$


Feynman rules for numerator
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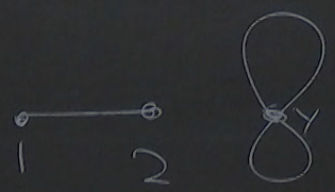
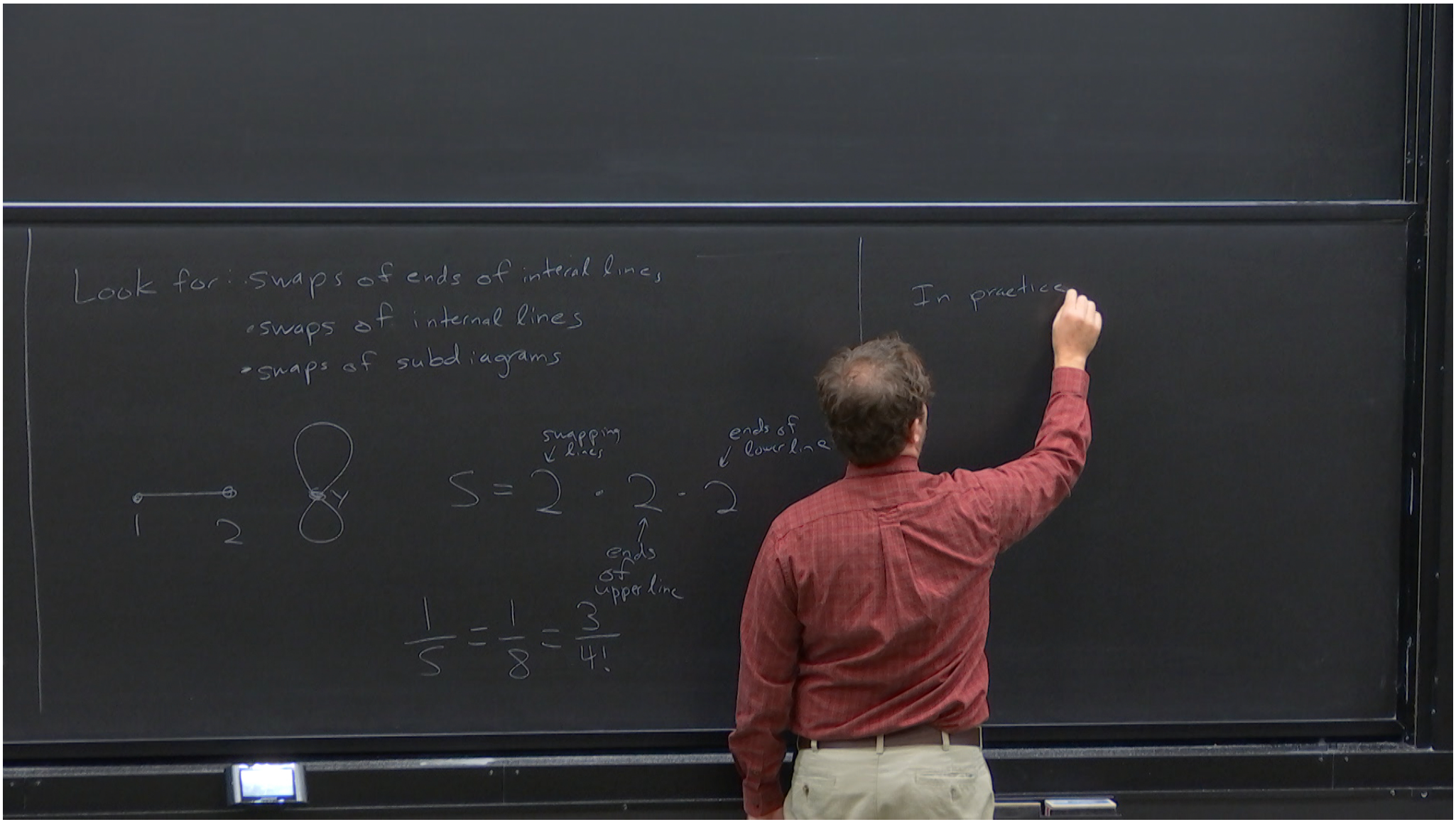
$\langle 0|T\varphi_1 \cdots \varphi_n \exp[i\int d^4x \mathcal{L}_{int}]|0\rangle \overset{!}{=}$ sum of all diagrams with n external points
1. x_1 x_2 $= \Delta_{12}$
2. x $= 1$ (in QFT I this will change)
3. x y $= -i\lambda\int d^4y$ (depends on theory)
if $\mathcal{L}_{int} = -g \frac{\varphi^{n_1} \Phi^{n_2} \rho^{n_3}}{n_1! \, n_2! \, n_3!}$
n_1
n_2
n_3
4. Divide by symmetry factor

