Title: Quantum Theory - Lecture 220927

Speakers: Dan Wohns

Collection: Quantum Theory (2022-2023)

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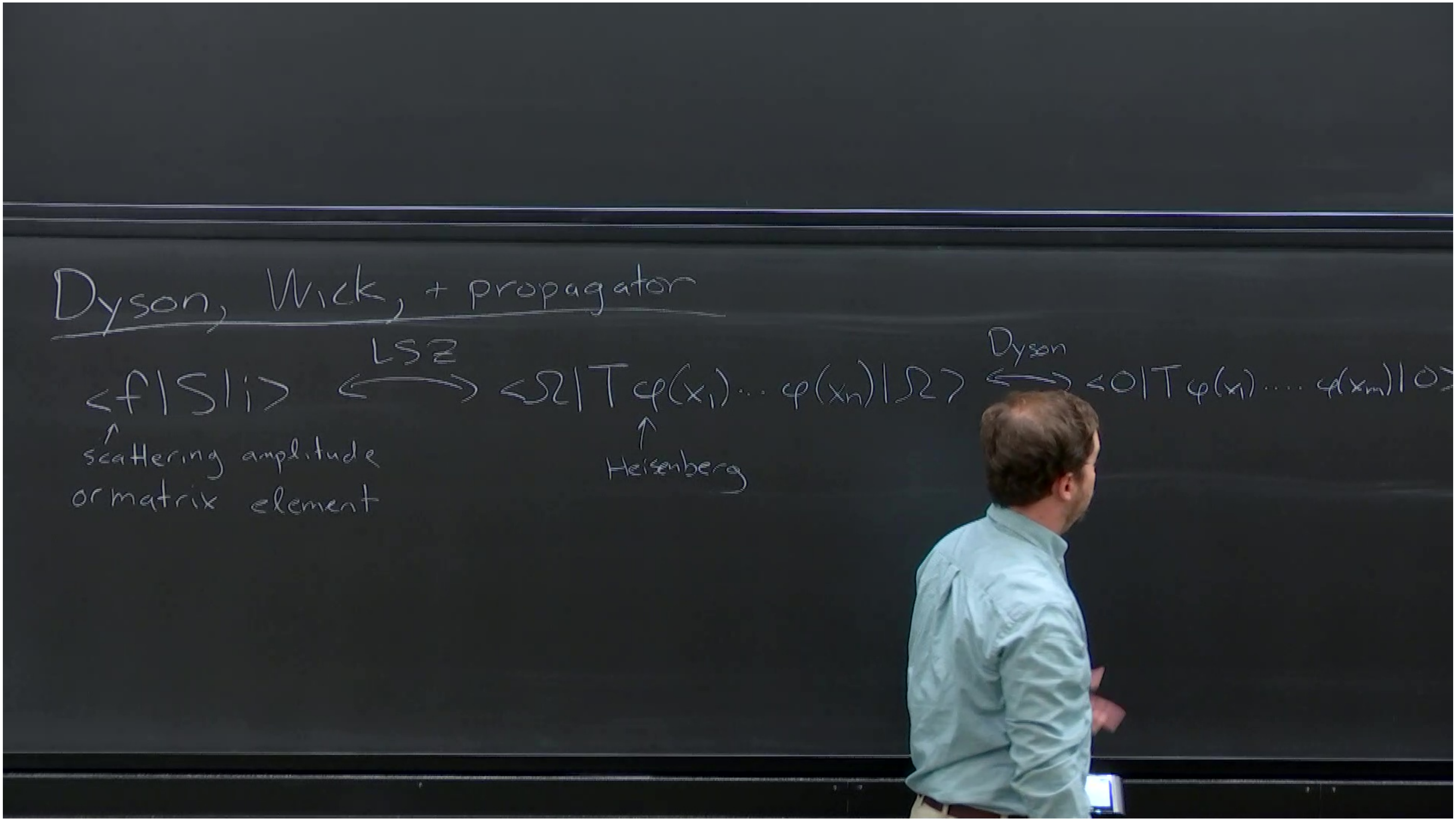
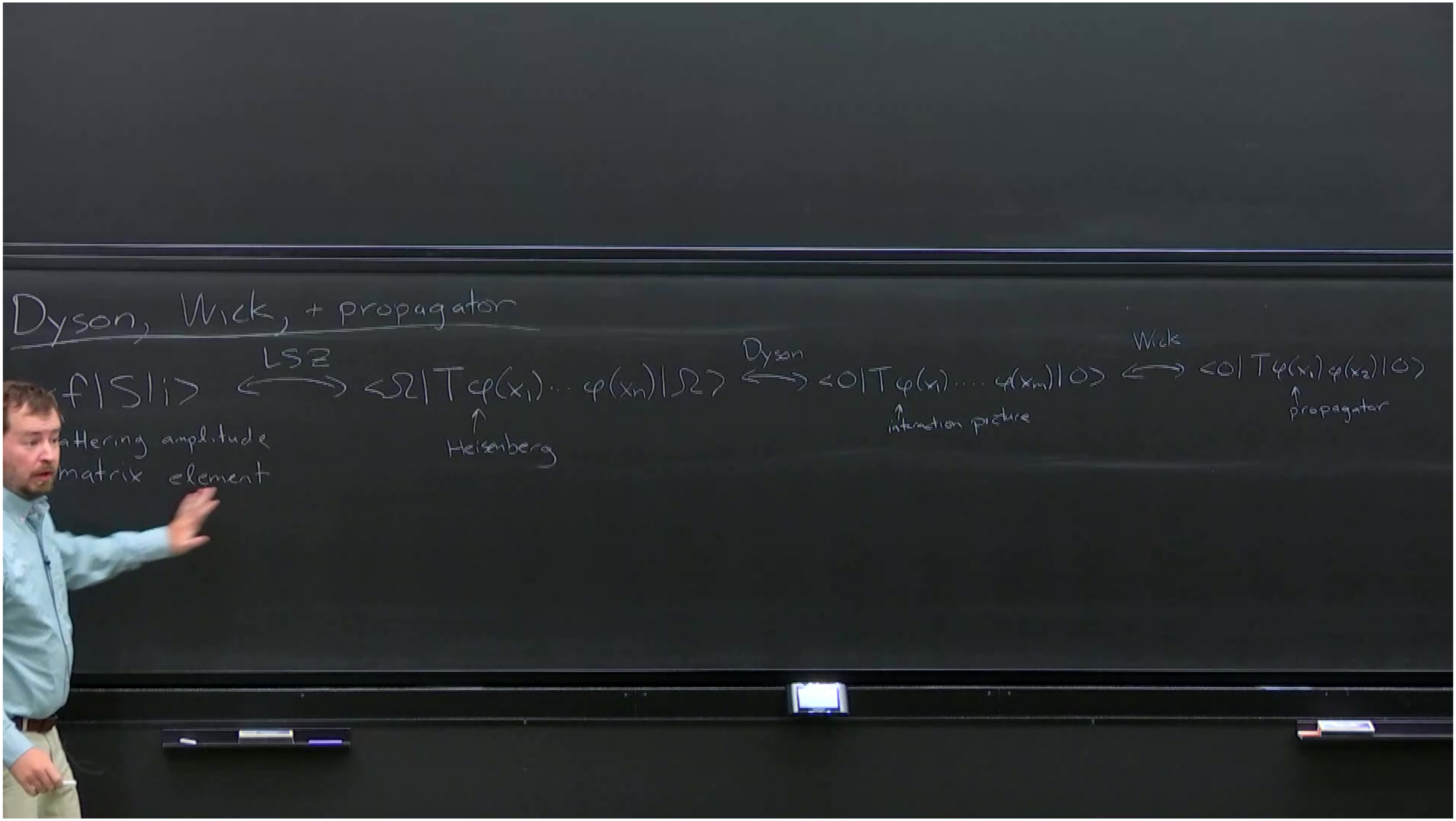
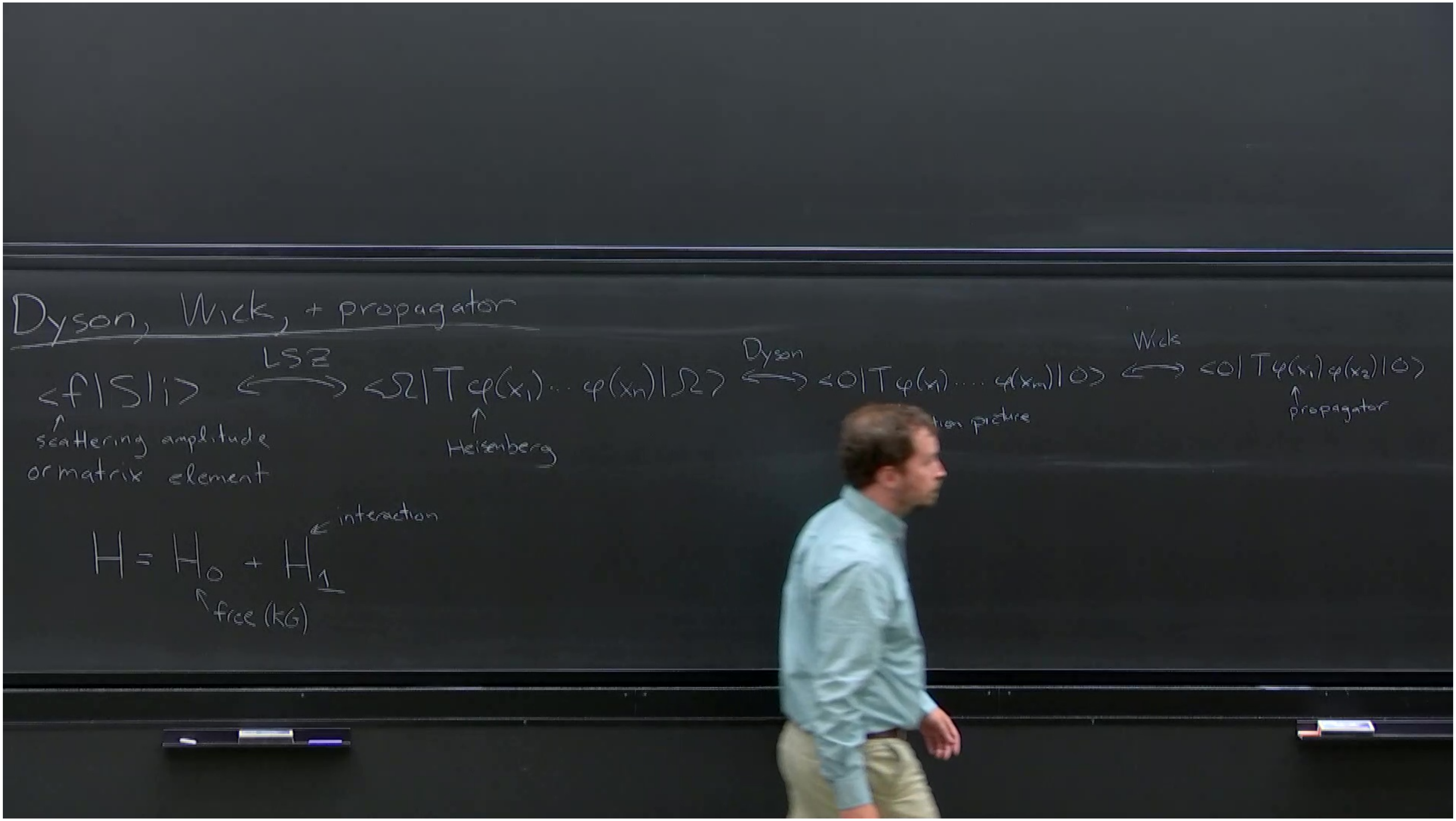
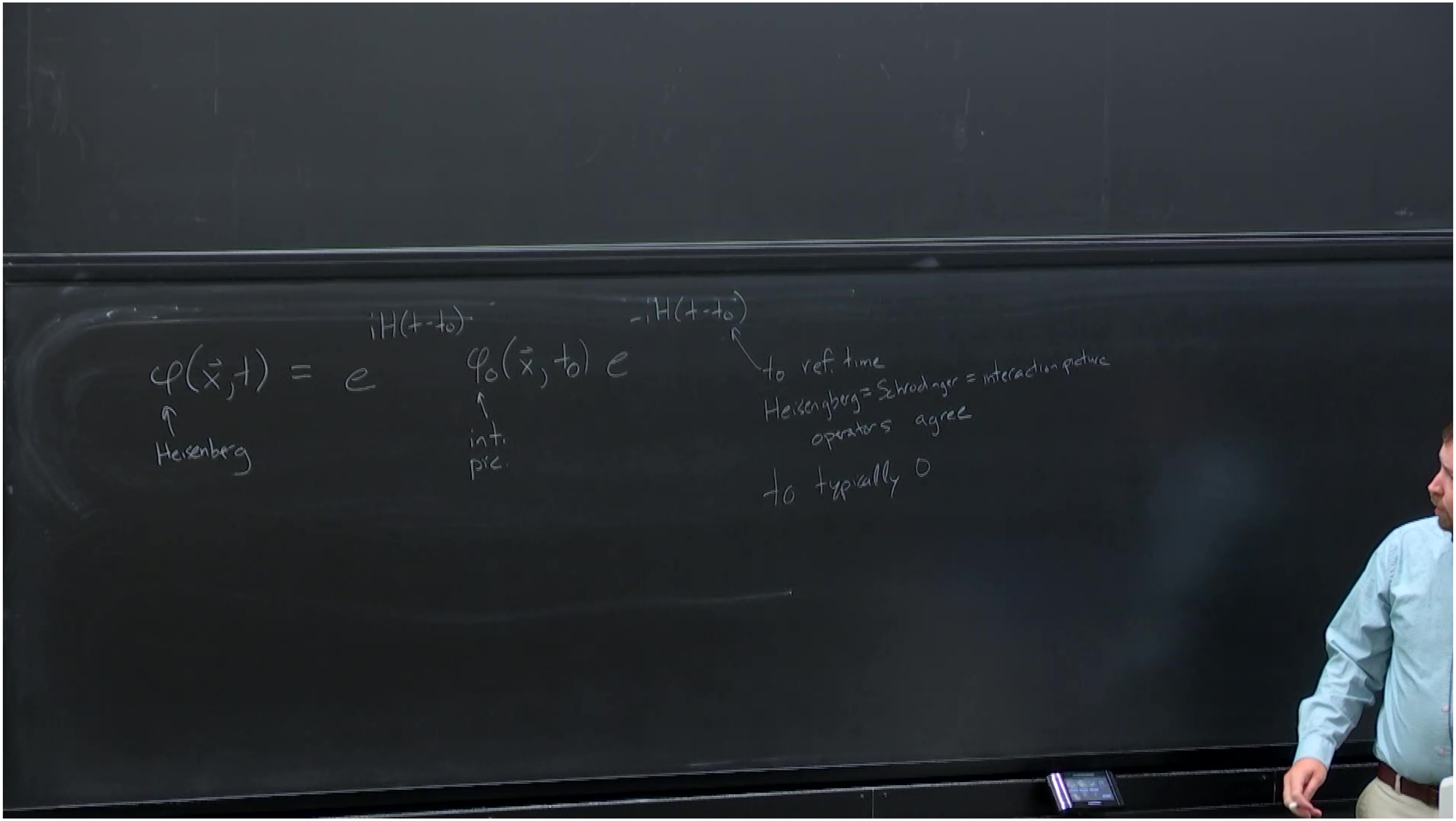
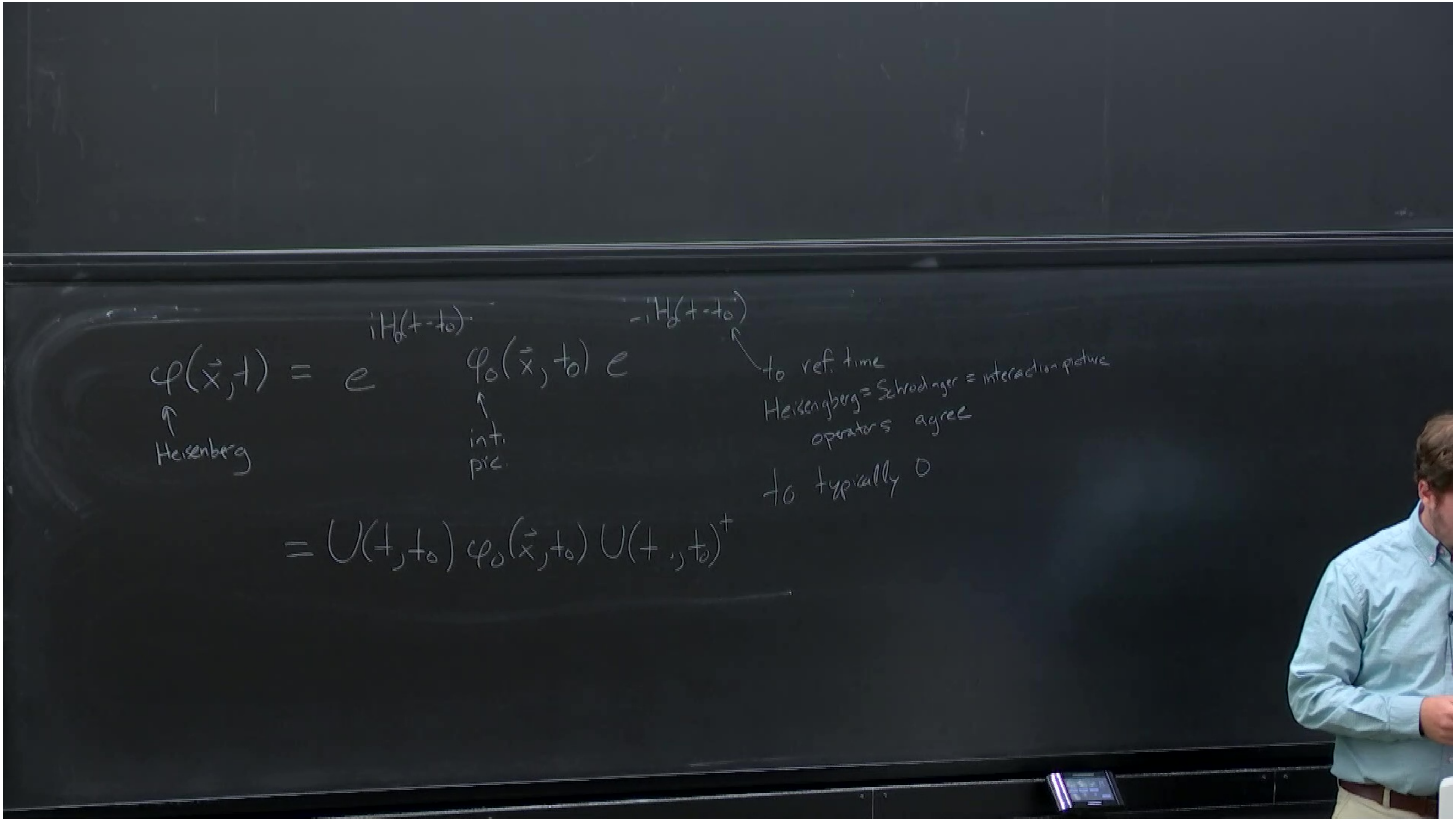