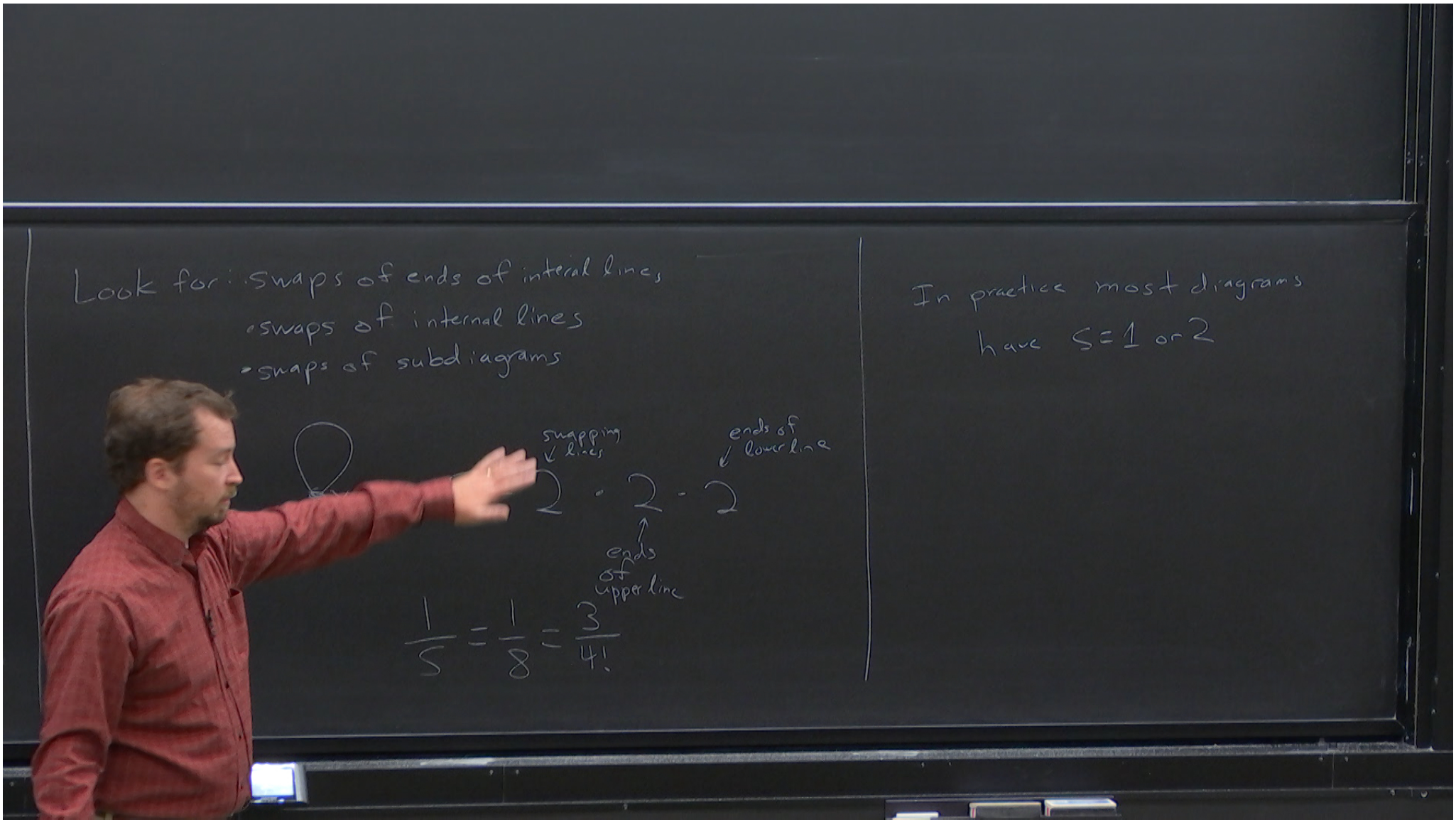
Look for: swaps of ends of interal lines
- swaps of internal lines
- swaps of subdiagrams