Dyson, Wick, + propagator
LSZ
Dyson
$\langle f|S|i\rangle \longleftrightarrow \langle \Omega | T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\cdots\varphi(x_m)|0\rangle$
scattering amplitude
or matrix element
Heisenberg

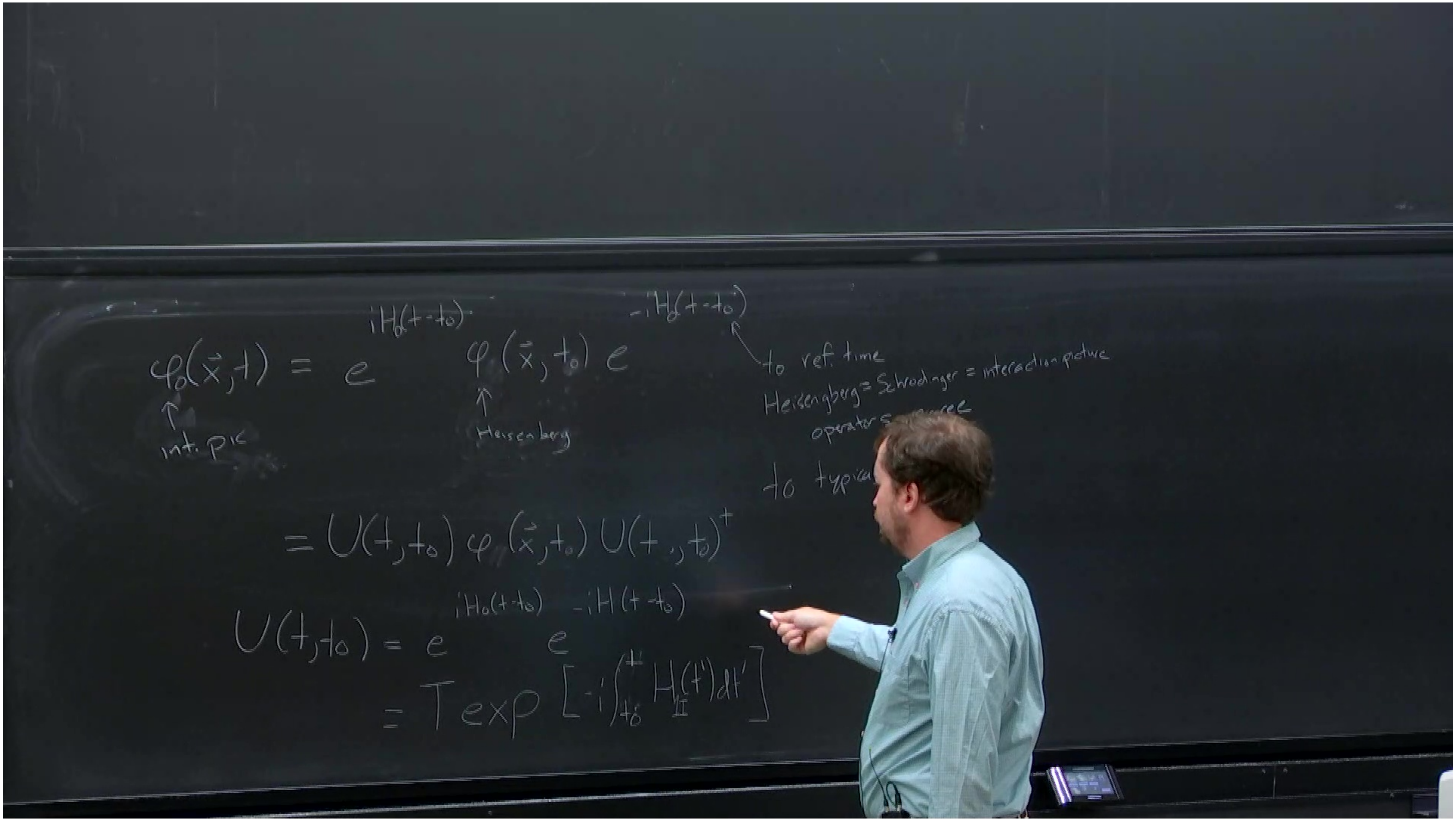


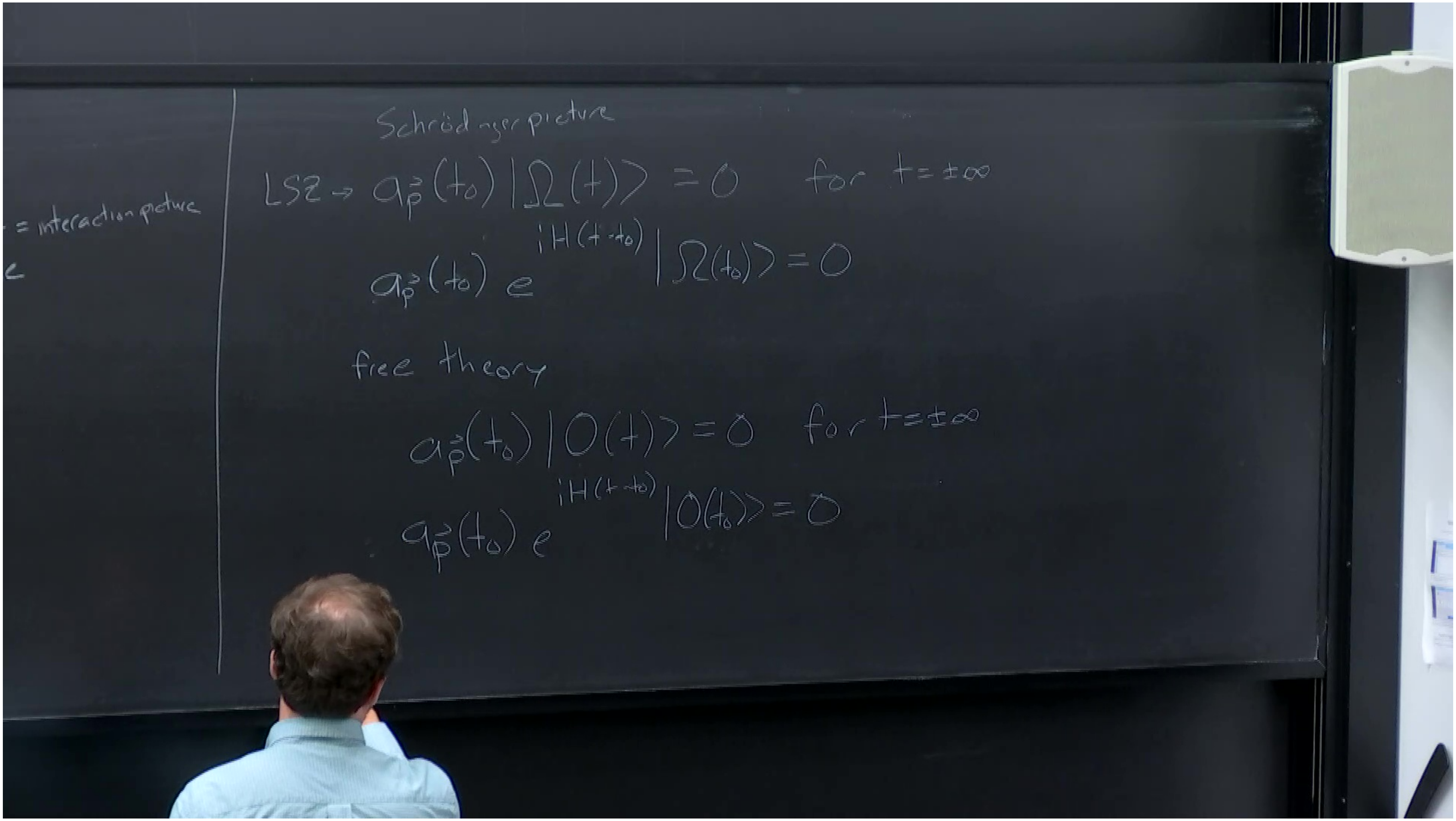
Dyson, Wick, + propagator
LSZ
Dyson
Wick
$\langle f|S|i\rangle \longleftrightarrow \langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\cdots\varphi(x_n)|0\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle$
attering amplitude
matrix element
Heisenberg
interaction picture
propagator