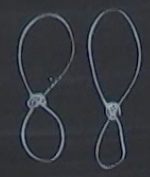
$$S = \underbrace{2}_{\text{swapping lines}} \cdot \underbrace{2}_{\text{ends of upper line}} \cdot \underbrace{2}_{\text{ends of lower line}}$$

$$\frac{1}{S} = \frac{1}{8} = \frac{3}{4!}$$

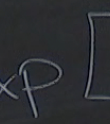

Look for: swaps of ends of interal lines
• swaps of internal lines
• swaps of subdiagrams
1
2
swapping lines
ends of lower line
S = 2 · 2 · 2
ends of upper line
1/S = 1/8 = 3/4!
In practice

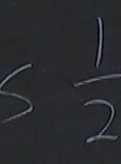

Look for: swaps of ends of interal lines
• swaps of internal lines
• swaps of subdiagrams
swapping lines
ends of lower line
2 · 2 · 2
ends of upper line
1/5 = 1/8 = 3/4!
In practice most diagrams
have S = 1 or 2

Denom: $\langle 0 | T \exp[i \int d^4x \mathcal{L}_I] | 0 \rangle$

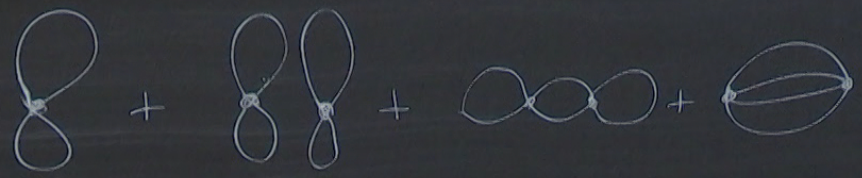
denom $= 1 +$ $+$  $+$ $+$ $+ \cdots$

$= \exp[$connected vacuum diagrams$]$

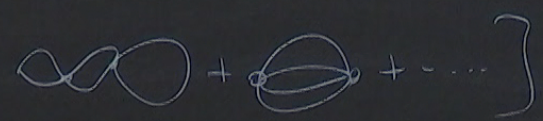
$= \exp[$  $+$ $+$ $+ \cdots]$

vs $\frac{1}{2!}$  ^{2}

Denom: $\langle 0 | T \exp[i \int d^4x \mathcal{L}_I] | 0 \rangle$

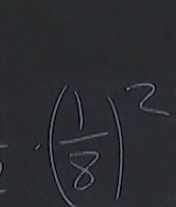
denom $= 1 +$  $+ \cdots$

$= \exp[\text{connected vacuum diagrams}]$

$= \exp[$  $+ \cdots]$

Swapping subdiagrams

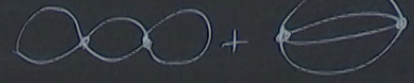
$\frac{1}{5} = \frac{1}{8^2} \frac{1}{2}$

vs $\frac{1}{2!}$  ^{2}

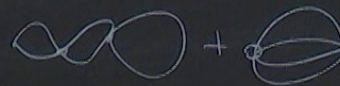
$\frac{1}{5} = \frac{1}{2} \cdot \left(\frac{1}{8}\right)^2$

Denom: $\langle 0 | T \exp[i \int d^4x \mathcal{L}_{int}] | 0 \rangle$

numerator = (

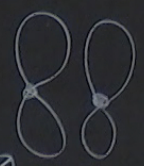
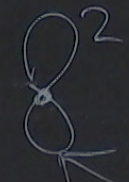
denom = 1 + + +  + ...

= exp[connected vacuum diagrams]

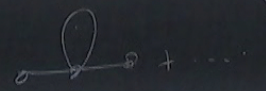
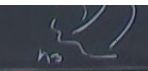
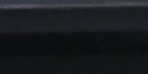
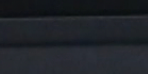
= exp[+  + ...]

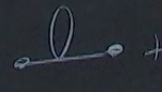
Swapping subdiagrams

$\frac{1}{5} = \frac{1}{8^2} \frac{1}{2}$

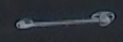
 vs $\frac{1}{2!}$  ^{2}

$\frac{1}{5} = \frac{1}{2} \cdot \left(\frac{1}{8}\right)^2$ ✓

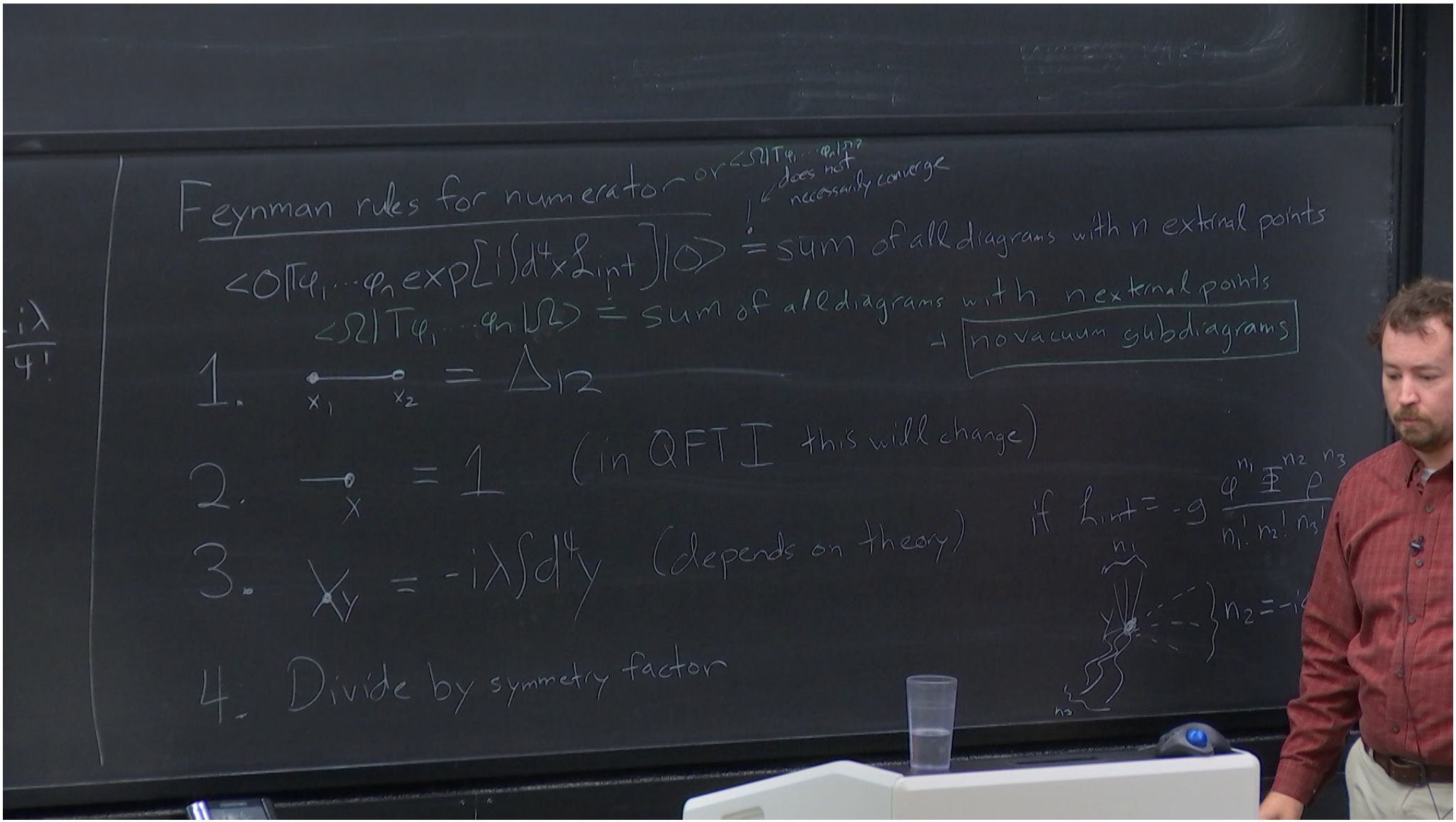
numerator = (+  + ⋯) (1 +  +  +  + + ⋯)