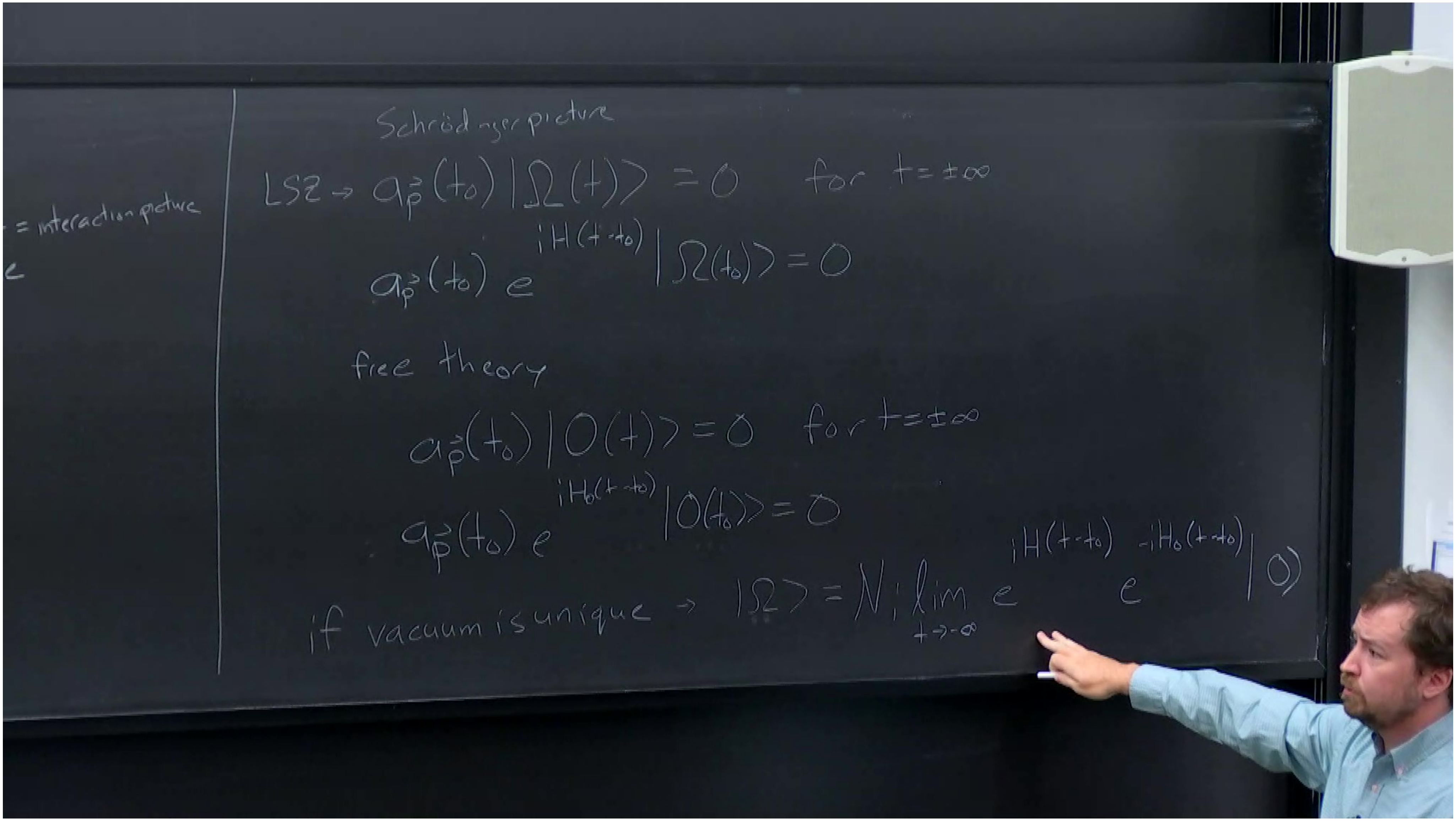

Dyson, Wick, + propagator
LSZ
<f|S|i>
scattering amplitude
or matrix element
<Ω|Tφ(x1)···φ(xn)|Ω>
Heisenberg
Dyson
<0|Tφ(x1)····φ(xn)|0>
picture
Wick
<0|Tφ(x1)φ(x2)|0>
propagator
H = H0 + H1
interaction
free (KG)


$\varphi(\vec{x},t) = e^{iH(t-t_0)} \varphi_0(\vec{x},t_0) e^{-iH(t-t_0)}$
Heisenberg
int. pic.
t_0 ref. time
Heisenberg = Schrodinger = interaction picture
operators agree
t_0 typically 0


$\varphi(\vec{x},t) = e^{iH_0(t-t_0)} \varphi_0(\vec{x},t_0) e^{-iH_0(t-t_0)}$
Heisenberg
int. pic.
t_0 ref. time
Heisenberg = Schrodinger = interaction picture
operators agree
t_0 typically 0
$= U(t,t_0) \varphi_0(\vec{x},t_0) U(t,t_0)^\dagger$

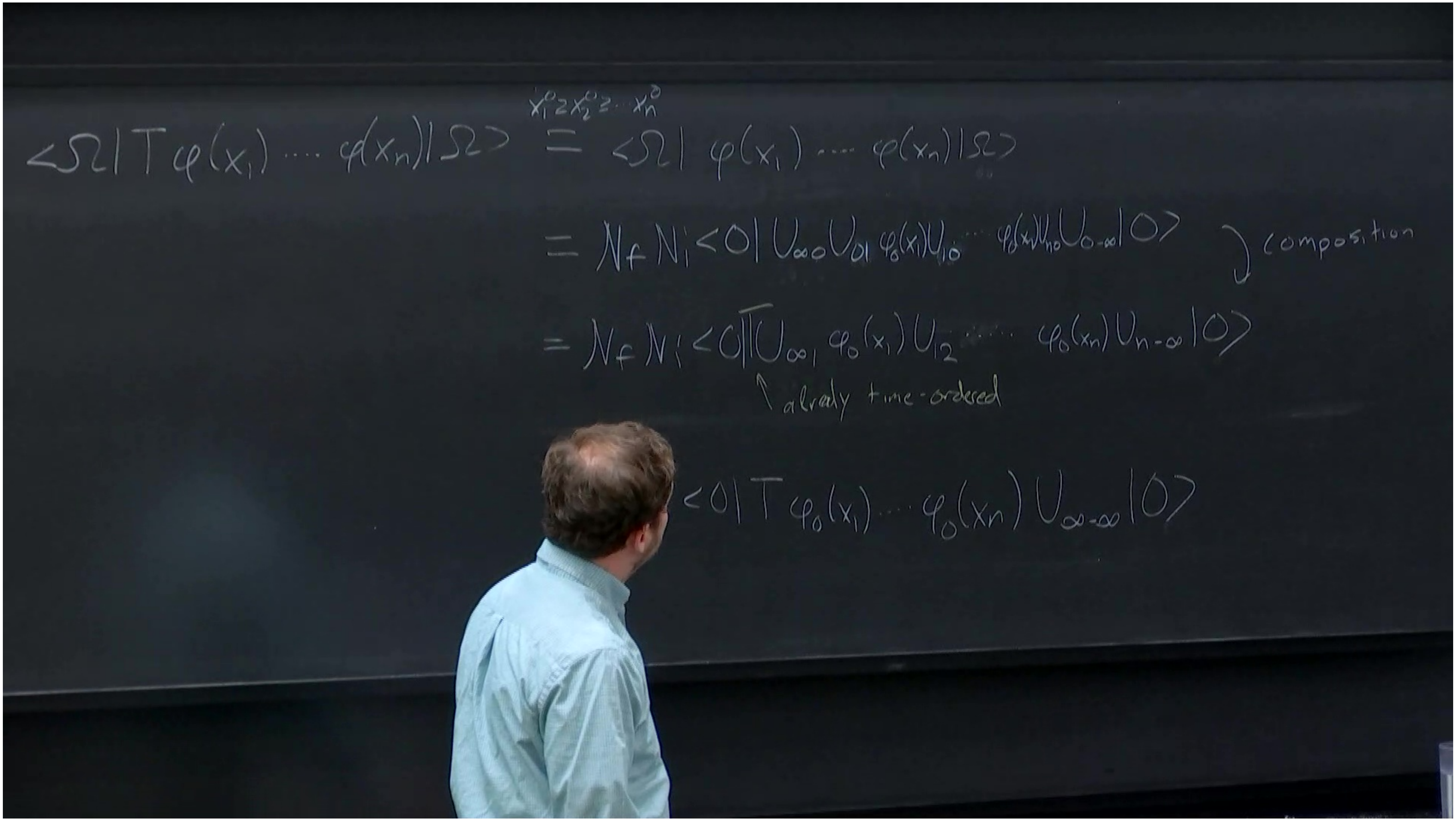

φ₀(x⃗,t) = e^{iH₀(t−t₀)} φ(x⃗,t₀) e^{−iH₀(t−t₀)}
int. pic
Heisenberg
t₀ ref. time
Heisengberg = Schrodinger = interaction picture
operators
t₀ typical
= U(t,t₀) φ(x⃗,t₀) U(t,t₀)†
U(t,t₀) = e^{iH₀(t−t₀)} e^{−iH(t−t₀)}
= T exp[−i ∫_{t₀}^{t} H_I(t′)dt′]


= interaction picture
Schrödinger picture
LSZ → $a_{\vec p}(t_0)|\Omega(t)\rangle = 0$ for $t = \pm\infty$
$a_{\vec p}(t_0)\, e^{iH(t-t_0)}|\Omega(t_0)\rangle = 0$
free theory
$a_{\vec p}(t_0)|0(t)\rangle = 0$ for $t = \pm\infty$
$a_{\vec p}(t_0)\, e^{iH(t-t_0)}|0(t_0)\rangle = 0$


= interaction picture
Schrödinger picture
LSZ → $a_{\vec{p}}(t_0)\,|\Omega(t)\rangle = 0$ for $t = \pm\infty$
$a_{\vec{p}}(t_0)\, e^{iH(t-t_0)}\,|\Omega(t_0)\rangle = 0$
free theory
$a_{\vec{p}}(t_0)\,|0(t)\rangle = 0$ for $t = \pm\infty$
$a_{\vec{p}}(t_0)\, e^{iH_0(t-t_0)}\,|0(t_0)\rangle = 0$
if vacuum is unique → $|\Omega\rangle = N_i \lim_{t\to-\infty} e^{iH(t-t_0)}\, e^{-iH_0(t-t_0)}\,|0\rangle$

$$|\Omega\rangle = \lim_{t\to+\infty} N_f \underset{\substack{\| \\ U(\infty,0)}}{U_{\infty 0}} |0\rangle$$