= (+  + ⋯) exp [connected vacuum diagrams]

cancels

$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle =$  + + ⋯

no vacuum subdiagrams


Feynman rules for numerator
or $\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle$?
does not necessarily converge
$\langle 0|T\varphi_1\cdots\varphi_n \exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle$ = sum of all diagrams with n external points
$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle$ = sum of all diagrams with n external points
+ no vacuum subdiagrams
1. x_1 x_2 = Δ_{12}
2. x = 1 (in QFT I this will change)
3. x y = $-i\lambda\int d^4y$ (depends on theory)
if $\mathcal{L}_{int} = -g\frac{\varphi^{n_1}\Phi^{n_2}\rho^{n_3}}{n_1!\,n_2!\,n_3!}$
n_1
y
n_3
$n_2 = -i$
4. Divide by symmetry factor
$\frac{i\lambda}{4!}$

$\mathcal{L}_{int}$

Feynman diagrams for $\langle f|S|i\rangle$

matrix element

$$\langle f|S|i\rangle = (2\pi)^4 \delta^{(4)}\left(\sum_{\text{initial}} p_i - \sum_{\text{final}} p_f\right) \cdot \mathcal{M}_{i\to f}$$

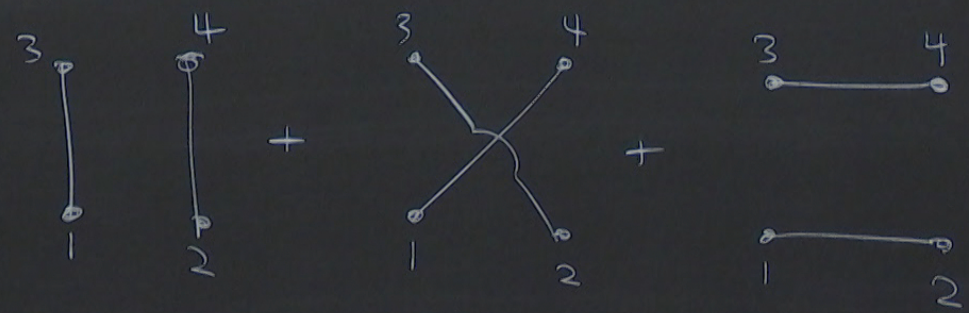
scattering amplitude

$$\langle f|S|i\rangle = i^4 \int \prod_{j=1}^{4} \left(d^4x_j \, e^{-i\lambda_j p_j \cdot x_j} (\partial_j^2 + m^2) \right) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

$$\langle f|S|i\rangle = i^4 \int \prod_{j=1}^{4} \left(d^4x_j \, e^{-i\lambda_j p_j \cdot x_j} \underbrace{(\partial_j^2 + m_{ph}^2)}_{\text{key feature}} \right) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

$$\mathcal{L}_{int} = -\frac{\lambda}{4!} \varphi^4$$

$$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$$



$$\langle f|S|i\rangle = i^4 \int \prod_{j=1}^{4} \left(d^4x_j \, e^{-i\lambda_j p_j \cdot x_j} \underbrace{(\partial_j^2 + m_{ph}^2)}_{\text{Key feature}} \right) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

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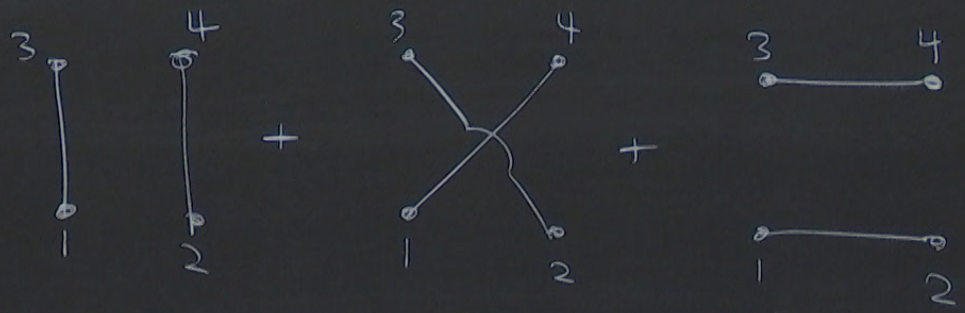

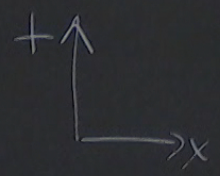

$$\langle f|S|i\rangle = i^4 \int \prod_{j=1}^{4} \left(d^4x_j \, e^{-i\lambda_j p_j \cdot x_j} \underbrace{(\partial_j^2 + m_{ph}^2)}_{\text{Key feature}} \right) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

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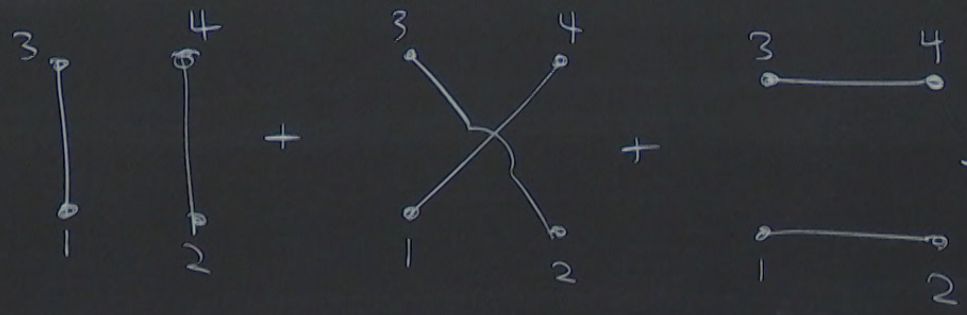
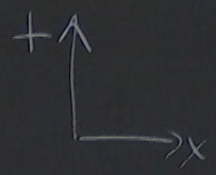


$$+ \mathcal{O}(\lambda)$$



$$\frac{1}{5} = \frac{1}{8} \cdot \frac{1}{2}$$

$$\frac{1}{5} = \frac{1}{2} \cdot \left(\frac{1}{8}\right)^2 \checkmark$$

$$F(x_1 - x_3) = (\partial_1^2 + m^2)(\partial_3^2 + m^2)\Delta_{13}$$

$$\frac{1}{5} = \frac{1}{8}\,\frac{1}{2} \nearrow \quad 00 \quad 2! \quad \frac{1}{5} = \frac{1}{2}\cdot\left(\frac{1}{8}\right)^2 \checkmark$$