$N_i \neq N_f$ in general


$x_1^0 \ge x_2^0 \ge \cdots x_n^0$
$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \langle\Omega|\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle$
$= N_f N_i \langle 0|U_{\infty 0}U_{01}\varphi_0(x_1)U_{10}\cdots\varphi_0(x_n)U_{n0}U_{0-\infty}|0\rangle$
composition
$= N_f N_i \langle 0|T U_{\infty 1}\varphi_0(x_1)U_{12}\cdots\varphi_0(x_n)U_{n-\infty}|0\rangle$
already time-ordered
$\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)U_{\infty-\infty}|0\rangle$

free (KG)

Assume $\langle \Omega | \Omega \rangle = 1$

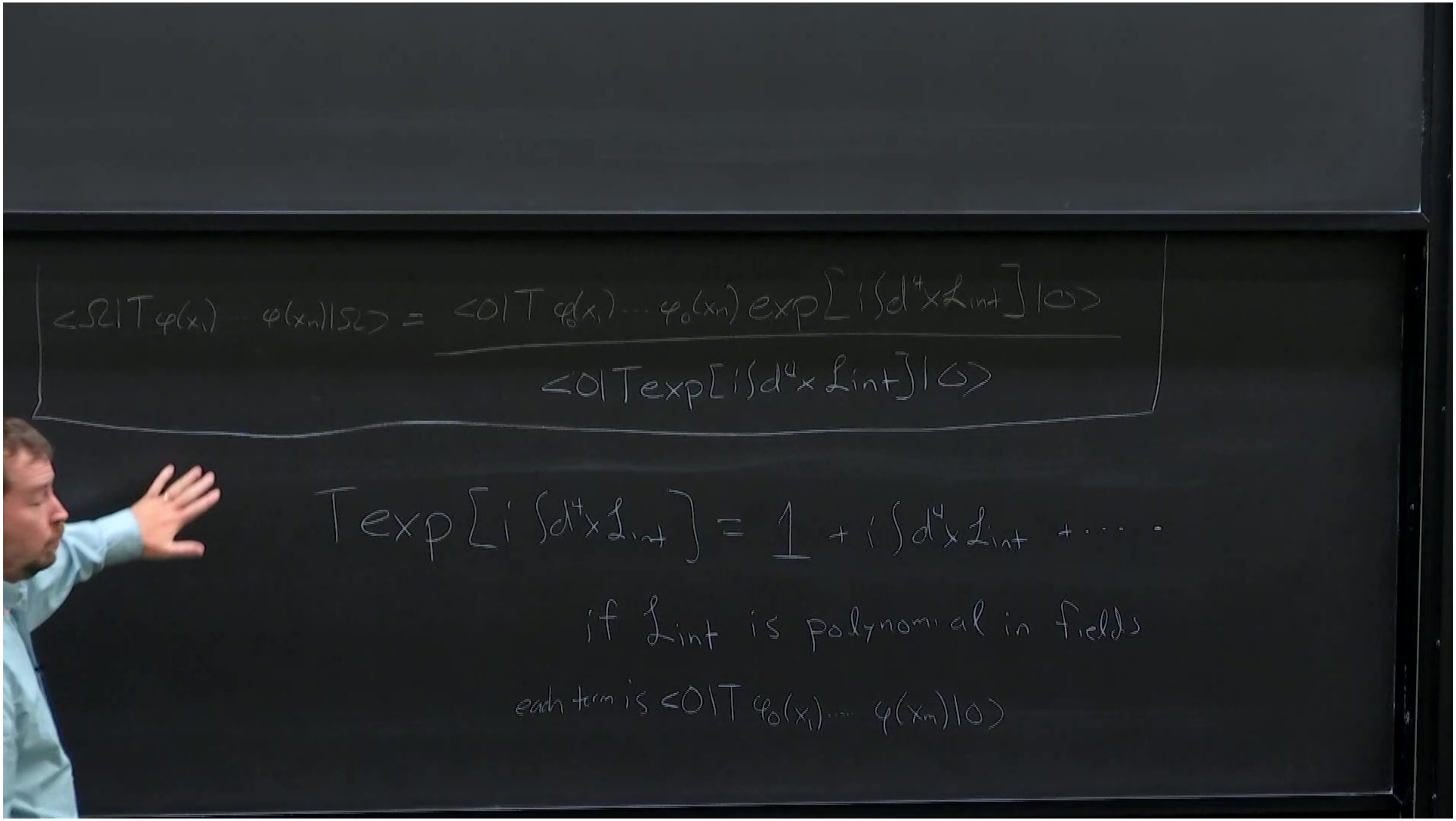
$$U^\dagger(t_1, t_2) = U(t_2, t_1)$$

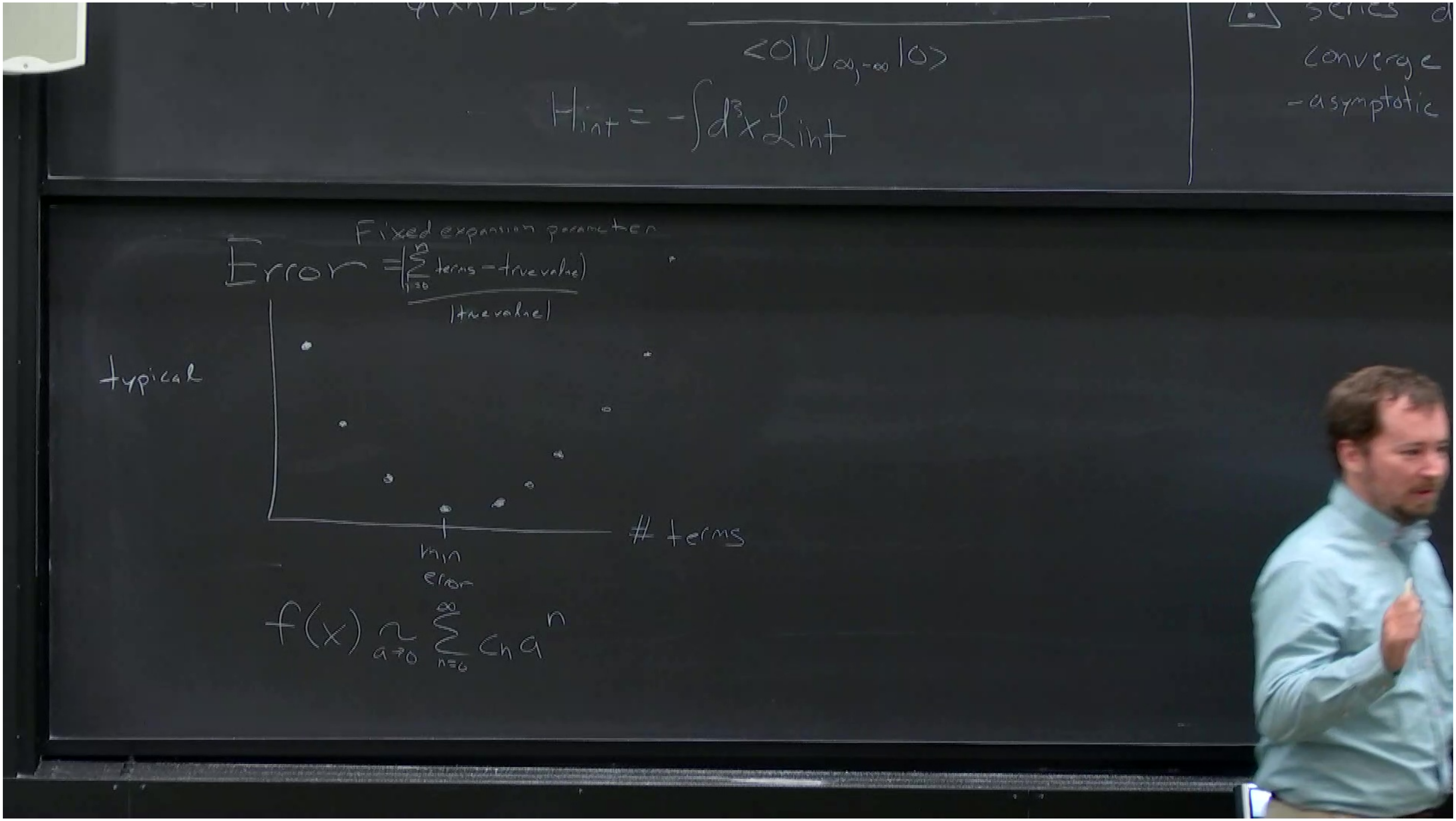
$$N_f N_i \langle 0 | U_{\infty 0} U_{0 -\infty} | 0 \rangle = 1$$