$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_{13}$$

$$\int d^4x_1\, d^4x_3\, F(x_1 - x_3)\, e$$

$$\langle f|S|i\rangle = i^4 \int \prod_{j=1}^{4} \left(d^4x_j \, e^{-i\lambda_j p_j \cdot x_j} \underbrace{(\partial_j^2 + m_{ph}^2)}_{\text{key feature}} \right) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

$$\mathcal{L}_{int} = -\frac{\lambda}{4!} \varphi^4$$

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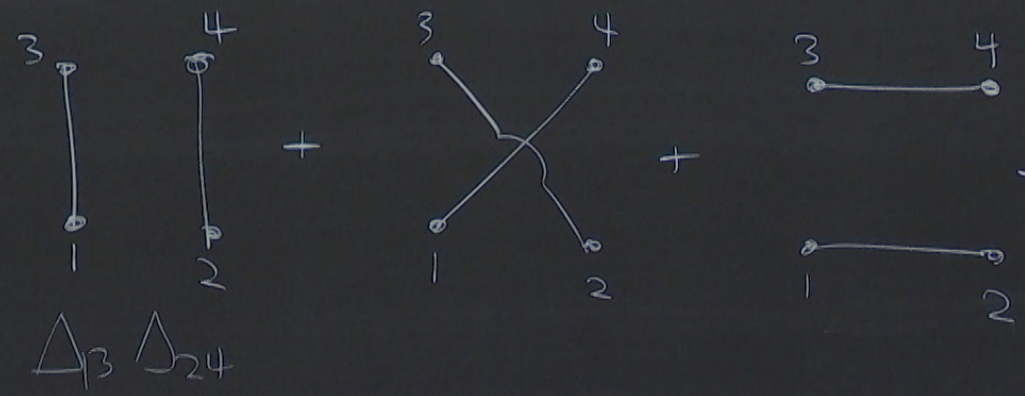
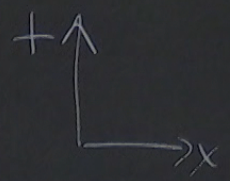
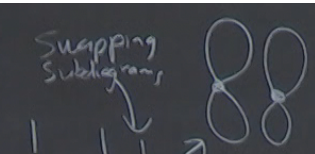


$+ \mathcal{O}(\lambda)$

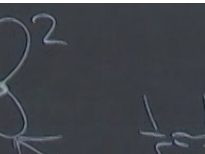


Swapping subdiagrams

$\frac{1}{5} = \frac{1}{8^2}\frac{1}{2}$



vs $\frac{1}{2^1}$



$\frac{1}{5} = \frac{1}{2}\cdot\left(\frac{1}{8}\right)^2$ ✓

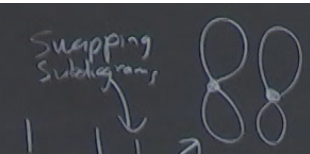
$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_{13}$$

$$\int d^4x_1\, d^4x_3\, F(x_1 - x_3)\, e^{i(x_1 \cdot p_1 - x_3 \cdot p_3)} \overset{\text{change of variables}}{=} \frac{1}{2^4}\int d^4x_{13}\, d^4\bar{x}_{13}\, F(\bar{x}_{13})\, e^{i(x_{13}\bar{p}_{13} - \bar{x}_{13}p_{13})}$$

$$x_{13} = x_1 + x_3 \qquad p_{13} = \frac{1}{2}(p_1 + p_3)$$

$$\bar{x}_{13} = x_1 - x_3 \qquad \bar{p}_{13} = \frac{1}{2}(p_1 - p_3)$$

Swapping subdiagrams,



$\frac{1}{5} = \frac{1}{8} \cdot \frac{1}{2}$

vs

$\frac{1}{2^1}$

$\frac{1}{5} = \frac{1}{2} \cdot \left(\frac{1}{8}\right)^2$ ✓

$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_{13}$$

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Swapping subdiagrams

$\frac{1}{5} = \frac{1}{8^2}\frac{1}{2}$

vs $\frac{1}{2^1}$

$\frac{1}{5} = \frac{1}{2}\cdot\left(\frac{1}{8}\right)^2$ ✓

$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_F(x_1 - x_3)$$

Jacobian

$$\int d^4x_1\, d^4x_3\, F(x_1 - x_3)\, e^{i(x_1 \cdot p_1 - x_3 \cdot p_3)} \overset{\text{change of variables}}{=} \frac{1}{2^4}\int d^4x_{13}\, d^4\bar{x}_{13}\, F(\bar{x}_{13})\, e^{i(x_{13}\bar{p}_{13} + \bar{x}_{13}p_{13})}$$

$$x_{13} = x_1 + x_3 \qquad p_{13} = \tfrac{1}{2}(p_1 + p_3)$$

$$\bar{x}_{13} = x_1 - x_3 \qquad \bar{p}_{13} = \tfrac{1}{2}(p_1 - p_3)$$

$$= \tilde{F}\left(\frac{p_1 + p_3}{2}\right)(2\pi)^4\,\delta^{(4)}(p_1 - p_3)$$

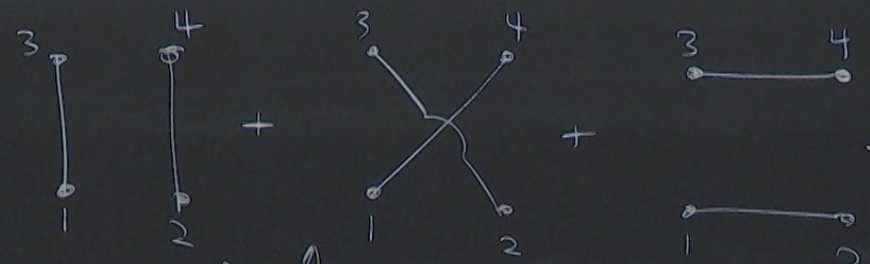
$$\frac{1}{5} = \frac{1}{8} = \frac{3}{4!}$$

element

$$\langle f|S|i\rangle = i^4 \int \prod_{j=1}^{4} \left(d^4x_j \, e^{-i\lambda_j p_j \cdot x_j} \underbrace{(\partial_j^2 + m_{ph}^2)}_{\text{key feature}} \right) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

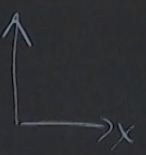
$$\mathcal{L}_{int} = -\frac{\lambda}{4!} \varphi^4$$

$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$



$+ \mathcal{O}(\lambda)$

$+$



$\Delta_{13} \Delta_{24}$

no scattering – 2 deltas

Swapping subdiagrams

$\frac{1}{5} = \frac{1}{8^2}\frac{1}{2}$ vs $\frac{1}{2!}$ $\frac{1}{5} = \frac{1}{2}\cdot\left(\frac{1}{8}\right)^2$ ✓

$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_F(x_1 - x_3)$$

Jacobian

$$\int d^4x_1\, d^4x_3\, F(x_1 - x_3)\, e^{i(x_1 \cdot p_1 - x_3 \cdot p_3)} \overset{\text{change of variables}}{=} \frac{1}{2^4}\int d^4x_{13}\, d^4\bar{x}_{13}\, F(\bar{x}_{13})\, e^{i(x_{13}\bar{p}_{13} + \bar{x}_{13}p_{13})}$$

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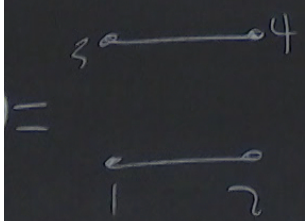
$$= \tilde{F}\left(\frac{p_1 + p_3}{2}\right)(2\pi)^4\,\delta^{(4)}(p_1 - p_3)$$

no scattering



$$\propto \delta^{(4)}(p_1 + p_2)\,\delta^{(4)}(p_3 + p_4) \qquad p_1^0 + p_2^0 \geq 2m > 0$$

$\bar{X}_{13} P_{13})$

$)$

ng

Disconnected diagrams

→ 1 delta per connected component

→ no interference between fully connected + disconnected diagrams