$$N_i N_f = \frac{1}{\langle 0 | U_{\infty -\infty} | 0 \rangle}$$

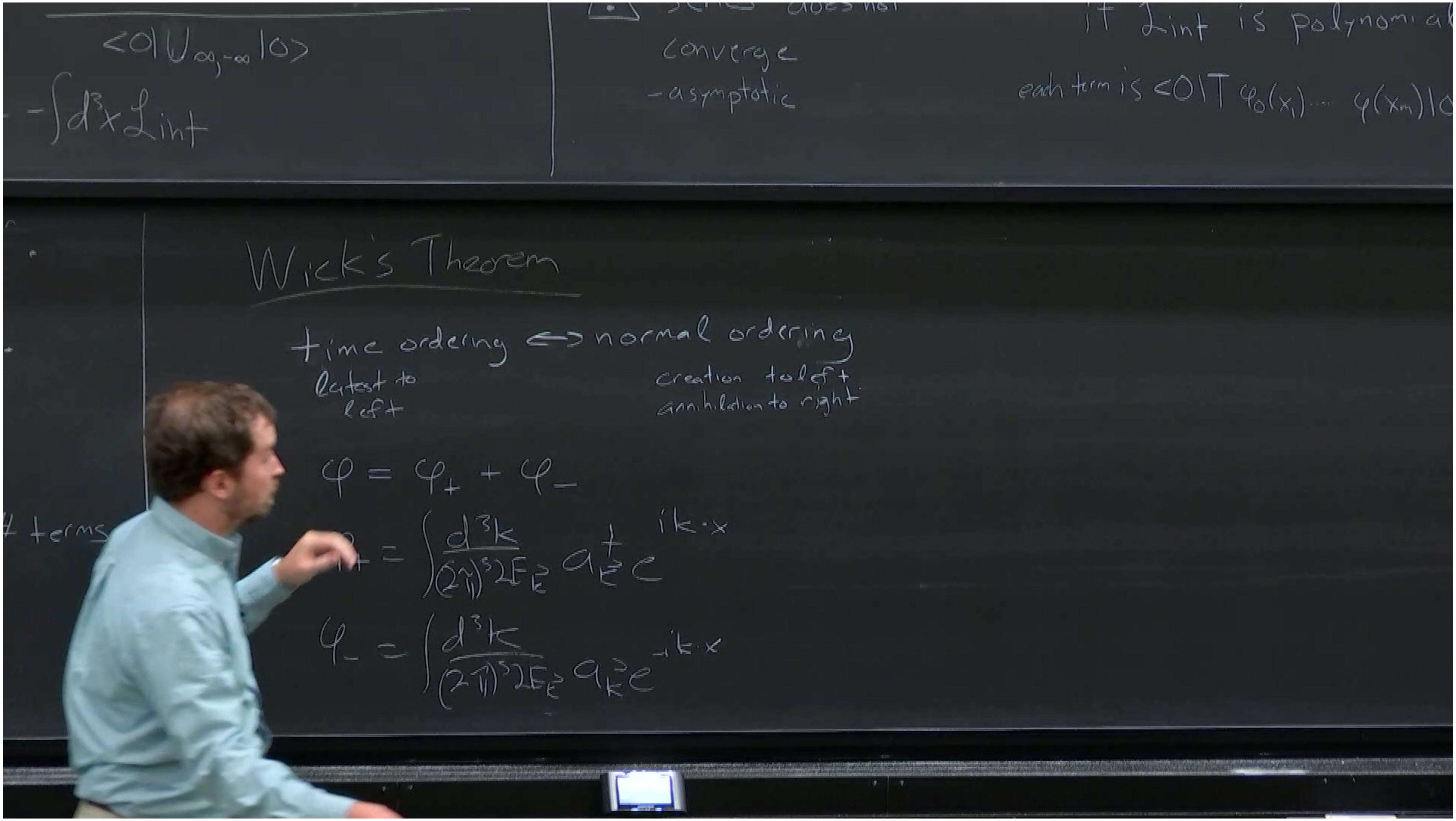
$$\langle \Omega | T \varphi(x_1) \cdots \varphi(x_n) | \Omega \rangle = \frac{\langle 0 | T \varphi_0(x_1) \cdots \varphi_0(x_n) U_{\infty -\infty} | 0 \rangle}{\langle 0 | U_{\infty, -\infty} | 0 \rangle}$$

$$H_{int} = -\int d^3x \, \mathcal{L}_{int}$$


$$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \frac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}$$
$$T\exp[i\int d^4x\,\mathcal{L}_{int}] = \underline{1} + i\int d^4x\,\mathcal{L}_{int} + \cdots .$$
if $\mathcal{L}_{int}$ is polynomial in fields
each term is $\langle 0|T\varphi_0(x_1)\cdots\varphi(x_m)|0\rangle$



<0|U∞,-∞|0>
Hint = -∫d³x Lint
converge
-asymptotic
Fixed expansion parameter
Error = |Σ terms - true value| / |true value|
typical
terms
min error
f(x) ~ Σ cn a^n


$\langle 0|U_{\infty,-\infty}|0\rangle$
$-\int d^3x \mathcal{L}_{int}$
converge
- asymptotic
if $\mathcal{L}_{int}$ is polynomi
each term is $\langle 0|T\,\varphi_0(x_1)\cdots\varphi(x_m)|0$
Wick's Theorem
time ordering ⟺ normal ordering
latest to left
creation to left
annihilation to right
$\varphi = \varphi_+ + \varphi_-$
$\varphi_+ = \int \frac{d^3k}{(2\pi)^3 2E_{\vec k}} a^\dagger_{\vec k} e^{ik\cdot x}$
$\varphi_- = \int \frac{d^3k}{(2\pi)^3 2E_{\vec k}} a_{\vec k} e^{-ik\cdot x}$
terms

series does not converge
- asymptotic

if $\mathcal{L}_{int}$ is polynomial in fields

each term is $\langle 0|T\,\varphi_0(x_1)\cdots\varphi(x_m)|0\rangle$

...eorem

...ring $\Leftrightarrow$ normal ordering
creation to left
annihilation to right

$+\varphi_-$

$\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}\,a^\dagger_{\vec{k}}\,e^{ik\cdot x}$

$\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}\,a_{\vec{k}}\,e^{-ik\cdot x}$

$$T\varphi(x)\varphi(y) \overset{x^0>y^0}{=} \varphi_+(x)\varphi_+(y)+\varphi_+(x)\varphi_-(y)$$

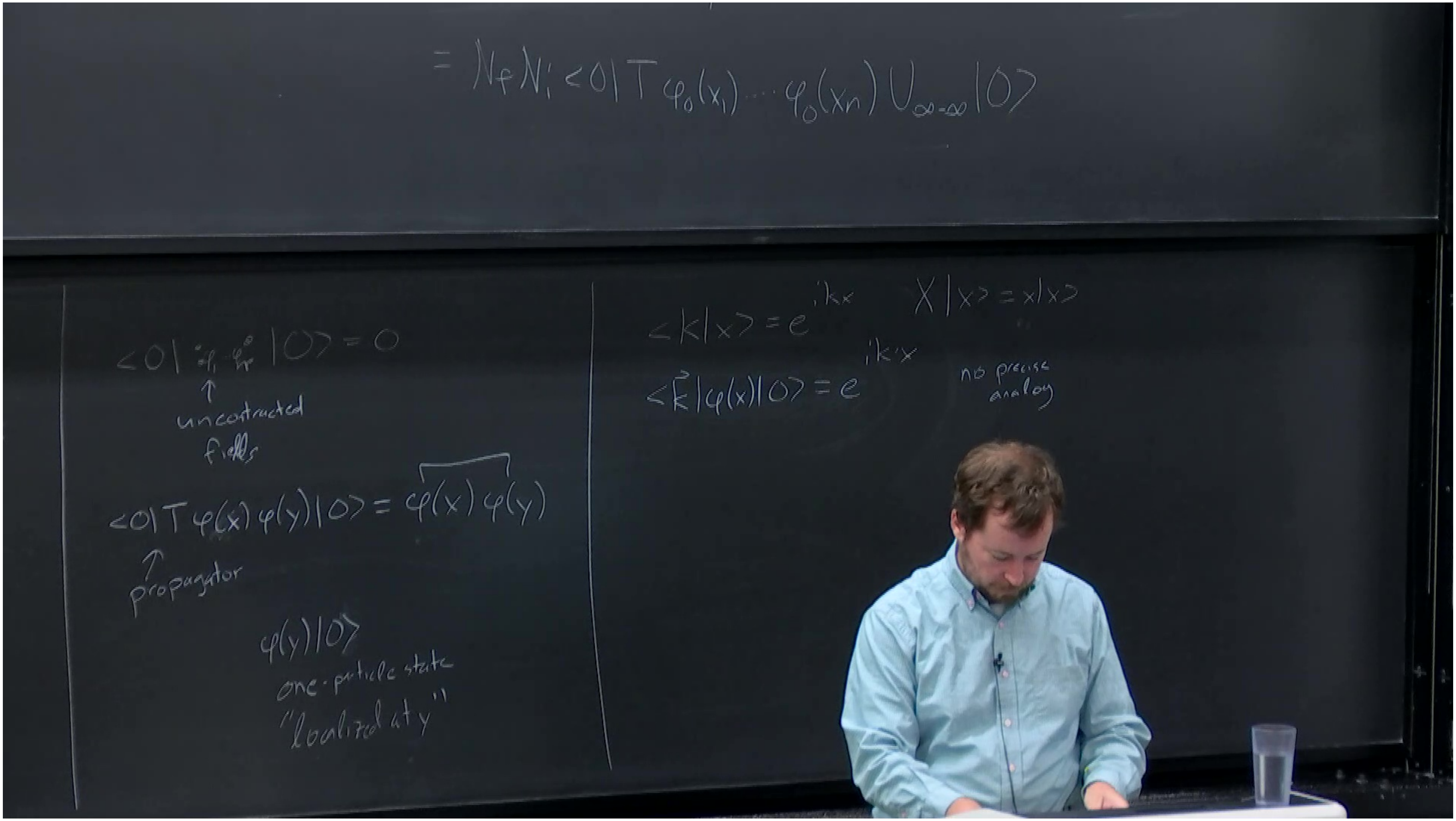
$$+\underbrace{\varphi_-(x)\varphi_+(y)}_{}+\varphi_-(x)\varphi_-(y)$$

$$\varphi_+(y)\varphi_-(y)+[\varphi_-(x),\varphi_+(y)]$$

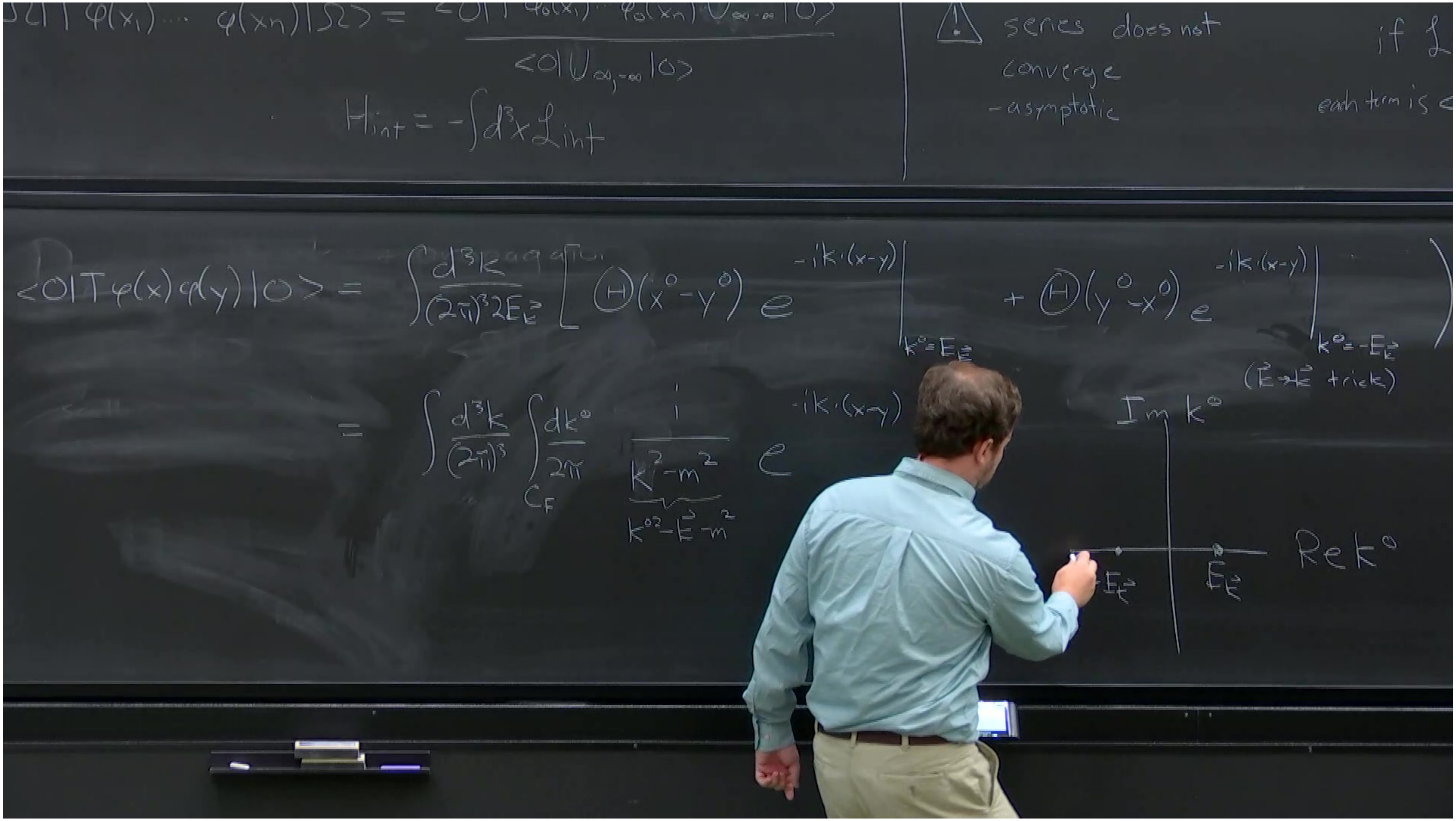
$$T\varphi(x)\varphi(y) = :\varphi(x)\varphi(y): + \boxed{\begin{aligned}&+\Theta(x^0-y^0)[\varphi_-(x),\varphi_+(y)]\\&+\Theta(y^0-x^0)[\varphi_-(y),\varphi_+(x)]\end{aligned}}$$

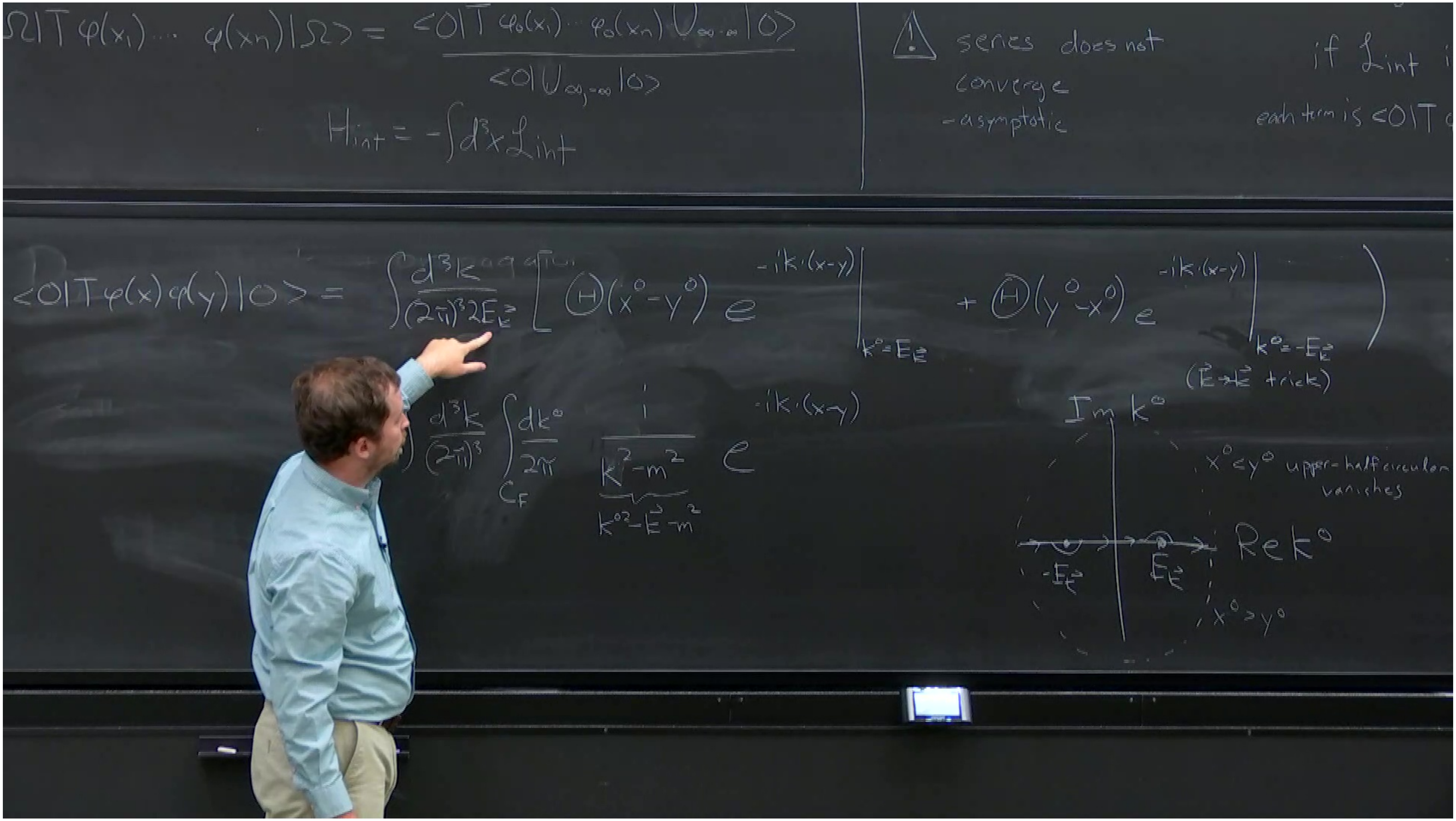
$$\square = \varphi(x)\varphi(y)$$

contraction

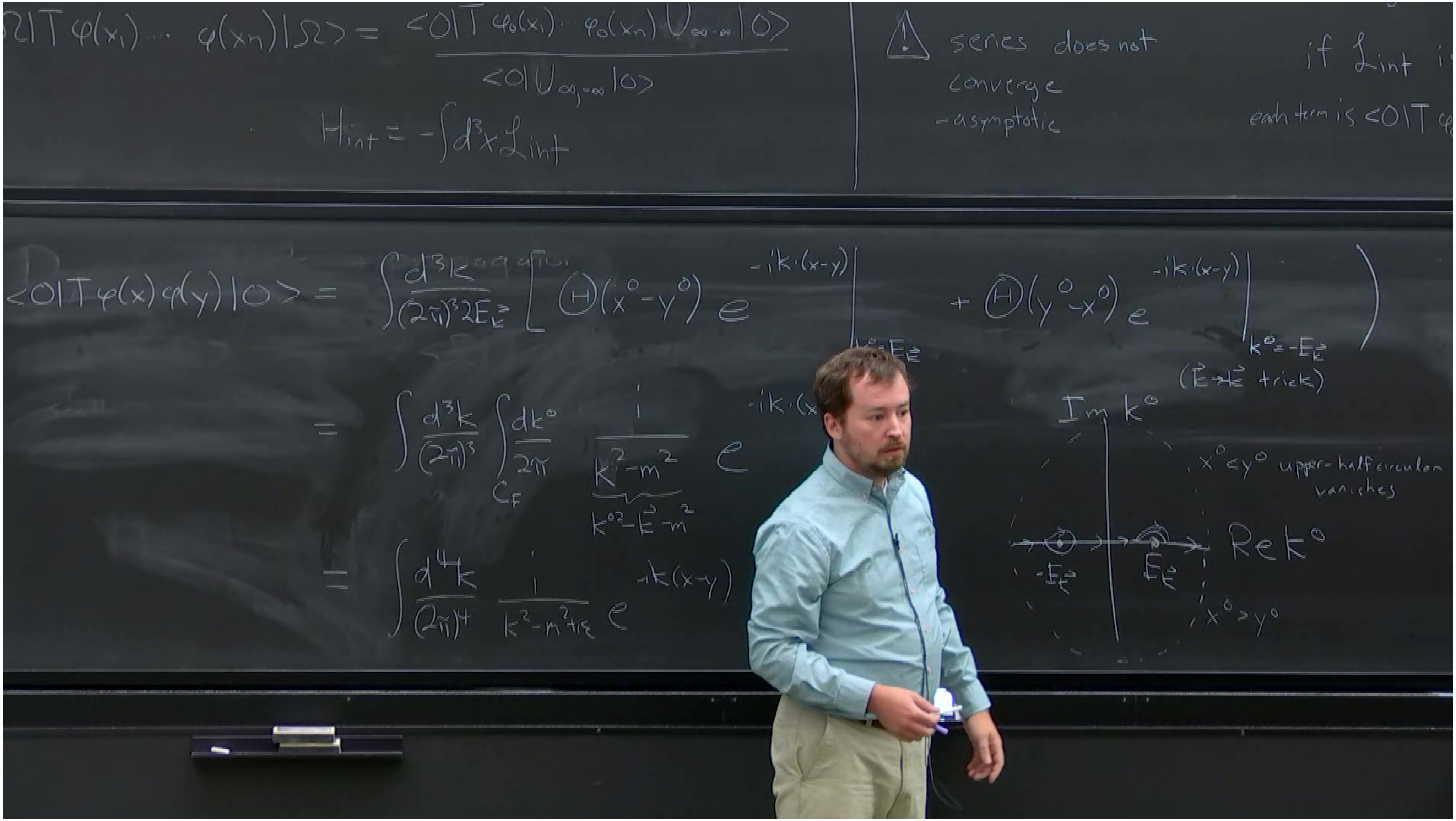


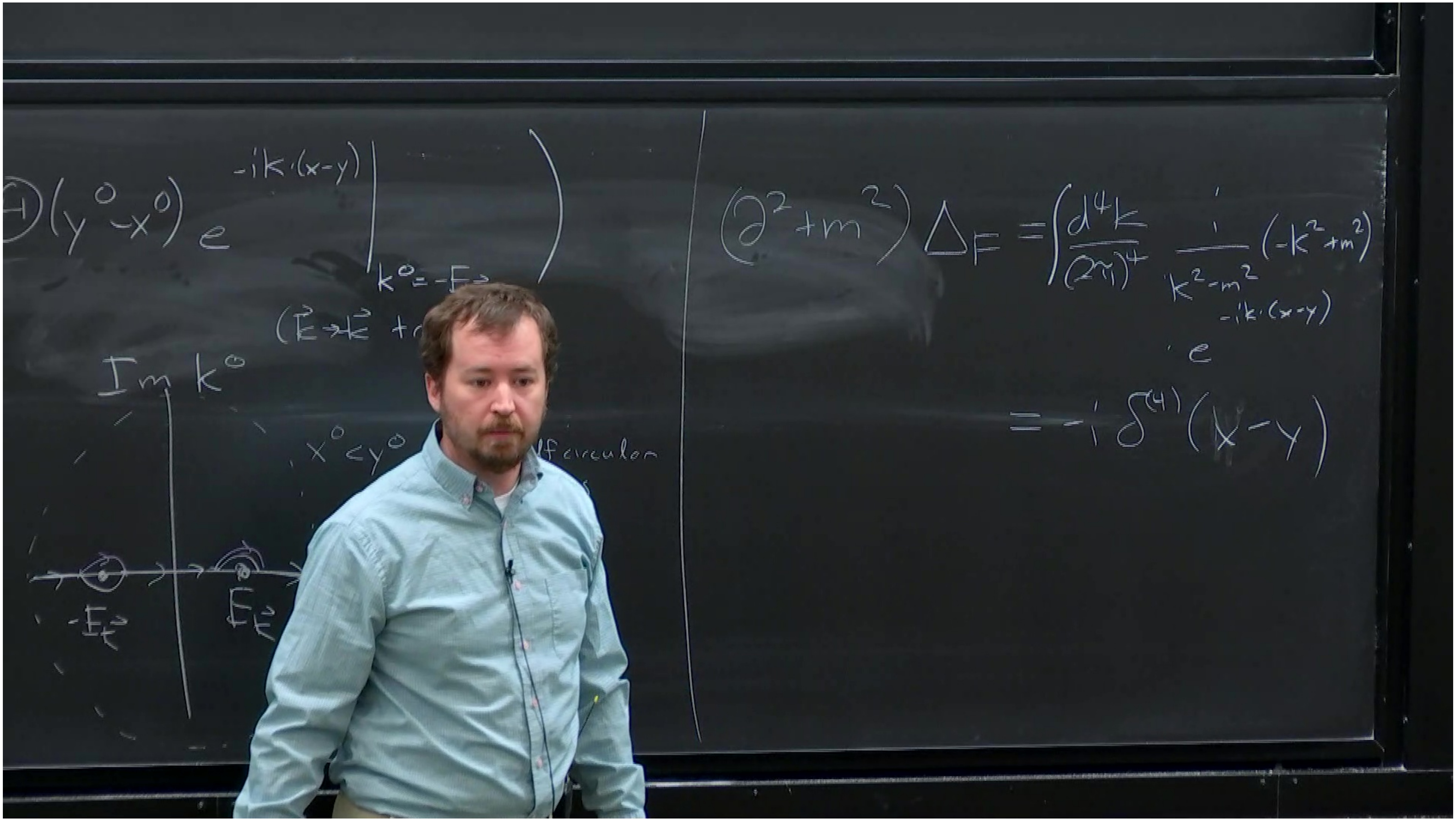
$= N_f N_i \langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)U_{\infty,-\infty}|0\rangle$
$\langle 0|\varphi_1^0 \cdots \varphi_n^0|0\rangle = 0$
uncontracted fields
$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \varphi(x)\varphi(y)$
propagator
$\varphi(y)|0\rangle$
one-particle state
"localized at y"
$\langle k|x\rangle = e^{ikx}$
$X|x\rangle = x|x\rangle$
$\langle \vec{k}|\varphi(x)|0\rangle = e^{ik\cdot x}$
no precise analogy


series does not converge
- asymptotic
each term is
$H_{int} = -\int d^3x \mathcal{L}_{int}$
$\langle 0 | U_{\infty,-\infty} | 0 \rangle$
$\langle 0 | T \varphi(x) \varphi(y) | 0 \rangle = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[\Theta(x^0 - y^0) e^{-ik\cdot(x-y)} \Big|_{k^0 = E_{\vec{k}}} + \Theta(y^0 - x^0) e^{-ik\cdot(x-y)} \Big|_{k^0 = -E_{\vec{k}}} \right]$
($\vec{k} \to -\vec{k}$ trick)
$= \int \frac{d^3k}{(2\pi)^3} \int_{C_F} \frac{dk^0}{2\pi} \frac{i}{\underbrace{k^2 - m^2}_{k^{02} - \vec{k}^2 - m^2}} e^{-ik\cdot(x-y)}$
Im k^0
Re k^0
$-E_{\vec{k}}$
$E_{\vec{k}}$



$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \frac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)U_{\infty,-\infty}|0\rangle}{\langle 0|U_{\infty,-\infty}|0\rangle}$
$H_{int} = -\int d^3x \mathcal{L}_{int}$
series does not converge
- asymptotic
if $\mathcal{L}_{int}$
each term is $\langle 0|T$
$\langle 0|T\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}\left[\Theta(x^0-y^0)e^{-ik\cdot(x-y)}\Big|_{k^0=E_{\vec{k}}} + \Theta(y^0-x^0)e^{-ik\cdot(x-y)}\Big|_{k^0=-E_{\vec{k}}}\right)$
($\vec{k}\to-\vec{k}$ trick)
$\int \frac{d^3k}{(2\pi)^3}\int_{C_F}\frac{dk^0}{2\pi}\frac{i}{\underbrace{k^2-m^2}_{k^{02}-\vec{k}^2-m^2}}e^{-ik\cdot(x-y)}$
Im k^0
$x^0<y^0$ upper-half circular vanishes
Re k^0
$-E_{\vec{k}}$
$E_{\vec{k}}$
$x^0>y^0$


Ω|T φ(x1) ... φ(xn)|Ω> = <0|T φ0(x1) ... φ0(xn) U∞,-∞|0> / <0|U∞,-∞|0>
Hint = -∫d³x ℒint
series does not converge
- asymptotic
if ℒint i
each term is <0|T φ
<0|T φ(x) φ(y)|0> = ∫ d³k/((2π)³ 2E_k) [Θ(x⁰-y⁰) e^{-ik·(x-y)} |k⁰=E_k + Θ(y⁰-x⁰) e^{-ik·(x-y)} |k⁰=-E_k]
(k → -k trick)
= ∫ d³k/(2π)³ ∫_{C_F} dk⁰/2π i/(k²-m²) e^{-ik·(x
k⁰²-k²-m²
= ∫ d⁴k/(2π)⁴ i/(k²-m²+iε) e^{-ik(x-y)}
Im k⁰
x⁰<y⁰ upper-half circular vanishes
Re k⁰
-E_k
E_k
x⁰>y⁰


$(\partial^2+m^2)\Delta_F = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2}(-k^2+m^2)\, e^{-ik\cdot(x-y)}$
$= -i\,\delta^{(4)}(x-y)$
$\mathrm{Im}\, k^0$
$x^0<y^0$